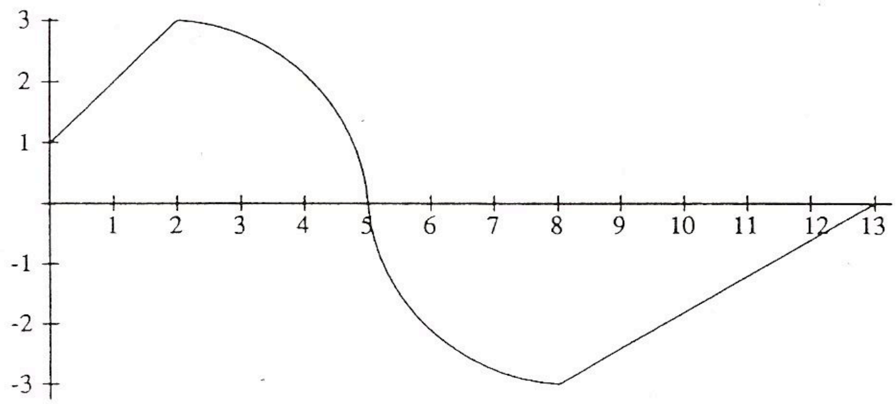
WORKSHEET 13 MATH 1300 April 17, 2008

Goal: To examine properties of the definite integral.

1. Let f be the function graphed below. Note: The graph of f consists of two straight line segments and two quarter-circles.



(a) Evaluate $\int_0^{13} f(x)dx$.

= Area of quadrilateral over [0,2] + Area of quarter-circle − Area quarter-circle − Area △ over [8,13]

$= [2+2] + \frac{1}{4}\pi\cdot(3)^2 - \frac{1}{4}\pi(3)^2 - \frac{1}{2}\cdot 5\cdot 3$

$= 4 - \frac{15}{2} = \frac{-7}{2}$

(b) Evaluate $\int_9^{12} f(x)dx$. To solve this problem we need the y-coords. of the pts with $x = 9$ and $x=12$. Assuming that the line passes through the point $(8,-3)$ the eqn of line under interval [8,13] is $y = \frac{3}{5}(x-13)$; so when $x=9$, $y=\frac{-12}{5}$ & when $x=12$, $y = \frac{-3}{5}$.

It follows that the integral equals: $x\left(-\frac{3}{5}\right) + \frac{1}{2}\left(-\frac{9}{5}\right) = \frac{}{5} - \frac{27}{10}$

$= \frac{-45}{10}$

(c) Evaluate $\int_0^{13} |f(x)|dx$.

This integral equals the sum of the absolute values of the 4 areas in (a).

2. Which of the following definite integrals is *not* zero, and why.

(a) $\int_{-\pi}^{\pi} \sin^3(x)dx$. The integrand is odd (i.e., $\sin^3(-x) = -\sin(x)$) so the integral equals 0.

(b) $\int_{-\pi}^{\pi} x^2 \sin(x)dx$. The integrand is odd (i.e., $(-x)^2 \sin(-x) = x^2 \sin(-x) = -\sin(x)$) so the integral equals 0.

(c) $\int_{-\pi}^{\pi} \cos^2(x)dx$. The function $\cos^2(x)$ is ≥ 0 for all points in the interval $[-\pi, \pi]$, and positive for some of these x-values so the integral is non zero (it is positive).

(d) $\int_{0}^{\pi} \cos(x)dx$. This integral equals 0 since $\int_0^{\pi/2} \cos(x)\,dx = -\int_{\pi/2}^{\pi} \cos(x)\,dx$

(e) $\int_{\pi}^{\pi} \cos(x)dx$. This integral equals 0 since it is over an interval consisting of a single point.

3. Calculate

$$\int_{-3}^{3} (x+5)\sqrt{9-x^2}\,dx.$$

Hint: Use $(x+5)\sqrt{9-x^2} = x\sqrt{9-x^2} + 5\sqrt{9-x^2}$ and THINK GEOMETRICALLY about the graphs of $y = x\sqrt{9-x^2}$ and $y = 5\sqrt{9-x^2}$

$$\int_{-3}^{3} 5\sqrt{9-x^2}\,dx = 5\underbrace{\int_{-3}^{3}\sqrt{9-x^2}\,dx}_{\text{area of semi-circle of radius 3}} = 5\cdot\left(\tfrac{1}{2}\pi(3)^2\right) = \frac{45}{2}\pi$$

$$\int_{-3}^{3} x\sqrt{9-x^2}\,dx$$

Since $(-x)\sqrt{9-(-x)^2} = -x\sqrt{9-x^2}$ this integral equals zero.

Ans. $0 + \frac{45}{2}\pi = \frac{45}{2}\pi$