MA 2550: Calculus I (Fall 2009)
Final Exam

NAME: (1 point!)

Instructions: Answer each of the following questions completely. To receive full credit, you must *justify* each of your answers (unless stated otherwise). How you reached your answer is more important than the answer itself. If something is unclear, or if you have any questions, then please ask. Good luck!

1. (2 points each) Consider the following function.

$$f(x) = \begin{cases} x^2 + 1, & x < 1 \\ x - 4, & x \geq 1 \end{cases}$$

For (a)–(e), evaluate the expression. If a limit does not exist, specify whether the limit equals ∞, $-\infty$, or simply does not exist (in which case, write DNE). For (a)–(d), you do *not* need to justify your answer.

(a) $\lim_{x \to 1^-} f(x) =$

(b) $\lim_{x \to 1^+} f(x) =$

(c) $\lim_{x \to 1} f(x) =$

(d) $f(1) =$

(e) Is f continuous at $x = 1$? Justify your answer.

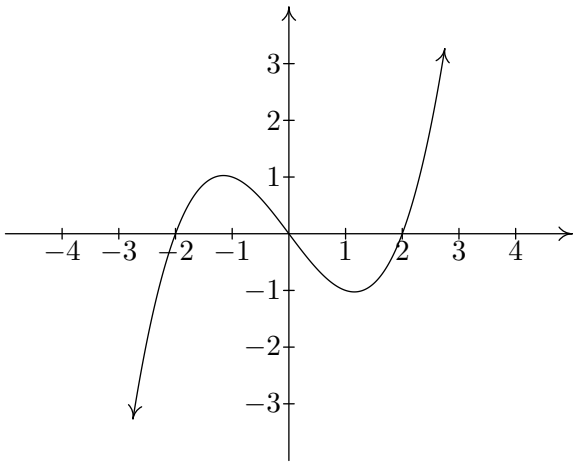
2. (4 points each) Evaluate each of the following limits. If a limit does not exist, specify whether the limit equals ∞, $-\infty$, or simply does not exist (in which case, write DNE). Sufficient work must be shown. Give *exact answers*.

   (a) $\lim_{x \to 4} \dfrac{\sqrt{x} - 2}{x - 4}$

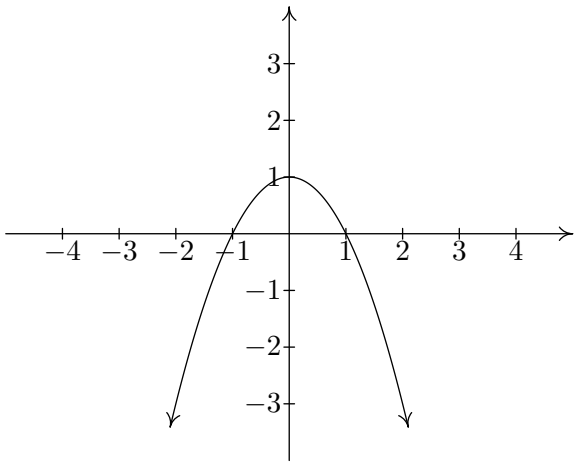
   (b) $\lim_{h \to 0} \dfrac{(x+h)^2 + 1 - (x^2 + 1)}{h}$ (Hint: your answer should be a function of x.)

   (c) $\lim_{x \to \infty} \dfrac{3x^2 - 5x + 1}{x(4 - x^2)}$ (You should briefly justify your answer.)

3. (2 points each) Using the graphs below, circle the correct, or approximate, value for each of the following expressions.



Graph of f



Graph of g

(a) $g(1)$

A. 0 B. $-\frac{3}{2}$ C. -1 D. 1

(b) $f'(0)$

A. 0 B. $\frac{1}{2}$ C. -1 D. 1

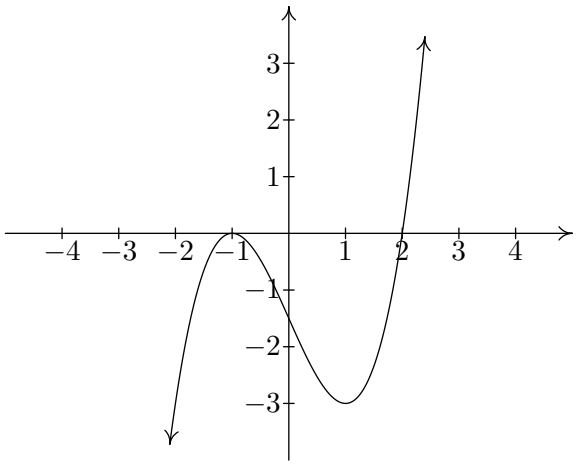
(c) Suppose $h(x) = f(g(x))$. Then $h'(1) =$

A. 0 B. -2 C. -1 D. 2

(d) At how many points does $f'(x) = 0$?

A. 0 B. 1 C. 2 D. 3

4. (2 points each) Let f be a differentiable function. Suppose that the following graph is the graph of the *derivative* of f (i.e., the graph of f').



Graph of f'

(a) Find the x-coordinates of all points on the graph of f where the tangent line is horizontal.

(b) Find the (open) intervals, if any, on which f is increasing.

(c) Find the (open) intervals, if any, on which f is decreasing.

(d) Find the x-coordinates, if any, where f attains a local max.

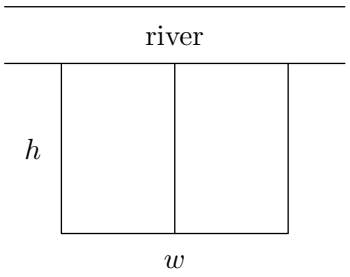
(e) Find the x-coordinates, if any, where f attains a local min.

5. (5 points) Find $\dfrac{dy}{dx}$ if $\sin^2 x + y^3 = xy$. You do *not* need to simplify your answer, but you should solve for $\dfrac{dy}{dx}$.

6. (5 points) Let $f(x) = \cos x$. Find the *equation* of the tangent line to the graph of f when $x = \pi/6$. It does not matter what form the equation of the line takes, but all coefficients should have exact values (i.e., no decimal approximations).

7. (5 points) The shock-waves from an earthquake on the ocean floor radiate out in the form of a circle on the surface of the ocean from its epicenter. If the radius of the shock-waves is increasing at a rate of 2 miles per second, what is the rate of change of the area enclosed by the radiating shock-waves when the radius is 5 miles? Give an exact answer. Your answer should be labeled with appropriate units.

8. A farmer has 1200 feet of fencing with which to enclose a pasture for grazing nuggets. The farmer only needs to enclose 3 sides of the pasture since the remaining side is bounded by a river (no, nuggets can't swim). In addition, some of the nuggets don't get along with some of the other nuggets. He plans to separate the troublesome nuggets by forming two adjacent corrals (see figure).



(a) (2 points) Let A represent the area of the rectangular pen. Find an equation for A that involves only the variable h.

(b) (2 points) Find the feasible domain for A. (Hint: how small can h be? How large can h be?)

(c) (5 points) Using your answers to (a) and (b), determine the *dimensions* that will maximize the area of the rectangular pen. (Justifying your answer will not only make sure that you receive full credit, but will also ensure that you don't make a mistake.)

9. (5 points each) Evaluate each of the following integrals. Sufficient work must be shown.

(a) $\displaystyle\int_0^1 x^2\sqrt{1-x^3}\,dx$

(b) $\displaystyle\int \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx$

10. (5 points) Setup (but do *not* evaluate) an integral that would determine the area of the region bounded by the graphs of $f(x) = 4x$ and $g(x) = x^2 - 5$.

11. (5 points) Setup (but do *not* evaluate) an integral that would determine the volume of the solid obtained by revolving the region bounded by the graphs of $y = 2 - x^2$ and $y = x^2$ about the x-axis.

12. (5 points) Setup (but do *not* evaluate) an integral that would determine the volume of the solid obtained by revolving the region bounded by the graphs of $f(x) = 1$ and $g(x) = x^2$ about the line $x = 2$.

13. (5 points) Let $f(x) = 4x - \dfrac{1}{x^2}$. Find the average value of f over the interval $[1, 3]$.

14. (5 points) A common theme this semester has been to start with an approximation for something that is seemingly difficult to compute and then take a limit to get an exact answer. Describe ONE such situation that we have discussed this semester. I'm looking for an intuitive understanding, but you should provide some detail using proper notation. (Using pictures to aid in your description will be very useful.)

15. **Bonus Question:** (5 points) What is it that we computed in problem 2b? Be as specific as possible.