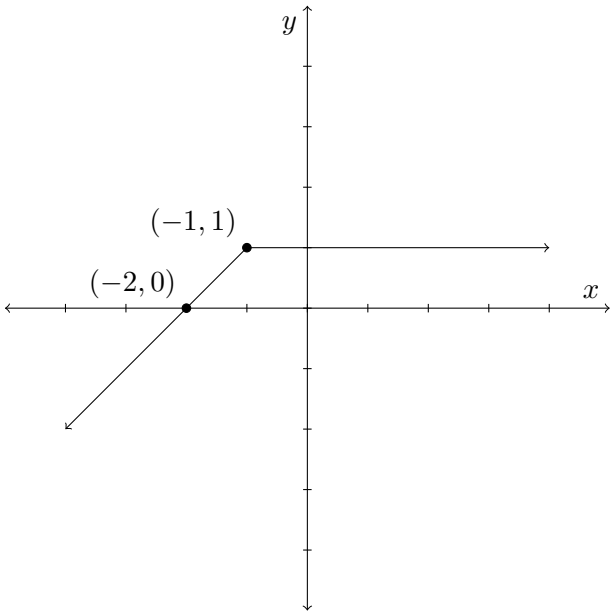
MA 2550: Calculus I (Spring 2009)
Exam 2

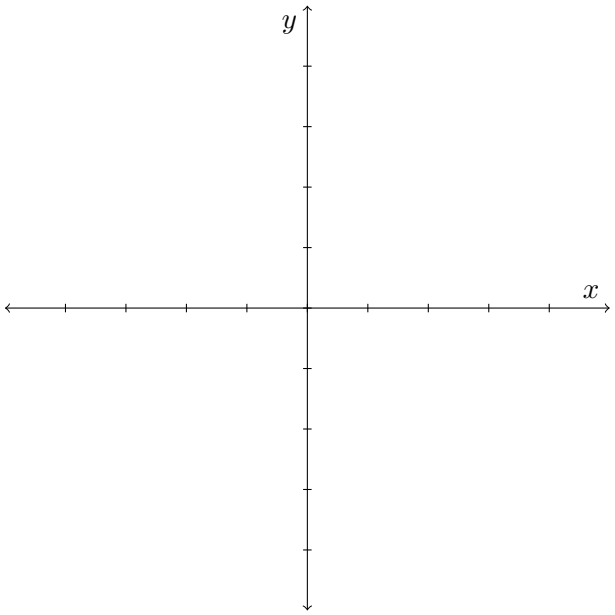
NAME: (1 point!)

Instructions: Answer each of the following questions completely. To receive full credit, you must *justify* each of your answers (unless stated otherwise). How you reached your answer is more important than the answer itself. If something is unclear, or if you have any questions, then please ask. Good luck!

1. (6 points) Suppose the graph of a function $y = f(x)$ looks like:



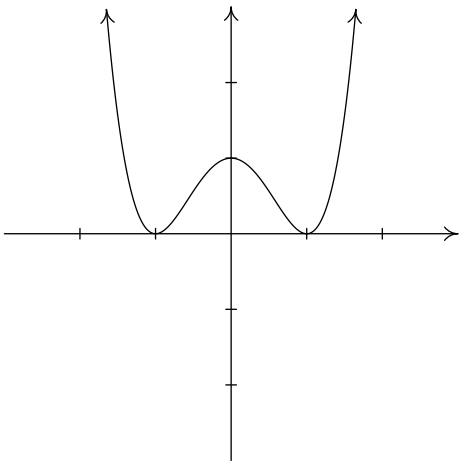
Using the axes provided, sketch the graph of the function $y = -f(x-1)$.



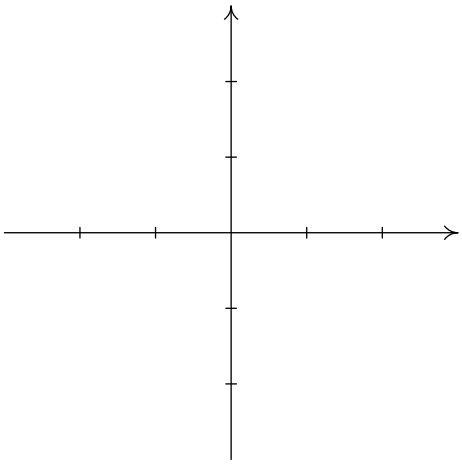
2. (8 points) Using the *limit definition of the derivative*, find the derivative of the following function. (No credit will be given for finding the derivative using another method.)

$$f(x) = x^2 - 3x + 1$$

3. (8 points) Using the graph of the function f given below, sketch a possible graph for the derivative of f.



Graph of f



Graph of f'

4. (8 points each) Differentiate each of the following functions. You do *not* need to simplify your answers, but sufficient work must be shown to receive full credit.

   (a) $f(x) = \dfrac{x}{2} + \sqrt{x} - \dfrac{1}{x} + \pi^2$

   (b) $y = \dfrac{x^2 - 3x + 1}{2 - x}$

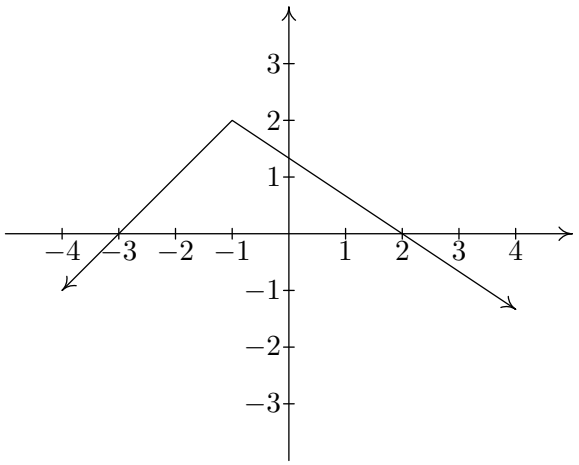
(c) $g(x) = \sec \dfrac{x}{2}$

(d) $h(x) = \cos^2 x$

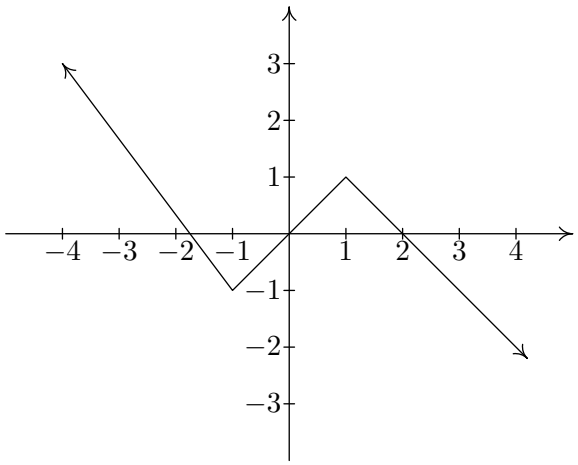
5. (8 points) Find $\dfrac{dy}{dx}$ if $x^2y + y^2 = x$. You do *not* need to simplify your answer, but you do need to solve for $\dfrac{dy}{dx}$.

6. (8 points) Find the *equation* of the tangent line to the graph of $f(x) = \sqrt{5 - x}$ at $x = 1$.

7. (3 points each) Consider the following graphs for functions f and g. Using the graphs, evaluate each of the following expressions. If an expression does not exist, write DNE.



Graph of f



Graph of g

(a) Find $g(0)$.

(b) Find $f'(0)$.

(c) Find $f'(-1)$.

(d) Find $g'(0)$.

(e) Suppose $h(x) = f(g(x))$. Find $h'(0)$.

8. (8 points) The shock-waves from an earthquake on the ocean floor radiate out in the form of a circle on the surface of the ocean from its epicenter. If the radius of the shock-waves is increasing at a rate of 2 miles per second, what is the rate of change of the area enclosed by the radiating shock-waves when the radius is 5 miles? Give an *exact* answer. Your answer should be labeled with appropriate units.

9. (6 points) A Tibetan monk leaves the monastery at 7:00AM and takes his usual path to the top of the mountain, arriving at 7PM. The following morning, he starts at 7:00AM at the top and take the same path back, arriving at the monastery at 7:00PM. Use the Intermediate Value Theorem to argue that there is a point on the path that the monk will cross at exactly the same time of day on both days. (Hint: let $p_u(t)$ be the position function for the monk on his way up and let $p_d(t)$ be the position function for the monk on his way down. You need to argue that there exists a $c \in (0, 12)$ such that $p_u(c) = p_d(c)$. Use the IVT on the function $P(t) = p_u(t) - p_d(t)$.)