MA 2560: Calculus II (Spring 2011)
Final Exam

NAME:	(2 points!)

Instructions: Answer each of the following questions completely. To receive full credit, you must show sufficient work for each of your answers (unless stated otherwise). How you reached your answer is more important than the answer itself. If something is unclear, or if you have any questions, then please ask. Good luck!

Here are some potentially useful formulas. You should *not* expect to use all of them.

$$\frac{d}{dx}[\sinh x] = \cosh x \qquad \frac{d}{dx}[\cosh x] = \sinh x$$

$$\frac{d}{dx}[\tanh x] = \text{sech}^{\,2} x \qquad \frac{d}{dx}[\text{sech } x] = -\text{sech } x \tanh x$$

$$\frac{d}{dx}[\arcsin x] = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}[\arccos x] = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}[\arctan x] = \frac{1}{1+x^2} \qquad \frac{d}{dx}[\text{arcsec } x] = \frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}[\sinh^{-1} x] = \frac{1}{\sqrt{1+x^2}} \qquad \frac{d}{dx}[\cosh^{-1} x] = \frac{1}{\sqrt{x^2-1}}$$

$$\frac{d}{dx}[\tanh^{-1} x] = \frac{1}{1-x^2} \qquad \frac{d}{dx}[\text{sech}^{\,-1} x] = \frac{-1}{x\sqrt{1-x^2}}$$

$$\int \sec x \, dx = \ln|\sec x + \tan x| + C \qquad \int \tan x \, dx = \ln|\sec x| + C$$

$$\int \sinh u \, du = \cosh u + C \qquad \int \cosh u \, du = \sinh u + C$$

$$\int \text{sech}^{\,2} u \, du = \tanh u + C \qquad \int \text{sech } u \tanh u \, du = -\text{sech } u + C$$

$$\int \frac{1}{\sqrt{a^2-u^2}} \, du = \arcsin\frac{u}{a} + C \qquad \int \frac{1}{u^2+a^2} \, du = \frac{1}{a}\arctan\frac{u}{a} + C$$

$$\int \frac{1}{u\sqrt{u^2-a^2}} \, du = \frac{1}{a}\text{arcsec } \frac{u}{a} + C \qquad \int \frac{1}{\sqrt{u^2+a^2}} \, du = \sinh^{-1}\frac{u}{a} + C$$

$$\int \frac{1}{\sqrt{u^2-a^2}} \, du = \cosh^{-1}\frac{u}{a} + C \qquad \int \frac{1}{a^2-u^2} \, du = \frac{1}{a}\tanh^{-1}\frac{u}{a} + C$$

 Written by D.C. Ernst

1. (6 points each) Evaluate each of the following integrals.

   (a) $\displaystyle\int \frac{e^x}{\sqrt{1-e^{2x}}}\,dx$

   (b) $\displaystyle\int \frac{1}{x^2\sqrt{x^2-9}}\,dx$

(c) $\displaystyle\int \sin^3 x \cos^2 x \; dx$

(d) $\displaystyle\int x \cos x \; dx$

(e) $\displaystyle\int \frac{x+5}{x^2+x-2}\,dx$

(f) $\displaystyle\int_2^\infty \frac{1}{4+x^2}\,dx$ (If the integral converges, give an *exact* answer.)

2. (6 points each) Setup (but do *not* evaluate) an integral that would determine the volume of the solid obtained by revolving the region bounded by the given graphs about the indicated line.

   (a) $y = 2 - x^2$ and $y = x^2$, about the x-axis.

   (b) $f(x) = 1$ and $g(x) = x^2$, about the line $x = 2$.

3. (6 points) Find the area of one loop of the graph of $r = 3\sin(5\theta)$.

4. (6 points) Consider the parametric curve given by

$$x = \frac{8}{3}t^{3/2}, \qquad y = 2t - t^2$$

Find the arc length for $1 \leq t \leq 2$.

5. Consider the sequence determined by

$$a_n = \frac{n^2 + 17}{2n(n+1)}.$$

   (a) (6 points) Does the sequence converge or diverge? If the sequence converges, find its limit.

   (b) (4 points) Does the corresponding series $\sum_{n=1}^{\infty} a_n$ converge or diverge? Explain your answer.

6. (6 points) Determine the *interval* of convergence of the following power series. (If necessary, don't forget to check convergence at the endpoints.)

$$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n (x-5)^n}{n5^n}$$

7. (6 points) Determine whether the following series *converges absolutely*, *converges conditionally*, or *diverges*. In order to receive full credit, you should clearly state what test(s) you are using and verify the appropriate hypotheses. The justification of your conclusion is vastly more important than the actual conclusion.

$$\sum_{n=1}^{\infty} \frac{(-1)^n n}{n^2+17}$$

8. Consider the function $f(x) = \dfrac{1}{x}$.

   (a) (6 points) Find the Taylor series for f centered at $a = 1$. (There are at least 2 ways to do this problem.)

   (b) (4 points) Find the *radius of convergence* for the Taylor series that you found in part (a).

9. (2 points each) We will approximate $\displaystyle\int_1^2 \frac{e^x - 1}{x}\, dx$.

   (a) Using the known Maclaurin series $\displaystyle e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$, find the 5th degree Taylor polynomial for $y = e^x$.

   (b) Using your answer from part (a), find a 4th degree polynomial that approximates $\displaystyle f(x) = \frac{e^x - 1}{x}$.

   (c) Using your answer from part (b), approximate $\displaystyle\int_1^2 \frac{e^x - 1}{x}\, dx$. (Round your answer to 3 decimal places.)

10. **Bonus Question:** We've repeatedly said that the alternating harmonic series converges. But what does it converge to? Well, let's see.

(a) (2 points) Write $y = \dfrac{1}{1+x}$ as a geometric series starting at $n = 1$.

(b) (2 points) Integrate your answer from part (a) over the interval $[0, 1]$. (Your answer should be a familiar looking series.)

(c) (2 points) Now, integrate the function $y = \dfrac{1}{1+x}$ (not the power series, but the original function) on the interval $[0, 1]$. (Your answer should be a number.)

(d) (2 points) What conclusion can you make?

