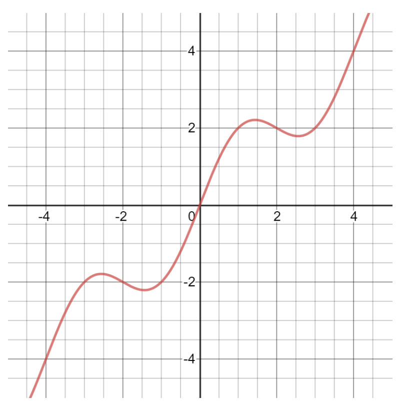
Directions:

- Do only the problems that you need to take and feel ready to take. If you have already earned Mastery on a Learning Target, do not attempt a problem for that Target! You can skip a Target if you need more time to practice with it, and take it on the next round.
- Each Learning Target problem is to be written up on a separate sheet, scanned to separate PDF files, and submitted to the appropriate Learning Target "assignment" on Blackboard. **Please do not submit more than one Learning Target in the same PDF, and make sure you are submitting it to the right Blackboard area.**
- If you are handwriting, submit your work by **scanning your work** using a scanning app or scanning device; **do not just take a picture** but scan your work to a clear, legible, black and white PDF file of size less than 100 MB. **Work submitted as an image file (JPG, PNG, etc.) will not be graded.**
- Please consult the grading criteria found in the Information on Learning Targets and Checkpoints document found in the *Learning Targets* area on Blackboard prior to submitting your work, to make sure your submission has met all the requirements.
- Please use the approved resources to double-check your work against errors prior to submitting your work.

Learning Target 1: *I can find the average rate of change of a function and the average velocity of an object on an interval.*

1. Let $f(x) = 3x^2 - \dfrac{1}{x}$. Find the average rate of change in f on the intervals $[1, 5]$ and $[3, 3.01]$. If you round, round your decimals to four places.
2. Let $g(x)$ be the graph shown below. Find the average rate of change in g on the intervals $[-2, 0]$ and $[1, 4]$.



3. A student is walking in a hallway, and their distance s (in feet) from the Math Department office at time t seconds is given by the following table:

Time	0	5	10	15	20
Distance	14	20	19	22	35

Find the student's average velocity from $t = 0$ to $t = 5$ and from $t = 5$ to $t = 20$.

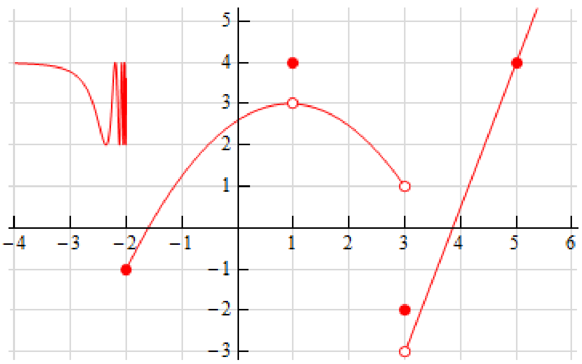
Learning Target 2 (Core): *I can find one- and two-sided limits of a function at a point and at infinity using numerical, graphical, and algebraic methods.*

1. Complete the table of values below using the function $f(x) = \dfrac{x^2 - 6x + 5}{x - 5}$. Then state the value of $\lim_{x \to 5} f(x)$ and explain your reasoning. You do not need to show your work on computing the table values, but they must all be correct.

x	4.5	4.9	4.99	5.01	5.1	5.5
$f(x)$						

2. Using only algebra (no graphs or tables), evaluate $\lim_{x \to 2} \dfrac{2x^2 - x - 6}{x - 2}$.

3. The function $h(x)$ is shown below. State the value of each limit shown below the graph. If the limit doesn't exist, write "does not exist" and then explain why.



(a) $\lim_{x \to -2^+} h(x)$

(b) $\lim_{x \to 1} h(x)$

(c) $\lim_{x \to 3} h(x)$

(d) $\lim_{x \to 3^-} h(x)$

Learning Target 3: *I can find the derivative of a function (both at a point and as a function) and the instantaneous velocity of an object using the definition of the derivative.*

Consider the function $f(x) = 10x - 2x^2$.

1. Write out the correct limit expression that would compute $f'(2)$.
2. Find the exact value of $f'(2)$ by computing the limit from part (a), using algebraic techniques.

Note: Your solution *must* begin with a correct statement of the limit. Your solution *can only* be found by evaluating the limit; no "shortcut" methods from later parts of this course are allowed (except in your notes to check your answer). *All significant algebra steps* must be shown and done correctly.

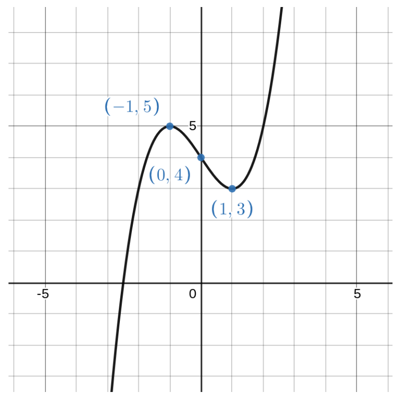
Learning Target 4 (Core): *I can use derivative notation correctly, state the units of a derivative, estimate the value of a derivative using difference quotients, and correctly interpret the meaning of a derivative in context.*

A wind turbine generates electricity which is then sold on the open market. The faster the wind blows, the more electricity is generated and therefore the more money is earned. The revenue earned (R, in dollars) is therefore a function of the wind speed (v in miles per hour). Denote this relationship $R = f(v)$.

1. Suppose $f'(10) = 5$. State the units of measurement for the numbers 10 and 5. (Clearly indicate which is which.)
2. Still assuming $f'(10) = 5$, explain the meaning of this statement in ordinary terms and without using any technical math jargon.

Learning Target 5: *Given information about f, f', or f'', I can correctly give information about f, f', or f'' and the increasing/decreasing behavior and concavity of f (and vice versa).*

Here is the graph of a function f with three of the points labelled:



1. On what interval or intervals is the derivative, f', positive? State your answer and give a clear, single sentence explaining your reasoning using the graph of f.
2. On what interval or intervals is the derivative, f', increasing? State your answer and give a clear, single sentence explaining your reasoning using the graph of f.

Important note: *Your explanations must refer to all functions explicitly by name.* Refer to f, f', or f''; or "the original", or "the first derivative", "the second derivative". Do not refer to "it", "the graph", "the function", "the line", etc. without also making it explicit which function you are referring to.

Learning Target 6 (Core): *I can compute basic-level derivatives using algebraic shortcut methods and solve simple application problems. (Functions involved will include constant, power, polynomial, exponential, and sine/cosine functions; applications include rates of change and slopes/equations of tangent lines).*

In each of the items below, use only the derivative computation rules found in Sections 2.1 and 2.2 of the text. **Do not use the limit definition or any rules not found in Sections 2.1 and 2.2**. Use of these will result in the work being marked as not having met the grading standards. **If algebra is needed to simplify the function before taking its derivative, show all your algebra work.**

1. Compute the derivatives of the following functions.
   (a) $y = 1 - 2x + 2^x + x^2 + 2^2$
   (b) $g(x) = 3\cos(x) - 2\sin(x)$
   (c) $h(x) = (x+2)^4$ (Remember, use only the rules from Sections 2.1 and 2.2.)
2. Find an equation for the tangent line to the graph of $y = 3\sin(x) + x^2$ at $x = 0$. Show all your work on this part.
3. The height (h, in meters) of a ball that is thrown straight up at time t seconds is $h(t) = 64 + 8t - 16t^2$. Find the instantaneous velocity of the ball at $t = 1$ seconds. Show all your work on this part.

Learning Target 7 (Core) : *I can compute derivatives involving the Product, Quotient, and Chain Rules.*

Find the derivatives of each of the following. **In each, state the rules you are using, in the correct order of use. You need only list when you use the Product, Quotient, or Chain Rules. In the case of the Chain Rule, clearly indicate the decomposition of the function by stating the "inner" and "outer" functions first. Correct answers given without this information will not meet the criteria for acceptability.** Show your work in a clear and logical order and circle/box off your answer. DO NOT simplify your answers.

1. $y = \dfrac{\sin(x)}{e^x}$
2. $y = e^x(x^2 + x + 1)$
3. $y = e^{x^2+x+1}$

Learning Target 8: *I can compute advanced-level derivatives using algebraic shortcut methods.*

Find the derivatives of each of the following. **In each, state the rules you are using, in the correct order of use. You need only list when you use the Product, Quotient, or Chain Rules. In the case of the Chain Rule, clearly indicate the decomposition of the function by stating the "inner" and "outer" functions first. Correct answers given without this information will not meet the criteria for acceptability**. Show your work in a clear and logical order and circle/box off your answer. DO NOT simplify your answers.

1. $y = \ln(x^2 + 100)$
2. $y = \sqrt{\ln(x) + e^x}$
3. $y = (\arctan(x))^2 + 1$

Learning Target 9: *I can find the critical values of a function, determine where the function is increasing and decreasing, and apply the First and Second Derivative Tests to classify the critical points as local extrema.*

Consider the function $f(x) = \frac{1}{2}x^3 - \frac{3}{2}x + 4$.

1. Find all the critical values of the function. Make your reasoning clear by giving all calculus and algebra steps clearly and neatly.
2. Make a first derivative sign chart for the function and use it to determine the intervals on which the function is increasing and decreasing. Your sign chart must be legible and formatted using the rules we discussed in class, and you must state which points you are using as test points.
3. Using either the First or Second Derivative Test (your choice), classify each critical value as a local maximum, local minimum, or neither and explain your reasoning.

Acceptable work requires showing ALL Calculus steps used to arrive at your answer; however, **you may use technology to perform any non-calculus task**, like factoring a polynomial or finding output values of functions, without showing the steps.

Learning Target 10: *I can determine the intervals of concavity of a function and find all of its points of inflection.*

Consider the function $f(x) = \frac{1}{2}x^3 - \frac{3}{2}x + 4$. Use calculus (not visual estimation from a graph) to find the intervals on which f is concave up and the intervals on which f is concave down, and state its inflection points.

If you construct a sign chart, make sure the chart has all the required properties that we have discussed. **Acceptable work requires showing ALL Calculus steps used** to arrive at your answer; however, you may use technology to perform any non-calculus task, like finding output values of functions, without showing the steps.

Learning Target 11: *I can use the Extreme Value Theorem to find the absolute maximum and minimum values of a continuous function on a closed interval.*

For each of the following, use Calculus (not visual estimation from a graph) to determine the absolute extreme values of the given function on the specified interval. You may assume that each function is continuous on the interval. **Acceptable work requires showing ALL Calculus steps used** to arrive at your answer; however, you may use technology to perform any non-calculus task, like finding output values of functions, without showing the steps.

1. $f(x) = \frac{1}{2}x^3 - \frac{3}{2}x + 4$ on $[-2, 0]$
2. $g(x) = e^{x^2-4x}$ on $[-2, 0]$

The first function above is the same function used in Learning Target 9. If you attempted that problem and showed work there, you do not need to do that work again here.

Learning Target 12 (Core): *I can set up and use derivatives to solve applied optimization problems.*

Set up and solve the optimization problem found below the bullet list. **In order for your work to meet quality standards, it needs to contain ALL of the following. Check carefully before turning in your work to make sure you've included each one.** A lengthy solution is not necessary, but the written solution for Exercise 3 that was posted earlier would be a good guide for you.

- A clear indication of what each variable in the solution represents;
- A clear statement of what quantity you are optimizing;
- A formula for the quantity you are optimizing and a clear indication of how you obtained it;
- If you use a constraint in the problem, a clear statement of what that constraint is and how you used it;
- The use of a derivative to find the input that optimizes your quantity;
- Reasoning that explains why your solution is correct — *don't just find a value but explain how you know that value optimizes the target quantity.*

Problem for this Learning Target: An open box is to be made out of an 8-inch by 15-inch piece of cardboard by cutting out squares of equal size from the four corners and bending up the sides. Find the dimensions of the resulting box that has the largest volume. (The answer must give all three dimensions of the box.)

Learning Target 13: *I can calculate the area under a curve, net change, and displacement using geometric formulas and Riemann sums.*

Consider the function $f(x) = \sqrt{x}$. Estimate the area under the curve, above the x-axis, and between $x = 1$ and $x = 4$, using the following Riemann sums. **On each one**: Clearly state the value of Δx, clearly state which points you are using to construct the rectangles, and show the setup of your calculation. **Keep all approximations to four (4) decimal places**.

1. L_6
2. M_4

Learning Target 14: *I can evaluate a definite integral using geometric formulas and the Properties of the Definite Integral.*

Compute the **exact value** of each of the three definite integrals below, by graphing the function being integrated first and then interpreting the integral as an area. Each answer needs to contain:

- A rough sketch of the graph of the function being integrated. You can produce the graph on Desmos if you want, and then make a simple but accurate hand-drawing of it on your work.
- Some indication of how you got the value of the integral, by showing work or giving a verbal explanation that explains all your steps.
- A correct answer that is **exact** with **no decimal approximations** used.

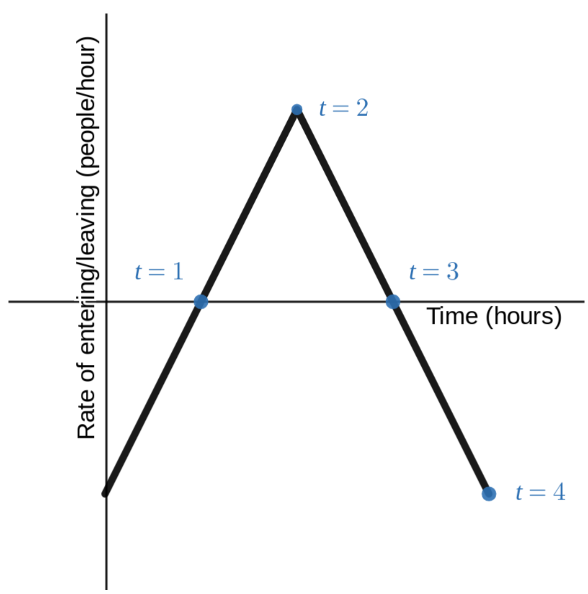
NOTE: You *must* arrive at your answer by interpreting the integral as an area and using geometry. You are not allowed to use antiderivatives or the Fundamental Theorem of Calculus on this Learning Target, even if it is possible and easy to do so. (Of course you can *check* your work with the FTC if you want.) Any use of antiderivatives or the FTC as the primary means of getting an answer will result in the work not meeting the standards.

1. $\int_{-2}^{1} (12 - 5x)\, dx$
2. $\int_{-3}^{3} 5 - \sqrt{9 - x^2}\, dx$
3. $\int_{1}^{6} |x - 3|\, dx$

(Note on the third integral: The vertical bars indicate the **absolute value** function. To get this on Desmos, just enter in vertical bars (above the back slash on your keyboard) or the text "abs"; for example `y = abs(x)` gives the function $y = |x|$.)

Learning Target 15: *I can explain the meaning of each part of the definition of the definite integral in terms of a graph, and interpret the definite integral in terms of areas, net change, and displacement.*

The graph below shows the rate at which people enter and leave the GVSU library, in people per hour, over a four-hour period ($0 \le t \le 4$). A positive rate at a particular time means that more people are entering than leaving at that moment; a negative rate means more people are leaving than entering. (Assume that there are a significant number of people already in the library at time $t = 0$.) Note the scaling on the vertical axis is not shown. However, four points in time are labelled on the graph.



1. Suppose the function shown here is $r(t)$. In specific, clear, and simple language, what does the definite integral $\int_{0}^{3} r(t)\, dt$ tell you in terms of people in the library?
2. At approximately what time during this 4-hour period was the number of people in the library at its maximum point? State your answer clearly then explain your reasoning.
3. How does the number of people in the library at $t = 4$ compare to the number of people in the library at $t = 0$? Are there more people in the library at $t = 4$, are there fewer people at $t = 4$, or is the number of people roughly the same? State your answer clearly then explain your reasoning.

Learning Target 16 (Core): *I can find antiderivatives of a function and evaluate a definite integral using the Fundamental Theorem of Calculus.*

Find the exact value of each of the following definite integrals by using the Fundamental Theorem of Calculus (not geometry or Riemann sums).

A correct solution must do the following:

1. Clearly state the antiderivative of the integrand;
2. Show the "Fundamental Theorem step" explained in class and on the discussion board; and
3. Give an **exact value** of the answer with *no decimal approximations* that is **fully simplified**; *and*
4. Give a decimal form of the exact answer that agrees with the exact form to four decimal places.

Two sample solutions of this form are found on Campuswire in a post from April 12. If any of the solutions are missing any of these four items, the work will be considered to not meet the grading standard.

1. $\displaystyle\int_{-2}^{1} \left(x^3 - 2x^2 + 10\right)\, dx$
2. $\displaystyle\int_{1}^{2} \left(\frac{1}{7z} + \frac{\sqrt[3]{z^2}}{4} - \frac{1}{2z^3}\right) dz$
3. $\displaystyle\int_{\frac{2\pi}{3}}^{\frac{\pi}{4}} 3\sin(v) + 8\csc(v)\cot(v)\ dv$