Atmospheric thermodynamics tutorial 5

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1 Vertical properties of the atmosphere

We assume a parcel of air moving upwards, that has constant specific humidity q_v. The saturation specific humidity decreases with height, and it reaches the level where $q_v = q_s$. This level is called Lifting Condensation Level (LCL). In order to calculate LCL, we can calculate LCL pressure p_{LCL}:

$$p_{LCL} = p\Big(\frac{T_{LCL}}{T}\Big)^{c_p/R}, \tag{1}$$

and in order to calculate T_{LCL}, we need to solve and equation that is a combination of first law of thermodynamics for entalphy and the Clausius-Clapeyron equation:

$$d(\ln f) = \frac{c_p}{R} d(\ln T) - \frac{L_{lv}}{R_v T} d(\ln T). \tag{2}$$

Integrating this equation from f to 1 and from T to T_{LCL} will give us an equation for T_{LCL}:

$$-\ln f = \frac{c_p}{R} \ln \frac{T_{LCL}}{T} + \frac{\epsilon L_{lv}}{R_v T}\Big(\frac{1}{T_{LCL}} - \frac{1}{T}\Big), \tag{3}$$

which needs to be solved numerically to obtain T_{LCL}

There is however, an estimate to the solution of equation above, given by Bolton (1980):

$$T_{LCL} = \frac{1}{\frac{1}{T-55} - \frac{\ln f}{2840}} + 55, \tag{4}$$

which validaty will be checked in the following analysis.

Assuming that the temperature changes with a dry adiabatic lapse rate, we can calculate the height of LCL. Assuming diffrent ground temperatures, we can plot the height of LCL as a function of relative humidity:

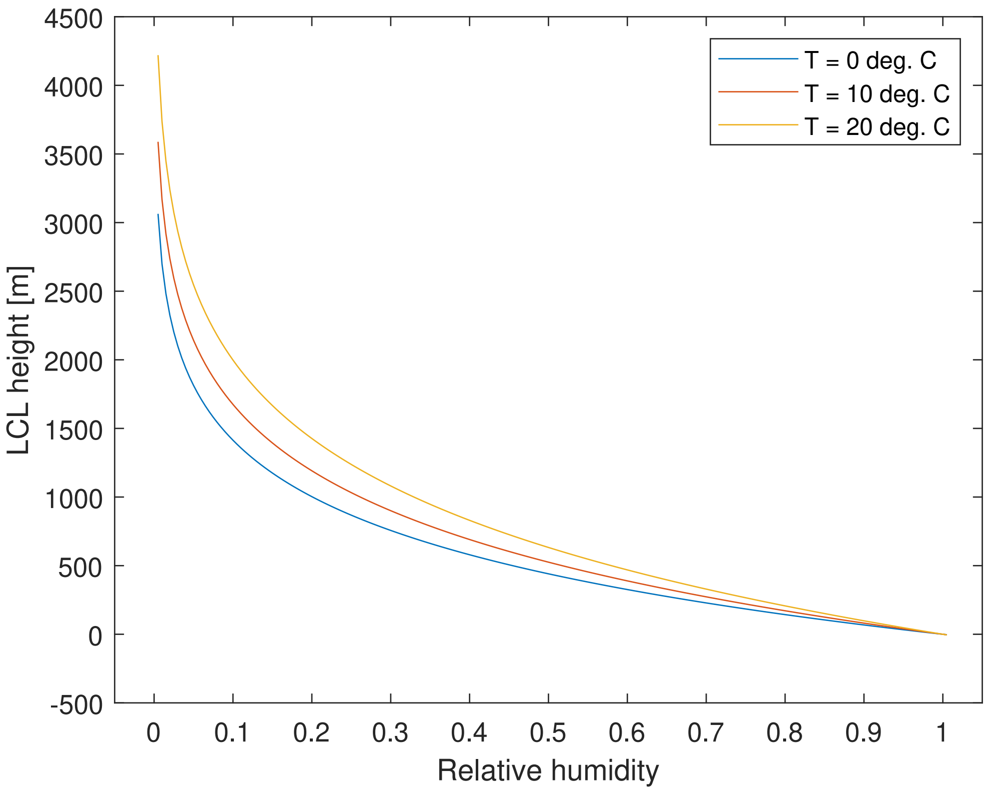

Figure 1: Height of the LCL as a function of relative humidity

We can see that the lcl height decreases with increasing relative humidity, and is higher for higher temperatures.

We can then plot the height of LCL as a function of temperature, for diffrent relative humidities:

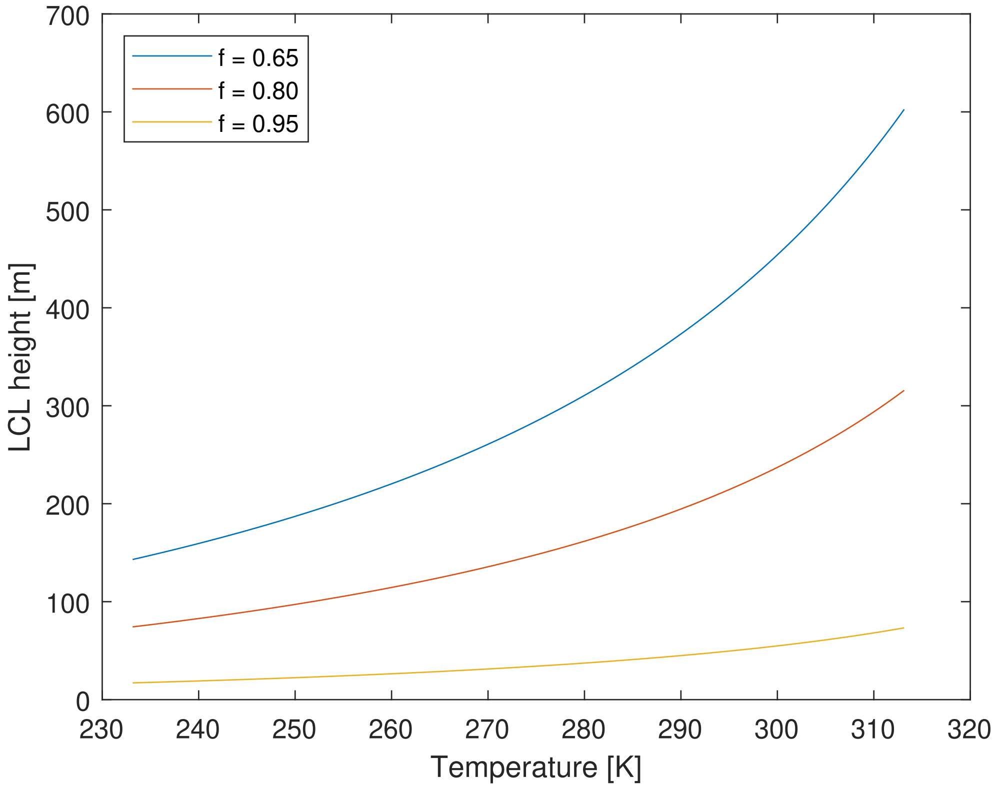

Figure 2: Height of the LCL as a function of temperature

We can see that the higher the temperature, the higher the LCL. Also, for smaller humidities, the LCL height is higher.

Next, we can calculate the diffrence between the T_{LCL} calculated using formulas (3) and (4) as a function of surface temperature and relative humidity:

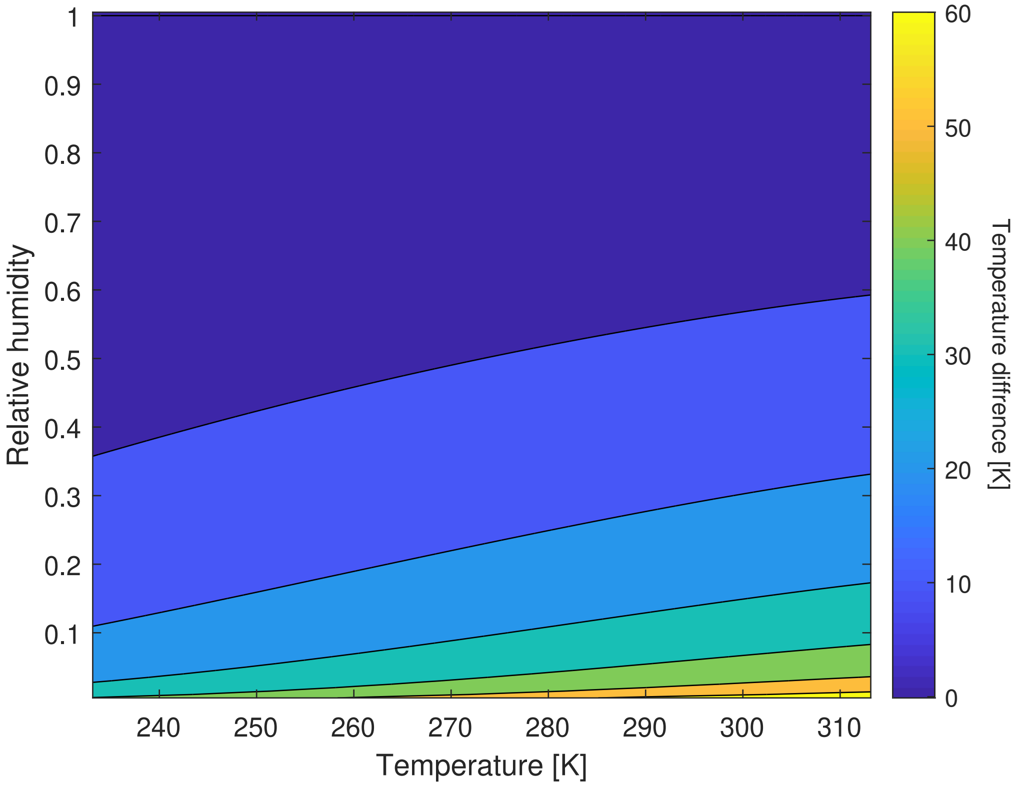

Figure 3: Diffrence between T_{LCL} calculated using formulas (3) and (4)

We can see that the smallest diffrence is for humidities bigger than 0.5, and for smaller temperatures.

Next, we can check the discrepancy between the height of LCL calculated using formula (3) and using an estimate $z_{LCL} = 120 \cdot (T_0 - T_d)$ for diffrent relative humidities and surface temperatures:

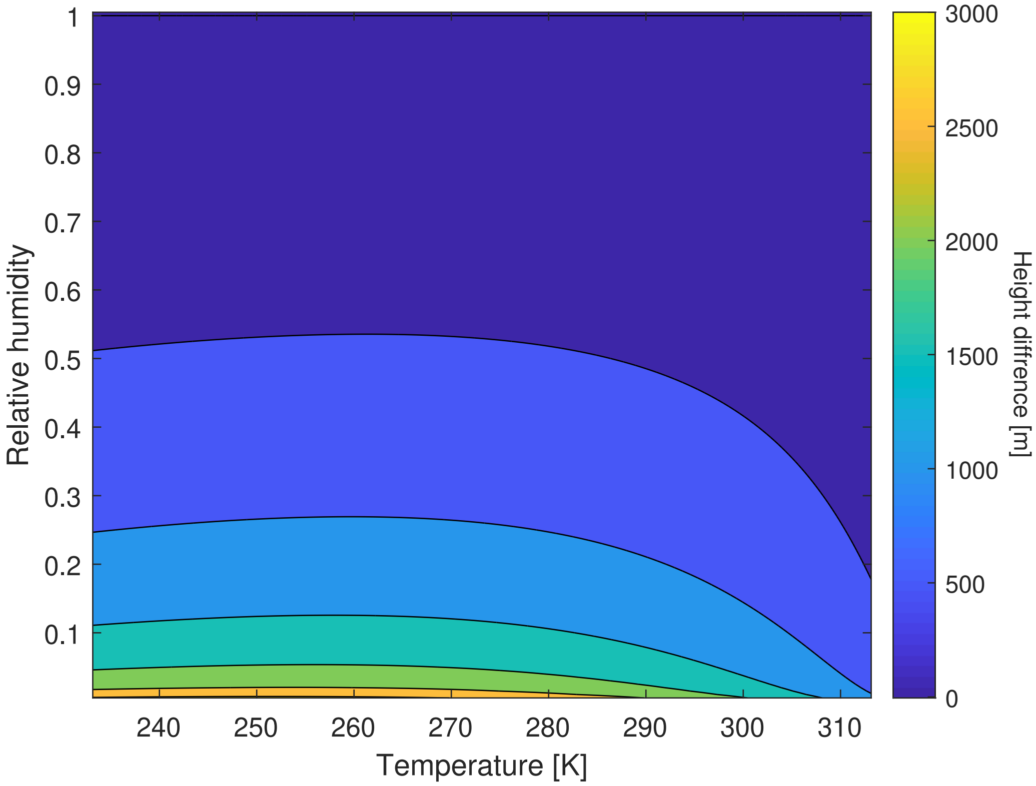

Figure 4: Diffrence between estimated height of LCL and the value calculated using (3)

We can see that the discrepancy gets very big for humidites smaller than 0.5 and for smaller temperatures.

Change of temperature with height is described by a temperature gradient. The dry adiabatic lapse rate, quantity describing change of temperature with height for adiabatic process for dry air is given by a relation:

$$\Gamma_d = \frac{g}{c_{pd}}. \tag{5}$$

The same quantity, but for moist, saturated air is given by:

$$\Gamma_s = \gamma \Gamma_d, \tag{6}$$

where:

$$\gamma = \frac{c_{pd}}{c_p} \frac{1 + q_s \beta_T \frac{R_v}{R}}{1 + q_s \beta_T \frac{L_{lv}}{c_p T}}, \tag{7}$$

where $\beta_T = L_{lv}/R_v T$. We can define a pseudo-adiabatic process, by assuming, that the system is open, and the condenset water is removed after it is produced. Then, we can define a pseudo-adiabatic lapse rate. Then, the equation for γ is given by a relation:

$$\gamma = \frac{1 + \frac{q_s L_{lv}}{R_d T}}{1 + \frac{q_s L_{lv}^2}{c_{pd} R_v T^2}}. \tag{8}$$

Note, that c_p and R in the formulas above are not the specific heat of water and universal gas constant, but rather are defined by following relations:

$$R = q_s R_v + q_d R_d, \tag{9}$$

$$c_p = q_d c_{pd} + q_s c_{pv} + q_l c_l. \tag{10}$$

By substituting $R- > R_d$ and $c_p- > c_{pd}$, we can obtain formula (8) from formula (7). In order to calculate γ, we need to solve those calculations numerically.

We can compare the two formulas, and plot the γ coefficient as a function of temperature:

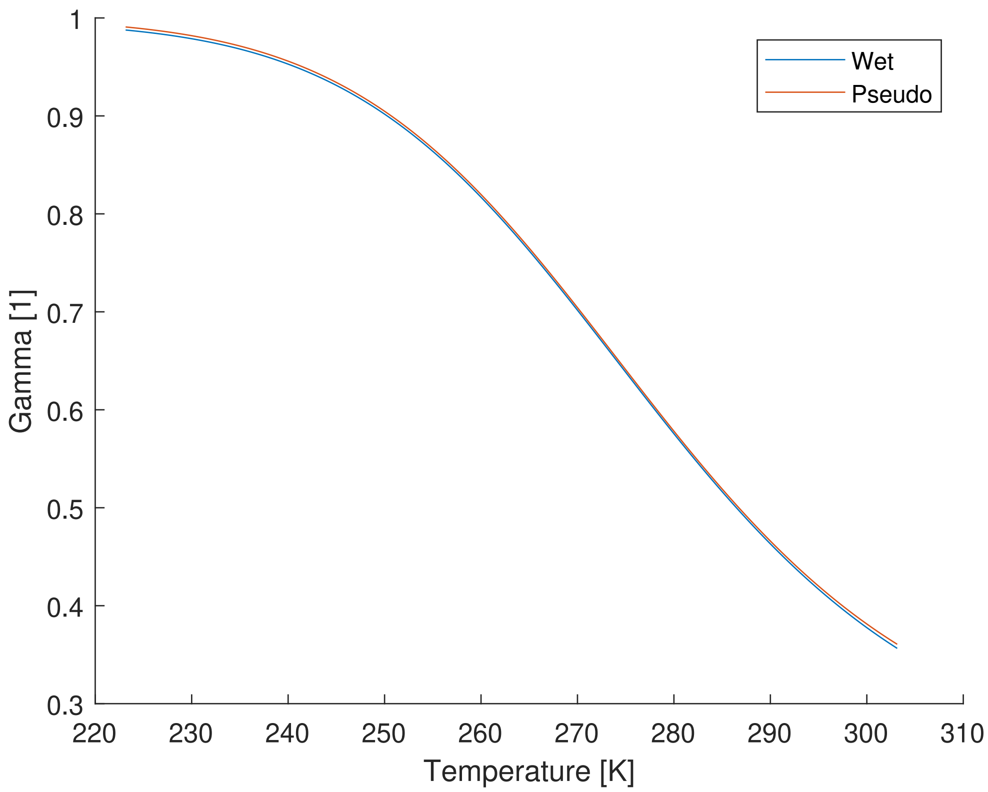

Figure 5: Values of gamma coefficient as a function of surface temperature

We can see that both coefficients are below 1, and are very close to each other.

Using the coefficients, we can then calculate the wet adiabatic lapse rate, and pseudoadiabatic lapse rate, and use them to calculate temperature as a fucnction of height:

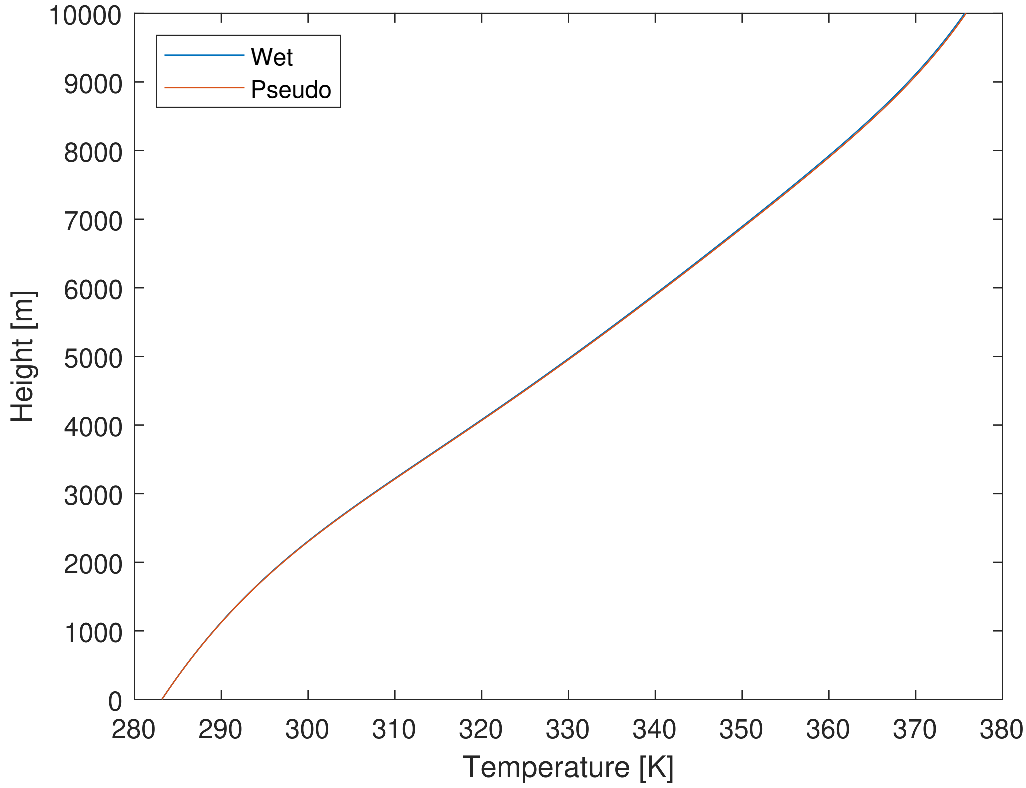

Figure 6: Change of temperature with height

We assumed surface temperature as 283.15 K. We can see that temperature increases steadily with height.

We can also calculate the amount of water condensed during the adiabatic process of vertical movement of air parcel using a formula:

$$q_l(h) = \frac{c_p}{L_{lv}}(\Gamma_d - \Gamma_s) \cdot h \tag{11}$$

Using the formula above, we can plot the amount of condensed water with height for the wet adiabatic lapse rate, and for pseudo adiabatic lapse rate:

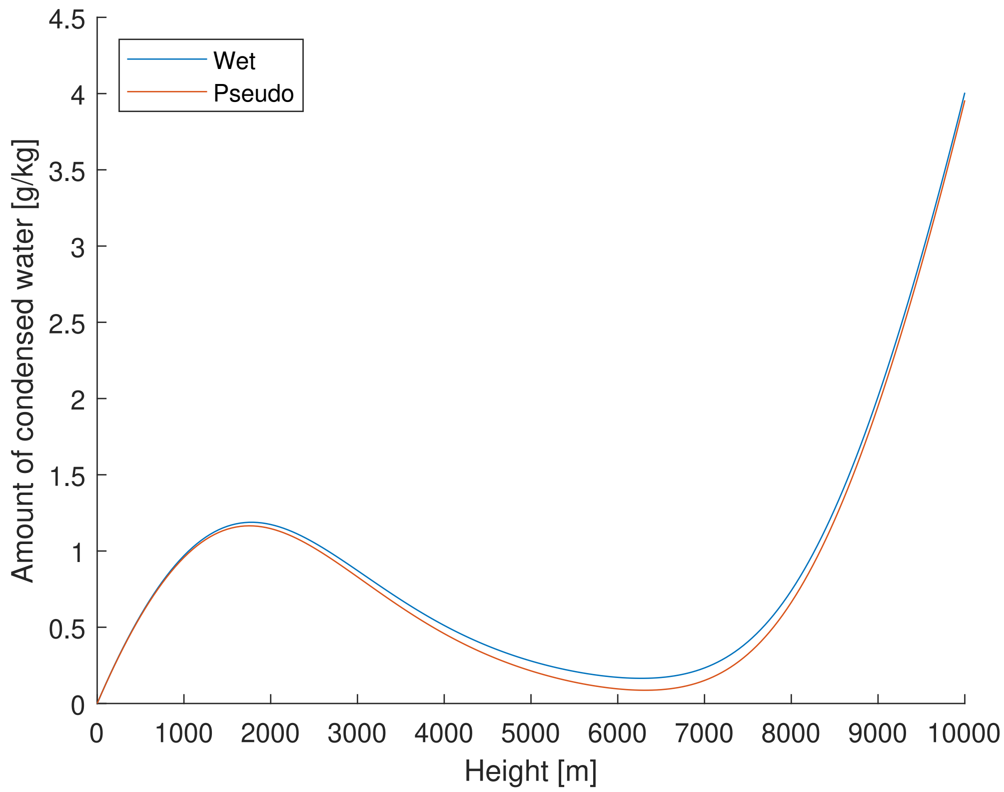

Figure 7: Amount of condensed water with height

We assume that the temperature is 273.15 K. We can see that the amount of water rises with height, and then decreases after about 2 km, and rises after 6 km. It is probably due to fact, that the scaling factor γ is close to 1 for small temperatures.