Section 9.10: Surface Area

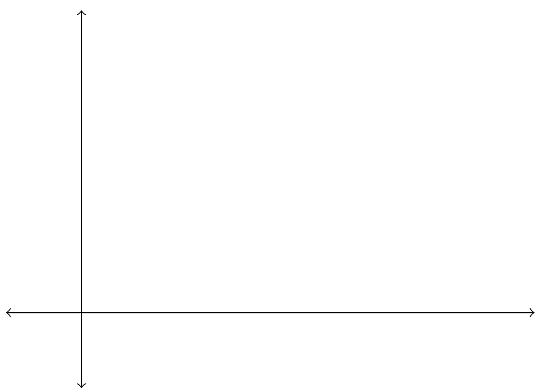
Goal

In this section, we will learn how integrals can be used to find the surface area of a solid of revolution.

Surface Area

As in the previous section, suppose that f is a "smooth" function on the interval $[a, b]$. We want to be able to find the surface area of the solid of revolution obtained by revolving $y = f(x)$ around the x-axis or y-axis. As usual, we first approximate and then take the limit.

Here's the picture:



The surface area of one of the approximating "slices" is equal to $2\pi r(c_i)\sqrt{1 + [f(c_i)]^2}\ \Delta x$. Adding up all of the "slices" and then taking the limit, we obtain

$$S = \text{surface area} = \lim_{n\to\infty} \sum_{i=1}^{n} 2\pi r(c_i)\sqrt{1 + [f(c_i)]^2}\ \Delta x.$$

Therefore, the surface area of the solid of revolution obtained by revolving the smooth curve $y = f(x)$ over the interval $[a, b]$ about the x-axis (respectively, y-axis) is given by

$$\boxed{S = 2\pi \int_a^b r(x)\sqrt{1 + [f'(x)]^2}\ dx = 2\pi \int_a^b r(x)\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\ dx,}$$

where $r(x) = f(x)$ (respectively, $r(x) = x$).

Alternatively, we may write

$$S = 2\pi \int_a^b r(x)\ ds.$$

More examples

Now, let's do a couple of surface area examples.

Example 1. Find surface area of the solid obtained by revolving the graph of $y = x^3$ on the interval $[0, 2]$ about the x-axis.

Example 2. Find surface area of the solid obtained by revolving the graph of $f(x) = \dfrac{x^5}{5} + \dfrac{1}{12x^3}$ on the interval $[1, 2]$ about the y-axis.