Atmospheric Thermodynamics - Tutorial 5

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1 Lifting Condensation Level

Considering a parcel of air with a constant specific humidity q_v rising adiabatically, eventually a height will be reached where q_v will equal the saturation specific humidity associated with the surrounding atmosphere. When this occurs, relative humidity is $f = 100\,\%$. The altitude at which this occurs is called the lifting condensation level z_{LCL}, and is associated with a temperature T_{LCL}, that is computed from:

$$F(T_{LCL}) = \ln f + \frac{c_p}{R} \ln\left(\frac{T_{LCL}}{T}\right) + \frac{\epsilon L_{lv}}{R}\left(\frac{1}{T_{LCL}} - \frac{1}{T}\right) = 0, \tag{1}$$

using the Netwon-Raphson root finding algorithm. Having solved the equation above, the T_{LCL} is seen to be a function dependent on initial ground temperature T and ground relative humidity f. With this result, the LCL height can be calculated from the equation:

$$T_{LCL} = T(z_{LCL}) = T - \Gamma z_{LCL}, \tag{2}$$

where Γ is the dry adiabatic lapse rate, taken to be $\Gamma = 10\,\frac{\mathrm{K}}{\mathrm{km}}$.

A second method for finding the z_{LCL} involves the lifting condensation level pressure p_{LCL} given by:

$$p_{LCL} = p\left(\frac{T_{LCL}}{T}\right)^{\frac{c_p}{R}}, \tag{3}$$

where p represents the ground pressure taken to be $p = 1000\,\mathrm{hPa}$ and R is the individual gas constant of dry air.

Then, using the barometric formula re-arranged in terms of z one gets:

$$z_{LCL} = \frac{T}{\Gamma}\left[1 - \left(\frac{p_{LCL}}{p}\right)^{\frac{R\Gamma}{g}}\right]. \tag{4}$$

Figures 1 and 2 should z_{LCL} as a function of initial relative humidity f, for several ground temperatures $T = [-20, -10, 0, 10, 20]$, as calculated by the two methods.

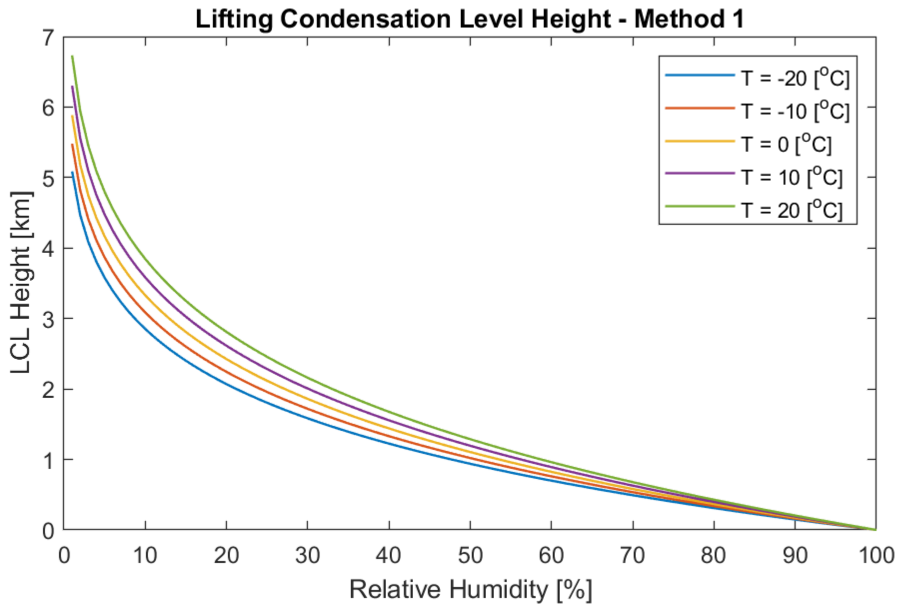

Figure 1. Dew-point deficit as a function of temperature for various values of relative humidity.

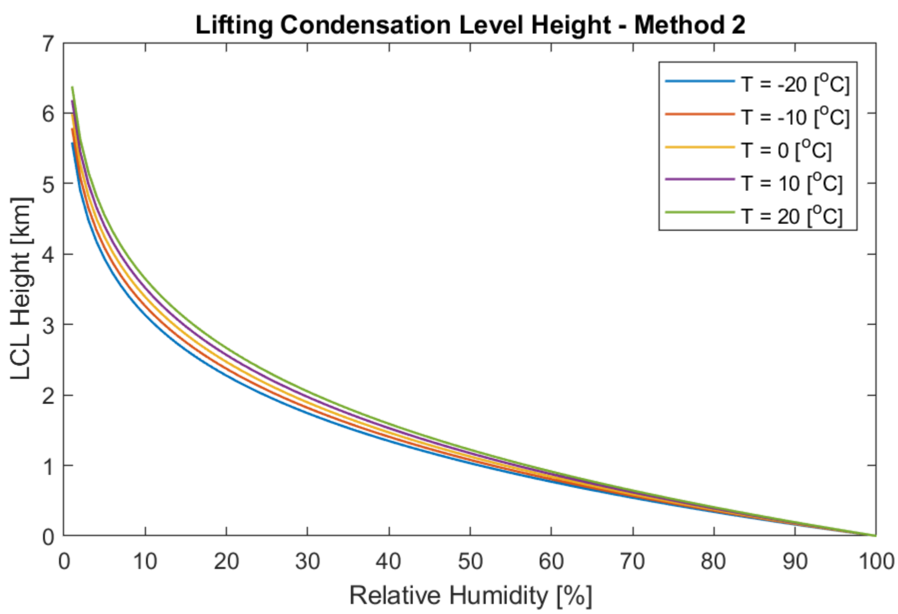

Figure 2. Dew-point deficit as a function of temperature for various values of relative humidity.

As can be seen, both models behave similarly and are convincing. The difference between each of these models and the model provided by Bolton(1980),

$$T_{LCL} = \frac{1}{\frac{1}{T-55} - \frac{\ln f}{2840}}, \tag{5}$$

is plotted in figures 3 and 4.

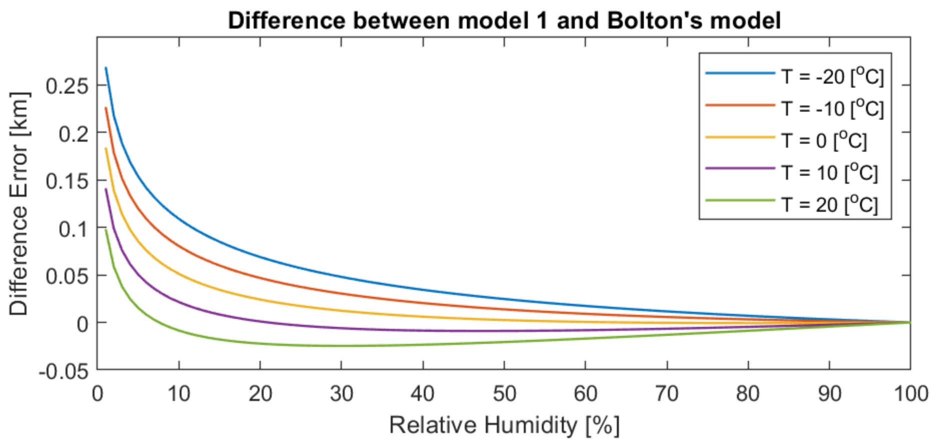

Figure 3. Difference between z_{LCL} results calculated from (2) and (5).

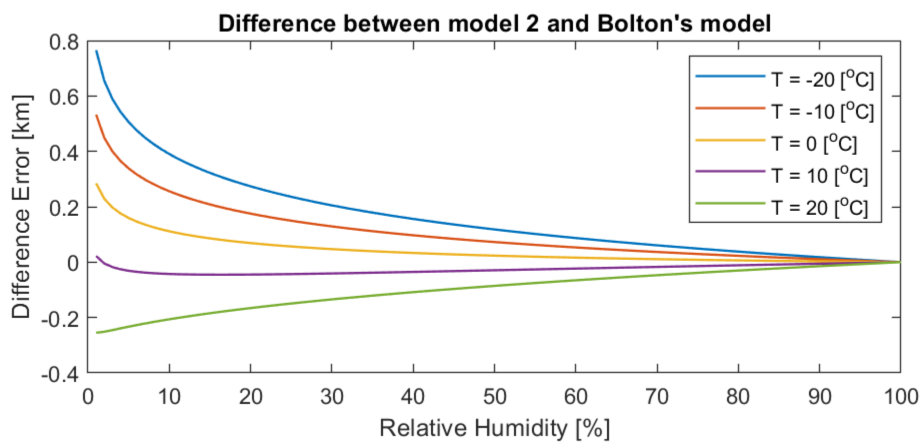

Figure 4. Difference between z_{LCL} results calculated from (4) and (5).

It is enough to compare the two difference error scales to see that model 1 replicated Bolton's model more successfully. In either case, the error grows sharply for low relative humidities and low temperatures.

An approximation of lifting condensation level is given by the formula:

$$z_{LCL} = 120(T - T_d). \tag{6}$$

When the absolute difference is taken between this value and the corresponding value generated by model 1, the following plot is generated.

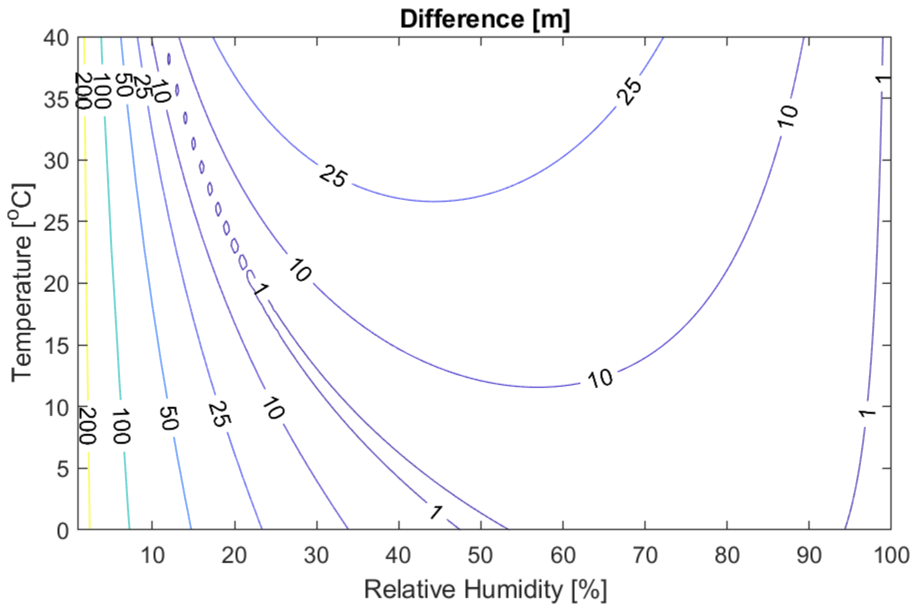

Figure 5. Difference between model 1 and the approximated value for z_{LCL}.

Figure 5 illustrates that the approximation is very good when the ground level conditions are humid and hot.

2 Adiabatic and Pseudo-Adiabatic Temperature Change

Just as the atmosphere's temperature profile can be represented via the dry adiabatic lapse rate Γ_d in the case of dry air, the moist adiabatic lapse rate Γ_s can be used to represent the case of moist air:

$$\Gamma_s = \gamma \Gamma_d. \tag{7}$$

In the pseudo adiabatic case, γ may be formulated as:

$$\gamma = \frac{1 + \frac{q_s L_{lv}}{R_d T}}{1 + \frac{q_s L_{lv}^2}{c_{pd} R_v T^2}}. \tag{8}$$

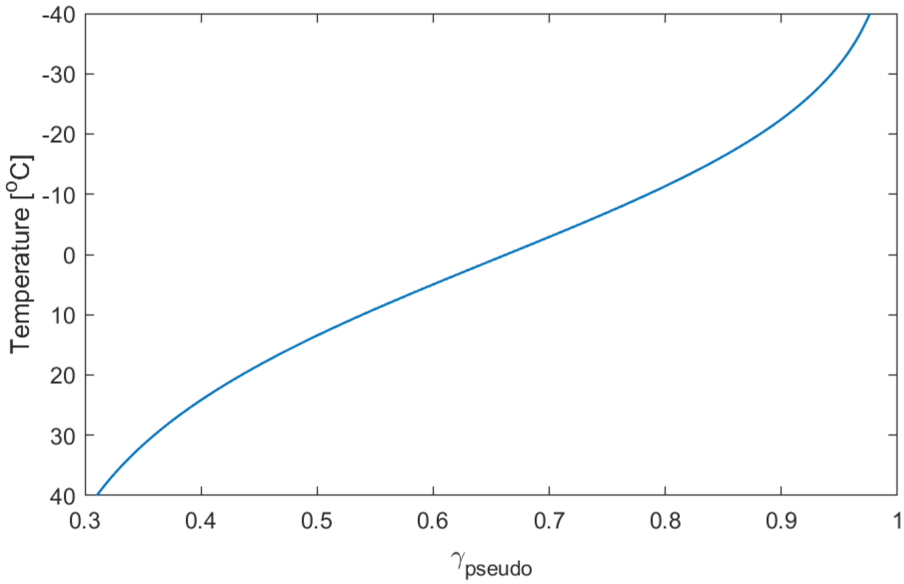

Figure 6. Difference between model 1 and the approximated value for z_{LCL}.