Atmospheric Thermodynamics - Tutorial 4

Robert Ruta

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1 Lifting Condensation Level

Considering a parcel of air with a constant specific humidity q_v rising adiabatically, eventually a height will be reached where q_v will equal the saturation specific humidity associated with the surrounding atmosphere. When this occurs, relative humidity is $f = 100\,\%$. The altitude at which this occurs is called the lifting condensation level z_{LCL}, and is associated with a temperature T_{LCL}, that is computed from:

$$F(T_{LCL}) = \ln f + \frac{c_p}{R} \ln\left(\frac{T_{LCL}}{T}\right) + \frac{\epsilon L_{lv}}{R}\left(\frac{1}{T_{LCL}} - \frac{1}{T}\right) = 0, \tag{1}$$

using the Netwon-Raphson root finding algorithm. Having solved the equation above, the T_{LCL} is seen to be a function dependent on initial ground temperature T and ground relative humidity f. With this result, the LCL height can be calculated from the equation:

$$T_{LCL} = T(z_{LCL}) = T - \Gamma z_{LCL}, \tag{2}$$

where Γ is the dry adiabatic lapse rate, taken to be $\Gamma = 10\,\frac{\mathrm{K}}{\mathrm{km}}$.

A second method for finding the z_{LCL} involves the lifting condensation level pressure p_{LCL} given by:

$$p_{LCL} = p\left(\frac{T_{LCL}}{T}\right)^{\frac{c_p}{R}}, \tag{3}$$

where p represents the ground pressure taken to be $p = 1000\,\mathrm{hPa}$ and R is the individual gas constant of dry air.

Then, using the barometric formula re-arranged in terms of z one gets:

$$z_{LCL} = \frac{T}{\Gamma}\left[1 - \left(\frac{p_{LCL}}{p}\right)^{\frac{R\Gamma}{g}}\right]. \tag{4}$$

Figures 1 and 2 should z_{LCL} as a function of initial relative humidity f, for several ground temperatures $T = [-20, -10, 0, 10, 20]$, as calculated by the two methods.

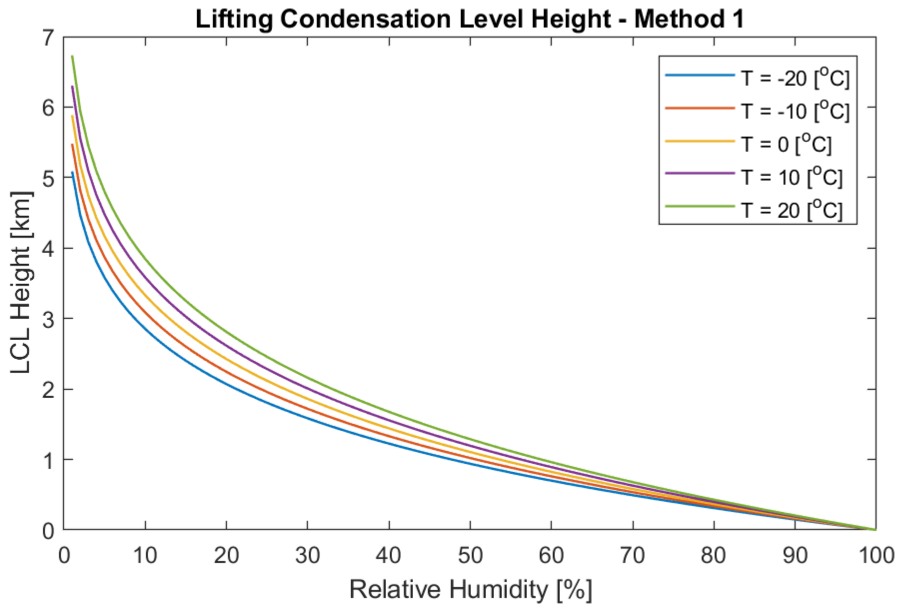

Figure 1. Dew-point deficit as a function of temperature for various values of relative humidity.

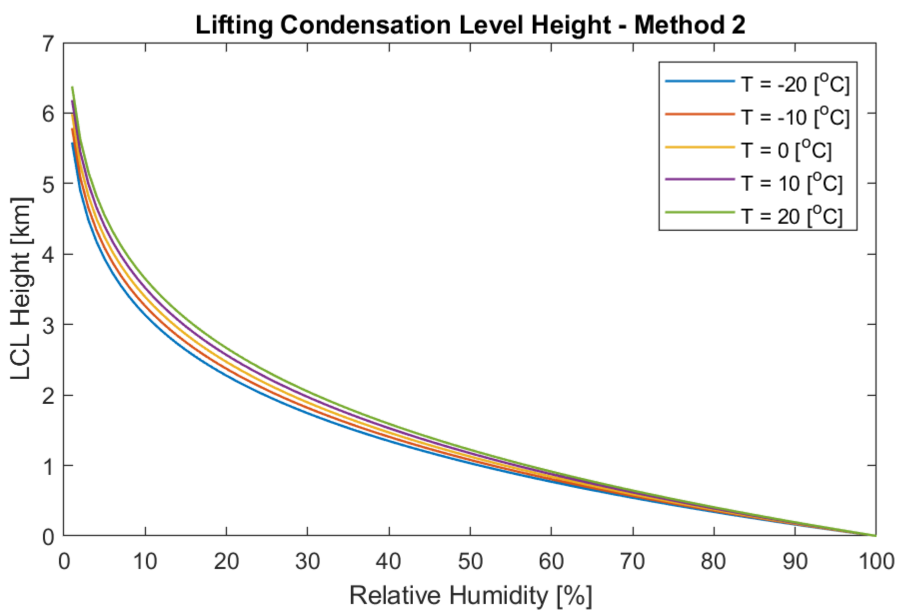

Figure 2. Dew-point deficit as a function of temperature for various values of relative humidity.

As can be seen, both models behave similarly and are convincing. The difference between each of these models and the model provided by Bolton(1980),

$$T_{LCL} = \frac{1}{\frac{1}{T-55} - \frac{\ln f}{2840}}, \tag{5}$$

is plotted in figures 3 and 4.

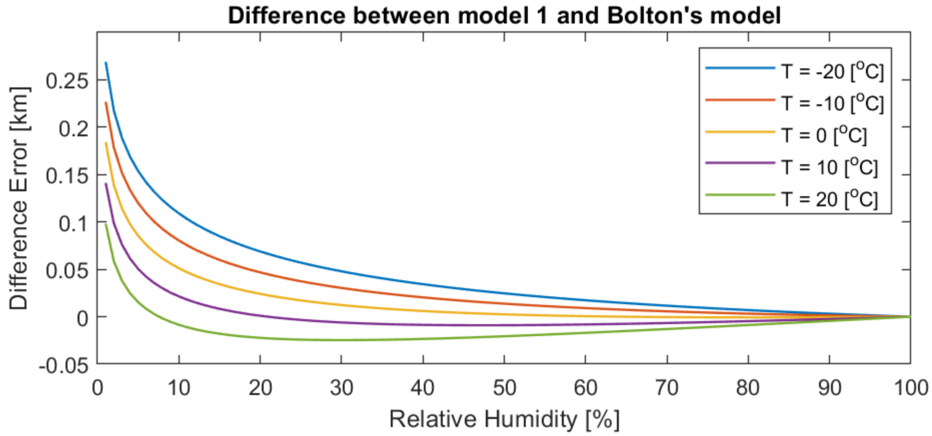

Figure 3. Difference between z_{LCL} results calculated from (2) and (5).

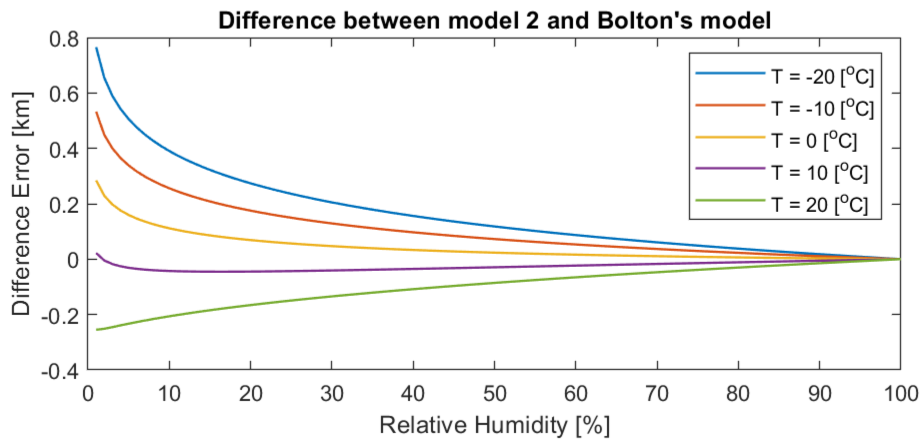

Figure 4. Difference between z_{LCL} results calculated from (4) and (5).