Section 6.5: Average Value of a Function

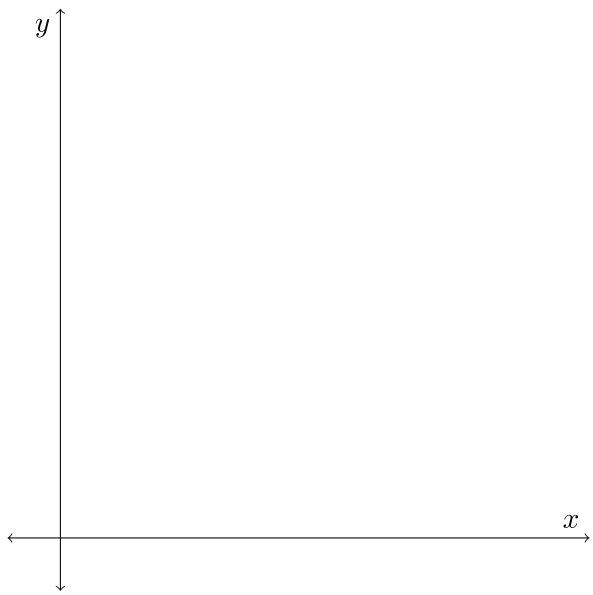
Goal

In this section, we will explore the technique for finding an average of infinitely many values of a function over an interval.

Average Value of a Function

If f is integrable on the closed interval $[a, b]$, then the average value of f on the interval is:

$$\boxed{f_{avg} = \frac{1}{b-a} \int_a^b f(x)\ dx}$$



Let's follow our ongoing theme of approximation to see if we can derive this formula.

The Mean Value Theorem for Integrals states that if f is continuous on the closed interval $[a, b]$, then there exists a number c in the closed interval $[a, b]$ such that

$$\boxed{\int_a^b f(x)dx = f(c)(b-a).}$$

We can quickly derive this from the formula for the Mean Value Theorem:

The formula for the average value can easily be found from the Mean Value Theorem for Integrals. So the value, $f(c)$, from the Mean Value Theorem for Integrals is the average value of the function on the given interval.

Examples

Let's do some examples.

Example 1.

(a) Find the average value of $f(x) = 3x^2 - 2x$ on the interval $[1, 4]$.

(b) Find the average height of $f(x) = \sin(x)$ on the interval $[\frac{\pi}{3}, \pi]$.

(c) Find the average value of $f(x) = \ln(x)$ on the interval $[1, e]$.

(d) Find a number, c, in $[1,3]$ for which $f(c)$ is the average value of $f(x) = \frac{1}{x}$ on the interval.

