Integration by Substitution

Motivation and Background

Currently, we do *not* have a technique for integrating most products, quotients, and compositions. Here are a couple that we can integrate:

$$\int \frac{x^2+x}{\sqrt{x}}\,dx, \quad \int \sec(x)\tan(x)\,dx$$

And here are some that we cannot currently integrate (unless you happen to see what the appropriate antiderivative is):

$$\int x\sqrt{x^2+1}\,dx, \quad \int \sin(x)\cos(x)\,dx, \quad \int \frac{x}{x^2+1}\,dx$$

To integrate functions like above, we will utilize a technique called *substitution*, which involves the use of dummy variable.

Important Note 1. Substitution is a technique that only works in special circumstances, which should become apparent after a little practice.

Important Note 2. If confronted with an integral of a product, quotient, or composition and you cannot integrate it straight away, then substitution may work. In most (but definitely not all) situations, you will pick u to be the inside of the more complicated part.

Examples

Example 3. Compute each of the following integrals.

1. $\int (3x-1)^{99}\,dx$

2. $\int 5x^2\sqrt{x^3-2}\,dx$

3. $\displaystyle\int xe^{x^2}\ dx$

4. $\displaystyle\int \sin^2(x)\cos(x)\ dx$

5. $\displaystyle\int \frac{x}{x^2+1}\ dx$

6. $\displaystyle\int \frac{x^2+1}{x}\ dx$

7. $\displaystyle\int x^2\sec^2(x^3)\ dx$

8. $\displaystyle\int \frac{x}{x^4+1}\,dx$

9. $\displaystyle\int x\sqrt{x-1}\,dx$

10. $\displaystyle\int \frac{\ln(x)}{x}\,dx$

11. $\displaystyle\int \frac{\arcsin(x)}{\sqrt{1-x^2}}\,dx$

12. $\displaystyle\int \frac{e^x}{e^{2x}+1}\,dx$