Plymouth State
UNIVERSITY

MA 2550: Calculus I

Section 4.5: Summary of Curve Sketching (Part 2)

Question: If $f(x) = p(x)/q(x)$ is a rational function such that $p(x)$ and $q(x)$ have no factors in common (i.e., the "fraction" is reduced), then when will $f(x)$ have a horizontal asymptote? When will it not?

Answer:

When the degree of the numerator is _____ than the degree of the denominator, other kinds of asymptotes are possible: *slant* (sometimes called *curvilinear* or *oblique*). To see what these new kinds of asymptotes are, we use polynomial long division.

Theorem: A rational function cannot have both a horizontal asymptote and a curvilinear (including slant) asymptote. Why?
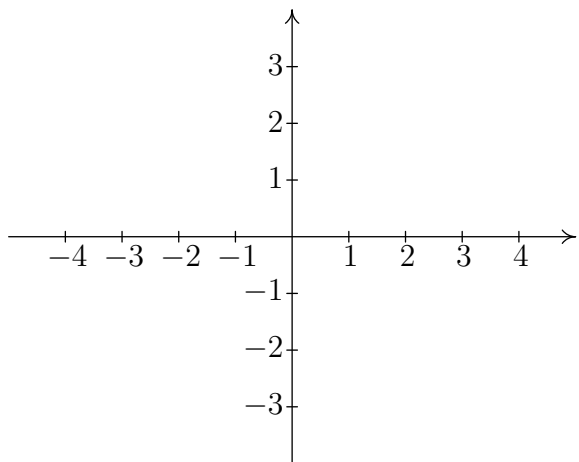
Example 1: Identify the curvilinear asymptote of the following function and sketch its graph.
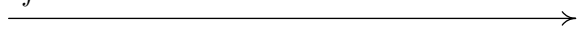
$$g(x) = \frac{x^3 - x^2 + 4}{x - 1}$$

f

f'

f''
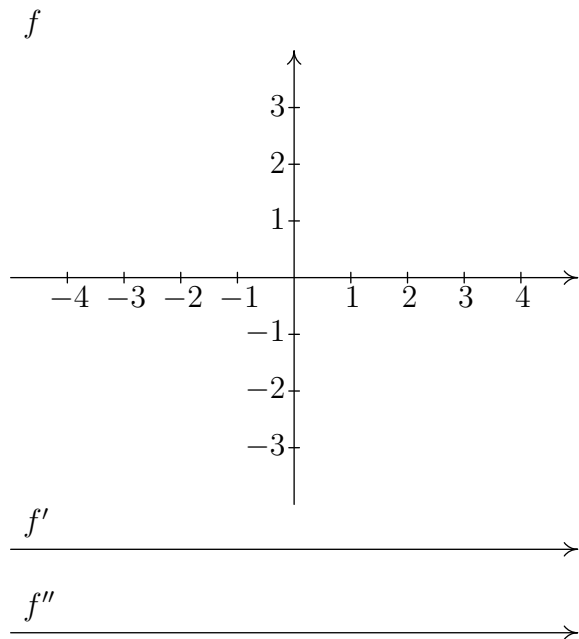
Example 2: Sketch the graph of the following function. $f(x) = \dfrac{x^2 + x + 1}{x - 1}$

f

f'

f''