MA 3110: Logic and Proof (Fall 2009)
Midterm Exam (take-home portion)

NAME:

Instructions: Prove any *four* of the following six theorems. If you turn in more than four proofs, I will only grade the first four that I see.

This portion of the Midterm Exam is worth 50 points. Each proof is worth 10 points. Your written presentation of the proofs (which includes spelling, grammar, punctuation, clarity, and ledgibility) is worth the remaining 10 points.

I expect your proofs to be *well-written, neat, and organized*. You should write in *complete sentences*. Do not turn in rough drafts. What you turn in should be the "polished" version of potentially several drafts. Feel free to type up your final version.

The simple rules for this portion of the exam are:

1. You may freely use any theorems that we have discussed in class, but you should make it clear where you are using a previous result and which result you are using. For example, if a sentence in your proof follows from Theorem 4.3.21, then you should say so.
2. You are NOT allowed to copy someone else's work.
3. You are NOT allowed to let someone else copy your work.
4. You are allowed to discuss the problems with each other and critique each other's work.

This portion of the Midterm Exam is due at the beginning of class on **Tuesday, 10.27**. You should turn in this cover page and the *four* proofs that you have decided to submit.

To convince me that you have read and understand the instructions, sign in the box below.

Signature:

Good luck and have fun!

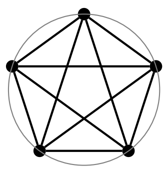
Theorem 1: If x is an odd integer, then 8 divides $x^2 - 1$.

Theorem 2: Let A and B be sets. Then $\mathcal{P}(A \cap B) = \mathcal{P}(A) \cap \mathcal{P}(B)$.*

*This is Theorem 2.5.5(b).

Theorem 3: Let $p_1, p_2, \ldots, p_n$ be n distinct points arranged on a circle. Then the number of line segments joining all pairs of points is $(n^2 - n)/2$.[†]

[†]Here is a picture of what things look like when $n = 5$.



Theorem 4: Let f_n denote the n^{th} Fibonacci number.[‡] Then for all $n \in \mathbb{N}$, $f_{n+6} = 4f_{n+3} + f_n$.

[‡]Recall that the Fibonacci numbers are defined by $f_1 = f_2 = 1$ and $f_n = f_{n-1} + f_{n-2}$ for $n \geq 3$.

Theorem 5: If R and S are both equivalence relations on a set A, then $R \cap S$ is an equivalence relation on A.

Theorem 6: Define the relation C on $\mathbb{R} \times \mathbb{R}$ via

$$(x, y)C(z, w) \text{ iff } x^2 + y^2 = z^2 + w^2.$$ [§]

Then C is an equivalence relation.

[§]This first sentence is a definition; it is something you should assume in your proof.