MTH 201 Checkpoint 5 Fall 2020

Directions:

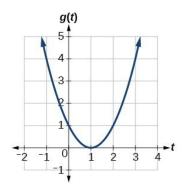
Please read this carefully, as some instructions have changed since Checkpoint 4.

• Do only the Checkpoint problems that you need to take and feel ready to take. If you have already earned Mastery on a Learning Target, do not attempt a problem for that Target! You can skip a Target if you need more time to practice with it, and take it on the next round.

- Do not put any work on this form; do all your work on separate pages. You may either handwrite or type up your work.
- New: When you have completed your work on a problem, please circle or box off the answer you wish me to check. The answer that you circle or box will be taken as your "official" final answer. Answers not boxed off, and work that is crossed out, will not be considered in the grading process.
- If you are handwriting, submit your work by **scanning your work** using a scanning app or scanning device; **do not just take a picture** but scan your work to a clear, legible, black and white PDF file of size less than 100 MB. Work submitted as an image file (JPG, PNG, etc.) will not be graded.
- Unless explicitly stated otherwise, you must show your work or explain your reasoning clearly on each item of each problem you do. Responses that consist of only answers with no work shown, or where the work is insufficient or difficult to read, or which have significant gaps or omissions (including parts left blank) will be given a grade of "x".
- New: The following are approved resources for use on this and any other Checkpoint: All documents and videos posted to the class Blackboard site; all work that you have submitted for grading, including previous Checkpoints; all videos used for Daily Prep assignments; and Wolfram|Alpha. You may also ask me (Talbert) questions at any time. All other resources, including classmates, are off-limits.

Learning Target F.2: I can find the average rate of change of a function on an interval.

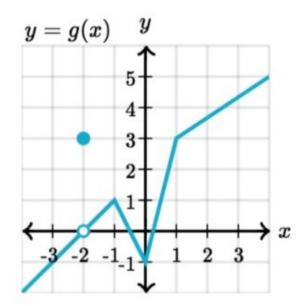
- 1. Let $f(x) = x^2 + x + 4$. Find the average rate of change in f on the intervals [0,3] and [1,1.01].
- 2. Let g(x) be the graph shown below. Find the average rate of change in g on the intervals [0, 2] and [1, 3].



Learning Target L.1 (Core): I can find the limit of a function at a point using numerical, graphical, and algebraic methods.

1. Complete the table of values below using the function $f(x) = \frac{x^2 - 4x - 5}{x + 1}$. Then state the value of $\lim_{x \to -1} f(x)$ and explain your reasoning.

- 2. Using only algebra (no graphs, tables, or estimations), evaluate $\lim_{x\to 2} \frac{x^2 x 2}{x^2 2x}$.
- 3. The function g(x) is shown below. State the value of $\lim_{x\to -2} g(x)$ and explain your reasoning.



Learning Target D.1 (Core): I can find the derivative of a function, both at a point and as a function, using the definition of the derivative.

Consider the function $f(x) = 3x^2 + 2x - 1$.

- 1. Set up, but do not evaluate, the limit that would compute f'(1) (the derivative of f at the point x = 1).
- 2. Set up, but do not evaluate, the limit that would compute f'(x) (the formula for the derivative of f at any point).
- 3. Choose one of the limits you set up and evaluate it to find either f'(1) or f'(x).

Note: Use of shortcut rules on this Learning Target is prohibited.

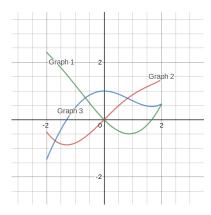
Learning Target D.2 (Core): I can use derivative notation correctly, state the units of a derivative, estimate the value of a derivative using difference quotients, and correctly interpret the meaning of a derivative in context.

At a home improvement store, you can purchase plastic coverings for lawn furniture at various prices. The price p (in dollars) is a function of the size S of the covering (measured in square feet).

- 1. Suppose S'(10) = 0.7. State the units of measurement for the numbers 10 and 0.7. (Clearly indicate which is which.)
- 2. Still assuming S'(10) = 0.7. Explain the meaning of this statement in ordinary terms (that is, in terms of price and size) and without using any technical jargon, including the terms "function", "derivative", "rate of change", "graph", "tangent", or "slope".
- 3. Suppose S(10) = 0.3, S(20) = 1.2, and S(30) = 0.04. Use forward, backward, and central differences to estimate S'(20). Clearly indicate which estimate is which.

Learning Target D.3 (Core): Given information about f, f', or f'', I can correctly give information about f, f', or f'' and the increasing/decreasing behavior and concavity of f (and vice versa).

Below are three graphs on the interval [-2, 2]: One of them is a function f, another is its first derivative f', and the the third is its second derivative f''. They are labelled Graph 1, Graph 2, and Graph 3, but not necessarily in the correct order of derivatives. State which graph is which function and provide a brief but complete, corret, and clear explanation.



Learning Target D.4: I can find the equation of the tangent line to a function at a point and use the tangent line to estimate values of the function.

Find each of the following. Make sure your answer is clearly indicated (for example by circling it). You are being graded on your processes as well as your answers, so show all your work.

- 1. Find an equation for the tangent line to the graph of $y = e^x + x 3$ at x = 0.
- 2. A function f(x) is such that f(5) = 1 and f'(5) = -10. Find an equation for the tangent line to the graph of f(x) at x = 5, and then use the line to estimate the value of f(4).

Learning Target DC.1: I can compute derivatives correctly for power, polynomial, and exponential functions and the sine and cosine functions, and basic combinations of these (constant multiples, sums, differences).

Find the derivatives of each of the following functions. Make sure your answer is clearly indicated (for example by circling it). You are being graded on your processes as well as your answers, so show all your work.

- 1. $y = 1 2x + 2^x + x^2 + 2^2$
- $2. \ g(x) = \sin(x) + 5\cos(x)$
- 3. $h(x) = (\sqrt{x} + x)(x^3 + 1)$ (Do NOT use the Product or Quotient Rules.)
- 4. $j(x) = \frac{1}{x^5} + \sqrt[5]{x}$

Learning Target DC.2 (Core): I can compute derivatives correctly for products, quotients, and composites of functions.

New instructions: Find the derivatives of each of the following. In each, state which rule you are using first. In the case of the Chain Rule, clearly indicate the decomposition of the function by stating the "inside" and "outside" functions first. Show your work in a clear and logical order and circle/box off your answer. DO NOT simplify your answer.

$$1. \ \ y = x^2 \cos x$$

$$2. \ \ y = \frac{x - x^2}{1 + 8x^2}$$

3.
$$v = e^{1 + \sin(x)}$$

4.
$$y = \sqrt[3]{1 - 8x}$$

Learning Target DC.3: I can compute derivatives correctly using multiple rules in combination.

New instructions: Find the derivatives of each of the following. In each, state ALL the rules you are using, in the correct order of use. You need only list when you use the Product, Quotient, or Chain Rules. In the case of the Chain Rule, clearly indicate the decomposition of the function by stating the "inside" and "outside" functions first. Show your work in a clear and logical order and circle/box off your answer. DO NOT simplify your answer.

1.
$$y = \sin(t \sin t)$$

$$2. \ \ y = \frac{e^{2x}}{\sqrt{1 + \cos(x)}}$$

Learning Target DC.4: I can compute the derivatives correctly for logarithmic, trigonometric, and inverse trigonometric functions.

New instructions: Find the derivatives of each of the following. In the case of the Chain Rule, clearly indicate the decomposition of the function by stating the "inside" and "outside" functions first. Show your work in a clear and logical order and circle/box off your answer. DO NOT simplify your answer.

- 1. $y = \ln(x^2 + 1)$
- $2. \ y = x^2 \cot(x)$
- 3. $y = \sqrt{\arcsin x}$

Learning Target DA.1 (Core): I can find the critical values of a function, determine where the function is increasing and decreasing, and apply the First and Second Derivative Tests to classify the critical points as local extrema.

Consider the function $g(w) = 2w^3 - 7w^2 - 3w - 2$

- 1. Find all the critical values of the function. Make your reasoning clear by giving all calculus and algebra steps clearly and neatly.
- 2. Make a first derivative sign chart for the function and use it to determine the intervals on which the function is increasing and decreasing. Your sign chart must be legible and formatted using the rules we discussed in class, and you must state which points you are using as test points.
- 3. Classify each critical value as a local maximum, local minimum, or neither and explain your reasoning.

Learning Target DA.2: I can determine the intervals of concavity of a function and find all of its points of inflection.

Consider the function $g(w) = 2w^3 - 7w^2 - 3w - 2$. Use calculus (not visual estimation from a graph) to find the intervals on which g is concave up and the intervals on which g is concave down, and state its inflection points. Show all your work; if you construct a sign chart, make sure the chart has all the required properties that we have discussed.

Learning Target DA.3: I can use the Extreme Value Theorem to find the absolute maximum and minimum values of a continuous function on a closed interval.

For each of the following, use Calculus (not visual estimation from a graph) to determine the absolute extreme values of the given function on the specified interval. You may assume that each function is continuous on the interval.

1.
$$g(w) = 2w^3 - 7w^2 - 3w - 2$$
 on $[-5, 3]$

2.
$$R(x) = \ln(x^2 + 8x + 14)$$
 on $[-4, 2]$