Atmospheric Thermodynamics - Tutorial 7

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1 Stability of the Atmosphere

Consider a parcel of air initially positioned at ground level $z = 0$ at a temperature of $T = 20\,^{\circ}\mathrm{C}$ and specific humidity $q'_v = 7.48\,\frac{\mathrm{g}}{\mathrm{kg}}$, that does not exchange matter with its environment, maintains a constant potential temperature θ, and its pressure $p'(z)$ is equivalent to the environmental pressure $p(z)$. Given such a parcel and the assumption that it rises vertically through the air, one can compute the key altitudes associated with cloud formation, namely, the Lifting Condensation (LCL) Level, Level of Free Convection (LFC), and Level of Neutral Buoyancy (LNB).

1.1 Lifting Condensation Level

The LCL is the altitude at which the parcel's specific humidity is equal to the saturation specific humidity of the environment. With the constraint of a constant specific humidity inside the air parcel, one only needs to look at the environmental saturation pressure curve, and see at which height it equals the parcel's $q'_v = 7.48\,\frac{\mathrm{g}}{\mathrm{kg}}$.

Given the Magnus-Tetens formula for saturation vapour pressure in terms of temperature in °C:

$$e_s(t) = e_{s0} \exp\left(\frac{17.27t}{t + 237.7}\right), \tag{1}$$

and using the relationship between e_s and q_s, one arrives at a result for the environmental temperature at the $T_{LCL} = 6.55\,^{\circ}\mathrm{C}$. Using this result and assuming a lapse rate $\Gamma = 8\,\frac{\mathrm{K}}{\mathrm{km}}$, one can easily find the height z_{LCL} and then use the barometric formula to compute the p_{LCL}, leading to the final result:

$$(T, p, z)_{LCL} = (6.55\,^{\circ}\mathrm{C}, 806\,\mathrm{hPa}, 1.68\,\mathrm{km}).$$

1.2 Level of Free Convection

The LFC is characterised as the altitude at which the parcel's temperature equates that of the environment. These two profiles cross because after crossing the LCL, the parcel no longer cools at the dry-adiabatic lapse rate Γ_d but rather at the moist-adiabatic lapse rate Γ_s. The moist-adiabatic lapse rate is given by:

$$\Gamma_s = \Gamma_d \gamma = \Gamma_d \frac{1 + \frac{q_s L_{lv}}{R_d T}}{1 + \frac{q_s L_{lv}^2 \epsilon}{c_{pd} R_d T^2}}. \tag{2}$$

Using this equation to iteratively compute the parcel's temperature from the initial conditions of $(T, p, z)_{LCL}$, graphically, the point of intersection between the parcel's temperature profile and the environmental profile occurs at the:

$$(T, p, z)_{LFC} = (0.70\,^{\circ}\mathrm{C}, 731.1\,\mathrm{hPa}, 2.41\,\mathrm{km}).$$

1.3 Level of Neutral Buoyancy

Since the the moist-adiabatic lapse rate given by (2) is not constant, the parcel's temperature crosses the environmental temperature profile once more. Computing the intersection point between the two temperature curves yields:

$$(T, p, z)_{LNB} = (-26.4\,^{\circ}\mathrm{C}, 451\,\mathrm{hPa}, 5.81\,\mathrm{km}).$$

1.4 CAPE

The convective available potential energy is computed via the equation:

$$CAPE = -R \int_{p_{LFC}}^{p_{LNB}} (T' - T) d(\ln p). \tag{3}$$

Plugging in the results from former sections yields the result:

$$CAPE = 0.5463\,\mathrm{kJ}.$$