

MTH 302: Skill Quiz 6

This quiz contains the **third and final attempt** at Skills LA.4 and LA.5, the **second attempt** at Skill LA.6, and the **first appearance** of Skill DE.1.

Instructions:

- If you had a "Success" mark on a skill from the first quiz, **do not do the problem for that skill again**. You only need one "Success" on each skill, then you're done.
 - You should only be working on the skills that you **need** to work on (because you've never tried them, or you did and got "Retry") *and* you feel **ready** to work on.
 - Make sure to consult the [Standards for Student Work in MTH 302](#) document before starting on your work, so you're clear on what is expected and what constitutes a "successful" attempt. Also check the *Success criteria* below each problem.
 - As before, you may hand-write your work on paper, hand-write it in a notes app, or type it up. But **please start a new page for each Skill**. If a Skill takes more than one page, that's OK, but **don't put two skills on the same page**.
 - When you are ready to submit your work: **Scan** your handwritten work to a clear, legible, black-and-white PDF using a scanner or scanning app -- **one PDF per problem**. So if you are doing both problems, you will have two PDFs: one for Skill LA.1 and another for Skill LA.2 (all parts).
 - Then, **upload each PDF to its designated folder** on Blackboard. For example the PDF for Skill LA.2 goes into the folder for Skill LA.2. **Please make sure you have put your work in the right folder, because work in the wrong folder significantly delays the grading process**.
 - Make sure to **click "Submit"** after uploading each item, before exiting.
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Foundational Skill LA.4

LA.4: I can add, subtract, and multiply matrices.

Let A , B , and C be the given matrices. In each of the parts, compute (by hand) the prescribed algebraic combination of A , B , and C whenever the operation is defined. If the operation is not defined, say so and explain why.

$$A = \begin{bmatrix} -3 & -5 & 2 \\ -1 & 6 & 0 \\ -3 & -3 & 8 \end{bmatrix} \quad B = \begin{bmatrix} -5 & 6 \\ -2 & 5 \\ -3 & -8 \end{bmatrix} \quad C = \begin{bmatrix} 5 \\ -6 \\ -5 \end{bmatrix}$$

(a) AB

(b) AB

(c) BC

(d) $C^T B$

Success criteria: All parts that are not defined are clearly indicated as such and a correct explanation is given on each. All parts that are defined, have correct answers with the exception of two "simple" errors allowed overall. (A "simple" error is one that does not pertain to the concept being assessed and which does not oversimplify the problem.) **You may use technology to do basic arithmetic, but nothing else.**

Foundational Skill LA.5

LA.5: I can determine if a matrix is invertible, using information about the matrix.

For each of the matrices below, determine whether the matrix is invertible using no further computation. (For example you may not type the matrix in to SymPy and have it find the inverse.) State clearly whether or not the matrix is invertible, and give an explanation for your reasoning. If there is not enough information to determine whether

the matrix is invertible, say so and then explain why.

1. A is a 3×3 matrix whose columns are the (3×1) vectors \mathbf{a} , \mathbf{b} , and $\mathbf{a} - 2\mathbf{b}$.

2. B is a 3×3 matrix with $B \cdot \begin{bmatrix} 1 \\ 7 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$ and $B \cdot \begin{bmatrix} -2 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$.

3. C is a 2×2 matrix that is row equivalent to $\begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$. (Note: "Row equivalent" means you can get the second matrix by taking the first and applying a sequence of elementary row operations.)

Success criteria: Each part has a correct answer (is invertible, is not invertible, or not enough information) along with a correct explanation for the answer.

Foundational Skill LA.6

LA.6: *I can find the eigenvalues and eigenvectors of a 2×2 and 3×3 matrix.*

For the matrix A below, find the eigenvalues, and find an eigenvector corresponding to each eigenvalue. **Show all work by hand except for basic arithmetic and row-reduction steps.**

$$A = \begin{bmatrix} -7 & -12 \\ 4 & 9 \end{bmatrix}$$

Success criteria: All work except for basic arithmetic and row-reduction steps is done by hand, and is present and legible with no gaps or missing steps. The work is correct and leads to the correct eigenvalues and eigenvectors. Up to two "simple" errors (copy errors, or basic arithmetic mistakes that do not oversimplify the problem) are allowed, but the resulting eigenvalues and eigenvectors must be consistent with the error or errors.

(*Warning:* Copy errors or arithmetic mistakes can greatly complicate your algebra work! You are strongly advised to double check all copying and computation often.) **You may use technology to do basic arithmetic and row-reduction, but nothing else.**

Foundational Skill DE.1

DE.1: I can solve a linear, homogeneous first-order differential equation using integration.

Solve the following differential equations or initial value problems:

1. $y' + t^2y = 0$
2. $y' + y \cos(t) = 0, y(0) = 1/2$

Show all integration work by hand. There is a template for a solution that we discussed in class, but on this skill quiz you will need to show all calculus steps.

Success criteria: All work except for basic arithmetic is done by hand, and is present and legible with no gaps or missing steps. The work is correct and leads to the correct solutions in each case. Up to two "simple" errors (copy errors, or basic arithmetic mistakes that do not oversimplify the problem) are allowed, but the resulting solutions must be consistent with the error or errors. (*Warning:* Copy errors or arithmetic mistakes can greatly complicate things! You are strongly advised to double check all copying and computation often.) **You may use technology to do basic arithmetic and row-reduction, but nothing else.**