MA 2560: Calculus II (Spring 2011)
Exam 2

NAME:

Instructions: Answer each of the following questions completely. To receive full credit, you must show sufficient work for each of your answers (unless stated otherwise). How you reached your answer is more important than the answer itself. If something is unclear, or if you have any questions, then please ask. Good luck!

Here are some potentially useful formulas. You should *not* expect to use all of them.

$$\frac{d}{dx}[\sinh x] = \cosh x \qquad \frac{d}{dx}[\cosh x] = \sinh x$$

$$\frac{d}{dx}[\tanh x] = \operatorname{sech}^2 x \qquad \frac{d}{dx}[\operatorname{sech} x] = -\operatorname{sech} x \tanh x$$

$$\frac{d}{dx}[\arcsin x] = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}[\arccos x] = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}[\arctan x] = \frac{1}{1+x^2} \qquad \frac{d}{dx}[\operatorname{arcsec} x] = \frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}[\sinh^{-1} x] = \frac{1}{\sqrt{1+x^2}} \qquad \frac{d}{dx}[\cosh^{-1} x] = \frac{1}{\sqrt{x^2-1}}$$

$$\frac{d}{dx}[\tanh^{-1} x] = \frac{1}{1-x^2} \qquad \frac{d}{dx}[\operatorname{sech}^{-1} x] = \frac{-1}{x\sqrt{1-x^2}}$$

$$\int \sec x \, dx = \ln|\sec x + \tan x| + C \qquad \int \tan x \, dx = \ln|\sec x| + C$$

$$\int \sinh u \, du = \cosh u + C \qquad \int \cosh u \, du = \sinh u + C$$

$$\int \operatorname{sech}^2 u \, du = \tanh u + C \qquad \int \operatorname{sech} u \tanh u \, du = -\operatorname{sech} u + C$$

$$\int \frac{1}{\sqrt{a^2-u^2}} \, du = \arcsin\frac{u}{a} + C \qquad \int \frac{1}{u^2+a^2} \, du = \frac{1}{a}\arctan\frac{u}{a} + C$$

$$\int \frac{1}{u\sqrt{u^2-a^2}} \, du = \frac{1}{a}\operatorname{arcsec}\frac{u}{a} + C \qquad \int \frac{1}{\sqrt{u^2+a^2}} \, du = \sinh^{-1}\frac{u}{a} + C$$

$$\int \frac{1}{\sqrt{u^2-a^2}} \, du = \cosh^{-1}\frac{u}{a} + C \qquad \int \frac{1}{a^2-u^2} \, du = \frac{1}{a}\tanh^{-1}\frac{u}{a} + C$$

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1. (10 points each) Integrate each of the following indefinite integrals. To receive full credit, you must show sufficient work for each of your answers.

   (a) $\displaystyle\int \frac{\sqrt{x^2-9}}{x}\,dx$

   (b) $\displaystyle\int \frac{x^3}{x^2-9}\,dx$

(c) $\displaystyle\int \frac{-2x^2+9}{x^3+9x}\,dx$

(d) $\displaystyle\int \frac{1}{x^2+2x+5}\,dx$

2. (10 points) Determine the volume of the solid obtained by revolving the region bounded by the graphs of $y = 2 - x^2$ and $y = x^2$ about the x-axis.

3. (10 points) Determine the volume of the solid obtained by revolving the region bounded by the graphs of $f(x) = 1$ and $g(x) = x^2$ about the line $x = 2$.

4. (10 points) Find the arc length of $y = \dfrac{1}{3}(x^2+2)^{3/2}$ on $[0,3]$. You should give an *exact* answer (i.e., *not* a decimal approximation using your calculator).

5. (10 points) Find the area of the surface obtained by revolving the graph of $y = x^2$ on the interval $[0, \sqrt{2}]$ about the y-axis. You should give an *exact* answer (i.e., *not* a decimal approximation using your calculator).

6. Consider the integral $\displaystyle\int_0^1 \frac{\sin^2 x}{\sqrt{x}}\, dx$.

   (a) (2 points) Explain why this integral is improper.

   (b) (8 points) It turns out that this integral is very difficult to compute by hand. However, if you look at the graph of $f(x) = \frac{\sin^2 x}{\sqrt{x}}$, you will see that it is entirely above the x-axis on the interval $[0, 1]$. If we can find a function g that is strictly larger on $[0, 1]$ than f such that

$$\int_0^1 g(x)\, dx$$

   converges, then we can conclude that the original integral converges, as well. Show that the original integral involving f converges by comparing with a larger function that converges on the same interval. (Hint: recall that $\sin^2 x \leq 1$. What can you do to this inequality to obtain f? Integrate the resulting function on the right hand side.)

7. (10 points) Determine whether the following integral converges or diverges. If the integral converges, determine its *exact* value. If the integral diverges, explain why.

$$\int_1^\infty \frac{\ln x}{x}\, dx$$

8. **Bonus Question:** (5 points) If f' is continuous on $[0, \infty)$ and $\lim_{x\to\infty} f(x) = 0$, show that

$$\int_0^\infty f'(x)\, dx = -f(0).$$

