# MA 2560: Calculus II (Spring 2011)
# Exam 3

**NAME:**

**Instructions:** Answer each of the following questions completely. To receive full credit, you must show sufficient work for each of your answers (unless stated otherwise). How you reached your answer is more important than the answer itself. If something is unclear, or if you have any questions, then please ask. Good luck!

Here are some potentially useful formulas. You should *not* expect to use all of them.

$$\frac{d}{dx}[\sinh x] = \cosh x \qquad \frac{d}{dx}[\cosh x] = \sinh x$$

$$\frac{d}{dx}[\tanh x] = \text{sech}^2 x \qquad \frac{d}{dx}[\text{sech}\ x] = -\text{sech}\ x \tanh x$$

$$\frac{d}{dx}[\arcsin x] = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}[\arccos x] = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}[\arctan x] = \frac{1}{1+x^2} \qquad \frac{d}{dx}[\text{arcsec}\ x] = \frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}[\sinh^{-1} x] = \frac{1}{\sqrt{1+x^2}} \qquad \frac{d}{dx}[\cosh^{-1} x] = \frac{1}{\sqrt{x^2-1}}$$

$$\frac{d}{dx}[\tanh^{-1} x] = \frac{1}{1-x^2} \qquad \frac{d}{dx}[\text{sech}^{-1} x] = \frac{-1}{x\sqrt{1-x^2}}$$

$$\int \sec x\ dx = \ln|\sec x + \tan x| + C \qquad \int \tan x\ dx = \ln|\sec x| + C$$

$$\int \sinh u\ du = \cosh u + C \qquad \int \cosh u\ du = \sinh u + C$$

$$\int \text{sech}^2 u\ du = \tanh u + C \qquad \int \text{sech}\ u \tanh u\ du = -\text{sech}\ u + C$$

$$\int \frac{1}{\sqrt{a^2-u^2}}\ du = \arcsin\frac{u}{a} + C \qquad \int \frac{1}{u^2+a^2}\ du = \frac{1}{a}\arctan\frac{u}{a} + C$$

$$\int \frac{1}{u\sqrt{u^2-a^2}}\ du = \frac{1}{a}\text{arcsec}\ \frac{u}{a} + C \qquad \int \frac{1}{\sqrt{u^2+a^2}}\ du = \sinh^{-1}\frac{u}{a} + C$$

$$\int \frac{1}{\sqrt{u^2-a^2}}\ du = \cosh^{-1}\frac{u}{a} + C \qquad \int \frac{1}{a^2-u^2}\ du = \frac{1}{a}\tanh^{-1}\frac{u}{a} + C$$



Written by D.C. Ernst

1. (10 points) Find the average value of $f(x) = \sin x$ over the interval $[0, \pi]$. Give a simplified and exact value.

2. Consider the differential equation $\dfrac{dy}{dt} = \dfrac{1}{1+t^2}$.

   (a) (8 points) Find the *general solution* of the above differential equation.

   (b) (6 points) Solve the *initial value problem*: $y(1) = 0$. Give simplified and exact values.

3. (8 points each) Consider the parametric curve given by

$$x = 2\sin 2t, y = 3\sin t.$$

This curve crosses itself at $(0, 0)$.

(a) Find all values of $t$ in $[0, 2\pi)$ such that $(x, y) = (0, 0)$.

(b) Find the slopes of all tangent lines at $(0, 0)$. Give simplified and exact values.

4. (12 points) Consider the parametric curve given by

$$x = \frac{8}{3}t^{3/2}, y = 2t - t^2.$$

Find the arc length for $1 \leq t \leq 2$. (Give an *exact* answer.)

5. (12 points) Find the area of one petal of the graph of $r = \cos(5\theta)$. Give a simplified and exact value.

 

6. (8 points each) Determine whether each of the following *sequences* converges or diverges. If the sequence converges, find its limit. Be sure to show sufficient justification.

(a) $a_n = \dfrac{84n^2}{2n^2 + 17\pi}$

(b) $a_n = \dfrac{84\cos^2(n)}{2n^2 + 17\pi}$

7. (10 points each) Determine whether each of the following *series* converges or diverges. If the series converges, find its sum. Be sure to show sufficient justification.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n 5^{n+1}}{7^{n-1} 2^{1-n}}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+n}$