Supplementary Homework Exercises for Section 11.3: The Integral Test

Exercises

Answer each of the following questions.

S1. Assume that f is a continuous positive decreasing function for $x \geq 1$ and $a_n = f(n)$. By drawing a picture, rank the following quantities in increasing order:

$$\int_1^{42} f(x)\, dx \qquad \sum_{n=1}^{41} a_n \qquad \sum_{n=2}^{42} a_n$$

S2. Determine whether each of the following series is convergent or divergent. You need to show sufficient justification and you can use any of our current tests for convergence.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt[5]{n}}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^5}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{\arctan n}{n^2+1}$

(d) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n \ln n}$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{n^3+1}$

(f) $\displaystyle\sum_{n=1}^{\infty} \frac{e^{1/n}}{n^2}$

(g) $1 + \frac{1}{8} + \frac{1}{27} + \frac{1}{64} + \frac{1}{125} + \cdots$

(h) $1 + \frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} + \frac{1}{4\sqrt{4}} + \frac{1}{5\sqrt{5}} + \cdots$

S3. Let's prove the theorem about the convergence of p-series. (Each of these is very short.)

(a) If $p < 0$, prove that $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^p}$ diverges by using the Divergence Test.

(b) If $p = 0$, prove that $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^p}$ diverges by using the Divergence Test.

(c) If $0 < p < 1$, prove that $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^p}$ diverges by using the Integral Test.

(d) If $p = 1$, prove that $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^p}$ diverges by using the Integral Test.

(e) If $p > 1$, prove that $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges by using the Integral Test.

