Math 1300: Calculus I, Spring 2008; Instructor: Dana Ernst
Section 5.3: Curve Sketching (Part 2)
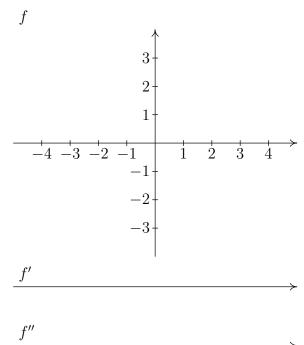
Question: If $f(x) = p(x)/q(x)$ is a rational function such that $p(x)$ and $q(x)$ have no factors in common (i.e., the "fraction" is reduced), then when will $f(x)$ have a horizontal asymptote? When will it not?

Answer:

When the degree of the numerator is _____ than the degree of the denominator, other kinds of asymptotes are possible: *oblique/slant, curvilinear.* To see what these new kinds of asymptotes are, we use polynomial long division.

Example 1: Identify the curvilinear asymptote of the following function and sketch its graph.
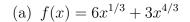
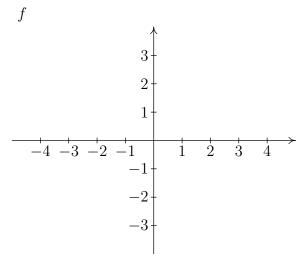$$g(x) = \frac{x^3 - x^2 + 4}{x - 1}$$

Theorem: A rational function cannot have both a horizontal asymptote and a curvilinear (including slant) asymptote. Why?

OK, we've dealt with polynomials and rational functions, how about some others? To sketch graphs of functions that are more complicated than rational functions, we follow the same set of guidelines that we wrote down in the first half of Section 5.3.

Example 2: Sketch the graphs of the following functions.

(a) $f(x) = 6x^{1/3} + 3x^{4/3}$

f

f'

f''

(b) $g(x) = e^{-x^2/2}$ (What is this graph?)

g

3
2
1

−4 −3 −2 −1   1   2   3   4

−1
−2
−3

g'

g''