The derivative of a function at a point

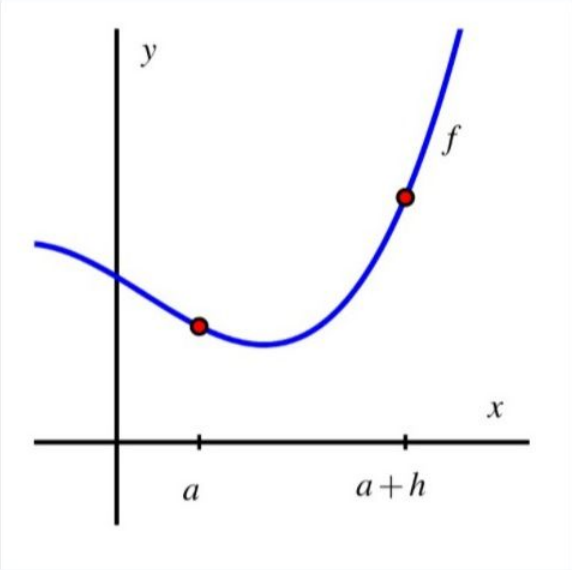
MTH 201 -- Module 2A

Today

- Activity + debrief: What the derivative is, and why we care
- Activity: Working with the derivative conceptually
- Activity (time permitting): Finding derivatives using the definition

What is the derivative?

The slope of the line that connects the two red points in this graph is the same thing as



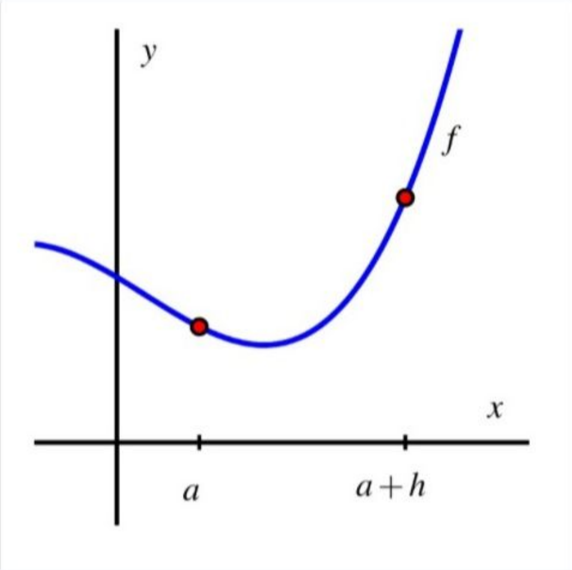
The instantaneous rate of change in f(x) at x = a+h

The instantaneous rate of change in f(x) at x = a

The average rate of change in f(x) from x = a to x = a+h

The average rate of change in f(x) from x = 0 to x = a

The slope of the line that connects the two red points in this graph is



$$1$$

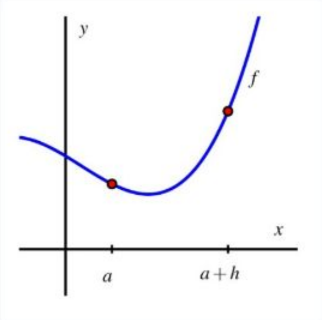
$$f(a+h)$$

$$f(a+h)-f(a)$$

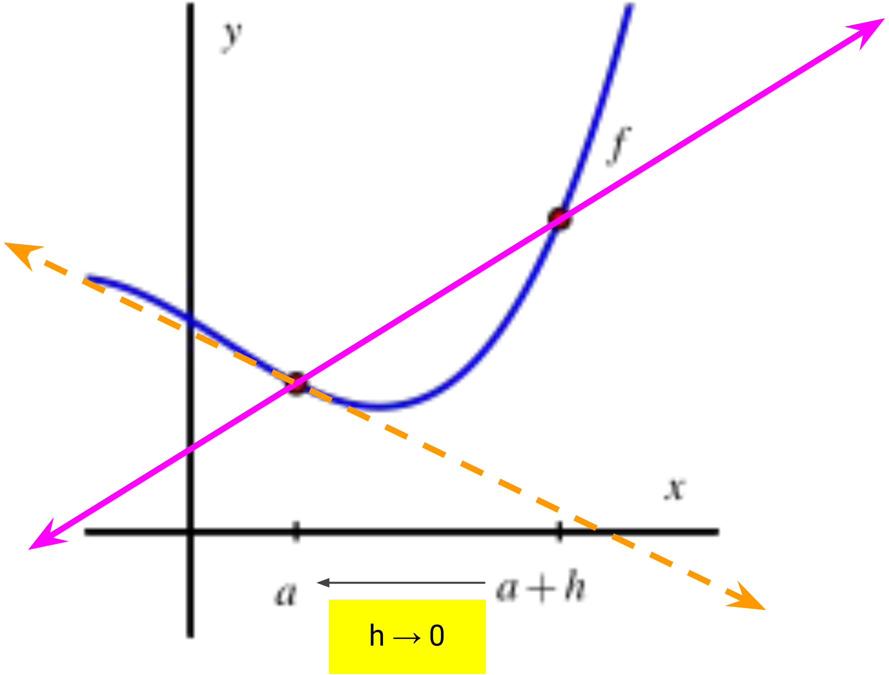
$$\frac{f(a+h)}{h}$$

$$\frac{f(a+h)-f(a)}{h}$$

To get the *derivative* of this function at $x = a$, we would need to take the slope of the line that connects the two red dots, and then



- Divide by a
- Take the limit as $a \to 0$
- Take the limit as $h \to 0$
- Set it equal to 0 and solve for a
- Nothing -- the derivative is the same thing as the slope of that line



Average rate of change = slope of the secant line between (a, f(a)) and (a+h, f(a+h))

$$\frac{f(a+h)-f(a)}{a+h-a} = \frac{f(a+h)-f(a)}{h}$$

Instantaneous rate of change = the number that the average rates of change approach as the interval shrinks

$$\lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$$

The **derivative** of the function y = f(x) at the point x = a:

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

The derivative of f(x) at x = a, f'(a), is all of the following things:

- The **instantaneous rate of change** in f(x) at x = a
- If f(t) is a position at time t, f'(a) is the **instantaneous velocity** at time t = a
- The **slope of the tangent line** to the graph of f(x) at x = a

Below is a statement about the derivative of a function f at x = 2. What, if anything, is wrong about this statement?

$$f'(2) = \frac{f(2+h) - f(2)}{h}$$

Answer Key

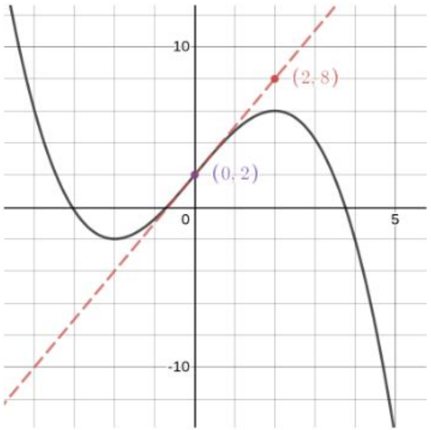
- Nothing, this is totally correct
  - 1 student
  - Thomas Fuller
- All the 2's should be a's instead
  - 0 students
- There should be $h \to 0$ underneath the $f'(2)$
  - 4 students
  - Gotthold Eisenstein, Pierre-Simon Laplace, Mary Jackson, Louise Hay
- There should be a limit as $h \to 0$ to the left of the $f'(2)$
  - 0 students
- There should be a limit as $h \to 0$ between $f'(2)$ and the equals sign
  - 1 student
  - John Urschel
- ✓ There should be a limit as $h \to 0$, between the equals sign and the fraction
  - 11 students
  - Heisuke Hironaka, Leonardo Fibonacci, Annie Easley, Alan Turing, Galileo Galilei, Shing-Tung Yau, Hipparchus, Katie Bouman, Dorothy Vaughan, Anna Pell Wheeler, Sophus Lie

$$\boxed{f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}}$$

Having h → 0 under f′(2) makes no semantic sense

Working with the derivative concept

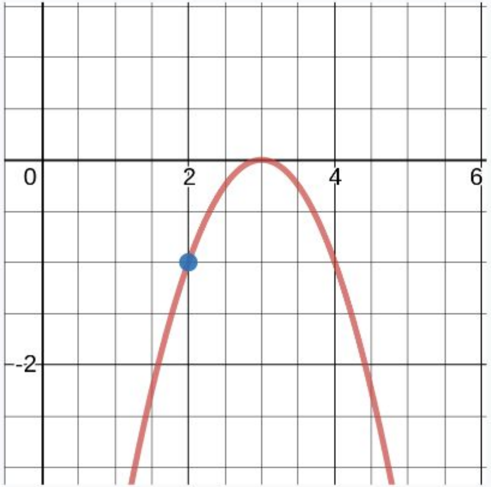
A function $y = A(x)$ is shown along with its tangent line at $x = 0$ and the points $(0, 2)$ and $(2, 8)$. Given this information, we can conclude



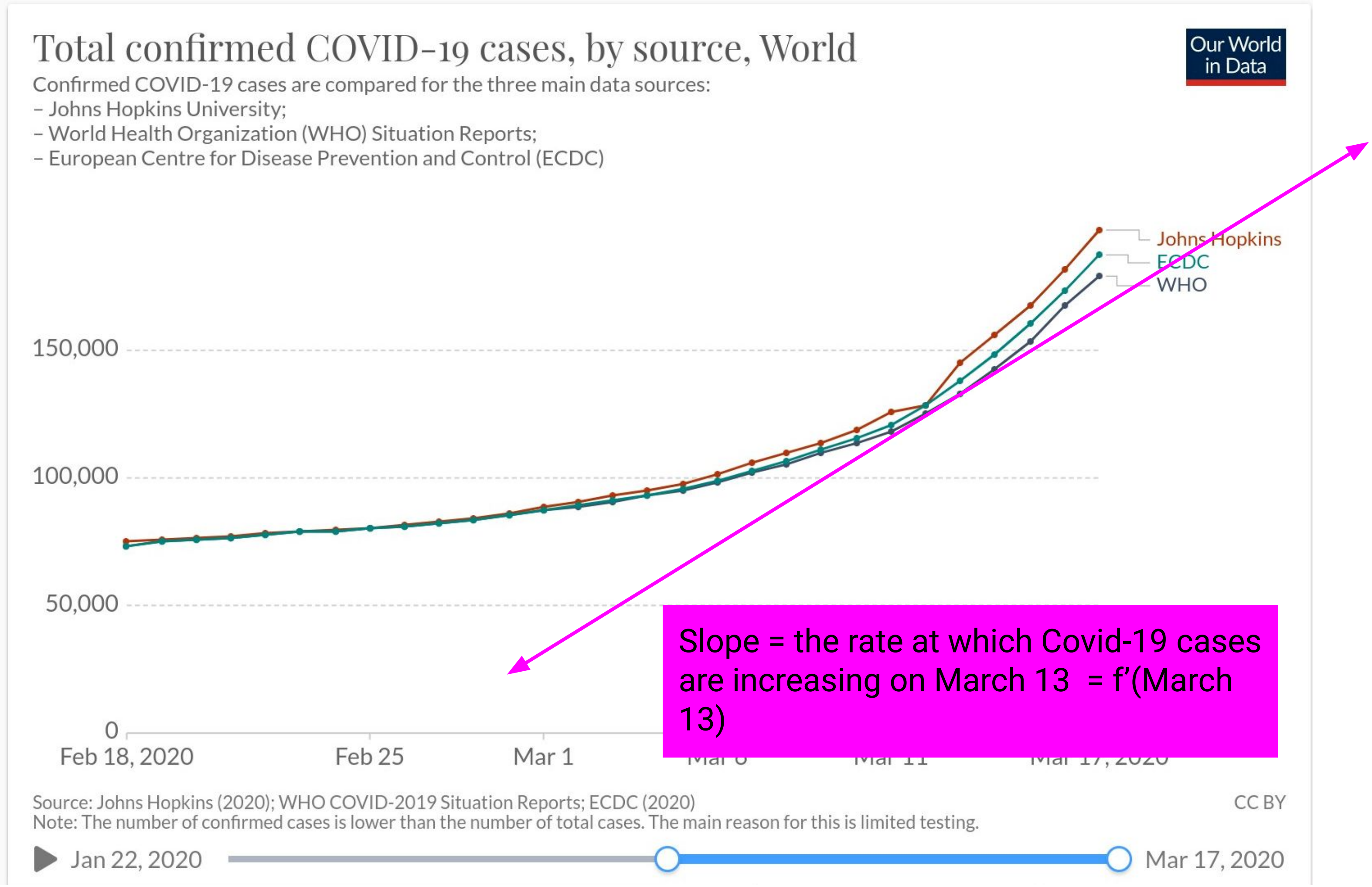
- $A(0) = 2$ and $A'(0) = 8$
- $A(0) = 2$ and $A(0) = 8$
- $A(0) = 2$ and $A'(0) = 6$
- $A'(0) = 2$ and $A'(2) = 6$
- $A(0) = 6$ and $A'(0) = 8$
- None of the above

To 0

Here's the graph of a function g(x) and the point on its graph at x = 2. Which of the following are true statements? Select ALL that apply.



- $g(2)$ is positive
- $g(2)$ is negative
- $g'(2)$ is positive
- $g'(2)$ is negative


Total confirmed COVID-19 cases, by source, World
Confirmed COVID-19 cases are compared for the three main data sources:
– Johns Hopkins University;
– World Health Organization (WHO) Situation Reports;
– European Centre for Disease Prevention and Control (ECDC)
Our World in Data
Johns Hopkins
ECDC
WHO
150,000
100,000
50,000
0
Feb 18, 2020
Feb 25
Mar 1
Slope = the rate at which Covid-19 cases are increasing on March 13 = f'(March 13)
Source: Johns Hopkins (2020); WHO COVID-2019 Situation Reports; ECDC (2020)
CC BY
Note: The number of confirmed cases is lower than the number of total cases. The main reason for this is limited testing.
Jan 22, 2020
Mar 17, 2020

A function $y = P(x)$ tells you the number of milligrams (y) of lead in x milliliters of water in a city's water supply. The units of measurement of $P'(100)$ would be

- Milligrams
- Milliliters
- Milligrams per milliliter
- Milliliters per milligram
- None of the above

Tc 0

Suppose y = g(x) is a function where the units of measurement of x are "milligrams" and the units of measurement of y are "meters". What are the units of measurement of g'(x)?

Answer Key

Meters

1 student

Anna Pell Wheeler

Milligrams

0 students

Milligrams x Meters

0 students

Milligrams per meter

7 students

Heisuke Hironaka, John Urschel, Thomas Fuller, Pierre-Simon Laplace, Dorothy Vaughan, Louise Hay, Sophus Lie

✓ Meters per milligram

10 students

Leonardo Fibonacci, Annie Easley, Alan Turing, Galileo Galilei, Shing-Tung Yau, Gotthold Eisenstein, Mary Jackson, Hipparchus, Katie Bouman, Vi Hart

$$\boxed{f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}}$$

The fraction is (Outputs)/(Inputs) so the units must do the same

Finding derivative values using the definition

Save for tomorrow if low on time

Recap

The derivative of a function y = f(x) at a point x = a:

- Is denoted f’(a) (“f prime of a”)
- Gives the instantaneous rate of change in f(x) at x = a
- Gives the instantaneous velocity of an object at time t = a if f(t) is position
- Gives the slope of the tangent line to the graph of f(x) at x = a
- Can be computed algebraically using a limit (below) if f(x) has a formula
- Has units equal to (Units of y) “per” (Units of x)

Next Up

- Day 2 of Module 2A -- Average rates of change and using them to find instantaneous rates of change
- No Daily Prep for day 2 on any module
- See calendar and Week 2 Guide for all due items and a schedule
- **Check Campuswire 1-2x per day to stay up to speed and not miss things**

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

Feedback:
http://gvsu.edu/s/1zJ