MA 2560: Calculus II (Spring 2009)
Exam 1

NAME:

Instructions: Answer each of the following questions completely. To receive full credit, you must *justify* each of your answers (unless stated otherwise). How you reached your answer is more important than the answer itself. If something is unclear, or if you have any questions, then please ask. Good luck!

Here are some potentially useful formulas. You should *not* expect to use all of them.

$$\frac{d}{dx}[\sinh x] = \cosh x \qquad \frac{d}{dx}[\cosh x] = \sinh x$$

$$\frac{d}{dx}[\tanh x] = \text{sech}^2 x \qquad \frac{d}{dx}[\text{sech } x] = -\text{sech } x \tanh x$$

$$\frac{d}{dx}[\sin^{-1} x] = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}[\cos^{-1} x] = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}[\tan^{-1} x] = \frac{1}{1+x^2} \qquad \frac{d}{dx}[\sec^{-1} x] = \frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}[\sinh^{-1} x] = \frac{1}{\sqrt{1+x^2}} \qquad \frac{d}{dx}[\cosh^{-1} x] = \frac{1}{\sqrt{x^2-1}}$$

$$\frac{d}{dx}[\tanh^{-1} x] = \frac{1}{1-x^2} \qquad \frac{d}{dx}[\text{sech}^{-1} x] = \frac{-1}{x\sqrt{1-x^2}}$$

$$\int \sinh u \, du = \cosh u + C \qquad \int \cosh u \, du = \sinh u + C$$

$$\int \text{sech}^2 u \, du = \tanh u + C \qquad \int \text{sech } u \tanh u \, du = -\text{sech } u + C$$

$$\int \frac{1}{\sqrt{a^2-u^2}} \, du = \sin^{-1} \frac{u}{a} + C \qquad \int \frac{1}{u^2+a^2} \, du = \frac{1}{a} \tan^{-1} \frac{u}{a} + C$$

$$\int \frac{1}{u\sqrt{u^2-a^2}} \, du = \frac{1}{a} \sec^{-1} \frac{u}{a} + C \qquad \int \frac{1}{\sqrt{u^2+a^2}} \, du = \sinh^{-1} \frac{u}{a} + C$$

$$\int \frac{1}{\sqrt{u^2-a^2}} \, du = \cosh^{-1} \frac{u}{a} + C \qquad \int \frac{1}{a^2-u^2} \, du = \frac{1}{a} \tanh^{-1} \frac{u}{a} + C$$

1. Let $f(x) = x + \arcsin e^x$. It turns out that f is a one-to-one function, which implies that f^{-1} exists (you do *not* need to show this).

   (a) (2 points) Find $f^{-1}\left(\frac{\pi}{2}\right)$.

   (b) (8 points) Find $\left(f^{-1}\right)'\left(\frac{\pi}{2}\right)$.

2. (8 points) Find an equation of the tangent line to the graph of $f(x) = \dfrac{e^{1/x}}{x}$ at the point $(1, e)$. (Your answer does not need to be in any particular form, but you should use exact values.)

3. (8 points each) Use logarithmic differentiation to find the derivative of each of the following functions. (You do *not* need to simplify your answer, but you do need to write $\frac{dy}{dx}$ as a function of x only.)

   (a) $y = (\tanh x)^x$

   (b) $y = \dfrac{\sqrt{1-x}}{x^2(x+5)^{3/2}}$

4. (8 points) Evaluate each of the following limits.

   (a) $\lim_{x\to\infty} x \sin\left(\frac{1}{x}\right)$

   (b) $\lim_{x\to\infty} x^{1/x}$

5. (10 points each) Integrate each of the following indefinite or definite integrals. For the definite integrals, you should give *exact* answers (i.e., not decimal approximations using your calculator).

   (a) $\displaystyle\int_0^1 \frac{\arctan x}{1+x^2}\,dx$

   (b) $\displaystyle\int \frac{1}{9x^2+25}\,dx$

(c) $\displaystyle\int \frac{1}{\sqrt{4-16x^2}}\ dx$

(d) $\displaystyle\int \frac{1}{\sqrt{e^{2x}-1}}\ dx$

(e) $\displaystyle\int_0^{\pi} x\cos x\ dx$