Übung 1 (a)

$$f(\rho) = \begin{pmatrix} f(\rho)(S) \\ f(\rho)(B) \end{pmatrix} = \begin{pmatrix} [\![\hat{[}B\hat{]}b]\!](\rho) \\ [\![Sb]\!](\rho) \end{pmatrix} = \begin{pmatrix} (\rho(B) \cup \{\varepsilon\}) \cdot \{b\} \\ \rho(S) \cdot \{b\} \end{pmatrix}$$

$$\begin{pmatrix} \emptyset \\ \emptyset \end{pmatrix} \mapsto \begin{pmatrix} \{b\} \\ \emptyset \end{pmatrix} \mapsto \begin{pmatrix} \{b\} \\ \{b^2\} \end{pmatrix} \mapsto \begin{pmatrix} \{b, b^3\} \\ \{b^2\} \end{pmatrix} \mapsto \begin{pmatrix} \{b, b^3\} \\ \{b^2, b^4\} \end{pmatrix}$$
$$\mapsto \begin{pmatrix} \{b, b^3, b^5\} \\ \{b^2, b^4\} \end{pmatrix}$$

Übung 1 (c)

$$
\begin{aligned}
[\![S]\!](\rho) &= [\![\widehat{\{}ba\widehat{\}}A]\!](\rho) \\
&= [\![\widehat{\{}ba\widehat{\}}]\!](\rho) \cdot [\![A]\!](\rho) \\
&= ([\![ba]\!](\rho))^* \cdot [\![A]\!](\rho) \\
&= ([\![b]\!](\rho) \cdot [\![a]\!](\rho))^* \cdot [\![A]\!](\rho) \\
&= (\{b\} \cdot \{a\})^* \cdot \rho(A) \\
&= (\{b\} \cdot \{a\})^* \cdot \{(ba)^n b \,|\, n \in \mathbb{N}\} \\
&= (\{ba\})^* \cdot \{(ba)^n b \,|\, n \in \mathbb{N}\} \\
&= \{(ba)^m \,|\, m \in \mathbb{N}\} \cdot \{(ba)^n b \,|\, n \in \mathbb{N}\} \\
&= \{(ba)^n b \,|\, n \in \mathbb{N}\} \\
&= \rho(S) \\
&= W(\mathcal{E}, S)
\end{aligned}
$$

Zusatzaufgabe 1 (b)

$$f(\rho) = \begin{pmatrix} f(\rho)(S) \\ f(\rho)(A) \end{pmatrix} = \begin{pmatrix} [\![aAb]\!](\rho) \\ [\![\widehat{(}\widehat{[}S\widehat{]}|b\widehat{)}]\!](\rho) \end{pmatrix} = \begin{pmatrix} \{a\} \cdot \rho(A) \cdot \{b\} \\ \rho(S) \cup \{\varepsilon, b\} \end{pmatrix}$$

$$\begin{pmatrix} \emptyset \\ \emptyset \end{pmatrix} \mapsto \begin{pmatrix} \emptyset \\ \{\varepsilon, b\} \end{pmatrix} \mapsto \begin{pmatrix} \{ab, abb\} \\ \{\varepsilon, b\} \end{pmatrix} \mapsto \begin{pmatrix} \{ab, abb\} \\ \{\varepsilon, b, ab, abb\} \end{pmatrix}$$

$$\mapsto \begin{pmatrix} \{ab, abb, aabb, aabbb\} \\ \{\varepsilon, b, ab, abb\} \end{pmatrix}$$

$$\mapsto \begin{pmatrix} \{ab, abb, aabb, aabbb\} \\ \{\varepsilon, b, ab, abb, aabb, aabbb\} \end{pmatrix}$$