MA 2560: Calculus II (Spring 2010)
Exam 3

NAME:

Instructions: Answer each of the following questions completely. To receive full credit, you must *justify* each of your answers (unless stated otherwise). How you reached your answer is more important than the answer itself. If something is unclear, or if you have any questions, then please ask. Good luck!

Here are some potentially useful formulas. You should *not* expect to use all of them.

$$\frac{d}{dx}[\sin^{-1} x] = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}[\cos^{-1} x] = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}[\tan^{-1} x] = \frac{1}{1+x^2} \qquad \frac{d}{dx}[\sec^{-1} x] = \frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}[\sinh^{-1} x] = \frac{1}{\sqrt{1+x^2}} \qquad \frac{d}{dx}[\cosh^{-1} x] = \frac{1}{\sqrt{x^2-1}}$$

$$\frac{d}{dx}[\tanh^{-1} x] = \frac{1}{1-x^2} \qquad \frac{d}{dx}[\text{sech}^{-1} x] = \frac{-1}{x\sqrt{1-x^2}}$$

$$\frac{d}{dx}[b^x] = b^x \ln b \qquad \log_b(x) = \frac{1}{x \ln b}$$

$$\int \sinh u \, du = \cosh u + C \qquad \int \cosh u \, du = \sinh u + C$$

$$\int \text{sech}^2 u \, du = \tanh u + C \qquad \int \text{sech } u \tanh u \, du = -\text{sech } u + C$$

$$\int \frac{1}{\sqrt{a^2-u^2}} \, du = \sin^{-1}\frac{u}{a} + C \qquad \int \frac{1}{u^2+a^2} \, du = \frac{1}{a}\tan^{-1}\frac{u}{a} + C$$

$$\int \frac{1}{u\sqrt{u^2-a^2}} \, du = \frac{1}{a}\sec^{-1}\frac{u}{a} + C \qquad \int \frac{1}{\sqrt{u^2+a^2}} \, du = \sinh^{-1}\frac{u}{a} + C$$

$$\int \frac{1}{\sqrt{u^2-a^2}} \, du = \cosh^{-1}\frac{u}{a} + C \qquad \int \frac{1}{a^2-u^2} \, du = \frac{1}{a}\tanh^{-1}\frac{u}{a} + C$$

$$\int b^u \, du = \frac{b^u}{\ln b} + C \qquad \int \sec x \, dx = \ln|\sec x + \tan x| + C$$

$$M_n = \sum_{i=1}^n f\left(\tfrac{1}{2}(x_{i-1}+x_i)\right)\Delta x \approx \int_a^b f(x)\, dx$$

$$T_n = \tfrac{\Delta x}{2}\left(f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n)\right) \approx \int_a^b f(x)\, dx$$

$$S_n = \tfrac{\Delta x}{3}\left(f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)\right) \approx \int_a^b f(x)\, dx$$

 Written by D.C. Ernst

1. (10 points) A population of nuggets increases at a rate of $r(t)$ nuggets per week. The following table gives values for $r(t)$ every 4 weeks for 24 weeks. Using Simpson's Rule (with $n = 6$), estimate the population of nuggets at the end of the 24 week period. Round your answer to 4 decimal places. (At the very least, I want to see the formula that you are plugging into your calculator.)

t (weeks)	$r(t)$ (nuggets/week)
0	0
4	200
8	3,000
12	11,500
16	4000
20	250
24	0

2. Consider the following integral.

$$\int_0^1 \frac{\sin^2 x}{\sqrt{x}}\, dx$$

   (a) (4 points) Explain why this integral is improper.

   (b) (8 points) It turns out that this integral is very difficult to compute by hand. Show that this integral converges by comparing with a larger function that converges on the same interval. (Hint: recall that $\sin^2 x \leq 1$.)

   (c) (8 points) Approximate the value of the original integral using the Midpoint Rule with $n = 4$. Round your answer to 4 decimal places. (At the very least, I want to see the formula that you are plugging into your calculator.)

3. (10 points) Determine whether the following integral converges or diverges. If the integral converges, determine its *exact* value. If the integral diverges, explain why.

$$\int_1^\infty \frac{\ln x}{x}\ dx$$

4. (10 points) Find the arc length of $y = \dfrac{1}{3}(x^2+2)^{3/2}$ on $[0,3]$. You should give an *exact* answer (i.e., *not* a decimal approximation using your calculator).

5. (10 points) Find the area of the surface obtained by revolving the graph of $y = x^2$ on the interval $[0, \sqrt{2}]$ about the y-axis. You should give an *exact* answer (i.e., *not* a decimal approximation using your calculator).

6. (6 points each) Consider the parametric curve given by

$$x = 2\sin 2t, y = 3\sin t.$$

This curve crosses itself at $(0, 0)$.

(a) Find all values of t in $[0, 2\pi)$ such that $(x, y) = (0, 0)$.

(b) Find the slopes of the tangent lines to the graph for the values of t that you found in part (a).

7. (10 points) Consider the parametric curve given by

$$x = \frac{8}{3}t^{3/2}, y = 2t - t^2.$$

Find the arc length for $1 \leq t \leq 2$. (Give an *exact* answer.)

8. (8 points) Find the points of intersections (as ordered pairs (x, y)) of $r = 2\sin(\theta)$ and $r = 1$.

9. (10 points) Find the area of one loop of the graph of $r = \cos(5\theta)$.

10. **Bonus Question 1:** (2 points) Explain why it does not make sense to approximate the original integral in problem 2.

11. **Bonus Question 2:** (5 points) If f' is continuous on $[0, \infty)$ and $\lim_{x \to \infty} f(x) = 0$, show that

$$\int_0^\infty f'(x)\, dx = -f(0).$$

