MA 3540: Calculus III-Spring 2012
Exam 3

NAME:	(2 points!)

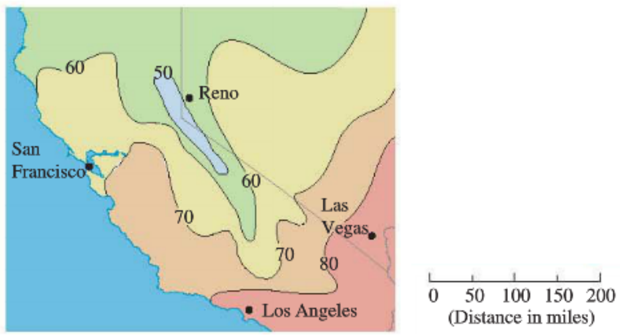
Instructions: Answer each of the following questions completely. To receive full credit, you must show sufficient work for each of your answers (unless stated otherwise). How you reached your answer is more important than the answer itself. If something is unclear, or if you have any questions, then please ask. Good luck!

1. (10 points) Find the equation of the tangent plane to $z = 2x^2y^3$ at the point $(1, 1, 2)$.

2. (10 points) Suppose $z = f(x, y)$ is a differentiable function such that $f(0, 1) = 2$, $f_x(0, 1) = 1$, and $f_y(0, 1) = -2$. Using this information, approximate $f(.1, .9)$. *Hint:* Use a linear approximation.

 Written by D.C. Ernst

3. (10 points) Using the weather map below, estimate the rate of change of the temperature at Reno in the direction of San Francisco. Assume that the contours are labeled in degrees Fahrenheit. When approximating distances, round to the nearest 50 miles. (Be sure to use the key when approximating distances.)



4. (12 points) Find the directional derivative of $f(x, y) = \ln(x^2 + y^2)$ at the point $(2, 1)$ in the direction of the vector $\langle -1, 2\rangle$. Provide an *exact* value.

5. (10 points) Suppose $(0,2)$ is a critical point of a function f with continuous second partial derivatives such that $f_{xx}(0,2) = -1$, $f_{yy}(0,2) = 2$, and $f_{xy}(0,2) = -8$. Determine whether f has a local maximum, a local minimum, or a saddle point at $(0,2)$. Be sure to justify your answer.

6. (12 points) Let $f(x,y) = 2x + 4y - x^2$ and let D be the region determined by the lines $x = 0$, $y = 5$, and $y = x$. Using the fact that $(1,4)$ is the only critical number of f, find the absolute maximum and minimum values of f over the region D.

7. (10 points) Consider the solid that lies below $z = e^{x^2y^2}$ and above the rectangle $[0,2] \times [0,2]$. Use a Riemann sum with $m = n = 2$ to approximate the volume of this solid. Use the lower lefthand corners for your sample points. Round your answer to two decimal places.

8. (12 points) Evaluate the following integral, where $D = \{(x, y) : 0 \leq x \leq 1, 0 \leq y \leq x^2\}$. Provide an *exact* value.

$$\iint_D \frac{y}{x^5 + 1} \, dA$$

9. (12 points) Evaluate the following integral by converting to polar coordinates, where D is the region bounded by the circle $x^2 + y^2 = 1$ in the first quadrant only. Provide an *exact* value.

$$\iint_D e^{-x^2 - y^2} \, dA$$