MTH 201 -- Calculus
Module 11B: Fundamental Theorem of Calculus first look

November 23-24, 2020

Placeholder for review -- wait and see what happens on DP

Suppose f and F are functions so that $f(x) = F'(x)$ (the derivative of F is f) and both functions are continuous and differentiable on $[a, b]$. Then according to the Fundamental Theorem of Calculus,

- $\int_a^b F(x)\,dx = f(b) - f(a)$
- $\int_a^b f(x)\,dx = f(b) - f(a)$
- $\int_a^b f(x)\,dx = f'(b) - f'(a)$
- $\int_a^b f(x)\,dx = F(b) - F(a)$
- None of the above

To 0

Which of the following is/are *antiderivatives* of the function $f(x) = \cos(x)$?

- $F(x) = \sin(x)$
- $F(x) = -\sin(x)$
- $F(x) = \sin(x) + 2$
- $F(x) = -\sin(x) + 2$

To 0

Which of the following is/are *antiderivatives* of the function $f(x) = \cos(x)$?

- $F(x) = \sin(x)$
- $F(x) = 2\sin(x)$
- $F(x) = \sin(x) - \pi$
- $F(x) = \sin(\pi x)$
- $F(x) = x + \sin(x)$
- $F(x) = x\sin(x)$

To 0

Fundamental Theorem of Calculus.

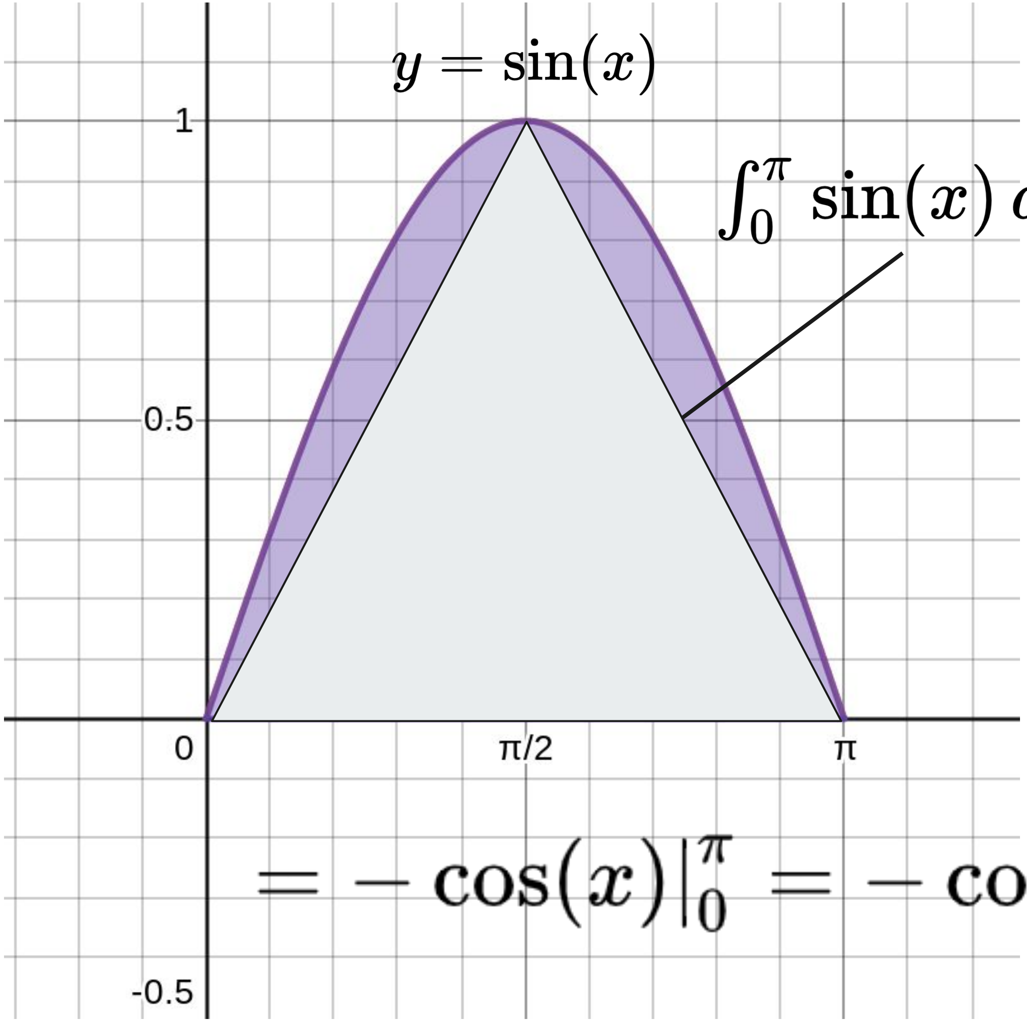
If f is a continuous function on $[a, b]$, and F is any antiderivative of f, then $\int_a^b f(x)\,dx = F(b) - F(a)$.

A common alternate notation for $F(b) - F(a)$ is

$$F(b) - F(a) = F(x)\big|_a^b,$$

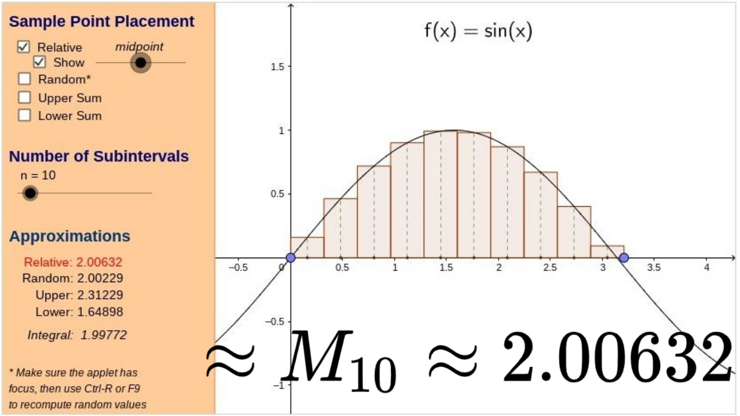
where we read the righthand side as "the function F evaluated from a to b." In this notation, the FTC says that

$$\int_a^b f(x)\,dx = F(x)\big|_a^b.$$



$$\int_0^{\pi} \sin(x)\,dx$$

$$\approx \frac{1}{2}\pi \cdot 1 = \pi/2 \approx 1.57$$



$$\approx M_{10} \approx 2.00632$$

$$= -\cos(x)\Big|_0^{\pi} = -\cos(\pi) - (-\cos(0)) = 2$$

Group activity -- Using the FTC to compute definite integrals [Jamboard]

What we learned/what's next

- The Fundamental Theorem of Calculus -- connects derivatives and integrals
- We can now compute a definite integral in three ways: Approximate by geometry, approximate by Riemann sums, find exact value via FTC/antiderivatives -- in increasing order of hardness

NEXT:

- Followup: Cataloguing common antiderviatives
- Thanksgiving Break!
- **100% online from now on**
- **Module 12:** More on using the FTC + applications of the FTC (Total Change Theorem)