

# MATH 1300: Calculus I, Spring 2008

## Review for Midterm Exam 2

Midterm Exam 2 covers material from sections 3.1, 3.2, 3.3, 3.4, 3.5, 3.6, 3.7, 4.1, 4.2, 4.3, and 4.4 of our textbook and material from Worksheets 4, 5, 6, and 7. Material covered on the previous exam is also fair game. In fact, one of the questions on Exam 2 is nearly identical to a question on Exam 1 (so, study Exam 1!). This review will give you a good indication of what you will be expected to know for the exam. However, you should not expect the exam to be identical to this review.

To study for the exam, you should do the following (in this order):

1. Study class notes and read the textbook
2. Review homework problems, worksheets, and homework associated with the worksheets
3. Attempt as many problems on this review that you feel are necessary to increase your understanding of the material
4. If you feel you need additional practice, attempt problems in the chapter reviews (that are similar to assigned homework problems) located at the end of chapters 3 and 4

### I. Concepts

True or False? Justify your answers.

- T** (a) If a function is not continuous at  $x = c$ , then it is not differentiable at  $x = c$ . a function must be cont in order to be diff.
- T** (b) The derivative with respect to  $x$  of a rational function  $f(x)$  is a rational function. ie, diff  $\Rightarrow$  cont
- F** (c)  $\frac{d}{dx}[f(cx)] = c \frac{d}{dx}[f(x)]$ .
- F** (d)  $\frac{d}{dx}\left[\frac{1}{f(x)}\right] = \frac{1}{f'(x)}$ .  $\frac{d}{dx}[(f(x))^{-1}] = \frac{-f'(x)}{[f(x)]^2}$
- F** ~~g~~ (e) If  $f$  and  $g$  are differentiable on an open interval containing  $c$ , except possibly at  $x = c$ , and  $\lim_{x \rightarrow c} \frac{f(x)}{g(x)}$  is an indeterminate form of type  $\infty/\infty$ , then  $\lim_{x \rightarrow c} \frac{f(x)}{g(x)} = \lim_{x \rightarrow c} \frac{f'(x)}{g'(x)}$ .
- F** (f)  $\frac{d}{dx}[(2x^3 + 5x^2 - 7)(3 \cos(x) + 13x)] = (6x^2 + 10x)(-3 \sin(x) + 13)$  Product Rule!

Counterexample:  
 $\lim_{x \rightarrow \infty} \frac{x + \sin x}{x}$

### II. Additional Problems

1. A representative from a certain publishing company is dropped from the top of a 220 foot tall building. The height  $h(t)$  in feet of the representative at  $t$  seconds is given by the position function  $h(t) = -16t^2 - 26t + 220$ .

## II. Additional Problems.

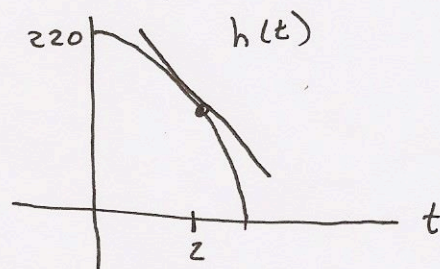
1.  $h(t) = -16t^2 - 26t + 220$ .

- a) Avg velocity. Technically, there's a problem with this question because the representative hits the ground in just under 3 seconds. However, if you ignore this minor detail (maybe he falls into a black hole and gets flung to another galaxy), then

$$v_{\text{avg}} = \frac{h(4) - h(2)}{4 - 2} = \frac{-140 - 104}{2} = -122 \text{ ft/sec}$$

b)  $v_{\text{inst}} = h'(2) = -32t - 26 \Big|_{t=2} = -90 \text{ ft/sec}$

- c) part (b) being negative means the distance between the representative and the ground (or the black hole) is decreasing. The dude is falling.



2.  $f(x) = x^2 - x$ . find eqn of tangent line at  $x = 2$ .

$f'(x) = 2x - 1$ , so  $f'(2) = 4 - 1 = 3$ .

also,  $f(2) = 2^2 - 2 = 4 - 2 = 2$ .

$$\boxed{y - 2 = 3(x - 2)}$$

3.  $y = x^2 \sin\left(\frac{\pi}{4}x\right)$

$$\frac{dy}{dx} = 2x \sin\left(\frac{\pi}{4}x\right) + \frac{\pi}{4} x^2 \cos\left(\frac{\pi}{4}x\right)$$



$$4. \frac{d}{dx} [\sec(x) \tan(x) + (\cos x)^2]$$

$$= \sec x \tan^2 x + \sec^3 x + 2 \cos x (-\sin x)$$

5.  $f(x) = \csc(2x) + \tan(x)$  . find eqn tangent line at  $(\frac{\pi}{4}, 2)$

$$f'(x) = -2 \csc(2x) \cot(2x) + \sec^2(x)$$

$$f'(\frac{\pi}{4}) = -2 \csc(\frac{\pi}{2}) \cot(\frac{\pi}{2}) + \sec^2(\frac{\pi}{4})$$

$$\csc(\frac{\pi}{2}) = \frac{1}{\sin(\frac{\pi}{2})} = \frac{1}{1} = 1$$

$$\cot(\frac{\pi}{2}) = \frac{\cos(\frac{\pi}{2})}{\sin(\frac{\pi}{2})} = \frac{0}{1} = 0$$

$$\sec^2(\frac{\pi}{4}) = \frac{1}{\cos^2(\frac{\pi}{4})} = \frac{1}{(\frac{\sqrt{2}}{2})^2} = \frac{1}{\frac{2}{4}} = 2$$

so  $f'(\frac{\pi}{4}) = -2(1)(0) + 2 = 2$

$$\boxed{y - 2 = 2(x - \frac{\pi}{4})}$$

6.  $(f \circ g \circ h)'(1) = \frac{d}{dx} f(g(h(x)))|_{x=1}$

$$= f'(g(h(1))) g'(h(1)) h'(1)$$

$$= f'(g(5)) g'(5) h'(1)$$

$$= f'(2) g'(5) h'(1)$$

$$= (3)(-2)(-8)$$

$$= 48$$

# 7. yay! Related Rates!

$$V = \frac{4}{3} \pi r^3$$

$$\frac{dV}{dt} = -3 \text{ in}^3/\text{sec} \leftarrow \text{decreasing}$$

$$r = 2 \text{ in} \Rightarrow D = 4 \text{ in}$$

$$V = \frac{4}{3} \pi \left(\frac{D}{2}\right)^3$$

$\leftarrow$  change to diameter

$$\frac{dD}{dt} = ?$$

$\leftarrow$  asking for rate diameter is decreasing!

$$V = \frac{4}{3} \pi \frac{D^3}{8}$$

so,

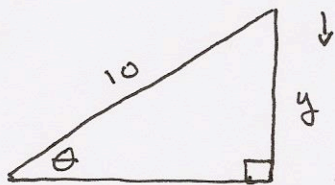
$$\frac{dV}{dt} = \frac{\pi}{2} D^2 \frac{dD}{dt}$$

$$V = \frac{\pi}{6} D^3$$

$$(-3) = \frac{\pi}{2} 4^2 \frac{dD}{dt}$$

$$\frac{dD}{dt} = \frac{-3}{8\pi} \text{ in/sec}$$

8.



$$\frac{dy}{dt} = -2 \text{ ft/sec}$$

$$y = 5 \text{ ft}$$

$$\frac{d\theta}{dt} = ?$$

$$\sin \theta = \frac{y}{10} \Rightarrow y = 10 \sin \theta$$

$$\frac{dy}{dt} = 10 \cos \theta \frac{d\theta}{dt}$$

$\uparrow$   
 need  $\theta$

$$\sin \theta = \frac{5}{10} = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}$$

$$-2 = 10 \cos\left(\frac{\pi}{6}\right) \frac{d\theta}{dt}$$

$$-2 = 10 \frac{\sqrt{3}}{2} \frac{d\theta}{dt}$$

$$\frac{d\theta}{dt} = \frac{-2}{5\sqrt{3}} \text{ rad/sec.}$$



9.  $y = \arcsin(x)$ . eqn tangent line at  $x = \frac{1}{2}$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} \Big|_{x=1/2} = \frac{1}{\sqrt{1-1/4}} = \frac{1}{\sqrt{3/4}} = \frac{2}{\sqrt{3}}$$

also,  $\arcsin(\frac{1}{2}) = \sin^{-1}(\frac{1}{2}) = \frac{\pi}{6}$

$$\boxed{y - \frac{\pi}{6} = \frac{2}{\sqrt{3}} \left(x - \frac{1}{2}\right)}$$

10.  $f(x) = \frac{x}{x^2+3}$

a)  $f'(x) = \frac{(x^2+3)(1) - x(2x)}{(x^2+3)^2} = \frac{x^2+3 - 2x^2}{(x^2+3)^2} = \frac{-x^2+3}{(x^2+3)^2}$

b)  $f(x) = (x)(x^2+3)^{-1}$

$$f'(x) = (x^2+3)^{-1} + x(-1)(2x)(x^2+3)^{-2}$$

$$= \frac{1}{x^2+3} - \frac{2x^2}{(x^2+3)^2}$$

$$= \frac{x^2+3 - 2x^2}{(x^2+3)^2} = \frac{-x^2+3}{(x^2+3)^2}$$

these should be the same!

c)  $y = \frac{x}{x^2+3}$

$$\ln y = \ln\left(\frac{x}{x^2+3}\right) = \ln x - \ln(x^2+3)$$

$$\frac{dy}{dx} \cdot \frac{1}{y} = \frac{1}{x} - \frac{2x}{x^2+3} = \frac{x^2+3 - 2x^2}{x(x^2+3)}$$

$$\frac{dy}{dx} = y \cdot \left( \frac{-x^2+3}{x(x^2+3)} \right) = \frac{x}{x^2+3} \cdot \frac{-x^2+3}{x(x^2+3)} = \frac{-x^2+3}{(x^2+3)^2}$$

same here!

11. find  $\frac{dy}{dx}$

a)  $y = \ln x^2 = 2 \ln x$

$$\frac{dy}{dx} = \frac{2}{x}$$

b)  $y = x \ln 2$

$$\frac{dy}{dx} = \ln 2$$

c)  $y = x^e$

$$\frac{dy}{dx} = e x^{e-1}$$

d)  $y = x^x$

$$\ln y = x \ln x$$

$$\frac{dy}{dx} \frac{1}{y} = \ln x + 1$$

$$\frac{dy}{dx} = x^x (\ln x + 1)$$

e)  $y = 2^x$

$$\frac{dy}{dx} = 2^x \ln(2)$$

f)  $y = \ln(\cos x)$   $\leftarrow$  I guess assuming  $\cos x > 0$ .

$$\frac{dy}{dx} = \frac{1}{\cos x} \cdot (-\sin x) = -\tan x$$

g)  $y = e^{3x+5}$

$$\frac{dy}{dx} = 3e^{3x+5}$$

12. Log Diff.

a)  $f(x) = \frac{(3-x)^{1/3} x^2}{(1-x)(3+x)^{2/3}}$

$$\ln y = \frac{1}{3} \ln(3-x) + 2 \ln x - \ln(1-x) - \frac{2}{3} \ln(3+x)$$

$$\frac{1}{y} \frac{dy}{dx} = \frac{-1}{3(3-x)} + \frac{2}{x} + \frac{1}{1-x} - \frac{2}{3(3+x)}$$

$$\frac{dy}{dx} = y \left( \frac{-1}{3(3-x)} + \frac{2}{x} + \frac{1}{1-x} \right)$$

$$= \frac{(3-x)^{1/3} x^2}{(1-x)(3+x)^{2/3}} \left( \frac{-1}{3(3-x)} + \frac{2}{x} + \frac{1}{1-x} \right)$$



$$12 \text{ b) } f(x) = (x+1)(e^{x^2} + 1)$$

$$\ln y = \ln(x+1) + \ln(e^{x^2} + 1)$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{1}{x+1} + \frac{2xe^{x^2}}{e^{x^2} + 1}$$

$$\frac{dy}{dx} = (x+1)(e^{x^2} + 1) \left( \frac{1}{x+1} + \frac{2xe^{x^2}}{e^{x^2} + 1} \right)$$

13. Limits !

$$a) \lim_{x \rightarrow \pi/2} \frac{1 - \sin x}{\cos x} \quad " \frac{0}{0} "$$

$$= \lim_{x \rightarrow \pi/2} \frac{-\cos x}{-\sin x} \quad (L'Hop)$$

$$= \frac{0}{1} = 0.$$

$$b) \lim_{x \rightarrow 0} \frac{e^x - 1}{x^3} \quad " \frac{0}{0} "$$

$$= \lim_{x \rightarrow 0} \frac{e^x}{3x^2} = +\infty$$

$$c) \lim_{x \rightarrow 0^+} x \ln x \quad " 0 \cdot (-\infty) "$$

can't use L'Hop yet!

$$= \lim_{x \rightarrow 0^+} \frac{\ln x}{1/x} \quad " \frac{-\infty}{\infty} "$$

$$= \lim_{x \rightarrow 0^+} \frac{1/x}{-1/x^2} \quad (L'Hop.)$$

$$= \lim_{x \rightarrow 0^+} \frac{-x^2}{x}$$

$$= 0$$

13.

$$d) \lim_{x \rightarrow 0^+} \left( \frac{1}{x} - \frac{1}{\sin x} \right)$$

" $\infty - \infty$ "  
can't use L'Hop yet!

$$= \lim_{x \rightarrow 0^+} \frac{\sin x - x}{x \sin x}$$

" $\frac{0}{0}$ "

$$= \lim_{x \rightarrow 0^+} \frac{\cos x - 1}{\sin x + x \cos x}$$

(L'Hop) " $\frac{0}{0}$ "

$$= \lim_{x \rightarrow 0^+} \frac{-\sin x}{2 \cos x - x \sin x} = 0.$$

$$e) \lim_{x \rightarrow 0^+} x^x$$

" $\frac{0}{0}$ "

can't use L'Hop yet!

$$y = \lim_{x \rightarrow 0^+} x^x$$

$$\ln y = \lim_{x \rightarrow 0^+} x \ln x$$

" $0 \cdot (-\infty)$ "  
again, not yet!

$$\ln y = \lim_{x \rightarrow 0^+} \frac{\ln x}{1/x}$$

$$\ln y = 0 \quad (\text{from part (c)})$$

$$y = e^0 = 1.$$

14.

$$\lim_{x \rightarrow 0^+} \frac{x}{\sin x} = 1$$

limit DNE

$$\lim_{x \rightarrow 0^-} \frac{-x}{\sin x} = -1$$

L'Hopital's Ahoy!

$$\text{but } \lim_{x \rightarrow 0^-} \frac{f'(x)}{g'(x)} = -1$$

So, we  
can't  
use L'H  
rule

Note that  $\lim_{x \rightarrow 0} \frac{f'(x)}{g'(x)}$  due since  $\lim_{x \rightarrow 0^+} \frac{f'(x)}{g'(x)} = 1$