# MA 3110: Logic and Proof (Fall 2009)
# Final Exam (take-home portion)

**NAME:**

**Instructions:** Prove any *four* of the following six theorems. If you turn in more than four proofs, I will only grade the first four that I see.

This portion of the Final Exam is worth 50 points. Each proof is worth 10 points. Your written presentation of the proofs (which includes spelling, grammar, punctuation, clarity, and legbility) is worth the remaining 10 points.

I expect your proofs to be *well-written, neat*, and *organized*. You should write in *complete sentences*. Do not turn in rough drafts. What you turn in should be the "polished" version of potentially several drafts. Feel free to type up your final version.

The simple rules for this portion of the exam are:

1. You may freely use any theorems that we have discussed in class or are in the sections of the text that we have covered, but you should make it clear where you are using a previous result and which result you are using. For example, if a sentence in your proof follows from Theorem 4.3.21, then you should say so.
2. You are NOT allowed to copy someone else's work.
3. You are NOT allowed to let someone else copy your work.
4. You are allowed to discuss the problems with each other and critique each other's work.

This portion of the Final Exam is due to my office by 5PM on **Friday, 12.18**. You should turn in this cover page and the *four* proofs that you have decided to submit.

To convince me that you have read and understand the instructions, sign in the box below.

**Signature:**

Good luck and have fun!

**Theorem 1:** Let $n \in \mathbb{N}$. For $a, b \in \mathbb{Z}$, define $a \equiv_n b$ iff $n$ divides $a - b$.* Then $\equiv_n$ is an equivalence relation.

*This is the definition of $\equiv_n$ and is not something that you should try to prove.

**Theorem 2:** If $f : A \to B$ and $g : B \to C$ are functions where $g \circ f : A \to C$ is 1-1, then $f$ is 1-1.

**Theorem 3:** Suppose that $f : \mathbb{R} \to \mathbb{R}$ is a function satisfying both

(i) $f(x + y) = f(x) + f(y)$;

(ii) $f(-x) = -f(x)$.[†]

Then $f$ is 1-1 if and only if $f^{-1}(\{0\}) = \{0\}$.

[†] Actually, all we really need to assume is (i) since (ii) follows from (i), but you do not need to prove that.

**Theorem 4:** Let $A$ be a set. Then $A$ contains a sequence of distinct terms if and only if $A$ can be put in 1-1 correspondence with a proper subset of itself.[‡]

[‡]This Theorem is a subset of Theorem 7.2.5.

**Theorem 5:** Suppose that $A$ and $B$ are sets such that card $A \leq$ card $B$. Then there exists a set $C \subseteq B$ such that card $C =$ card $A$.[§]

[§]This is Problem 7.2 on page 176.

**Theorem 6:** If $A$ and $B$ are countable sets, then $A \cup B$ is countable.[¶]

[¶]This is Exercise 7.3.9.