MA 4140: Algebraic Structures (Spring 2010)
Exam 2 (take-home portion)

NAME:

Instructions

Prove any *three* of the following theorems.

This portion of Exam 2 is worth 40 points. Each of the three proofs that you complete is worth 10 points. Your written presentation of the proofs (which includes spelling, grammar, punctuation, clarity, and legibility) is worth the remaining 10 points.

I expect your proofs to be *well-written, neat, and organized*. You should write in *complete sentences*. Do not turn in rough drafts. What you turn in should be the "polished" version of potentially several drafts. Feel free to type up your final version.

The LaTeX source file of this exam is also available if you are interested in typing up your solutions using LaTeX. I'll be happy to help you do this.

The simple rules for this portion of the exam are:

1. You may freely use any theorems that we have discussed in class, but you should make it clear where you are using a previous result and which result you are using. For example, if a sentence in your proof follows from Proposition 2.9, then you should say so.
2. You cannot use any results from the book or otherwise that we have not covered, unless you prove them.
3. You are NOT allowed to copy someone else's work.
4. You are NOT allowed to let someone else copy your work.
5. You are allowed to discuss the problems with each other and critique each other's work.

This portion of Exam 2 is due by 5PM on Friday, April 23. You should turn in this cover page and the three proofs that you have decided to submit.

To convince me that you have read and understand the instructions, sign in the box below.

Signature:

Good luck and have fun!

Theorem 1. Let G be a group and let $H \leq G$ such that all of the left cosets of H are equal to the right cosets of H. Then for all $a, b \in G$, if $x \in aH$ and $y \in bH$, then $xy \in abH$.

Theorem 2. Let G be a group and let H and K be subgroups of G such that for all $g \in G$, $gH = Hg$ and $gK = Kg$. Then for all $g \in G$, $g(H \cap K) = (H \cap K)g$.*

*Recall that $H \cap K$ is a subgroup by Exercise 2.43.

Theorem 3. Let $(G, \cdot)$ and $(H, \circ)$ be two groups. Suppose $f : G \to H$ satisfies $f(a \cdot b) = f(a) \circ f(b)$ for all $a, b \in G$. Define $K = \{a \in G : f(a) = e'\}$, where e' is the identity in H. Then $K \leq G$.[†]

[†]It is important to notice that we are *not* assuming that f is an isomorphism, but only that f respects the operations of both groups.

Theorem 4. Let $(G, \cdot)$ and $(H, \circ)$ be isomorphic groups. If G has a subgroup of order n, then H must also have a subgroup of order n.

Theorem 5. Let G be a group and let $g \in G$. Define $\psi : G \to G$ via

$$\psi(x) = gxg^{-1}$$

for all $x \in G$. Then ψ is an isomorphism of G to itself.