

## Übung 1 (a)

$$f(\rho) = \begin{pmatrix} f(\rho)(S) \\ f(\rho)(B) \end{pmatrix} = \begin{pmatrix} \llbracket \hat{B}b \rrbracket(\rho) \\ \llbracket Sb \rrbracket(\rho) \end{pmatrix} = \begin{pmatrix} (\rho(B) \cup \{\varepsilon\}) \cdot \{b\} \\ \rho(S) \cdot \{b\} \end{pmatrix}$$

$$\begin{pmatrix} \emptyset \\ \emptyset \end{pmatrix} \mapsto \begin{pmatrix} \{b\} \\ \emptyset \end{pmatrix} \mapsto \begin{pmatrix} \{b\} \\ \{b^2\} \end{pmatrix} \mapsto \begin{pmatrix} \{b, b^3\} \\ \{b^2\} \end{pmatrix} \mapsto \begin{pmatrix} \{b, b^3\} \\ \{b^2, b^4\} \end{pmatrix} \\ \mapsto \begin{pmatrix} \{b, b^3, b^5\} \\ \{b^2, b^4\} \end{pmatrix}$$

## Übung 1 (c)

$$\begin{aligned}\llbracket S \rrbracket(\rho) &= \llbracket \{\hat{ba}\hat{A}\} \rrbracket(\rho) \\ &= \llbracket \{\hat{ba}\hat{\phantom{A}}\} \rrbracket(\rho) \cdot \llbracket A \rrbracket(\rho) \\ &= (\llbracket ba \rrbracket(\rho))^* \cdot \llbracket A \rrbracket(\rho) \\ &= (\llbracket b \rrbracket(\rho) \cdot \llbracket a \rrbracket(\rho))^* \cdot \llbracket A \rrbracket(\rho) \\ &= (\{b\} \cdot \{a\})^* \cdot \rho(A) \\ &= (\{b\} \cdot \{a\})^* \cdot \{(ba)^n b \mid n \in \mathbb{N}\} \\ &= (\{ba\})^* \cdot \{(ba)^n b \mid n \in \mathbb{N}\} \\ &= \{(ba)^m \mid m \in \mathbb{N}\} \cdot \{(ba)^n b \mid n \in \mathbb{N}\} \\ &= \{(ba)^n b \mid n \in \mathbb{N}\} \\ &= \rho(S) \\ &= W(\mathcal{E}, S)\end{aligned}$$

## Zusatzaufgabe 1 (b)

$$f(\rho) = \begin{pmatrix} f(\rho)(S) \\ f(\rho)(A) \end{pmatrix} = \begin{pmatrix} \llbracket aAb \rrbracket(\rho) \\ \llbracket \widehat{([S]|\widehat{b})} \rrbracket(\rho) \end{pmatrix} = \begin{pmatrix} \{a\} \cdot \rho(A) \cdot \{b\} \\ \rho(S) \cup \{\varepsilon, b\} \end{pmatrix}$$

$$\begin{aligned} \begin{pmatrix} \emptyset \\ \emptyset \end{pmatrix} &\mapsto \begin{pmatrix} \emptyset \\ \{\varepsilon, b\} \end{pmatrix} \mapsto \begin{pmatrix} \{ab, abb\} \\ \{\varepsilon, b\} \end{pmatrix} \mapsto \begin{pmatrix} \{ab, abb\} \\ \{\varepsilon, b, ab, abb\} \end{pmatrix} \\ &\mapsto \begin{pmatrix} \{ab, abb, aabb, aabbb\} \\ \{\varepsilon, b, ab, abb\} \end{pmatrix} \\ &\mapsto \begin{pmatrix} \{ab, abb, aabb, aabbb\} \\ \{\varepsilon, b, ab, abb, aabb, aabbb\} \end{pmatrix} \end{aligned}$$