MA 2560: Calculus II (Fall 2009)
Exam 3

NAME:

Instructions: Answer each of the following questions completely. To receive full credit, you must *justify* each of your answers (unless stated otherwise). How you reached your answer is more important than the answer itself. If something is unclear, or if you have any questions, then please ask. Good luck!

Here are some potentially useful formulas. You should *not* expect to use all of them.

$$\frac{d}{dx}[\sinh x] = \cosh x \qquad \frac{d}{dx}[\cosh x] = \sinh x$$

$$\frac{d}{dx}[\tanh x] = \text{sech}^2 x \qquad \frac{d}{dx}[\text{sech } x] = -\text{sech } x \tanh x$$

$$\frac{d}{dx}[\sin^{-1} x] = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}[\cos^{-1} x] = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}[\tan^{-1} x] = \frac{1}{1+x^2} \qquad \frac{d}{dx}[\sec^{-1} x] = \frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}[\sinh^{-1} x] = \frac{1}{\sqrt{1+x^2}} \qquad \frac{d}{dx}[\cosh^{-1} x] = \frac{1}{\sqrt{x^2-1}}$$

$$\frac{d}{dx}[\tanh^{-1} x] = \frac{1}{1-x^2} \qquad \frac{d}{dx}[\text{sech}^{-1} x] = \frac{-1}{x\sqrt{1-x^2}}$$

$$\int \sinh u \, du = \cosh u + C \qquad \int \cosh u \, du = \sinh u + C$$

$$\int \text{sech}^2 u \, du = \tanh u + C \qquad \int \text{sech } u \tanh u \, du = -\text{sech } u + C$$

$$\int \frac{1}{\sqrt{a^2-u^2}} \, du = \sin^{-1}\frac{u}{a} + C \qquad \int \frac{1}{u^2+a^2} \, du = \frac{1}{a}\tan^{-1}\frac{u}{a} + C$$

$$\int \frac{1}{u\sqrt{u^2-a^2}} \, du = \frac{1}{a}\sec^{-1}\frac{u}{a} + C \qquad \int \frac{1}{\sqrt{u^2+a^2}} \, du = \sinh^{-1}\frac{u}{a} + C$$

$$\int \frac{1}{\sqrt{u^2-a^2}} \, du = \cosh^{-1}\frac{u}{a} + C \qquad \int \frac{1}{a^2-u^2} \, du = \frac{1}{a}\tanh^{-1}\frac{u}{a} + C$$

1. (10 points) Find the arc length of $y = \dfrac{1}{3}(x^2+2)^{3/2}$ on $[0,3]$. You should give an *exact* answer (i.e., *not* a decimal approximation using your calculator).

2. (10 points) Find the area of the surface obtained by revolving the graph of $y = x^2$ on the interval $[0, \sqrt{2}]$ about the y-axis. You should give an *exact* answer (i.e., *not* a decimal approximation using your calculator).

3. (6 points each) Consider the parametric curve given by

$$x = 2\sin 2t, y = 3\sin t.$$

This curve crosses itself at $(0, 0)$.

(a) Find all values of t in $[0, 2\pi)$ such that $(x, y) = (0, 0)$.

(b) Find the slopes of the tangent lines to the graph for the values of t that you found in part (a).

4. (10 points) Find the area of one loop of the graph of $r = \cos(5\theta)$.

5. (10 points) Consider the parametric curve given by

$$x = \frac{8}{3}t^{3/2}, y = 2t - t^2.$$

Find the arc length for $1 \leq t \leq 2$. (Give an *exact* answer.)

6. (8 points each) Determine whether each of the following sequences *converge* or *diverge*. If the sequence converges, find its limit. If the sequence diverges, you need to show sufficient work to justify your answer.

   (a) $a_n = \dfrac{\sqrt{n}}{\ln(n)}$

   (b) $a_n = \sin(n\pi)$

7. (6 points each) Consider the sequence $\{1, -\frac{2}{3}, \frac{4}{9}, -\frac{8}{27}, \ldots\}$ (assume the pattern continues).

   (a) Find a closed form formula for the nth term a_n.

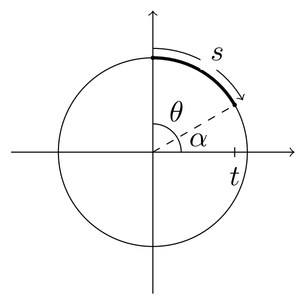
   (b) Determine whether the sequence converges or diverges. If the sequence converges, find its limit. If the sequence diverges, you need to show sufficient work to justify your answer.

8. (10 points each) Determine whether each of the following series *converge* or *diverge*. If the series converges, find its sum. If the series diverges, you need to show sufficient work to justify your answer.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n 2^{n+1}}{5^{n-1}}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{2}{n(n+2)}$

9. **Bonus Question:** Have you ever wondered why the inverse of sine is also referred to as arcsine? Don't lie; of course you have! Well, it's about time, you found out. Read on. Consider the following picture of the unit circle.



Recall that $y = \sqrt{1-x^2}$ is the equation for the top half of the unit circle.

(a) (1 point) Find $\dfrac{dy}{dx}$ for the top half of the unit circle.

(b) (3 points) Find the arc length of the top half of the unit circle from $x = 0$ to $x = t$. (Hint: simplify the integrand by getting common denominators and then evaluate the integral using an appropriate formula).

(c) (1 point) Notice that $t = \cos(\alpha)$. The "co" in cosine stands for complementary because $\cos(\alpha) = \sin(\pi/2 - \alpha)$. Also, notice that $\theta = \pi/2 - \alpha$. What is the relationship between t, θ, and sine?

(d) (1 point) Recall that an angle measured in radians is equal to the arc length divided by the radius. We're on the unit circle; so the radius is 1. What is the relationship between s, θ, and sine?

If you answered all of these questions correctly, you should see why the inverse of sine is also referred to as arcsine.