Supplementary Homework Exercises for Section 11.5: Comparison Tests

Exercises

Answer each of the following questions.

S1. Suppose $\sum a_n$ and $\sum b_n$ are series with positive terms such that $\sum b_n$ converges.

(a) If $a_n > b_n$ for all n, what can you say about $\sum a_n$? Explain.

(b) If $a_n < b_n$ for all n, what can you say about $\sum a_n$? Explain.

S2. Suppose $\sum a_n$ and $\sum b_n$ are series with positive terms such that $\sum b_n$ diverges.

(a) If $a_n > b_n$ for all n, what can you say about $\sum a_n$? Explain.

(b) If $a_n < b_n$ for all n, what can you say about $\sum a_n$? Explain.

S3. Determine whether each of the following series is convergent or divergent. You need to show sufficient justification and you can use any of our current tests for convergence.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{2n^3+1}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos^2 n}{n^2+1}$

(c) $\displaystyle\sum_{n=2}^{\infty} \frac{\sqrt{n}}{n-1}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{2+(-1)^n}{n^{3/2}}$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{e^{1/n}}{n}$

(f) $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{n^n}$

(g) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+1}}$

(h) $\displaystyle\sum_{n=1}^{\infty} \frac{1+5^n}{1+4^n}$

(i) $\displaystyle\sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right)$

S4. Provide an example of a pair of series $\sum a_n$ and $\sum b_n$ where $\displaystyle\lim_{n\to\infty} \frac{a_n}{b_n} = 0$ and $\sum b_n$ diverges, but $\sum a_n$ converges.

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