Plymouth State UNIVERSITY

MA 2550: Calculus I, Fall 2008

EXAM 1

NAME: (1 point)

Instructions: Answer each of the following questions completely. To receive full credit, you must *justify* each of your answers (unless stated otherwise). How you reached your answer is more important than the answer itself. If something is unclear, or if you have any questions, then please ask. Good luck!

1. (4 points each) Consider the following function.

$$f(x) = \begin{cases} \frac{x+1}{x-2}, & x < 1 \\ \frac{-1}{x-3}, & x \geq 1 \end{cases}$$

For (a)–(d), evaluate the expression. If an expression does not exist, write DNE.

(a) $\lim_{x \to 1^-} f(x)$

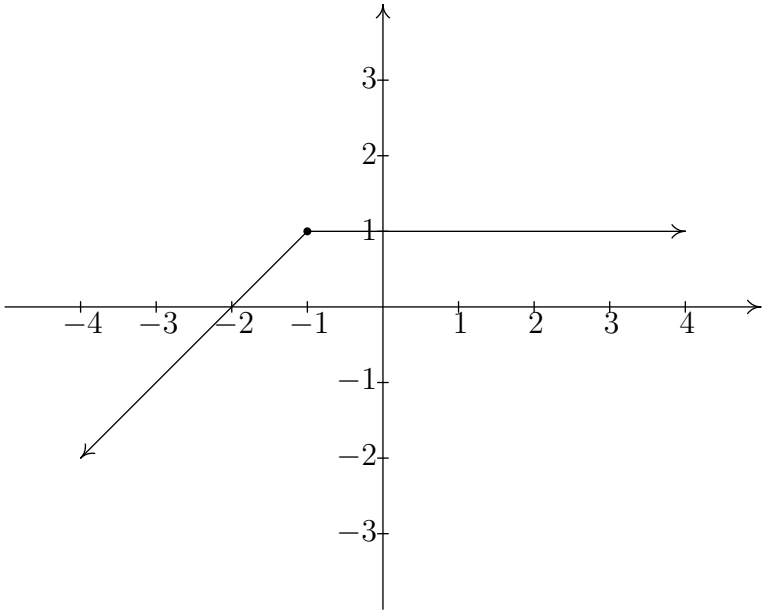
(b) $\lim_{x \to 1^+} f(x)$

(c) $\lim_{x \to 1} f(x)$

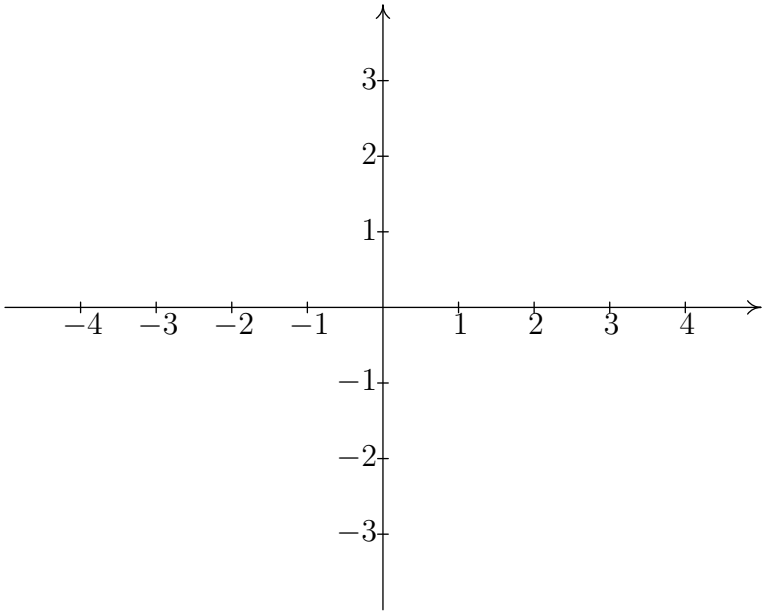
(d) $f(1)$

(e) Identify any x-values where f has discontinuities. You do *not* need to justify your answer.

2. (8 points) Suppose the graph of $f(x)$ looks like:



Using the axes provided, sketch the graph of the function $y = 2f(x + 1)$.



3. (8 points each) Evaluate each of the following limits. If a limit does not exist, specify whether the limit equals ∞, $-\infty$, or simply does not exist (in which case, write DNE). Sufficient work must be shown. Give *exact answers*.

(a) $\lim_{x \to -3} \dfrac{x+3}{x^2+9}$

(b) $\lim_{x \to 4} \dfrac{\sqrt{x}-2}{x-4}$

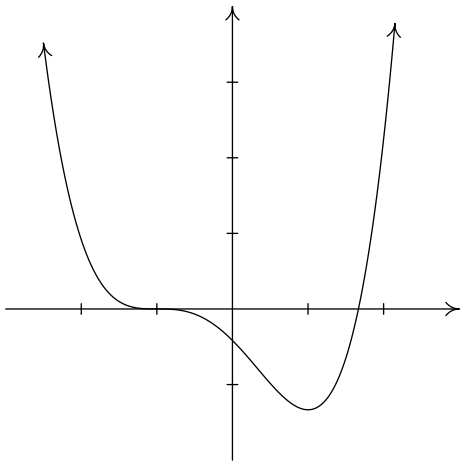
(c) $\lim_{x \to \frac{\pi}{2}} \dfrac{\cos^2 x}{1-\sin(x)}$

4. (8 points) Using the *limit definition of the derivative*, find the derivative of the following function. (No credit will be given for finding the derivative using another method.)

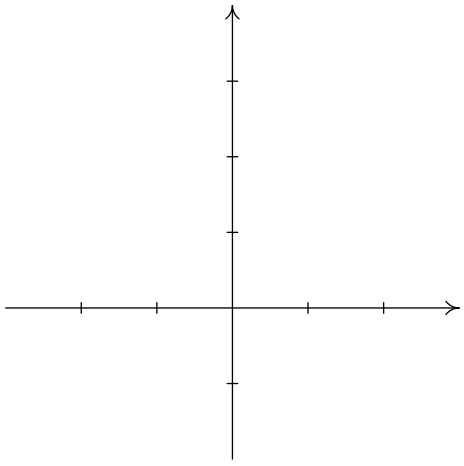
$$f(x) = x^2 - 3x$$

5. (8 points) Given that $f'(x) = \cos x$ when $f(x) = \sin x$, find the *equation* of the tangent line to the graph of f when $x = \pi/6$. It does not matter what form the equation of the line takes, but all coefficients should have exact values (i.e., no decimal approximations).

6. (8 points) Using the graph of the function f given below, sketch the graph for the derivative of f.



Graph of f



Graph of f'

7. (8 points) Using the ϵ-δ definition of limit, complete the proof that

$$\lim_{x \to 2} 3x - 1 = 5.$$

Proof: Let _____ > 0. Choose $\delta =$ _____ and assume that

$$0 < |x - \_\_\_\_\_| < \_\_\_\_\_.$$

Then we see that

$$\begin{aligned} |f(x) - \_\_\_\_\_| &= |\_\_\_\_\_| \\ &= 3\,|\_\_\_\_\_| \\ &< \_\_\_\_\_ \cdot \_\_\_\_\_ \\ &= \_\_\_\_\_, \end{aligned}$$

which shows that

$$|f(x) - \_\_\_\_\_| < \_\_\_\_\_.$$

□

8. (5 points each) Provide an example of each of the following. You should have three separate answers and you do *not* need to justify your answer.

   (a) An *equation* of a function f that is continuous everywhere, but not differentiable at $x = 1$.

   (b) An *equation* of a function g such that $\lim_{x \to 1} g(x)$ exists, but $g(1)$ does not exist.

   (c) An *equation* of a function h such that $\lim_{x \to 1^-} h(x) \neq \lim_{x \to 1^+} h(x)$.