MA 2550: Calculus I (Fall 2010)
Exam 3

NAME:	(2 points!)

Instructions: Answer each of the following questions completely. To receive full credit, you must show sufficient work for each of your answers (unless stated otherwise). How you reached your answer is more important than the answer itself. If something is unclear, or if you have any questions, then please ask. Good luck!

1. (10 points) Suppose f is a function with the following properties.

   (a) $f(-3) = 0$, $f(-2) = -2$, and $f(0) = 0$

   (b) $\lim_{x \to 2^-} f(x) = \infty$ and $\lim_{x \to 2^+} f(x) = -\infty$

   (c) $\lim_{x \to \infty} f(x) = 0$ and $\lim_{x \to -\infty} f(x) = \infty$

   (d) $f'(-2) = 0$ and $f'(0) = 0$

   (e) $f'(x) > 0$ on $(-2, 0)$, $(0, 2)$, and $(2, \infty)$

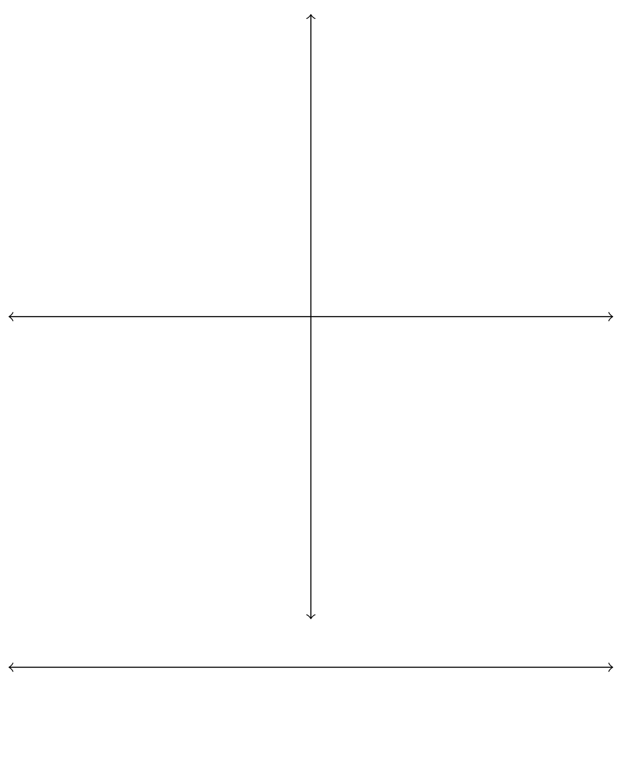
   (f) $f'(x) < 0$ on $(-\infty, -2)$

   (g) $f''(-1) = 0$ and $f''(0) = 0$

   (h) $f''(x) > 0$ on $(-\infty, -1)$ and $(0, 2)$

   (i) $f''(x) < 0$ on $(-1, 0)$ and $(2, \infty)$

   Using the above information, make a sketch of the graph of f. You do *not* need to justify your answer.



2. (5 points each) Consider the function $f(x) = 3x^4 - 4x^3$.

   (a) Find all critical numbers of f.

   (b) Using your answer(s) from part (a), determine whether each critical number determines a local maximum, local minimum, or neither. (You must show sufficient work to justify your answer.)

3. (8 points) Find *all* asymptotes (vertical and horizontal) of the following function.

$$f(x) = \frac{x}{x^2 - 9}$$

4. (5 points each) Consider the function $f(x) = \dfrac{x^5}{20} - \dfrac{x^4}{6} + \dfrac{x^3}{6} + 5x + 1$.

   (a) Find the second derivative of f.

   (b) Using your answer from part (a), find the x-values of all inflection points for f.

5. (8 points) Use appropriate calculus techniques to find the absolute maximum and absolute minimum values of the function $f(x) = 2x^3 - 3x^2 - 12x + 1$ on the interval $[1, 4]$. Sufficient work must be shown.

6. (8 points) Find $\dfrac{dy}{dx}$ if $\sin(x+y) = xy$. (You do *not* need to simplify your answer.)

7. (8 points) Let
$$f(x) = \frac{x}{x+2}.$$
Verify that f satisfies the hypotheses of the Mean Value Theorem on the interval $[1, 4]$, and then find all the numbers c that the Mean Value Theorem guarantees exist.

8. (8 points) Given $f(x) = xe^{\sin x}$, find the differential dy. (You do *not* need to simplify your answer.)

9. (8 points) Using L'Hôpital's Rule, evaluate the following limit.

$$\lim_{x \to 0^+} \frac{\ln x}{x}$$

10. (8 points) The shock-waves from an earthquake on the ocean floor radiate out in the form of a circle on the surface of the ocean from its epicenter. If the radius of the shock-waves is increasing at a rate of 7 kilometers per second, what is the rate of change of the area enclosed by the radiating shock-waves when the radius is 4 kilometers? Give an *exact* answer, not a decimal approximation. Your answer should be labeled with appropriate units.

11. You have 100 feet of fence to make a rectangular play area for you cute baby nugget. The wall of the house bounds one side.

    (a) (4 points) Let A represent the area of the rectangular play area. Find an equation for A that involves only a single variable.

    (b) (2 points) Find the feasible domain for A.

    (c) (6 points) Using your answers to (a) and (b), find the *dimensions* that will maximize the area of the rectangular play area. (Justifying your answer will not only make sure that you receive full credit, but will also ensure that you don't make a mistake.)

