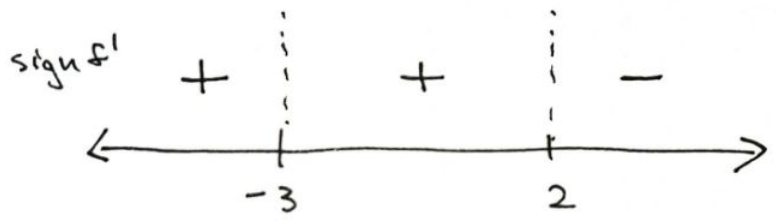
MTH 201 -- Calculus
Module 8B: The Extreme Value Theorem part 2

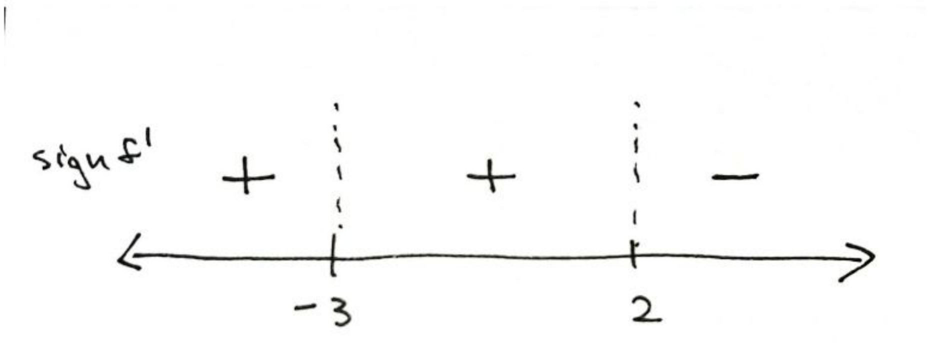
November 2-3, 2020

Here's a partial first derivative sign chart for a function f. The only things this sign chart doesn't show are the test points used, and the behavior of f. What are the critical numbers of f?



- $x = -3$
- $x = 0$
- $x = 2$
- All of the above
- Both $x = -3, 2$ but not $x = 0$
- Not enough information

Here's the partial first derivative sign chart for a function f again. According to the First Derivative Test, [select all that apply]



- f has a local maximum at $x = -3$
- f has a local minimum at $x = -3$
- f has a local maximum at $x = 2$
- f has a local minimum at $x = 2$
- None of the above

To 0

Suppose that the function f from the first two questions is continuous; and on $[-5,5]$ we know that $f(-5)=6$, $f(5)=3$, $f(-3)=2$, and $f(2)=1$. Then according to the Extreme Value Theorem, [select all that apply]

- f has an absolute maximum at $x=-5$
- f has an absolute maximum at $x=-3$
- f has an absolute minimum at $x=-3$
- f has an absolute minimum at $x=2$
- f has an absolute maximum at $x=5$
- None of the above

To 0

Extreme Value Theorem

If f is a continuous function on a closed interval [a,b], then f has both an absolute max and absolute min value on [a,b].

Note 3.3.2. Thus, we have the following approach to finding the absolute maximum and minimum of a continuous function f on the interval $[a, b]$:

1 • find all critical numbers of f that lie in the interval;

2 • evaluate the function f at each critical number in the interval and at each endpoint of the interval;

3 • from among those function values, the smallest is the absolute minimum of f on the interval, while the largest is the absolute maximum.

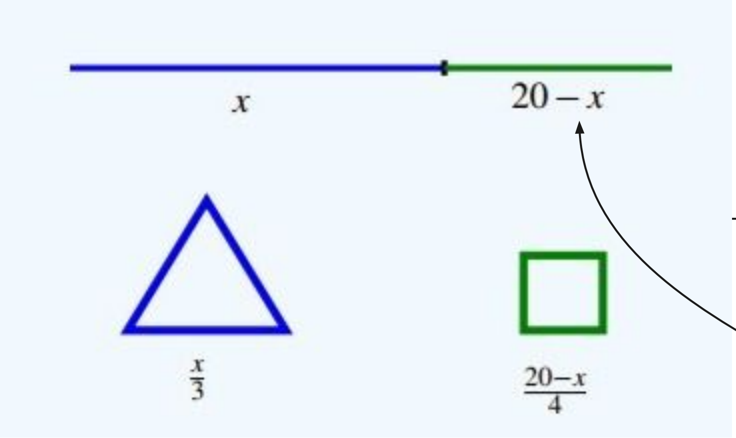
Note: This really simplifies things -- no sign charts necessary for instance

Building mathematical models

How to build a model

1. Examine the situation carefully
2. Draw a diagram
3. Identify the independent and dependent variables
4. Build a formula that relates the variables, based on the diagram -- simplify if needed
5. Turn the formula into a function -- input variable goes in, output comes out
6. Figure out the domain of your function (could be limited because of practical concerns)
7. Graph the function on the domain to examine its behavior

You now have a model that you can do calculus with!



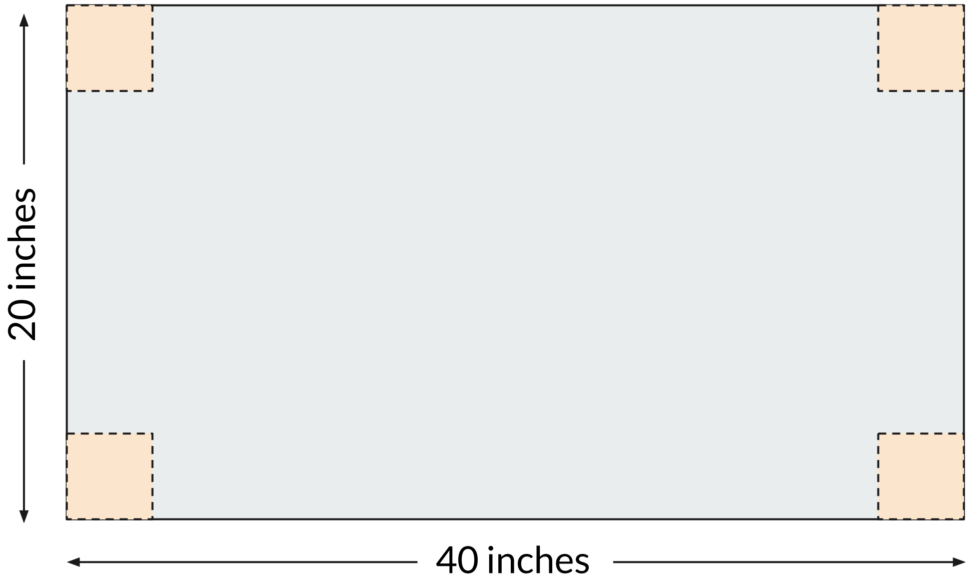
$$A = A_{\triangle} + A_{\square} = \frac{\sqrt{3}x^2}{36} + \left(\frac{20-x}{4}\right)^2.$$

Domain is not all real numbers --- just [0,20] because you only have 20cm of wire available.

Total area is a function of how much wire you give to the triangle / the location of the cut

Formula involved using geometry

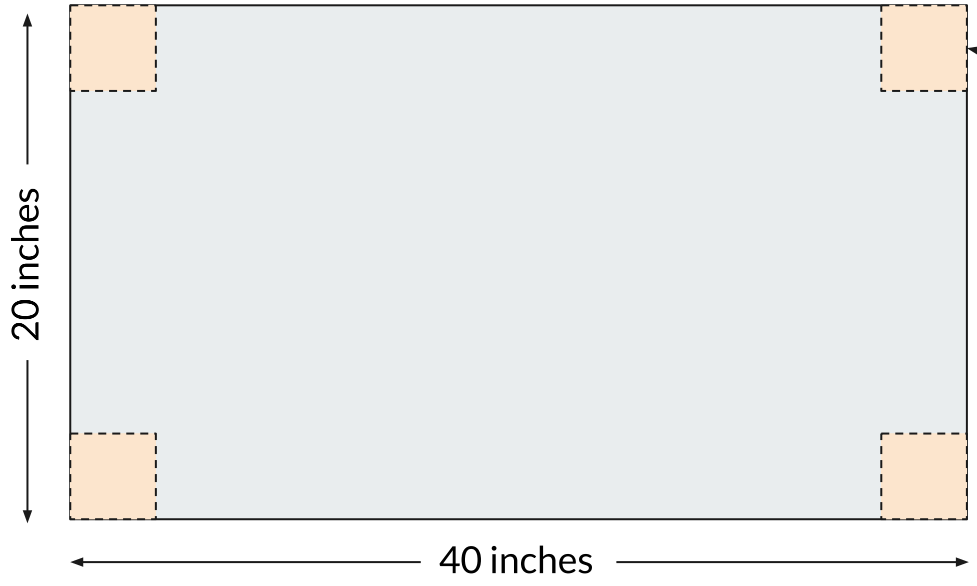
Practice: Optimizing the volume of a box



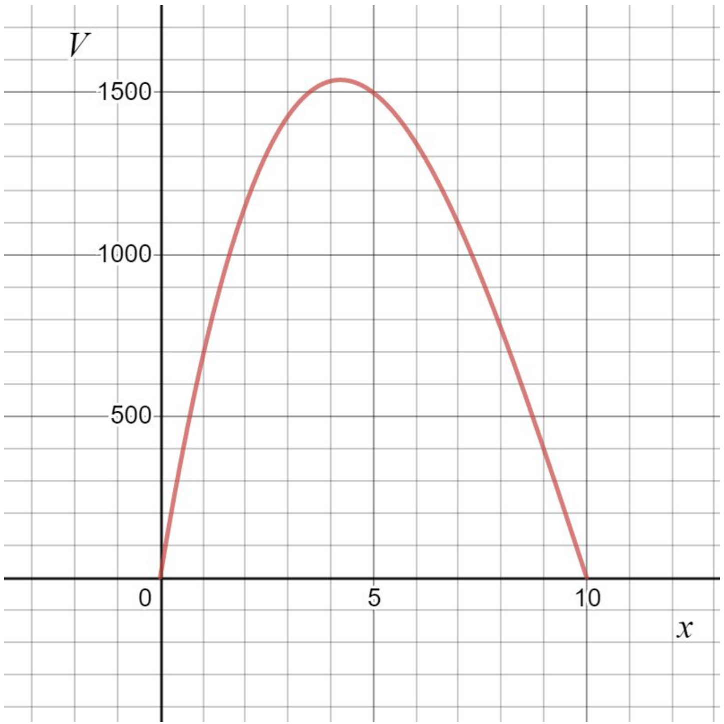
You can build a box by taking a 20in x 40in sheet of cardboard, then cutting off squares of equal size from the corners, then folding up the edges.

How does the volume of the box relate to the size of the squares you cut off?

→ Build a function that relates these two, and find its domain.



What size of square gives us the maximum possible volume?



Alice wants to find the absolute maximum value of $f(x) = -x^3 + 2x + 5$ on $[0, 2]$. So she plugs in $x = 0, 1, 2$ into f to get $f(0) = 5, f(1) = 6, f(2) = 1$ and concludes that the max value happens at $x = 1$.

- There's nothing wrong with this logic, and Alice's answer is correct.
- Alice's answer is correct, but there's a flaw in her logic. (She's correct but only by coincidence.)
- Alice's answer is not correct, and there's a flaw in her logic.

To 0