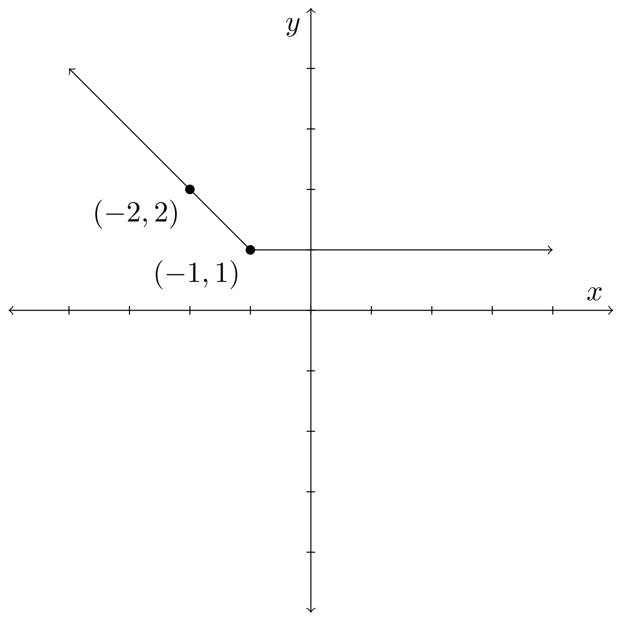
MA 2550: Calculus I (Spring 2009)
Exam 1

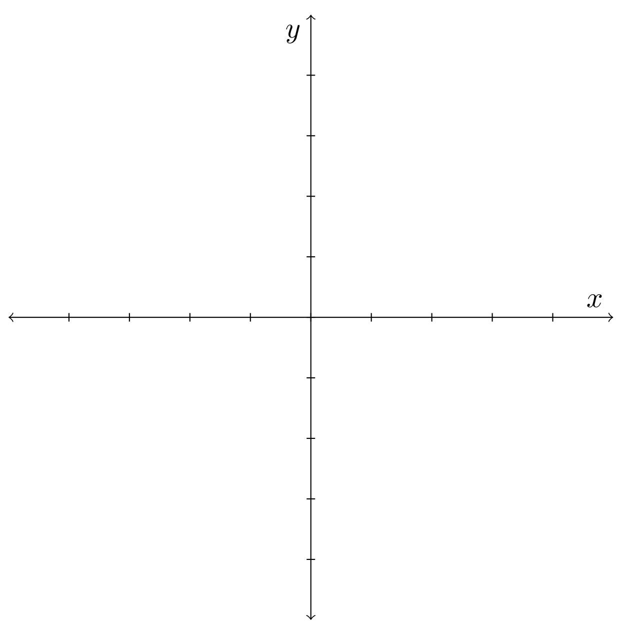
NAME:

Instructions: Answer each of the following questions completely. To receive full credit, you must *justify* each of your answers (unless stated otherwise). How you reached your answer is more important than the answer itself. If something is unclear, or if you have any questions, then please ask. Good luck!

1. (6 points) Suppose the graph of a function $y = f(x)$ looks like:



Using the axes provided, sketch the graph of the function $y = -f(x+1)$.

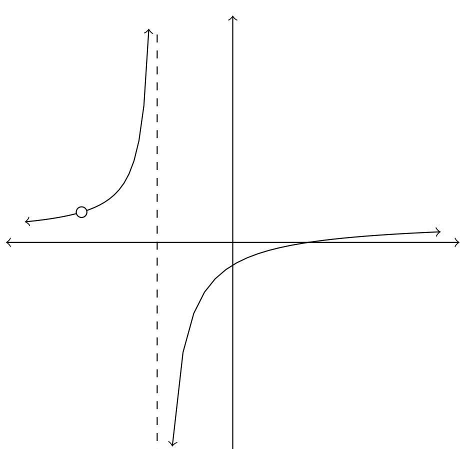


2. (3 points each) Let a, b, c and d be real numbers. Match each function with the only possible correct graph.

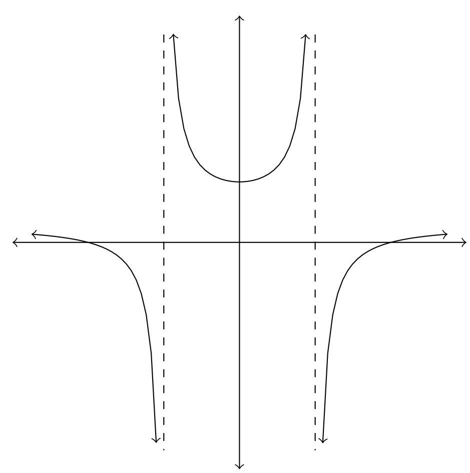
   (a) $f(x) = \dfrac{(x-a)(x-b)}{(x-a)(x-c)}$ Graph: _______

   (b) $g(x) = \dfrac{(x-a)(x-b)}{(x-a)(x-b)}$ Graph: _______

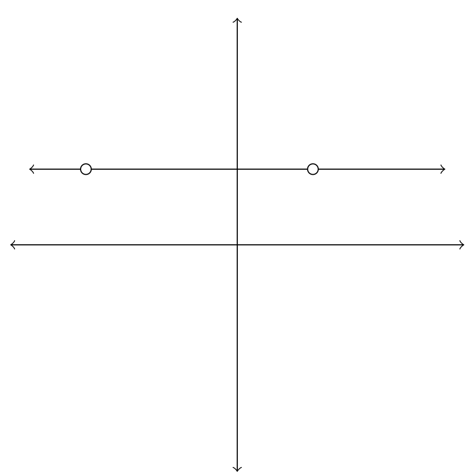
   (c) $h(x) = \dfrac{(x-a)(x-b)}{(x-c)(x-d)}$ Graph: _______



(a) Graph A



(b) Graph B



(c) Graph C

3. (6 points) Let $f(x) = 3x^2 - x + 5$ and $g(x) = 2x - 1$. Find $f \circ g(x)$ and simplify your answer.

4. (6 points each) Evaluate each of the following limits. If a limit does not exist, specify whether the limit equals ∞, $-\infty$, or simply does not exist (in which case, write DNE). Sufficient work must be shown. Give *exact answers*.

   (a) $\displaystyle\lim_{x \to -2} \frac{x+2}{x^2+4}$

   (b) $\displaystyle\lim_{x \to 4} \frac{\sqrt{x}-2}{x-4}$

(c) $\displaystyle\lim_{x\to\frac{\pi}{2}} \frac{\cos^2 x}{1-\sin(x)}$ (Hint: use a well-known trig identity.)

(d) $\displaystyle\lim_{x\to 0} \frac{\frac{1}{x+4}-\frac{1}{4}}{x}$

5. (3 points each) Consider the following function.

$$f(x) = \begin{cases} \dfrac{-1}{x-2}, & x > -1 \\ x^2 + 1, & x \leq -1 \end{cases}$$

For (a)–(h), evaluate the given expression. If an expression does not exist, specify whether it equals ∞, $-\infty$, or simply does not exist (in which case, write DNE). You do *not* need to justify your answers.

(a) $\lim\limits_{x \to -1^-} f(x)$

(b) $\lim\limits_{x \to -1^+} f(x)$

(c) $\lim\limits_{x \to -1} f(x)$

(d) $f(-1)$

(e) $\lim\limits_{x \to 2^-} f(x)$

(f) $\lim\limits_{x \to 2^+} f(x)$

(g) $\lim\limits_{x \to 2} f(x)$

(h) $f(2)$

(i) Identify any x-values where f has discontinuities.

6. (8 points) Use the Squeeze Theorem to evaluate $\lim_{x\to 0} x^4 \cos\left(\frac{2}{x}\right)$.

7. (8 points) Using the ϵ-δ definition of limit, complete the proof that

$$\lim_{x\to 1} 5x + 2 = 7$$

by filling in the blanks.

Proof: Let _____ > 0. Choose $\delta =$ _____. Assume that

$$0 < \underline{\hspace{3cm}} < \delta.$$

Then

$$\begin{aligned} |f(x) - \underline{\hspace{1.5cm}}| &= |\underline{\hspace{3cm}}| \\ &= 5\,|\underline{\hspace{3cm}}| \\ &< 5 \cdot \underline{\hspace{1.5cm}} \\ &= 5 \cdot \underline{\hspace{1.5cm}} \\ &= \underline{\hspace{1.5cm}}. \end{aligned}$$

This shows that

$$|f(x) - \underline{\hspace{1.5cm}}| < \underline{\hspace{1.5cm}}$$

whenever

$$0 < \underline{\hspace{3cm}} < \delta.$$

$\square$

8. (4 points each) Provide an example of each of the following. Each question should have its own separate answer and you do *not* need to justify your answer. (Providing a correct example of a graph but not an equation will be worth 3 points.)

   (a) An *equation* in the variables x and y that does not represent y as a function of x.

   (b) An *equation* of a function f such that $\lim_{x \to 1} f(x) = \infty$.

   (c) An *equation* of a function h such that $\lim_{x \to 1^-} h(x) \neq \lim_{x \to 1^+} h(x)$.