Chapter 1: Logic
Sections 1.6–1.9

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Symbolically, we write implications as $A \implies B$, which may also be read "A implies B." Like we did with implications, we can summarize the truth and falsehood of other compound statements in truth tables.

Recall that if A and B are predicates (i.e., contain a free variable), then $A \implies B$ is true if all possible values of the free variable(s) make the truth values for A and B not occur in the 2nd row of the truth table.

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- conjunction: "A and B" denoted $A \wedge B$
- disjunction: "A or B" (inclusive or) denoted $A \vee B$
- negation: "Not A" denoted $\sim A$
- equivalence: "A if and only if B denoted $A \iff B$ (we usually abbreviate "if and only if" as "iff")

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By the way, why are there 4 rows in the truth table above? What if we had a more complicated compound statement involving 3 different statements?

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Observations? A compound statement that is always false (regardless of the truth value of the simpler statements involved) is called a contradiction.

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Obervations? A statement that is true (regardless of the truth value of the simpler statements involved) is called a tautology.

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Certainly, there are situations with specific A and B, where an implication and its converse are equivalent, but this does not happen in general. When would an implication and its converse be equivalent anyway?

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If A and B are predicates and $A \iff B$ is true for all possible substitutions of variables (from the appropriate universe), then we say that A and B are equivalent.

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Let’s take a look at an example.

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The phrases "A is equivalent to B", "A iff B", and "A is necessary and sufficient for B" can be used interchangeably.

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We can always write "It is not true that A" for $\sim A$, but often is more useful to rephrase a negation in terms of what is true rather than what is not true.

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This example illustrates what happens in general.

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Using these symbols along with the appropriate quantifiers, we can rewrite the statement as

$$(\text{For all } x, M(x) \implies (S(x) \wedge B(x))) \vee D$$

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> *There exists a Martian that is either tall or has some hair, and my name is Darth Vader.*

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So, the negation of "If $A(x)$, then $B(x)$" is "There exists x such that $A(x) \wedge \sim B(x)$."

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There exists a real number x such that $x^3 = x$.

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Recall the second statement from our thought experiment in Section 1.1:

There exists a real number x such that $x^3 = x$.

This is an example of an existence theorem.

1.9 Existence Theorems

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Recall the second statement from our thought experiment in Section 1.1:

There exists a real number x such that $x^3 = x$.

This is an example of an existence theorem. But how do we prove it?

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Suppose that we wanted to prove "There exists a clacking waggler."

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Suppose that we wanted to prove "There exists a clacking waggler." We prove existence theorems in two steps.

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1. We produce a candidate. That is, we describe an object that should be a clacking waggler. (This happens behind the scenes and is often the hard part. The proof that gets written down does not necessarily reflect all the work that went into finding the candidate.)
2. We show that our candidate actually is what we claim. In this case, we show that it is a waggler and clacks.