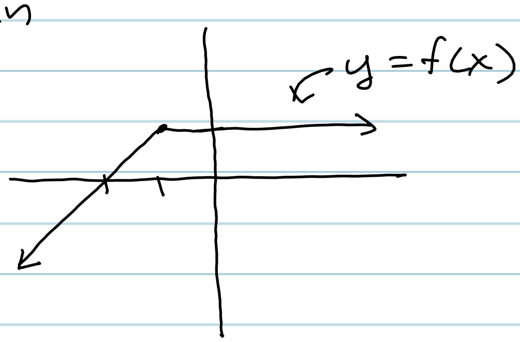
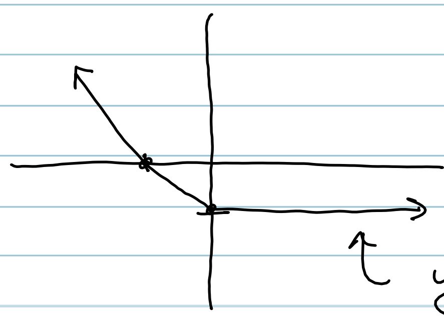
Partial Solutions to Exam 2

1. Given





rt 1

$$y = -f(x-1)$$

reflect over x-axis

2. Let $f(x) = x^2 - 3x + 1$. Then

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 - 3(x+h) + 1 - x^2 + 3x - 1}{h}$$

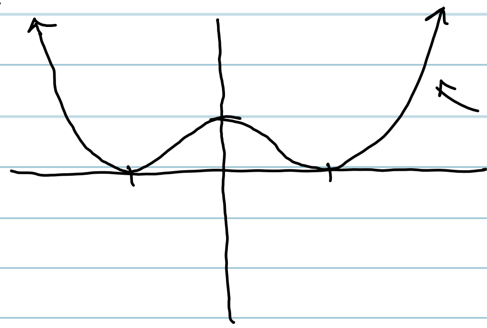
$$= \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 - \cancel{3x} - 3h + \cancel{1} - \cancel{x^2} + \cancel{3x} - \cancel{1}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(2x + h - 3)}{\cancel{h}}$$

$$= \lim_{h \to 0} (2x + h - 3)$$

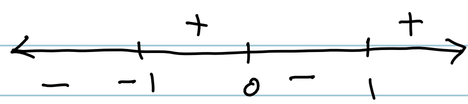
$$= \boxed{2x - 3}$$

3. Given:



graph of f

graph of f'



get this info 1st

4. (a) $f(x) = \frac{x}{2} + \sqrt{x} - \frac{1}{x} + \pi^2$

rewrite

$$= \frac{1}{2}x + x^{1/2} - x^{-1} + \pi^2$$

$$f'(x) = \frac{1}{2} + \frac{1}{2}x^{-1/2} - (-1)x^{-2}$$

$$= \boxed{\frac{1}{2} + \frac{1}{2\sqrt{x}} + \frac{1}{x^2}}$$

(b) $y = \dfrac{x^2 - 3x + 1}{2 - x}$

$$\frac{dy}{dx} = \boxed{\frac{(2x-3)(2-x) - (x^2-3x+1)(-1)}{(2-x)^2}}$$

used quotient rule

(c) $g(x) = \sec\left(\dfrac{x}{2}\right)$

$\overset{\text{rewrite}}{=} \sec\left(\tfrac{1}{2}x\right)$

out in

$$g'(x) = \underbrace{\sec\left(\tfrac{1}{2}x\right)\tan\left(\tfrac{1}{2}x\right)}_{\text{out}'} \cdot \underbrace{\tfrac{1}{2}}_{\text{in}'}$$

$$= \boxed{\tfrac{1}{2}\sec\left(\tfrac{x}{2}\right)\tan\left(\tfrac{x}{2}\right)}$$

(d) $h(x) = \cos^2 x$

$\overset{\text{rewrite}}{=} (\cos x)^2$

out

in

$$h'(x) = 2(\cos x)^1 \cdot (-\sin x)$$

$$= \boxed{-2\cos x \sin x}$$

5. Given: $x^2y + y^2 = x$

$$\frac{d}{dx}\left[x^2y + y^2\right] = \frac{d}{dx}\left[x\right]$$

$$2xy + x^2\frac{dy}{dx} + 2y\frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \boxed{\frac{1-2xy}{x^2+2y}}$$

6. $f(x) = \sqrt{5-x} = (5-x)^{1/2}$ @ $x = 1$

$f'(x) = \frac{1}{2}(5-x)^{-1/2}(-1)$ by chain rule

$m = f'(1) = \dfrac{-1}{2\sqrt{5-1}} = \dfrac{-1}{2 \cdot 2} = \dfrac{-1}{4}$

$x_0 = 1$

$y_0 = f(1) = \sqrt{5-1} = 2$

So, using $y - y_0 = m(x - x_0)$, we get

$$\boxed{y - 2 = \frac{-1}{4}(x-1)}$$

7. (a) $g(0) = \boxed{0}$

(just read off of graph)

(b) $f'(0) = \boxed{-\frac{2}{3}}$

(just find slope of corresponding line segment)

(c) $f'(-1)$ $\boxed{\text{DNE}}$

(there's a sharp turn @ $x=-1$)

(d) $g'(0) = \boxed{1}$

(just find slope of corresponding line segment)

(e) $h'(0) = f'(g(0)) \cdot g'(0)$

$= f'(0) \cdot 1$

$= -\frac{2}{3} \cdot 1$

$= \boxed{-\frac{2}{3}}$

8. $A = \pi r^2$

$\left.\frac{dA}{dt}\right|_{r=5} = ?$

$r = 5$

$\left.\frac{dr}{dt}\right|_{r=5} = 2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$\left.\frac{dA}{dt}\right|_{r=5} = 2\pi \cdot 5(2)$

$= \boxed{20\pi \text{ mi}^2/\text{sec}}$

9. Let $P(t) = p_u(t) - p_d(t)$. Then

(1) P is cont

(2) $P(0) = 0 - \text{top} = \text{neg } \#$

(3) $P(12) = \text{top} - 0 = \text{pos } \#$

So, by IVT, there exists a $c \in (0, 12)$ s.t. $P(c) = 0$. This implies that

$$p_u(c) = p_d(c).$$