Supplementary Homework Exercises for Section 11.10: Taylor Series

Exercises

Answer each of the following questions.

S1. Suppose that f is a continuous function that is increasing on $(-\infty, 3)$ and decreasing on $(3, \infty)$. Explain why the series $1 - 0.5(x-2) + 0.4(x-2)^2 - 0.3(x-2)^3 + \cdots$ cannot be the Taylor series for f centered at $x = 2$.

S2. If $f^{(n)}(0) = (n+1)!$ for $n = 0, 1, 2, \ldots$, find the Maclaurin series for f and its radius of convergence.

S3. Using the definition, find the Maclaurin series for each of the following functions.

(a) $f(x) = \ln(1+x)$

(b) $f(x) = e^{5x}$

S4. Using the definition, find the Taylor series for each of the following functions centered at the indicated point.

(a) $f(x) = \dfrac{1}{x}$, $a = -1$

(b) $f(x) = \sin x$, $a = \pi/2$

S5. Let $f(x) = x\cos(x^3)$.

(a) Using a know Macaularin series (see table from notes), obtain a Maclaurin series for f.

(b) Evaluate $\displaystyle\int x\cos(x^3)\, dx$ as an infinite series.

(c) Approximate $\displaystyle\int_0^1 x\cos(x^3)\, dx$ using a degree 4 Taylor polynomial for $y = \cos(x)$. (*Note:* the polynomial that you end up integrating should have degree 13.)

S6. Using a known Maclaurin series, determine which function has $\displaystyle\sum_{n=0}^{\infty}(-1)^n\frac{x^{4n}}{n!}$ as its Maclaurin series.