MTH 201 -- Calculus
Module 10A: Determining distance traveled from velocity

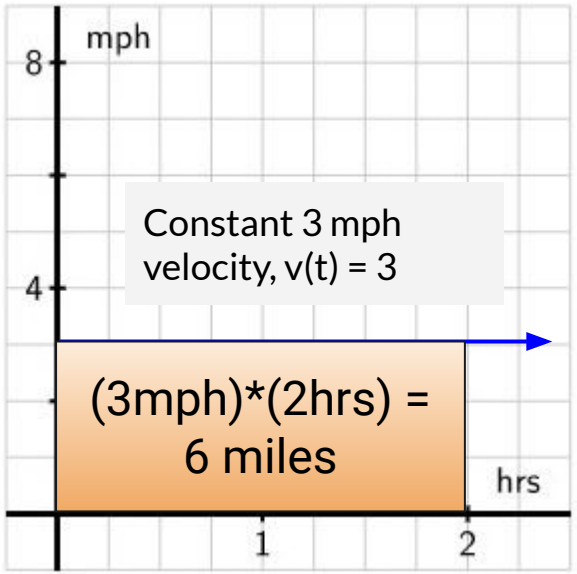
November 11-12, 2020

Agenda

- Review from Daily Prep 10A
- Minilecture: Doing area sums to find distance traveled
- Practice with area sums
- Minilecture: Finding distance traveled with antiderivatives
- Practice with antiderivatives
- Summing up and intro to the followup

Placeholder for review -- wait for Daily Prep responses

Given a moving object's velocity, can we reconstruct how far it travels?



Time	0	1	2	3	4	5
Distance	0	3	6	9	12	15

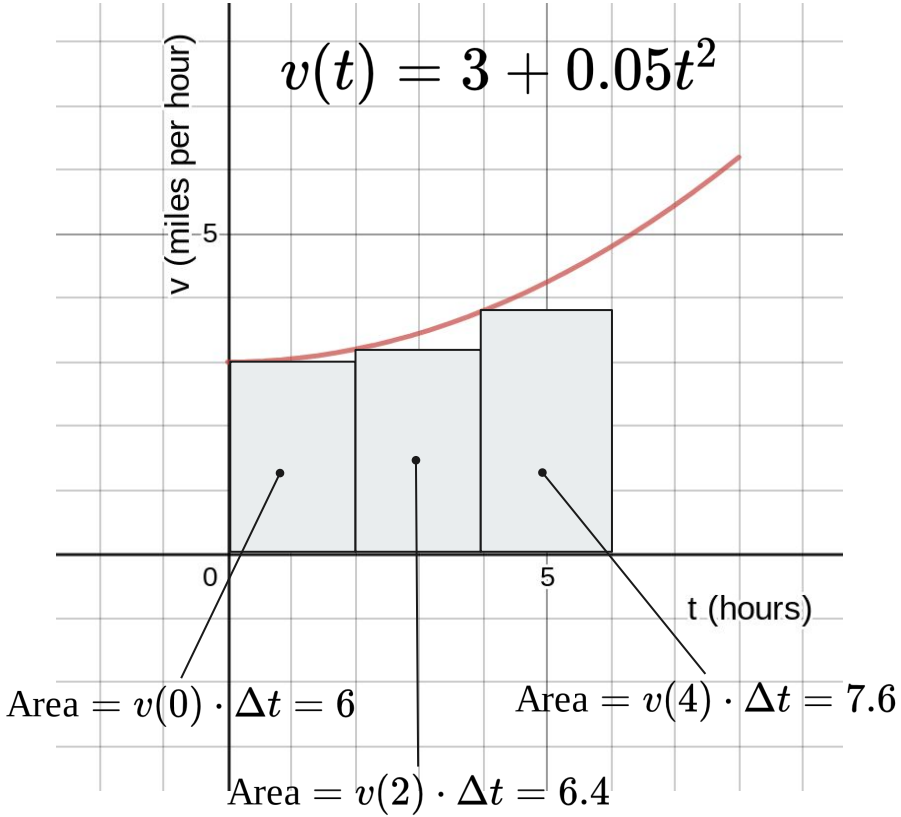
Position is NOT constant but has a constant rate of change

$s(t) = 3t$

Big idea #1:

We can reconstruct distance traveled from velocity using areas

Q: But what if the function is curvy and we can't fit a box under its graph?
A: Approximate the area by putting *several* boxes underneath its graph.



Set up 3 boxes → each has a width of Δt = 2 (hours)
Upper-left of the box touches the graph

t	v(t)	Area
0	3	6
2	3.2	6.4
4	3.8	7.6
	SUM	20

Total distance travelled is about 20 miles.
This is **SMALLER** than the actual distance travelled because not all the area is picked up by the boxes.

Activity 4.1.2. Suppose that a person is walking in such a way that her velocity varies slightly according to the information given in Table 4.1.4 and graph given in Figure 4.1.5.

Table 4.1.4. *Velocity data for the person walking.*

t	$v(t)$
0.00	1.500
0.25	1.789
0.50	1.938
0.75	1.992
1.00	2.000
1.25	2.008
1.50	2.063
1.75	2.211
2.00	2.500

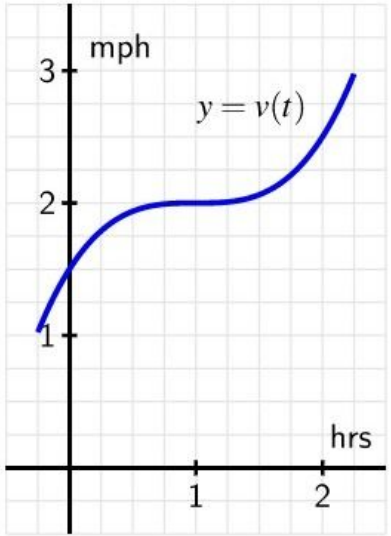
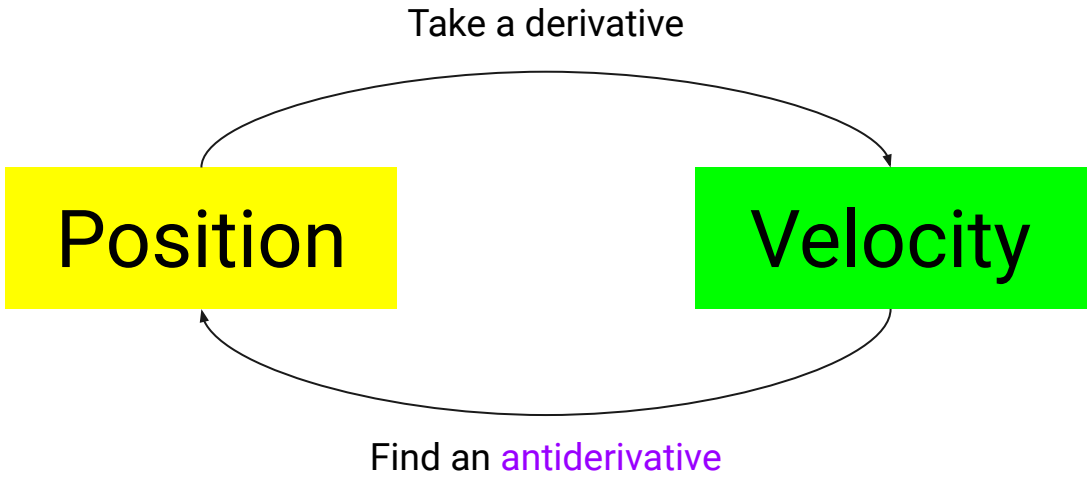


Figure 4.1.5. The graph of $y = v(t)$.

a. Using the grid, graph, and given data appropriately, estimate the distance traveled by the walker during the two hour interval from $t = 0$ to $t = 2$. You should use time intervals of width $\Delta t = 0.5$, choosing a way to use the function consistently to determine the height of each rectangle in order to approximate distance traveled.

Big idea #2:

We can also reconstruct distance traveled from velocity using antiderivatives



$$s(t) = 64 - 16(t-1)^2 \longrightarrow v(t) = s'(t) = -32(t-1)$$

$$??? \longleftarrow v(t) = 3 + 0.05t^2$$

Activity for you:

1. Using whatever means you have (including guesswork) come up with an antiderivative for $v(t) = 3 + 0.05t^2$.
2. Now come up with another one. (Hint: It may not be that different from the first.)
3. Use that antiderivative to find the total distance traveled from t = 0 to t = 6. It should be close to, but greater than 20 based on the area-based estimate we did. (Hint: That distance traveled is the difference between the starting and ending positions.)

Summary

- We're answering the question, *Given the velocity of a moving object, how far does it travel over a time interval?*
- Approach #1: Estimate the area under the graph of the velocity using rectangular boxes and the sum of their area. (This is *simple* but not *accurate*)
- Approach #2: Find an antiderivative for the velocity and find the difference in starting and ending positions (This is *accurate* but not always *simple*)
- **In the followup:** Working with velocity graphs that are "piecewise linear"; what to do about negative velocity values.