MTH 201: Calculus

Module 2A: The derivative of a function at a point

Prof. Talbert

GVSU

August 4, 2020

Agenda for today

- Review of Daily Prep assignment, and Q+A

Agenda for today

- Review of Daily Prep assignment, and Q+A
- Short lecture: Computing a derivative with limits

Agenda for today

- Review of Daily Prep assignment, and Q+A
- Short lecture: Computing a derivative with limits
- Practice with computing derivatives using limits

Agenda for today

- Review of Daily Prep assignment, and Q+A
- Short lecture: Computing a derivative with limits
- Practice with computing derivatives using limits
- Polling activity on the concept of the deriative

Agenda for today

- Review of Daily Prep assignment, and Q+A
- Short lecture: Computing a derivative with limits
- Practice with computing derivatives using limits
- Polling activity on the concept of the deriative
- For next time: Followup activities and things to do

Review and Q+A

Go to `www.menti.com` and use code ?

Computing derivatives

The definition of the derivative

Let f be a function and $x = a$ a value in the function's domain. We define the **derivative of f with respect to x at evaluated at $x = a$**, denoted $f'(a)$, by the formula

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

provided the limit exists.

What the formula means:

- $\frac{f(a+h)-f(a)}{h}$ is the *average rate of change* in f on an interval starting at $x = a$ and ending at $x = a + h$ (h is the length of the interval)
- $\lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$ is what happens to those average rates as the length of the interval shrinks to 0.

Example

Example

Let $f(x) = x^2 - 2x + 1$. Find the value of $f'(2)$ using the definition.
(See whiteboard for solution)

Example

Example

Let $f(x) = x^2 - 2x + 1$. Find the value of $f'(2)$ using the definition. (See whiteboard for solution)

$$\begin{aligned} f'(2) &= \lim_{h\to 0} \frac{f(2+h) - f(2)}{h} \\ &= \lim_{h\to 0} \frac{((2+h)^2 - 2(2+h) + 1) - (2^2 - 2(2) + 1)}{h} \\ &= \lim_{h\to 0} \frac{(4 + 4h + h^2 - 4 - 2h + 1) - (1)}{h} \\ &= \lim_{h\to 0} \frac{2h + h^2}{h} \\ &= \lim_{h\to 0} (2 + h) \\ &= 2. \end{aligned}$$

Desmos: Does the answer make sense?

Bonus: $f'(1)$

Replace all the 2's with 1's, basically

$$\begin{aligned}
f'(1) &= \lim_{h\to 0} \frac{f(1+h) - f(1)}{h} \\
&= \lim_{h\to 0} \frac{((1+h)^2 - 2(1+h) + 1) - (1^2 - 2(1) + 1)}{h} \\
&= \lim_{h\to 0} \frac{(1 + 2h + h^2 - 2 - 2h + 1) - (0)}{h} \\
&= \lim_{h\to 0} \frac{h^2}{h} \\
&= \lim_{h\to 0} h \\
&= 0.
\end{aligned}$$

Desmos: Does the answer make sense?

In groups

Let $f(x) = 3 - 2x$.

1. Set up the limit that would compute $f'(5)$.
2. Before calculating the limit, go to Desmos and graph f. What *should* the value of $f'(5)$ be, and why?
3. Now go through the limit computation step by step with your partner(s). Does your result verify your guess?

Bonus practice

This will appear in your follow-up activity. If you have time, you can get started here.

Velocity

A water balloon is tossed vertically in the air from a window. The balloon's height in feet at time t in seconds after being launched is given by $s(t) = -16t^2 + 16t + 32$.

- Set up the limit that will compute the instantaneous velocity of the balloon at time $t = 1$.
- Graph $s(t)$ on Desmos and estimate what the value of the instantaneous velocity should be.
- Now compute the limit you set up to find the *exact* value of the velocity.

Next

All due dates are on the Course Calendar

- Complete Followup Activities