MA 2550: Calculus I (Fall 2010)
Exam 1

NAME:

Instructions: Answer each of the following questions completely. To receive full credit, you must show sufficient work for each of your answers (unless stated otherwise). How you reached your answer is more important than the answer itself. If something is unclear, or if you have any questions, then please ask. Good luck!

1. (8 points) Find the equation of the line through $(1, 1)$ and $(-5, -3)$. (It does not matter what form your final answer takes, but if you make a mistake in an intermediate step while putting your equation in a particular form, it will count against you.)

2. Let $f(x) = x^2 + 2x + 1$ and $g(x) = \sqrt{x} - 1$.

   (a) (6 points) Find $f \circ g(x)$ and simplify your answer.

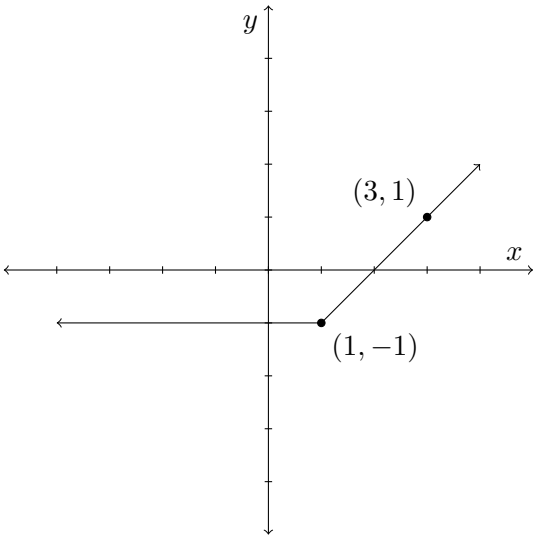
   (b) (4 points) Find the domain of $f \circ g$. (*Be careful!*)

3. A farmer wants to build a fence along a river. She has 500 feet of fencing and wants to enclose a rectangular pen on all three sides with the river as the fourth side.

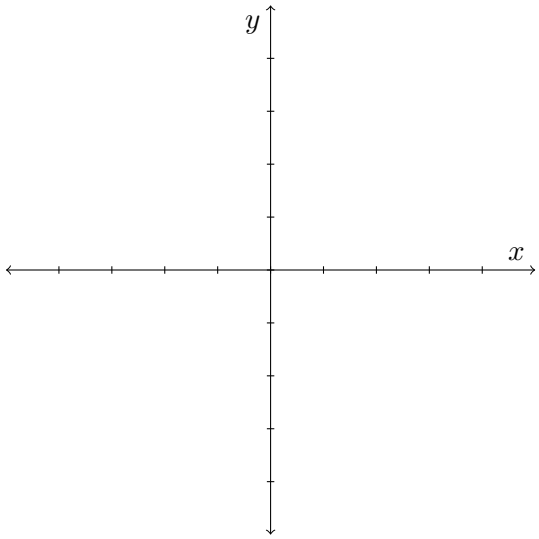
   (a) (6 points) If x is the length of the side perpendicular to the river, determine the area of the pen as a function of x.

   (b) (4 points) Determine the domain of the function you found in part (a).

4. (8 points) Suppose the graph of a function $y = f(x)$ is given in Figure (a) below. Using the axes provided in Figure (b), sketch the graph of the function $y = -f(x+1) + 2$.



(a) Graph of $y = f(x)$



(b) $y = -f(x+1) + 2$

5. (3 points) Let f be a function and suppose that you are trying to guess $\lim_{x\to 0} f(x)$. You plug in $x = 0.1, 0.01, 0.001, \ldots$ and get $f(x) = 0$ for all these values. In fact, you are told that for all $n = 1, 2, \ldots$, $f\left(\frac{1}{10^n}\right) = 0$.

   True or **False** (circle the correct answer): We can conclude that $\lim_{x\to 0} f(x) = 0$.

   Note: You do *not* need to justify your answer.

6. (3 points each) Let $f(x) = \dfrac{x^2 - 1}{x - 1}$ and $g(x) = x + 1$.

   (a) **True** or **False** (circle the correct answer): The functions f and g are equal.

   (b) **True** or **False** (circle the correct answer): $\lim_{x\to -1} f(x) = \lim_{x\to -1} g(x)$.

   *Note:*You do *not* need to justify your answers.

7. (3 points) If $\lim_{x\to 3} f(x) = 0$ and $\lim_{x\to 3} g(x) = 0$, then $\lim_{x\to 3} \dfrac{f(x)}{g(x)}$ (circle the correct answer)

   (a) does not exist.

   (b) must exist.

   (c) not enough information.

   Note: You do *not* need to justify your answer.

8. (8 points) Suppose a nugget is dropped from a height of 10 meters on planet Nuggeton and assume that the height of the nugget after t seconds is given by $h(t) = 10 - 6t^2$, where $h(t)$ is measured in meters. Find the average velocity of the nugget between 2 seconds and 2.1 seconds. (Please round your final answer to *two decimal places* and be sure to label your answer with the appropriate units.)

9. Let $f(x) = x^2 + 1$.

   (a) (6 points) Use algebra to find a simple formula for the slope of the chord between $(1, f(1))$ and $(1 + \Delta x, f(1 + \Delta x))$. Simplify as much as possible.

   (b) (4 points) Determine what happens as Δx approaches 0.

   (c) (4 points) What does your answer to part (b) represent?

10. (6 points) Using the ϵ-δ definition of limit, complete the proof that

$$\lim_{x \to 1} 5x + 2 = 7$$

by filling in the blanks.

Proof. Let $\epsilon > 0$. Choose $\delta =$ _____. Assume that

$$0 < |x - 1| < \delta.$$

Then

$$\begin{aligned} |f(x) - \_\_\_\_\_| &= |5x - 5| \\ &= 5\,|\_\_\_\_\_| \\ &< 5 \cdot \delta \\ &= 5 \cdot \_\_\_\_\_ \\ &= \epsilon. \end{aligned}$$

□

11. (8 points each) Evaluate each of the following limits. If a limit does not exist, write DNE. Sufficient work must be shown and proper notation should be used. In particular, you should write limits where appropriate. Give *exact answers*.

(a) $\lim_{x \to -2} \dfrac{x^2 + 2x}{x^2 - 4}$

(b) $\lim_{x\to 9} \dfrac{\sqrt{x}-3}{x-9}$

(c) $\lim_{x\to 0} \dfrac{\frac{1}{x-2}+\frac{1}{2}}{x}$

12. **Bonus Question:** (5 points, missing this question will not count against you) Using the $\epsilon-\delta$ definition of the limit, prove that

$$\lim_{x\to 0} x\sin\left(\frac{1}{x}\right) = 0.$$

(Hint: use the fact that $|\sin a| < 1$ for any real number a.)

