Action Grammars: Grammar Induction-Based Learning of Temporal Abstractions

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Abstract

Hierarchical Reinforcement Learning algorithms have been successfully applied to large-scale problems with sparse reward signals. By operating at multiple time scales, the Reinforcement Learning agent is able to overcome difficulties in exploration and value information propagation. However, all of these algorithms face one of three unsatisfying properties: They either require manual specification of hierarchical structures, lack clear interpretability or can hardly be justified in a comparative fashion. This work combats all of these shortcomings in a fully automated and end-to-end fashion. By treating an on-policy trajectory as a sentence sampled from the policy-conditioned language of the environment, we are able to apply powerful ideas from computational linguistics to the sub-structure discovery problem. We identify hierarchical constituents with the help of unsupervised grammatical inference.

Rob: Only make repo public once analysis is done.

1 Introduction

Learning a policy over temporally-extended actions allows the Hierarchical Reinforcement Learning (HRL) agent to combat the uncertainty induced by single time-step decision making. The agent overcomes exploration problems, by restricting her decision process in a syntactically meaningful way. The biggest challenge of HRL is the actual identification of a meaningful substructure specification. As of yet, this challenge has not been successfully addressed. Current approaches require the optimization of many hyperparameters (e.g. the number of desired options, network architecture) and make strong assumptions regarding the initiation set of the subtask (e.g. the complete state space). This can easily lead to misspecification and results in a significant slow-down of learning and exploration. While providing a fully end-to-end approach, "deep" attempts lack interpretability and often times require severe amounts of pre-training.

Human infants, on the other hand, learn seemingly unstructured patterns in nature and from observing role models. They are incredibly well equipped to infer hierarchical rule-based structures from language, visual input as as well as auditory stimuli [19, 13, 18]. By observing an expert, they get a head-start in their learning process and are able to learn over higher level sequences of low level control elements. Furthermore, there is convincing evidence from several MEG and fMRI studies that indicates a form of hierarchical language comprehension in the brain [10, 14, 4, 23] and a parallelism to motor control [27, 33]. Inspired by such observations this work overcomes the identified weaknesses by merging HRL with the field of computational linguistics. More specifically, we propose the usage of grammatical inference algorithms to extract hierarchical structures from trajectory sentences with the ultimate aim to deploy them in the HRL process. Thereby, the original RL problem is split into two alternating stages:

[*]This work was done while R.T.L. was a Master's student in the FaisalLab & Imperial College London.
[†]GitHub repository: `https://github.com/RobertTLange/action-grammars-hrl`.

1. **Grammar Learning**: Given episodic trajectories we treat the time-series as a sentence sampled from the language of the policy-conditioned environment. The language in turn was generated by the grammar induced by the current policy. Using grammar induction the agent extracts hierarchical constituents of the current policy. Based on this estimate she constructs temporally-extended actions which convey hierarchical syntactic meaning. Afterwards, we augment the agent's action space with such actions.
2. **Action Learning**: Using the grammar-augmented action space, the agent acquires new value information from interacting with the environment and refines his action-value estimates using Semi-Markov-Decision-Process-Q-Learning [3]. Afterwards, we sample simulated "sentences" from the improved policy by rolling out observations in the environment.

The overall learning procedure consists of alternating updates of the grammar estimate and a refinement of the corresponding "action grammar"-value estimates (see right part of figure 1).

Rob: Add viz of 3 contributions - Expert, Transfer, Online

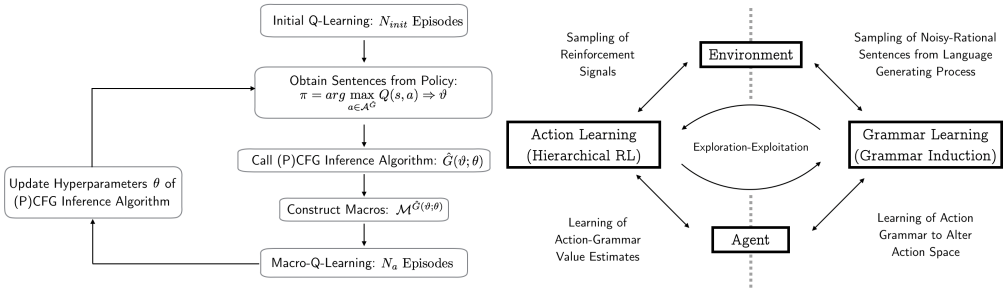


Figure 1: Action Grammars for HRL. **Left.** Applicability of expert action grammars for imitation and transfer learning. **Right.** Online-inferred action grammars alternation loop.

We proceed in the following manner: First, we summarize the current state of research on substructure discovery for HRL and review the required technical background. Afterwards, we introduce our proposed framework and outline how grammar-based temporal abstractions are able to provide efficient, effective and interpretable sub-structures. Our experiments highlight the usability of the action grammars framework for imitation learning and transfer learning given an expert grammar. Furthermore, we display strong results of an online version which iteratively refines grammar and value estimates.

2 Related Work

We are not the first to infer hierarchical structure in subgoal achievement problems. More specifically, the option discovery problem deals with the question of how to construct an option set that captures the hierarchical structure between sub-regions of the core MDP. Often times this task is limited to defining the initiation set and the termination condition. The intra-option policy can afterwards be easily learned (e.g. by introducing a pseudo-reward for reaching the termination state).

Roughly speaking the current state-of-the-art approaches can be categorized in three main pillars: First, graph theoretic [15, 22, 17, 28] and visitation-based [20, 32, 28] approaches aim to identify bottlenecks. Bottlenecks are regions in the state space which characterize successful trajectories. Gradient-based approaches, on the other hand, discover parametrized options by iteratively optimizing an objective function such as the estimated expected value of the log likelihood with respect to the latent variables in a probabilistic setting [8] or simply the expected cumulative reward in a policy gradient context [1, 31]. Finally, multi-layer [2, 35, 12] approaches attempt to split the goal discovery and goal achievement across different stages and layers of the learning architecture. Usually, the top level of the hierarchy specifies goals in the environment while the lower levels have to achieve such.

3 Technical Background

Temporally-Extended Actions. Semi-Markov Decision Processes (SMDP) extend the classical Markov Decision Process setting to incorporate not only environmental uncertainty but also time uncertainty. Instead of dealing with a Dirac waiting distribution, the time between individual decisions is modeled as a random variable, $\tau \in \mathbb{Z}_{++}$. It is described by the probability distribution $P(s', \tau | s, m)$ which characterizes the the joint likelihood of transitioning from state $s \in \mathcal{S}$ into state s' in τ time steps given action m was pursued. Thereby, SMDPs allow one to elegantly model the execution of actions which extend over multiple time-steps (e.g. sequences of primitive actions or sub-policy execution). Multiple different hierarchical action structures have been proposed [21, 34, 25, 9]. In this work we focus on the most simplest, namely macro-actions. Simply put, a macro-action, $m \in \mathcal{M}$ specifies the sequential and deterministic execution of multiple (τ_m) primitive actions. Value estimates can then be updated using SMDP-Q-Learning [26] in a model-free manner:

$$Q(s,m)_{k+1} = (1-\alpha)Q(s,m)_k + \alpha \left(\sum_{i=1}^{\tau_m} \gamma^{i-1} r_{t+i} + \gamma^{\tau_m} \max_{m' \in \mathcal{A} \cup \mathcal{M}} Q(s', m')_k \right)$$

Furthermore, the DQN objective can easily be refined to the case of macro-actions:

$$L(\theta) := \mathbb{E}_{s,m,r^{\tau_m},s',\tau \sim D_{\tau_m}} [(r^{\tau_m} + \gamma^{\tau_m} \max_{m'} Q(s', m'; \theta^-) - Q(s, m; \theta))^2]$$

Context-Free Grammars. Formal grammars and the theory of computational linguistics study both generating and accepting systems that underlie a language. Given a start symbol S, a formal grammar $(\Sigma, \mathbb{N}, S, \mathcal{P})$ produces an output which is a string of words. The terminal vocabulary Σ is a set of terminal elements used to construct the sentences of a language. $\mathbb{N}$ denotes the non-terminal vocabulary which is a set of elements only used in the process of deriving a sentence. The production rules $\mathcal{P}$ are ordered pairs of strings such that $\alpha \rightarrow \beta, \alpha \in V^+, \beta \in V^\star$. A type-2 grammar [6, 7], also known as context-free grammar (CFG) is such that the production rules have the following form:

$$A \rightarrow \beta \text{ , where } \beta \neq \lambda \text{ or } |\beta| \neq 0$$

Since production rules either map from one-to-one, one-to-none or one-to-many they are called context-free. The context of a non-terminal symbol does not influence the production rule [27]. A context-free grammar that is non-branching and loop-free is called a straight-line grammar [30]. Such grammars are restrictive since they are only capable of generating a single sentence.

The process of inferring a grammar for a language that is consistent with a given sample of sentences is called grammatical inference or grammar induction [16]. The smallest grammar problem [5, 30] formalizes the problem of finding the smallest CFG which compresses a string generated by a straight-line grammar. This problem turns out to be NP-hard [5]. Two greedy approximations to the smallest CFG are provided by Sequitur [24] and G-Lexis [29]. Given a single sentence of the language, Sequitur sequentially reads in all symbols and collects repeating subsequences of symbols into a production rule. Therewhile, the final encoded string is only allowed to have unique bigrams (*Digram Uniqueness*, [24]) and production rules must be used more than once in the derivation of the string (*Rule Uniqueness*, [24]). In order to overcome Sequitur's problem of noise overfitting, k-Sequitur [33] has been proposed. Instead of replacing a bigram with a rule if the bigram occurs twice, it has to occur at least k times. As k increases the discovered CFG grammar becomes less and less sensitive to overfitting noise and the resulting grammar is more parsimonious in terms of productions. Lexis [29] provides an optimization-based alternative which iteratively constructs a directed acyclic graph (DAG), the so-called Lexis-DAG. Starting from a trivial graph which connects a set of target sentences with the set of elements in the terminal vocabulary, the Lexis-DAG is constructed by adding intermediate nodes. The indirect objective is to minimize a cost function (e.g. number of concatenations or DAG edges) while imposing that the constructed graph satisfies a set of Lexis-DAG properties. Again, this problem by itself is NP-hard. G-Lexis, the greedy algorithmic implementation, searches for substrings that will lead to a maximal reduction in the cost, when added as new intermediate node.

4 Context-Free Action Grammars

Action sequences as well as communication by the means of words both convey meaning and are goal-directed. Both consist of hierarchical structures and are conditioned by the environment in which they are uttered in. The crucial assumption that connects linguistics with eager behavior is as follows:

Assumption 1. *Observed episodic behavior (with trajectory $\vartheta = \{\vartheta_1, \ldots, \vartheta_T\}$ where $\vartheta_t = \{s_t, a_t\}$) can be equivalently viewed as sentences sampled from the language, $L(G)$ with $G \sim \pi|E$.*

By treating sequential behavior as sentences, we are able to extract temporally-extended actions by applying grammatical inference techniques. In general, the language of actions depends on two factors. First, trajectories of a RL agent are generated by a policy π resulting from the current value estimates. Second, the environment E (i.e. the start and goal location as well as the transition success rate) dictates the length of the sentences as well as the allowed sub-sequences. Hence, the sampled sentences or trajectories encode valuable information. Not only do they convey self-information but they also encode structures within the environment.

Let us assume that the optimal policy of a Reinforcement Learning agent is hierarchically structured for a specific environment E. The optimal policy $\pi^\star$ then consists of a hierarchy of subgoal achievements which increase in sequential difficulty when moving up the hierarchy. We define the terminal vocabulary Σ to consist of the primitive action space $\mathcal{A}$, hence $\Sigma = \mathcal{A}$. A trajectory obtained from traversing the current policy π is viewed as a sample from the language generated by the grammar $L(\pi|E)$. We write $\vartheta^i \sim L(\pi|E)$ for $i = 1, \ldots N_g$ trajectories. Since we assume an episodic reinforcement learning task in which an episode ends with the achievement of the goal, each trajectory has an individual length denoted by T_i.

Given a set of trajectories, $\vartheta^1, \ldots \vartheta^{N_g}$, we construct a grammar training set from which we infer a (probabilisitic) context-free grammar. Thereby we obtain a grammar estimate $\hat{G}$. Afterwards, we transform this grammar into temporally-extended actions such as macros ($\mathcal{M}^{\hat{G}}$) or options ($\mathcal{O}^{\hat{G}}$). We augment the action space of the HRL agent, e.g. $\mathcal{A}^{\hat{G}} = \mathcal{A} \cup \mathcal{M}^{\hat{G}}$. The HRL agent can then use this new action space in his further learning.

Rob: Formalize grammar buffer and value transfer

Grammar learning thereby introduces a reflective period in which the agent takes a bird's eye-view on the observed behavior. The grammar inference process identifies repeating patterns that led to successful goal achieving experiences. By extracting these patterns and redefining them as temporally-extended actions, we additionally *save* the progress made not only in the value estimate but also in the augmented action space.

Rob: Add one figure which summarizes the network architecture, grammar buffer and learning loop.

5 Experiments

The following experiments intend to answer this set of questions:

- Does a grammar learned on optimal policy traces allow for rapid imitation learning?
- Can CFG grammars be used in order to enhance curriculum as well as transfer learning?
- Is online grammar inference and action space adaptation able to structure the exploration process of the HRL agent?

In order to illustrate our results we chose the Towers of Hanoi (ToH) environment.

Rob: Add Finn-style paragraph on which questions we are trying to answer with our experiments.

Rob: Introduce RL formulation of Towers of Hanoi

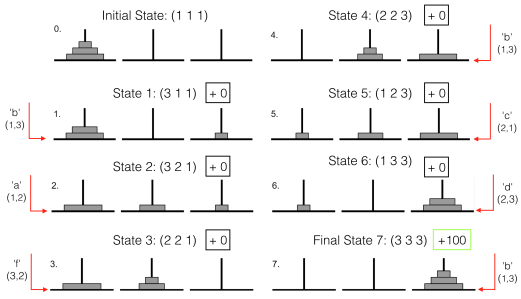
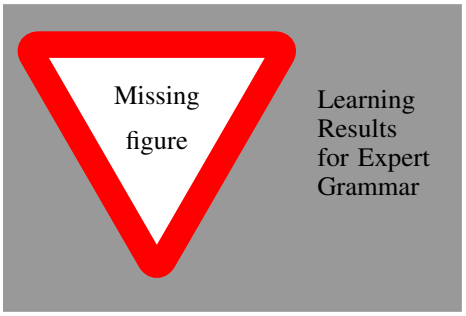


Figure 2: RL Formulation of the ToH Problem

Policy (ϑ): bafbcdbafecfbafbcdbcfecdbafbcdb				
Algorithm	2-Sequitur		G-Lexis	
ϑ^{enc}	BCDfBEfDdBb		BbafecfBbcfecdBb	
$\frac{\lvert\vartheta^{enc}\rvert}{\lvert\vartheta\rvert}$	0.355		0.516	
$\mathbb{N}$	PR	$\mathcal{M}^{2-Seq}$	PR	$\mathcal{M}^{G-Lex}$
B	CEd	bafbcd	-	bafbcd
C	-	baf		
D	-	ec		
E	-	bc		

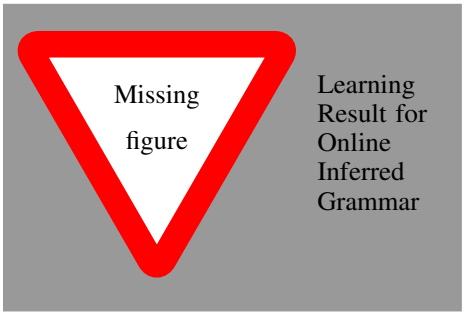
Table 1: ToH (5 disks) Grammar-Macro Construction

5.1 Learning with Expert Grammar

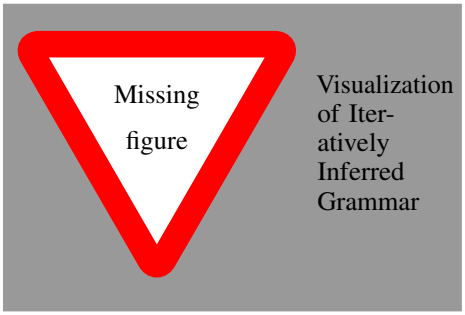


5.2 Learning with Transfer Grammar

5.3 Learning with Online Inferred Grammar



Learning Result for Online Inferred Grammar



Visualization of Iteratively Inferred Grammar

6 Discussion & Outlook

Motivated by a parallelism between the hierarchical generating processes of language and motion, we have derived multiple algorithmic approaches which exploit powerful grammatical inference frameworks to identify temporally-extended actions. At the center of this analysis was the formal notion of Semi-Markov Decision Processes and their capability to model stochastic waiting times between decisions. By sensibly defining temporally-extended actions and abstracting away unnecessary decision points, one is able to overcome the curse of dimensionality. Learning volatility is decreased by efficient exploration that incorporates domain knowledge. In order to overcome the necessity to manually articulate this knowledge, we proposed to turn to grammatical inference. By inferring the hierarchical structure of agent-environment interactions, we were able to fully automate the Hierarchical Reinforcement Learning pipeline.

In order to validate our proposed framework, we tested both approaches for an imitation learning as well as an online RL task in multiple environments. Our contributions can be summarized as follows:

- The CFAG approach to macro-actions extraction from flat production rules performs very well in both imitation and online learning. The agent can easily generalize from the inferred hierarchical structure and is able to increase the action learning speed drastically. Especially, the first grammars updates prove to be powerful in reducing the dimensionality of the problem.
- Alternating between grammar updates and learning action values is an effective way of both online learning of an optimal grammar as well as an optimal policy. The first grammar extraction and action space augmentation has the largest significant effect in the further learning procedure of the agent.

Future work is going to include:

- Grammar buffer as form of meta/multi-task learning solution. Rediscovery might indicate which task is currently active.
- Talk about surprisal and model-based extensions.

In future work we are interested in testing and extending our approach to both visual (pixel-based state representations, e.g. ATARI games) and physical (joint and velocity-bases state representations, e.g. MoJuCo) domains. Formal grammars are especially useful for languages with large terminal vocabulary. So far we have only experimented with small action spaces and single agents. Pastra and Aloimonos [27] note that social interactions of more than one agent can also be formulated within the notion of tool use. Hence, we are interested in possible applications to multi-agent RL and testing the scalability of our approach to real-life domains.
Another important question that we have not been able to fully address, is how to learn initiation sets for options. Most common approaches allow options to be executed starting from every state in the state space. Since not all sub-routines can be efficiently executed from every state, such a crude assumption can slow down learning. We saw that learning a grammar on sequences of state transitions might provide a good first step. More experiments are needed.
Furthermore, our approach has only attempted to merge grammatical inference with two HRL algorithms. There remain many other promising frameworks such as the Hierarchies of Abstract Machines (HAMs, Parr [26], Parr and Russell [25]). HAMs define a hierarchy over finite state machines. This could naturally lead itself to automated identification via Hierarchical Hidden Markov Models [11].
Future work also has to further analyze the development of the inferred grammar throughout the learning process. Edit distances such as the Levenshtein and Jaro-Winkler distance provide two measures of string similarity which might be used to efficiently monitor the development of the inferred flat productions compared with the optimal grammar.
Ultimately, we envision a form of dictionary of action which provides an expandable library of skills for Hierarchical Reinforcement Learning agents which act in diverse naturalistic environments. This could provide a mayor contribution to a key endeavor in general artificial intelligence: life-long learning.

Todo list

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Supplementary Material

Bayesian Optimization Hyperparameter Spaces

MLP Hyperparameter Search Space:		
Hyperparameter	Range	Description
Batchsize	Integer: $[50, 500]$	Number of data points in mini-batch
Learning Rate	Float: $[0.0001, 0.05]$	SGD learning rate
# Hidden Layers	Integer: $[1, 6]$	
# Hidden Layer Units	Integer: $[30, 500]$	

Notes on Reproduction

Please clone the repository `https://github.com/RobertTLange/action-grammars-hrl` and follow the instructions outlined below: