MG 5860: Visual Group Theory (Summer 2009)
Quiz 4 (20 points)

NAME:

Instructions: Answer each of the following questions completely. If something is unclear, or if you have any questions, then please ask. Good luck!

1. (5 points) What is our second definition of a group?

2. (5 points) Consider the following multiplication table (not necessarily for a group) that displays a binary operation.

$*$	e	a	b
e	e	a	b
a	a	a	b
b	b	b	b

Does this binary operation possess inverses? If the answer is "yes," explain why. If the answer is "no," then provide a specific example to illustrate.

3. The multiplication table for the group of symmetries of an equilateral triangle (denoted S_3 or D_3) is give below, where h represents horizontal flip (assuming triangle oriented so that one tip is pointing up) and r represents rotation by $120°$ clockwise.

$*$	e	r	h	r^2	rh	hr
e	e	r	h	r^2	rh	hr
r	r	r^2	rh	e	hr	h
h	h	hr	e	rh	r^2	r
r^2	r^2	e	hr	r	h	rh
rh	rh	h	r	hr	e	r^2
hr	hr	rh	r^2	h	r	e

(a) (6 points) Using the table, find the inverses of the 6 group elements of D_3.

$e^{-1} = $ _____

$r^{-1} = $ _____

$h^{-1} = $ _____

$(r^2)^{-1} = $ _____

$(rh)^{-1} = $ _____

$(hr)^{-1} = $ _____

(b) (4 points) Using your answers to part (a), solve the following group equation in D_3 for x. (*Warning*: remember that whatever you do to one side of the equation, you need to do the *exact* same thing to the other side.) You should simplify your answer if necessary.

$$r^2 xh = r$$