Sec 3.1 (10)

a) $m_{SEC} = \dfrac{f(2)-f(1)}{2-1} = \dfrac{2^3-1^3}{1} = \boxed{7}$

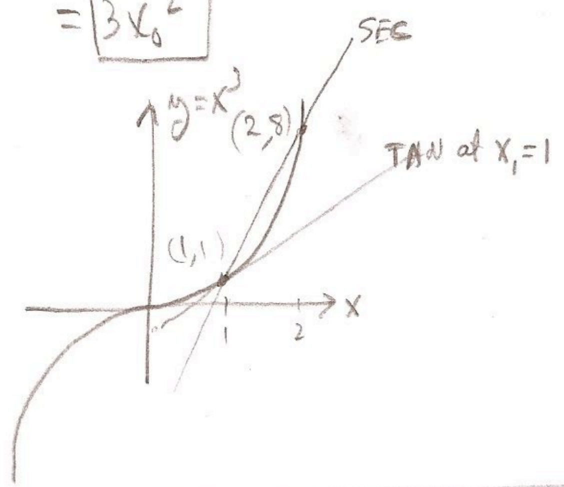
b) $m_{TAN} = \lim\limits_{x_1\to 1} \dfrac{f(x_1)-f(1)}{x_1-1} = \lim\limits_{x_1\to 1} \dfrac{x_1^3-1}{x_1-1} = \lim\limits_{x_1\to 1} \dfrac{(x_1-1)(x_1^2+x_1+1)}{x_1-1}$

$= \lim\limits_{x_1\to 1} (x_1^2+x_1+1) = \boxed{3}$

c) $m_{TAN} = \lim\limits_{x_1\to x_0} \dfrac{f(x_1)-f(x_0)}{x_1-x_0} = \lim\limits_{x_1\to x_0} \dfrac{x_1^3-x_0^3}{x_1-x_0} = \lim\limits_{x_1\to x_0} (x_1^2+x_1x_0+x_0^2)$

$= \boxed{3x_0^2}$

d)

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Sec 3.1 (20)

a) THE ROCK WILL HIT THE GROUND WHEN $16t^2=576$

$t^2=36 \Rightarrow \boxed{t=6 \text{ SECS}}$

(ONLY $t\geq 0$ IS MEANINGFUL)

b) $V_{AVG} = \dfrac{16(6^2)-16(0^2)}{6-0} = \boxed{96 \text{ ft/sec}}$

c) $V_{AVG} = \dfrac{16(3^2)-16(0^2)}{3-0} = \boxed{48 \text{ ft/sec}}$

d) $V_{INST} = \lim\limits_{t_1\to 6} \dfrac{16t_1^2-16(6^2)}{t_1-6} = \lim\limits_{t_1\to 6} \dfrac{16(t_1^2-36)}{t_1-6}$

$= \lim\limits_{t_1\to 6} 16(t_1+6) = 16(6+6) = \boxed{192 \text{ ft/sec}}$

SEC 3.2

(7) $y - y_0 = m(x - x_0)$

$y - f(3) = f'(3)(x-3)$ for $x_0 = 3$

$$\boxed{\begin{aligned} y - (-1) &= 5(x-3) \\ \text{OR} \quad y &= 5x - 16 \end{aligned}}$$

(22) $$\frac{dV}{dr} = \frac{d}{dr}[V] = \lim_{h \to 0} \frac{V(r+h) - V(r)}{h} = \lim_{h \to 0} \frac{\frac{4}{3}\pi(r+h)^3 - \frac{4}{3}\pi r^3}{h}$$

$$= \lim_{h \to 0} \frac{\frac{4}{3}\pi(r^3 - 3r^2h + 3rh^2 + h^3 - r^3)}{h}$$

$$= \lim_{h \to 0} \frac{4}{3}\pi(3r^2 + 3rh + h^2) = \boxed{4\pi r^2}$$

FROM THE DEFINITION OF DERIVATIVE

WITH THE POWER RULE: $\frac{d}{dr}[V] = \frac{d}{dr}\left[\frac{4}{3}\pi r^3\right]$

(WHICH WASN'T INTRODUCED UNTIL SEC 3.3)

$$= \frac{4}{3}\pi \cdot 3r^2 = \boxed{4\pi r^2}$$

(23) (a) D

(b) F

(c) B

(d) C

(e) A

(f) E

SEC 3.3

(5) $\frac{d}{dx}[\pi^3] = \boxed{0}$ SINCE $\pi^3$ IS A CONSTANT

(11) $f'(x) = \frac{d}{dx}\left[-3x^{-8} + 2x^{1/2}\right]$

$$= -3(-8x^{-9}) + 2\left(\tfrac{1}{2}x^{-1/2}\right)$$

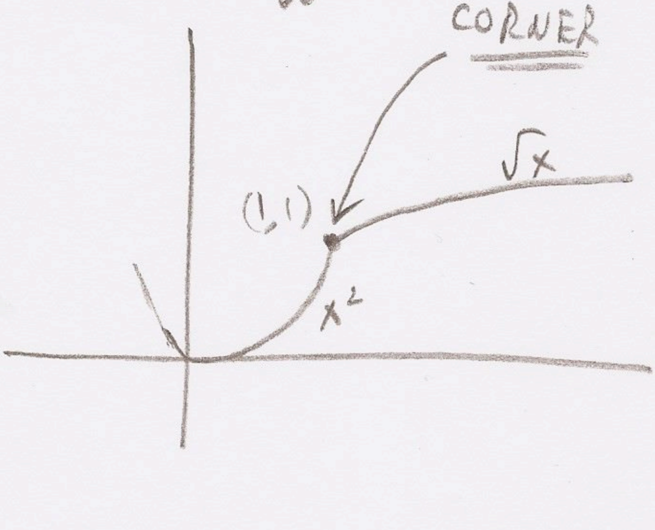
$$= \boxed{+24x^{-9} + \frac{1}{\sqrt{x}}}$$

(63) $\lim\limits_{x\to 1^-} f'(x) = \lim\limits_{x\to 1^-} (2x) = 2$ $\left[f'(x) = \frac{d}{dx}[x^2] = 2x \text{ for } x \le 1\right]$

$\lim\limits_{x\to} f'(x) = \lim\limits_{x\to 1^+} \frac{1}{2\sqrt{x}} = \frac{1}{2}$ $\left[f'(x) = \frac{d}{dx}[\sqrt{x}] = \frac{d}{dx}[x^{1/2}] = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}} \text{ for } x > 1\right]$

so $\lim\limits_{x\to 1} f'(x) = \text{DNE}$ because the left- and right-hand limits are different.

So $f$ is NOT differentiable at $x = 1$.



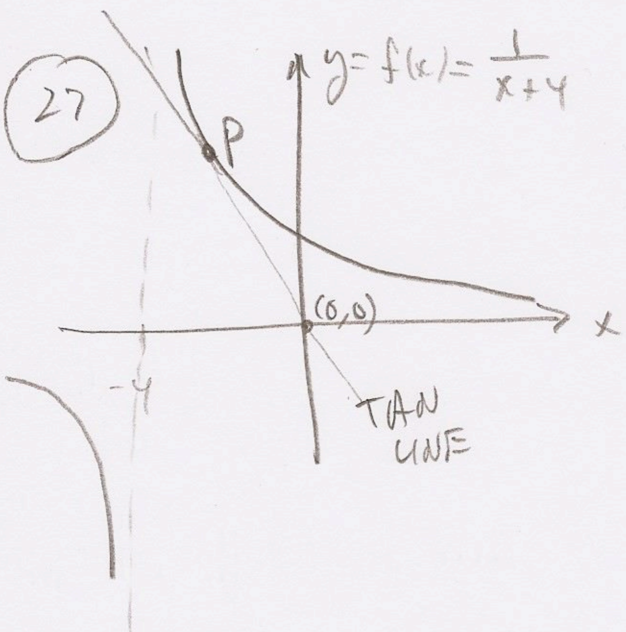
SEC 3.4

(13) $\left.\dfrac{dy}{dx}\right|_{x=1} = \dfrac{d}{dx}\left[\dfrac{2x-1}{x+3}\right]_{x=1} \overset{\text{QUO}}{=} \left.\dfrac{(x+3)\cdot\frac{d}{dx}[2x-1] - (2x-1)\cdot\frac{d}{dx}[x+3]}{(x+3)^2}\right|_{x=1}$

$= \left.\dfrac{(x+3)(2) - (2x-1)(1)}{(x+3)^2}\right|_{x=1} = \dfrac{(1+3)(2) - (2-1)(1)}{(1+3)^2}$

$= \dfrac{8-1}{16} = \boxed{\dfrac{7}{16}}$

(27)




Let P = POINT OF TANGENCY TO $f(x) = \frac{1}{x+4}$ WHERE THE TAN LINE PASSES THRU THE ORIGIN.

SUPPOSE P HAS 1st COORDINATE $x_0$. THEN THE y-COORD OF P IS $f(x_0) = \frac{1}{x_0+4}$

$P\left(x_0, \frac{1}{x_0+4}\right)$

THE TAN LINE PASSES THRU THE PTS $\left(x_0, \frac{1}{x_0+4}\right)$ AND $(0,0)$

SO $m_{TAN} = \dfrac{\Delta y}{\Delta x} = \dfrac{\frac{1}{x_0+4} - 0}{x_0 - 0} = \dfrac{1}{x_0(x_0+4)}$ EQUAL

BUT ALSO $m_{TAN} = \left.\dfrac{d}{dx}\left[\dfrac{1}{x+4}\right]\right|_{x=x_0} \overset{\text{QUO}}{=} \left.\dfrac{-1}{(x+4)^2}\right|_{x=x_0} = -\dfrac{1}{(x_0+4)^2}$

SO $\dfrac{1}{x_0(x_0+4)} = -\dfrac{1}{(x_0+4)^2} \implies \dfrac{1}{x_0} = \dfrac{-1}{x_0+4}$

$\implies x_0 + 4 = -x_0 \implies 2x_0 = -4 \implies \boxed{x_0 = -2}$

SEC 3.5

(7) $f'(x) = \frac{d}{dx}\left[\sec x - \sqrt{2}\tan x\right]$

$= \frac{d}{dx}\left[\sec x\right] - \sqrt{2}\cdot\frac{d}{dx}\left[\tan x\right]$

$= \boxed{\sec x\tan x - \sqrt{2}\sec^2 x}$

(15) $f'(x) = \frac{d}{dx}\left[\sin^2 x + \cos^2 x\right] = 2\sin x\cdot\frac{d}{dx}\left[\sin x\right] + 2\cos x\cdot\frac{d}{dx}\left[\cos x\right]$

$= 2\sin x\cos x + 2\cos x(-\sin x)$

$= \boxed{0}$

ALTERNATE METHOD (EASIER): $\frac{d}{dx}\left[\sin^2 x + \cos^2 x\right] = \frac{d}{dx}\left[1\right] = \boxed{0}$

(25) Let $f(x) = \tan x$. Then $f'(x) = \sec^2 x$

(a) at $x_0 = 0$, $m_{TAN} = f'(0) = \sec^2 0 = \frac{1}{\cos^2 0} = \frac{1}{1\cdot 1} = 1$

and $y_0 = f(0) = \tan 0 = 0$

So $y - y_0 = m(x - x_0) \Rightarrow y - 0 = 1(x - 0) \Rightarrow \boxed{y = x}$

(b) at $x_0 = \pi/4$, $m_{TAN} = f'(\pi/4) = \sec^2 \pi/4 = \frac{1}{\cos^2 \pi/4} = \frac{1}{\left(\frac{\sqrt{2}}{2}\right)^2}$

$= \frac{1}{1/2} = 2$

and $y_0 = f(\pi/4) = \tan \pi/4 = 1$

So $y - y_0 = m(x - x_0) \Rightarrow y - 1 = 2(x - \pi/4) \Rightarrow \boxed{y = 2x - \pi/2 + 1}$

(c) at $x_0 = -\pi/4$, $m_{TAN} = f'(-\pi/4) = \sec^2(-\pi/4) = 2$

and $y_0 = f(-\pi/4) = \tan(-\pi/4) = -1$

So $y - y_0 = m(x - x_0) \Rightarrow y - (-1) = 2(x - (-\pi/4)) \Rightarrow \boxed{y = 2x + \pi/2 - 1}$

SEC 3.6

(13) $f'(x) = \frac{d}{dx}\left[\sqrt{4+\sqrt{3x}}\right] = \frac{d}{dx}\left[\left(4+(3x)^{1/2}\right)^{1/2}\right]$

$= \frac{1}{2}\left[\left(4+(3x)^{1/2}\right)^{-1/2}\right] \cdot \frac{d}{dx}\left[4+(3x)^{1/2}\right]$

$= \frac{1}{2} \cdot \frac{1}{\sqrt{4+\sqrt{3x}}} \cdot \frac{1}{2}(3x)^{-1/2} \cdot \frac{d}{dx}\left[3x\right]$

$= \frac{1}{2} \cdot \frac{1}{\sqrt{4+\sqrt{3x}}} \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{3x}} \cdot 3 = \boxed{\frac{3}{4\sqrt{3x} \cdot \sqrt{4+\sqrt{3x}}}}$

ANSWER IN BACK OF BOOK IS WRONG.
THEIR ANSWER IS THE DERIVATIVE
OF $f(x) = \sqrt{4+3\sqrt{x}}$

(21) $f'(x) = \frac{d}{dx}\left[2\left(\sec(x^7)\right)^2\right] = 2 \cdot 2\sec(x^7) \cdot \frac{d}{dx}\left[\sec(x^7)\right]$

$= 4\sec(x^7) \cdot \sec(x^7) \cdot \tan(x^7) \cdot \frac{d}{dx}\left[x^7\right]$

$= 4\sec(x^7) \cdot \sec(x^7) \cdot \tan(x^7) \cdot 7x^6$

$= \boxed{28x^6 \sec^2(x^7)\tan(x^7)}$

(33) $\frac{dy}{dx} = \frac{d}{dx}\left[\cos^3(\sin 2x)\right] = \frac{d}{dx}\left[\left(\cos(\sin 2x)\right)^3\right]$

$= 3\left(\cos(\sin 2x)\right)^2 \cdot \frac{d}{dx}\left[\cos(\sin 2x)\right]$

$= 3\cos^2(\sin 2x) \cdot \left[-\sin(\sin 2x)\right] \cdot \frac{d}{dx}\left[\sin 2x\right]$

$= 3\cos^2(\sin 2x) \cdot \left[-\sin(\sin 2x)\right] \cdot \cos 2x \cdot \frac{d}{dx}\left[2x\right]$

$= 3\cos^2(\sin 2x) \cdot \left[-\sin(\sin 2x)\right] \cdot \cos 2x \cdot 2$

$= \boxed{-6 \cdot \cos^2(\sin 2x) \cdot \sin(\sin 2x) \cdot \cos 2x}$

SEC 3.7

(15) AIR REMOVED = VOLUME DECREASING

RADIUS IS DECREASING

FIND $\left.dV/dt\right|_{r=9\text{cm}}$ GIVEN $\frac{dr}{dt} = -15$ cm/min

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = \frac{d}{dt}\left[\frac{4}{3}\pi r^3\right] = \frac{4}{3}\pi \cdot 3r^2 \frac{dr}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$\left.\frac{dV}{dt}\right|_{r=9\text{cm}} = 4\pi (9\text{cm})^2 (-15\text{cm/min}) = \boxed{-4860\pi \text{ cm}^3/\text{min}}$$

(37) FIND $\left.dy/dt\right|_{\substack{x=1\\y=2}}$ GIVEN $\frac{dx}{dt} = 6$ units/sec

$\frac{xy^3}{1+y^2} = \frac{8}{5}$. FOR CONVENIENCE IN DIFFERENTIATING, REWRITE AS $xy^3 = \frac{8}{5}(1+y^2)$

NOW $\frac{d}{dt}\left[xy^3\right] = \frac{d}{dt}\left[\frac{8}{5}(1+y^2)\right]$

$$x \cdot 3y^2\, dy/dt + y^3 \cdot \frac{dx}{dt} = \frac{8}{5}(2y \cdot dy/dt)$$

NOW LET $x=1$, $y=2$, AND $\frac{dx}{dt} = 6$

$$1 \cdot 3 \cdot 2^2\, dy/dt + 2^3 \cdot 6 = \frac{8}{5}(2 \cdot 2 \cdot dy/dt)$$

$$12\, dy/dt + 48 = \frac{32}{5}\, dy/dt$$

$$12\, dy/dt - \frac{32}{5}\, dy/dt = -48$$

$$\frac{28}{5}\, dy/dt = -48 \Rightarrow dy/dt = \frac{-48}{28/5} = \frac{-240}{28} = \boxed{-\frac{60}{7} \text{ units/sec}}$$

THE PARTICLE IS $\boxed{\text{FALLING}}$, SINCE $dy/dt < 0$

SEC 4.1

(12) $x^3 + y^3 = 3xy^2$

$$\frac{d}{dx}\left[x^3 + y^3\right] = \frac{d}{dx}\left[3xy^2\right]$$

$$3x^2 + 3y^2 y' = 3\left[x \cdot 2yy' + y^2\right]$$

$$x^2 + y^2 y' = 2xyy' + y^2$$

$$y^2 y' - 2xyy' = y^2 - x^2$$

$$y'\left[y^2 - 2xy\right] = y^2 - x^2$$

$$\boxed{y' = \frac{y^2 - x^2}{y^2 - 2xy}}$$

(23) $x^3 y^3 - 4 = 0$

$$\frac{d}{dx}\left[x^3 y^3 - 4\right] = \frac{d}{dx}\left[0\right]$$

$$x^3 \cdot 3y^2 y' + y^3 \cdot 3x^2 = 0$$

$$x^3 y^2 y' = -x^2 y^3$$

$$y' = -\frac{x^2 y^3}{x^3 y^2} = -\frac{y}{x}$$

$$y'' = (y')' = \left(-\frac{y}{x}\right)' = \frac{d}{dx}\left[-\frac{y}{x}\right]$$

$$\overset{\text{quo}}{=} -\frac{xy' - y}{x^2} = -\frac{x(-y/x) - y}{x^2}$$

$$= -\frac{-y - y}{x^2}$$

$$= \boxed{\frac{2y}{x^2}}$$

Sec 4.2

(5) $y = \ln|x^2-1|$

$dy/dx = \frac{1}{x^2-1} \cdot 2x = \boxed{\frac{2x}{x^2-1}}$

(13) $y = x \cdot \ln x$

$dy/dx = x \cdot \frac{1}{x} + \ln x = \boxed{1 + \ln x}$

(21) $y = \ln(\tan x)$

$dy/dx = \frac{1}{\tan x} \cdot \frac{d}{dx}[\tan x] = \boxed{\frac{1}{\tan x} \cdot \sec^2 x}$

$= \boxed{(\cot x)(\sec^2 x)} = \frac{\cos x}{\sin x} \cdot \frac{1}{\cos^2 x} = \frac{1}{(\sin x)(\cos x)}$

$= (\csc x)(\sec x)$

(29) $\frac{d}{dx}\left[\ln \frac{\cos x}{(4-3x^2)^{1/2}}\right] = \frac{d}{dx}\left[\ln \cos x - \ln(4-3x^2)^{1/2}\right]$

$= \frac{d}{dx}\left[\ln \cos x - \frac{1}{2}\ln(4-3x^2)\right] = \frac{1}{\cos x}(-\sin x) - \frac{1}{2}\left(\frac{1}{4-3x^2}\right)(-6x)$

$= \boxed{-\tan x + \frac{3x}{4-3x^2}}$

(39) $m_{TAN} = \frac{d}{dx}[\ln x]\Big|_{x=e^{-1}} = \frac{1}{x}\Big|_{x=e^{-1}} = \frac{1}{e^{-1}} = e$

at $x_0 = e^{-1}$: $y = \ln(e^{-1}) = -1 \ln e = -1$

So $y - y_0 = m(x - x_0) \Rightarrow y - (-1) = e(x - e^{-1}) \Rightarrow y + 1 = ex - 1$

$\boxed{y = ex - 2}$

Sec 4.3

(19) $y = e^{(x - e^{3x})}$

$dy/dx = e^{(x-e^{3x})} \cdot \frac{d}{dx}\left[x - e^{3x}\right]$

$= e^{(x-e^{3x})} \cdot \left(1 - \frac{d}{dx}\left[e^{3x}\right]\right)$

$= e^{(x-e^{3x})} \left[1 - e^{3x} \cdot 3\right]$

$= \boxed{\left(1 - 3e^{3x}\right) e^{(x-e^{3x})}}$

(33) $y = \sin^{-1}(3x)$

$dy/dx = \frac{1}{\sqrt{1-(3x)^2}} \cdot \frac{d}{dx}\left[3x\right] = \boxed{\frac{3}{\sqrt{1-9x^2}}}$

(37) $y = \tan^{-1}(x^3)$

$dy/dx = \frac{1}{1+(x^3)^2} \cdot \frac{d}{dx}\left[x^3\right] = \boxed{\frac{3x^2}{1+x^6}}$

(44) $y = x^2 \left(\sin^{-1}x\right)^3$

$dy/dx \overset{\text{PROD}}{=} x^2 \cdot \frac{d}{dx}\left[\left(\sin^{-1}x\right)^3\right] + \left(\sin^{-1}x\right)^3 \cdot \frac{d}{dx}\left[x^2\right]$

$= x^2 \cdot 3\left(\sin^{-1}x\right)^2 \cdot \frac{d}{dx}\left[\sin^{-1}x\right] + 2x\left(\sin^{-1}x\right)^3$

$= x^2 \cdot 3\left(\sin^{-1}x\right)^2 \cdot \frac{1}{\sqrt{1-x^2}} + 2x\left(\sin^{-1}x\right)^3$

$= \boxed{\frac{3x^2\left(\sin^{-1}x\right)^2}{\sqrt{1-x^2}} + 2x\left(\sin^{-1}x\right)^3}$

## Sec 4.4

(7) $$\lim_{\theta \to 0} \frac{\tan\theta}{\theta} \overset{\text{L'HOP}}{=} \lim_{\theta \to 0} \frac{\sec^2\theta}{1} = \lim_{\theta \to 0} \frac{1}{\cos^2\theta} = \frac{1}{1^2} = \boxed{1}$$

(21) $$\lim_{x \to \infty} \left(x \cdot \sin \pi/x\right) = \lim_{x \to \infty} \frac{\sin \pi/x}{\frac{1}{x}}$$

$$\overset{\text{L'HOP}}{=} \lim_{x \to \infty} \frac{(\cos \pi/x) \cdot \frac{d}{dx}(\pi/x)}{\frac{d}{dx}\left[\frac{1}{x}\right]} = \lim_{x \to \infty} \frac{\left(\cos \frac{\pi}{x}\right) \cdot \pi \cdot \left(-\frac{1}{x^2}\right)}{-\frac{1}{x^2}}$$

$$= \lim_{x \to \infty} \left(\cos \frac{\pi}{x}\right) \cdot \pi = \pi \cdot \lim_{x \to \infty} \left(\cos \frac{\pi}{x}\right) = \pi \cdot \cos 0 = \boxed{\pi}$$

(35) $$\lim_{x \to \infty} \left[x - \ln(x^2 - 1)\right] = \lim_{x \to \infty} \left[\ln(e^x) - \ln(x^2 - 1)\right]$$

$$= \lim_{x \to \infty} \left[\ln\left(\frac{e^x}{x^2 - 1}\right)\right] = \ln\left[\lim_{x \to \infty} \frac{e^x}{x^2 - 1}\right]$$

[CAN TAKE $\lim\limits_{x \to \infty}$ INSIDE $\ln$ BECAUSE $\ln$ IS CONTINUOUS: SEE THEOREM 2.5.5 ON p. 148]

$$\overset{\text{L'HOP}}{=} \ln\left[\lim_{x \to \infty} \frac{e^x}{2x}\right] = \ln\left[\underbrace{\lim_{x \to \infty} \frac{e^x}{2}}_{+\infty \text{ as } x \to \infty}\right] = \boxed{+\infty}$$