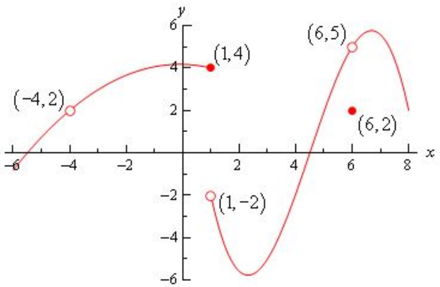
Interpreting, estimating, and using the derivative

MTH 201 -- Module 3A

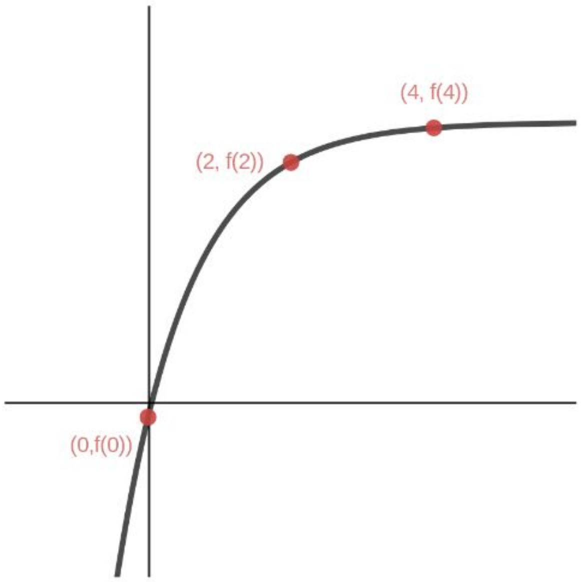
Retrieval practice

The function is shown. The value of



- Equals 2
- Equals 5
- Equals 6
- Does not exist

The graph of a function f is shown along with three points on the graph. Rank the following quantities in order from smallest to largest.



The average rate of change in f on $[0, 2]$

$f'(0)$

$f'(2)$

$f'(4)$

The **derivative** of the function y = f(x) at the point x = a:

$$f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$$

The derivative of f(x) at x = a, f’(a), is all of the following things:

- The **instantaneous rate of change** in f(x) at x = a
- If f(t) is a position at time t, f’(a) is the **instantaneous velocity** at time t = a
- The **slope of the tangent line** to the graph of f(x) at x = a

The **derivative** of the function y = f(x) ***as a function***:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

- Whereas f’(a) (for example, f’(2) or f’(-1)) is a **number**, f’(x) is a **function**.
- It accepts a number as an input, and its output is the derivative of f at that point.
- Since f’(x) is a function, we can *graph it, do algebra* with it, *or make a table* out of it.

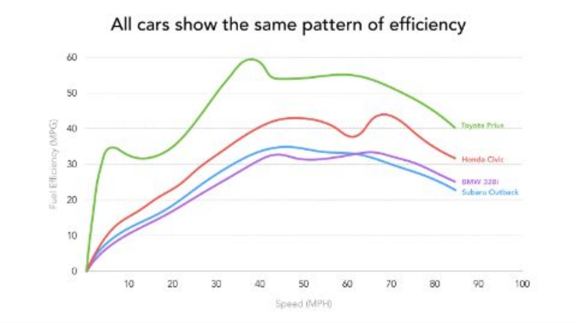
COMPUTING f'(x) is important but not the main point.

Knowing **how and when to apply** f’(x) and then **explain the results** is the main point.

This involves...

- Correctly stating the **units of measurement** of a derivative
- Being able to **estimate the value** of a derivative **from data** (without formulas)
- **Explaining the meaning** of a function value and its derivative values at a point, in a way that makes sense and doesn't have a lot of technical jargon
- **Drawing conclusions about derivatives** based on a real-world situation
- **Drawing conclusions about real-world situations** based on derivative values

The function shown in this graph gives the fuel efficiency (miles per gallon) of different models of cars as a function of the speed at which they are driven (miles per hour). The units of the derivative are:



- Miles per hour
- Miles per gallon
- Miles per gallon per hour
- Miles per hour per gallon
- Miles per gallon per miles per hour
- Miles per hour per miles per gallon

To 0

The function $g(x)$ has the following values: $g(1) = 3$, $g(3) = 5, g(5) = 0$. Using a *central difference* approximation, the value of $g'(3)$ is

- -1.5
- -1.3333
- -0.75
- 0.75
- 1.333
- 1.5
- None of the above

To 0

The function $g(x)$ has the following values: $g(1) = 3$, $g(3) = 5, g(5) = 0$. Which of the following approximations to $g'(3)$ is a positive number? Select ALL that apply.

- Backward difference
- Forward difference
- Central difference
- None of the above

To 0

A company manufactures rope, and the total cost of producing r feet of rope is $C(r)$ dollars. The units of $C'(r)$ are

- Feet
- Dollars
- Dollars per foot
- Feet per dollar

To 0

A company manufactures rope, and the total cost of producing r feet of rope is $C(r)$ dollars. It costs \$800 to produce 2000 feet of rope. Another way of saying this is

- $C(800) = 2000$
- $C(2000) = 800$
- $C'(800) = 2000$
- $C'(2000) = 800$
- None of the above

To 0

A company manufactures rope, and the total cost of producing r feet of rope is $C(r)$ dollars. Suppose we know that $C'(1500) = -0.50$. In practical terms, this would mean that

- When .50 feet of rope is manufactured, the price of manufacturing the rope is dropping.
- When 1500 feet of rope is manufactured, the price of manufacturing the rope is dropping.
- When the price is set at $1500, we have to reduce the amount of rope we're manufacturing.
- When the price is set at $0.50, we have to reduce the amount of rope we're manufacturing.

To 0

A little more on interpretation

Cost of producing *r* feet of rope is C(*r*) dollars.

$$C'(1500) = -0.50$$

Q: What are the units?
A: Dollars per foot.

Incorrect:
- When we produce 1500 feet of rope it costs \$0.50.
- When we produce 1500 feet of rope the cost drops by \$0.50.

Correct but jargon-y: **When we produce 1500 feet of rope, the instantaneous rate of change in the cost is -\$0.50.**

Better: **When we produce 1500 feet of rope, the cost is decreasing at a rate of \$0.50 per foot.**

The derivative is a rate of change, not an *amount* of change.

Suppose that $C(2000) = 800$ and $C'(2000) = 0.35$. A reasonable estimate for $C(2100)$ would be

- $35
- $765
- $800.35
- $835
- None of the above

The temperature H (in degrees Celsius) of a cup of coffee placed on the kitchen counter is given by $H = f(t)$, where t is the number of minutes since it was placed there. Then $f'(t)$ is

- A positive number
- A negative number
- Zero
- There's not enough information to say anything about f'(t)

To 0

Poll locked. Responses not accepted.

The temperature (H in degrees celsius) of a cup of coffee is given by $H = f(t)$ where t is measured in minutes. The units of $f'(35)$ are

- Degrees per minute
- Temperature per minute
- Minutes per degree
- Degrees
- Minutes
- None of the above

To 0

Suppose $f'(35) = -1.5$ and $f(35) = 68$. What can we conclude from this information? (Select all that apply)

- The temperature of the coffee after 35 minutes is 68 degC
- The temperature of the coffee is decreasing after 35 minutes of sitting out
- The temperature after 45 minutes of sitting out will be approximately 53 degC
- The temperature of the coffee has decreased by 1.5 degC over the last 35 minutes

Remember what’s important:

- Correctly stating the **units of measurement** of a derivative
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Feedback:
http://gvsu.edu/s/1zJ