AEP 6: Exploring logistic functions

Overview

This AEP puts some of our derivative rules to work in exploring a class of functions used in population and financial modeling.

Learning Targets associated with this AEP:

- **DC.1 (CORE)**: I can compute derivatives correctly for power, polynomial, and exponential functions and the sine and cosine functions, and basic combinations of these (constant multiples, sums, differences).
- **DC.2 (CORE)**: I can compute derivatives correctly for products, quotients, and composites of functions.
- **DC.3**: I can compute derivatives correctly using multiple rules in combination.
- **DA.2**: I can determine the intervals of concavity of a function and find all of its points of inflection.

Technology Background

For this AEP, you'll need to know how to graph functions on Desmos. You'll also need to know how to type up professional-looking mathematics in a word processor, including fractions and exponents; you'll get some tutorials on this but it will be up to you to learn how to use the tools.

AEP Description and Tasks

What this AEP is about

A lot of real-world quantitative situations involve data that behave like this:

- The data start with small values
- There is initially rapid, exponential growth
- However at some point, the data stop growing exponentially — the values continue to grow, but approaching some upper limit.

This happens, for example, when studying a population living in an environment with outside constraints like predators or physical limitations; or a university that experiences initially fast enrollment growth but then the enrollment growth slows as limitations on dorm spaces and classroom facilities are reached.

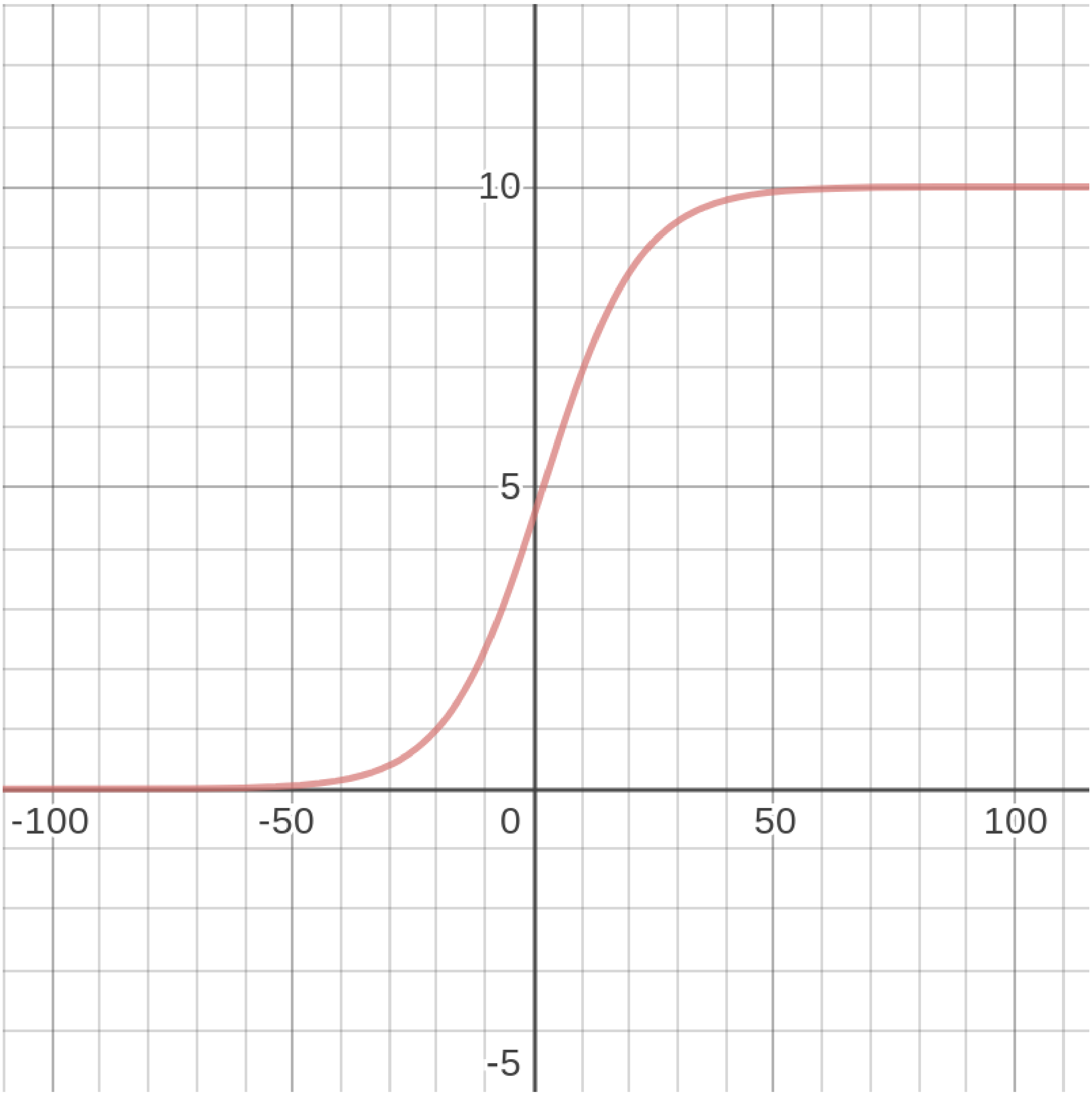
A class of functions that models behaviors like this is called **logistic functions**. Logistic functions all have the same form:

$$f(x) = \frac{L}{1 + e^{-k(x-a)}}$$

In this function,

- x is the input variable; it often represents time
- L is the upper limit that the data approach as x gets very large (L must be positive)
- k is a growth rate (k must also be positive)
- a is a horizontal shift value
- e is the usual number, $e \approx 2.71828...$

For example, here's the logistic function $f(x) = \frac{10}{1+e^{-0.1(x-2)}}$:



Notice the "S" shape of the curve. This shape is the distinctive feature of logistic functions, and the reason they are so widely used in modeling real data.

Here's a Desmos graph that allows you to play with sliders to see the effects of changing the parameters in a logistic function: https://www.desmos.com/calculator/kf3dod6jwn You might need to adjust the viewing window to get a good look at the graph, depending on how you set the sliders.

Setup (do this first)

You're going to build your own random logistic function to work with on this AEP, so that we're not all using the same one. Go to Wolfram|Alpha and generate the following:

- A random number between 5 and 50, and let this be L.
- A random number between 0.05 and 0.35, and let this be k.
- A random number between -5 and 5, and let this be a.

To generate a random number between two bounds a and b, just enter "random number between a and b" and hit enter. For example, here's how to generate a random number between 10 and 10000.

Tasks for this AEP

1. List the values of L, k, and a that you generated (and state which is which). Then go to Desmos.com, enter your logistic function with those parameter values, and adjust the viewing window so that the "S" shape is clearly visible. (Here's an example of a logistic function with a bad viewing window; here's the same function in a much better window.) In the writeup, share a link to your Desmos graph.
2. Using the derivative rules of Chapter 2, **find the first derivative of your logistic function**. You'll need to show all the steps on this and simplify completely. To make this neat and professional: First do all the calculus separately and make sure your answer is correct; then type up your work *using correct notation*. For this AEP, "correct notation" means in particular to use actual fractions and exponents, and don't type up the work using basic text input from the keyboard. For example, `20/(1+e^(-0.2(x-2)))` all in one line is not correctly formatted; writing it as $\frac{20}{1+e^{0.2(x-2)}}$ is correct. You'll be given some tutorials for how to do this in Google Docs and Word; if you need help for another app, please ask on Campuswire.
3. Does your logistic function have any critical values? If so, find them. If not, explain why not.
4. Now **find the second derivative of your logistic function**. For this, you can use technology to find the second derivative, but you need to state the second derivative fully simplified and formatted as described earlier. Then **make a second derivative sign chart** for your function and **find the interval on which it's concave up and the interval where it's concave down**, and **find the exact x- and y-coordinate of the inflection point**.
5. Look at the y-coordinate of the inflection point and compare it to the value of L, the upper limit that the function approaches. What do you notice about these two numbers?
6. Finally, think about a real-world situation that your function might model. What is the practical significance of the inflection point in that situation?

Assignment Expectations and Grading Criteria

AEPs are graded using the "EMRN" rubric found in the syllabus. Please note, it is the case with all AEP's that **your grade is primarily based on your explanations and writing, and only secondarily on the precision and correctness of your computations.** Correct computations with insufficient explanation will need to be revised and may receive an "N" grade.

Also, **significant incompleteness will result in a grade of "N".** This includes the following:

- **Giving answers with no explanations.** As mentioned above, you are being graded on explanations and writing, not so much on answers. Submissions that contain items where there is an answer with no explanation or insufficient explanation, will be graded "N" and returned without comment.
- Leaving items blank (even accidentally)
- Leaving large gaps in computations (skipping important steps)
- Giving only partial attempts at tasks (for example, working down to a certain point in a solution and then stopping because you need help)

A grade of E or M requires all of the following to be met:

- All the links you provide work.
- Both derivatives must be correct, fully simplified, and formatted as described above. (Putting your function as a single line of text, like `20/(1+e^(-0.2(x-2)))`, will result in an "R" grade and you'll need to resubmit with the right formatting.
- All work needs to be shown *and* your thought processes clearly expressed in all of the tasks of the assignment.
- All the information needed for an "outsider" to understand your work needs to be self-contained within the work. **The reader should not have to do any work to fill in gaps.**
- Explanations must be given in clearly written and grammatically correct English. Multiple instances of failure to capitalize beginnings of sentences, subject-verb agreement, misuse of punctuation, etc. will result in a grade of R or N.
- Some simple mistakes in calculations are allowed, but significant errors and those that lead to incorrect explanations will probably result in a grade or R or N.

There are some additional formatting requirements in the "Submitting your work" section below.

A grade of "E" is given if all of the above expectations are met, and the work is of superior quality in terms of the clarity of explanations and work, appearance of the writeup, and precision of the mathematics.

Submitting your work

AEP submissions must be typewritten and saved as either a PDF or MS Word file. No part of your submission may involve handwriting; work that is submitted that contains handwriting will be graded N and returned without feedback. This includes electronic handwritten documents, for example using a stylus and a note-taking app. To type up your work, you can use MS Word or Google Docs (both of which have equation editors for mathematical notation) or any other computer-based math typesetting tool. Just make sure you save your work as a Word document or PDF (no `.odt`, `.rtf`, or other file extensions are allowed).

When you have your work typed up, double-check it for neatness, correctness, and clarity. Then, go to Blackboard, then **Assignments**, then **AEP**, then **AEP 6**. Clicking on the text “AEP 6” will take you to a place on Blackboard where you can upload your work. All grading and revisions of labs are done entirely on Blackboard. **Do not email your work to the professor** – only Blackboard submissions are accepted.

Getting Help

Please note that according to the syllabus, for AEP’s **“no interactions at all with another person or with unauthorized sources on the internet is allowed.”** Violations of this rule include *any* consultation with other students or former students, including Math Center tutors; using work from another student or former student; submitting the problem set to an online help site such as Chegg or Coursehero; or asking for help in an online forum. All such violations will be treated as academic dishonesty and will result in a grade of “N” and being banned from revising the work.

You **may** ask me (Talbert) for help on this assignment in the form of **specific mathematical or technical questions**. If I cannot answer a question because it would give too much away, I’ll tell you so. **However please note: I will not “look over your work” before you submit it to give you feedback on the overall submission**; the expectations are clearly laid out above, so just follow those directions and submit your best work, and you’ll be allowed to revise it if needed.

You can ask technology related questions to anyone at any time. For example if you need help with Desmos, or with figuring out how to type up your work, there are no restrictions on that. I recommend the `#tech` channel on Campuswire so that you’ll reach a large audience.