MTH 201 -- Calculus
Module 12A: Review and practice with the Fundamental Theorem

November 30-December 1, 2020

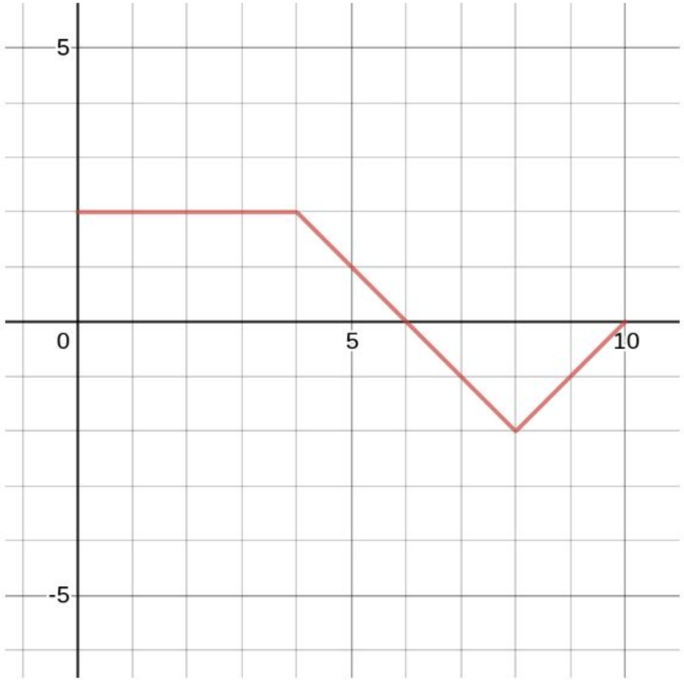
Announcements/Agenda

+ **Review the calendar and watch Campuswire+Blackboard** for end-of-course deadlines and announcements

Agenda:

- Loose ends from 11B
- Review of FTC
- Practicing with computations/applications

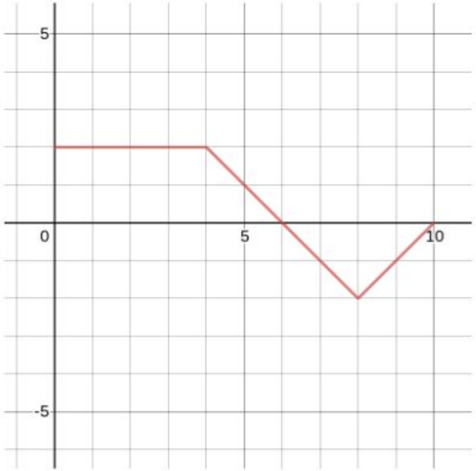
Here's the graph of $y = f(x)$. The exact value of $\int_0^{10} f(x)\,dx$ is



- 6
- 14
- 26
- Finite and possible to calculate but it's not any of the above
- Infinite
- Impossible to calculate without finding an antiderivative first
- Impossible to calculate, so we should use a Riemann sum to approximate it

To 0

Here's the graph of $y = f(x)$ again, and suppose that it represents the velocity (in miles per hour) of a moving object over a 10-hour period. At the end of 10 hours, the object



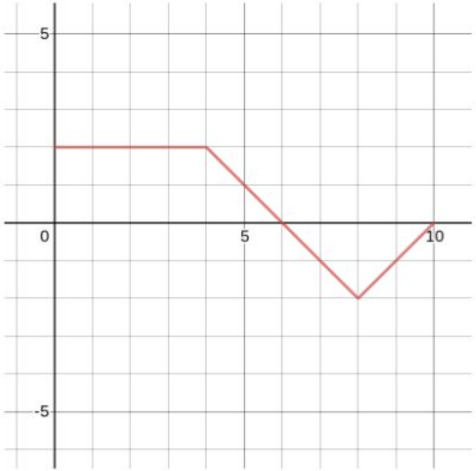
Has traveled a total of 6 miles

Is 6 miles ahead of the point where it started

Is 6 miles behind the point where it started

To 0

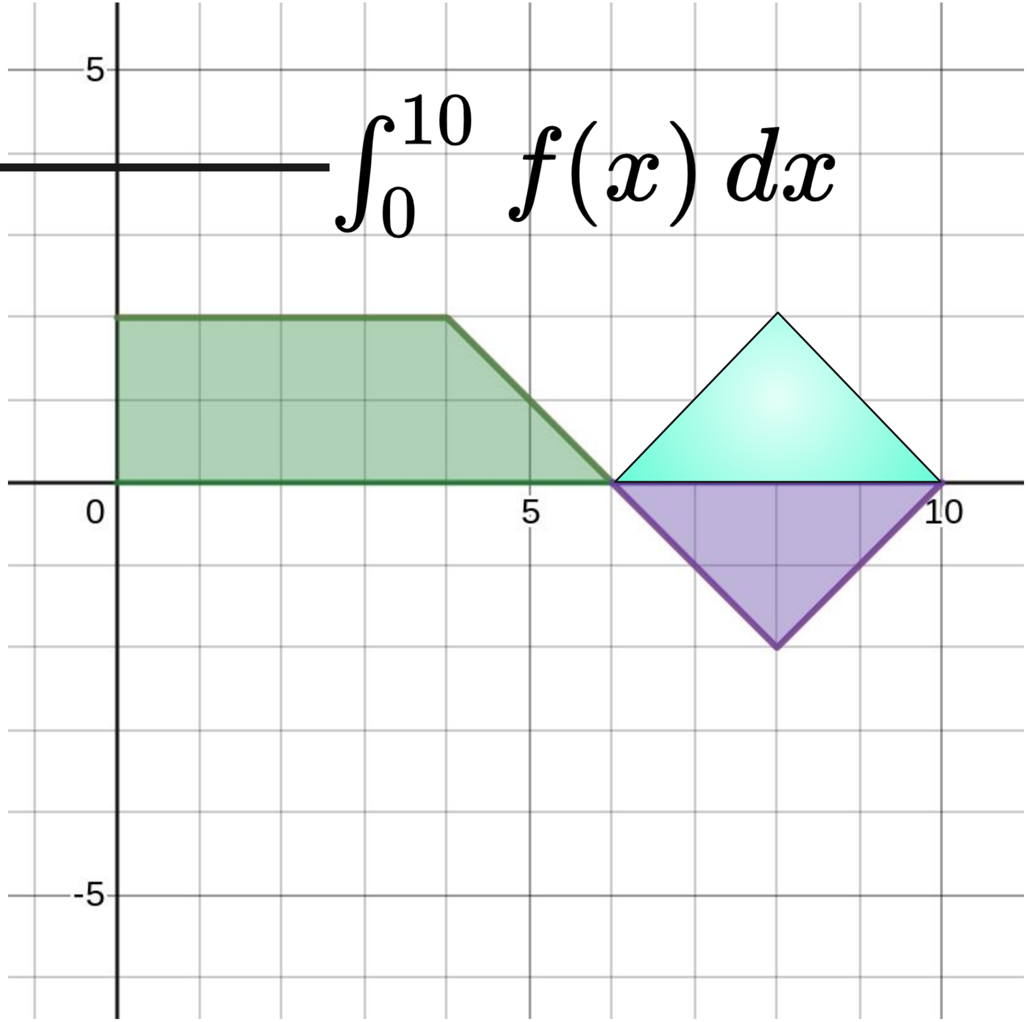
Here's the graph of $y = f(x)$ again, and suppose that it represents the velocity (in miles per hour) of a moving object over a 10-hour period. At the end of 10 hours, how many miles has the object traveled?



- 6 miles
- 14 miles
- 18 miles
- 26 miles
- None of the above

- The **exact area that's between 0 and 10, and between the graph of f(x) and the x-axis**
- Area below the x-axis is considered negative; area above is positive
- Integral gives the net amount of area -- negative cancels out positive
- If f(x) = velocity, integral gives the **total change in position** (not always the same as "total distance traveled"
- Treating negative area as positive gives total distance traveled

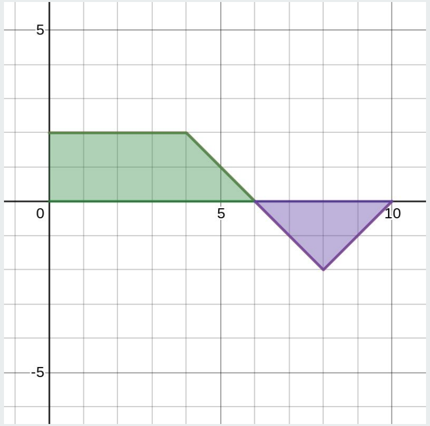
$$\int_0^{10} f(x)\,dx$$



Three ways to compute a definite integral

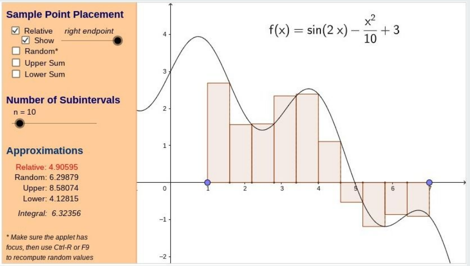
Use geometry

<u>Pro</u>: Simple; <u>Con</u>: Rare



Estimate with a Riemann sum

<u>Pros</u>: Simple, works on any function; <u>Cons</u>: Only an estimate; lots of computation



Use an antiderivative

<u>Pro</u>: 100% exact, not an estimate; <u>Con</u>: Often very difficult to compute

$$\int_0^\pi \sin(x)\,dx = -\cos(\pi) - (-\cos(0)) = 2$$

The best choice for computing $\int_0^2 (2 - 3x)\,dx$ is probably

Geometry

Estimation with a Riemann sum

Antiderivatives/FTC

Tc 0

The best choice for computing $\int_0^2 \arctan(2-3x)\,dx$ is probably

Geometry

Estimation with a Riemann sum

Antiderivatives/FTC

To 0

The best choice for computing $\int_0^2 (2 - 3x^2)\,dx$ is probably

Geometry

Estimation with a Riemann sum

Antiderivatives/FTC

Tc 0

The best choice for computing $\int_0^2 \sqrt{4-x^2}\,dx$ is probably

Geometry

Estimation with a Riemann sum

Antiderivatives/FTC

To 0

Work on WeBWorK for Module 12

FTC questions start with question 5

An antiderivative for $\frac{5x+1}{x}$ is

- $5x + 1$
- $5x + x$
- $5x + \ln(x)$
- $\frac{(5/2)x^2+x}{(1/2)x^2}$

What we learned/what's next

- The Fundamental Theorem of Calculus -- connects derivatives and integrals
- We can now compute a definite integral in three ways: Approximate by geometry, approximate by Riemann sums, find exact value via FTC/antiderivatives -- in increasing order of hardness

NEXT:

- Followup: Cataloguing common antiderviatives
- Thanksgiving Break!
- **Module 12:** More on using the FTC + applications of the FTC (Total Change Theorem)