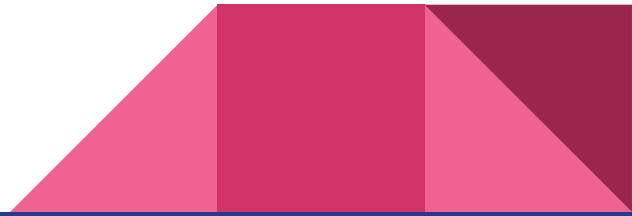
The derivative of a function at a point

MTH 201 -- Module 2A

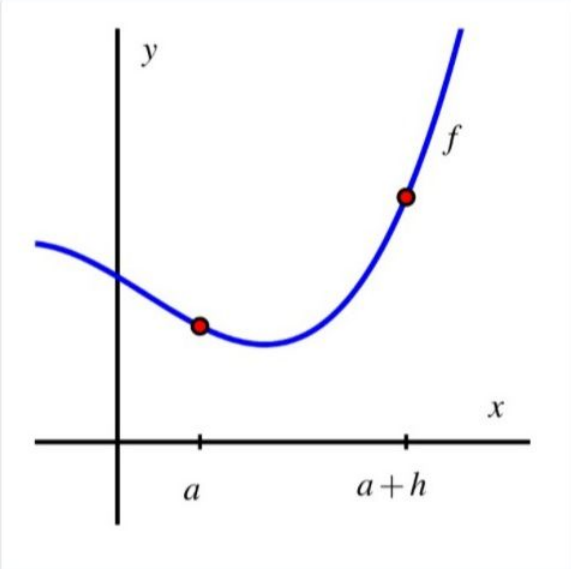
Today

- Activity + debrief: What the derivative is, and why we care
- Activity: Working with the derivative conceptually
- Activity (time permitting): Finding derivatives using the definition



What is the derivative?

The slope of the line that connects the two red points in this graph is the same thing as



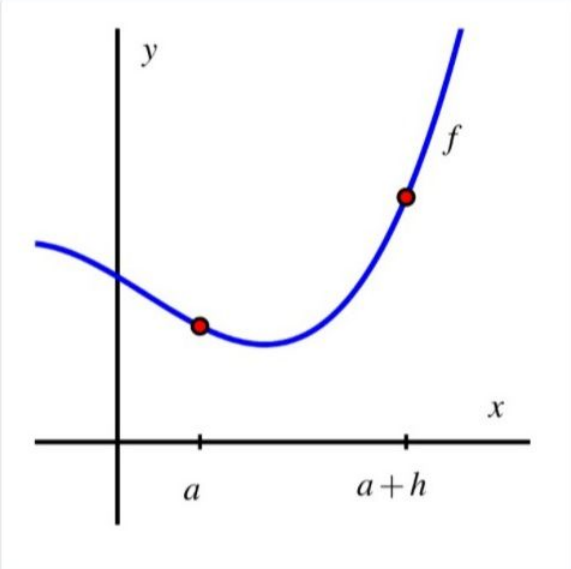
The instantaneous rate of change in f(x) at x = a+h

The instantaneous rate of change in f(x) at x = a

The average rate of change in f(x) from x = a to x = a+h

The average rate of change in f(x) from x = 0 to x = a

The slope of the line that connects the two red points in this graph is



1

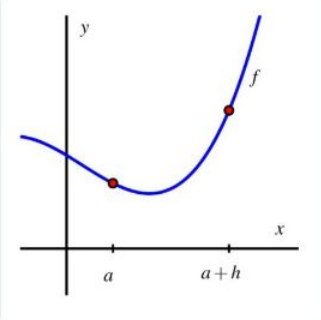
$f(a+h)$

$f(a+h)-f(a)$

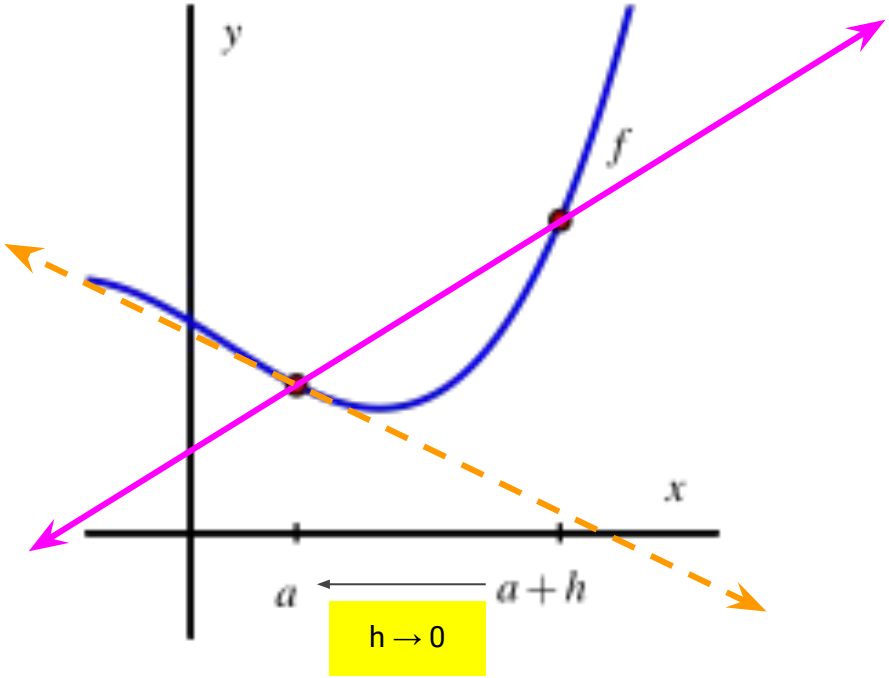
$\frac{f(a+h)}{h}$

$\frac{f(a+h)-f(a)}{h}$

To get the *derivative* of this function at $x = a$, we would need to take the slope of the line that connects the two red dots, and then



- Divide by a
- Take the limit as $a \to 0$
- Take the limit as $h \to 0$
- Set it equal to 0 and solve for a
- Nothing -- the derivative is the same thing as the slope of that line



Average rate of change = slope of the secant line between (a, f(a)) and (a+h, f(a+h))

$$\frac{f(a+h)-f(a)}{a+h-a} = \frac{f(a+h)-f(a)}{h}$$

Instantaneous rate of change = the number that the average rates of change approach as the interval shrinks

$$\lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$$

The **derivative** of the function y = f(x) at the point x = a:

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

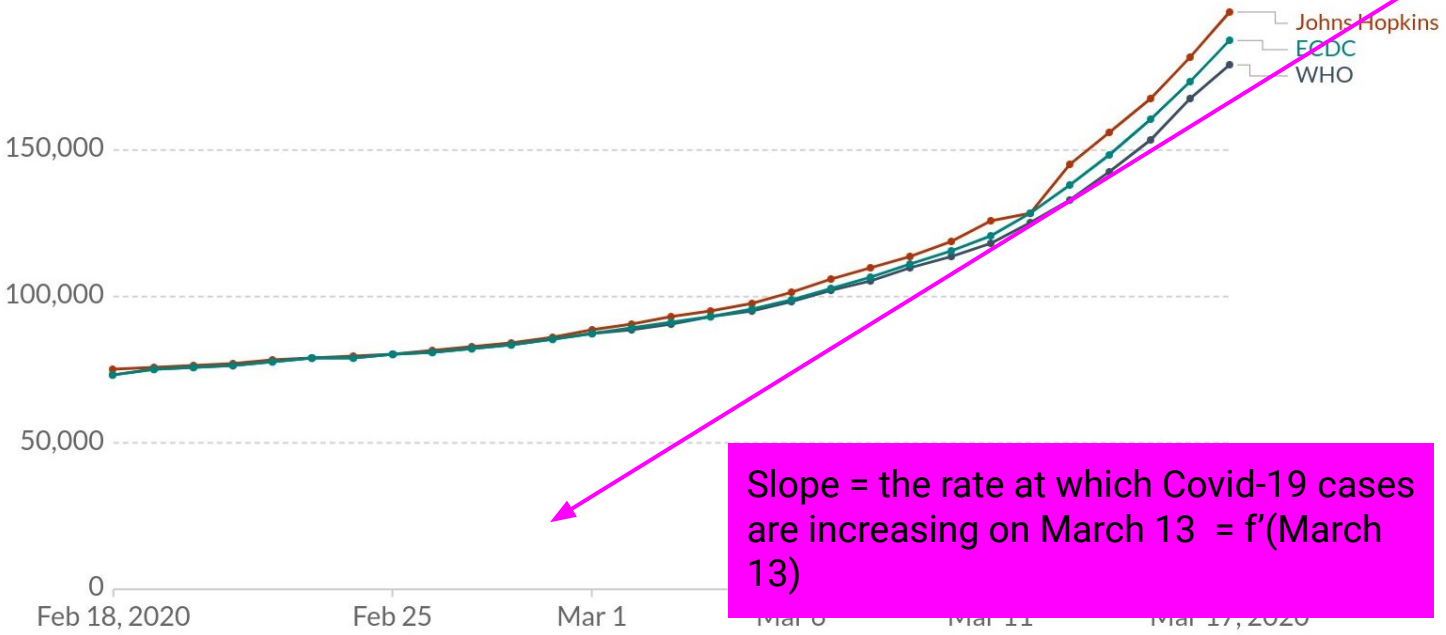
The derivative of f(x) at x = a, f'(a), is all of the following things:

- The **instantaneous rate of change** in f(x) at x = a
- If f(t) is a position at time t, f'(a) is the **instantaneous velocity** at time t = a
- The **slope of the tangent line** to the graph of f(x) at x = a

Total confirmed COVID-19 cases, by source, World

Our World in Data

Confirmed COVID-19 cases are compared for the three main data sources:
– Johns Hopkins University;
– World Health Organization (WHO) Situation Reports;
– European Centre for Disease Prevention and Control (ECDC)

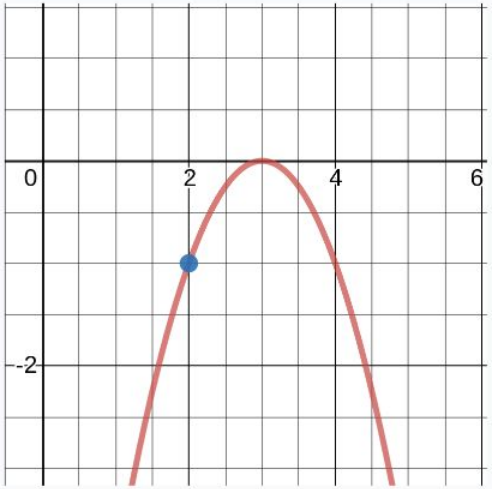


Source: Johns Hopkins (2020); WHO COVID-2019 Situation Reports; ECDC (2020)
Note: The number of confirmed cases is lower than the number of total cases. The main reason for this is limited testing.

Jan 22, 2020 — Mar 17, 2020

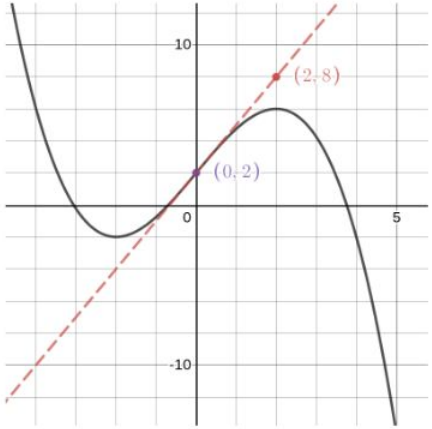
Working with the derivative concept

Here's the graph of a function g(x) and the point on its graph at x = 2. Which of the following are true statements? Select ALL that apply.



- $g(2)$ is positive
- $g(2)$ is negative
- $g'(2)$ is positive
- $g'(2)$ is negative

A function $y = A(x)$ is shown along with its tangent line at $x = 0$ and the points $(0, 2)$ and $(2, 8)$. Given this information, we can conclude



- $A(0) = 2$ and $A'(0) = 8$
- $A(0) = 2$ and $A(0) = 8$
- $A(0) = 2$ and $A'(0) = 6$
- $A'(0) = 2$ and $A'(2) = 6$
- $A(0) = 6$ and $A'(0) = 8$
- None of the above

To 0

A function $y = P(x)$ tells you the number of milligrams (y) of lead in x milliliters of water in a city's water supply. The units of measurement of $P'(100)$ would be

- Milligrams
- Milliliters
- Milligrams per milliliter
- Milliliters per milligram
- None of the above

Finding derivative values using the definition

Save for tomorrow if low on time

Recap

The derivative of a function y = f(x) at a point x = a:

- Is denoted f’(a) (“f prime of a”)
- Gives the instantaneous rate of change in f(x) at x = a
- Gives the instantaneous velocity of an object at time t = a if f(t) is position
- Gives the slope of the tangent line to the graph of f(x) at x = a
- Can be computed algebraically using a limit (below) if f(x) has a formula
- Has units equal to (Units of y) “per” (Units of x)

Next Up

- Day 2 of Module 2A -- Average rates of change and using them to find instantaneous rates of change
- No Daily Prep for day 2 on any module
- See calendar and Week 2 Guide for all due items and a schedule
- **Check Campuswire 1-2x per day to stay up to speed and not miss things**

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

Feedback:
http://gvsu.edu/s/1zJ