Solutions

REVIEW WORKSHEET 2 MATH 1300 MAY 01,2008

1. Find the following limits:

(a) $\lim_{x\to\infty} xe^{-x}$ $= \lim_{x\to\infty} \frac{x}{e^x}$ has form $\frac{\infty}{\infty}$, use L'Hopital's Rule.

$= \lim_{x\to\infty} \frac{1}{e^x}$

$= \boxed{0}$

(b) $\lim_{x\to 0^+} \frac{\cot(x)}{\ln(x)}$ $\frac{\infty}{\infty}$ $= \lim_{x\to 0^+} \frac{-\csc^2 x}{1/x} \overset{\lim}{=} \frac{-(\sin^2 x)^{-1}}{x^{-1}} = \lim_{x\to 0^+} \frac{-x}{\sin^2 x} \to \frac{0}{0}$

repeated L'Hopital

$= \lim_{x\to 0^+} \frac{-1}{2\sin x\cos x}$

$= -\infty$

(c) $\lim_{x\to-\infty} \frac{x^5-4x^2+10x}{x^2-4}$ $= \lim_{x\to-\infty} \frac{5x^4-8x+10}{2x}$ (By L'Hopital's Rule)

$= \lim_{x\to-\infty} \frac{20x^3-8}{2}$

$= \boxed{-\infty}$

2. Find the relative maxima and minima, any inflection points, asymptotes and y-intecepts of the following functions and graph.

(a) $\frac{x}{x^2-4}$

y-intercepts when $x=0$:

$y = \frac{0}{0^2-4} = \underline{0}$

$\underline{(0,0)}$

Vertical Asymptotes when $x^2-4=0$

so $(x-2)(x+2)=0$

$\underline{x=2}, \underline{x=-2}$

non vertical Asymptotes: $\lim_{x\to\pm\infty} \frac{x}{x^2-4}$

$= \lim_{x\to\pm\infty} \frac{1}{2x} = 0$

so $y=0$ is a Horizontal Asymptote.

Max, mins:

$f'(x) = \frac{-x^2-4}{(x^2-4)^2} = 0$

no solutions so there are no maxima or minima

~~Inflection pts.~~

More max, mins

$f'(x) = \frac{-x^2-4}{(x^2-4)^2}$ is undefined when $(x^2-4)^2=0$

~~or~~ so $(x-2)^2(x+2)^2=0$

$x = 2, -2$

But these are asymptotes so no maxima or minima!

Inflection pts

$f''(x) = \frac{2x(x^2-12)}{(x^2-4)^3} = 0$

when $2x(x^2-12)=0$

$2x(x-2\sqrt{3})(x+2\sqrt{3})=0$

$x = 0, -2\sqrt{3}, +2\sqrt{3}$

$x < -2\sqrt{3} \Rightarrow f''(x) < 0$

$0 < x < +2\sqrt{3} \Rightarrow f''(x) < 0$

$-2\sqrt{3} < x < 0 \Rightarrow f''(x) > 0$

$x > 2\sqrt{3} \Rightarrow f''(x) > 0$

So inflection pts at

$x = 0, -2\sqrt{3}, 2\sqrt{3}$.

(b) $\frac{x^3+1}{x^3-1}$ $= \frac{(x+1)(x^2-x+1)}{(x-1)(x^2+x+1)}$

y intercepts

$y = \frac{0^3+1}{0^3-1} = -1$

$(0,-1)$

vertical asymptote at $\underline{x=1}$

non vertical asymptote:

$\lim_{x\to\pm\infty} \frac{x^3+1}{x^3-1} = 1$

Horizontal Asymptote at $y=1$

$f'(x) = \frac{-6x^2}{(x^3-1)^2} = 0$

so $\underline{x=0}$

$f'(x)$ is undefined when $x=1$, This is an asymptote!

$f''(x) = \frac{-12x(4x^3-1)}{(x^3-1)^3} = 0$

when $\underline{x = 0, \frac{1}{\sqrt[3]{4}}}$

if $x<0$, $f''(x)>0$

if $0 < x < \frac{1}{\sqrt[3]{4}}$, $f''(x) < 0$

if $x > \frac{1}{\sqrt[3]{4}}$, $f''(x) < 0$

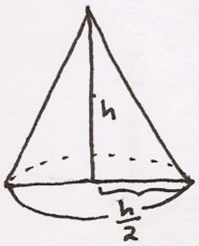
so $x=0$ is not a relative maxima or minima

so $x=0$ inflection pt.

$\underline{(0,0)}$

3. Sand pouring from a chute forms a conical pile whose height is always equal to its diameter. The rate at which is height is increasing is a constant 5 ft/min. At what rate is the sand pouring when thepile is 10ft high?

The rate the sand is pouring is the rate at which The Volume is increasing, So we need to find $\frac{dV}{dt}$ where V is the volume of the pile



We Know: $\frac{1}{2}h = r$ ①

$V = \frac{1}{3}\pi r^2 h$ ②

$\frac{dh}{dt} = 5$ ③

h = height
r = radius of The base
V = Volume

Using ① and ② we get:

$$V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h = \frac{1}{12}\pi h^3$$

By Implicit Differentiation:

$$\frac{dV}{dt} = \frac{1}{12}\pi\left(3h^2\frac{dh}{dt}\right) = \frac{1}{4}\pi h^2\frac{dh}{dt}$$

when h = 10,

$$\frac{dV}{dt} = \frac{1}{4}\pi(10)^2(5) = \boxed{125\pi\ \frac{ft}{min}}$$

4. For the function $\sqrt{25-x^2}$ on the interval $[-5,3]$, find the c guaranteed by the mean value theorem.

The MVT guarantees that there is a value $c \in -5, 3$ such that

$$f'(c) = \frac{f(3)-f(-5)}{3-(-5)}$$

$f(3) = \sqrt{25-9} = \sqrt{16} = 4$

$f(-5) = \sqrt{25-25} = 0$

So $\frac{f(3)-f(-5)}{3-(-5)} = \frac{4-0}{3+5} = \frac{4}{8} = \frac{1}{2}$

$$f'(x) = \frac{-x}{\sqrt{25-x^2}}$$

So set $f'(c) = \frac{1}{2}$:

$\frac{-x}{\sqrt{25-x^2}} = \frac{1}{2}$ square both sides

$\frac{x^2}{25-x^2} = \frac{1}{4}$ cross multiply

$4x^2 = 25 - x^2$

$5x^2 = 25$

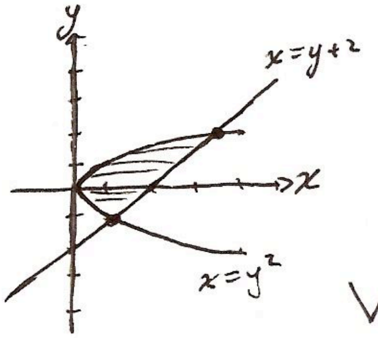
$x^2 = 5$

$\underline{x = \sqrt{5}}$

so $\boxed{c = \sqrt{5}}$

5. Find the volumes of the following solids of revolution using either the disk/washer method or the shell method (first sketch the regions)

(a) The solid formed by rotating the region bounded by the functions $x = y^2$ and $x = y + 2$ about the y-axis.



use Washers.

Limits of integration: $y^2 = y + 2$

$y^2 - y - 2 = 0$

$(y-2)(y+1) = 0 \quad y = \underline{-1, 2}$

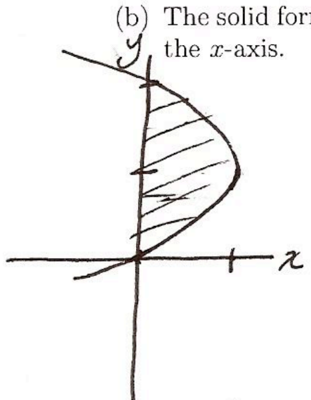
$$V = \pi \int_{-1}^{2} (\underbrace{y+2}_{R_1})^2 - (\underbrace{y^2}_{R_2})^2 \, dy$$

$$= \pi \left[\frac{1}{3}(y+2)^3 - \frac{1}{5}y^5 \right]_{-1}^{2}$$

$$= \pi \left[\left(\left(\frac{64}{3}\right) - \left(\frac{32}{5}\right) \right) - \left(\frac{1}{3} + \frac{1}{5} \right) \right]$$

$$= \pi \left(21 - \frac{33}{5} \right) = \boxed{\frac{72}{5}\pi}$$

(b) The solid formed by rotating the region bounded by the function $x = 2y - y^2$ and the y-axis about the x-axis.



use Shells

Limits of integration:

$2y - y^2 = 0$

$y(2-y) = 0 \quad y = \underline{0, 2}$

$$V = 2\pi \int_0^2 y(2y - y^2)\, dy$$

$$= 2\pi \int_0^2 2y^2 - y^3 \, dy$$

$$= 2\pi \left[\frac{2}{3}y^3 - \frac{1}{4}y^4 \right]_0^2 = 2\pi \left[\frac{16}{3} - \frac{16}{4} \right] = \boxed{\frac{8\pi}{3}}$$