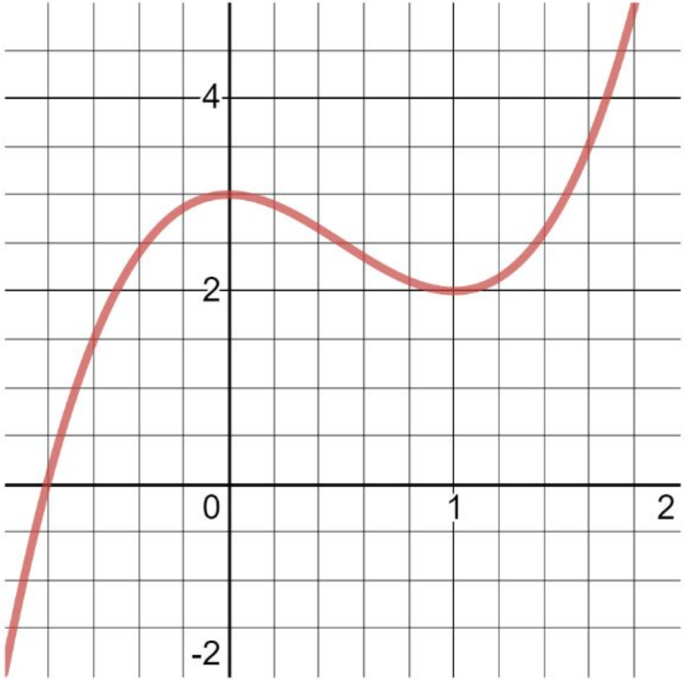
MTH 201 -- Calculus
Module 3A: Interpreting and estimating derivatives

September 21-22, 2020

Agenda for today

- Polling activity over Daily Preparation
- Q&A time
- Minilecture: Connecting concavity to the second derivative
- Activity: Card sorting
- Quick (ungraded) quiz
- Feedback time

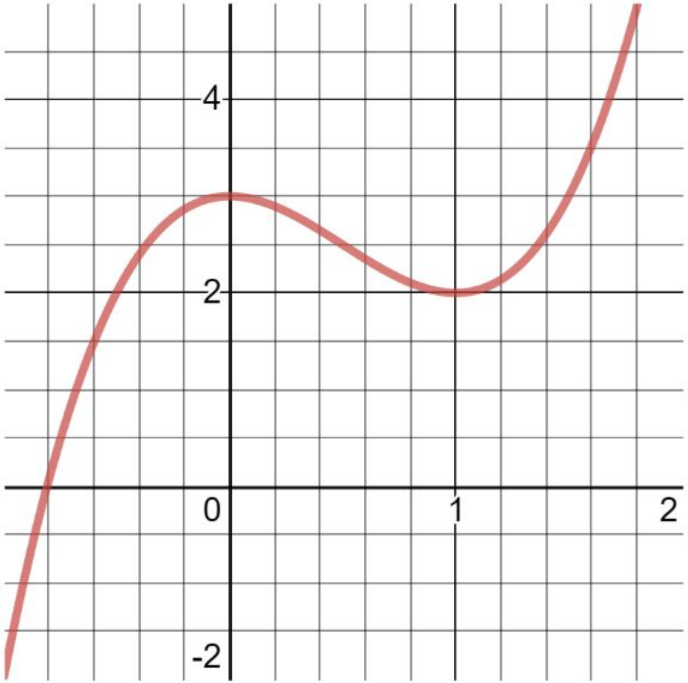
The graph of a function $f(x)$ is shown. The *derivative* of $f(x)$ (that is, $f'(x)$) is positive on the interval



- $(1, \infty)$
- $(-0.8, \infty)$
- $(0.5, \infty)$
- $(-\infty, 0) \cup (1, \infty)$
- None of the above

To 0

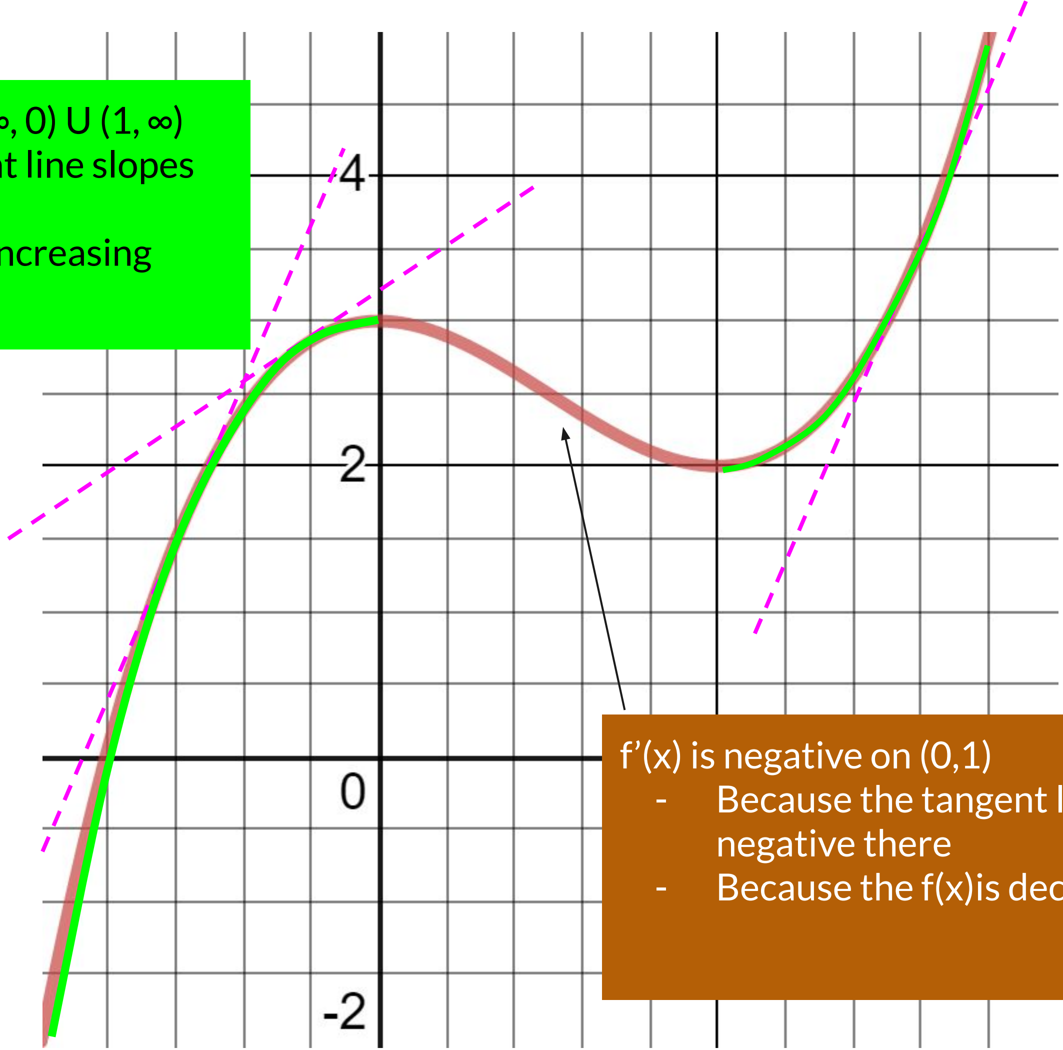
The graph of a function $f(x)$ is shown. The function is *concave up* on the interval



- $(1, \infty)$
- $(-0.8, \infty)$
- $(0.5, \infty)$
- $(-\infty, 0) \cup (1, \infty)$
- None of the above

To 0

f’(x) is positive on (-∞, 0) U (1, ∞)
- Because the tangent line slopes are all positive there
- Because the f(x) is increasing there



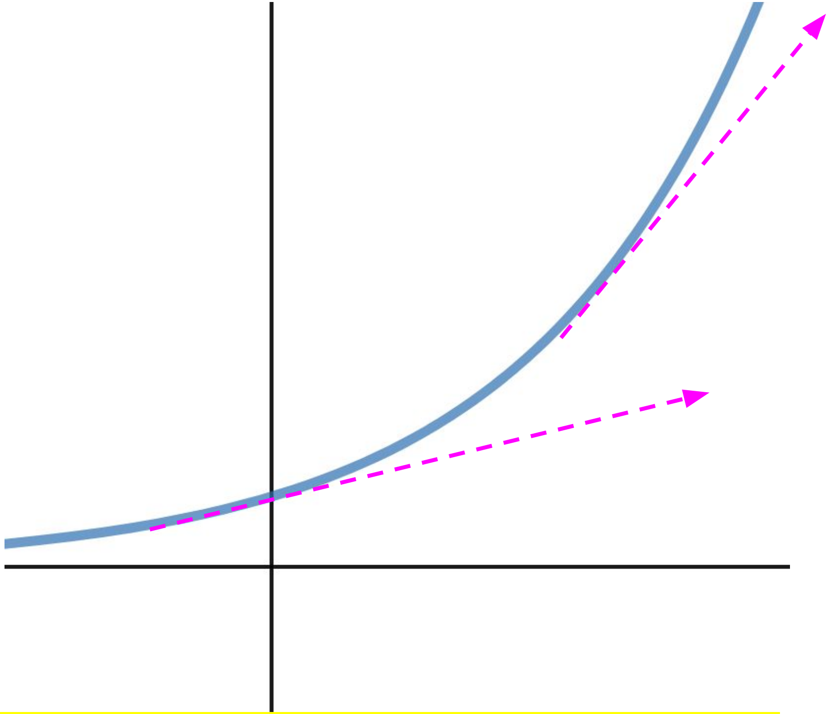
f’(x) is negative on (0,1)
- Because the tangent line slopes are all negative there
- Because the f(x)is decreasing there

$f'(x) > 0$ means **$f(x)$ is increasing**

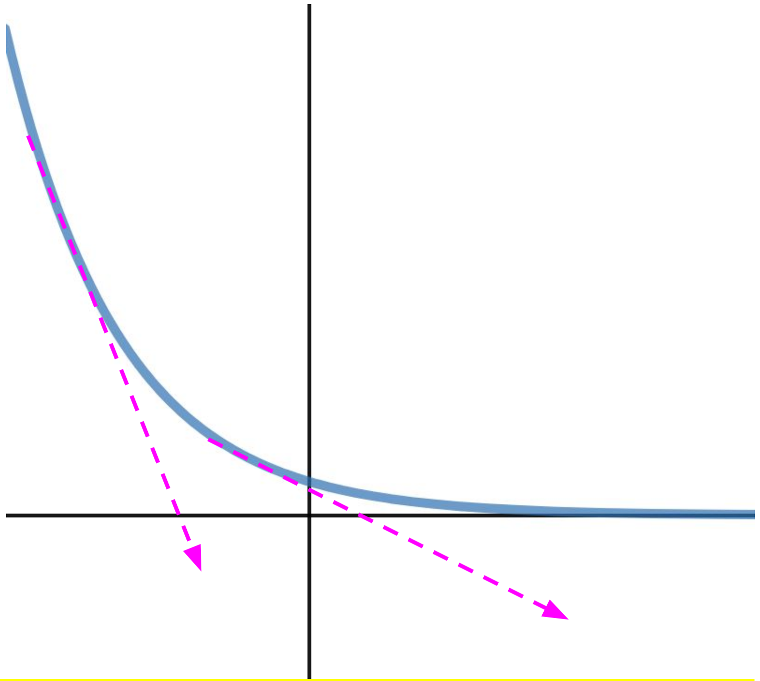
$f'(x) < 0$ means **$f(x)$ is decreasing**

$f'(x) = 0$ means... we'll discuss later

A function is concave up on an interval if its rate of change is increasing on that interval.



Increasing and concave up



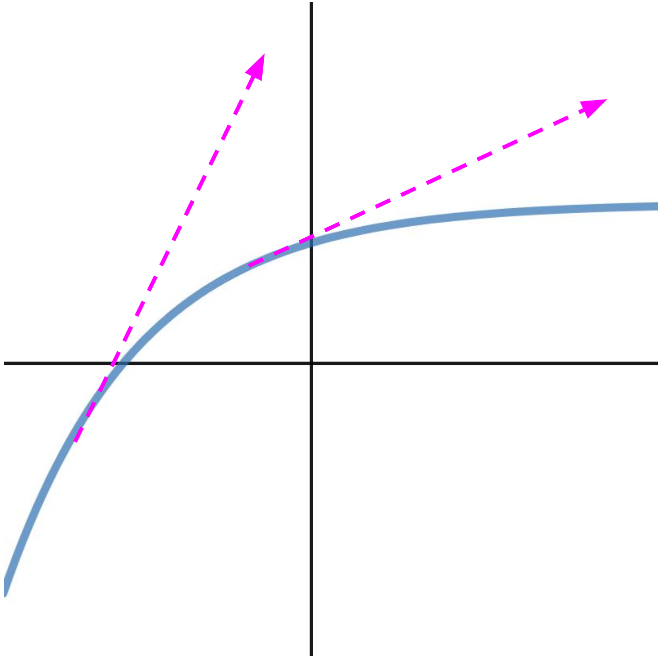
Decreasing and concave up

In both cases: Tangent line slopes are increasing

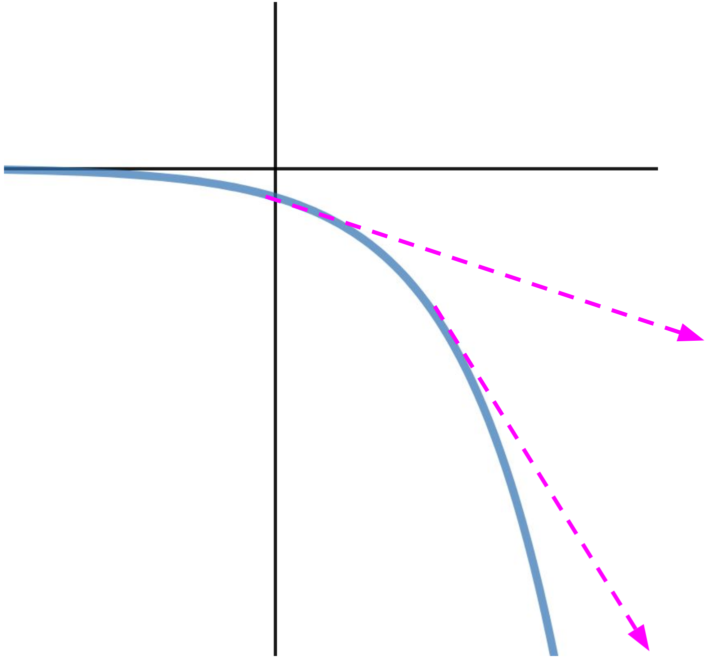
Concave up → Rate of change is increasing → f'(x) is increasing

The original f(x) might not be increasing.

A function is concave down on an interval if its rate of change is decreasing on that interval.



Increasing and concave down

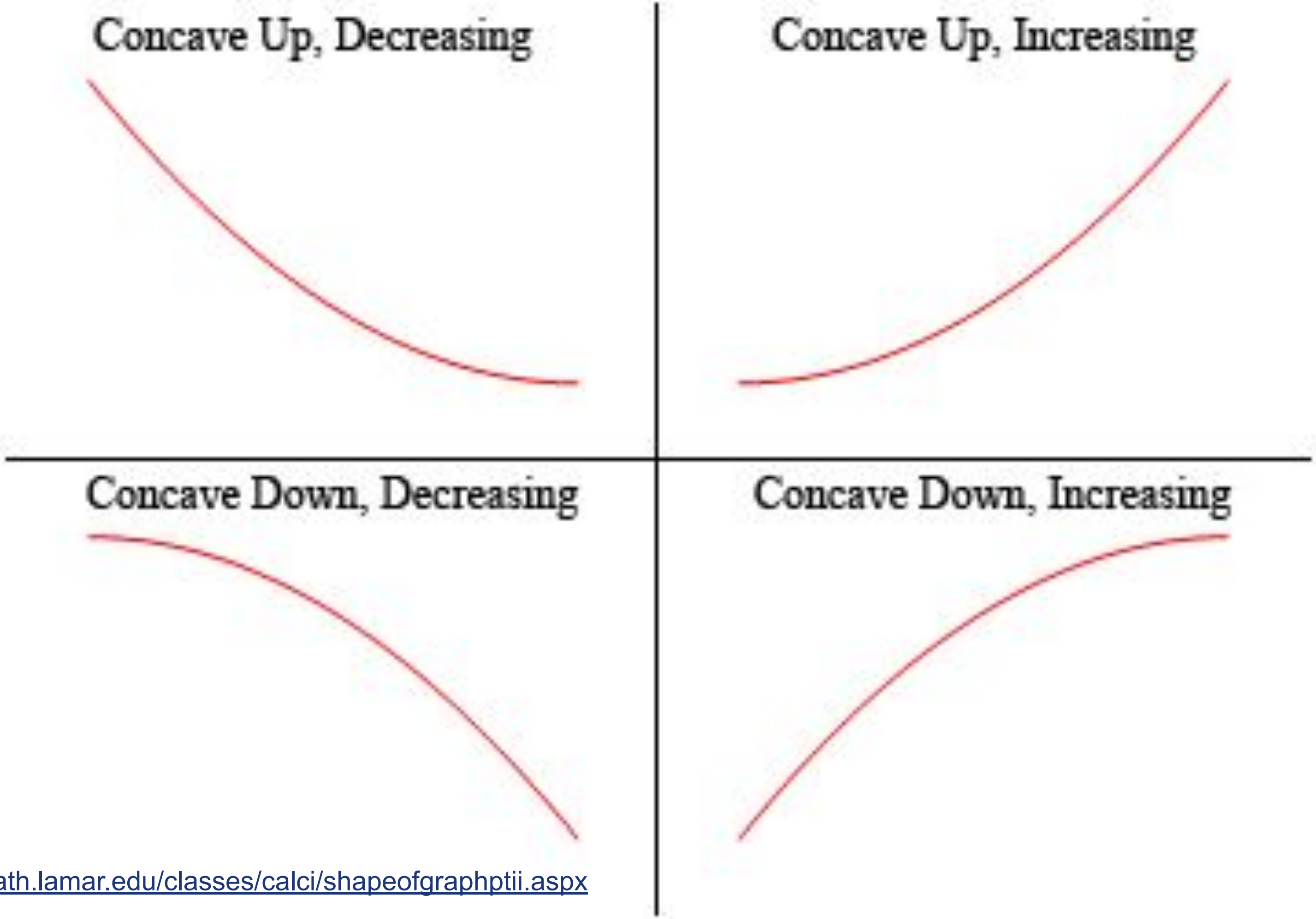


Decreasing and concave down

In both cases: Tangent line slopes are decreasing

Concave down →
Rate of change is decreasing →
f'(x) is decreasing

The original f(x) might <u>not</u> be decreasing.



https://tutorial.math.lamar.edu/classes/calci/shapeofgraphptii.aspx

Connecting this to the second derivative

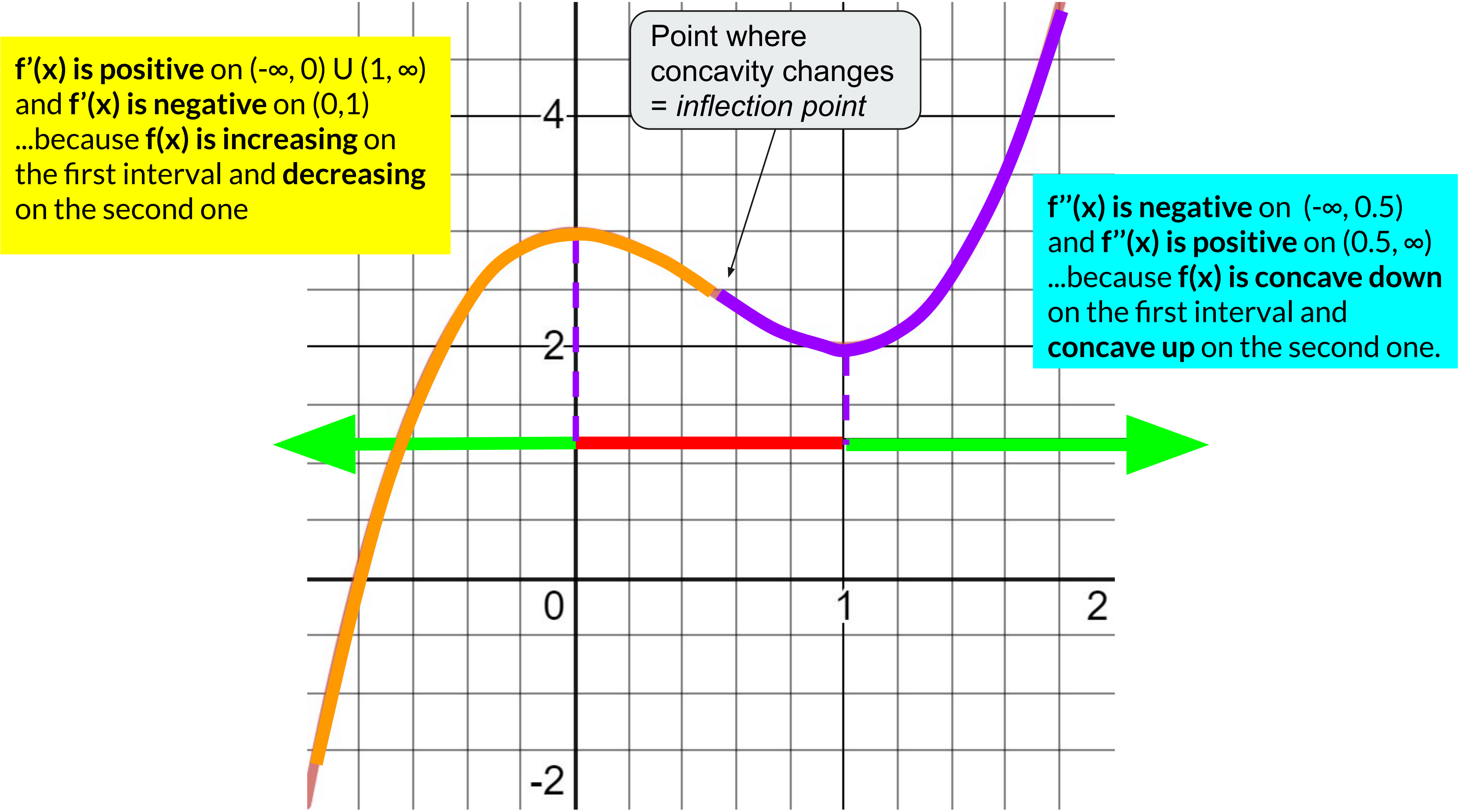
The second derivative $f''(x)$ of a function $f(x)$ is...

→ The derivative of $f'(x)$

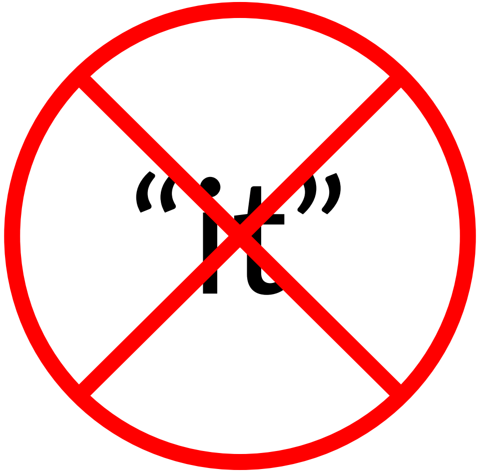
→ The rate at which $f'(x)$ is changing

Concave up →
Rate of change is increasing →
f'(x) is increasing →
f''(x) is positive

Concave down →
Rate of change is decreasing →
f’(x) is decreasing →
f’’(x) is negative



f’(x) is positive on (-∞, 0) U (1, ∞) and f’(x) is negative on (0,1) ...because f(x) is increasing on the first interval and decreasing on the second one
Point where concavity changes = inflection point
f’’(x) is negative on (-∞, 0.5) and f’’(x) is positive on (0.5, ∞) ...because f(x) is concave down on the first interval and concave up on the second one.
4
2
0
1
2
-2



"The function is increasing because it's positive"... "It's negative, so it's concave down"

"The function is increasing because its derivative is positive"... "The second derivative f"(x) is negative, so f(x) is concave down"

Activity: Sorting out f, f', and f''
→ Desmos

Feedback:
http://gvsu.edu/s/1rx

Add sticky notes for comments, ideas, and questions.