AEP 2: Derivatives and real-world data

Overview: In this AEP, youâll seek out data on a topic of interest to you, analyze the data and its first two derivatives, and think about what the information from the derivatives mean in context.

Learning Targets associated with this AEP:

- D.2 (CORE): I can use derivative notation correctly, state the units of a derivative, estimate the value of a derivative using difference quotients, and correctly interpret the meaning of a derivative in context.
- D.3 **(CORE)**: Given information about f, f', or f'', I can correctly give information about f, f', or f'' and the increasing/decreasing behavior and concavity of f (and vice versa).

Deadlines: AEPs do not have fixed deadlines; simply work on this item until you are ready to submit it.

AEP description

Data sets that involve two variables, and one of the variables is related to the other, are called *two-variable data sets*. An example of a two-variable data set is the US Census Bureau data used in AEP 1, in which the population of a city was given in terms of the year in which the data were collected. There is an implied relationship between one variable and the other. For example, in the population data, there is an implied relationship between time and population – as time passes, population changes. Time is like the input variable to a function, and population is like the output; we call time the independent variable and population the dependent variable because population depends on time. Population also depends on a host of other variables (income levels, unemployment rates, cost of living, etc.) but in our data set, time is the independent variable we focus on.

Note that there is no explicit formula that perfectly describes this relationship — if there were, we wouldnât need to collect census data! Instead, to tell the story of the data, we rely on two kinds of analysis techniques:

- Numerical, where we simply use the data themselves; and
- Graphical, where we use regression techniques (discussed in Lab 1) to find an explicit function that provides a reasonable, but imperfect fit to the data and then use the formula.

For this AEP, you are going to go search up a two-variable data set on some topic that interests you and then analyze it — tell its story — using the concepts of the derivative and the numerical derivative approximation techniques weâve studied.

AEP setup

Think of a topic that interests you – sports, politics, weather, etc, Then go to the internet and search for a two-variable data set that relates two quantities involved with that topic. You may use any kind of data you like as long as:

- It doesnât involve populations and time, because this is the subject of AEP 1.
- The set involves only two variables, and there are at least five pairs of data in the set.
- The data set is real (not made up by you) and you can provide a link to the source.
- The data set has some kind of personal relevance to you, that you can explain. You are especially encouraged to look for data that relate to your academic major in some way.

For example, I (Prof. Talbert) like to cook, so I began to wonder how the prices of some of the ingredients I use have changed over time. Doing a Google search on "price of a pound of ground beef over time" led me to this site, which shows the price in US dollars of a pound of beef in each year from 1995 to 2017. (Here the independent variable is time in years, and the dependent variable is price in US dollars.)

https://www.statista.com/statistics/236776/retail-price-of-ground-beef-in-the-united-states/

Once you have found a data set you like, **place your data into a Google Spreadsheet with the independent variable data in column A and the dependent variable data in column B.** IMPORTANT NOTE: If you are using data for years, rescale the data so that time = 0 corresponds to your first year. This will avoid issues with the regression analysis later. For example, here are my ground beef data, entered in with 1995 set as year 0: `https://i.imgur.com/WK4nxxs.jpg`

If your data set is very long, over 30 pairs, you are allowed to use just a subset of the data consisting of 30 pairs. Most of the calculations in this lab are done with technology, so donât worry about the number of hand computations. But donât cut out more than this, or you will lose accuracy (and this is considered bad data analysis).

Creating a Google Spreadsheet is the same process as creating a Google Doc – just choose Spreadsheets instead of Documents when creating. If you are unfamiliar with this process, please contact the professor or friend, or ask a question on the `tech` channel on Slack.

AEP tasks

1. Provide a link to the source of your data, and write a short paragraph explaining why you chose it and why itâs of interest to you personally.
2. Enter your data into a Google Spreadsheet as described above. Then get a link to share that spreadsheet, and make sure that the link allows **Anyone with the link** (not just Grand Valley State University people) to **Edit** (not just view or comment). Here is a short video showing how to do this (some of the specifics may be different for you, but the process is the same): https://drive.google.com/a/mail.gvsu.ec DaJiDTGNPMhqlhEBdnPLYLbeCTEn/view?usp=drivesdk I will be checking your data directly in your spreadsheet, so itâs important that I can edit the work youâve done. In your writeup, just paste the link for this task.
3. Either in your writeup or in the spreadsheet (your choice), construct a table of values for both the first and second derivatives of the data you have. You will need to use the numerical approximation techniques we discussed in Section 1.5 of the textbook for this. You will also need to think about which of the three approximation techniques weâve studied is the best choice for a given data point. For example, at the far ends of the table, you cannot use a central difference approximation, so what technique will you use? In addition to showing the completed table either in

the writeup or in the Google Spreadsheet, write a 2-3 sentence paragraph explaining your techniques; then show your work on one computation for the first derivative and one computation for the second derivative. Here is an example of the completed spreadsheet for my ground beef price example (explanation and work not shown): https://i.imgur.com/CS8KTmA.jpg

4. Using your computations, identify the intervals on which each of the following behaviors occur. You can phrase your intervals in interval notation (for example, $[0, 5]$) or as inequalities (for example $0 \leq t \leq 5$). Note that depending on your data, some of these intervals may be empty — if that's the case, say so. For each part, give an explanation for how you used the data in your tables to arrive at you answer. Do not use graphs here – just the data in your table. (âI looked at the dataâ is not enough!)

   (a) The data are increasing and concave up

   (b) The data are increasing and concave down

   (c) The data are decreasing and concave up

   (d) The data are decreasing and concave down

5. Use your data — including the original data along with the first and second derivatives — to give a non-technical, descriptive story about your data. Avoid technical words like "function", "derivative", "concave up", and "concave down" — instead, interpret the increasing/decreasing and concave up/down behavior into terms that are understandable and accessible to a person who has not studied any calculus.

   Hereâs part of an example of a good description for part 5, using my ground beef data:

   For most years from 1995 to 2017, the price of ground beef was increasing. The exceptions were in 1996-1997, 2006, 2008-2009, and 2015-2016. However the growth was not always occurring at the same rate. For example, from 2003 through 2006, the price of ground beef was increasing, but at a slowing pace. In some years, for example 1998-1999 and 2012-2013, the price of ground beef was increasing at an accelerating pace. At the end of this set of data, the price of a pound of ground beef reached a turning point, changing from decreasing in 2016 to increasing, at an increasing rate, in 2017.

Evaluation criteria

Grading procedure: AEPs are graded using the "EMRN" rubric found in the syllabus. **A grade of E or M requires all of the following to be met:**

- There must be a working link that can be used to go to the source of the data.
- The data must satisfy the four criteria given in the "AEP Setup" section.
- There must be a working link to your Google Spreadshseet containing the data set, and the data must be formatted as described above.
- All first and second derivative values must be correct in the table.
- All other calculations are correct and the reasoning is clearly and carefully explained.

  Each of the four grades (E, M, R, N) are assigned as follows:

Grade	Work that earns this grade
E	All of the criteria above are met; there are no significant errors; the writeup is professional and attractive.

M |
R Nearly all of the criteria for an "M" are met; some significant errors present.
N At least one item is left blank or has significant incompletions; or the responses are guesses; or significant errors across the response

Please note: Work that receives a grade of "N" can only be revised after spending one token.

Submitting your work

In MTH 201, Lab work must be typewritten and saved as either a PDF or MS Word file. No part of your submission may involve handwriting; work that is submitted that contains handwriting will be graded N and returned without feedback. To type up your work, you can use MS Word or Google Docs (both of which have equation editors for mathematical notation) or any other computer-based math typesetting tool. Just make sure you save your work as a Word document or PDF (no `.odt`, `.rtf`, or other file extensions are allowed).

When you have your work typed up, double-check it for neatness, correctness, and clarity. Then, go to Blackboard, then **Assignments**, then **AEP**, then **AEP 1**. Clicking on the text "AEP 1" will take you to a place on Blackboard where you can upload your work. All grading and revisions of labs are done entirely on Blackboard. **Do not email your work to the professor** – only Blackboard submissions are accepted.