MA 2560: Calculus II (Spring 2009)
Final Exam

NAME: (2 points)

Instructions: Answer each of the following questions completely. To receive full credit, you must *justify* each of your answers (unless stated otherwise). How you reached your answer is more important than the answer itself. If something is unclear, or if you have any questions, then please ask. Good luck!

1. (8 points) Evaluate the following limit.

$$\lim_{x \to 0^+} (\cos x)^{1/x}$$

2. (8 points each) Evaluate each of the following integrals.

   (a) $\displaystyle\int \frac{e^x}{\sqrt{1 - e^{2x}}}\, dx$

(b) $\displaystyle\int \frac{1}{x^2\sqrt{x^2-9}}\, dx$

(c) $\displaystyle\int \sin^3 x \cos^2 x \; dx$

(d) $\displaystyle\int \frac{x+5}{x^2+x-2}\,dx$

(e) $\displaystyle\int_1^\infty \frac{1}{4+x^2}\,dx$ (If the integral converges, give an *exact* answer.)

3. (10 points) Find the surface area of the surface obtained by revolving the graph of $y = x^2$ on the interval $[0, \sqrt{2}]$ about the y-axis. You should give an *exact* answer (i.e., not a decimal approximation using your calculator).

4. (10 points) Find the area of one loop of the graph of $r = 3\sin(5\theta)$.

5. (10 points) Consider the parametric curve given by

$$x = \frac{8}{3}t^{3/2} \qquad y = 2t - t^2$$

Find the arc length for $1 \leq t \leq 2$.

6. (5 points each) Consider the sequence determined by

$$a_n = \frac{n^2 + 17}{2n(n+1)}.$$

(a) Does the sequence converge or diverge? If the sequence converges, find its limit.

(b) Does the corresponding series $\sum_{n=1}^{\infty} a_n$ converge or diverge? Explain your answer.

7. (10 points each) Determine whether each of the following series *converge absolutely*, *converge conditionally*, or *diverge*. In order to receive full credit, you should clearly state what test(s) you are using and verify the appropriate hypotheses. The justification of your conclusion is vastly more important than the actual conclusion.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n}{n^2+17}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos^2(n)}{n^2}$

8. (10 points) Determine the *interval* of convergence of the following power series.

$$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n (x-5)^n}{n5^n}$$

9. (10 points) Find a power series representation for the following function and state its *radius* of convergence.

$$f(x) = \frac{2x}{3-x^2}$$

10. (5 points each) Consider the function $f(x) = xe^x$.

(a) Using a known Maclaurin series, find a power series for f.

(b) Integrate the power series that you found in part (a) over the interval $[0, 1]$.

(c) Now, using integration by parts, find $\displaystyle\int_0^1 xe^x\ dx$.

(d) Using the information obtained in parts (b) and (c), find the sum of the series $\displaystyle\sum_{n=0}^{\infty} \frac{1}{(n+2)n!}$.

11. **Bonus Question 1:** (5 points) Integrate the following.

$$\int (x-1)\sqrt{2x-x^2}\, dx$$

12. **Bonus Question 2:** We've repeatedly said that the alternating harmonic series converges. But what does it converge to?

   (a) (2 points) Write $y = \dfrac{1}{1+x}$ as a geometric series starting at $n = 1$.

   (b) (2 points) Integrate your answer from part (a) on the interval $[0, 1]$. (Your answer should be a familiar looking series.)

   (c) (2 points) Now, integrate the function $y = \dfrac{1}{1+x}$ on the interval $[0, 1]$. (Your answer should be a number.)

   (d) (2 points) What conclusion can you make?