Solutions to Selected Problems from 8.3

23. $\int \sqrt{5+4x-x^2}\,dx$

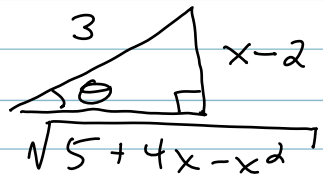
$= \int \sqrt{-(x^2-4x+4)+5+4}\,dx$

$= \int \sqrt{9-(x-2)^2}\,dx$

$a = 3$

$u = x-2 = 3\sin\theta$

$dx = 3\cos\theta\,d\theta$



$= \int \sqrt{9-9\sin^2\theta}\;3\cos\theta\,d\theta$

$= 3\int \sqrt{1-\sin^2\theta}\cdot 3\cos\theta\,d\theta$

$= 9\int \cos^2\theta\,d\theta$

$= 9\cdot\frac{1}{2}\int 1+\cos 2\theta\,d\theta$

$= \frac{9}{2}\left[\theta + \frac{1}{2}\sin 2\theta\right] + C$

$= \frac{9}{2}\left[\theta + \frac{1}{2}\cdot 2\sin\theta\cos\theta\right] + C$

$= \boxed{\frac{9}{2}\left[\arcsin\left(\frac{x-2}{3}\right) + \frac{x-2}{3}\cdot\frac{\sqrt{5+4x-x^2}}{3}\right] + C}$

24. $\int \frac{1}{\sqrt{t^2-6t+13}}\,dt$

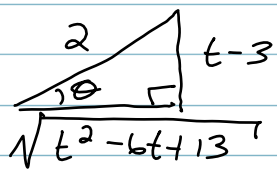
$= \int \frac{1}{\sqrt{t^2-6t+9+13-9}}\,dt$

$= \int \frac{1}{\sqrt{(t-3)^2+4}}\,dt$

$a = 2$

$u = t-3 = 2\tan\theta$

$du = 2\sec^2\theta\,d\theta$



$= \int \frac{1}{\sqrt{4\tan^2\theta+4}}\,2\sec^2\theta\,d\theta$

$= \int \frac{1}{2\sqrt{\tan^2\theta+1}}\,2\sec^2\theta\,d\theta$

$= \int \frac{1}{\sqrt{\sec^2\theta}}\,\sec^2\theta\,d\theta$

$= \int \sec\theta\,d\theta$

$= \ln|\sec\theta+\tan\theta| + C$

$= \boxed{\ln\left|\frac{2}{\sqrt{t^2-6t+13}} + \frac{t-3}{\sqrt{t^2-6t+13}}\right| + C}$

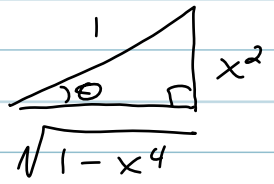
29. $\int x\sqrt{1-x^4}\,dx$

$a = 1$

$u = x^2 = \sin\theta$

$2x\,dx = \cos\theta\,d\theta$

$dx = \dfrac{\cos\theta\,d\theta}{2x}$



$= \int x\sqrt{1-\sin^2\theta}\,\dfrac{\cos\theta\,d\theta}{2x}$

$= \dfrac{1}{2}\int \sqrt{\cos^2\theta}\,\cos\theta\,d\theta$

$= \dfrac{1}{2}\int \cos^2\theta\,d\theta$

$= \dfrac{1}{2}\cdot\dfrac{1}{2}\int 1+\cos 2\theta\,d\theta$

$= \dfrac{1}{4}\left[\theta + \dfrac{1}{2}\sin 2\theta\right] + C$

$= \dfrac{1}{4}\left[\theta + \dfrac{1}{2}\cdot 2\sin\theta\cos\theta\right] + C$

$= \boxed{\dfrac{1}{4}\left[\arcsin(x^2) + x^2\cdot\sqrt{1-x^4}\right] + C}$