In [ ]:
```
from sympy import *
init_printing()
```

MTH 302: Linear Algebra and Differential Equations

Linear differential equations

2023 February 16

Housekeeping

- Pre-February 14 Class Preps are out of whack
- If you did the second problem that was intended for Application/Analysis 2
- New tutorial on using SymPy to solve DE's (will use today)

Today's Goals

- Develop shortcut for homogeneous linear DEs
- Practice solving homogeneous linear DEs
- Practice solving non-homogeneous linear DE's using integrating factors
- See how SymPy does this
- Skill Quiz

Review of Class Prep activities

Linear differential equations

$$\frac{dy}{dt} + p(t)y = f(t)$$

- A function y related to its derivative $y'(t)$ in an equation
- The only thing being done to y and y' are multiplication by constants *or functions of* t, then adding/subtracting.

Homogeneous linear DEs

$$\frac{dy}{dt} + p(t)y = 0$$

Solve by separating the y and t parts then integrating.

Try these:

- $y' + 5y = 0$
- $y' + \frac{2}{t}y = 0$
- $y' = -y\cos(t)$

At the board

Start with the generic homogeneous DE $y' + p(t)y = 0$ and solve it to come up with a "shortcut" for solving *any* homogeneous linear DE.

Then use the result to (quickly!) solve $ty' + 2y = 0$.

Non-homogeneous linear DEs

Start with: $y' + p(t)y = f(t)$

- Find an **integrating factor** by integrating $p(t)$: $P(t) = \int p(t)\,dt$. Integrating factor is $e^{P(t)}$.
- Multiply DE by integrating factor: $e^{P(t)}y' + e^{P(t)}p(t)y = f(t)e^{P(t)}$

- Left side is the derivative of $e^{P(t)}y$
- Therefore $e^{P(t)}y = \int e^{P(t)} f(t)\, dt$
- Therefore

$$y = e^{-P(t)} \int e^{P(t)} f(t)\, dt$$

Example

Solve $y' + 5y = t^2$.

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1. $p(t) = 5$. So $P(t) = 5t$. Integrating factor is e^{5t}.
2. Therefore $y = e^{-5t} \int e^{5t} t^2\, dt$
3. Evaluate the integral (using integration by parts) then multiply by e^{-5t}.

In [ ]:
```
t = var("t")

# Is SymPy right about this?
exp(-5*t) * integrate(exp(5*t)* t**2, t)
```

In [ ]:
```
y = Function("y")
de = Eq(diff(y(t),t) + 5*y(t), t**2)
dsolve(de, y(t))
```

Mixed practice

At your group, solve the IVP that matches the number of your group:

1. $y' + 2y = 2t,\ y(1) = 0$
2. $y' + ty = 10t,\ y(0) = 5$
3. $y' + \frac{2}{t}y = e^t,\ y(1) = 4$

4. $y' = 0.03 - \frac{2}{100-t}y,\ y(0) = 1$

Next time

- Applications of linear DE's

Skill Quiz

- Third/final appearance of LA.3
- Second versions of LA.4 and LA.5
- First appearance of LA.6