Plymouth State UNIVERSITY

MA 2550: Calculus I, Fall 2008

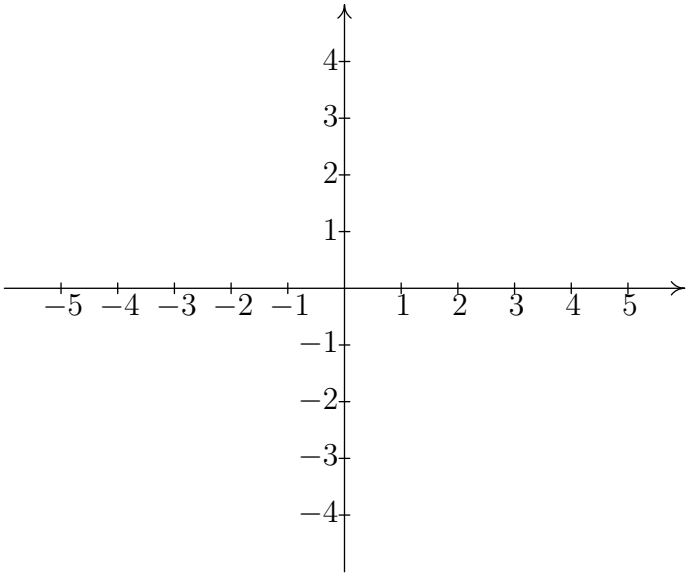
EXAM 3

NAME:

Instructions: Answer each of the following questions completely. To receive full credit, you must *justify* each of your answers (unless stated otherwise). How you reached your answer is more important than the answer itself. If something is unclear, or if you have any questions, then please ask. Good luck!

1. (10 points) Suppose f is a function with the following properties.
   (a) $f(-4) = 0$, $f(-2) = 0$, and $f(1.75) = 0$
   (b) $f(-3) = -1$, $f(-1) = 3$, and $f(0) = 4$
   (c) $\lim_{x \to 2^-} f(x) = -\infty$ and $\lim_{x \to 2^+} f(x) = \infty$
   (d) $\lim_{x \to \infty} f(x) = 0$ and $\lim_{x \to -\infty} f(x) = \infty$
   (e) $f'(-3) = 0$ and $f'(0) = 0$
   (f) $f'(x) > 0$ on $(-3, 0)$
   (g) $f'(x) < 0$ on $(-\infty, -3)$, $(0, 2)$, and $(2, \infty)$
   (h) $f''(-1) = 0$
   (i) $f''(x) > 0$ on $(-\infty, -1)$ and $(2, \infty)$
   (j) $f''(x) < 0$ on $(-1, 2)$

   Using the above information, make a sketch of the graph of f.



2. (8 points) Find all critical numbers of the following function AND then determine whether each critical number determines a local maximum, local minimum, or neither.

$$f(x) = 5x^{2/3} + x^{5/3}$$

3. (8 points) Find the coordinates of all inflection points for the following function.

$$f(x) = \frac{x^5}{20} - \frac{x^4}{6} + \frac{x^3}{6} + 5x + 1$$

4. (8 pts) Find *all* aymptotes of the following function.

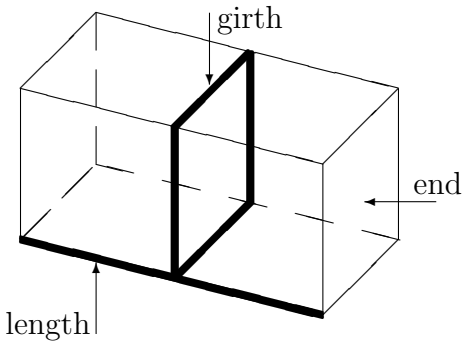
$$f(x) = \frac{2x^3 - 2x^2 - 1}{x^2 - 1}$$

5. (6 points each) Provide an example of each of the following. You do *not* need to justify your answers.

   (a) An *equation* of a rational function f such that f has a horizontal asymptote at $y = 0$.

   (b) An *equation* of a function g such that $g''(0) = 0$, but g does *not* have an inflection point at $x = 0$.

6. (8 points) The U.S. Postal Service will accept a box for domestic shipment only if the sum of its length and girth (distance around) does not exceed 108 inches. What dimensions (length and width) will give a box with a square end the largest possible volume? (You must show sufficient work to justify that your answer is the correct one. In particular, you should consider the domain of the function that you are maximizing.)



7. (8 points) At this time, we do *not* know how to evaluate the following definite integral using either a limit of Riemann sums or the Fundamental Theorem of Calculus.

$$\int_0^{\pi} \sin^2 x \, dx$$

However, we can approximate this integral. Approximate the above integral using 4 equal width rectangles and right endpoints. (You should give an exact answer for your approximation.)

8. (8 points) Evaluate the following definite integral using a limit of Riemann sums and right endpoints.

$$\int_0^2 x^2 \, dx$$

You may find some of the following formulas useful:

$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$	$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$	$\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$	$\sum_{i=1}^{n} 1 = n$

9. (6 points each) Evaluate each of the following integrals. Sufficient work must be shown.

(a) $\displaystyle\int \frac{4+x^3}{x^2}\,dx$

(b) $\displaystyle\int \sqrt{x}(1-x)\,dx$

(c) $\displaystyle\int_0^{2\pi} \cos x - \sin x\,dx$

10. (6 points each) Let

$$A(x) = \int_0^x t - t^2 \, dt.$$

(a) Find $A(2)$.

(b) Find $A'(2)$ (Hint: Find $A'(x)$ first)

11. **Bonus Question:** (5 points) For A as in the previous problem, where does A attain a maximum value on the interval $[0, \infty)$? Justify your answer; argument by picture is sufficient.