

MTH 201: Calculus – Module 1B

The concept of the limit (AC 1.2)

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Agenda

Review of Daily Prep

Polling for today

Go to `www.mentimeter.com` and enter code 32 04 90

Or go to `https://www.menti.com/4d48wn64k6`

Q+A

Limits using algebra

Finding limits using algebra

Alert

Finding a limit of a function at a point, is not always the same thing as evaluating the function at that point.

Example

$$\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2} = 4$$



But $\frac{x^2 - 4}{x - 2}$ evaluated at $x = 2$ is undefined ($0/0$).

Using algebra first

Pro Tip

Always try to simplify the algebra first.

Example

$$\begin{aligned}\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2} &= \lim_{x \rightarrow 2} \frac{(x - 2)(x + 2)}{x - 2} \\ &= \lim_{x \rightarrow 2} (x + 2)\end{aligned}$$

As x gets closer to 2, $x + 2$ gets closer to 4.

Therefore $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2} = 4$.

Practice with the concept

Go to:

<http://gvsu.edu/s/1pq>

Find the value of the limit using algebra simplification first. Spoiler: The answer is 12.

Instantaneous velocity using limits

Average velocity

Consider a moving object whose position function is given by $s(t) = t^2$ where t is time in seconds, s is position in meters.



dg5.png

Finding average and instantaneous velocity

The **average velocity** on the small time interval from $t = 3$ to $t = 3 + h$ is

$$\frac{s(3 + h) - s(3)}{(3 + h) - 3} = \frac{s(3 + h) - s(3)}{h}$$

On Mentimeter: If we wanted to find the *instantaneous velocity* at $t = 3$, how would we modify this?

Working out the velocity

We get the instantaneous velocity at $t = 3$ by taking the **average velocity** expression and **evaluating its limit as $h \rightarrow 0$** :

$$IV_{t=3} = \lim_{h \rightarrow 0} \frac{s(3+h) - s(3)}{h}$$

But we can now work out the value with algebra! Work with your partner to do this:

<http://gvsu.edu/s/1pt>

Spoiler: The answer is 6 meters/second.

Getting the instantaneous velocity

$$\begin{aligned} IV_{t=3} &= \lim_{h \rightarrow 0} \frac{s(3+h) - s(3)}{h} \\ &= \lim_{h \rightarrow 0} \frac{(3+h)^2 - 9}{h} \\ &= \lim_{h \rightarrow 0} \frac{9 + 6h + h^2 - 9}{h} \\ &= \lim_{h \rightarrow 0} \frac{6h + h^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{h(6+h)}{h} \\ &= \lim_{h \rightarrow 0} (6+h) \\ &= 6 \text{ meters/second} \end{aligned}$$

What to do next

Coming up next

- Online followup activities: Due 11:59pm ET, Friday 9/11 (Link to be posted and in email)
- Daily Prep for Module 2A: Due 11:59pm ET, Sunday 9/13 (Blackboard)
- WeBWork 1B: Due 11:59pm ET, Sunday 9/13 (WeBWork)

Continue to check email and announcements every day — ask questions and give help on CampusWire too.