1. (4 points each) Match each function with the correct graph. (Note that there are more graphs than functions.)

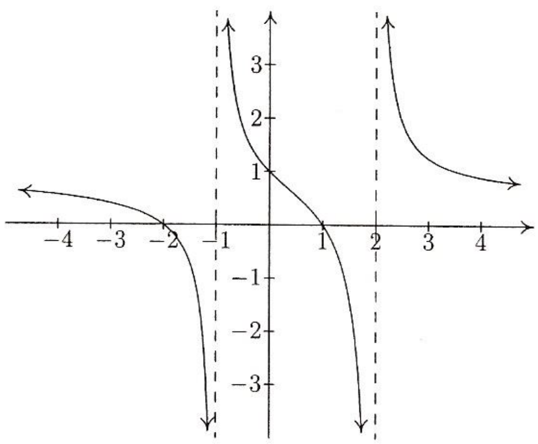
(a) $g(x) = \dfrac{x^2+2x+1}{(x-2)(x+1)} = \dfrac{(x+1)(x+1)}{(x-2)(x+1)}$ Graph: D

(b) $f(x) = \dfrac{x^2+x-2}{(x-2)(x+1)} = \dfrac{(x+2)(x-1)}{(x-2)(x+1)}$ Graph: A

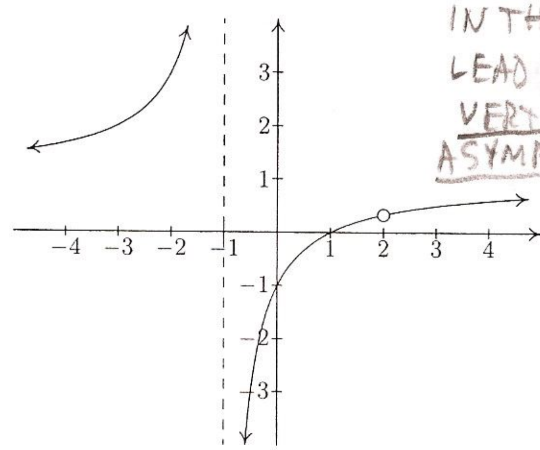
(c) $h(x) = \dfrac{3x^2-3x-6}{(x-2)(x+1)} = \dfrac{3(x-2)(x+1)}{(x-2)(x+1)}$ Graph: C

COMMON FACTORS IN NUMERATOR AND DENOMINATOR LEAD TO HOLES IN THE GRAPH.

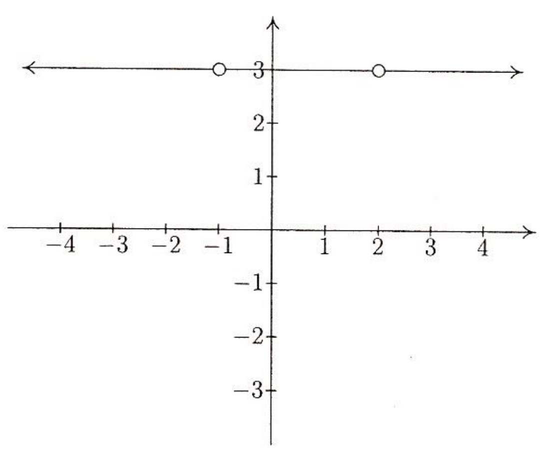
FACTORS IN THE DENOMINATOR WITH NO MATCHING FACTOR IN THE NUMERATOR LEAD TO VERTICAL ASYMPTOTES



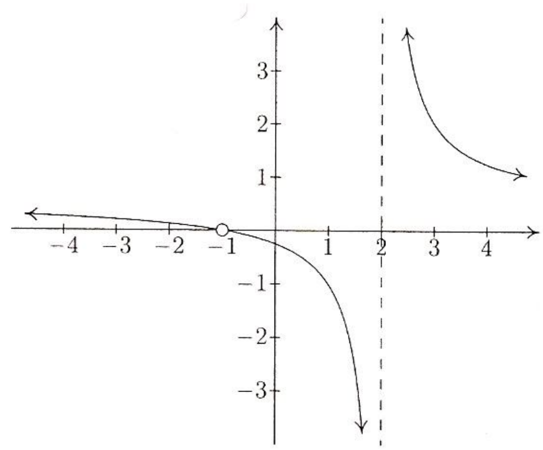
Graph A



Graph B

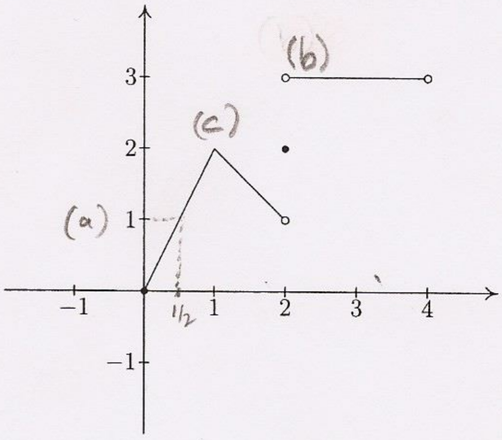


Graph C

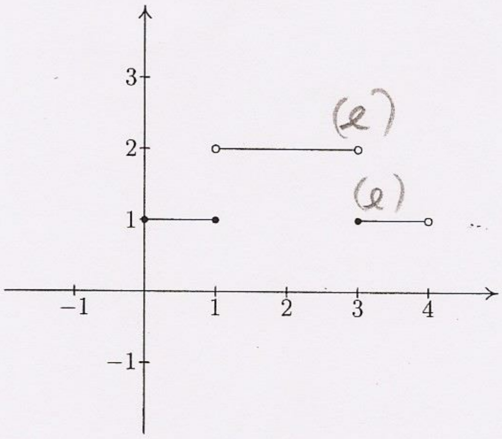


Graph D

2. (3 points each) Using the graphs below, evaluate each of the following expressions or answer the question. When you answer the two questions, (d) and (g) below, state your reasoning.



Graph of f



Graph of g

(a) $f(1/2)$ $= \boxed{1}$

(b) $\lim_{x\to 2^+} f(x)$ $= \boxed{3}$

(c) $\lim_{x\to 1} f(x)$ $= \boxed{2}$

(d) Is $f(x)$ continuous at $x = 1$? Explain your answer. YES. $\boxed{\lim_{x\to 1} f(x) = f(1)}$

INFORMAL: CAN DRAW GRAPH THRU (1,2) WITHOUT LIFTING PENCIL FROM PAPER.

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^-} f(x) = 2 = f(1)$

(e) $\lim_{x\to 3} g(x)$ $= \boxed{DNE}$

$\lim_{x\to 3^+} g(x) = 1 \neq 2 = \lim_{x\to 3^-} f(x)$

(f) $g(f(2.5))$

$= g(3) = \boxed{1}$

(g) Does $g(x)$ have an inverse function? Explain your answer.

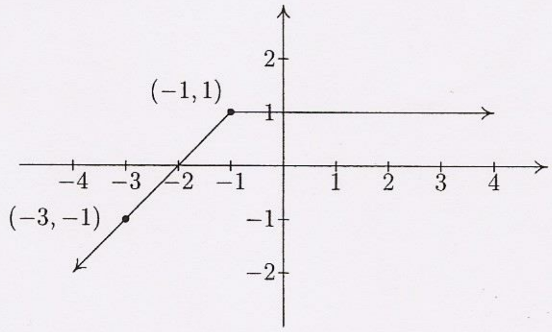
NO. $g(x)$ FAILS THE HORZONTAL LINE TEST

SO $g(x)$ IS NOT 1-1

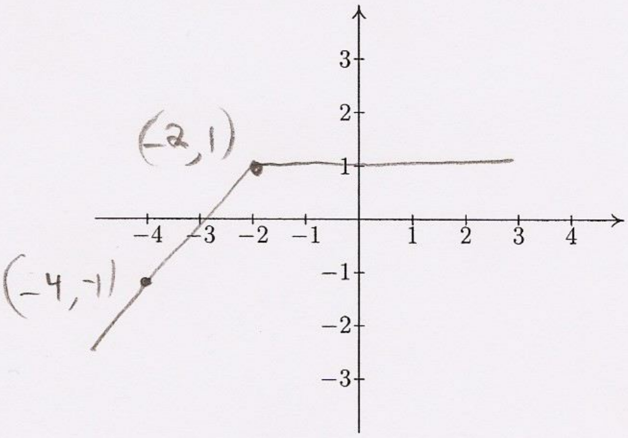
SO $g(x)$ DOES NOT HAVE AN INVERSE.

IT IS POSSIBLE TO DRAW A HORZ LINE THAT CUTS THE GRAPH OF g IN MORE THAN ONE PLACE

3. (3 points each) Suppose the graph of $f(x)$ looks like:

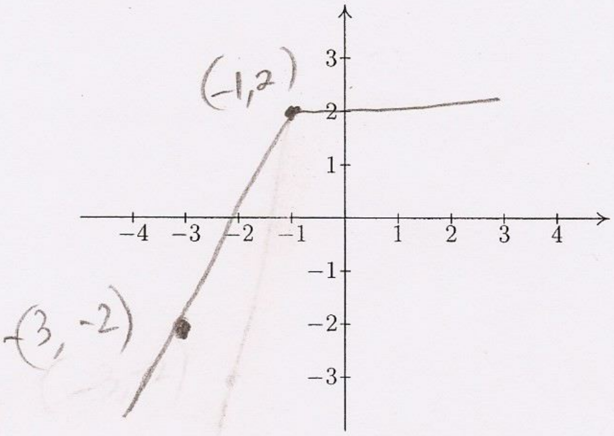


Using the axes provided, sketch the graph of each function.



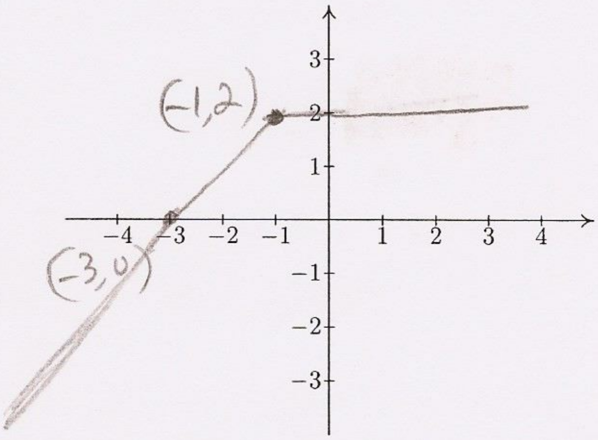
Graph of $y = f(x+1)$

SHIFT LEFT BY 1



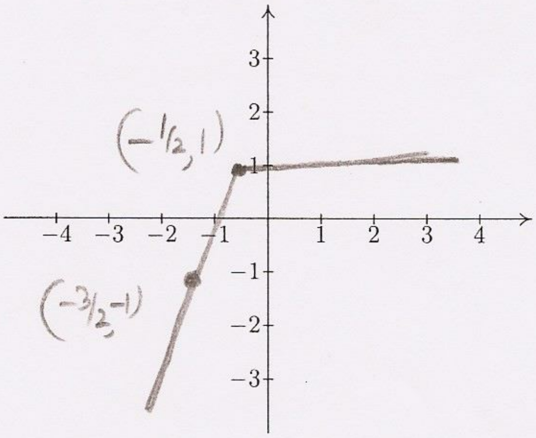
Graph of $y = 2f(x)$

VERTICAL STRETCH BY FACTOR OF 2



Graph of $y = f(x) + 1$

SHIFT UP BY 1



Graph of $y = f(2x)$

HORZ COMPRESS BY FACTOR OF 2

4. (4 points each) For each of the following, find ALL values of x which satisfy the given equation.

(a) $\log_2(x+1) - \log_2(x) = 1$

$$\log_2\left(\frac{x+1}{x}\right) = 1$$

$$\exp_2\left(\log_2\left(\frac{x+1}{x}\right)\right) = \exp_2(1)$$

$$\frac{x+1}{x} = 2^1$$

$$\frac{x+1}{x} = 2$$

$$x+1 = 2x$$

$$\boxed{1 = x}$$

(b) $4^x - 7(2^x) - 8 = 0$ (Hint: $4^x = 2^{2x}$)

$$2^{2x} - 7(2^x) - 8 = 0$$

$$(2^x)^2 - 7(2^x) - 8 = 0$$

Let $u = 2^x$

$$u^2 - 7u - 8 = 0$$

$$(u-8)(u+1) = 0$$

$u = 8$ | $u = -1$

For $u = 8$: $2^x = 8$

$$\boxed{x = 3}$$

For $u = -1$: $2^x = -1$
IMPOSSIBLE
SINCE $2^x > 0$ for all x
SO NO X-SOLUTION FOR $u = -1$

(c) $2\sin(x) - 1 = 0$

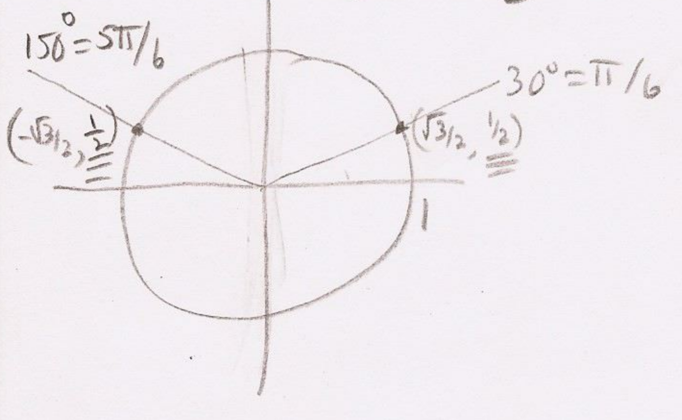
$$2 \sin x = 1$$

$$\sin x = \frac{1}{2}$$

so

$$\boxed{\begin{aligned} x &= \pi/6 \pm 2n\pi \\ x &= \frac{5\pi}{6} \pm 2n\pi \end{aligned}}$$

$n = 0, 1, 2, 3, \ldots$



AROUND THE CIRCLE n TIMES IN BOTH POSITIVE AND NEGATIVE DIRECTIONS PICKS UP ALL POSITIVE AND NEGATIVE X's SUCH THAT $\sin x = \frac{1}{2}$

5. (4 points each) Evaluate each of the following limits. If a limit does not exist, specify whether the limit equals ∞, $-\infty$, or simply does not exist (in which case, write DNE). Sufficient work must be shown.

(a) $\lim_{x\to\infty} \frac{6-x^2}{2x^2+6}$ $= \boxed{\frac{-1}{2}}$

RATIO OF LEADING COEFFICIENTS, WHEN deg(NUM) = deg(DENOM)

TECHNIQUE OF CONJUGATES

(b) $\lim_{x\to 4} \frac{x-4}{\sqrt{x}-2}$ $= \lim_{x\to 4}\left[\frac{x-4}{\sqrt{x}-2}\cdot\frac{\sqrt{x}+2}{\sqrt{x}+2}\right] = \lim_{x\to 4}\frac{(x-4)(\sqrt{x}+2)}{x-4}$

$= \lim_{x\to 4}(\sqrt{x}+2) = \sqrt{4}+2 = 2+2 = \boxed{4}$

$\sin^2 x + \cos^2 x = 1$

$\cos^2 x = 1 - \sin^2 x$

(c) $\lim_{x\to\frac{\pi}{2}} \frac{\cos^2 x}{1-\sin(x)}$ $= \lim_{x\to\pi/2}\frac{1-\sin^2 x}{1-\sin x} = \lim_{x\to\frac{\pi}{2}}\frac{(1-\sin x)(1+\sin x)}{1-\sin x}$

$= \lim_{x\to\pi/2}(1+\sin x) = 1+\sin \pi/2 = 1+1 = \boxed{2}$

(d) $\lim_{t\to 1^-} \frac{1}{1-t}$ $= \boxed{+\infty}$

AS $t\to 1^-$, t TAKES ON VALUES LESS THAN 1, SUCH AS .9, .99, ... AND FOR THESE t's, $1-t>0$ AND $1-t\to 0$.

(e) $\lim_{t\to 0} \frac{\sin(3t)}{2t}$

$= \frac{1}{2}\lim_{t\to 0}\left(\frac{\sin 3t}{t}\right) = \frac{1}{2}\lim_{t\to 0}\left(\frac{3\sin 3t}{3t}\right) = \frac{3}{2}\lim_{t\to 0}\left(\frac{\sin 3t}{3t}\right)$

$= \frac{3}{2}\cdot 1 = \boxed{\frac{3}{2}}$

6. (5 points) Use the Squeezing Theorem to evaluate the following limit. Sufficient work must be shown.

$$\lim_{x\to\infty} \frac{1}{x}\cos(x)$$

$-1 \le \cos x \le 1$ for all x

$-\frac{1}{x} \le \frac{1}{x}\cos x \le \frac{1}{x}$ (MULTIPLY THRU BY $\frac{1}{x} > 0$ FOR $x \to +\infty$)

OBSERVE:

$\lim_{x\to\infty}\left(-\frac{1}{x}\right) = 0$ and $\lim_{x\to\infty}\left(\frac{1}{x}\right) = 0$

SO: $\lim_{x\to\infty} \frac{1}{x}\cos x = \boxed{0}$ BY THE SQUEEZING THM.

7. (6 points) Using the limit definition of the slope of the tangent line (which is denoted $m_{\tan}$ in the book), find the *slope* of the tangent line to $y = x^2 + 2$, at the point where $x = 2$.

USE $x_0 = 2$

$m_{\text{TAN AT } P(x_0, f(x_0))} \overset{\text{DEF}}{=} \boxed{\lim_{h\to 0} \frac{f(x_0+h) - f(x_0)}{h}} = \lim_{h\to 0} \frac{f(2+h) - f(2)}{h}$

$= \lim_{h\to 0} \frac{[(2+h)^2 + 2] - [2^2 + 2]}{h} = \lim_{h\to 0} \frac{4 + 4h + h^2 + 2 - 4 - 2}{h}$

$= \lim_{h\to 0} \frac{4h + h^2}{h} = \lim_{h\to 0} \frac{h(4+h)}{h} = \lim_{h\to 0}(4+h) = \boxed{4}$

USING THE OTHER FORMULATION FOR m_{TAN}:

USE $x_0 = 2$

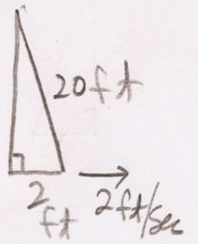
$m_{\text{TAN AT } P(x_0, f(x_0))} \overset{\text{DEF}}{=} \boxed{\lim_{x\to x_0} \frac{f(x) - f(x_0)}{x - x_0}} = \lim_{x\to 2} \frac{f(x) - f(2)}{x - 2}$

$= \lim_{x\to 2} \frac{[x^2 + 2] - [2^2 + 2]}{x - 2} = \lim_{x\to 2} \frac{x^2 + 2 - 4 - 2}{x - 2} = \lim_{x\to 2} \frac{x^2 - 4}{x - 2}$

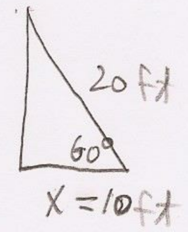
$= \lim_{x\to 2} \frac{(x-2)(x+2)}{x-2} = \lim_{x\to 2}(x+2) = 2 + 2 = \boxed{4}$

8. (3 points each) A 20 foot ladder is leaning against a wall with its base 2 feet from the wall. The bottom of the ladder begins to slide away from the wall at 2 feet per second.

(a) After how many seconds is the angle that the base of the ladder makes with the ground equal to $60° = \pi/3$?



$\Rightarrow$



$\cos 60° = \frac{1}{2} = \frac{adj}{hyp} = \frac{X}{20}$

So $\frac{X}{20} = \frac{1}{2}$

$X = 10$ ft

THE BASE HAS TO SLIDE 10 ft − 2ft = 8ft FROM ITS INITIAL POSITION UNTIL THE BASE IS 10 ft LONG.

AT 2ft/sec, THIS SLIDING WILL TAKE $\frac{8 \text{ ft}}{2 \text{ ft/sec}} = \boxed{4 \text{ sec}}$

(b) After how many seconds does the top of the ladder reach the ground?

THE TOP OF THE LADDER REACHES THE GROUND AFTER THE BASE SLIDES 20ft − 2ft = 18ft, AT WHICH TIME THE LADDER IS LYING ON THE GROUND AND THE BASE EQUALS THE LENGTH OF THE LADDER.

AT 2ft/sec, THIS SLIDING WILL TAKE $\frac{18 \text{ ft}}{2 \text{ ft/sec}} = \boxed{9 \text{ sec}}$

9. (3 points each) Let f be function that has an inverse, denoted by f^{-1}. Use facts about inverse functions to answer the following questions.

(a) Suppose that $f(2) = 3$ and $f(4) = 6$. Find the equation of the secant line (also called chord) to the graph of f^{-1} through the pair of points whose x-coordinates are $x = 3$ and $x = 6$.

IF $f(2)=3$, THEN $f^{-1}(f(2)) = f^{-1}(3)$
$2 = f^{-1}(3)$

AND IF $f(4)=6$, THEN $f^{-1}(f(4)) = f^{-1}(6)$
$4 = f^{-1}(6)$

SINCE $f^{-1}(3)=2$, THE POINT $(3,2)$ LIES ON THE GRAPH OF f^{-1}
SINCE $f^{-1}(6)=4$, THE POINT $(6,4)$ LIES ON THE GRAPH OF f^{-1}
THE SLOPE OF THE SECANT LINE THRU $(3,2)$ AND $(6,4)$ IS:

$m = \frac{\Delta y}{\Delta x} = \frac{4-2}{6-3} = \frac{2}{3}$, SO THE EQUATION IS: $\boxed{y-4=\frac{2}{3}(x-6)}$
OR $\boxed{y-2=\frac{2}{3}(x-3)}$

BOTH OF WHICH REDUCE TO $\boxed{y = \frac{2}{3}x}$

(b) Explain why the graph of f and the graph of f^{-1} are symmetric about the line $y = x$.

IF (a,b) IS ON THE GRAPH OF f,
THEN (b,a) IS ON THE GRAPH of f^{-1}!

$\left[(a,b) \text{ IS ON } f \overset{\text{(IMPLIES)}}{\Longrightarrow} b = f(a) \Rightarrow f^{-1}(b) = f^{-1}(f(a)) \right.$
$\Rightarrow f^{-1}(b) = a$
$\left. \Rightarrow (b,a) \text{ IS ON } f^{-1} \right]$

THE POINTS (a,b) AND (b,a) ARE SYMMETRIC IN THE LINE $y=x$. SO EVERY POINT (a,b) ON THE GRAPH OF f CORRESPONDS TO A SYMMETRICALLY PLACED POINT (b,a) ON THE GRAPH OF f^{-1}. SO THE ENTIRE GRAPHS OF f AND f^{-1} ARE SYMMETRIC ABOUT THE LINE $y=x$.