**Directions**:

- Do only the Checkpoint problems that you need to take and feel ready to take. If you have already earned Mastery on a Learning Target, do not attempt a problem for that Target! You can skip a Target if you need more time to practice with it, and take it on the next round.

- Do not put any work on this form; do all your work on separate pages. You may either handwrite or type up your work.

- When you have completed your work on a problem, please circle or box off the answer you wish me to check. The answer that you circle or box will be taken as your "official" final answer. **Work that is crossed out**, **will not be considered in the grading process**.

- If you are handwriting, submit your work by **scanning your work** using a scanning app or scanning device; **do not just take a picture** but scan your work to a clear, legible, black and white PDF file of size less than 100 MB. Work submitted as an image file (JPG, PNG, etc.) will not be graded.

- Unless explicitly stated otherwise, you must show your work or explain your reasoning clearly on each item of each problem you do. Responses that consist of only answers with no work shown, or where the work is insufficient or difficult to read, or which have significant gaps or omissions (including parts left blank) will be given a grade of "x".

- The following are approved resources for use on this and any other Checkpoint: All documents and videos posted to the class Blackboard site; all work that you have submitted for grading, including previous Checkpoints; all videos used for Daily Prep assignments; and Wolfram|Alpha. You may also ask me (Talbert) questions at any time. All other resources, including classmates, are off-limits.

Learning Target F.2: *I can find the average rate of change of a function on an interval.*

1. Let $f(x) = x^2 + x$. Find the average rate of change in f on the intervals $[0, 4]$ and $[3, 3.5]$.
2. Let $g(x)$ be the graph shown below. Find the average rate of change in g on the intervals $[-1, 3]$ and $[0, 3]$.
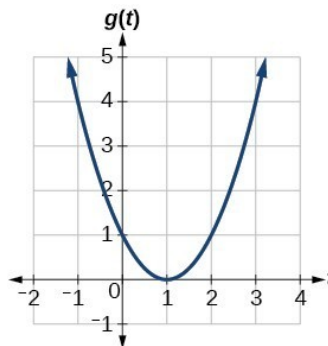
Learning Target L.1 (Core): *I can find the limit of a function at a point using numerical, graphical, and algebraic methods.*

1. Complete the table of values below using the function $f(x) = \dfrac{x^4 - 1}{x + 1}$. Then state the value of $\lim_{x \to -1} f(x)$ and explain your reasoning.

x	-1.5	-1.1	-1.01	-.99	-.9	-.5
$f(x)$						

2. Using only algebra (no graphs or tables), evaluate $\lim\limits_{x \to 1} \dfrac{x^4 - 1}{x - 1}$.

3. The function $y = h(x)$ is shown below. State the value of $\lim\limits_{x \to 4} h(x)$ and explain your reasoning.
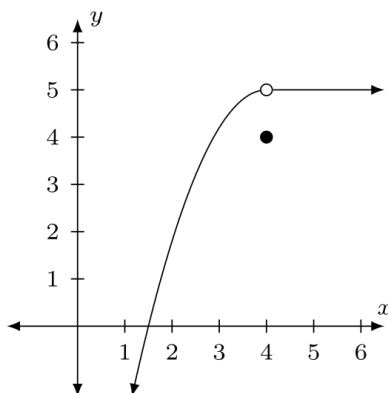
Learning Target D.1 (Core): *I can find the derivative of a function, both at a point and as a function, using the definition of the derivative.*

Consider the function $f(x) = -7x^2 - 6x - 3$.

1. Set up, but do not evaluate, the limit that would compute $f'(1)$ (the derivative of f at the point $x = 1$).

2. Set up, but do not evaluate, the limit that would compute $f'(x)$ (the formula for the derivative of f at any point).

3. Choose one of the limits you set up and evaluate it to find either $f'(1)$ or $f'(x)$.

Learning Target D.2 (Core): *I can use derivative notation correctly, state the units of a derivative, estimate the value of a derivative using difference quotients, and correctly interpret the meaning of a derivative in context.*

In some recipes that involve adding liquids to the dish you're cooking, you have to heat up the liquid first by mixing the liquid with another liquid that's already hot. (This is called "tempering" the liquid.) Last night Prof. Talbert made a soup where he had to temper 2 cups of milk by slowly mixing in hot water to the milk. The temperature T of the milk-water mixture (in degrees Celsius) was a function of the number of milliliters (L) of the hot liquid that was added to the milk. Denote the relationship by $T = f(L)$.

1. Suppose $T'(50) = 2$. State the units of measurement for the numbers 50 and 2. (Clearly indicate which is which.)

2. Still assuming $T'(50) = 2$, explain the meaning of this statement in ordinary terms (that is, in terms of angles and height or altitude) and without using any technical jargon — in a way that an ordinary cook with no calculus background could easily understand.

3. Suppose $T(20) = 5$, $T(30) = 3$, and $T(40) = 2.5$. Use forward, backward, and central differences to estimate $T'(30)$. Clearly indicate which estimate is which.

Learning Target D.3 (Core): *Given information about f, f', or f'', I can correctly give information about f, f', or f'' and the increasing/decreasing behavior and concavity of f (and vice versa).*

Suppose we are given a function $y = f(x)$ and we know that $f(2) = 4$ and $f''(2) = 1$. For each of the following questions, answer **YES**, **NO**, or **NOT ENOUGH INFORMATION** and then give a 1-3 sentence explanation for your answers. *You'll be evaluated on both the correctness of the answers and the correctness and clarity of the explanations.*

1. Is the graph of f above the x-axis at $x = 2$?

2. Is f increasing at $x = 2$?

3. Is the slope of f increasing at $x = 2$?

Learning Target D.4: *I can find the equation of the tangent line to a function at a point and use the tangent line to estimate values of the function.*

Find each of the following. Make sure your answer is clearly indicated (for example by circling it). You are being graded on your processes as well as your answers, so show all your work.

1. Find an equation for the tangent line to the graph of $y = \sin(x) + x$ at $x = 0$.

2. A function $f(x)$ is such that $f(2) = -3$ and $f'(2) = 5$. Find an equation for the tangent line to the graph of $f(x)$ at $x = 2$, and then use the line to estimate the value of $f(2.01)$.

Learning Target DC.1 (Core): *I can compute derivatives correctly for power, polynomial, and exponential functions and the sine and cosine functions, and basic combinations of these (constant multiples, sums, differences).*

Find the derivatives of each of the following functions. Make sure your answer is clearly indicated (for example by circling it). You are being graded on your processes as well as your answers, so show all your work.

1. $y = 2 + 2^\pi + x^{2\pi} + 2^x$

2. $g(x) = -\sin(x) + 3\cos(x)$

3. $h(x) = \dfrac{x^2 + 3x}{x^2}$ (Do NOT use the Product, Quotient, or Chain Rules.)

Learning Target DC.2 (Core): *I can compute derivatives correctly for products, quotients, and composites of functions.*

New instructions: Find the derivatives of each of the following. **In each, state which rule you are using first. In the case of the Chain Rule, clearly indicate the decomposition of the function by stating the "inside" and "outside" functions first**. Show your work in a clear and logical order and circle/box off your answer. DO NOT simplify your answer.

1. $y = x^2 \sin(x)$

2. $y = \sin(x^2)$

3. $y = \dfrac{\sin(x)}{x^2}$

4. $y = \sin^2 x$

Learning Target DC.3: *I can compute derivatives correctly using multiple rules in combination.*

New instructions: Find the derivatives of each of the following. **In each, state ALL the rules you are using, in the correct order of use. You need only list when you use the Product, Quotient, or Chain Rules. In the case of the Chain Rule, clearly indicate the decomposition of the function by stating the "inside" and "outside" functions first**. Show your work in a clear and logical order and circle/box off your answer. DO NOT simplify your answer.

1. $y = x^2 \cos(x^2 + x)$

2. $y = \dfrac{e^{3x}}{1 + e^{2x}}$

Learning Target DC.4: *I can compute the derivatives correctly for logarithmic, trigonometric, and inverse trigonometric functions.*

New instructions: Find the derivatives of each of the following. **In the case of the Chain Rule**, **clearly indicate the decomposition of the function by stating the "inside" and "outside" functions first**. Show your work in a clear and logical order and circle/box off your answer. DO NOT simplify your answer.

1. $y = \ln(x^2 - x + 1)$
2. $y = x^2 \ln(x)$
3. $y = x^2 \arcsin(x)$

Learning Target DA.1 (Core): *I can find the critical values of a function, determine where the function is increasing and decreasing, and apply the First and Second Derivative Tests to classify the critical points as local extrema.*

Consider the function $f(x) = 3x^4 - 4x^3$.

1. Find all the critical values of the function. Make your reasoning clear by giving all calculus and algebra steps clearly and neatly.

2. Make a first derivative sign chart for the function and use it to determine the intervals on which the function is increasing and decreasing. Your sign chart must be legible and formatted using the rules we discussed in class, and you must state which points you are using as test points.

3. Classify each critical value as a local maximum, local minimum, or neither and explain your reasoning.

Learning Target DA.2: I can determine the intervals of concavity of a function and find all of its points of inflection.

Consider the function $h(x) = 3x^4 - 4x^3$. Use calculus (not visual estimation from a graph) to find the intervals on which h is concave up and the intervals on which h is concave down, and state its inflection points. Show all your work; if you construct a sign chart, make sure the chart has all the required properties that we have discussed.

Learning Target DA.3: I can use the Extreme Value Theorem to find the absolute maximum and minimum values of a continuous function on a closed interval.

Use Calculus (not visual estimation from a graph) to determine the absolute extreme values of the function $f(x) = 3x^4 - 4x^3$ on the interval $[1, 3]$.

Learning Target DA.4 (Core): I can set up and use derivatives to solve applied optimization problems.

Set up and solve the following optimization problem. **In order for your work to meet quality standards, it needs to contain ALL of the following. Check carefully before turning in your work to make sure you've included each one**. A lengthy solution is not necessary, but the written solution for Exercise 3 that was posted earlier would be a good guide for you.

- A clear indication of what each variable in the solution represents;
- A clear statement of what quantity you are optimizing;
- A formula for the quantity you are optimizing and a clear indication of how you obtained it;
- If you use a constraint in the problem, a clear statement of what that constraint is and how you used it;
- The use of a derivative to find the input that optimizes your quantity;

- Reasoning that explains why your solution is correct (for example, don't just find a value but explain how you know that value optimizes the target quantity).

Use the list above as a checklist before you turn in your work.

Problem for DA.4: A box with an open top has vertical sides, a square bottom, and a volume of 108 cubic meters. If the box has the least possible surface area, find its dimensions. (*Remember to give all three dimensions of the box, and double check to make sure all items on the checklist above have been included in your solution.*)


Learning Target INT.1: I can calculate the area between curves, net change, and displacement using geometric formulas and Riemann sums.

Consider the function $f(x) = -3x^4 + 4x^3$. Estimate the area under the curve, above the x-axis, and between $x = 0$ and $x = 1$, using the following Riemann sums. **On each one**: Clearly state the value of Δx, clearly state which points you are using to construct the rectangles, and show the setup of your calculation. **Keep all approximations to four (4) decimal places**.

1. L_2
2. M_5

Learning Target INT.2: I can explain the meaning of each part of the definition of the definite integral in terms of a graph, and interpret the definite integral in terms of areas, net change, and displacement.

The graph below shows the rate at which patients were admitted to an intensive care unit (ICU) at a hospital, measured in "people per day", during a one-week period (seven days). A negative rate indicates that more patients were leaving the ICU than were entering.
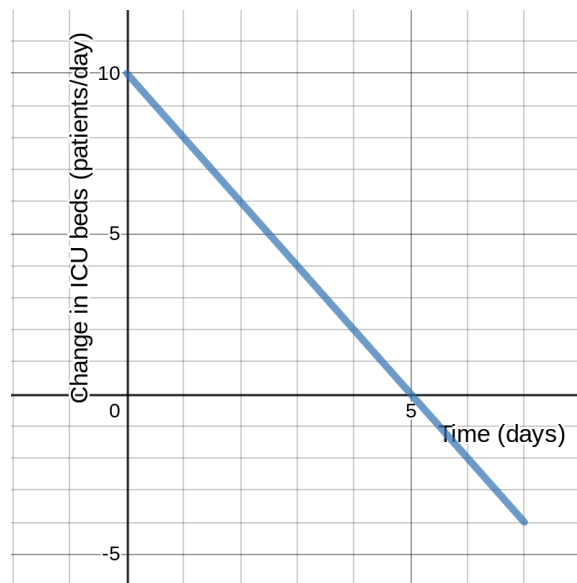
1. If the function whose graph is shown is called $r(t)$, what are the units of the definite integral $\int_0^7 r(t)\, dt$? State your answer and briefly explain.

2. At the end of the week shown here, were there *more* patients in the ICU than at the beginning of the week, were there *fewer* patients, or was the number of patients the same? State your answer clearly and give a clear, well-reasoned explanation.

3. If there were 30 patients in the ICU at the beginning of the week, how many are there at the end of the week? State your answer clearly and make your reasoning clear.

Learning Target INT.3: I can evaluate a definite integral using geometric formulas and the Properties of the Definite Integral.

The graph of $f(x)$ is shown below. It is made up of line segments and parts of circles. **Using only the graph,** evaluate the exact value — no decimal approximations — of each of the integrals shown below. **Note: Antidifferentiation is not allowed in solutions to this problem, nor are decimal approximations.** Also, **show all your work** — answers with insufficient work or no work will not meet the grading criteria.

$y = f(x)$
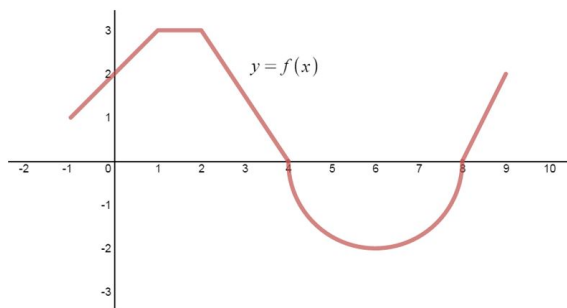
1. $\displaystyle\int_{-1}^{4} f(x)\, dx$

2. $\displaystyle\int_{4}^{8} f(x)\, dx$

3. $\displaystyle\int_{-1}^{9} f(x)\, dx$

Learning Target INT.4 (Core): I can evaluate a definite integral using the Fundamental Theorem of Calculus.

Find the exact value of each of the following definite integrals by using the Fundamental Theorem of Calculus (not geometry or Riemann sums).

Each solution must **show all steps** and **give the answer in both exact form that is *fully simplified*, *and* decimal form**. Leaving an answer unevaluated will be counted as a "simple error". See the December 10 Campuswire post for further explanation.

1. $\displaystyle\int_{0}^{1} \left(-3x^4 + 4x^3\right) dx$

2. $\displaystyle\int_{1}^{4} \left(\frac{1}{x^2} + \frac{1}{x}\right) dx$

3. $\displaystyle\int_{0}^{\pi/4} (x - \sin(x))\, dx$

4. $\displaystyle\int_{1}^{2} \frac{x^2 + x}{x}\, dx$

Learning Target INT.5: I can correctly antidifferentiate basic functions and identify antiderivatives.

State an antiderivative for each of the following functions.

1. $f(x) = \dfrac{1}{x^4} + \dfrac{1}{\sqrt{1 - x^2}}$

2. $g(x) = 3 + x^2 + \sqrt[4]{x}$

3. $h(x) = \cos(x) - \csc(x)\cot(x)$