$$\frac{dx}{dt} = 4x - 2y + 3t$$

$$\frac{dy}{dt} = 3x - 3y - t$$

$\lambda_1 = -2 \quad \vec{v}_1 = \begin{bmatrix} 1/3 \\ 1 \end{bmatrix}$

$\lambda_2 = 3 \quad \vec{v}_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

Homogeneous solution: $\vec{x}_h = c_1 e^{-2t}\begin{bmatrix} 1/3 \\ 1 \end{bmatrix} + c_2 e^{3t}\begin{bmatrix} 2 \\ 1 \end{bmatrix}$

Nonhomogeneous solution: Try $\vec{x}_p = \begin{bmatrix} At + B \\ Ct + D \end{bmatrix}$.

$\vec{x}_p' = \begin{bmatrix} A \\ C \end{bmatrix}$

These two have to equal each other

$$\begin{bmatrix} 4 & -2 \\ 3 & -3 \end{bmatrix}\begin{bmatrix} At + B \\ Ct + D \end{bmatrix} + \begin{bmatrix} 3t \\ -t \end{bmatrix} = \begin{bmatrix} 4At + 4B - 2Ct - 2D + 3t \\ 3At + 3B - 3Ct - 3D - t \end{bmatrix}$$

so...

$$4At + 4B - 2Ct - 2D + 3t = A$$
$$3At + 3B - 3Ct - 3D - t = C$$

Notice, no "t" terms here

Collect like terms

$$(4A - 2C + 3)t + (4B - 2D) = A$$
$$(3A - 3C - 1)t + (3B - 3D) = C$$

Set like terms equal

$$\begin{aligned} 4A - 2C + 3 &= 0 \\ 4B - 2D &= A \\ 3A - 3C - 1 &= 0 \\ 3B - 3D &= C \end{aligned} \quad \Longrightarrow \quad \begin{aligned} 4A - 2C &= -3 \\ -A + 4B - 2D &= 0 \\ 3A - 3C &= 1 \\ 3B - C - 3D &= 0 \end{aligned}$$

$$\left[\begin{array}{cccc|c} 4 & 0 & -2 & 0 & -3 \\ -1 & 4 & 0 & -2 & 0 \\ 3 & 0 & -3 & 0 & 1 \\ 0 & 3 & -1 & -3 & 0 \end{array}\right]$$

RREF $\Rightarrow$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -11/6 \\ 0 & 1 & 0 & 0 & -7/36 \\ 0 & 0 & 1 & 0 & -13/6 \\ 0 & 0 & 0 & 1 & 19/36 \end{array}\right] \begin{array}{l} A \\ B \\ C \\ D. \end{array}$$

THEREFORE... the particular solution is

$$\vec{x}_p = \begin{bmatrix} -\frac{11}{6}t - \frac{7}{36} \\ -\frac{13}{6}t + \frac{19}{36} \end{bmatrix}$$

So the general overall solution is ...

$$\vec{x}(t) = c_1 e^{-2t}\begin{bmatrix} 1/3 \\ 1 \end{bmatrix} + c_2 e^{3t}\begin{bmatrix} 2 \\ 1 \end{bmatrix} + \begin{bmatrix} -\frac{11}{6}t - \frac{7}{36} \\ -\frac{13}{6}t + \frac{19}{36} \end{bmatrix}$$