Name: ____________________________

Instructions: You may use a 3×5 notecard with notes on it as well as a calculator. Except on multiple choice questions, you need to show all work in a clear and complete way to receive credit. The Assessment will end promptly at 12:50pm unless you have made alternate arrangements.

Items 1—10 are multiple choice questions that address a variety of learning objectives. Please circle the ONE response you believe is most correct. You do not need to justify your answer.

1. (2 points) Which of the following statements is always true, for any continuous function f whose domain is the entire real number line (i.e. not confined to a closed interval)?
   (a) If f has a local extreme value at $x = c$, then f has a critical number at $x = c$.
   (b) If f has a critical number at $x = c$, then f has a local extreme value at $x = c$.
   (c) If f is such that $f'(c) = 0$, then $f''(c) = 0$ too.
   (d) All of the above
   (e) Just (a) and (c)

2. (2 points) Which of the following statements is always true, for any continuous function f whose domain is the entire real number line?
   (a) If f has a local extreme value at $x = c$, then f has a global extreme value at $x = c$.
   (b) If f has a global extreme value at $x = c$, then f has a local extreme value at $x = c$.
   (c) If f has a local extreme value at $x = c$, then f has an inflection point at $x = c$.
   (d) All of the above
   (e) Just (a) and (c)

3. (2 points) Suppose f is a function whose derivative is $f'(x) = \dfrac{x^2 - 1}{x^2}$. Then
   (a) f has no critical values
   (b) f has one critical value
   (c) f has two critical values
   (d) f has three critical values
   (e) f has more than three critical values

4. (2 points) Suppose $k(x)$ is a function such that $k'(4) = 0$ and $k(4) = 3$. Then
   (a) The graph of k touches the x-axis at $x = 4$
   (b) k has a local minimum at $x = 4$
   (c) k has a local maximum at $x = 4$
   (d) k has neither a local minimum nor a local maximum at $x = 4$
   (e) There is not enough information present to determine the behavior of k at $x = 4$

5. (2 points) Suppose g is a function such that $g'(3) = 0$ and $g''(3) > 0$. Then
   (a) g has a local minimum at $x = 3$
   (b) g has a local maximum at $x = 3$
   (c) g has an inflection point at $x = 3$
   (d) All of the above
   (e) None of the above

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6. (2 points) Which of the following is an antiderivative for the function $f(x) = x + \dfrac{1}{x}$?
   (a) $F(x) = 1 - \frac{1}{x^2}$
   (b) $F(x) = x^2 + \ln(x)$
   (c) $F(x) = \frac{x^2}{2} + \ln(x) - 10$
   (d) Both (b) and (c)
   (e) None of the above

7. (2 points) A continuous function whose domain is a closed interval
   (a) Must have a critical number inside the interval
   (b) Must have a local extreme value inside the interval
   (c) Must have a global extreme value inside the interval or at the endpoints of the interval
   (d) All of the above
   (e) Just (a) and (c)

8. (2 points) The family of functions $f(x) = cx^2 + x + 1$, where c is a parameter,
   (a) Is increasing for all x when $c > 0$ and decreasing for all x when $c < 0$
   (b) Is decreasing for all x when $c > 0$ and increasing for all x when $c < 0$
   (c) Is concave up for all x when $c > 0$ and concave down for all x when $c < 0$
   (d) Is concave down for all x when $c > 0$ and concave up for all x when $c < 0$
   (e) None of the above

9. (2 points) In the process of calculating the Riemann sum M_8 for the function $f(x)$ on the interval $[1, 2]$, the value of Δx
   (a) Equals 1/4
   (b) Equals 1/2
   (c) Equals 2
   (d) Equals 16
   (e) Cannot be determined without more information about f

10. (2 points) Under which of the following conditions will the Riemann sum L_8 be an overestimate for the area between the graph of a function f and the x-axis on the interval $[a, b]$?
    (a) f is increasing on $[a, b]$
    (b) f is decreasing on $[a, b]$
    (c) f is concave up on $[a, b]$
    (d) f is concave down on $[a, b]$
    (e) None of the above

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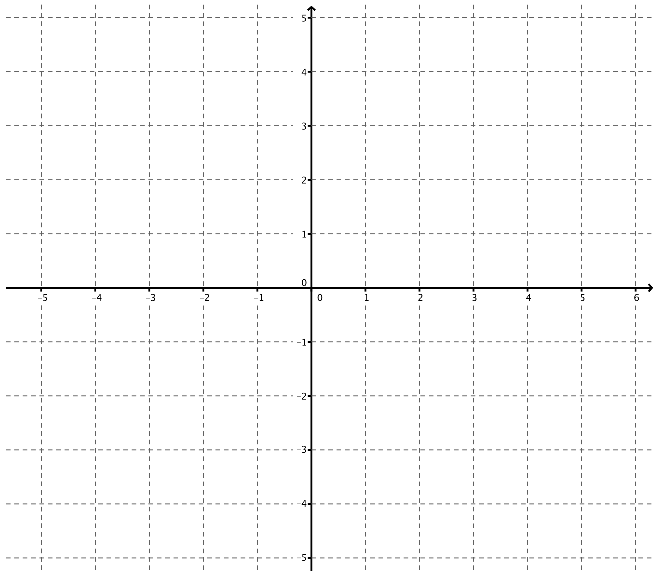
The next several items are problems to solve. Be sure to give complete, clear, and correct solutions to each, not just answers unless clearly specified.

11. (20 points) For some function $f(x)$, whose formula you do not have, it is known that

$$f'(x) = -xe^{-x}(x-2) \qquad \text{and} \qquad f''(x) = e^{-x}(x^2-4x+2)$$

Construct well-labeled first and second derivative sign charts for $f(x)$, clearly labeling all critical points, relative extremes, and points of inflection, as well as the relevant behavior of f in each appropriate interval. Use your work to construct a well-labeled possible graph of $f(x)$ on the axes provided, using the additional given fact that $f(0) = 2$.



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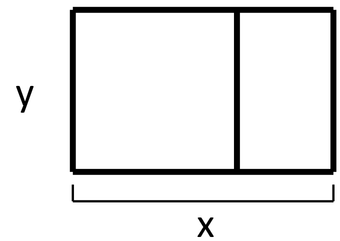
12. Suppose you want to make a fenced-in region like the one below:



(a) (4 points) What is the total length of the fence (including the fence on the inside) in terms of x and y?

(b) (4 points) What is the area of the fenced-in region in terms of x and y?

(c) (12 points) If you have to make the area of the fenced-in region 24 square feet, how should you build the fence in order to minimize the amount of fencing used? Make sure you justify that your answer actually produces a minimum.

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13. (20 points) Work EXACTLY ONE of the following problems. Clearly indicate which one you are doing and which one you are not doing; submitting significant work on both will result in a grade of “0” on this problem. Show all work and organize your solution clearly, as a narrative that explains the correctness to the reader using mathematics as support. You will lose credit for disorganized solutions.

- If two resistors with resistances R_1 and R_2 are connected in parallel, as in the figure, then the total resistance R, measured in Ohms (Ω), is given by:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

  If R_1 and R_2 are increasing at rates of $.3\,\Omega/s$ and $.2\,\Omega/s$, respectively, how fast is R increasing when $R_1 = 80\,\Omega$ and $R_2 = 100\,\Omega$?

- At noon, ship A is $150km$ west of ship B. Ship A is sailing east at $35km/h$, and ship B is sailing north at $25km/h$. How fast is the distance between the ships changing at 4:00 P.M.?

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14. The speed of a runner increased steadily during the first three seconds of a race. Her speed at half-second intervals is given in the following table:

t (s)	0	0.5	1.0	1.5	2.0	2.5	3.0
v (ft/s)	0	6.2	10.8	14.9	18.1	19.4	20.2

(a) (8 points) Calculate the Riemann sums L_6 and R_6. Show all work.

(b) (6 points) Calculate the Riemann sum M_3. Why can't we calculate M_6 in this case?

(c) (6 points) State upper and lower estimates for the distance, in feet, that the runner traveled during this 3-second period and explain why your underestimate is an underestimate, and similarly for the overestimate.