MA 2550: Calculus I (Fall 2009)
Exam 3

NAME: (2 points!)

Instructions: Answer each of the following questions completely. To receive full credit, you must *justify* each of your answers (unless stated otherwise). How you reached your answer is more important than the answer itself. If something is unclear, or if you have any questions, then please ask. Good luck!

1. (8 points) Suppose f is a function with the following properties.

   (a) $f(-3) = 0$, $f(-2) = -2$, and $f(0) = 0$

   (b) $\lim_{x \to 2^-} f(x) = \infty$ and $\lim_{x \to 2^+} f(x) = -\infty$

   (c) $\lim_{x \to \infty} f(x) = 0$ and $\lim_{x \to -\infty} f(x) = \infty$

   (d) $f'(-2) = 0$ and $f'(0) = 0$

   (e) $f'(x) > 0$ on $(-2, 0)$, $(0, 2)$, and $(2, \infty)$

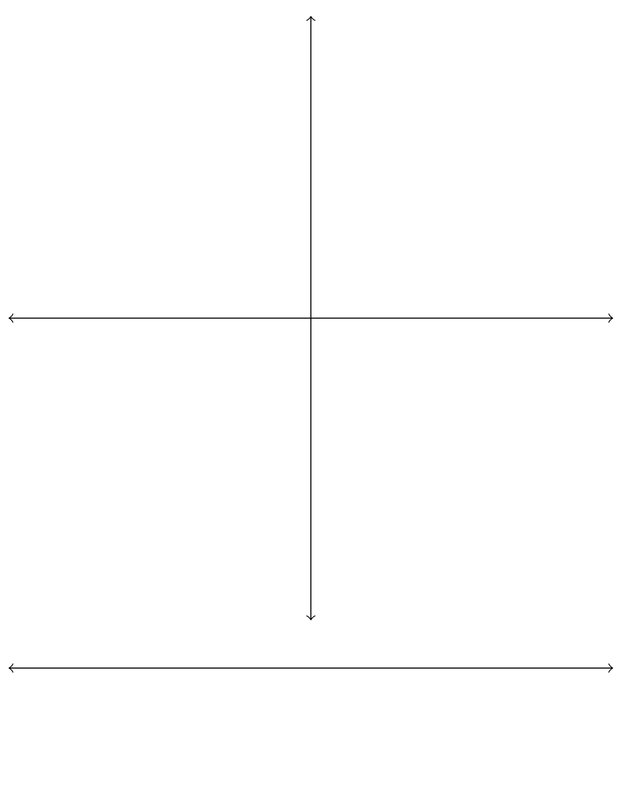
   (f) $f'(x) < 0$ on $(-\infty, -2)$

   (g) $f''(-1) = 0$ and $f''(0) = 0$

   (h) $f''(x) > 0$ on $(-\infty, -1)$ and $(0, 2)$

   (i) $f''(x) < 0$ on $(-1, 0)$ and $(2, \infty)$

   Using the above information, make a sketch of the graph of f. You do *not* need to justify your answer.



2. (4 points each) Consider the following function.

$$f(x) = 5x^{2/3} + x^{5/3}$$

(a) Find all critical numbers of f.

(b) Using your answer(s) from part (a), determine whether each critical number determines a local maximum, local minimum, or neither. (You must show sufficient work to justify your answer.)

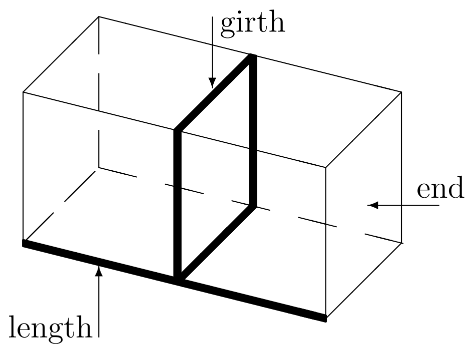
3. (4 points each) Consider the following function.

$$f(x) = \frac{x^5}{20} - \frac{x^4}{6} + \frac{x^3}{6} + 5x + 1$$

(a) Find the second derivative of f.

(b) Using your answer from part (a), find the x-values of all inflection points for f.

4. The U.S. Postal Service will accept a box for domestic shipment only if the sum of its length and girth (distance around) does not exceed 108 inches. What dimensions (length and width) will give a box with a *square* end the largest possible volume? (You must show sufficient work to justify that your answer is the correct one. In particular, you should consider the domain of the function that you are maximizing.) Hint: volume is maximized when length plus girth is equal to 108.



(a) (4 points) Let V represent the volume of the box (with a square end). Find an equation for V that involves only a single variable.

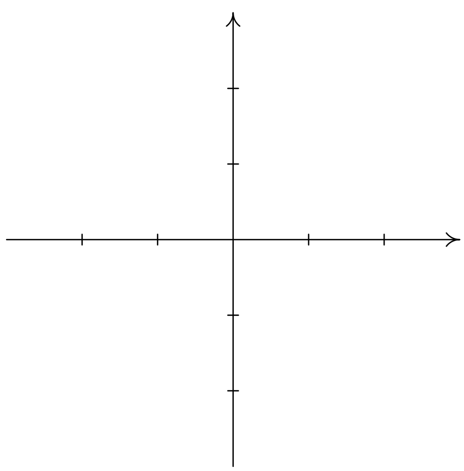
(b) (4 points) Find the feasible domain for V.

(c) (6 points) Using your answers to (a) and (b), find the *dimensions* that will maximize the volume of the box. (You must justify that your answer is actually correct.)

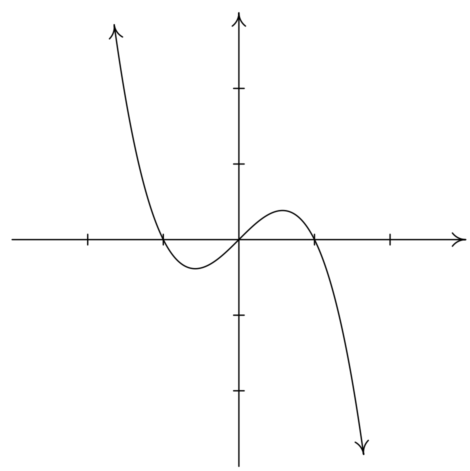
5. (8 points) Find *all* asymptotes of the following function.

$$f(x) = \frac{2x^3 - 2x^2 - 1}{x^2 - 1}$$

6. (8 points) Using the graph of the function f given below, sketch a possible graph for the *antiderivative* of f, denoted by F.



Graph of f



Graph of F

7. (8 points each) Evaluate each of the following indefinite integrals. Sufficient work must be shown.

(a) $\displaystyle\int \frac{1+x^2}{3x^5}\,dx$

(b) $\displaystyle\int \sqrt{x}(1-x)\,dx$

(c) $\displaystyle\int \sec x(\tan x+\sec x)\,dx$

8. (8 points) At this time, we do *not* know how to evaluate the following definite integral using a limit of Riemann sums.

$$\int_0^{\pi} \cos^2 x \; dx$$

However, we can approximate this integral. Approximate the above integral using 4 equal width rectangles and right endpoints. (You should give an *exact* answer for your approximation.)

9. (8 points) Evaluate the following definite integral using a limit of Riemann sums and right endpoints.

$$\int_0^1 x^2 + 1 \; dx$$

You may find some of the following formulas useful:

$\displaystyle\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$	$\displaystyle\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$	$\displaystyle\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$	$\displaystyle\sum_{i=1}^{n} 1 = n$

$\displaystyle\int_a^b f(x)\; dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x$	$\displaystyle\Delta x = \frac{b-a}{n}$	$x_i = a + i\Delta x$

10. (4 points) Provide an example of an *equation* of a function f such that f has a critical number at $x = 0$, but f does not have a local maximum or local minimum at $x = 0$.

11. **Bonus Question:** (5 points) Find the exact value of the following definite integral by interpreting it in terms of net signed area. (Be sure to explain your answer.)

$$\int_0^{2\pi} \sin x \; dx$$