AEP 5: Applying derivative rules to real data

Overview

In this AEP, you'll construct regression models
for a data set, use the techniques of Chapter 2 to find the first and second derivatives of the data at a point, and then explain what the derivative information tells you about how the relationship is being affected by change.

Learning Targets associated with this AEP:

- **D.2 (CORE)**: I can use derivative notation correctly, state the units of a derivative, estimate the value of a derivative using difference quotients, and correctly interpret the meaning of a derivative in context.
- **D.3 (CORE)**: Given information about f, f', or f'', I can correctly give information about f, f', or f'' and the increasing/decreasing behavior and concavity of f (and vice versa).
- **DC.1 (CORE)**: I can compute derivatives correctly for power, polynomial, and exponential functions and the sine and cosine functions, and basic combinations of these (constant multiples, sums, differences).

Technology Background

For ths Miniproject, you will need to know how to **plot data points** in Desmos and how to **fit data with polynomial and exponential regressions** in Desmos. These procedures are explained in AEP 1 and AEP 2. (You do not need to complete those AEPs in order to do this one, however.)

AEP Description and Tasks

What this AEP is about

Data sets that involve two variables that are linked together somehow are unsurprisingly called *two-variable data set*. An example of a two-variable data set would be the Census Bureau population data for a city, where the population of a city is linked with the year when the population is measured. There is an implied relationship between one variable and the other. For example, in the population data, there is an implied relationship between time and population – as time passes, population changes. Time is like the input variable to a function, and population is like the output; we call time the **independent variable** and population the **dependent variable** because population depends on time.

AEP 5 is about modeling a two-variable data set in the real world with a function, then using the rules of differentiation that we've learned to analyze and understand the change behavior of the model on a deep level, and interpret the meaning of that behavior to a general audience.

Setup (do this first)

For this Miniproject, you are to go out onto the Internet and find a two-variable data set and analyze it. You can use any kind of data you wish, as long as:

- It doesn't have to do with population and time, because that's been don already (it's the subject of AEP 1).
- The set has at least five pairs of data in it.
- The data set is real (not made up) and you can provide a link to a reference for it.
- The data set has some kind of personal relevance to you. Otherwise this assignment won't be as interesting.
- There must be some kind of potential relationship between the data. For example if you gather information about the temperature in Tampa, Florida on a given day and the stock value of Apple on that day, it's highly unlikely that those will be related, so the model you build later on will be irrelevant.
- **Most importantly:** The data must be *numerical*. By "numerical", we mean data that are measured using numbers that are not just labels. For example, the number of students at GVSU, the number of Covid-19 cases reported on a given day, and the temperature on Mars are all numerical data; but data like names, countries, and so on are not numerical data. **Some data that use numbers are not numerical data:** For example ZIP codes, phone numbers, and rankings (first place, second place, etc.) might *use* numbers but they are not actually numerical data — the numbers are just labels and have no mathematical meaning. **Avoid data like that.** (In statistics, we'd say that we want only *interval* or *ratio* data, not *nominal* or *ordinal* data.)

Also, please use a different data set than any of the ones you have used on earlier AEP's, if you've done any earlier AEP's.

Please note that submissions in which the data don't meet the above requirements will be returned with a grade of "N".

Place your data into a spreadsheet, such as Excel or a Google Spreadsheet with the independent variable in Column A and the dependent variable in Column B. **Note:** If the numbers in your data are large, you should consider rescaling your data using some sensible method. For example, if you are looking at household incomes that range from $50,000 to $500,000, then consider measuring your incomes instead in "tens of thousands" and let the data range from 5 to 50 instead.

Then copy and paste your data from the spreadsheet into a Desmos page. This will tell Desmos to produce a data table, and the data will be plotted in Desmos with the zooming level set appropriately.

At this point you're ready to begin the Miniproject tasks.

Tasks for this AEP

1. (This is a warmup exercise to make sure you are familiar with the basic concepts of in this miniproject.) Suppose that $V(t) = 24 \cdot 1.07^t + 6\sin(t)$ represents the value of a person's investment portfolio (in thousands of dollars) in year t, where $t = 0$ corresponds to January 1, 2010.

   (a) At what instantaneous rate is the portfolio's value changing on January 1, 2012? Do your work by hand; show all your work, and put correct units of measurement on the answer.

   (b) Calculate the value of $V''(2)$. What are the units on this quantity? What does the sign (positive/negative/zero) of this quantity tell you about how the portfolio's value is changing? Show all your work by hand, and use clear and correct English.

2. Now go out to the web and find a set of two-variable data.

   (a) Write a paragraph that includes a link to the source of your data. Use the paragraph to explain what the data represent, why you found the data interesting enough to use for this AEP, and who might find these data important in a real-world job.

   (b) On Desmos, plot the data points (by entering them into a spreadsheet and then copy/pasting them as described above). Then fit the data with a third-degree polynomial regression, a fourth-degree polynomial regression, and an exponential regression. When done, give a link to your graphs.

   c) Give the precise formulas for each of your regression models. The values of the parameters are given on the graph. For example, look at the sample results here: https://goo.gl/0U8atf. The quadratic model has three parameters: a, b, and c; their values are shown where it says "Parameters". This means that the formula for this model is

   $$y_1 = 57.689x_1^2 - 7594.8x_1 + 244380$$

   For each of your three models, use the techniques we learned in Chapter 2 of *Active Calculus* to calculate both the first and second derivatives of each of the three models.

3. Look through the four models you created and find the one that has the highest R^2 value. This number R^2 (it might be lower case, r^2) is a statistic that measure how well the model fits the data. The closer to 1 this number is, the better the fit. Once you identify the model that has the best fit, choose a data point in your data set and give the values of the first and second derivatives at that point. In 1–3 well-constructed sentences, explain exactly what these two numbers tell you about how your data set is changing instantaneously right at that point. **This explanation is to be targeted toward a person who is smart, but does not know a single thing about calculus**. Therefore you are forbidden to use jargon such as the terms *function*, *derivative*, *tangent line*, and so on (that list is not all-inclusive). I (Prof. Talbert) will act as your secretary and return your memo to you for editing if it gets too jargony. The words *data* and *graph* are fairly common and therefore OK. As a starting point for your explanations you might consider the concepts of a function "increasing at an increasing rate", "increasing at a decreasing rate", and so on – but don't rely too much on these stock phrases but rather make sure to give the memo recipient usable, actionable information that he/she can

understand.

Assignment Expectations and Grading Criteria

AEPs are graded using the "EMRN" rubric found in the syllabus. Please note, it is the case with all AEP's that **your grade is primarily based on your explanations and writing, and only secondarily on the precision and correctness of your computations.** Correct computations with insufficient explanation will need to be revised and may receive an "N" grade.

Also, **significant incompleteness will result in a grade of "N".** This includes the following:

- **Giving answers with no explanations.** As mentioned above, you are being graded on explanations and writing, not so much on answers. Submissions that contain items where there is an answer with no explanation or insufficient explanation, will be graded "N" and returned without comment.
- Leaving items blank (even accidentally)
- Leaving large gaps in computations (skipping important steps)
- Giving only partial attempts at tasks (for example, working down to a certain point in a solution and then stopping because you need help)

A grade of E or M requires all of the following to be met:

- All the links you provide work.
- The explanation memo in the final task must be substantive and appropriately targeted to the audience described there, and it must be professionally written using correct spelling and grammar.
- All work needs to be shown *and* your thought processes clearly expressed in all of the tasks of the assignment. The results also need to be correct.
- All the information needed for an "outsider" to understand your work needs to be self-contained within the work. **The reader should not have to do any work to fill in gaps.**
- Explanations must be given in clearly written and grammatically correct English. Multiple instances of failure to capitalize beginnings of sentences, subject-verb agreement, misuse of punctuation, etc. will result in a grade of R or N.
- Some simple mistakes in calculations are allowed, but significant errors and those that lead to incorrect explanations will probably result in a grade or R or N.

There are some additional formatting requirements in the "Submitting your work" section below.

A grade of "E" is given if all of the above expectations are met, and the work is of superior quality in terms of the clarity of explanations and work, appearance of the writeup, and precision of the mathematics.

Submitting your work

AEP submissions must be typewritten and saved as either a PDF or MS Word file. No part of your submission may involve handwriting; work that is submitted that contains handwriting will be graded N

and returned without feedback. This includes electronic handwritten documents, for example using a stylus and a note-taking app. To type up your work, you can use MS Word or Google Docs (both of which have equation editors for mathematical notation) or any other computer-based math typesetting tool. Just make sure you save your work as a Word document or PDF (no `.odt`, `.rtf`, or other file extensions are allowed).

When you have your work typed up, double-check it for neatness, correctness, and clarity. Then, go to Blackboard, then **Assignments**, then **AEP**, then **AEP 5**. Clicking on the text "AEP 5" will take you to a place on Blackboard where you can upload your work. All grading and revisions of labs are done entirely on Blackboard. **Do not email your work to the professor** – only Blackboard submissions are accepted.

Getting Help

Please note that according to the syllabus, for AEP's **"no interactions at all with another person or with unauthorized sources on the internet is allowed."** Violations of this rule include *any* consultation with other students or former students, including Math Center tutors; using work from another student or former student; submitting the problem set to an online help site such as Chegg or Coursehero; or asking for help in an online forum. All such violations will be treated as academic dishonesty and will result in a grade of "N" and being banned from revising the work.

You **may** ask me (Talbert) for help on this assignment in the form of **specific mathematical or technical questions**. If I cannot answer a question because it would give too much away, I'll tell you so. **However please note: I will not "look over your work" before you submit it to give you feedback on the overall submission**; the expectations are clearly laid out above, so just follow those directions and submit your best work, and you'll be allowed to revise it if needed.

You can ask technology related questions to anyone at any time. For example if you need help with Desmos, or with figuring out how to type up your work, there are no restrictions on that. I recommend the `#tech` channel on Campuswire so that you'll reach a large audience.