

Übung 1

$$mA_G = \begin{pmatrix} 0 & 2 & 5 & \infty & 4 \\ \infty & 0 & 2 & \infty & 1 \\ \infty & \infty & 0 & 3 & \infty \\ 3 & \infty & \infty & 0 & \infty \\ \infty & \infty & \infty & 9 & \infty \end{pmatrix} = D_G^{(0)}$$

Änderungen von $D_G^{(0)}$ zu $D_G^{(2)}$:

$(4, 2, 5), (1, 3, 4), (1, 5, 3), (4, 3, 7), (4, 5, 6)$

Änderungen von $D_G^{(2)}$ zu $D_G^{(5)}$:

$(1, 4, 7), (2, 4, 5), (2, 1, 8), (3, 1, 6), (3, 2, 8),$
 $(3, 5, 9), (5, 1, 12), (5, 2, 14), (5, 3, 16)$

Übung 2

$$(S, \oplus, \odot, 0, 1) = (\mathbb{N} \cup \{\infty\}, \max, \min, 0, \infty)$$

$$mA_G = \begin{pmatrix} \infty & 5 & 3 & 0 & 0 \\ 0 & \infty & 4 & 3 & 0 \\ 0 & 4 & \infty & 0 & 2 \\ 0 & 0 & 0 & \infty & 4 \\ 0 & 0 & 0 & 0 & \infty \end{pmatrix} = D_G^{(0)}$$

$D_G^{(1)}$: keine Änderung

$D_G^{(2)}$: $(1, 3, 4), (3, 4, 3), (1, 4, 3)$

$D_G^{(3)}$: $(1, 5, 2), (2, 5, 2)$

$D_G^{(4)}$: $(1, 5, 3), (2, 5, 3), (3, 5, 3)$

$D_G^{(5)}$: keine Änderung

Übung 3

$$(S, \oplus, \odot, 0, 1) = (\mathcal{P}(\Sigma^*), \cup, \cdot, \emptyset, \{\varepsilon\})$$

$$mA_G = \begin{pmatrix} \{\varepsilon, d\} & \{a\} & \emptyset \\ \emptyset & \{\varepsilon\} & \{b\} \\ \{c\} & \emptyset & \{\varepsilon\} \end{pmatrix} = D_G^{(0)}$$

Übung 3

$$\begin{aligned}D_G^{(1)}(1, 1) &= D_G^{(0)}(1, 1) \cup (D_G^{(0)}(1, 1) \cdot (D_G^{(0)}(1, 1))^* \cdot D_G^{(0)}(1, 1)) \\&= \{\varepsilon, d\} \cup (\{\varepsilon, d\} \cdot \{\varepsilon, d\}^* \cdot \{\varepsilon, d\}) \\&= \{d\}^*\end{aligned}$$

$$\begin{aligned}D_G^{(1)}(1, 2) &= D_G^{(0)}(1, 2) \cup (D_G^{(0)}(1, 1) \cdot (D_G^{(0)}(1, 1))^* \cdot D_G^{(0)}(1, 2)) \\&= \{a\} \cup (\{\varepsilon, d\} \cdot \{\varepsilon, d\}^* \cdot \{a\}) \\&= \{d\}^* \{a\}\end{aligned}$$

$$\begin{aligned}D_G^{(1)}(1, 3) &= D_G^{(0)}(1, 3) \cup (D_G^{(0)}(1, 1) \cdot (D_G^{(0)}(1, 1))^* \cdot D_G^{(0)}(1, 3)) \\&= \emptyset \cup (\{\varepsilon, d\} \cdot \{\varepsilon, d\}^* \cdot \emptyset) \\&= \emptyset\end{aligned}$$

Übung 3

$$\begin{aligned}D_G^{(1)}(3, 1) &= D_G^{(0)}(3, 1) \cup (D_G^{(0)}(3, 1) \cdot (D_G^{(0)}(1, 1))^* \cdot D_G^{(0)}(1, 1)) \\&= \{c\} \cup (\{c\} \cdot \{\varepsilon, d\}^* \cdot \{\varepsilon, d\}) \\&= \{c\}\{d\}^*\end{aligned}$$

$$\begin{aligned}D_G^{(1)}(3, 2) &= D_G^{(0)}(3, 2) \cup (D_G^{(0)}(3, 1) \cdot (D_G^{(0)}(1, 1))^* \cdot D_G^{(0)}(1, 2)) \\&= \emptyset \cup (\{c\} \cdot \{\varepsilon, d\}^* \cdot \{a\}) \\&= \{c\}\{d\}^*\{a\}\end{aligned}$$

$$\begin{aligned}D_G^{(1)}(3, 3) &= D_G^{(0)}(3, 3) \cup (D_G^{(0)}(3, 1) \cdot (D_G^{(0)}(1, 1))^* \cdot D_G^{(0)}(1, 3)) \\&= \{\varepsilon\} \cup (\{c\} \cdot \{\varepsilon, d\}^* \cdot \emptyset) \\&= \{\varepsilon\}\end{aligned}$$

Übung 3

$$D_G^{(1)} = \begin{pmatrix} \{d\}^* & \{d\}^*\{a\} & \emptyset \\ \emptyset & \{\varepsilon\} & \{b\} \\ \{c\}\{d\}^* & \{c\}\{d\}^*\{a\} & \{\varepsilon\} \end{pmatrix}$$

$$\begin{aligned} D_G^{(2)}(3,3) &= D_G^{(1)}(3,3) \cup (D_G^{(1)}(3,2) \cdot (D_G^{(1)}(2,2))^* \cdot D_G^{(1)}(2,3)) \\ &= \{\varepsilon\} \cup (\{c\}\{d\}^*\{a\} \cdot \{\varepsilon\}^* \cdot \{b\}) \\ &= \{\varepsilon\} \cup \{c\}\{d\}^*\{ab\} \end{aligned}$$

$$\begin{aligned} D_G^{(3)}(3,3) &= D_G^{(2)}(3,3) \cup (D_G^{(2)}(3,3) \cdot (D_G^{(2)}(3,3))^* \cdot D_G^{(2)}(3,3)) \\ &= (\{\varepsilon\} \cup \{c\}\{d\}^*\{ab\})^* \\ &= (\{c\}\{d\}^*\{ab\})^* \end{aligned}$$