## Integration by Substitution

## Motivation and Background

Currently, we do not have a technique for integrating most products, quotients, and compositions. Here are a couple that we can integrate:

$$\int \frac{x^2 + x}{\sqrt{x}} \ dx, \quad \int \sec(x) \tan(x) \ dx$$

And here are some that we cannot currently integrate (unless you happen to see what the appropriate antiderivative is):

$$\int x\sqrt{x^2+1} \ dx, \quad \int \sin(x)\cos(x) \ dx, \quad \int \frac{x}{x^2+1} \ dx$$

To integrate functions like above, we will utilize a technique called *substitution*, which involves the use of dummy variable.

**Important Note 1.** Substitution is a technique that only works in special circumstances, which should become apparent after a little practice.

**Important Note 2.** If confronted with an integral of a product, quotient, or composition and you cannot integrate it straight away, then substitution may work. In most (but definitely not all) situations, you will pick u to be the inside of the more complicated part.

## Examples

Example 3. Compute each of the following integrals.

1. 
$$\int (3x-1)^{99} dx$$

2. 
$$\int 5x^2\sqrt{x^3-2} \ dx$$

$$3. \int xe^{x^2} dx$$

$$4. \int \sin^2(x) \cos(x) \ dx$$

$$5. \int \frac{x}{x^2 + 1} \ dx$$

$$6. \int \frac{x^2 + 1}{x} \, dx$$

$$7. \int x^2 \sec^2(x^3) \ dx$$

$$8. \int \frac{x}{x^4 + 1} \ dx$$

9. 
$$\int x\sqrt{x-1} \ dx$$

$$10. \int \frac{\ln(x)}{x} \ dx$$

11. 
$$\int \frac{\arcsin(x)}{\sqrt{1-x^2}} \ dx$$

$$12. \int \frac{e^x}{e^{2x} + 1} \ dx$$