Introduction to Linear Algebra, Spring 2007
MATH 3130, Section 001

EXAM 3

YOUR NAME:

Instructions: Answer each of the following questions completely. To receive full credit, you must *justify* each of your answers (unless stated otherwise). How you reached your answer is more important than the answer itself. If something is unclear, or if you have any questions, then please ask. Good luck!

1. Let A be a matrix that is row equivalent to one of the following matrices.

$$M_1 = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \qquad M_2 = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \qquad M_3 = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 5 & 6 \\ 0 & 0 & 1 & 7 \end{bmatrix}$$

For each of the following statements, state whether A is row equivalent to M_1, M_2, or M_3. You should write either M_1, M_2, or M_3 in each blank provided. You do *not* need to justify your answers. [**5 points each**]

_____ Rank$A$ is 2.

_____ 0 is an eigenvalue of A.

_____ Nul$A = \{\mathbf{0}\}$.

_____ The columns of A form a basis for $\mathbb{R}^3$.

_____ The dimension of Row$A$ is 3 and dimension of $(\text{Row}A)^\perp$ is 1.

_____ The first and third columns of A form a basis for Col$A$.

1. Let $\mathbf{p}_1(t) = 1 + t^2, \mathbf{p}_2(t) = t + t^2, \mathbf{p}_3(t) = 1 + 2t + t^2 \in \mathbb{P}_2$. Determine whether $\{\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3\}$ is linearly independent in $\mathbb{P}_2$. You must justify your answer. [**12 points**]

2. Consider the following matrix.

$$A = \begin{bmatrix} 1 & -2 & 2 & 3 \\ 0 & 0 & 1 & 2 \\ 2 & -4 & 5 & 8 \end{bmatrix}$$

Find a basis for Nul$A$. [**12 points**]

3. Consider the following matrix.

$$A = \begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix}$$

Show that A is *not* diagonalizable. You must sufficiently justify your answer. [**12 points**]

4. Consider the following matrix.

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

Diagonalize A. That is, find the invertible matrix P and the diagonal matrix D such that $A = PDP^{-1}$. You do *not* need to find P^{-1}. [**12 points**]

5. Let A be an $n \times n$ matrix and assume $\mathbf{x}$ is an eigenvector for A with corresponding eigenvalue λ.

   (a) Prove that $\mathbf{x}$ is an eigenvector for A^2. In your proof, you should state what eigenvalue of A^2 corresponds to $\mathbf{x}$. *Hint:* Do NOT try to diagonalize A; A might not even be diagonalizable. Note that this result holds for any arbitrary power of A. [**12 points**]

   (b) Now, assume that A is 2×2, $\lambda = 3$, and λ has multiplicity 1. Denote the eigenspace for λ by E_λ. Geometrically describe what the eigenspace E_λ looks like. Also, geometrically describe what A *and* A^2 do to E_λ. [**10 points**]