EESC GR6922: Atmospheric Radiation, Fall 2022
Homework 4, due Dec 16

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1 Drop size distributions

Drops within a cloud come in a variety of sizes droplet size distribution $n(r)$. The drop size distribution is often modeled using a Gamma distribution:

$$n(r) = N \frac{b^{\alpha+1}}{\Gamma(\alpha+1)} r^{\alpha} e^{-br}$$

where N is the particle concentration (number per volume), Γ is the Euler gamma function, and α, b are parameters related to the mean and width of the distribution.

Show that the effective radius for a gamma distribution is given by $r_e = (\alpha + 3)/b$

2 Single scattering by spheres

This problem relies on the ability to do Mie calculations of single-scattering properties – extinction, scattering, and absorption efficiencies and scattering phase functions – for spheres. Scott Prahl's `miepython` package looks pretty great. It can be installed via `pip` or download from Github (`https://github.com/scottprahl/miepython`). The Github page links to thorough documentation including examples. A table of the complex index of refraction of water is bundled with the code (`data/segelstein81_index.txt`).

1. Plot the extinction and absorption cross-sections for a drop of pure water with radius r 10 µm across the solar spectrum (0.2 - 4 µm). It may be interesting to show the size parameter $x = \frac{2\pi r}{\lambda}$ in addition to the wavelength. Make a second plot showing extinction and absorption efficiencies. Make a second set of plots for the water vapor window(8-12 µm), where the atmosphere is transparent enough that clouds can impact the surface and top-of-atmosphere radiation fields. Remember that the complex index of refraction varies with wavelength.
2. Plot the scattering phase function for a drop with radius $r = 10$ µm at wavelength $\lambda = .5$µm. Show the delta-scaled phase function on the same plot. You may but need not assume that $f = g^2$, and you'll need to make a choice of the angular width over which to replace the phase function with a delta function. Explain your choices.
3. Plot the extinction and absorption cross-sections and efficiences for droplets ranging from 4 - 20 µm for wavelength $\lambda = .5$µm. Do the same for a Gamma distribution of cloud drops with r_e varying over the same range. You can set effective variance $\nu_e \equiv 1/\alpha + 3 = 0.1$. How and why do the curves differ?

3 Two-stream solutions

Using the solutions to the two-stream equations we developed in class

1. Plot the reflectance, transmittance, and absorptance of a single layer as a function of total optical depth τ^* and single scattering albedo. Let experience guide the range of values over which you show the values.

2. Use Mie calculations to compute the single-scattering albedo and phase functions for Gamma distributions at $r_e = [2, 4, 8, 12, 16, 24]\mu\mathrm{m}$ at wavelengths $\lambda_{\text{vis}} = .65\mu\mathrm{m}$ and $\lambda_{\text{nir}} = 2.16\mu\mathrm{m}$. Plot the reflectance at λ_{nir} as a function of the reflectance at λ_{vis} for $\tau^* = 4, 6, 8, 12, 16, 24, 32$. Connect lines of constant τ^* with lines; connect lines of constant r_e with lines of another type/color. Explain how measurements at these two wavelengths might be used to estimate τ^* and r_e. In what parameter range would it be possible to estimate one of these quantities from a single measurement, rather than both quantities from the two measurements?

4 Optional problem for the curious

As we learned when discussing geometric optics, the rainbow arises from backward scattering near 128° whose angular position varies slightly because the real part of the index of refraction varies slightly across the visible spectrum.

Estimate the angular width of the rainbow using Mie calculations. This will require computing the phase function for a variety of wavelengths (canonically, those corresponding to red, orange, yellow, green, blue, indigo, and violet light), identifying the maximum in the phase function near the rainbow angle, and assessing how this angle varies with wavelength. It would be interesting to do this for cloud drops ($\mathcal{O}(10)\mu\mathrm{m}$), drizzle drops ($\mathcal{O}(100\mu\mathrm{m}$), and raindrops ($\mathcal{O}(1\mathrm{mm})$) for both single particles and drop size distributions. (Precipitation usually follows an exponential rather than a Gamma size distribution.)