MTH 201: Calculus

Module 2B: The derivative of a function at a point

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Agenda for today

- Review of Daily Prep assignment, and Q+A

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- Short lecture: The derivative as a function

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- Short lecture: The derivative as a function
- Practice with computing derivatives using limits

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- Review of Daily Prep assignment, and Q+A
- Short lecture: The derivative as a function
- Practice with computing derivatives using limits
- Polling activity: Identifying derivative graphs
- For next time: Followup activities and things to do

Review and Q+A

Go to `www.menti.com` and use code ?

The derivative as a function

Last time: A formula for finding the derivative of a function at a single point

The definition of the derivative

Let f be a function and $x = a$ a value in the function's domain. We define the **derivative of f with respect to x at evaluated at $x = a$**, denoted $f'(a)$, by the formula

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

provided the limit exists.

$f'(a)$ tells us:

- The slope of the tangent line to the graph of f at $x = a$
- The instantaneous rate of change in f at $x = a$
- If f is a position: $f'(a)$ is the instantaneous velocity at time $x = a$

The derivative is a function on its own

Key insight

The derivative itself changes as a changes. So not only is f a function of x, so is f'.

Example

Let $f(x) = x - x^2$. Then:

a	-3	-2	-1	0	1	2	3
$f(a)$	-12	-6	-2	0	0	-2	-6
$f'(a)$	7	5	3	1	-1	-3	-5

$$f'(3) = \lim_{h\to 0} \frac{f(3+h) - f(3)}{h} = \lim_{h\to 0} \frac{((3+h) - (3+h)^2) - (-6 - (-6)^2)}{h}$$

Do we have to recalculate $f'(a)$ using a limit *every time*? Also do you see the pattern in the $f'(a)$ values?

Computing derivative FORMULAS

It looks like, if $f(x) = x - x^2$ then $f'(x) = 1 - 2x$. Does this always work?

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$$
\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h\to 0} \frac{((x+h) - (x+h)^2) - (x - x^2)}{h} \\
&= \lim_{h\to 0} \frac{(x + h - (x^2 + 2xh + h^2)) - (x - x^2)}{h} \\
&= \lim_{h\to 0} \frac{x + h - x^2 - 2xh - h^2 - x + x^2}{h} \\
&= \lim_{h\to 0} \frac{h - 2xh - h^2}{h} \\
&= \lim_{h\to 0} (1 - 2x - h) \\
&= 1 - 2x
\end{aligned}
$$

The definition of the derivative (as a formula)

Let f be a function and $x = a$ a value in the function's domain. We define the **derivative of f with respect to** x, denoted $f'(x)$, by the formula

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

provided the limit exists.

The result is a formula that accepts x as an input and gives the rate of change in f at this input, as the output.

Demo: https://www.desmos.com/calculator/rwjzrvo9an

Practice

Let $f(x) = x^2 - 2x + 1$. Find a formula for $f'(x)$ using the definition.

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$$
\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h\to 0} \frac{((x+h)^2 - 2(x+h) + 1) - (x^2 - 2x + 1)}{h} \\
&= \lim_{h\to 0} \frac{(x^2 + 2xh + h^2 - 2(x+h) + 1) - (x^2 - 2x + 1)}{h} \\
&= \lim_{h\to 0} \frac{x^2 + 2xh + h^2 - 2x - 2h + 1 - x^2 + 2x - 1}{h} \\
&= \lim_{h\to 0} \frac{2xh + h^2 - 2h}{h} \\
&= \lim_{h\to 0} (2x + h - 2) \\
&= 2x - 2
\end{aligned}
$$

Desmos: Does the answer make sense?

Information about this idea

- We will eventually shorten this process considerably with shortcut methods.
- But you still need to know the definition, because not all functions are formulas.
- Being able to do these computations is a Core Learning Target (D.1).

Derivatives of formulas as graphs

Back to Menti: Can you identify the graph of the derivative given the graph of the function and vice versa?

Next

All due dates are on the Course Calendar

- Complete Followup Activities