This writeup would probably get a grade of 6. It's quite good and has no mathematical errors, but there are writing and stylistic errors that prevent it from being truly excellent. The lack of "why" explanations is the main culprit.

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MTH 201-02
Solution for Section 1.1, Exercise 1

A bungee jumper dives from a tower at time $t = 0$. Her height h (measured in feet) at time t (in seconds) is given by the graph in Figure 1.3 on page 7 of the textbook and by the function $s(t) = 100\cos(0.75t) \cdot e^{-0.2t} + 100$.

(a) What is the change in vertical position of the bungee jumper between $t = 0$ and $t = 15$?

Solution:

$$\begin{aligned} s(15) - s(0) &= (100\cos(0.75 \cdot 15) \cdot e^{-0.2 \cdot 15} + 100) - (100\cos(0.75 \cdot 0) \cdot e^{-0.2 \cdot 0} + 100) \\ &= (100\cos(11.25) \cdot e^{-3} + 100) - (100\cos(0) \cdot e^{0} + 100) \\ &= (100\cos(11.25) \cdot e^{-3} + 100) - (100 \cdot 1 \cdot 1 + 100) \\ &= (100\cos(11.25) \cdot e^{-3} + 100) - (100 \cdot 1 \cdot 1 + 100) \\ &= (100\cos(11.25) \cdot e^{-3} + 100) - 200 \\ &\approx -94.7469 \text{ feet} \end{aligned}$$

Correct math, but no setup or explanation. Also should lose the negative sign in the answer.

(b) Estimate the jumper's average velocity on each of the following time intervals: $[0, 15]$, $[0, 2]$, $[1, 6]$, and $[8, 10]$. Include units on your answers.

Solution: For each interval $[a, b]$, we calculate the change in distance $s(b) - s(a)$ divided by the change in time $b - a$:

$$\begin{aligned} \text{Average velocity on [0,15]} &= \frac{s(15) - s(0)}{15 - 0} \\ &= \frac{-94.7469}{15 - 0} \\ &\approx -6.58313 \end{aligned}$$

Note: The work for $s(15) - s(0)$ is done in part (a). The computations for the remaining intervals proceed similarly:

$$\text{Average velocity on [0,2]} = \frac{s(2) - s(0)}{2 - 0} = \frac{-95.2583}{2} \approx -47.6292$$

$$\text{Average velocity on [1,6]} = \frac{s(6) - s(1)}{6 - 1} = \frac{-66.2547}{2} \approx -13.2509$$

$$\text{Average velocity on [8,10]} = \frac{s(10) - s(8)}{10 - 8} = \frac{-14.6943}{2} \approx -7.3472$$

Better job of setup/explanation but no units on the answers.

(c) On what time interval(s) do you think the bungee jumper achieves her greatest average velocity? Why?

Solution: The greatest average velocity would occur on the interval where the distance changes the most over the shortest period of time. From the graph, this appears to be the interval $[0, 4]$.

(d) Estimate the jumper's instantaneous velocity at $t = 5$. Show your work and explain your reasoning, and include units on your answer.

Solution: First, recall from the discussion in the textbook that the average velocity on the interval $[a, a + h]$ is given by

$$AV_{[a,a+h]} = \frac{s(a+h) - s(a)}{h}$$

In this problem, $a = 5$, and h is some small nonzero number. We first note that

$$s(5) = 100\cos(0.75 \cdot 5) \cdot e^{-0.2 \cdot 5} + 100 = 100\cos(3.75) \cdot e^{-1} \approx 69.8133$$

Now we will use the average velocity formula above to find the average velocities from $t = 5$ to $t = 5 + h$ for increasingly small values of h. We start arbitrarily with $h = 0.25$:

$$AV_{[5,5.25]} = \frac{s(5.25) - s(5)}{0.25} = \frac{75.5171 - 69.8133}{0.25} = \frac{5.70376}{0.25} \approx 22.815 \text{ feet per second}$$

Now we continue similar calculations for values of h that are approaching, but not equal to zero:

Value of h	Average velocity from $t = 5$ to $t = 5 + h$
0.1	22.2523 feet per second
0.01	21.8543 feet per second
0.001	21.812 feet per second
0.0001	21.8077 feet per second

Based on this information, we can estimate that the instantaneous velocity of the jumper at time $t = 5$ is approximately 21.8 feet per second.

(e) Among the average and instantaneous velocities you computed in earlier questions, which are positive and which are negative? What does negative velocity indicate?

Solution: All of the average velocities in part (b) were negative, while the instantaneous velocity in part (d) was positive. The negative velocity indicates a downward motion.

Writer doesn't explain WHY the negative velocity should indicate downward motion. Note that the problem does not explicitly ask for this explanation, but correct explanations should always explain WHY even if not explicitly prompted.