

Class Activities: The Product and Quotient Rules

Get into groups of 2–4 and work through all of the following activities. These are not to be turned in, and they will not be graded. Instead, record your group’s work on your copy and keep it for notes. I will be coming to each group one by one as you work to observe what you’re doing, answer questions, and catch any misconceptions that are happening. We will stop with about 10 minutes remaining to debrief the main ideas.

1 Focus questions

- If f and g are differentiable functions, then

$$\frac{d}{dx} [f(x) \cdot g(x)] =$$

and

$$\frac{d}{dx} \left[\frac{f(x)}{g(x)} \right] =$$

- For any real number c , if $f(x) = c$ for all x then $f'(x) =$ _____.
- For any nonzero real number n , if $f(x) = x^n$, then $f'(x) =$ _____.
- For any positive real number a , if $f(x) = a^x$, then $f'(x) =$ _____.
- For any real number k , if $f(x)$ is a differentiable function with derivative $f'(x)$, then $\frac{d}{dx} [k \cdot f(x)] =$ _____.
- If $f(x)$ and $g(x)$ are differentiable functions with derivatives $f'(x)$ and $g'(x)$ respectively, then $\frac{d}{dx} [f(x) + g(x)] =$ _____.
- $\frac{d}{dx} [\sin(x)] =$ _____
- $\frac{d}{dx} [\cos(x)] =$ _____

2 Computation practice

Use the rules we learned in this section to determine the derivative of each of the following functions. For each, state your answer using full and proper notation, labeling the derivative with its name. For example, if you are given a function $h(z)$, you should write “ $h'(z) =$ ” or “ $\frac{dh}{dz} =$ ” as part of your response. **Do not simplify your results algebraically for now.**

- Let $m(w) = 3w^{17}4^w$. Find $m'(w)$.

Continued \rightarrow

- Let $h(t) = (\sin(t) + \cos(t))t^4$. Find $h'(t)$.

- Let $r(z) = \frac{3^z}{z^4 + 1}$. Find $r'(z)$.

- Let $v(t) = \frac{\sin(t)}{\cos(t) + t^2}$. Find $v'(t)$.

3 Computation without complete information

Suppose that $f(x)$ and $g(x)$ are differentiable functions and that we know the following data about f and g :

$$f(3) = -2 \quad f'(3) = 7 \quad g(3) = 4 \quad g'(3) = -1$$

- Let $p(x) = f(x) \cdot g(x)$. Find the value of $p'(3)$. (*Guidance:* What would be a formula for $p'(x)$, for any x value?)

- Let $q(x) = \frac{f(x)}{g(x)}$. Find the value of $q'(3)$.

Continued \rightarrow

4 Applications

1. Determine the equation of the tangent line to the graph of $f(x) = e^x \sin x$ at $x = 1$.

2. When a camera flashes, the intensity I of light seen by the eye is given by the function

$$I(t) = \frac{100t}{e^t}$$

where I is measured in “candles” and t is measured in milliseconds. Compute $I'(0.5)$, $I'(2)$, and $I'(5)$. State the units of these quantities and explain the meaning of each.

✂

What was the least clear point from today’s class?