Biologically Plausible Deep Learning: A Critical Review of Guerguiev *et al.* (2017) [1]

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February 7, 2019

[1] Guerguiev, J., Lillicrap, T. P., & Richards, B. A. (2017). Towards deep learning with segregated dendrites. ELife, 6, e22901.

[2] Code: github.com/RobertTLange/Bio-Plausible-DeepLearning

Biologically Implausible Backpropagation

$\rightarrow$ MLP: $h_l := f(h_{l-1}; \theta_l) = \sigma_l(W_l h_{l-1} + b_l), \;\; l = 1, \ldots, L$

Biologically Implausible Backpropagation

$\rightarrow$ MLP: $h_l := f(h_{l-1};\theta_l) = \sigma_l(W_l h_{l-1} + b_l),\ \ l = 1,\dots,L$

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial \theta_l} &= \left(\frac{dh_l}{d\theta_l}\right)^T \frac{\partial \mathcal{L}}{\partial h_l} = \left(\frac{dh_l}{d\theta_l}\right)^T \left(\frac{dh_{l+1}}{dh_l}\right)^T \frac{\partial \mathcal{L}}{\partial h_{l+1}} \\
&= \left(\frac{dh_l}{d\theta_l}\right)^T \underbrace{\left(W_{l+1} diag\left(\sigma'_{l+1}(W_{l+1}h_l + b_{l+1})\right)\right)^T}_{:=\delta_{l+1}} \frac{\partial \mathcal{L}}{\partial h_{l+1}} \\
&= \left(\frac{dh_l}{d\theta_l}\right)^T \left(\prod_{i=l+1}^{L} \delta_i\right) \frac{\partial \mathcal{L}}{\partial h_L}
\end{aligned}$$

Biologically Implausible Backpropagation

→ MLP: $h_l := f(h_{l-1}; \theta_l) = \sigma_l(W_l h_{l-1} + b_l), \ l = 1, \dots, L$

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&= \left(\frac{dh_l}{d\theta_l}\right)^T \underbrace{\left(\boxed{W_{l+1}} diag\left(\sigma'_{l+1}(\boxed{W_{l+1}} h_l + b_{l+1})\right)\right)^T}_{:=\delta_{l+1}} \frac{\partial \mathcal{L}}{\partial h_{l+1}} \\
&= \left(\frac{dh_l}{d\theta_l}\right)^T \left(\prod_{i=l+1}^{L} \delta_i\right) \frac{\partial \mathcal{L}}{\partial h_L}
\end{aligned}$$

Weight Transport Problem

Biologically Implausible Backpropagation

→ MLP: $h_l := f(h_{l-1}; \theta_l) = \sigma_l(W_l h_{l-1} + b_l), \; l = 1, \ldots, L$

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial \theta_l} &= \left(\frac{dh_l}{d\theta_l}\right)^T \frac{\partial \mathcal{L}}{\partial h_l} = \left(\frac{dh_l}{d\theta_l}\right)^T \left(\frac{dh_{l+1}}{dh_l}\right)^T \frac{\partial \mathcal{L}}{\partial h_{l+1}} \\
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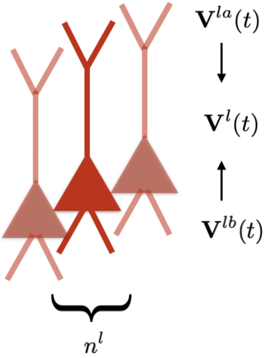
&= \left(\frac{dh_l}{d\theta_l}\right)^T \left(\prod_{i=l+1}^{L} \delta_i\right) \frac{\partial \mathcal{L}}{\partial h_L}
\end{aligned}
$$

Global Signed Error

A Solution - Electrical Segregation of ↓ and ↑ Info

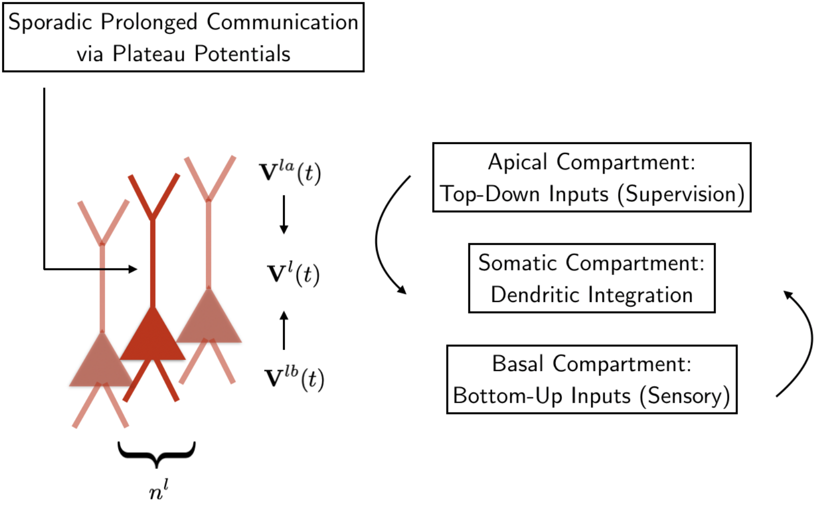


Apical Compartment:
Top-Down Inputs (Supervision)

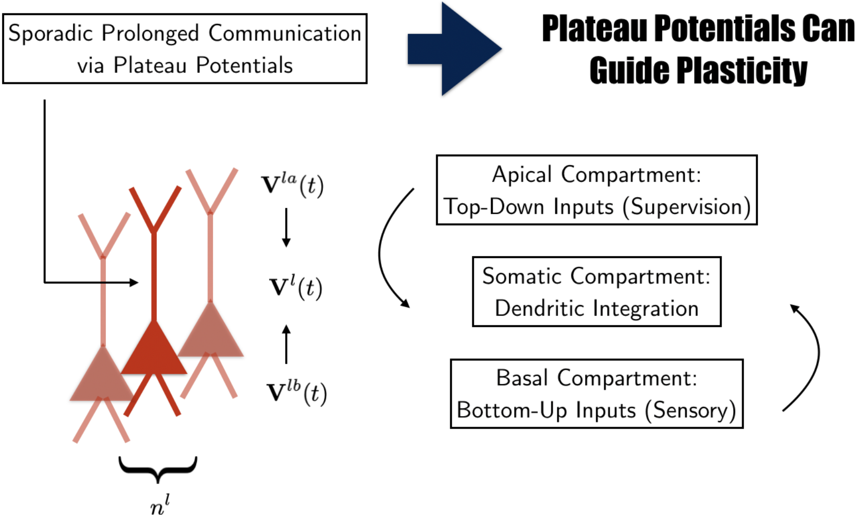
Somatic Compartment:
Dendritic Integration

Basal Compartment:
Bottom-Up Inputs (Sensory)

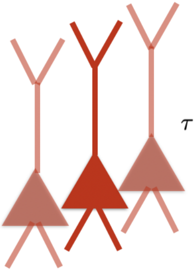
A Solution - Electrical Segregation of ↓ and ↑ Info



A Solution - Electrical Segregation of ↓ and ↑ Info



Guerguiev *et al.* (2017) - Hidden Layer Structure



n^l

$$\mathbf{V}^{la}(t) \longleftarrow \mathbf{Y}^l \mathbf{s}^{l+1}$$

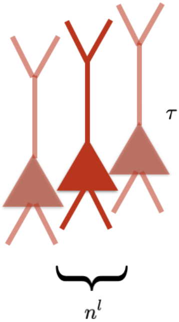
$$\downarrow$$

$$\tau \frac{dV_i^l(t)}{dt} = -V_i^l(t) + \frac{g_b}{g_{leak}}\left(V_i^{lb}(t) - V_i^l(t)\right) + \frac{g_a}{g_{leak}}\left(V_i^{la}(t) - V_i^l(t)\right)$$

$$\uparrow$$

$$\mathbf{V}^{lb}(t) \longleftarrow \mathbf{W}^l \mathbf{s}^{l-1} + \mathbf{b}^l$$

Guerguiev *et al.* (2017) - Hidden Layer Structure

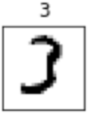


$$\mathbf{V}^{la}(t) \leftarrow \mathbf{Y}^l \mathbf{s}^{l+1} \longleftrightarrow s^{l+1} \sim Poisson\left(\phi_{max}\sigma(V^{l+1}(t))\right)$$

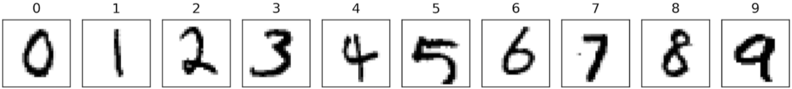
$$\tau \frac{dV_i^l(t)}{dt} = -V_i^l(t) + \frac{g_b}{g_{leak}}\left(V_i^{lb}(t) - V_i^l(t)\right) + \frac{g_a}{g_{leak}}\left(V_i^{la}(t) - V_i^l(t)\right)$$

$$\mathbf{V}^{lb}(t) \leftarrow \mathbf{W}^l \mathbf{s}^{l-1} + \mathbf{b}^l$$

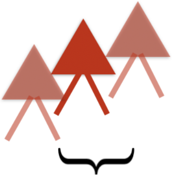
$$s_0 :\sim Poisson\left(\mathbf{x}\right)$$



Guerguiev *et al.* (2017) - Output Layer Structure



$$\phi_i^L(t) = \phi_{max}\sigma(V_i^L(t)) \text{ with } I_i(t) = 0,\ \forall\ i = 1, \dots, n_L$$

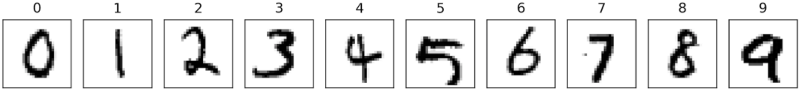


n^L

$$\tau \frac{dV_i^L(t)}{dt} = -V_i^L(t) + \frac{g_d}{g_{leak}}\left(V_i^{Lb}(t) - V_i^L(t)\right) + I_i(t)$$

$$\mathbf{V}^{Lb}(t) \leftarrow W^L s^{L-1} + b^L$$

Guerguiev *et al.* (2017) - Output Layer Structure



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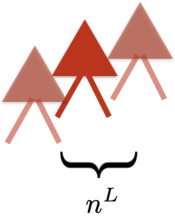


$$\mathbf{V}^{Lb}(t) \leftarrow W^L s^{L-1} + b^L$$

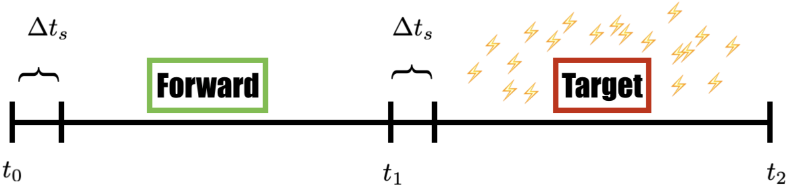
Guerguiev *et al.* (2017) - Credit Assignment Signals

No Supervision

$$I_i(t) = 0,\ \forall i = 1, \ldots, n^L$$

Supervision

$$I_i(t) = \begin{cases} \phi_{max}, & \text{for } i = y^{label} \\ 0, & \text{else} \end{cases}$$



Guerguiev *et al.* (2017) - Credit Assignment Signals

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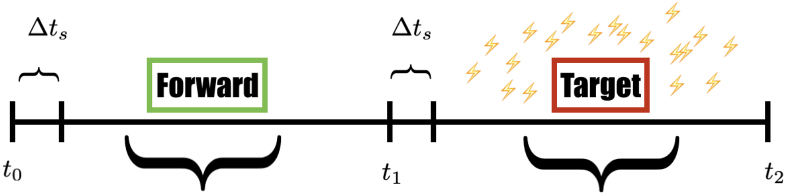

$$\sigma\left(\frac{1}{\Delta t_1}\int_{t_1-\Delta t_1}^{t_1} V_i^{la}(t)dt\right) := \alpha^{lf}(t)$$

$$\sigma\left(\frac{1}{\Delta t_2}\int_{t_2-\Delta t_2}^{t_2} V_i^{la}(t)dt\right) := \alpha^{lt}(t)$$

Forward Plateau Potential

Target Plateau Potential

Guerguiev *et al.* (2017) - Learning

Forward phase activity $\Leftrightarrow$ Target phase activity

Guerguiev *et al.* (2017) - Learning

Forward phase activity $\Leftrightarrow$ Target phase activity

$\Rightarrow$ Output Layer:

- Target firing rates: $\phi_i^{L\star} = \frac{1}{\Delta t_2} \int_{t_1+\Delta t_s}^{t_2} \phi_i^L(t) dt$
- Loss function:

$$L^L = ||\phi^{L\star} - \bar{\phi}^{Lf}||_2^2 = ||\frac{1}{\Delta t_2} \int_{t_1+\Delta t_s}^{t_2} \phi^L(t) dt - \frac{1}{\Delta t_1} \int_{t_0+\Delta t_s}^{t_1} \phi^L(t) dt||_2^2$$

Guerguiev *et al.* (2017) - Learning

Forward phase activity $\Leftrightarrow$ Target phase activity

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$\Rightarrow$ Hidden Layer:
- Target firing rates: $\phi_i^{l\star} = \bar{\phi}_i^{lf} + \alpha_i^{lt} - \alpha_i^{lf}$
- Loss function:

$$L^l = ||\phi^{l\star} - \bar{\phi}^{lf}||_2^2 = ||\alpha^{lt} - \alpha^{lf}||_2^2$$

Guerguiev *et al.* (2017) - Learning

Forward phase activity $\Leftrightarrow$ Target phase activity

$\Rightarrow$ Output Layer:

- Target firing rates: $\phi_i^{L\star} = \frac{1}{\Delta t_2} \int_{t_1+\Delta t_s}^{t_2} \phi_i^L(t)dt$
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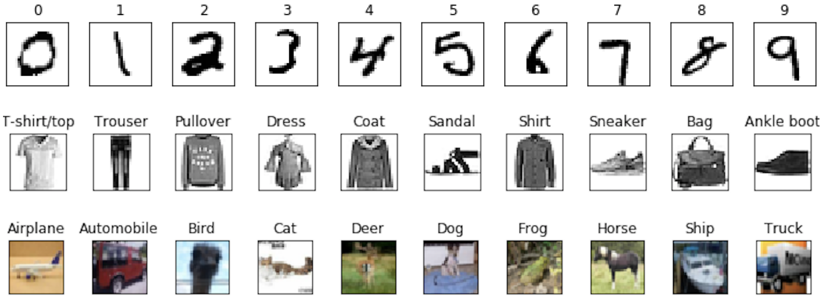
$\Rightarrow$ Hidden Layer:

- Target firing rates: $\phi_i^{l\star} = \bar{\phi}_i^{lf} + \alpha_i^{lt} - \alpha_i^{lf}$
- Loss function:

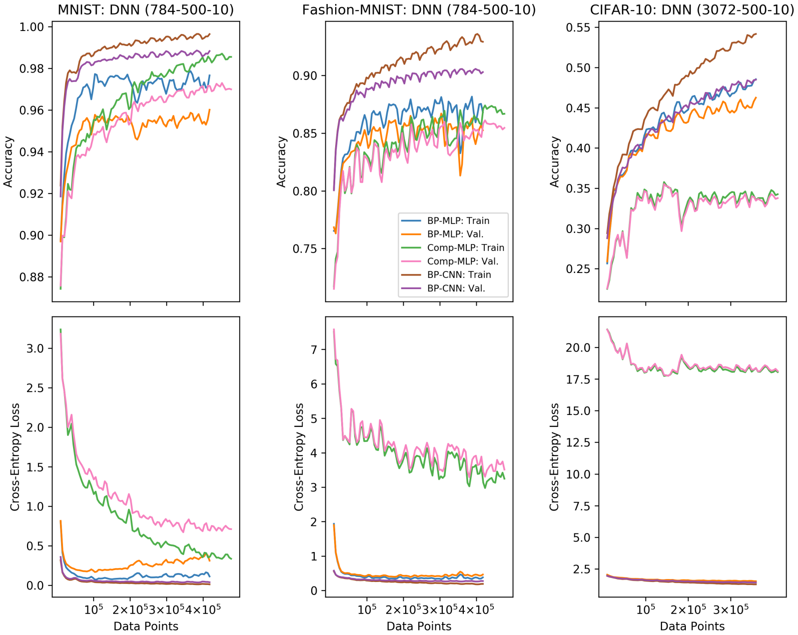
$$L^l = ||\phi^{l\star} - \bar{\phi}^{lf}||_2^2 = ||\alpha^{lt} - \alpha^{lf}||_2^2$$

$\Rightarrow$ Local error minimization via gradient descent

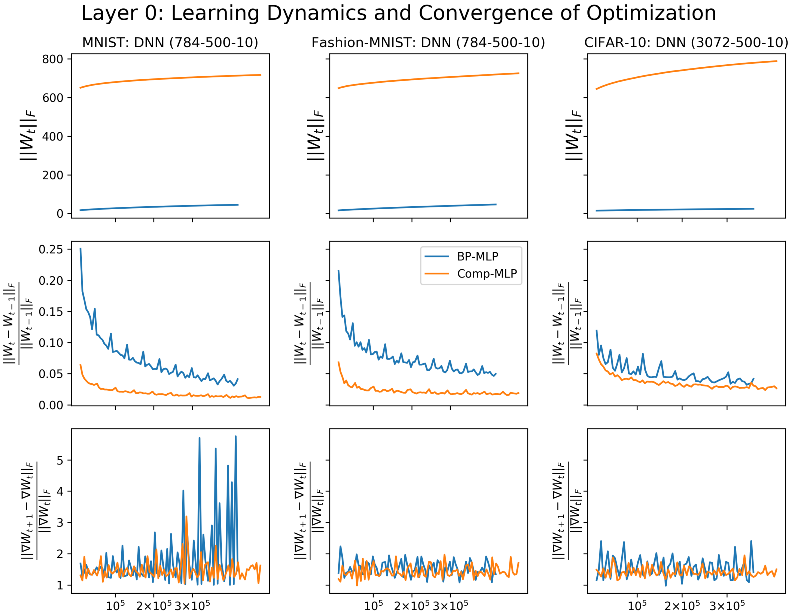
Empirical Investigations



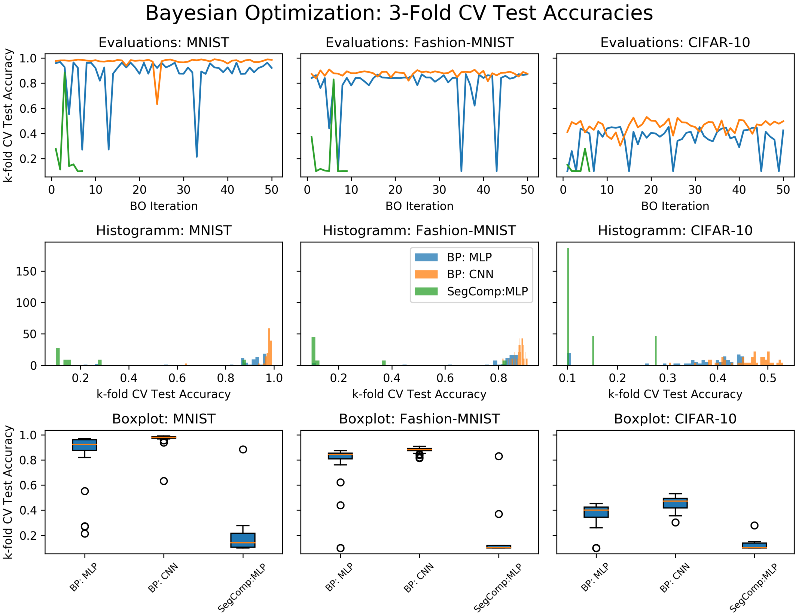
State-of-the-Art Performance



Fast convergence or Overfitting?



Well... Robustness?



Guerguiev *et al.* (2017) - Accomplishments/Problems

- ☑ Physiological plausible electrical segregation
- ☑ Signal can be used to exploit depth in near-continuous time
- ☒ **Computational** Problems
  - → Expensive/slow training
  - → Non-robust!
- ☒ **Physiological** Problems
  - → How is the teaching signal internally generated - Mismatch neurons?
  - → 2 global phases? - Length sampled from inverse Gaussian
  - → Stoch. gen. of plateau potentials - apical calcium spikes

Where to go from here?

→ Sacramento *et al.* (2018): Local error from mismatch with local **interneurons**

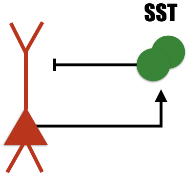
⇒ Lateral ⇔ Apical

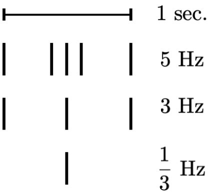
⇒ No separate phases

→ Naud & Sprekeler (2018): Â **Multiplexing**

⇒ Burst Fraction ⇔ Apical ⇓

⇒ Event Rate ⇔ Somatic ⇑





References I

Guerguiev, Jordan, Lillicrap, Timothy P, & Richards, Blake A. 2017. Towards deep learning with segregated dendrites. *ELife*, **6**, e22901.

Naud, Richard, & Sprekeler, Henning. 2018. Sparse bursts optimize information transmission in a multiplexed neural code. *Proceedings of the National Academy of Sciences*, 201720995.

Sacramento, João, Costa, Rui Ponte, Bengio, Yoshua, & Senn, Walter. 2018. Dendritic cortical microcircuits approximate the backpropagation algorithm. *Pages 8735–8746 of: Advances in Neural Information Processing Systems*.