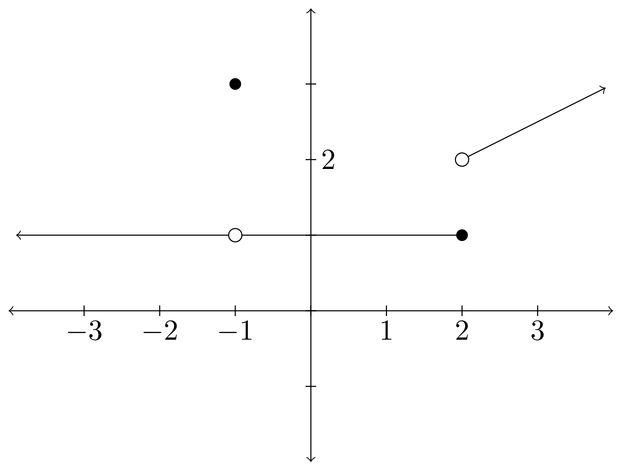
MA 2550: Calculus I (Fall 2010)
Exam 2

NAME:

Instructions: Answer each of the following questions completely. To receive full credit, you must show sufficient work for each of your answers (unless stated otherwise). How you reached your answer is more important than the answer itself. If something is unclear, or if you have any questions, then please ask. Good luck!

1. (2 points each) Suppose a function f has the following graph.



For (a)–(h), evaluate the given expression. If an expression does not exist, then write DNE. You do *not* need to justify your answers.

(a) $\lim_{x \to -1^-} f(x) =$ ____________

(b) $\lim_{x \to -1^+} f(x) =$ ____________

(c) $\lim_{x \to -1} f(x) =$ ____________

(d) $f(-1) =$ ____________

(e) $\lim_{x \to 2^-} f(x) =$ ____________

(f) $\lim_{x \to 2^+} f(x) =$ ____________

(g) $\lim_{x \to 2} f(x) =$ ____________

(h) $f(2) =$ ____________

Written by D.C. Ernst

2. (8 points) Complete the following proof that $\lim_{x\to 0} x^2 \sin\left(\frac{17}{x}\right) = 0$ by filling in the blanks.

Proof. First, note that

$$\underline{\hspace{3cm}} \leq \sin\left(\frac{17}{x}\right) \leq \underline{\hspace{3cm}}.$$

This implies that

$$\underline{\hspace{3cm}} \leq x^2 \sin\left(\frac{17}{x}\right) \leq \underline{\hspace{3cm}}.$$

Also, note that

$$\lim_{x\to 0} \underline{\hspace{3cm}} = 0$$

and

$$\lim_{x\to 0} \underline{\hspace{3cm}} = 0.$$

Therefore, by the ____________ Theorem, it must be the case that

$$\lim_{x\to 0} x^2 \sin\left(\frac{17}{x}\right) = \underline{\hspace{3cm}}.$$

$\square$

3. (8 points) Evaluate the following limit: $\lim_{x\to 0} \frac{\tan(42x)}{x}$. (Sufficient work must be shown.)

4. (8 points) Using the *limit definition of the derivative*, find the derivative of $f(x) = x^2 - 4x$. (No credit will be given for finding the derivative using another method.)

5. (10 points each) Differentiate each of the following functions. You do *not* need to simplify your answers, but sufficient work must be shown to receive full credit. If you make a mistake in an intermediate step while simplifying, it will count against you.

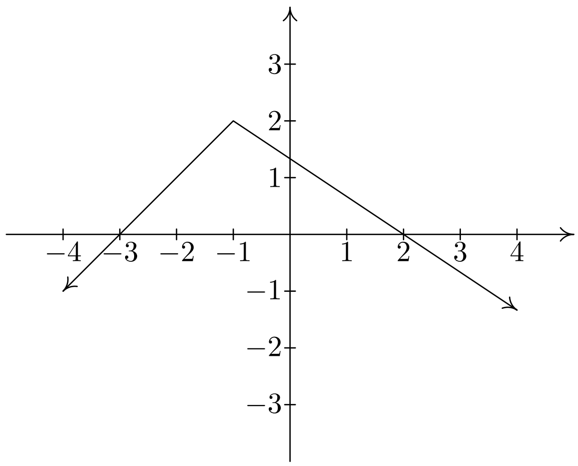
   (a) $f(x) = \dfrac{x^2}{2} + \sqrt{x} - \dfrac{3}{x} + \pi^{99}$

   (b) $y = \dfrac{x^2 - 3x + 1}{2 - x}$

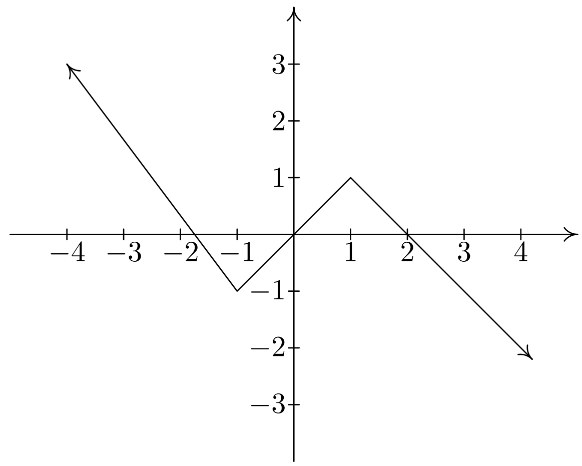
   (c) $g(x) = x\sqrt{1 - x^2}$

   (d) $g(x) = \sec \dfrac{x}{5}$

6. (2 points each) Consider the following graphs for functions f and g. Using the graphs, evaluate each of the following expressions. If an expression does not exist, write DNE.



Graph of f



Graph of g

(a) $g(0) =$ ____________.

(b) $f'(0) =$ ____________.

(c) $f'(-1) =$ ____________.

(d) $g'(0) =$ ____________.

(e) Suppose $h(x) = f(g(x))$. Then $h'(0) =$ ____________. (*Hint:* Use some of your answers from above and think Chain Rule.)

7. (10 points) Find an *equation* of the tangent line to the graph of $f(x) = \sin^2 x$ at $x = \pi/3$. It does not matter what form the equation takes, but all coefficients should be simplified and have exact values (i.e., no decimal approximations).

8. **Bonus Question:** (5 points, missing this question will not count against you) Using the limit definition of the derivative, prove that $f'(0)$ does not exist when $f(x) = |x|$.