

Class Activities: Velocity

Today we'll start by working through parts of Activity 1.1 in Section 1.1, followed by a quick debrief. Then we'll do the following, which is a remixed version of Activity 1.3.

Consider the function $s(t) = 64 - 16(t - 1)^2$, which gives the position of an object at time t .

1. Calculate the value of $s(2)$.

$$s(2) = 64 - 16(2-1)^2 = 64 - 16 = 48.$$

2. Calculate the expression $s(2 + h)$ and simplify completely.

$$\begin{aligned} s(2+h) &= 64 - 16((2+h)-1)^2 && \rightarrow = 64 - 16h^2 - 32h - 16 \\ &= 64 - 16(h+1)^2 && = -16h^2 - 32h + 48. \\ &= 64 - 16(h^2 + 2h + 1) \end{aligned}$$

3. Calculate the value of $s(2 + h) - s(2)$ and simplify completely, including factoring out any common factors.

$$\begin{aligned} s(2+h) - s(2) &= -16h^2 - 32h + 48 - 48 \\ &= -16h^2 - 32h. \end{aligned}$$

4. Set up and simplify the expression for the average velocity of an object with position $s(t)$ on the interval $[2, 2 + h]$. The result should be an expression whose only variable is h , and there should be no fractions in the result.

$$\begin{aligned} AV_{[2, 2+h]} &= \frac{s(2+h) - s(2)}{h} = \frac{-16h^2 - 32h}{h} \\ &= \frac{\cancel{h}(-16h - 32)}{\cancel{h}} = -16h - 32. \end{aligned}$$

✂

$$h = 0.5$$

$$h = -0.1$$

5. Use the result of (d) to find the average velocity of an object with position $s(t)$ on the interval $[2, 2.5]$ and then on the interval $[1.9, 2]$. (Hint: What is the value of h each time?)

$$AV_{[2, 2.5]} = -16(0.5) - 32$$

$$= -8 - 32$$

$$= -40$$

$$AV_{[1.9, 2]} = -16(-0.1) - 32$$

$$= 1.6 - 32$$

$$= -30.4$$

6. In your own words, what is happening as h approaches 0?

As h approaches 0, the time interval $[a, a+h]$ is getting shorter and shorter. So the average velocity is getting closer and closer to instantaneous velocity.

7. Use the result of (4) to find the *instantaneous* velocity of an object with position $s(t)$ at $t = 2$. What does the sign (positive/negative) of the answer indicate? And what are the units of the answer?

Take the average velocity and let h go to 0:

$$\begin{aligned} -16h - 32 &\longrightarrow -16(0) - 32 \\ &= -32 \text{ ft/s} \end{aligned}$$

8. How would you adjust the above processes if you were using the same position function for s but wanted to find the instantaneous velocity at $t = 1$? And by just looking at the graph, what do you think is the value of the instantaneous velocity at $t = 1$?

Calculate $\frac{s(1+h) - s(1)}{h}$, simplify, then let h go to 0.



What was the muddiest point from today's class?