Lab 1: Review of Calculus I

Names:

Goal

The goal of this lab is to review some of the main topics from Calculus I. The purpose of this assignment is to get your brain thinking about calculus again. Don't panic if there are a couple questions that you don't remember how to do. However, if you find yourself struggling significantly, then we should talk.

Directions

In groups of 2–4 (I do not want anyone working alone), answer each of the following questions in the space provided. You only need to turn in one lab per group (make sure you put everyone's name on this sheet). Feel free to consult your notes and textbook. The lab is due by 5PM on **Wednesday, February 2** and is worth 10 points.

Exercises

For you those of you that had me last semester, you should recognize most of the following problems from your final exam.

1. Consider the following function.
$$f(x) = \begin{cases} x^2+1, & x<1 \\ x-4, & x \geq 1 \end{cases}$$
For (a)–(e), evaluate the given expression. If a limit does not exist, specify whether the limit equals ∞, $-\infty$, or simply does not exist (in which case, write DNE). For (a)–(d), you do *not* need to justify your answer.
    (a) $\lim_{x\to 1^-} f(x) =$ ____________
    (b) $\lim_{x\to 1^+} f(x) =$ ____________
    (c) $\lim_{x\to 1} f(x) =$ ____________
    (d) $f(1) =$ ____________
    (e) Is f continuous at $x = 1$? Justify your answer.

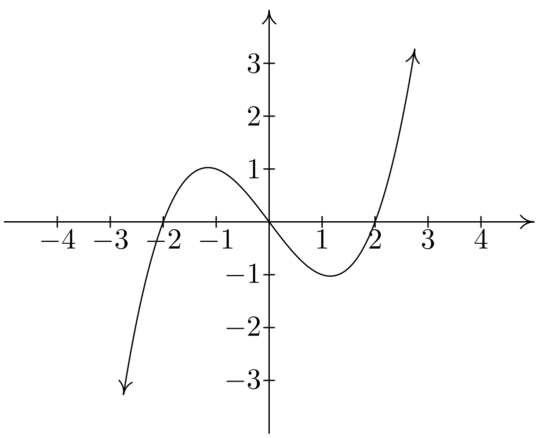
2. Evaluate each of the following limits. If a limit does not exist, specify whether the limit equals ∞, $-\infty$, or simply does not exist (in which case, write DNE). Sufficient work must be shown. Give *exact answers*.
    (a) $\lim_{x\to 4} \frac{\sqrt{x}-2}{x-4}$

 Written by D.C. Ernst

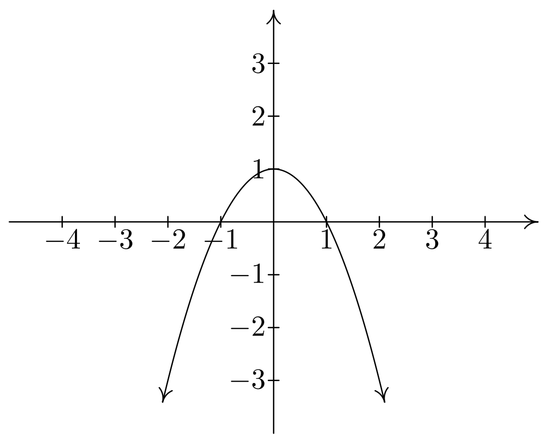
(b) $\displaystyle\lim_{h\to 0}\frac{(x+h)^2+1-(x^2+1)}{h}$ (*Hint:* your answer should be a function of x.)

(c) $\displaystyle\lim_{x\to\infty}\frac{3x^2-5x+1}{x(4-x^2)}$ (You should briefly justify your answer.)

3. Using the graphs below, circle the correct, or approximate, value for each of the following expressions.



Graph of f



Graph of g

(a) $g(1) =$

A. 0 B. $-\frac{3}{2}$ C. -1 D. 1

(b) $f'(0) =$

A. 0 B. $\frac{1}{2}$ C. -1 D. 1

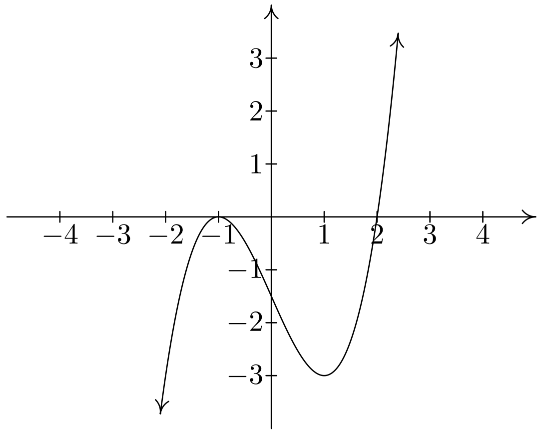
(c) Suppose $h(x) = f(g(x))$. Then $h'(1) =$

A. 0 B. -2 C. -1 D. 2

(d) At how many points does $f'(x) = 0$?

A. 0 B. 1 C. 2 D. 3

4. Let f be a differentiable function. Suppose that the following graph is the graph of the *derivative* of f (i.e., the graph of f').



Graph of f'

(a) Find the x-coordinates of all points on the graph of f where the tangent line is horizontal.

(b) Find the (open) intervals, if any, on which f is increasing.

(c) Find the (open) intervals, if any, on which f is decreasing.

(d) Find the x-coordinates, if any, where f attains a local max.

(e) Find the x-coordinates, if any, where f attains a local min.

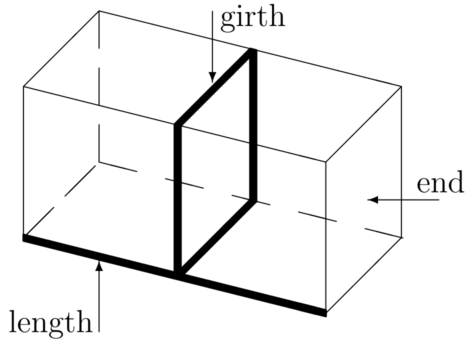
5. Find $\dfrac{dy}{dx}$ if $\sin^2 x + y^3 = xy$. You do *not* need to simplify your answer, but you should solve for $\dfrac{dy}{dx}$.

6. Consider the following function.
$$f(x) = \frac{\ln x}{x}$$
Find the *equation* of the tangent line to the graph of f when $x = 1$. It does not matter what form the equation of the line takes, but all coefficients should have exact values (i.e., no decimal approximations).

7. A large spherical meteor-nugget is speeding towards Earth. If the radius of the meteorite is decreasing at a rate of 1/4 kilometer per day, what is the rate of change in the volume of the meteorite when the radius is 5 kilometers? Give an *exact* answer (i.e., not a decimal approximation). Your answer should be labeled with appropriate units. (Hint: the volume of a sphere of radius r is $V = \frac{4}{3}\pi r^3$.)

8. The shock-waves from an earthquake on the ocean floor radiate out in the form of a circle on the surface of the ocean from its epicenter. If the radius of the shock-waves is increasing at a rate of 2 miles per second, what is the rate of change of the area enclosed by the radiating shock-waves when the radius is 5 miles? Give an exact answer. Your answer should be labeled with appropriate units.

9. The U.S. Postal Service will accept a box for domestic shipment only if the sum of its length and girth (distance around) does not exceed 108 inches. What dimensions (length and width) will give a box with a *square* end the largest possible volume? (*Hint:* volume is maximized when length plus girth is equal to 108.)



   (a) Let V represent the volume of the box (with a square end). Find an equation for V that involves only a single variable and find the feasible domain for V

   (b) Find the *dimensions* that will maximize the volume of the box. (You must justify that your answer is actually correct.)

10. At this time, we do *not* know how to evaluate the following definite integral.

$$\int_0^\pi \cos^2 x \; dx$$

However, we can approximate this integral. Approximate the above integral using 4 equal width rectangles and right endpoints. (You should give an *exact* answer for your approximation.)

11. Evaluate the following definite integral using a limit of Riemann sums and right endpoints.

$$\int_0^1 x^2 + 1 \; dx$$

12. Evaluate each of the following integrals. Sufficient work must be shown.

    (a) $\displaystyle\int \frac{\sqrt{x} - 5x}{x^{3/2}}\, dx$

    (b) $\displaystyle\int_0^1 x^2\sqrt{1 - x^3}\, dx$

13. Set up (but do *not* evaluate) an integral that determines the total area of the region bounded by the graphs of $f(x) = x - x^2$ and $g(x) = x^2$.

14. Suppose the velocity function for the position of a tornado moving in a straight line path in a valley is given by $v(t) = -t^2 + 4$, where t is time measured in minutes and position is measured in miles.*

    (a) Find the net distance traveled by the tornado during the first 3 minutes.

    (b) Set up (but do *not* evaluate) an integral that determines the total distance traveled by the tornado during the first 3 minutes.

*I think this tornado is moving faster than your average tornado.

15. Provide an example of each of the following. You do *not* need to justify your answer.

    (a) An *equation* of a function f that is continuous everywhere, but not differentiable at $x = 0$.

    (b) An *equation* of a function g such that g has a critical number at $x = 0$, but g does not have a local maximum or local minimum at $x = 0$.

    (c) An *equation* of a function h such that h has a local maximum at $x = 0$, but $h'(0) \neq 0$.

    (d) An *equation* of a function k such that $k''(0) = 0$, but k does *not* have an inflection point at $x = 0$.

16. A common theme in Calculus I is to start with an approximation for something that is seemingly difficult to compute and then take a limit to get an exact answer. Describe ONE such situation that was discussed in Calculus I. I'm looking for an intuitive understanding, but you should provide some detail using proper notation. (Using pictures to aid in your description will be very useful.)

17. (State in detail (i.e. include appropriate hypotheses) each of the following theorems:

    (i) Mean Value Theorem

    (ii) Fundamental Theorem of Calculus, part 1

    (iii) Fundamental Theorem of Calculus, part 2

