Biologically Plausible Deep Learning: A Critical Review of Guerguiev *et al.* (2017) [1]

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[1] Guerguiev, J., Lillicrap, T. P., & Richards, B. A. (2017). Towards deep learning with segregated dendrites. ELife, 6, e22901.
[2] Code: github.com/RobertTLange/Bio-Plausible-DeepLearning

Motivation - Backpropagation

$\rightarrow$ MLP: Composition of layers $\{h_l\}_{l=1}^{L}$, $h_0 = x$, $\theta_l = \{W_l, b_l\}$ and "Learn" synaptic weights, $\Theta = \{\theta_l\}_{l=1}^{L}$ iteratively.

$$h_l := f(h_{l-1}; \theta_l) = \sigma_l(W_l h_{l-1} + b_l)$$

$$\min_{\theta} \mathcal{L}(h_L|\Theta) = -\sum_{y} q(y|x) \log p(y|h_L; \Theta)$$

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→ Backpropagation:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial \theta_l} &= \left(\frac{dh_l}{d\theta_l}\right)^T \frac{\partial \mathcal{L}}{\partial h_l} = \left(\frac{dh_l}{d\theta_l}\right)^T \left(\frac{dh_{l+1}}{dh_l}\right)^T \frac{\partial \mathcal{L}}{\partial h_{l+1}} \\
&= \left(\frac{dh_l}{d\theta_l}\right)^T \underbrace{\left(W_{l+1} diag\left(\sigma'_{l+1}(W_{l+1} h_l + b_{l+1})\right)\right)^T}_{:=\delta_{l+1}} \frac{\partial \mathcal{L}}{\partial h_{l+1}} \\
&= \left(\frac{dh_l}{d\theta_l}\right)^T \delta_{l+1} \delta_{l+2} \frac{\partial \mathcal{L}}{\partial h_{l+2}} = \dots
\end{aligned}$$

Motivation - Problems with Backpropagation

$$\frac{\partial \mathcal{L}}{\partial \theta_l} = \left(\frac{dh_l}{d\theta_l}\right)^T \left(\prod_{i=l+1}^{L} \delta_i\right) \frac{\partial \mathcal{L}}{\partial h_L}$$

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- ☒ **Global signed error signal**

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- ☐ Alternatives:
  - Feedback Alignment (Lillicrap *et al.* , 2016)
  - Target Propagation (Lee *et al.* , 2014)

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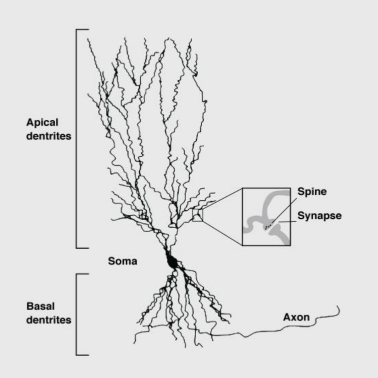
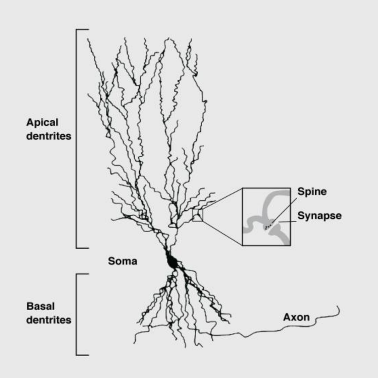
- ☐ Problems with Alternatives:
  - Need form of info transmission to determine local errors
  - Not possible in single compartment neurons without feedback pathway

Motivation - Electrical Segregation of $\downarrow$ and $\uparrow$ Info



- $\rightarrow$ Körding & König (2001): Local error computation via electrical segregation
- $\rightarrow$ Multi-compartmental segregation avoids need for feedback pathway
- $\rightarrow$ Apical dendrites ($\Downarrow$)
- $\rightarrow$ Basal dendrites ($\Uparrow$)

Motivation - Electrical Segregation of ↓ and ↑ Info



→ Körding & König (2001): Local error computation via electrical segregation

→ Multi-compartmental segregation avoids need for feedback pathway

→ Apical dendrites (⇓)

→ Basal dendrites (⇑)

→ **Plateau Potentials**:

- Apical ⇒ soma via voltage-gate Ca^{2+} channels
- Prolonged upswing in MP due to events in apical shaft

⇒ Can guide plasticity in pyramidal neurons

Guerguiev *et al.* (2017) - Neuron and Network Model

$\rightarrow$ 3 Compartment Hidden Layer: $\mathbf{V}^{0a}(t), \mathbf{V}^{0b}(t), \mathbf{V}^{0}(t) \in \mathbb{R}^m$

$$\tau \frac{dV_i^0(t)}{dt} = -V_i^0(t) + \frac{g_b}{g_l}\left(V_i^{0b}(t) - V_i^0(t)\right) + \frac{g_a}{g_l}\left(V_i^{0a}(t) - V_i^0(t)\right)$$

$$V_i^{0b} = \sum_{j=1}^{l} W_{ij}^0 s_j^{input}(t) + b_i^0 \quad \text{and} \quad V_i^{0a} = \sum_{j=1}^{n} Y_{ij} s_j^1(t)$$

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$\rightarrow$ 2 Compartment Output Layer: $\mathbf{V}^{1b}(t), \mathbf{V}^{1}(t) \in \mathbb{R}^n$

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$$V_i^{1b} = \sum_{j=1}^{l} W_{ij}^1 s_j^0(t) + b_i^1$$

$\rightarrow$ $s_j^{input}(t) = \sum_k \kappa(t - t_{jk}^{input})$ with κ response kernel

Guerguiev *et al.* (2017) - Credit Assignment Signals

- → **Forward** $(t_0 + \Delta t_s \rightarrow t_1)$: $I_i(t) = 0, \ \forall i = 1, \ldots, n$
  - At t_1: $\alpha_i^f = \sigma\left(\frac{1}{\Delta t_1} \int_{t_1 - \Delta t_1}^{t_1} V_i^{0a}(t) dt\right)$
- → **Target** $(t_1 + \Delta t_s \rightarrow t_2)$: $I_k(t) = \phi_{max}$ for $y_{sample} = k$
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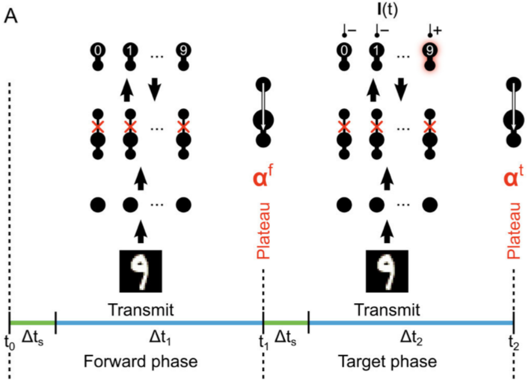

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Forward phase dynamics $\Leftrightarrow$ Target phase dynamics

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$\Rightarrow$ Somatic compartments generate Poisson process spikes:

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- Loss function: $L^1 = ||\phi_i^{1\star} - \bar{\phi}_i^{1f}||_2^2 =$ $||\frac{1}{\Delta t_2}\int_{t_1+\Delta t_s}^{t_2} \phi_i^1(t)dt - \frac{1}{\Delta t_1}\int_{t_0+\Delta t_s}^{t_1} \phi_i^1(t)dt||_2^2$

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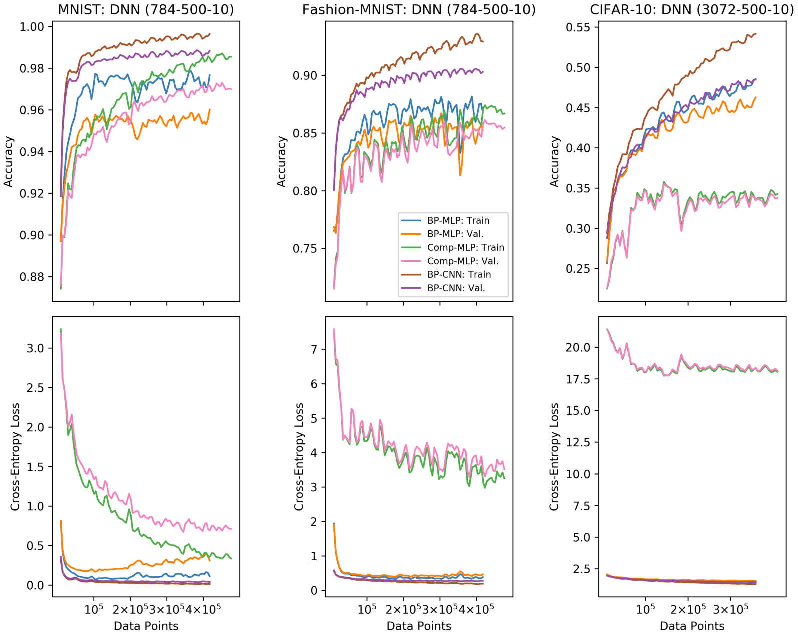
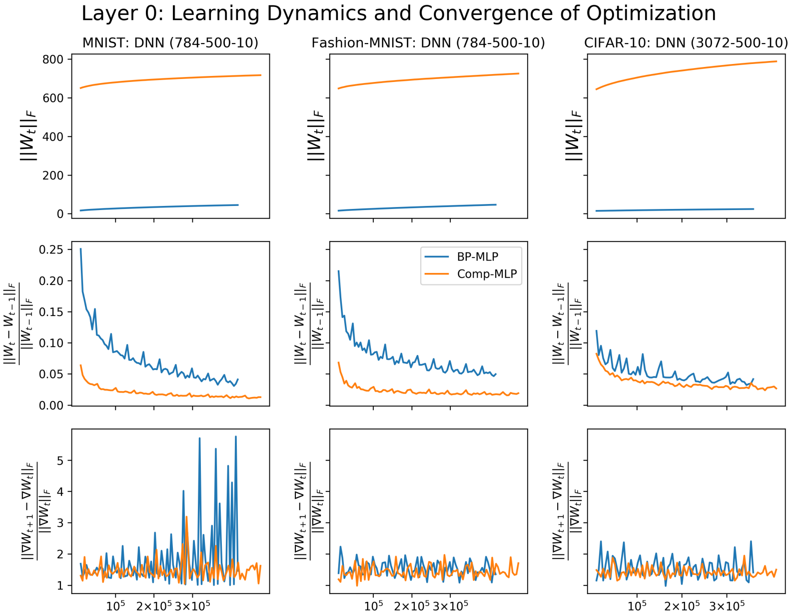
⇒ Local error minimization via SGD

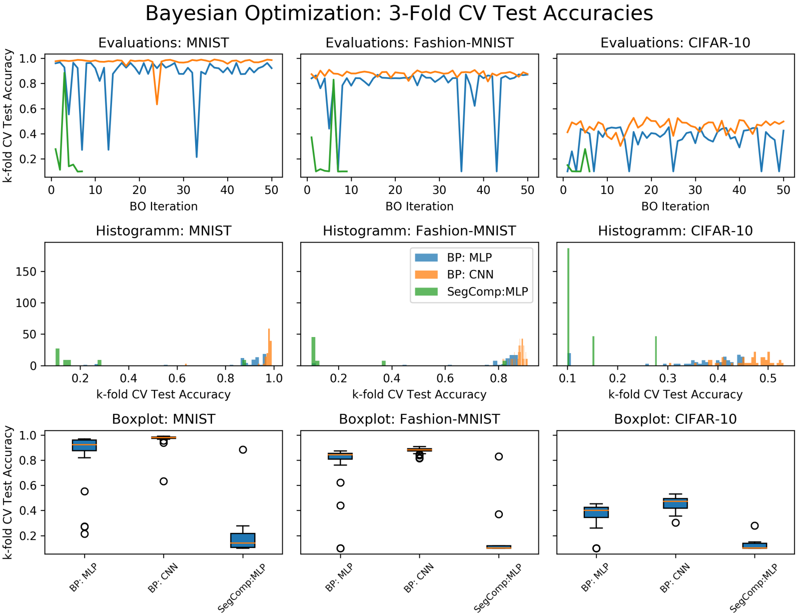
Experiments - Learning Dynamics: Performance



Experiments - Learning Dynamics: Dynamics



Experiments - Learning Dynamics: Robustness



Guerguiev *et al.* (2017) - Accomplishments/Problems

- ☑ Segregated compartments generate local targets that act as credit assignment signals in a physiologically plausible manner
- ☑ Signal can be used to exploit depth in near-continuous time
- ☒ **Computational** Problems
  - $\rightarrow$ Huge hyperparameter space $\rightarrow$ most likely not robust!
- ☒ **Physiological** Problems
  - $\rightarrow$ How is the teaching signal internally generated?
  - $\rightarrow$ 2 global phases? - Length sampled from inverse Gaussian
  - $\rightarrow$ Stoch. gen. of plateau potentials - apical calcium spikes

$\Rightarrow$ Sacramento *et al.* (2018): Neocortical micro-circuits and inhibitory interneurons might act synchronizing.

References I

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