MA 2550: Calculus I (Fall 2010)
Final Exam

NAME:

Instructions: Answer each of the following questions completely. To receive full credit, you must show sufficient work for each of your answers (unless stated otherwise). How you reached your answer is more important than the answer itself. If something is unclear, or if you have any questions, then please ask. Good luck!

1. (2 points each) Consider the following function.

$$f(x) = \begin{cases} x^2+1, & x<1 \\ x-4, & x \geq 1 \end{cases}$$

For (a)–(e), evaluate the given expression. If a limit does not exist, specify whether the limit equals ∞, $-\infty$, or simply does not exist (in which case, write DNE). For (a)–(d), you do *not* need to justify your answer.

(a) $\lim\limits_{x\to 1^-} f(x) =$ ____________

(b) $\lim\limits_{x\to 1^+} f(x) =$ ____________

(c) $\lim\limits_{x\to 1} f(x) =$ ____________

(d) $f(1) =$ ____________

(e) Is f continuous at $x=1$? Justify your answer.

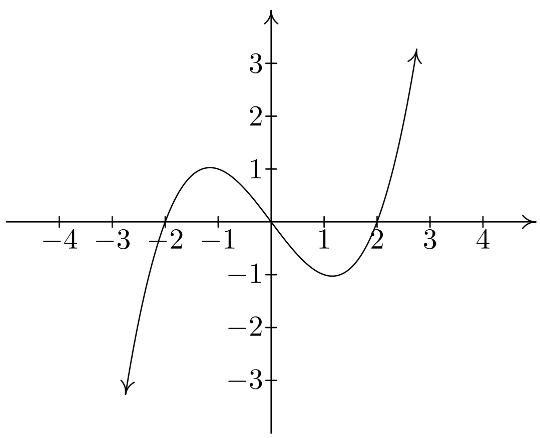
2. (4 points each) Evaluate each of the following limits. If a limit does not exist, specify whether the limit equals ∞, $-\infty$, or simply does not exist (in which case, write DNE). Sufficient work must be shown. Give *exact answers*.

(a) $\lim\limits_{x\to 4} \dfrac{\sqrt{x}-2}{x-4}$

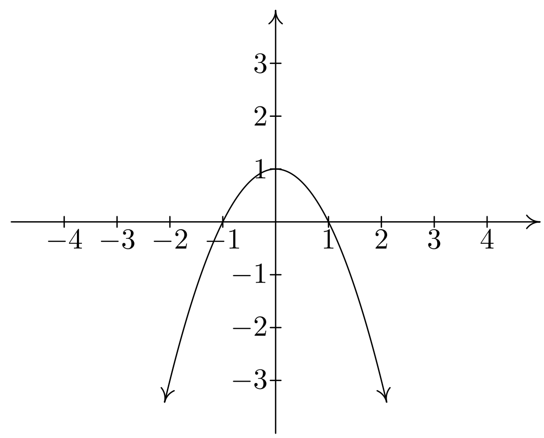
(b) $\displaystyle\lim_{h\to 0} \frac{(x+h)^2+1-(x^2+1)}{h}$ (*Hint:* your answer should be a function of x.)

(c) $\displaystyle\lim_{x\to\infty} \frac{3x^2-5x+1}{x(4-x^2)}$ (You should briefly justify your answer.)

3. (2 points each) Using the graphs below, circle the correct, or approximate, value for each of the following expressions.



Graph of f



Graph of g

(a) $g(1) =$

A. 0 B. $-\frac{3}{2}$ C. -1 D. 1

(b) $f'(0) =$

A. 0 B. $\frac{1}{2}$ C. -1 D. 1

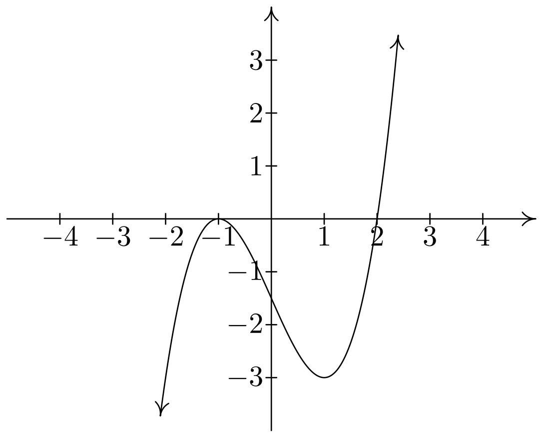
(c) Suppose $h(x) = f(g(x))$. Then $h'(1) =$

A. 0 B. -2 C. -1 D. 2

(d) At how many points does $f'(x) = 0$?

A. 0 B. 1 C. 2 D. 3

4. (2 points each) Let f be a differentiable function. Suppose that the following graph is the graph of the *derivative* of f (i.e., the graph of f').



Graph of f'

(a) Find the x-coordinates of all points on the graph of f where the tangent line is horizontal.

(b) Find the (open) intervals, if any, on which f is increasing.

(c) Find the (open) intervals, if any, on which f is decreasing.

(d) Find the x-coordinates, if any, where f attains a local max.

(e) Find the x-coordinates, if any, where f attains a local min.

5. (5 points) Find $\dfrac{dy}{dx}$ if $\sin^2 x + y^3 = xy$. You do *not* need to simplify your answer, but you should solve for $\dfrac{dy}{dx}$.

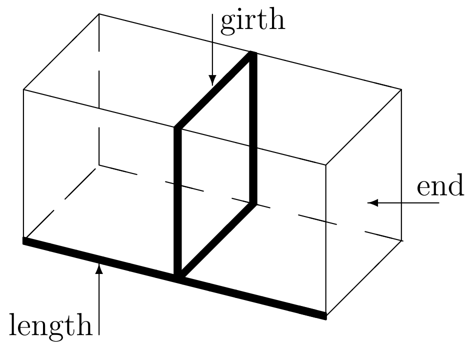
6. (5 points) Consider the following function.

$$f(x) = \frac{\ln x}{x}$$

Find the *equation* of the tangent line to the graph of f when $x = 1$. It does not matter what form the equation of the line takes, but all coefficients should have exact values (i.e., no decimal approximations).

7. (5 points) A large spherical meteor-nugget is speeding towards Earth. If the radius of the meteorite is decreasing at a rate of 1/4 kilometer per day, what is the rate of change in the volume of the meteorite when the radius is 5 kilometers? Give an *exact* answer (i.e., not a decimal approximation). Your answer should be labeled with appropriate units. (Hint: the volume of a sphere of radius r is $V = \frac{4}{3}\pi r^3$.)

8. The U.S. Postal Service will accept a box for domestic shipment only if the sum of its length and girth (distance around) does not exceed 108 inches. What dimensions (length and width) will give a box with a *square* end the largest possible volume? (*Hint:* volume is maximized when length plus girth is equal to 108.)



(a) (2 points) Let V represent the volume of the box (with a square end). Find an equation for V that involves only a single variable and find the feasible domain for V

(b) (4 points) Find the *dimensions* that will maximize the volume of the box. (You must justify that your answer is actually correct.)

9. (5 points) Evaluate the following definite integral using a limit of Riemann sums and right endpoints.

$$\int_0^1 x^2 + 1 \, dx$$

You may find some of the following formulas useful:

$\displaystyle\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$	$\displaystyle\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$	$\displaystyle\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$	$\displaystyle\sum_{i=1}^{n} 1 = n$

$\displaystyle\int_a^b f(x)\, dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x$	$\Delta x = \dfrac{b-a}{n}$	$x_i = a + i\Delta x$

10. (5 points each) Evaluate each of the following integrals. Sufficient work must be shown.

(a) $\displaystyle\int \frac{\sqrt{x} - 5x}{x^{3/2}}\, dx$

(b) $\displaystyle\int_0^1 x^2\sqrt{1 - x^3}\, dx$

11. (5 points) Set up (but do *not* evaluate) an integral that determines the total area of the region bounded by the graphs of $f(x) = x - x^2$ and $g(x) = x^2$.

12. (5 points each) Suppose the velocity function for the position of a tornado moving in a straight line path in a valley is given by $v(t) = -t^2 + 4$, where t is time measured in minutes and position is measured in miles.*

    (a) Find the net distance traveled by the tornado during the first 3 minutes.

    (b) Set up (but do *not* evaluate) an integral that determines the total distance traveled by the tornado during the first 3 minutes.

*I think this tornado is moving faster than your average tornado.

13. (2 points each) Provide an example of each of the following. You do *not* need to justify your answer.

    (a) An *equation* of a function f that is continuous everywhere, but not differentiable at $x = 0$.

    (b) An *equation* of a function g such that g has a critical number at $x = 0$, but g does not have a local maximum or local minimum at $x = 0$.

    (c) An *equation* of a function k such that $k''(0) = 0$, but k does *not* have an inflection point at $x = 0$.

14. (3 points) **Choose your own adventure:** Certainly, there are some things that you studied that I didn't ask you about. Make up a problem that covers a concept that you studied, but didn't appear on this exam and then solve the problem. (Make your problem as difficult or easy as you like.)

15. **Bonus Question:** (5 points) Let $A(x) = \displaystyle\int_0^x t - t^2 \, dt$. Determine where A attains a maximum value on the interval $[0, \infty)$. Justify your answer; argument by picture is sufficient.

