MTH 302: Linear Algebra and Differential Equations

Inverses and Determinants

2023 February 7

Housekeeping

- Progress on grading (especially Miniprojects)
- Application Analysis 2 (Blackboard)
- Quiz Thursday has four skills on it

Today's Goals

- Review matrix inverses
- Solve systems with matrix inverses
- Compute determinants of 2×2 and 3×3 matrices by hand, and any size with SymPy
- Explain the parts of the Invertible Matrix Theorem
- (**Skill LA.5**) Given information about a matrix, determine if it's invertible (and vice versa)

Review of Class Prep

Activity part 1

Work in your groups.

- Exercise 1 is a review of how to use SymPy to find matrix inverses.
- Exercise 2 is an important application of matrix inverses and is part of Application Analysis 3.
- Exercises 3 and 4 go together and connect matrix inverses back to something you learned earlier in the course.

Break time

Activity part 2

Again in groups:

- Exercise 1 is practice finding 3×3 determinants by hand
- Exercise 2 uses SymPy to find a shortcut for determinants for a special kind of matrix
- Exercises 3 and 4 are experiment-conjecture activities about determinants of products and inverses. Exercise 3 is part of Application Analysis 3.

The Invertible Matrix Theorem

Theorem 1.9.2 (Invertible Matrix Theorem) Let $\mathbf{A}$ be an $n \times n$ matrix. The following statements are equivalent:

a. $\mathbf{A}$ is invertible.
b. The columns of $\mathbf{A}$ are linearly independent.
c. The columns of $\mathbf{A}$ span $\mathbb{R}^n$.
d. $\mathbf{A}$ has a pivot position in every column.
e. $\mathbf{A}$ has a pivot position in every row.
f. $\mathbf{A}$ is row equivalent to $\mathbf{I}_n$.
g. For each $\mathbf{b} \in \mathbb{R}^n$, the equation $\mathbf{Ax} = \mathbf{b}$ has a unique solution.
h. $\det(\mathbf{A}) \neq 0$.

A quick way to test for invertibility

In [ ]:
```
from sympy import *
init_printing()
```

In [ ]:
```
# Is this matrix invertible?
A = randMatrix(6,6,-10,10)
A
```

In [ ]:
```
# At least two "computational" ways to find out
```

How to use the IVT

- Given: Some partial information about a system or a matrix
- Determine: As many conclusions about that system or matrix as possible.

⟶ **Poll Everywhere**

In [ ]:
```
A = Matrix(5,5,[1,2,3,4,5, 0,0,1,2,3, 0,0,7,8,9, 0,0,0,-1,-2, 0,0,0,0,1])
A
```

In [ ]:
```
B = randMatrix(10,10,-100,100)
B.rref(pivots=False)
```

In [ ]:
```

```