MA 2560: Calculus II (Spring 2009)
Exam 2

NAME:

Instructions: Answer each of the following questions completely. To receive full credit, you must *justify* each of your answers (unless stated otherwise). How you reached your answer is more important than the answer itself. If something is unclear, or if you have any questions, then please ask. Good luck!

Here are some potentially useful formulas. You should *not* expect to use all of them.

$$\frac{d}{dx}[\sinh x] = \cosh x \qquad \frac{d}{dx}[\cosh x] = \sinh x$$

$$\frac{d}{dx}[\tanh x] = \text{sech}^2 x \qquad \frac{d}{dx}[\text{sech}\, x] = -\text{sech}\, x \tanh x$$

$$\frac{d}{dx}[\sin^{-1} x] = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}[\cos^{-1} x] = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}[\tan^{-1} x] = \frac{1}{1+x^2} \qquad \frac{d}{dx}[\sec^{-1} x] = \frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}[\sinh^{-1} x] = \frac{1}{\sqrt{1+x^2}} \qquad \frac{d}{dx}[\cosh^{-1} x] = \frac{1}{\sqrt{x^2-1}}$$

$$\frac{d}{dx}[\tanh^{-1} x] = \frac{1}{1-x^2} \qquad \frac{d}{dx}[\text{sech}^{-1} x] = \frac{-1}{x\sqrt{1-x^2}}$$

$$\int \sinh u \; du = \cosh u + C \qquad \int \cosh u \; du = \sinh u + C$$

$$\int \text{sech}^2 u \; du = \tanh u + C \qquad \int \text{sech}\, u \tanh u \; du = -\text{sech}\, u + C$$

$$\int \frac{1}{\sqrt{a^2-u^2}} \; du = \sin^{-1}\frac{u}{a} + C \qquad \int \frac{1}{u^2+a^2} \; du = \frac{1}{a}\tan^{-1}\frac{u}{a} + C$$

$$\int \frac{1}{u\sqrt{u^2-a^2}} \; du = \frac{1}{a}\sec^{-1}\frac{u}{a} + C \qquad \int \frac{1}{\sqrt{u^2+a^2}} \; du = \sinh^{-1}\frac{u}{a} + C$$

$$\int \frac{1}{\sqrt{u^2-a^2}} \; du = \cosh^{-1}\frac{u}{a} + C \qquad \int \frac{1}{a^2-u^2} \; du = \frac{1}{a}\tanh^{-1}\frac{u}{a} + C$$

$$\int_a^b f(x)\; dx \approx \sum_{i=1}^{n} f\left(\frac{1}{2}(x_{i-1}+x_i)\right)\Delta x = M_n$$

$$\int_a^b f(x)\; dx \approx \frac{\Delta x}{2}\left(f(x_0)+2f(x_1)+2f(x_2)+\cdots+2f(x_{n-1})+f(x_n)\right) = T_n$$

$$\int_a^b f(x)\; dx \approx \frac{\Delta x}{3}\left(f(x_0)+4f(x_1)+2f(x_2)+4f(x_3)+\cdots+2f(x_{n-2})+4f(x_{n-1})+f(x_n)\right) = S_n$$

1. (10 points each) Integrate each of the following indefinite or definite integrals. For the definite integrals, you should give *exact* answers (i.e., *not* decimal approximations using your calculator).

   (a) $\displaystyle\int \frac{\sin^3 x}{\sqrt{\cos x}}\,dx$

   (b) $\displaystyle\int_0^{\sqrt{8}} \frac{x}{\sqrt{9-x^2}}\,dx$

(c) $\displaystyle\int \frac{\sqrt{x^2-9}}{x}\,dx$

(d) $\displaystyle\int \frac{x^3}{x^2-9}\,dx$

(e) $\displaystyle\int \frac{2x^2+3}{x^3+x}\,dx$

2. (10 points each) For each of the following improper integrals, state why the integral is improper and then determine whether the integral converges or diverges. If the integral converges, determine its *exact* value.

   (a) $\displaystyle\int_0^{\infty} \frac{1}{9x^2+25}\,dx$

(b) $\displaystyle\int_{-1}^{1} \frac{1}{x^2}\, dx$ *

3. (10 points) Find the arc length of $y = \dfrac{1}{3}(x^2+2)^{3/2}$ on $[0, 3]$. You should give an *exact* answer (i.e., *not* a decimal approximation using your calculator).

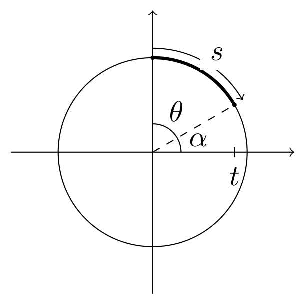
*Observe that if you had naively tried to integrate by just finding the antiderivative of $y = \dfrac{1}{x^2}$, plugging in 1 and -1, and then subtracting, you'd get the wrong answer.

4. (10 points) Find the area of the surface obtained by revolving the graph of $y = x^3$ on the interval $[0, 2]$ about the x-axis. You should give an *exact* answer (i.e., *not* a decimal approximation using your calculator).

5. (10 points) A population of nuggets increases at a rate of $r(t)$ nuggets per week. The following table gives values for $r(t)$ every 4 weeks for 24 weeks. Using Simpson's Rule (with $n = 6$), estimate the population of nuggets at the end of the 24 week period. Round your answer to 4 decimal places. (At the very least, I want to see the formula that you are plugging into your calculator.)

t (weeks)	$r(t)$ (nuggets/week)
0	0
4	200
8	3,000
12	11,500
16	4000
20	250
24	0

6. **Bonus Question:** Have you ever wondered why the inverse of sine is also referred to as arcsine? Don't lie; of course you have! Well, it's about time, you found out. Read on. Consider the following picture of the unit circle.



Recall that $y = \sqrt{1-x^2}$ is the equation for the top half of the unit circle.

(a) (1 point) Find $\dfrac{dy}{dx}$ for the top half of the unit circle.

(b) (3 points) Find the arc length of the top half of the unit circle from $x = 0$ to $x = t$. (Hint: simplify the integrand by getting common denominators and then evaluate the integral using an appropriate formula).

(c) (1 point) Notice that $t = \cos(\alpha)$. The "co" in cosine stands for complementary because $\cos(\alpha) = \sin(\pi/2 - \alpha)$. Also, notice that $\theta = \pi/2 - \alpha$. What is the relationship between t, θ, and sine?

(d) (1 point) Recall that an angle measured in radians is equal to the arc length divided by the radius. We're on the unit circle; so the radius is 1. What is the relationship between s, θ, and sine?

If you answered all of these questions correctly, you should see why the inverse of sine is also referred to as arcsine.