This writeup would get a grade of 8. There are no math errors; the solution is complete and clear; and the writing abides by all the guidelines for grammar and style.

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MTH 201-02
Solution for Section 1.1, Exercise 1

A bungee jumper dives from a tower at time $t = 0$. Her height h (measured in feet) at time t (in seconds) is given by the graph in Figure 1.3 on page 7 of the textbook and by the function $s(t) = 100\cos(0.75t) \cdot e^{-0.2t} + 100$.

(a) What is the change in vertical position of the bungee jumper between $t = 0$ and $t = 15$?

Solution: We find the change in vertical position by finding the difference between the jumper's height at time $t = 15$ and her height at time $t = 0$:

$$\begin{aligned}
s(15) - s(0) &= (100\cos(0.75 \cdot 15) \cdot e^{-0.2 \cdot 15} + 100) - (100\cos(0.75 \cdot 0) \cdot e^{-0.2 \cdot 0} + 100) \\
&= (100\cos(11.25) \cdot e^{-3} + 100) - (100\cos(0) \cdot e^{0} + 100) \\
&= (100\cos(11.25) \cdot e^{-3} + 100) - (100 \cdot 1 \cdot 1 + 100) \\
&= (100\cos(11.25) \cdot e^{-3} + 100) - (100 \cdot 1 \cdot 1 + 100) \\
&= (100\cos(11.25) \cdot e^{-3} + 100) - 200 \\
&\approx -94.7469 \text{ feet}
\end{aligned}$$

Since we only care about the absolute distance travelled in this question, the total change in vertical position is approximately 94.7469 feet.

(b) Estimate the jumper's average velocity on each of the following time intervals: $[0, 15]$, $[0, 2]$, $[1, 6]$, and $[8, 10]$. Include units on your answers.

Solution: For each interval $[a, b]$, we calculate the change in distance $s(b) - s(a)$ divided by the change in time $b - a$:

$$\begin{aligned}
\text{Average velocity on [0,15]} &= \frac{s(15) - s(0)}{15 - 0} \\
&= \frac{-94.7469}{15 - 0} \\
&\approx -6.58313 \text{ feet per second}
\end{aligned}$$

Note: The work for $s(15) - s(0)$ is done in part (a). The computations for the remaining intervals proceed similarly:

$$\begin{aligned}
\text{Average velocity on [0,2]} &= \frac{s(2) - s(0)}{2 - 0} = \frac{-95.2583}{2} \approx -47.6292 \text{ feet per second} \\
\text{Average velocity on [1,6]} &= \frac{s(6) - s(1)}{6 - 1} = \frac{-66.2547}{2} \approx -13.2509 \text{ feet per second} \\
\text{Average velocity on [8,10]} &= \frac{s(10) - s(8)}{10 - 8} = \frac{-14.6943}{2} \approx -7.3472 \text{ feet per second}
\end{aligned}$$

(c) On what time interval(s) do you think the bungee jumper achieves her greatest average velocity? Why?

Solution: The greatest average velocity would occur on the interval where the distance changes the most over the shortest period of time. From the graph, this appears to be the interval $[0, 4]$.

(d) Estimate the jumper's instantaneous velocity at $t = 5$. Show your work and explain your reasoning, and include units on your answer.

Solution: First, recall from the discussion in the textbook that the average velocity on the interval $[a, a + h]$ is given by

$$AV_{[a,a+h]} = \frac{s(a + h) - s(a)}{h}$$

In this problem, $a = 5$, and h is some small nonzero number. We first note that

$$s(5) = 100\cos(0.75 \cdot 5) \cdot e^{-0.2 \cdot 5} + 100 = 100\cos(3.75) \cdot e^{-1} \approx 69.8133$$

Now we will use the average velocity formula above to find the average velocities from $t = 5$ to $t = 5 + h$ for increasingly small values of h. We start arbitrarily with $h = 0.25$:

$$AV_{[5,5.25]} = \frac{s(5.25) - s(5)}{0.25} = \frac{75.5171 - 69.8133}{0.25} = \frac{5.70376}{0.25} \approx 22.815 \text{ feet per second}$$

Now we continue similar calculations for values of h that are approaching, but not equal to zero:

Value of h	Average velocity from $t = 5$ to $t = 5 + h$
0.1	22.2523 feet per second
0.01	21.8543 feet per second
0.001	21.812 feet per second
0.0001	21.8077 feet per second

Based on this information, we can estimate that the instantaneous velocity of the jumper at time $t = 5$ is approximately 21.8 feet per second.

(e) Among the average and instantaneous velocities you computed in earlier questions, which are positive and which are negative? What does negative velocity indicate?

Solution: All of the average velocities in part (b) were negative, while the instantaneous velocity in part (d) was positive. The negative velocity indicates a downward motion because while time is increasing, the jumper's height goes from a large value to a small value.