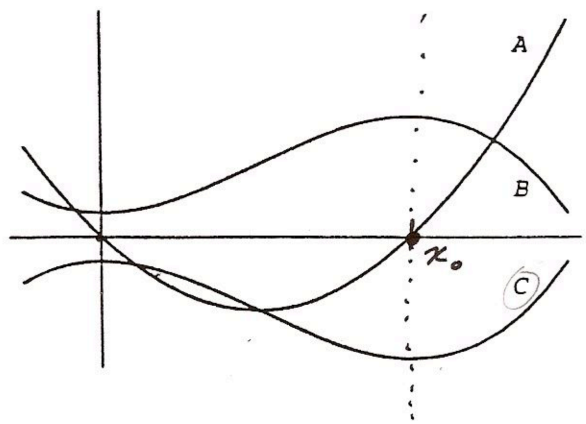
Solutions

WORKSHEET 11 **MATH 1300** **April 3, 2008**

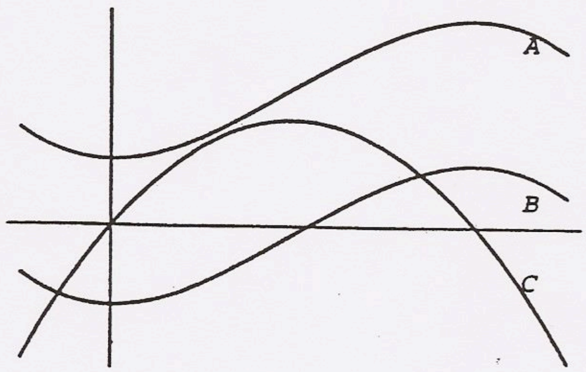
Goal: To develop a better understanding of the relationship between a function and its antiderivatives.

1. Consider the three graphs in the figure below. If the graph labeled A is the graph of a function f, then which graph is an antiderivative of f? (You must be able to explain the reasoning you used to obtain your answer.)



C. if A is the derivative graph of a function, g, then g must be increasing (have positive slope) on the intervals $(-\infty, 0)$ and (x_0, ∞) and g must be decreasing (negative slope) on the interval $(0, x_0)$. This matches graph C.

2. In this problem the graph labeled C is the graph of a function f. Which graph represents an antiderivative of f?

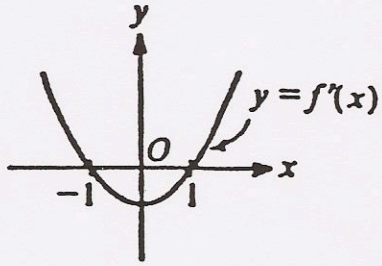


Both

Graph B is the same as graph A shifted down so

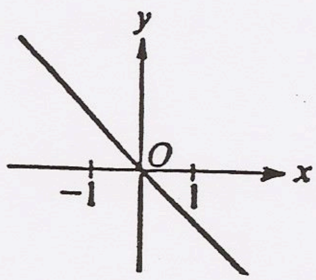
Graphs A and B will have the same derivative graph.

3. The graph of the derivative of f is shown in the figure below.

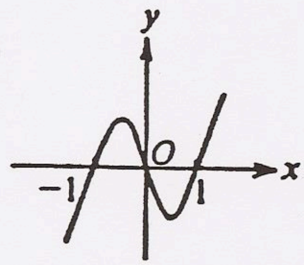


Explain why each graph below cannot be the graph of f.

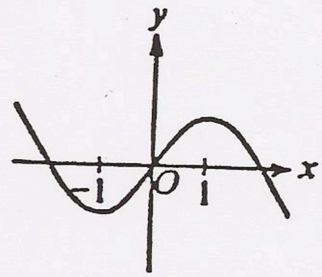
a.



b.



c.



The graph of f' shows that f is increasing on the intervals $(-\infty, -1)$ and $(1, \infty)$. and decreasing on $(-1, 1)$

But...

(a) is decreasing everywhere.

(b) is increasing on $(-\infty, -\frac{1}{2})$ and $(\frac{1}{2}, \infty)$.

(c) is decreasing on $(-\infty, -1)$.

4. **(a)** Confirm that $\frac{d}{dx}\sin^2 x = 2\sin x\cos x = \sin 2x$.

$$\frac{d}{dx}(\sin x)^2 = 2\sin x \frac{d}{dx}(\sin x)$$
$$= 2\sin x\cos x$$
$$= \sin x\cos x + \cos x\sin x$$
$$= \sin(x+x)$$
$$= \sin(2x)$$

$\sin(A+B) = \sin A\cos B + \cos A\sin B$

(b) Your work in (a) verifies that one antiderivative of $\sin 2x$ is $\sin^2 x$. Find an antiderivative of $\sin 2x$ that involves the function $\cos 2x$ and explain how $\sin 2x$ can have these two different antiderivatives.

$$\frac{d}{dx}\cos(2x) = -2\sin 2x$$

So $$\frac{d}{dx}\left[-\frac{1}{2}\cos(2x)\right] = -\frac{1}{2}\frac{d}{dx}\cos(2x)$$
$$= -\frac{1}{2}(-2\sin 2x)$$
$$= \sin 2x$$

So $-\frac{1}{2}\cos(2x)$ is an antiderivative of $\sin(2x)$.

$$-\frac{1}{2}\cos(2x) = -\frac{1}{2}(\cos^2 x - \sin^2 x)$$
$$= -\frac{1}{2}(1-\sin^2 x - \sin^2 x)$$
$$= -\frac{1}{2}(1-2\sin^2 x)$$
$$= -\frac{1}{2} + \sin^2 x.$$

$\Rightarrow$ $-\frac{1}{2}\cos(2x) = \sin^2 x + (-\frac{1}{2})$

The two functions differ By a constant, $(-\frac{1}{2})$, so They have the same derivative.