MA 2560: Calculus II (Spring 2009)
Exam 3

NAME:

Instructions: Answer each of the following questions completely. To receive full credit, you must *justify* each of your answers (unless stated otherwise). How you reached your answer is more important than the answer itself. If something is unclear, or if you have any questions, then please ask. Good luck!

1. (8 points) Find the area of one loop of the graph of $r = 2\cos(3\theta)$.

2. (8 points) Consider the parametric curve given by

$$x = 1 + 3t^2, y = 4 + 2t^3.$$

   Find the arc length for $0 \leq t \leq 1$. (Give an *exact* answer.)

3. (8 points each) Consider the parametric curve given by

$$x = 2\sin 2t, y = 3\sin t.$$

This curve crosses itself at $(0, 0)$.

(a) Find all values of t in $[0, 2\pi)$ such that $(x, y) = (0, 0)$.

(b) Find the slopes of the tangent lines to the graph for the values of t that you found in part (a).

4. (4 points each) Determine whether each of the following sequences *converge* or *diverge*. If a sequence converges, find its limit. (You must show sufficient work to justify your answers.)

(a) $a_n = \dfrac{\sqrt{n}}{\ln(n)}$

(b) $a_n = \sin(n\pi)$

(c) $\{1, -\frac{2}{3}, \frac{4}{9}, -\frac{8}{27}, \ldots\}$ (Assume the pattern continues.)

5. (10 points each) Determine whether each of the following series *converge* or *diverge*. If a series involves negative terms and converges, state whether the series *converges absolutely* or *converges conditionally*. In order to receive full credit, you should clearly state what test(s) you are using and verify the appropriate hypotheses. The justification of your conclusion is vastly more important than the actual conclusion.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{e^{1/n}}{n^2}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n}{n+1}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n}{n^2+1}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n^3 5^n}{n!}$

6. (8 points each) Given that each of the following series converge, find the sum of each series.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n 3^{n+1}}{7^{n-1}}$

(b) $\displaystyle\sum_{n=2}^{\infty} \frac{2}{n^2-1}$ (Hint: write as a telescoping series using partial fractions.)

7. **Bonus Question:** Imagine that you are stacking an infinite number of spherical nuggets of decreasing radii on top of each other. The radii of the spheres are 1 cm, $\frac{1}{\sqrt{2}}$ cm, $\frac{1}{\sqrt{3}}$ cm, $\frac{1}{\sqrt{4}}$ cm, etc.

   (a) (4 points) Does the infinite stack of spherical nuggets have finite or infinite height? (You must justify your answer.)

   (b) (4 points) Does the infinite stack of spherical nuggets have finite or infinite total volume? Recall that the volume of a sphere of radius r is $V = \frac{4}{3}\pi r^3$. (You must justify your answer.)