MA 3540: Calculus III-Spring 2012
Exam 1

NAME:

Instructions: Answer each of the following questions completely. To receive full credit, you must show sufficient work for each of your answers (unless stated otherwise). How you reached your answer is more important than the answer itself. If something is unclear, or if you have any questions, then please ask. Good luck!

1. (3 points each) Let $\vec{a}$, $\vec{b}$, and $\vec{c}$ be vectors in V_3. Determine whether each of the following expressions is meaningful. If an expression is meaningful, determine whether the expression is a vector or scalar. Circle the correct answer.

   (a) $(\vec{a} \cdot \vec{b}) \cdot \vec{c}$ (*Note:* both "dots" represent dot product)

        meaningless   vector   scalar

   (b) $|\vec{a}|(\vec{b} \cdot \vec{c})$

        meaningless   vector   scalar

   (c) $\vec{a} \cdot \vec{b} + \vec{c}$

        meaningless   vector   scalar

   (d) $(\vec{a} \cdot \vec{b})\vec{c}$

        meaningless   vector   scalar

   (e) $\vec{a} \cdot (\vec{b} + \vec{c})$

        meaningless   vector   scalar

   (f) $|\vec{a}| \cdot (\vec{b} + \vec{c})$

        meaningless   vector   scalar

   (g) $\vec{a} \cdot (\vec{b} \times \vec{c})$

        meaningless   vector   scalar

   (h) $(\vec{a} \cdot \vec{b}) \times (\vec{a} \cdot \vec{c})$

        meaningless   vector   scalar
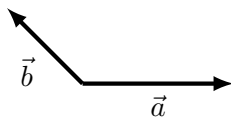
   (i) $(\vec{a} \times \vec{b}) \cdot (\vec{a} \times \vec{c})$

        meaningless   vector   scalar

   (j) $\vec{a} \times (\vec{b} \times \vec{c})$

        meaningless   vector   scalar

2. (6 points) Consider the following vectors $\vec{a}$ and $\vec{b}$ and assume that they are in standard position. Draw the vector $2\vec{a} - \vec{b}$ in *standard position.*

3. (6 points each) Let $\vec{a} = \langle 3, 0, 1 \rangle$, $\vec{b} = \langle 1, -2, 3 \rangle$, $c = -2$, and $d = 4$.

   (a) Find $|c\vec{a} + d\vec{b}|$.

   (b) Find the angle between $\vec{a}$ and $\vec{b}$. Write your answer in radians and round your answer to two decimal places.

   (c) Find the projection of $\vec{b}$ onto $\vec{a}$.

4. (8 points) Let $\vec{u} = \vec{i} - \vec{j} - 3\vec{k}$ and $\vec{v} = 2\vec{i} - \vec{j} + \vec{k}$. Determine whether $\vec{u}$ and $\vec{v}$ are orthogonal, parallel, or neither.

5. (6 points each) Suppose $\vec{a}$ and $\vec{b}$ are two vectors in V_3 such that $|\vec{a}| = 2$, $|\vec{b}| = 3$, and the angle between $\vec{a}$ and $\vec{b}$ is $\pi/6$. Give *exact answers* to the questions below.

   (a) Find $\vec{a} \cdot \vec{b}$.

   (b) Find $|\vec{a} \times \vec{b}|$. What does your answer represent?

6. (8 points) Find the area of the triangle determined by the points $P(1, 0, 1)$, $Q(-2, 1, 3)$, and $R(4, 2, 5)$.

7. (8 points) Determine whether the following vectors are coplanar. You must sufficiently justify your answer.

$$\vec{u} = \vec{\imath} + 5\vec{\jmath} - 2\vec{k}$$
$$\vec{v} = 3\vec{\imath} - \vec{\jmath}$$
$$\vec{w} = 5\vec{\imath} + 9\vec{\jmath} - 4\vec{k}$$

8. Consider the line L passing through the points $(-8, 1, 4)$ and $(3, -2, 4)$.

   (a) (6 points) Find the vector equation of L.

   (b) (4 points) Find a third point on L.

Written by D.C. Ernst