MA 3540: Calculus III-Spring 2012
Exam 2

NAME:

Instructions: Answer each of the following questions completely. To receive full credit, you must show sufficient work for each of your answers (unless stated otherwise). How you reached your answer is more important than the answer itself. If something is unclear, or if you have any questions, then please ask. My favorite word is nugget. Good luck!

1. (6 points each) Use your awesome artistic skills to sketch the graph of each of the following surfaces in $\mathbb{R}^3$.

   (a) $2x + 3y + z = 0$

   (b) $z = y^2$

   (c) $y^2 = z^2 + x^2$

2. (6 points each) Use your awesome artistic skills to make a rough sketch of the contour plot in $\mathbb{R}^2$ for each of the following functions of two variables. For your contour plot, draw several level curves that correspond to equally spaced z-values. You do *not* need to label the z-values of the level curves.

   (a) $g(x, y) = x + y$

   (b) $h(x, y) = x^2$

3. (8 points) Find an equation of the plane that passes through the point $(2, 3, -1)$ and contains the line given by $x = 1 + 3t, y = -2t, z = 2 - t$.

4. (8 points) The following limit does *not* exist. Explain why.

$$\lim_{(x,y)\to(0,0)} \frac{xy}{x^2 + y^2}$$

5. (8 points) The following limit does exist. Find its limit. You must sufficiently justify your answer.

$$\lim_{(x,y)\to(0,0)} \frac{x^2 y}{x^2 + y^2}$$

6. Consider the curve given by $\vec{r}(t) = \langle \cos t, \sin t, t \rangle$.

   (a) (4 points) Show that the curve lies on the cylinder $x^2 + y^2 = 1$.

   (b) (6 points) Use (a) to help sketch the graph of this curve.

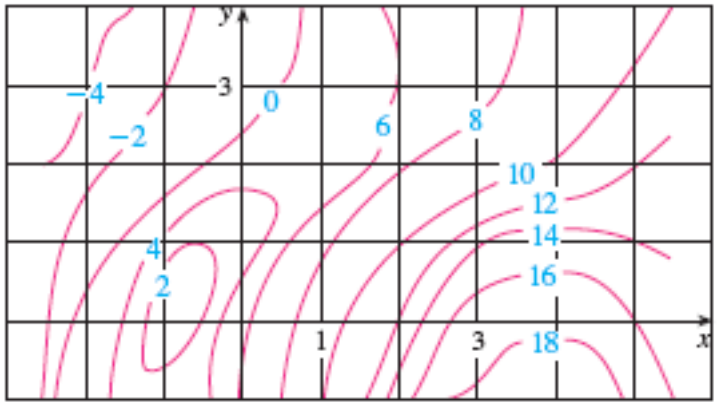
   (c) (6 points) Find the unit tangent vector $\vec{T}(t)$ at $t = 0$ and then add this vector to your picture in part (b) so that the tail of the vector sits at the point corresponding to $t = 0$.

(d) (6 points) Find the arc length of the curve from $t = 0$ to $t = \pi$.

(e) (6 points) Find the curvature at any point on the curve.

7. (6 points) Let $f(x,y) = x^2y^3 + 2xy^2$. Find $f_{xy}(1,1)$.

8. (4 points each) A contour plot is given for a function f of two variables. Use it to answer each of the following questions.



(a) Determine the sign (positive or negative) of $f_x(2,1)$. Justify your answer.

(b) Determine the sign (positive or negative) of $f_y(2,1)$. Justify your answer.

(c) How does $f_x(2,1)$ compare to $f_x(2,0)$? Justify your answer.

9. **Bonus Question** (2 points) What does your answer to problem 7 represent?

10. **Bonus Question** (1 point) What is Dana's favorite word?

