Atmospheric thermodynamics tutorial 1

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1 Latent heats

A formula for latent heat of condensation:

$$L_{lv}(T) = L_{lv0} - (c_l - c_{pv})(T - T_0), \tag{1}$$

and a formula for latent heat of sublimation:

$$L_{iv}(T) = L_{iv0} - (c_i - c_{pv})(T - T_0), \tag{2}$$

both describe a behavior of latent heat with temperature, and are a linear function of temperature. For purposes of this tutorial we assume:

$L_{lv0} = 2.501 \frac{MJ}{K}, L_{iv0} = 2.834 \frac{MJ}{K}$,
$c_{pv} = 1870 \frac{J}{kgK}, c_{pl} = 4193 \frac{J}{kgK}, c_{pi} = 2107 \frac{J}{kgK}$.

We can calculate a relative error, based on a constant value L_{nv0}, where $n = l, i$:

$$\epsilon = \frac{L_{nv0} - L_{nv}}{L_{nv}} \tag{3}$$

The formula above will be useful to check if certain assupmtions are correct. Either one constant value or values from the table will be used as a reference.

The behavior of latent heats with temperature is presented in figure 1:

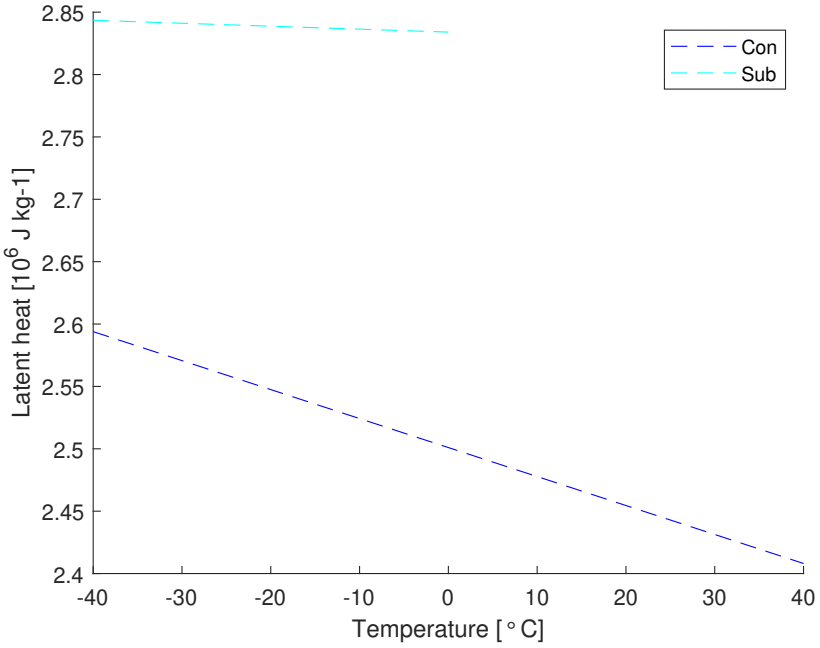

Figure 1: Behavior of latent heats with temperature.

Both lines are a picture of linear behavior of latent heat, with the slope of the line being the diffrence between specific heats of diffrent states of matter. The sublimation line has smaller slope, therefore its value does not change as much with temperature, as it is for condensation.

In the figure below, error described by equation 3 is shown:

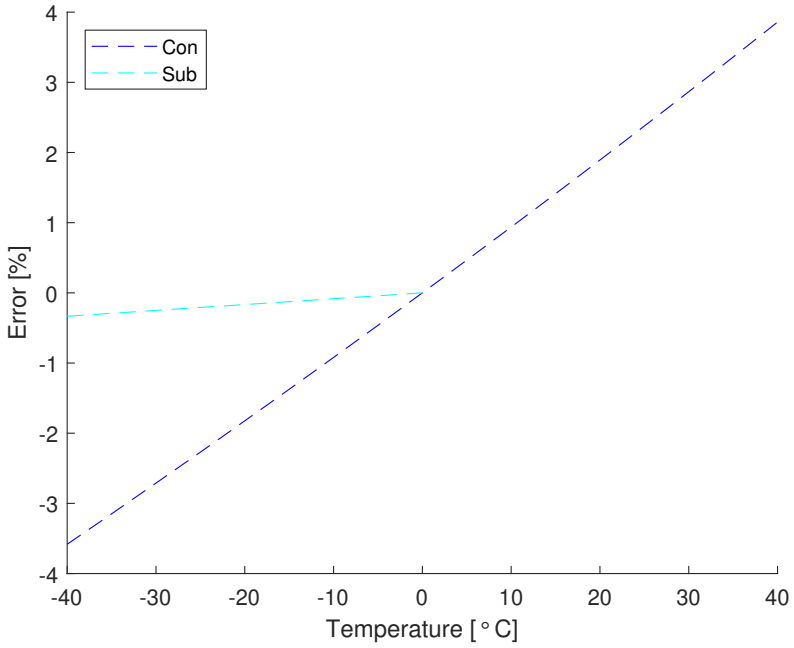

Figure 2: Relative error of latent heats with temperature.

In order to answer the question if the assumption of constant latent heat is a good assumption, we need to define what is a good assumption. Both of errors are in range between -3.5% and 3.5%. However the error for latent heat of sublimation is between -0.3% and 0%, which is ten times less than the error for latent heat of condensation, therefore in this frame of reference, only the assumption for sublimation is correct.

Additionally, we can calculate an error between Smithsonian Meteorological Table latent heat values and values calculated using constant specific heats assuption. The values of the errors are presented in the fig 3

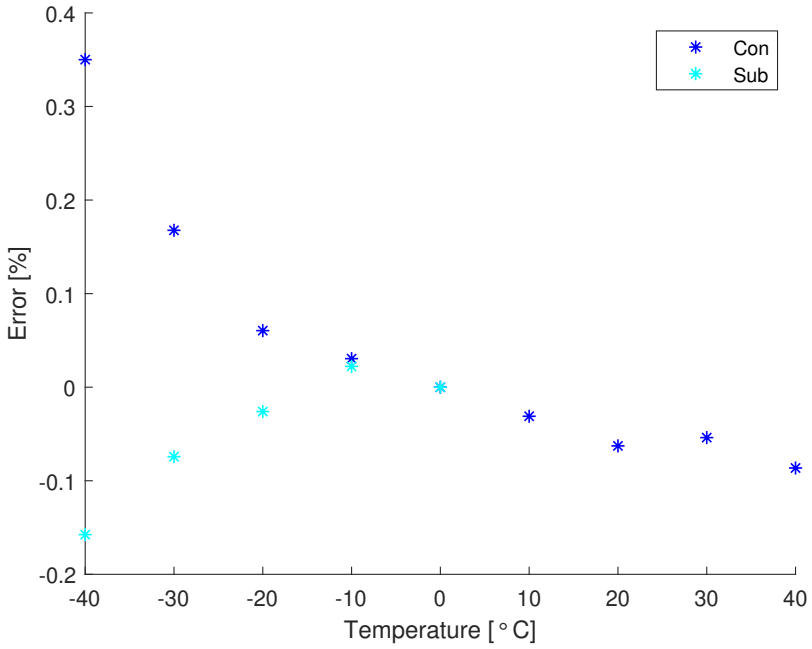

Figure 3: Relative error of latent heats with temperature.

We can see, that the errors are of the order of fraction of a percent, therefore we can conclude that the assumption of constant specific heats is correct.