Plymouth State University

MA 2550: Calculus I, Fall 2008

FINAL EXAM

NAME: (1 point!)

Instructions: Answer each of the following questions completely. To receive full credit, you must *justify* each of your answers (unless stated otherwise). How you reached your answer is more important than the answer itself. If something is unclear, or if you have any questions, then please ask. Good luck!

1. (3 points each) Consider the following function.

$$f(x) = \begin{cases} x^2 + 1, & x < 1 \\ x - 4, & x \geq 1 \end{cases}$$

For (a)–(e), evaluate the expression. If a limit does not exist, specify whether the limit equals ∞, $-\infty$, or simply does not exist (in which case, write DNE).

(a) $\lim\limits_{x \to 1^-} f(x) =$

(b) $\lim\limits_{x \to 1^+} f(x) =$

(c) $\lim\limits_{x \to 1} f(x) =$

(d) $f(1) =$

(e) Is f continuous at $x = 1$? Justify your answer.

2. (4 points each) Evaluate each of the following limits. If a limit does not exist, specify whether the limit equals ∞, $-\infty$, or simply does not exist (in which case, write DNE). Sufficient work must be shown. Give *exact answers.*

(a) $\displaystyle\lim_{x\to 2} \frac{x-2}{x^2-4}$

(b) $\displaystyle\lim_{h\to 0} \frac{(x+h)^2+1-(x^2+1)}{h}$ (Hint: your answer should be a function of x.)

(c) $\displaystyle\lim_{x\to\infty} \frac{3x^2-5x+1}{4-x^2}$

3. (5 points each) Differentiate each of the following functions. You do *not* need to simplify your answers, but sufficient work must be shown to receive full credit.

(a) $f(x) = \dfrac{3x^4}{2} + x^2\sqrt{3} - \dfrac{3}{x^2} + \sqrt{2}$

(b) $y = \sqrt{\sin 3x}$

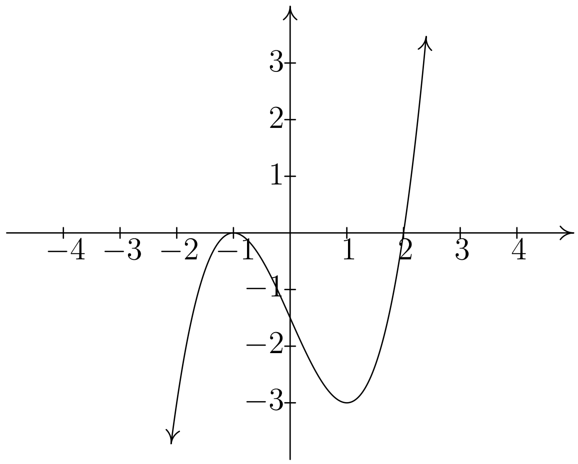
(c) $g(x) = \dfrac{x}{1 - x^2}$

(d) $A(x) = \displaystyle\int_0^x \cos t \; dt$

4. (5 points) Find $\dfrac{dy}{dx}$ if $x^3 + y^3 = xy$. You do *not* need to simplify your answer.

5. (5 points) Let $f(x) = \cos x$, find the *equation* of the tangent line to the graph of f when $x = \pi/6$. It does not matter what form the equation of the line takes, but all coefficients should have exact values (i.e., no decimal approximations).

6. (3 pts each) Let $f(x)$ be continuous over $(-\infty, \infty)$. The following is the graph of the *derivative* of $f(x)$:



Graph of f'

(a) Find the x-coordinates of all points on the graph of $f(x)$ where the tangent line is horizontal.

(b) Find the intervals, if any, on which $f(x)$ is increasing.

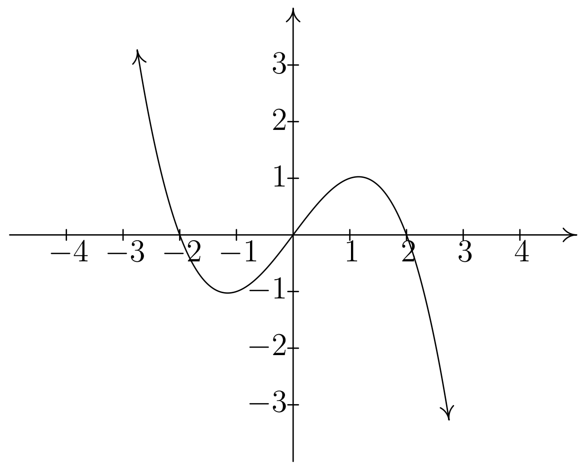
(c) Find the intervals, if any, on which $f(x)$ is decreasing.

(d) Find the x-coordinates, if any, where $f(x)$ attains a local max.

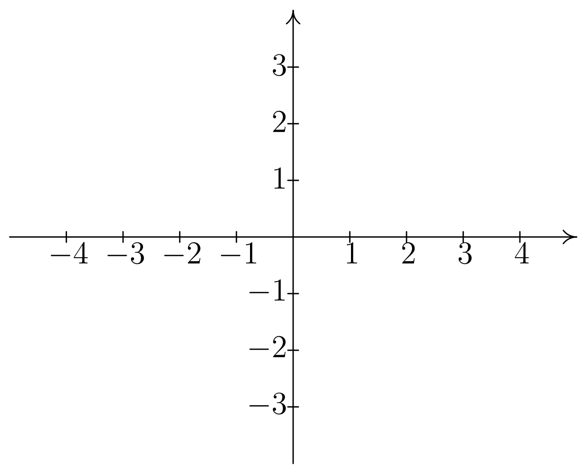
(e) Find the x-coordinates, if any, where $f(x)$ attains a local min.

(f) Find the x-coordinates, if any, of all of the inflection points of $f(x)$.

7. (5 points) Assume that f is a continuous function and that the graph of the function F given below is an antiderivative of f. Sketch the graph of f. (Hint: Recall that $F'(x) = f(x)$.)



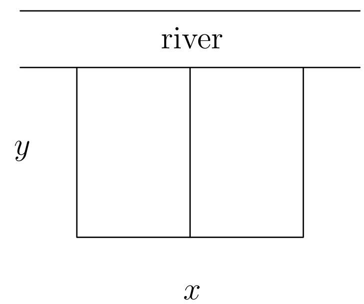
Graph of F



Graph of f

8. (6 points) A large spherical meteorite is speeding towards Earth. If the radius of the meteorite is decreasing at a rate of 1/4 mile per day, what is the rate of change in the volume of the meteorite when the radius is 5 miles? Give an exact answer. Your answer should be labeled with appropriate units. (Hint: the volume of a sphere of radius r is $V = \frac{4}{3}\pi r^3$.)

9. (6 points) A farmer has 1200 feet of fencing with which to enclose a pasture for grazing nuggets. The farmer only needs to enclose 3 sides of the pasture since the remaining side is bounded by a river (no, nuggets can't swim). In addition, some of the nuggets don't get along with some of the other nuggets. He plans to separate the troublesome nuggets by forming two adjacent corrals (see figure). Determine the dimensions that would yield the maximum area for the pasture. (You must show sufficient work to justify that your answer is the correct one. In particular, you should consider the domain of the function that you are maximizing.)



10. (5 points each) Evaluate each of the following integrals. Sufficient work must be shown.

(a) $\displaystyle\int \frac{4-5x^3}{x^2}\,dx$

(b) $\displaystyle\int x^2\sqrt{4-5x^3}\,dx$

(c) $\displaystyle\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$

11. (5 points) Find the area of the region bounded by the graphs of $f(x) = 4x$ and $g(x) = x^2 - 5$.

12. (5 points) Setup (but do *not* evaluate) an integral that would determine the volume of the solid obtained by revolving the region bounded by the graphs of $y = 2 - x^2$ and $y = x^2$ about the x-axis.

13. (5 points) Setup (but do *not* evaluate) an integral that would determine the volume of the solid obtained by revolving the region bounded by the graphs of $f(x) = 1$ and $g(x) = x^2$ about the line $x = 2$.

14. (5 points each) Consider the following function.

$$f(x) = 4x - x^2$$

(a) Find the average value of f on the interval $[0, 3]$.

(b) Find at least one value c in the interval $[0, 3]$ that the Mean Value Theorem for Integrals guarantees exists.

15. (3 points each) Provide an example of each of the following. You do *not* need to justify your answer.

(a) An *equation* of a function f that is continuous everywhere, but not differentiable at $x = 0$.

(b) An *equation* of a function g such that g has a critical number at $x = 0$, but g does not have a local maximum or local minimum at $x = 0$.

(c) An *equation* of a function h such that h has a local maximum at $x = 0$, but $h'(0) \neq 0$.

(d) An *equation* of a function k such that $k''(0) = 0$, but k does *not* have an inflection point at $x = 0$.

16. (5 points) A common theme this semester has been to start with an approximation for something that is seemingly difficult to compute and then take a limit to get an exact answer. Describe ONE such situation that we have discussed this semester. I'm looking for an intuitive understanding, but you should provide some detail using proper notation. (Using pictures to aid in your description will be very useful.)

17. **Bonus Question 1:** (3 points) What is it that we computed in problem 2b? Be as specific as possible.

18. **Bonus Question 2:** (3 points) Describe the function $A(x)$ given in problem 3d? Drawing the appropriate picture will convince me you understand what this function is. (Note: I want you to describe $A(x)$, not $A'(x)$.)

19. **Bonus Question 3:** (5 points) Explain how the Mean Value Theorem for Integrals is related to the ordinary Mean Value Theorem.