Hall Effect Lab Report

Robert J. Pierrard

Advanced Lab PHYS 4321, Professor de Heer
Georgia Institute of Technology

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Abstract

We use two different magnets to produce magnetic fields for measuring the characteristics of the Hall Effect for a Hall sensor with unknown physical properties - material and geometry. With these data we calculate a material constant which we call the Hall constant. To aid these calculations, we also probe the geometry of the magnetic field produced by the larger of the two magnets and determine a relationship between the supplied voltage and produced magnetic field. With our large magnet we compare our Hall voltage to the input voltage; and with our small magnet we compare our Hall voltage directly to the produced magnetic field. In both experiments we find the Hall constant to have a value of $0.02A^{-1}$. The agreement of our experimental results strengthen the confidence of our results.

1 Introduction

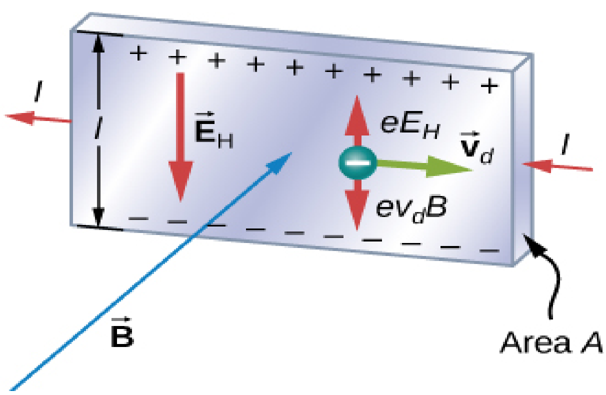
The Hall Effect is a common lab experiment where a voltage difference is produced across an electrical conductor transverse to both an applied electric current and a magnetic field, which are also perpendicular to each other (Figure 1). It was first discovered by Edwin Hall in 1879, and ever since it has been a staple experiment for demonstrating material properties of the Hall sensor, including the density of charge carriers - the magnitude of the produced voltage difference is directly dependent on this parameter [1].

In this lab we use two different magnets to produce magnetic fields to measure the characteristics of the Hall Effect. In addition to these measurements, we probe the geometry of the magnetic field produced by the larger of the two magnets and calculate a relationship between its supplied voltage and magnetic field magnitude. Our end goal is to calculate a physical constant of the material within our Hall sensor which relates the applied magnetic field to the Hall voltage. By understanding the characteristics of the magnetic field as a whole, we hope to better understand/qualify our results.

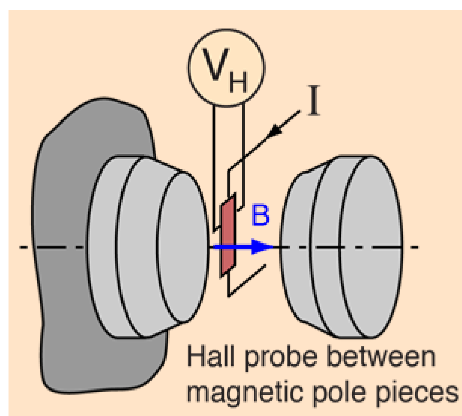
2 Theoretical Considerations

Investigating the Hall Effect is fundamentally an investigation on the motion of charge carriers through a material. In a conductor, charge carriers can be characterized as traveling in straight lines with occasional collisions causing them to randomly change direction. In the absence of an electric field, the mean displacement of all charge carriers is approximately zero. However, when an electric field is applied, the mean displacement is skewed in (or opposite to) the direction of the electric field - on average the charge carriers move with (or against) the direction of the electric field, we call the velocity at which they move the drift velocity. Further, we know that when a charge moves through a magnetic field it experiences a Lorentz force. So, on average the charge carriers will feel a force in the direction specified by the cross product of their average velocity and the magnetic field [4].

The effects these behaviors can be observed in a simplified 2D setup, where an electric current is driven across one direction of a conducting plane, and a magnetic field is applied perpendicular to the plane. Consequently, the magnetic forces felt by the charges will result in polarization in the direction perpendicular to both the electric and magnetic fields. This behavior is summarized in the simple illustration of Figure 1a.



(a)



(b)

Figure 1: A simplified pictorial demonstration of (a) the hall effect circuit configuration and (b) the hall effect circuit configuration within its an apparatus producing a magnetic field. Images sourced from [2] and [5]

Once the film becomes polarized there is an additional electrostatic force due to the polarization. Following [5] we can derive the Hall voltage in terms of other parameters. In the steady-state, this electrostatic force will balance the magnetic force such that

$$F_e = F_B$$

The magnetic force they feel is

$$F_B = ev_dB \tag{1}$$

where e is the charge of an electron, v_d is the drift velocity, and B is the magnetic field. Conversely, the elctric force is

$$F_e = eE \tag{2}$$

Equating these two terms we can clearly see that solving for the drift velocity results in

$$v_d = \frac{E}{B} \tag{3}$$

where E is the electric field generated from the charge polarization. The current can be represented as a few simple parameters, the number of charge carriers per volume n and the cross sectional area of the strip A.

$$I = nev_dA \tag{4}$$

Combining these two equations we can get the current in terms of the electric and magnetic fields.

$$I = ne(\frac{E}{B})A \tag{5}$$

The field E is related to the potential difference V between the edges of the strip

$$E = V/l \tag{6}$$

In this context the potential difference between the edges of the strip is known as the Hall Potential or Hall voltage. It can be measured directly with a voltmeter.Combining equations 5 and 6 together yields an equation for the Hall voltage.

$$V_{hall} = \frac{IBl}{neA} \tag{7}$$

and combining equations 3 and 6 yields

$$V_{hall} = Blv_d \tag{8}$$

Using these equations we have a direct relation between the Hall voltage and our applied magnetic field, given that they are perpendicular to one another as depicted in Figure 1a.

3 Experimental Methods

The entirety of this lab follows the instructions given in [3]. We will explain the overarching process, for step-by-step instruction read [3]. This lab can easily be divided into four different parts, enumerated as different subsections below.

3.1 Mapping Magnetic Field

The first part of this lab is dedicated to becoming familiar with our large magnetic field apparatus and our instrumentation. Our goal is to map the strength of the produced magnetic field as a function of position.

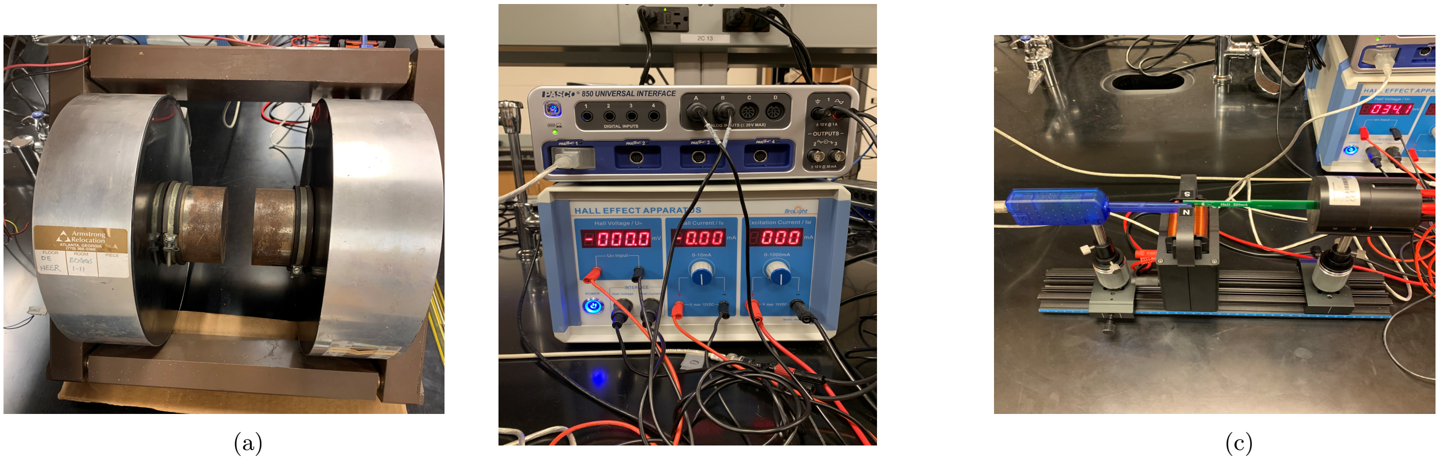
Our magnetic field is produced by the device in Figure (2a). Using the Pasco device, shown in the top part of figure (2b), and its corresponding software we can provide a specified voltage to this magnet. However, this device requires a large voltage to produced substantial magnetic fields. Therefore, rather than sending our voltage signal directly to the magnet, we sent it to a 3.97x voltage multiplier and then send the output of this multiplier to the magnet. With this in mind, when powering the large magnet we send up to a maximum of 10V to the voltage multiplier, supplying our magnet with at most 39.7V.

After providing a constant voltage of 39.7V to our magnet, mapping the produced magnetic field is a straightforward process. We simply position our magnetic field sensor at various locations around the magnet. We want to take advantage of available symmetries. It is clear that the magnet is in a cylindrical configuration and so it is radially symmetric, as such it produces a radially symmetric magnetic field. If we let the axis of this symmetry be the Z axis, we take measurements at different Z positions and at different radial positions. We reference the direction along the Z axis as the "perpendicular direction" and the radial direction as the "axial direction." Our magnetic field sensor takes measurements which also correspond to the perpendicular and axial directions. After taking various measurement we create a complete picture of our magnetic field.

The actual measurement process is quite straightforward within the Pasco environment. Following the directions in [3] we configure our Pasco device and recording environment. Through the Pasco function generator we supply the desired steady-state 10V to the voltage multiplier. To collect our data we simply click record and stop. Given that we are taking two measurements (perpendicular and axial magnetic field strength) at various positions specified by two coordinates, we find it easiest to manually store our measurements in an Excel file. This file can then be converted to a CSV file which can easily be analyzed to produce figures in Python or other coding languages.

3.2 Magnetic Field Variation from Voltage Sweeps

The second part of this lab concerns measuring the magnetic field at a single location while varying the voltage applied to the magnet. The setup is simple, we insert the magnetic field sensor (the blue device in Figure 2c) into the center of the large magnet. Following the convention defined when mapping the magnetic field and shown in Figure 4, this position corresponds to an axial position of 0 cm and a perpendicular position of 17.0 cm.



(a) (b) (c)

Figure 2: Three images depicting the experimental setup. (2a) depicts a large apparatus which creates a magnetic field between, and around, its two plates when powered. (2b) shows our measuring and control devices, their use is detailed throughout the Experimental Methods section. In the center of (2c) we see a small apparatus which produces a magnetic field. To the left of this image we have an electric field sensor, and to the right we have a Hall Effect Sensor, both of which are used throughout this lab.

With the magnetic field sensor in place, we vary the strength of the field by controlling the input voltage into the magnet. As when mapping the magnetic field, this is done by sending a voltage out from the Pasco box to the voltage multiplier via the Pasco interface on our computers. Through this interface we send out a triangular voltage signal which ramps up and down from 0V to 10V, corresponding to 0V to 39.7V supplied to the magnet after the voltage multiplier. The frequency of this oscillation is set to be very low, 0.01 Hz, so that the effects of hysteresis (which are very prevalent when working with magnets) can be minimized. With everyhing set in place, we record both the perpendicular and axial magnetic field components for an entire output voltage cycle. Lastly, we export our recorded values to a CSV file for analysis.

3.3 Hall Effect Variation from Voltage Sweeps

Now, for measuring the Hall Effect, we must use a special device (the green device in Figure 2c). Internally, this device is constructed similar to Figure 1a. A current is applied across a thin plane of metal. The device is inserted into a magnetic field oriented normal to the metal plane. Then,a voltage which we call the Hall voltage is measured perpendicular to the directions of both the magnetic field and the current. This measurement characterizes the Hall Effect.

In a similar fashion to measuring the magnetic field in response to voltage sweeps, we insert our Hall Effect sensor into the center of the magnetic field produced by the large magnet. We supply a constant current to our Hall Sensor and record our output Hall voltage while we vary our magnet voltage like before, through an entire cycle. We repeat this process with a total of 7 different Hall currents (0.5mA-2mA in 0.25mA increments) and export our measurements to a CSV file.

3.4 Hall Effect from Variation of Magnetic Field in Small Magnet

For the last part of this lab, we are again measuring the Hall voltage. However, now we are using a much smaller magnet than what has been used thus far (Figure 2a). Instead, we are using the configuration in Fig 2c. Further, we have the necessary space to measure both the magnetic field and the Hall voltage simultaneously. By directly comparing the magnetic field and Hall voltage, we will not have any hysteresis. We repeat the process done for measuring the Hall voltage while using the big magnet. However, as stated, now we record the magnetic field rather than the output voltage. Again, we take our measurements with the same 7 Hall currents and export our data to a CSV file.

4 Data Analysis

Each of the following subsections have a direct correspondence with the Experimental Methods section.

4.1 Mapping Magnetic Field

After collecting data for the perpendicular and axial components of the magnetic field at various perpendicular and axial locations, it is relatively straight forward to produce a vector plot displaying the data. We use python and the matplotlib package to generate our figures. Our magnetic field vector plot can be seen in Figure 4.

Looking at this figure, the most striking feature is the uniformity of the magnetic field in the interior ($<\approx 5cm$). In this regime, the magnetic field is entirely dominated by the perpendicular component, pointing straight from one plate to the other. Further there is minimal variation in field strength along the perpendicular direction. On the contrary, at larger axial distances, the shape of the magnetic field seems to become curved and its magnitude drops off rapidly. As a takeaway, it is clear that if one would like to work within a uniform magnetic field, then it is essential to work at a low axial position. However, the precise position within this regime is not so important.

4.2 Magnetic Field Variation from Voltage Sweeps

As with generating the vector plot, producing figures for the magnetic field as a function of applied voltage using matplotlib is quite straightforward and our results can be seen in Figure 5. We expect for the magnetic field to have a direct dependence on the voltage supplied to the magnet. Looking at the axial magnetic field, this appears to be the case. We can clearly see a gap between the ascending and descending voltage curves, indicating some hysteresis, but this is mostly negligible. It is apparent that in this scope the axial magnetic field is linearly proportional to the supplied Voltage.

$$\frac{\Delta B_{axial}}{\Delta V} = \frac{145-5}{10-0} = 14G/V$$

Assuming that $B_{axial} = 0$ when $V = 0$

$$\Delta B_{axial} = 14\Delta V \tag{9}$$

With this equation we could provide the required voltage for any desired output axial magnetic field in the range of approximately 0 to 140G.

While our treatment for the axial magnetic field is quite simple, our data for the perpendicular magnetic field are much more troublesome. We do not see a linear relationship similar to the axial direction. Rather, we see a sort of step function with an apparent maximum (in magnitude). The effects of hysteresis are similar and can also be neglected. We will assume that this stepwise pattern is a consequence of the electronics of the system. It is difficult to produce a uniform linear ramp in voltage. Further, the maximum achieved value (where the trend flatlines) is difficult to explain physically. It is more likely that our sensor is being saturated, 1600G may be the maximum value it can read. Looking at our vector field data, 1600G is also the maximum value recorded there. With this consideration, the vector field may not be as uniform as initially thought. There are most likely additional variations that are undetectable with our sensor. Despite these issues, we can still visualize a linear regression of best fit from a voltage of 0 to 4V. Following the same procedure as above.

$$\frac{\Delta B_{perp}}{\Delta V} = \frac{1400-200}{4-0} = 300G/V$$

Assuming that $B_{perp} = 0$ when $V = 0$

$$\Delta B_{perp} = 300V \tag{10}$$

4.3 Hall Effect Variation from Voltage Sweeps

When measuring the Hall Effect, it should be evident whether the magnetic field is saturating at a certain value, or if our magnetic field sensor itself is saturating. We again expect a linear relationship between the measured Hall voltage and the voltage supplied to the magnet (voltage multiplier). Following formulas 7 and 8 we see that the Hall voltage is linearly proportional to the magnetic field. We have already showed above that the magnetic field is linearly proportional to the supplied voltage. Therefore, the Hall voltage is linearly proportional to the supplied voltage.

Figure 3 shows that our predictions are correct. The Hall voltage is linearly dependent on the supplied magnet voltage. Further, our measurements are repeated for various values of the Hall current. Looking at 7 we see that the Hall voltage should also be linearly dependent on this value. This relationship appears to be confirmed. Scaling the Hall current seems to scale the Hall voltage by the same value. For example, at a Supplied Voltage of 10V and a Hall current of 0.5A, our Hall voltage is just under 30mV. If we scale the Hall current by 4 so that our Hall current is 2.0A then the Hall voltage also approximately quadruples to about 115mV. These values are consistent with our equations. Assuming the equations are otherwise correct, the slope of each line should be equal to

$$\frac{V_{hall}}{B} = \frac{Il}{neA} \tag{11}$$

rearranging and applying 10

$$\frac{V_{hall}}{V} = 300 \cdot I_{hall}\frac{l}{neA} \tag{12}$$

With 12 we can solve for the constants of our system $\frac{l}{neA}$ for each regression. They should be in agreement, we get the results for our "Hall constant" in Table 1 below.

Hall Current (A)	Hall Constant (A^{-1})
0.5	0.019
0.75	0.019
1.0	0.019
1.25	0.019
1.5	0.018
1.75	0.019
2.0	0.019

Table 1: Hall constant values corresponding to various Hall currents. Calculations made based on data in Figure 5 following the procedure outlined in the text. Calculated values are extremely consistent with each other. Although three digits are shown, only two are significant.

Our results are extraordinarily consistent using estimates to the single digit for our Hall Voltages. Therefore, we have 2 significant digits and can confidently say for our system that

$$\frac{l}{neA} = 0.02A^{-1} \tag{13}$$

Without having knowledge of the internal structure of our Hall Voltage sensor it is difficult to determine whether these results seem physically reasonable. We do not know the values for l, n, or A. The negative sign indicates the polarity of our charge carriers since the other three parameters must be positive.

4.4 Hall Effect from Variation of Magnetic Field in Small Magnet

Now, for our small magnet, we create similar figures to those above. The data analysis required to produce these figures is again relatively straight forward. We simply plot the Hall voltage against the magnetic field strength. All data for this section was collected in one recording (rather than breaking it up into different recordings for each Hall current). As a result, we have to separate the different Hall currents digitally. Given that we also recorded the Hall currents directly, we simply check the Hall current corresponding to each measurement and divide the measurements accordingly.

Our Hall voltage versus magnetic field strength measurements for the small magnet are captured in Figure 6. We again see the same staircase effect as was present in Figure 5. In both instances, we are directly measuring the perpendicular magnetic field strength. This staircase effect is not present in other measurements. Further, the steps increase precisely every 100G. Therefore, we assume that the perpendicular component of the magnetic field sensor can only measure discrete values of multiples of 100G. By plotting our data as a scatter plot (without connecting points) we can confirm this assumption (Figure 7). For the sake of extracting information, Figure 6 is more clear, but we must keep the discontinuity of the magnetic field sensor in mind. Applying similar logic as was done for calculating the Hall constant in the large magnet, we can use equation 11 again. However, this time we do not need to include a factor of -300 to convert from magnetic field strength to output voltage. Therefore we get...

$$\frac{V_{hall}}{B} = I_{hall}\frac{l}{neA} \tag{14}$$

Using this equation we can again calculate the value for the Hall constant of our sensor. We expect for our calculated value from these data to match that from the big magnet, since our three unknown factors are all properties of the Hall sensor. For consistency we will calculate this value using the midpoint of the vertical bars of our data. As an example, for calculating the Hall constant using the 2.0A data we get

$$\frac{l}{neA} = \frac{V_{hall}}{BI_{hall}} = \frac{32.5 - 0}{(800 - 0) \cdot 2.0} = 0.02A^{-1}$$

Calculating the value for each of the other Hall currents, we get Table 2. Our values calculated in this section have more variability than those in Table 1, but both data are very much in agreement with each other. Our value for the Hall constant with significant figures is again equal to 0.02. Given the way that these values were extracted from Figure 6, there is undoubtedly some error. This error comes directly from the discontinuous nature of the magnetic field sensor. For example, with the 2.0A regression, if we use the lower bound of 30.5V instead of using the midpoint value of 32.5V at 800G, our calulated value of $\frac{l}{neA}$ shifts to 0.019 - in better agreement with 1 but irrelevant. These discrepancies are so small that they can be ignored, considering our significant figures we still get $\frac{l}{neA} = 0.02$.

Hall Current (A)	Hall Constant (A^{-1})
0.5	0.020
0.75	0.021
1.0	0.022
1.25	0.021
1.5	0.021
1.75	0.019
2.0	0.020

Table 2: Hall constant values corresponding to various Hall currents. Calculations made based on data in Figure 6 following the procedure outlined in the text. Calculated values are very consistent with each other. Although three digits are shown, only two are significant.

5 Conclusion

Throughout this lab we have conducted various measurements highlighting the relationships between magnetic fields, electric fields, and their interactions with a conductive material. To begin we mapped the magnetic field within and around a large electromagnet. We are able to conclude that the electromagnet produces a uniform magnetic field in the space between its two plates. Extending our picture axially away from the center of the magnet, the magnetic field is no longer exclusively directed in the perpendicular direction. Rather, it bows outward and diminishes with greater axial distance, as seen in Figure 4. Our conclusion of uniformity within the center of the magnet is later called into question with the discovery that the resolution of our magnetic field sensor is restricted to multiples of $100G$. Therefore, we conclude that this uniformity is accurate to within $\pm 50G$.

Continuing with our characterization of the magnetic field, we measure its magnitude at the center of the electromagnet as a function of the applied voltage. In the measurements of the perpendicular magnetic field, we see the first evidence of the discontinuities in our magnetic field sensor. Despite this effect, we are able to fit a linear regression consistent with our theoretical calculations within the voltage range that we observe. Additionally, We find that despite being in the center of the magnet, there is still an axial component to our magnetic field which also scales linearly. Therefore, we can conclude that as the applied voltage increases, the magnitude of the magnetic field increases but the direction does not change.

With the linear relationship between our applied voltage and the produced magnetic field established, we continue on to measure the Hall effect using a Hall sensor. Despite some hysteresis, we again see the linear relationship that we expect from theory. With these data we are able to calculate the Hall constant, a material parameter $\frac{l}{neA}$ where e is the charge of an electron, l and A are geometric characteristics of our Hall sensor, and n is a material characteristic, the free electron number density.

In the final part of our experiments we repeat a similar procedure for measuring the Hall constant and we again yield the same results, further strengthening the validity of our calculations and measurements.

Outside of the scope of this report, our value for the Hall constant is consistent with a poorly conducting metal - but not a semiconductor. This claim is made from backhand calculations where the order of magnitude of the geometry of the Hall sensor material is approximated. It would be unscrupulous to include these calculation due to the amount of assumptions that must be made.

References

[1] G S Leadstone. The discovery of the hall effect. *Phys Educ. Vol 14*, 1979.

[2] R Nave. Hall effect, hall probe, 2021.

[3] Grant Nunn. Hall effect guide extras, 2021.

[4] Evan Rule. Hall effect, 2013.

[5] Bil Moebs Samuel J. Ling, Jeff Sanny. 11.7: The hall effect, 2020.

Appendix

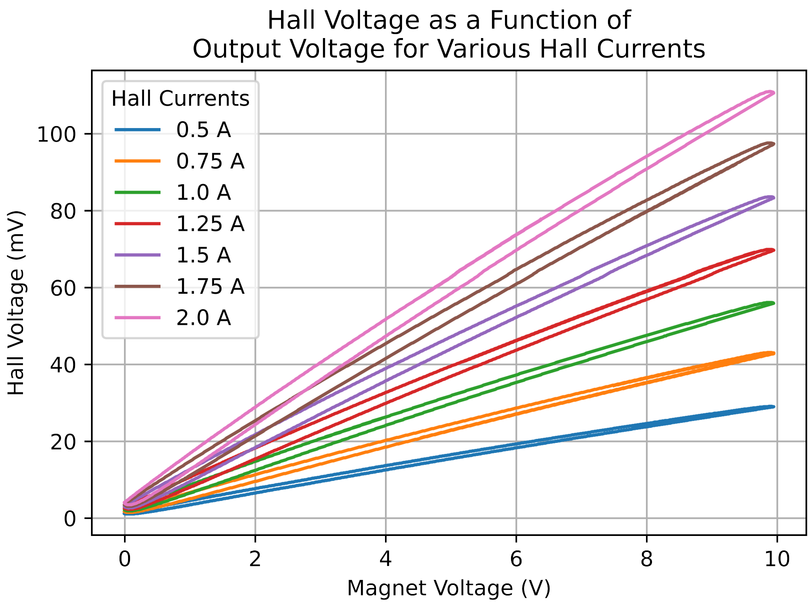


Figure 3: Measured results for the Hall voltage as a function of the voltage supplied to the voltage multiplier for various Hall currents when using the large magnet. Results show clear linear dependencies. There is no staircasing or saturation effects because we are measuring the supplied voltage directly rather than using the magnetic field sensor. Regressions are used to calculate the Hall constants in Table 1.

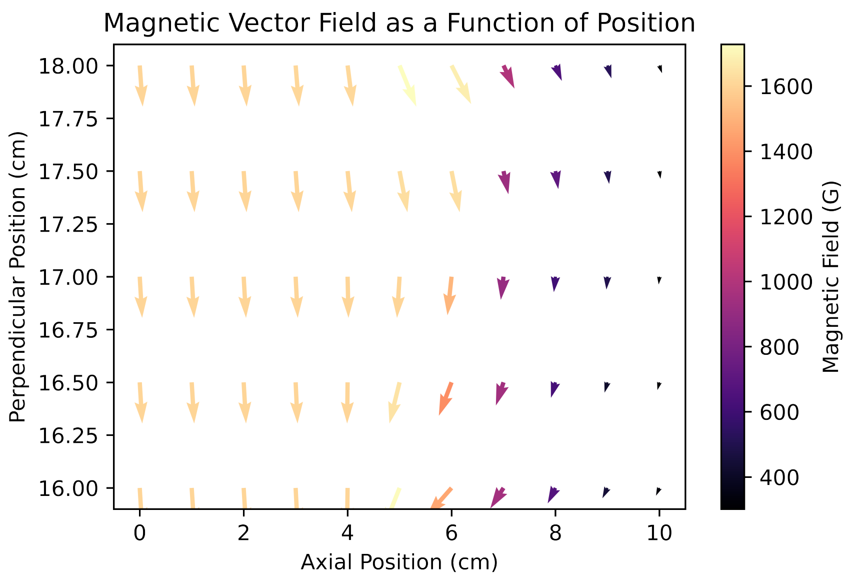


Figure 4: 2D vector plot showing the geometry of the magnetic field produced by the large magnet. A radial position of 0 cm corresponds to the center of the magnet. A radial position of 17 cm corresponds to midway between the two faces of the magnet. The total spacing between the two faces of the magnet is approximately 2cm. Note the uniformity of the field in the perpendicular direction at low radial positions.

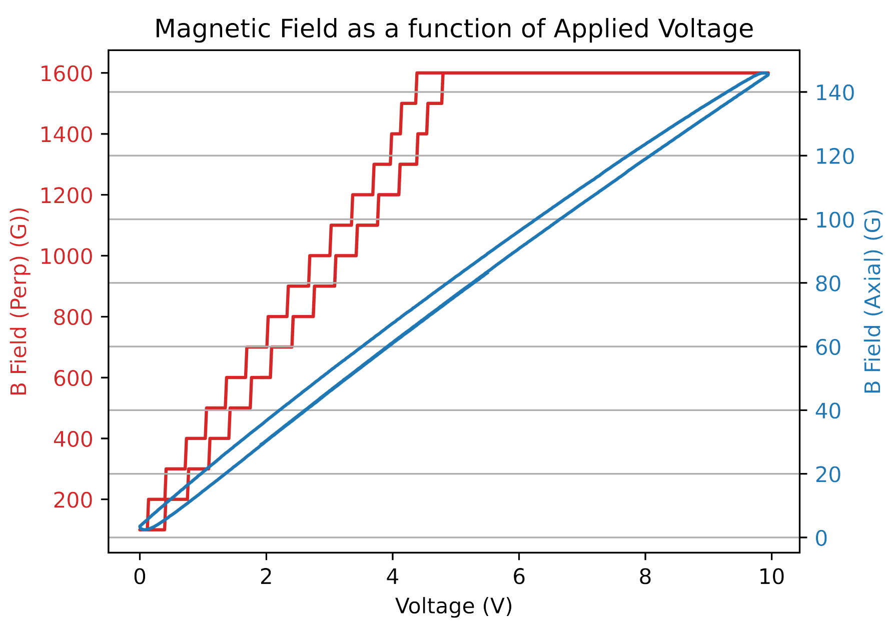


Figure 5: Measured relationship between the produced magnetic field and the voltage supplied to the voltage multiplier. Measurements are taken with the magnetic field sensor positioned approximately at the center of the large magnet. Despite this, there is still an axial component to the magnetic field, as expected with an imperfect magnet and imperfect placement at the center of the magnet. The step-like and flatlining behavior of the perpendicular field highlight the limitations of the magnetic field sensor used. Despite this, a linear relationship can still be extracted.

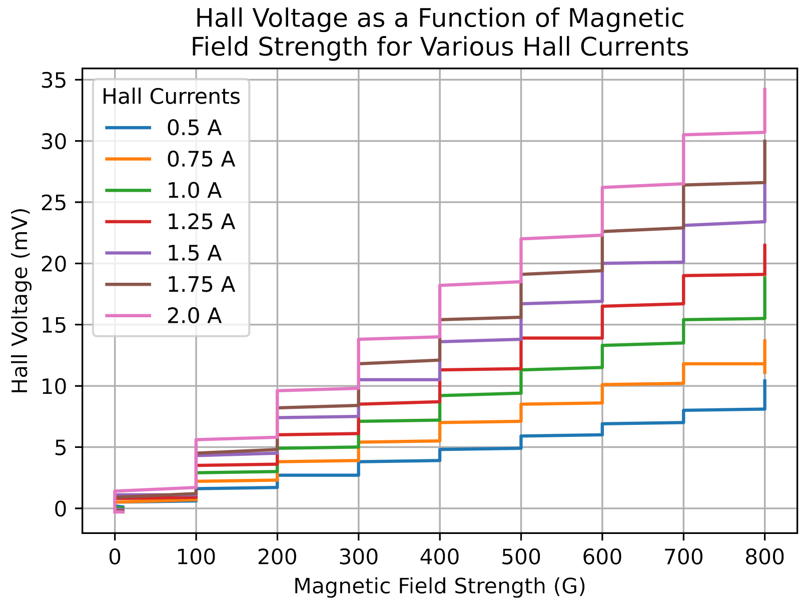


Figure 6: Measured results for the Hall voltage as a function of the magnetic field for various Hall currents when using the small magnet. Results are somewhat obscured by a staircasing effect, yet the linear nature is still apparent. Staircasing effects are due to the limitations of the magnetic field sensor. Regressions are used to calculate the Hall constants in Table 2

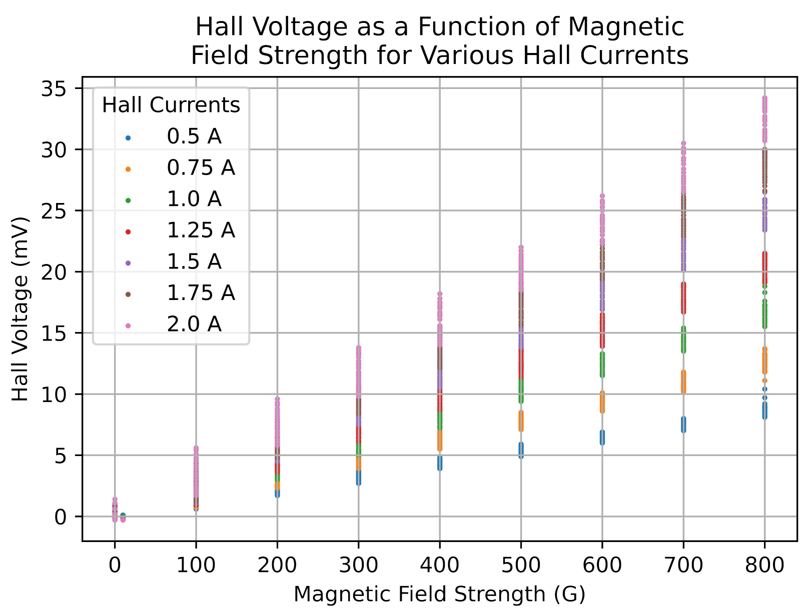


Figure 7: Same results as Figure 6 but without connecting data points. Used as an illustration to show the discrete nature of the perpendicular magnetic field strength measurements. It is clear these measurements are all multiples of 100G.