MA 2550: Calculus I (Spring 2009)
Exam 3

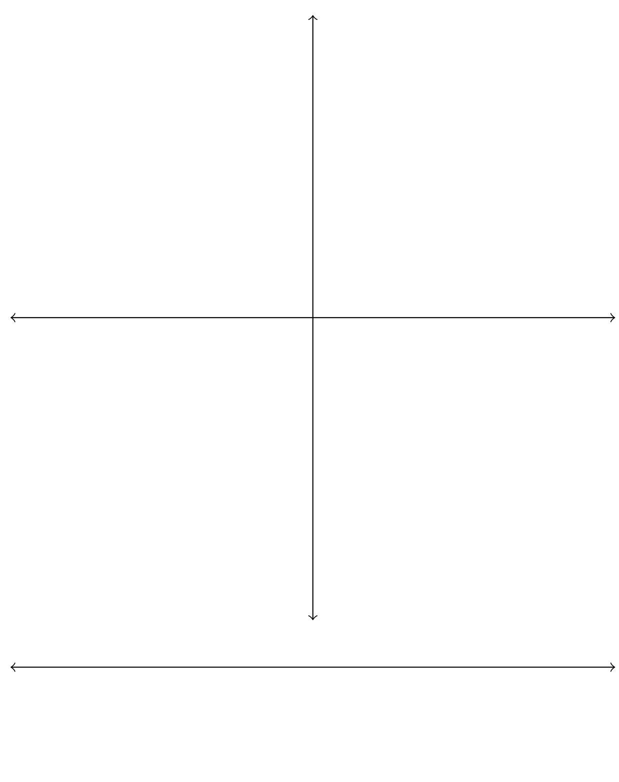
NAME:	(2 points!)

Instructions: Answer each of the following questions completely. To receive full credit, you must *justify* each of your answers (unless stated otherwise). How you reached your answer is more important than the answer itself. If something is unclear, or if you have any questions, then please ask. Good luck!

1. (8 points) Suppose f is a function with the following properties.

   (a) $f(-4) = 0$, $f(-2) = 0$, and $f(1.75) = 0$
   (b) $f(-3) = -1$, $f(-1) = 3$, and $f(0) = 4$
   (c) $\lim_{x\to 2^-} f(x) = -\infty$ and $\lim_{x\to 2^+} f(x) = \infty$
   (d) $\lim_{x\to \infty} f(x) = 0$ and $\lim_{x\to -\infty} f(x) = \infty$
   (e) $f'(-3) = 0$ and $f'(0) = 0$
   (f) $f'(x) > 0$ on $(-3, 0)$
   (g) $f'(x) < 0$ on $(-\infty, -3)$, $(0, 2)$, and $(2, \infty)$
   (h) $f''(-1) = 0$
   (i) $f''(x) > 0$ on $(-\infty, -1)$ and $(2, \infty)$
   (j) $f''(x) < 0$ on $(-1, 2)$

   Using the above information, make a sketch of the graph of f. You do *not* need to justify your answer.



2. (6 points each) Consider the following function.

$$f(x) = 5x^{2/3} + x^{5/3}$$

(a) Find all critical numbers of f.

(b) Using your answer(s) from part (a), determine whether each critical number determines a local maximum, local minimum, or neither. (You must show sufficient work to justify your answer.)

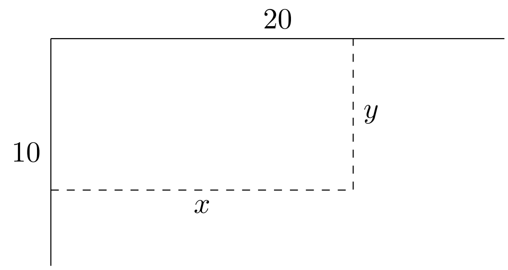
3. (8 points) Find the x-values of all inflection points for the following function.

$$f(x) = \frac{x^5}{20} - \frac{x^4}{6} + \frac{x^3}{6} + 5x + 1$$

4. (8 points) Find *all* aymptotes of the following function.

$$f(x) = \frac{5x^3 - 2x^2 - 1}{x^2 - 4}$$

5. Suppose you want to build a enclosed, rectangular pen for your cute baby nugget. For two sides of the pen you are going to use two perpendicular stone walls in your backyard, whose total lengths are 10 ft and 20 ft, respectively, and for the other two sides you are going to use 24 ft of fencing (See drawing).

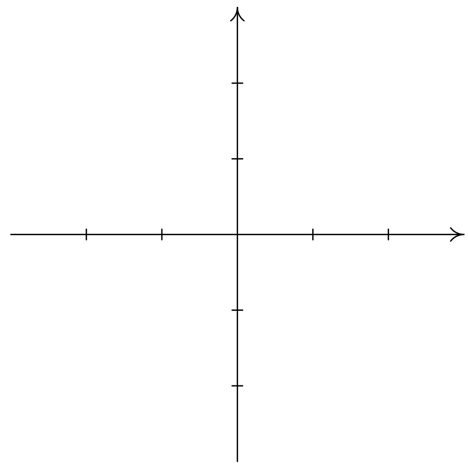


   (a) (4 points) Let A represent the area of the rectangular pen. Find an equation for A that involves only the variable x.

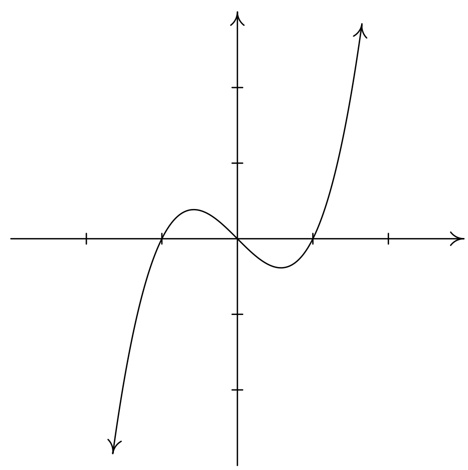
   (b) (4 points) Find the feasible domain for A. (Hint: how small can x be? How large can x be?)

   (c) (6 points) Using your answers to (a) and (b), find the *dimensions* that will maximize the area of the rectangular pen. (Justifying your answer will not only make sure that you receive full credit, but will also ensure that you don't make a mistake.)

6. (8 points) Using the graph of the function f given below, sketch a possible graph for the *antiderivative* of f, denoted by F.



Graph of F



Graph of f

7. (8 points each) Evaluate each of the following indefinite integrals. Sufficient work must be shown.

(a) $\displaystyle\int \frac{4+x^3}{x^2}\,dx$

(b) $\displaystyle\int \sec x(\tan x + \sec x)\,dx$

8. (8 points) At this time, we do *not* know how to evaluate the following definite integral using a limit of Riemann sums.

$$\int_0^{\pi} \cos^2 x \; dx$$

However, we can approximate this integral. Approximate the above integral using 4 equal width rectangles and right endpoints. (You should give an *exact* answer for your approximation.)

9. (8 points) Evaluate the following definite integral using a limit of Riemann sums and right endpoints.

$$\int_0^2 x^2 \, dx$$

You may find some of the following formulas useful:

$\displaystyle\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$	$\displaystyle\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$	$\displaystyle\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$	$\displaystyle\sum_{i=1}^{n} 1 = n$

$\displaystyle\int_a^b f(x) \, dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x$	$\displaystyle\Delta x = \frac{b-a}{n}$	$x_i = a + i\Delta x$

10. (4 points each) Provide an example of each of the following. You do *not* need to justify your answers.

    (a) An *equation* of a function f such that f has a critical number at $x = 0$, but f does not have a local maximum or local minimum at $x = 0$.

    (b) An *equation* of a function g such that g has a local minimum at $x = 0$, but $g'(0) \neq 0$.

11. **Bonus Question:** (5 points) A truck driver handed in a ticket at a toll booth showing that in 2 hours he had covered 158 mi on a toll road with speed limit 70 mph. The driver was cited for speeding. Use the Mean Value Theorem to explain why. Be sure to argue that the appropriate hypotheses of the Mean Value Theorem are satisfied.