MAT 411: Introduction to Abstract Algebra
Final Exam (Take-Home Portion)

Your Name:

Names of Any Collaborators:

Instructions

This portion of the Final Exam is worth a total of 16 points and is due by **5pm on Thursday, May 12**. Your total combined score on the in-class portion and take-home portion is worth 20% of your overall grade.

I expect your solutions to be *well-written, neat, and organized.* Do not turn in rough drafts. What you turn in should be the "polished" version of potentially several drafts.

Feel free to type up your final version. The LaTeX source file of this exam is also available if you are interested in typing up your solutions using LaTeX. I'll gladly help you do this if you'd like.

The simple rules for the exam are:

1. You may freely use any theorems that we have discussed in class, but you should make it clear where you are using a previous result and which result you are using. For example, if a sentence in your proof follows from Theorem 1.41, then you should say so.
2. Unless you prove them, you cannot use any results from the course notes that we have not yet covered.
3. You are **NOT** allowed to consult external sources when working on the exam. This includes people outside of the class, other textbooks, and online resources.
4. You are **NOT** allowed to copy someone else's work.
5. You are **NOT** allowed to let someone else copy your work.
6. You are allowed to discuss the problems with each other and critique each other's work.

I will vigorously pursue anyone suspected of breaking these rules.

You should **turn in this cover page** and all of the work that you have decided to submit. **Please write your solutions and proofs on your own paper.**

To convince me that you have read and understand the instructions, sign in the box below.

Signature:

Good luck and have fun!

1. (4 points each) Prove any **three** of the following theorems. You are allowed to use an earlier theorem in the proof of a later theorem (even if you did not prove the earlier theorem). However, you may not use a later theorem in the proof of an earlier theorem. This includes the theorems in the notes. *Note:* The definitions of maximal and prime appear in the course notes.

   Theorem 1. Let I be an ideal of R. Then $I = R$ iff I contains a unit. *Note:* This is Theorem 10.49 in the course notes.

   Theorem 2. Assume R is commutative. Then R is a field iff its only ideals are 0 and R. *Note:* This is Theorem 10.50 in the course notes.

   Theorem 3. If R is a field, then every nonzero ring homomorphism from R into another ring is an injection. *Note:* This is Corollary 10.51 in the course notes.

   Theorem 4. Assume R is commutative. The ideal M is maximal iff R/M is a field. *Note:* This is Theorem 10.55 in the course notes.

   Theorem 5. Assume R is a commutative ring. Then the ideal P is a prime ideal in R iff the quotient ring R/P is an integral domain. *Note:* This is Theorem 10.60 in the course notes.

   Theorem 6. Assume R is a commutative ring. Every maximal ideal of R is a prime ideal. *Note:* This is Corollary 10.61 in the course notes.

2. (4 points) Prove **one** of the following theorems.

   Theorem 7. Let R be a commutative ring with 1. Then the principal ideal (x) in the polynomial ring $R[x]$ is a prime ideal iff R is an integral domain.

   Theorem 8. Let R be a commutative ring with 1. Then the principal ideal (x) in the polynomial ring $R[x]$ is a maximal ideal iff R is a field.

