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HOMEWORK FOR WORKSHEET 9 **MATH 1300** **DUE March 14, 2008**

1. Use the method from Worksheet 9 to find any non-vertical asymptotes for the function

$$f(x) = \frac{2x^3 + 2x + 1}{x^2 + 1},$$

and explain *how* you know that your answer is correct. (Hint: Look at parts (d) and (e) of Case 3 from the worksheet.)

$$\begin{array}{r|l} & 2x \\ \hline x^2+1 & 2x^3+2x+1 \\ & 2x^3+2x \\ \hline & 0+1 \end{array}$$

$$\Rightarrow f(x) = \frac{2x^3+2x+1}{x^2+1} = 2x + \frac{1}{x^2+1}$$

Note that $\frac{1}{x^2+1} \longrightarrow 0$ as $|x| \longrightarrow \infty$. This implies that $f(x) \longrightarrow 2x$ as $|x| \longrightarrow \infty$.

Furthermore, we see that

$$\lim_{|x|\to\infty} \frac{f(x)}{2x} = \lim_{|x|\to\infty} \frac{2x^3+2x+1}{x^2+1} \cdot \frac{1}{2x}$$

$$= \lim_{|x|\to\infty} \frac{2x^3+2x+1}{2x^3+2x} = \lim_{|x|\to\infty} \frac{2x^3}{2x^3} = 1.$$

Therefore, $\boxed{y = 2x}$ is a (slant) asymptote of f.

In Worksheet 9 we did not consider the fourth possibility, that $f(x) = \dfrac{P(x)}{Q(x)}$ where the degree of $Q(x)$ is more than 1 less than the degree of $P(x)$. We examine this case in the next two exercises.

2. Use the same method as you did in answering problem 1 to find any *hidden* asymptotes for the function

$$f(x) = \frac{x^3 + x + 1}{x - 1}.$$

$$\begin{array}{r|l} & x^2 + x + 2 \\ \hline x - 1 & x^3 + x + 1 \\ & \underline{x^3 - x^2} \\ & x^2 + x + 1 \\ & \underline{x^2 - x} \\ & 2x + 1 \\ & \underline{2x - 2} \\ & 3 \end{array}$$

$$\Longrightarrow f(x) = x^2 + x + 2 + \frac{3}{x-1}$$

Note that $\dfrac{3}{x-1} \longrightarrow 0$ as $|x| \longrightarrow \infty$. This implies that $f(x) \longrightarrow x^2 + x + 2$ as $|x| \to \infty$.

Furthermore, we see that

$$\lim_{|x| \to \infty} \frac{f(x)}{x^2+x+2} = \lim_{|x| \to \infty} \frac{x^3+x+1}{x-1} \cdot \frac{1}{x^2+x+2}$$

$$= \lim_{|x| \to \infty} \frac{x^3}{x^3} = 1.$$

Therefore, $\boxed{y = x^2 + x + 2}$ is a curvilinear asymptote of f.

3. Use the same reasoning as in problems 1 and 2 to find all asymptotes for the function

$$g(x) = \frac{-x^5 + x^2 - 8}{2x^2 - 1}.$$

$$\begin{array}{r|l} & -\frac{1}{2}x^3 - \frac{1}{4}x + \frac{1}{2} \\ \hline 2x^2-1 & -x^5 + x^2 - 8 \\ & -x^5 + \frac{1}{2}x^3 \\ \hline & -\frac{1}{2}x^3 + x^2 - 8 \\ & -\frac{1}{2}x^3 + \frac{1}{4}x \\ \hline & x^2 - \frac{1}{4}x - 8 \\ & x^2 - \frac{1}{2} \\ \hline & -\frac{1}{4}x - \frac{15}{2} \end{array}$$

$$\Rightarrow g(x) = -\frac{1}{2}x^3 - \frac{1}{4}x + \frac{1}{2} + \frac{-\frac{1}{4}x - \frac{15}{2}}{2x^2-1}.$$

Note that $\dfrac{-\frac{1}{4}x - \frac{15}{2}}{2x^2-1} \longrightarrow 0$ as $|x| \longrightarrow \infty$.

Then $g(x) \longrightarrow -\frac{1}{2}x^3 - \frac{1}{4}x + \frac{1}{2}$ as $|x| \to \infty$.

Furthermore, we see that

$$\lim_{|x|\to\infty} \frac{g(x)}{-\frac{1}{2}x^3 - \frac{1}{4}x + \frac{1}{2}} = \lim_{|x|\to\infty} \frac{-x^5 + x^2 - 8}{2x^2 - 1} \cdot \frac{1}{-\frac{1}{2}x^3 - \frac{1}{4}x + \frac{1}{2}}$$

$$= \lim_{|x|\to\infty} \frac{-x^5}{-x^5} = 1.$$

So, $\boxed{y = -\frac{1}{2}x^3 - \frac{1}{4}x + \frac{1}{2}}$ is a curvilinear asymptote.