

## Section 7.3: Trigonometric Substitutions

### Goal

In this section, we will introduce a technique of integration called *trigonometric substitution*. This technique is useful for dealing with functions containing the forms:  $a^2 - u^2$ ,  $a^2 + u^2$ , and  $u^2 - a^2$ .

### The 3 basic trig substitutions

form	substitution	identity	triangle
$a^2 - u^2$	$u = a \sin \theta$	$1 - \sin^2 \theta = \cos^2 \theta$	
$a^2 + u^2$	$u = a \tan \theta$	$1 + \tan^2 \theta = \sec^2 \theta$	
$u^2 - a^2$	$u = a \sec \theta$	$\sec^2 \theta - 1 = \tan^2 \theta$	

#### Important Note 1.

1. The pattern of constant<sup>2</sup>, variable<sup>2</sup>, and sign matches each identity.
2. You should always try  $u$ -sub before attempting trig sub. Also, sometimes you may already know a formula for evaluating the integral.

### Examples

OK, let's jump in and do a bunch of examples.

**Example 2.** Integrate.

(a)  $\int \frac{x^3}{\sqrt{1-x^2}} dx$

(b)  $\int \frac{1}{(4x^2+9)^2} dx$

(c)  $\int \frac{\sqrt{x^2 - 25}}{x} dx$

(d)  $\int \frac{1}{\sqrt{3 - 2x - x^2}} dx$

(e)  $\int \sqrt{x^2 + 1} \, dx$

(f)  $\int \frac{x}{\sqrt{x^2 - 25}} \, dx$