Chapter 2: Sets
Sections 2.1–2.3

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We can also use interval notation to denote sets (of real numbers).

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Notice that the statement "If $x \in S$, then $x \in A$" is an implication. So, if we want to prove that $S \subseteq A$, what do we need to do?

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We will prove this theorem for homework and it will definitely be one of the ones that I ask you to present.

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If B is a subset of X and $B \neq X$, then we say that B is a proper subset of X.

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Let A and B be sets.

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(For now, an intuitive understanding of indexing will suffice.)

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so that $I_n = [0, 1/n]$. In this case, we say that the sets we are working with are indexed by $\mathbb{N} = \{1, 2, 3, \ldots\}$. We can write

$$A = \{I_1, I_2, I_3, \ldots\} = \{I_n : n \in \mathbb{N}\} = \{I_n\}_{n \in \mathbb{N}} = \{I_n\}_{n=1}^{\infty}.$$

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We see that

$$\bigcup_{n \in \mathbb{N}} I_n = [0, \infty) = \{x \in \mathbb{R} : x \geq 0\}.$$

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