# Section 8.3: Trigonometric Substitutions

## Goal

In this section, we will introduce a technique of integration called *trigonometric substitution*. This technique is useful for dealing with functions containing the forms: $a^2 - u^2$, $a^2 + u^2$, and $u^2 - a^2$.

## The 3 basic trig substitutions

| form | substitution | identity | triangle |
|---|---|---|---|
| $a^2 - u^2$ | $u = a\sin\theta$ | $1 - \sin^2\theta = \cos^2\theta$ | |
| $a^2 + u^2$ | $u = a\tan\theta$ | $1 + \tan^2\theta = \sec^2\theta$ | |
| $u^2 - a^2$ | $u = a\sec\theta$ | $\sec^2\theta - 1 = \tan^2\theta$ | |

**Important Note 1.**

1. The pattern of constant$^2$, variable$^2$, and sign matches each identity.
2. You should always try $u$-sub before attempting trig sub. Also, sometimes you may already know a formula for evaluating the integral.

## Examples

OK, let's jump in and do a bunch of examples.

 

**Example 2.** Integrate.

(a) $\displaystyle\int \frac{x^3}{\sqrt{1-x^2}}\,dx$

(b) $\displaystyle\int \frac{1}{(4x^2+9)^2}\,dx$

 

(c) $\displaystyle\int \frac{\sqrt{x^2-25}}{x}\,dx$

(d) $\displaystyle\int \frac{1}{\sqrt{3-2x-x^2}}\,dx$

 

(e) $\displaystyle\int \sqrt{x^2+1}\, dx$

(f) $\displaystyle\int \frac{x}{\sqrt{x^2-25}}\, dx$

 