MA 2560: Calculus II (Spring 2010)
Exam 2

NAME:

Instructions: Answer each of the following questions completely. To receive full credit, you must *justify* each of your answers (unless stated otherwise). How you reached your answer is more important than the answer itself. If something is unclear, or if you have any questions, then please ask. Good luck!

Here are some potentially useful formulas. You should *not* expect to use all of them.

$$\frac{d}{dx}[\sinh x] = \cosh x \qquad \frac{d}{dx}[\cosh x] = \sinh x$$

$$\frac{d}{dx}[\tanh x] = \text{sech}^2 x \qquad \frac{d}{dx}[\text{sech } x] = -\text{sech } x \tanh x$$

$$\frac{d}{dx}[\sin^{-1} x] = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}[\cos^{-1} x] = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}[\tan^{-1} x] = \frac{1}{1+x^2} \qquad \frac{d}{dx}[\sec^{-1} x] = \frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}[\sinh^{-1} x] = \frac{1}{\sqrt{1+x^2}} \qquad \frac{d}{dx}[\cosh^{-1} x] = \frac{1}{\sqrt{x^2-1}}$$

$$\frac{d}{dx}[\tanh^{-1} x] = \frac{1}{1-x^2} \qquad \frac{d}{dx}[\text{sech}^{-1} x] = \frac{-1}{x\sqrt{1-x^2}}$$

$$\frac{d}{dx}[b^x] = b^x \ln b \qquad \log_b(x) = \frac{1}{x \ln b}$$

$$\int \sinh u \; du = \cosh u + C \qquad \int \cosh u \; du = \sinh u + C$$

$$\int \text{sech}^2 u \; du = \tanh u + C \qquad \int \text{sech } u \tanh u \; du = -\text{sech } u + C$$

$$\int \frac{1}{\sqrt{a^2-u^2}} \; du = \sin^{-1}\frac{u}{a} + C \qquad \int \frac{1}{u^2+a^2} \; du = \frac{1}{a}\tan^{-1}\frac{u}{a} + C$$

$$\int \frac{1}{u\sqrt{u^2-a^2}} \; du = \frac{1}{a}\sec^{-1}\frac{u}{a} + C \qquad \int \frac{1}{\sqrt{u^2+a^2}} \; du = \sinh^{-1}\frac{u}{a} + C$$

$$\int \frac{1}{\sqrt{u^2-a^2}} \; du = \cosh^{-1}\frac{u}{a} + C \qquad \int \frac{1}{a^2-u^2} \; du = \frac{1}{a}\tanh^{-1}\frac{u}{a} + C$$

$$\int b^u \; du = \frac{b^u}{\ln b} + C \qquad \int \sec x \; dx = \ln|\sec x + \tan x| + C$$

1. (2 points each) Determine whether each of the following statements is TRUE or FALSE (circle the correct answer).

   (a) Every elementary function has an elementary derivative. TRUE FALSE

   (b) Every elementary function has an elementary antiderivative. TRUE FALSE

2. (12 points) Evaluate each of the following limits. If a limit does not exist, specify whether it equals ∞, $-\infty$, or simply does not exist (in which case, write DNE). If the limit does exist, you should give an *exact* answer, as opposed to a decimal approximations.

   (a) $\lim_{x\to 0} \frac{1-\cos x}{x^2}$

   (b) $\lim_{x\to\infty} x^{1/x}$

3. (12 points each) Evaluate **any 6 of the 7 following integrals**. Put a big X through the problem that you do *not* want me to grade. If you complete all 7 problems, I will only grade the first 6. For the definite integrals, you should give *exact* answers (i.e., *not* decimal approximations using your calculator).

   (a) $\displaystyle\int \frac{\cos^3 x}{\sin x}\, dx$

   (b) $\displaystyle\int \sec^4 x \tan x\, dx$

(c) $\int_0^{\pi} x \cos x \ dx$

(d) $\int \frac{x}{\sqrt{x^2 - 9}} \ dx$

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(e) $\displaystyle\int \frac{\sqrt{x^2-9}}{x}\,dx$

(f) $\displaystyle\int \frac{x^2}{x^2-9}\,dx$

(g) $\displaystyle\int \frac{2x^2 - x + 6}{x^3 + 9x}\, dx$

4. **Bonus Question:** (5 points) Use an integral (and one of our recent techniques) to prove that the area of a circle of radius 1 is π. (Hint: Consider the portion of the circle in the first quadrant and multiply your answer by 4.)