Diagram Calculus for the Temperley–Lieb Algebra

Dana Ernst, University of Colorado at Boulder

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Abstract: The Temperley–Lieb Algebra TL_n (invented by Temperley and Lieb in 1971) is a certain finite dimensional associative algebra, depending on a parameter δ, which arose in the context of Potts models in statistical mechanics. The algebra TL_n can be realized as a certain diagram algebra. In 1987, V.F.R. Jones showed that TL_n occurs naturally as a quotient of the Hecke algebra arising from a Coxeter system of type A_{n-1} (whose underlying group is the symmetric group, S_n). In this talk, we will introduce the diagram calculus for TL_n, show that TL_n has dimension equal to the nth Catalan number, and show that if we make a special choice for the parameter ($\delta = 2$), TL_n is isomorphic to a quotient of the group algebra $\mathbb{Z}[S_n]$.