MTH 201 -- Calculus
Module 8A: The Extreme Value Theorem part 1

October 28-29, 2020

If $f'(3) > 0$, then

$f(3)$ is above the horizontal axis on the graph

f is increasing at $x = 3$

f is concave up at $x = 3$

f is increasing at an increasing rate at $x = 3$

To 0

The Extreme Value Theorem states that if f is continuous on a closed interval $[a, b]$ then

- f cannot change concavity anywhere in $[a, b]$
- f must have an inflection point somewhere in $[a, b]$
- f must change from increasing to decreasing or vice versa somewhere in $[a, b]$
- f must have an absolute maximum and an absolute minimum somewhere in $[a, b]$

To find the absolute max or min values of a continuous function on a closed interval, the first step according to what was discussed on the videos is to

- Find the second derivative of the function
- Plug the endpoints of the interval into the function
- Find the critical values of the function that are inside the interval
- Plug the endpoints of the interval into the first derivative of the function

Tc 0

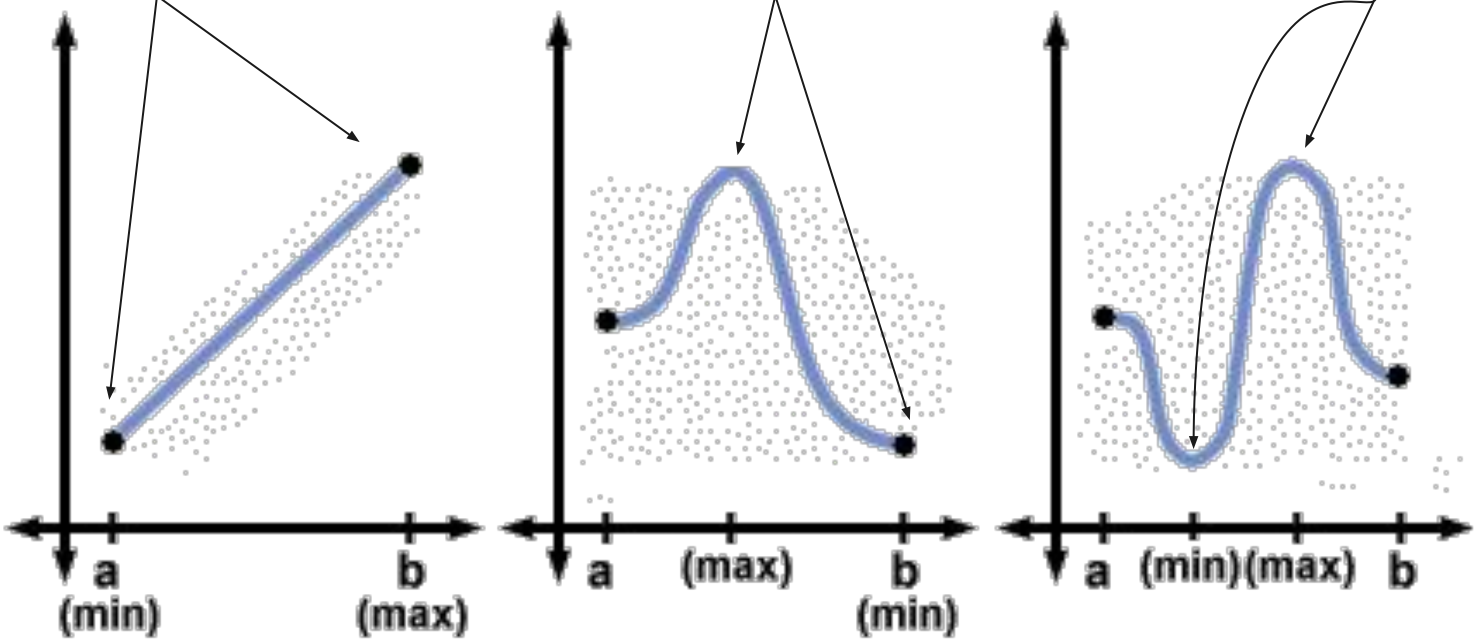
Extreme Value Theorem

If f is a continuous function on a closed interval [a,b], then f has both an absolute max and absolute min value on [a,b].

The extrema could happen at the endpoints

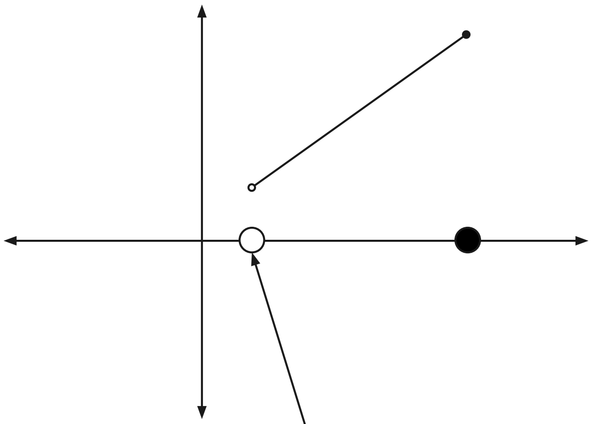
Or one at an endpoint and the other at a critical value

Or both at critical values

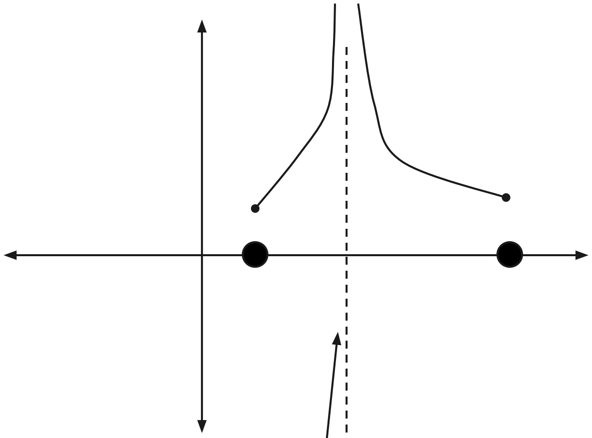


...but both the absolute max and absolute min MUST be there, and endpoints/critical values are the only places they can be.

https://origin-www.sparknotes.com/math/calcab/applicationsofthederivative/section3/page/2/



If the function were continuous but the interval wasn't closed...



...or if the interval was closed but the function wasn't continuous...

...then the function might fail to have both absolute max and min values.

But continuous + closed interval = both absolute max and min exist! 😀😀

Note 3.3.2. Thus, we have the following approach to finding the absolute maximum and minimum of a continuous function f on the interval $[a, b]$:

1 • find all critical numbers of f that lie in the interval;

2 • evaluate the function f at each critical number in the interval and at each endpoint of the interval;

3 • from among those function values, the smallest is the absolute minimum of f on the interval, while the largest is the absolute maximum.

Note: This really simplifies things -- no sign charts necessary for instance

Demo and practice at the Jamboard

Alice wants to find the absolute maximum value of $f(x) = -x^3 + 2x + 5$ on $[0, 2]$. So she plugs in $x = 0, 1, 2$ into f to get $f(0) = 5, f(1) = 6, f(2) = 1$ and concludes that the max value happens at $x = 1$.

- There's nothing wrong with this logic, and Alice's answer is correct.
- Alice's answer is correct, but there's a flaw in her logic. (She's correct but only by coincidence.)
- Alice's answer is not correct, and there's a flaw in her logic.

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