## MA 3110: Logic, Proof, and Axiomatic Systems (Fall 2009)

TR 3:30-4:45PM, Hyde 314

#### **Instructor Information**

**Instructor:** Dr. Dana Ernst

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### Course Information and Policies

**Prerequisites:** A satisfactory grade in MA 2550 or MA 2490.

Course description: Mathematical logic is introduced and used in developing techniques of writing proofs in mathematical settings. Although the variety of settings may vary each semester, stresses proof development and includes mathematical induction and relations.

**Purpose:** The primary objective of this course is to develop skills necessary for effective proof writing. Students will improve their ability to read and write mathematics. Successful completion of MA 3110 provides students with the background necessary for upper division mathematics courses. Also, the purpose of any mathematics course is to challenge and train the mind. Learning mathematics enhances critical thinking and problem solving skills.

**Text:** Chapter Zero: Fundamental Notions of Abstract Mathematics, by Carol Schumacher, 2nd edition, (Addison-Wesley).

Comments about this course and expectations: This course will be different than any other math class that you have ever taken for two main reasons. First, you are used to being asked to do things like: "solve for x," "take the derivative of this function," "integrate this function," etc. Accomplishing tasks like these usually amount to mimicking examples that you have seen in class or in your textbook. The steps you take to "solve" problems like these are always justified by mathematical facts (theorems), but rarely are you paying explicit attention to when you are actually using these facts. Furthermore, justifying (i.e., proving) the mathematical facts you use may have been omitted by the instructor. And, even if the instructor did prove a given theorem, you may not have taken the time or have been able to digest the content of the proof. Unlike previous courses, this course is all about "proof." Mathematicians are in the business of proving theorems and this is exactly our endeavor. For the first time, you will be exposed to what "doing" mathematics is really all about. This will most likely be a shock to your system. Considering the number of math courses that you have taken before you arrived here, one would think that you have some idea what mathematics is all about. You must be prepared to modify your paradigm. The second reason why this course will be different for you is that the method by which the class will run and the expectations that I have of you will be different. In a typical course, math or otherwise, you sit and listen to a lecture. (Hopefully) These lectures are polished and well-delivered. You may have often been lured into believing that the instructor has opened up your head and is pouring knowledge into it. I absolutely love lecturing and I do believe there is value in it, but I also believe that the reality is that most students do not learn by simply listening. You must be active in the learning you are doing. I'm sure each of you have said to yourselves, "Hmmm, I understood this concept when the professor was going over

it, but now that I am alone, I am lost." In order to promote a more active participation in your learning, we will incorporate ideas from an educational philosophy called the Moore method (after R.L. Moore, a former professor of mathematics at the University of Texas, Austin). Modifications of the Moore method are also referred to as inquiry-based learning (IBL) or discovery-based learning. In this course, *everyone* will be required to

- 1. regularly interact with the textbook (i.e., read and take notes) on your own;
- 2. write up quality proofs to assigned problems;
- 3. present proofs on the board to the rest of the class;
- 4. participate in discussions centered around a student's presented proof.

As the semester progresses, it should become clear to you what the expectations are. This will be new to probably everyone and there may be some growing pains associated with it. The plan will be to start the semester with me lecturing one day a week while the other day will be reserved for student presentations. The hope is to slowly move towards less lecturing and more student presentations, but we will adjust as necessary. For more information, see Class Presentations below.

Goals: (adopted from Chapter Zero Instructor Resource Manual) Aside from the obvious goal of wanting you to learn how to write rigorous mathematical proofs, one of my principal ambitions is to make you the student independent of me. Nothing else that I teach you will be half so valuable or powerful as the ability to reach conclusions by reasoning logically from first principles and being able to justify those conclusions in clear, persuasive language (either oral or written). Furthermore, I want you to experience the unmistakable feeling that comes when one really understands something thoroughly. Much "classroom knowledge" is fairly supercial, and students often find it hard to judge their own level of understanding. For many of us, the only way we know whether we are "getting it" comes from the grade we make on an exam. I want you to become less reliant on such externals. When you can distinguish between really knowing something and merely knowing about something, you will be on your way to becoming an independent learner. Lastly, it is my sincere hope that all of us (myself included) will improve our oral and written communications skills.

Writing Connection/Writing in the Discipline (WRCO): MA 3110 satisfies the Writing Connection (WRCO) requirement of the PSU General Education Program. One of the major goals of the course is for students to become competent and confident in reading and writing technical prose that occurs in the discipline of mathematics. The course develops methods of reasoning required to prove theorems and explain solutions to abstract mathematical problems. Students also gain proficiency in the language of abstract mathematical proofs. Writing proofs of theorems or other statements allows students the opportunity to practice logical thinking and document rigorous logical arguments. As students become increasingly skilled in thinking clearly and ordering their thoughts, they should gain greater aptitude in writing clearly and concisely. Students will complete daily homework assignments in which mathematical writing composes the majority of the work. On average, students should expect to be writing a total of 2–3 pages per week. Students are expected to use proper grammar and write in complete sentences. When writing proofs, all underlying assumptions need to be explicitly stated. Mathematical writing comprises over 50% of a student's grade.

Class presentations: (adopted from *Chapter Zero Instructor Resource Manual*) Though the atmosphere in this class should be informal and friendly, what we do in the class is serious business. In particular, the presentations made by students are to be taken very seriously since they spearhead the work of the class. Here are some of my expectations:

1. In order to make the presentations go smoothly, the presenter needs to have written out the proof in detail and gone over the major ideas and transitions, so that he or she can make clear the path of the proof to others.

- 2. The purpose of class presentations is not to prove to me that the presenter has done the problem. It is to make the ideas of the proof clear to the other students.
- 3. Presenters are to write in complete sentences, using proper English and mathematical grammar.
- 4. Presenters should explain their reasoning as they go along, not simply write everything down and then turn to explain.
- 5. Fellow students are allowed to ask questions at any point and it is the responsibility of the person making the presentation to answer those questions to the best of his or her ability.
- 6. Since the presentation is directed at the students, the presenter should frequently make eyecontact with the students in order to address questions when they arise and also be able to see how well the other students are following the presentation.

I will be grading presentations both on the content of their proof and on the quality of the presentation. I will probably grade the quality more harshly as the semester wears on, since I expect your presentation skills to develop with practice. I will always ask for volunteers to present proofs, but when no volunteers come forward, I will call on someone to present their proof. Each student is expected to be engaged in this process. The problems chosen for presentation will come from the assigned homework. Your grade on your own presentations, as well as your level of interaction during other's presentations, will be worth 30% of your overall grade.

Homework: Homework will be assigned regularly. I will always make it clear when a given homework assignment is due (so, there should never be any confusion). Your homework will always be graded for completion and I will usually select a couple of problems to grade carefully and provide feedback on. Each homework assignment will be worth 10 points while your overall grade on homework will be worth 30% of your overall grade. At the end of the semester, I will drop (at least) 3 of your lowest homework scores. In general, late homework will not be accepted. However, you are allowed to turn in up to 3 homework assignments late with no questions asked. Unless you have made arrangements in advance with me, homework turned in after class will be considered late.

Exams: There will be 2 exams: one midterm exam and a cumulative final exam. Both exams will consist of an in-class part and a take-home part. The in-class portion of the midterm will take place on **Thursday, October 22** while the in-class portion of the final exam will take place on **Thursday, December 17** at **2:30–5:00pm**. Each of these exams will be worth 20% of your overall grade. Calculators will be completely unnecessary on the in-class portions of the exams and will not be allowed. Make-up exams will only be given under extreme circumstances, as judged by me. In general, it will be best to communicate conflicts ahead of time.

**Attendance:** I am not explicitly grading attendance, but regular participation is required, which you cannot accomplish if you are not present. You should communicate absences ahead of time whenever possible.

#### Course Evaluation

Grading: You will be graded on the quality of your

- 1. written work on homework and exams (content and quality of written presentation);
- 2. oral presentations (content and quality of oral presentation);
- 3. participation in in-class discussions and activities.

Basis for Evaluation: Your final grade will be determined by the scores of your homework, oral presentations, general participation, and exams.

Homework	30%
Presentations/participation	30%
Midterm Exam	20%
Final Exam	20%

**Grade Determination:** Grades may be "massaged" at the end of the semester, but in general this is what you should expect:

93 – 100%	A	7376%	$\mathbf{C}$
9092%	A-	7072%	C-
8789%	B+	6769%	D+
8386%	В	6366%	D
80 – 82%	B-	60 – 62%	D-
77 – 79%	C+	0-59%	$\mathbf{F}$

#### Additional Information

Math Center: This student-run organization provides peer tutoring services for most 1000 and 2000 level math courses and some 3000 level courses. Tutors are typically math majors interested in teaching math and practicing their instructional skills. The Math Center is located in Hyde Hall room 351. You can drop in anytime during open hours. More information can be found at:

http://www.plymouth.edu/math/resources/center.html

Student Handbook: The PSU Student Handbook addresses policies pertaining to students with disabilities, religious observation, honor code, general conduct, etc. The Handbook can be found at: http://www.plymouth.edu/stulife/handbook/handbook.html

**ACT for Growth:** All teacher education majors are subject to the Areas of Concern/Targets for Growth policy, which is located at:

http://www.plymouth.edu/education/act.html

# **Closing Remarks**

When does the learning happen? It might happen in class, but most likely it happens when you sit down to do your homework. Most of you can follow what I or one of your classmates does on the board, but the question is, can you do it on your own? To learn best, you must struggle with mathematics on your own. It is supposed to be difficult. However, if you are struggling too much, then there are resources available for you. I am always happy to help you. If my office hours don't work for you, then we can probably find another time to meet. You can also get help from each other. Get a study buddy! Help each other learn. Go to the Math Center (see above). It is your responsibility to be aware of how well you understand the material. Don't wait until it is too late if you need help. Ask questions!