Disc / Washer Method

Revolving around x-axis:

- Thickness of each disc is always Δx, so will always integrate with respect to x.
- Radius of each disc is always $f(x)$ (or $f(x) - g(x)$ for washer method), so area is always $A(x) = \pi (f(x))^2$ (or $A(x) = \pi \left[(f(x))^2 - (g(x))^2 \right]$ for washer method).
- $V = \int_{x=a}^{x=b} A(x)\,dx = \int_{x=a}^{x=b} \pi (f(x))^2\,dx$ for disc method

$V = \int_{x=a}^{x=b} A(x)\,dx = \int_{x=a}^{x=b} \pi \left[(f(x))^2 - (g(x))^2 \right]$ for washer method

Revolving around y-axis:

- Thickness of each disc is always Δy, so will always integrate with respect to y.
- Radius of each disc is always $f(y)$ (or $f(y) - g(y)$ for washer method), so area is always $A(y) = \pi (f(y))^2$ (or $A(y) = \pi \left[(f(y))^2 - (g(y))^2 \right]$ for washer method).

- $V = \int_{y=c}^{y=d} A(y)\,dy = \int_{y=c}^{y=d} \pi (f(y))^2\,dy$ for disc method

$$V = \int_{y=c}^{y=d} A(y)\,dy = \int_{y=c}^{y=d} \pi \left[(f(y))^2 - (g(y))^2 \right]$$ for washer method

Shell Method:

Revolving around x-axis:

- Depth of each shell is always Δy, so will always integrate with respect to y.
- Height of each shell is always $f(y)$ (or $f(y) - g(y)$ for 2 curves)
- Length of each shell (when opened up) is always $2\pi y$.
- Thus, the area of the face of each shell (when opened up) is $A(y) = 2\pi y \cdot f(y)$ (or $A(y) = 2\pi y [f(y) - g(y)]$ for 2 curves).

- $V = \int_{y=c}^{y=d} 2\pi y \cdot f(y)\,dy$ for one curve

$$V = \int_{y=c}^{y=d} 2\pi y [f(y) - g(y)]\,dy$$ for 2 curves

Revolving around y-axis:

- Depth of each shell is always Δx, so will always integrate with respect to x.
- Height of each shell is always $f(x)$ (or $f(x)-g(x)$ for 2 curves)
- Length of each shell (when opened up) is always $2\pi x$.
- Thus, the area of the face of each shell (when opened up) is $A(x) = 2\pi x \cdot f(x)$ (or $A(x) = 2\pi x [f(x)-g(x)]$ for 2 curves)
- $V = \int_{x=a}^{x=b} 2\pi x \cdot f(x)\, dx$ for one curve

$$V = \int_{x=a}^{x=b} 2\pi x [f(x) - g(x)]\, dx \quad \text{for 2 curves.}$$

Always Remember:

When using the disc or washer method we integrate with respect to the variable of the axis we're revolving around.

When using the shell method we integrate with respect to the variable of the axis we're <u>not</u> revolving around.