The Circle-Dot System*

Welcome to Circle-Dot! The object of the game is to construct a circle-dot word made entirely of o's and .'s. The words in this toy logical system do not mean anything, but the system teaches us some of the fundamentals of formal proofs.

You do not have any words to begin with. On each of your turns you can apply one of the 5 available axioms/rules to your current list of constructed circle-dot words to produce a new word. It may require some study and investigation on your part to come up with an efficient strategy for beating this game at the higher levels of difficulty.

In the Circle-Dot System, we do work that is very much like a formal proof, in that we have axioms to start with, rules to make progress with, and a goal for which we aim. They are summarized here; note that o and . are valid symbols in the system, and w and v are variables that stand for any sequence of o's and .'s.

The axioms

Here are the axioms for Circle-Dot:

> **Axiom 1:** o.

> **Axiom 2:** .o

At any time in your proof, you may quote an axiom.

The rules

Here are the rules for generating new statements from known statements:

> **Rule 1:** Given wv and vw, conclude w

> **Rule 2:** Given w and v, conclude $w.v$

> **Rule 3:** Given $wv.$, conclude wo

Some theorems

Try to prove the following theorems using the axioms and rules.

(1) . (just a single dot)

(2) ...

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*This handout is based on the NES MAA Fall 2008 Workshop on *Lurch*. For a complete introduction to the Circle-Dot System, together with an interesting theoretical question about provability in the system, see `http://math.scranton.edu/monks/courses/math448/ToyProofsV2.html` on the website of Ken Monks from the University of Scranton.

(3) ..o

(4) .oo

(5) o..o

(6) oooo

(7) .o.

(8) .ooo