Class Activities: The notion of limit

Get into groups of 2–4 and work through all of the following activities. These are not to be turned in, and they will not be graded. Instead, record your group's work on your copy and keep it for notes. I will be coming to each group one by one as you work to observe what you're doing, answer questions, and catch any misconceptions that are happening. We will stop with about 10 minutes remaining to debrief the main ideas.

1   Taking limits numerically and algebraically

1. Consider the function $f(x) = \dfrac{x^2 - 1}{x - 1}$. Does the value of $f(1)$ exist? Why/why not?

2. Using the same function $f(x)$ as above, use a calculator or spreadsheet to fill in the blanks of the following table. Some of these have already been done for you; your calculations should not be drastically different from the ones already here. For example a calculation equal to 22.5 is very likely incorrect.

x	$f(x)$	x	$f(x)$
0.5	1.5	1.5	
0.9		1.1	2.1
0.99	1.99	1.01	
0.999		1.001	2.001

   Based on the table, estimate the value of $\displaystyle\lim_{x \to 1} \frac{x^2 - 1}{x - 1}$.

3. Use algebra to simplify the fraction $\dfrac{x^2 - 1}{x - 1}$. Use the result to find the *exact* value of $\displaystyle\lim_{x \to 1} \frac{x^2 - 1}{x - 1}$.

2   Instantaneous velocity and limits

Consider a moving object whose position function is given by $s(t) = t^2$ where s is measured in meters and t is measured in minutes.

1. Find an expression for the average velocity of the object on the interval $[4, 4 + h]$. Simplify the expression algebraically as much as possible. (There should be no fractions present once the expression is fully simplified, just a simple expression that involves the variable h.)
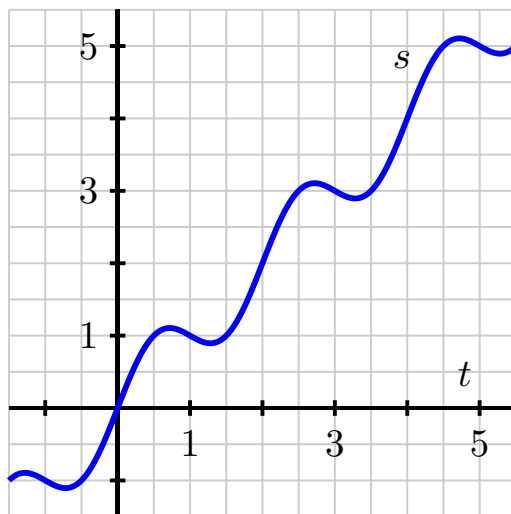
2. Determine the average velocity of the object on the interval $[4, 4.2]$. Include units on your answer. (Hint: What's the value of h in this case?)

3. Determine the instantaneous velocity of the object when $t = 4$ by taking a limit of the average velocity formula you found in question (1). Include units on your answer. Important question: When you take a limit, what's your variable and what should it be approaching?

4. How would you go about finding the object's instantaneous velocity at $t = 10$ seconds? What would do differently in the above process?

3 Average velocity, instantaneous velocity, and slopes

A moving object has position s at time t given by the graph below:

Assume s is measured in feet and t is in seconds.

1. Use the graph to estimate the average velocity of the object on the intervals $[2, 3]$, $[2, 2.5]$, and $[2, 2.25]$. For each interval, draw a line on the graph whose slope is the average velocity on that interval. For example, the average velocity on the interval $[1, 2]$ is $\frac{s(2)-s(1)}{2-1}$, which is also the slope of the line connecting $(1, s(1))$ and $(2, s(2))$

2. Use the graph to estimate the instantaneous velocity of the object when $t = 2$.

3. Is the instantaneous velocity of the object at $t = 2$ greater than, less than, or equal to the average velocity on the interval $[1.5, 2.5]$? Explain.

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What was the least clear point from today's class?