MTH 302: Linear Algebra and Differential Equations

Matrix Algebra and Introduction to Inverses

2023 February 2

Housekeeping

- Change to quiz setup for Class Preps next week
- Where do you find due dates?
- What's in the announcements?

Today's Goals

- Add, subtract, rescale, and multiply matrices by hand
- Do all of this in SymPy as well
- Make conjectures about matrix arithmetic
- Find the inverse of a square matrix (if it exists) using RREF
- Make conjectutes about matrix inverses (connect to linear independence, etc.)

Debriefing Class Prep

No big questions but some items to clear up

Practice

Work through **Exercise 1.7 #1 (and #2 if time)** by hand.

Note: This is Foundational Skill LA.4

> LA.4: I can add, subtract, and multiply matrices.

We will debrief this using SymPy.

In [ ]:
```
from sympy import *
init_printing()
A = Matrix(2,3,[3,-5,2,-1,5,-4])
B = Matrix(3,2,[-6,10,2,11,-3,-2])
C = Matrix(3,2,[5,3,-1,0,2,-4])
```

In [ ]:
```
A.transpose()
```

In [ ]:
```
# Check arithmetic here
```

Activity

In groups, work on Part 1 of tonight's activity.

Demo for the first item is on the next slide.

In [ ]:
```
# Randomly generate two 4x4 matrices
A = randMatrix(4,4,-15,15)
B = randMatrix(4,4,-15,15)
(A,B)
```

In [ ]:
```
(A+B, B+A)
```

Does $A + B$ appear to be equal to $B + A$ all the time? If so, what's a general explanation? If not, what's an example of failure and an example of success?

Break time!

Inverses

If $\mathbf{A}$ is $n \times n$, then $\mathbf{A}$ is **invertible** if there is another matrix $\mathbf{B}$ such that $\mathbf{AB} = \mathbf{I}_n$ and $\mathbf{BA} = \mathbf{I}_n$.

Example: $\mathbf{A} = \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$ is invertible:

$$\begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} \frac{2}{3} & -\frac{1}{3} \\ \frac{1}{6} & \frac{1}{6} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

(Check that this also works if you reverse the order.)

Not all matrices are invertible

Let $\mathbf{A} = \begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix}$. Is there a way to fill in the entries?

$$\begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

In [ ]:
```
a,b,c,d = var("a b c d")
A = Matrix(2,2,[1,2,-1,-2])
X = Matrix(2,2,[a,b,c,d])
A*X
```

Finding inverses using RREF

- Form an augmented matrix with $\mathbf{A}$ on the left and $\mathbf{I}_n$ on the right

- RREF the entire thing
- If pivot position in every row: Get $\mathbf{I}_n$ on the left and $\mathbf{A}^{-1}$ on the right
- Otherwise $\mathbf{A}$ is not invertible

In [ ]:
```
# Example
A = randMatrix(3,3,-10,10)
I = eye(3)   # Produces 3x3 identity matrix
M = A.col_insert(4,I)
M
```

In [ ]:
```
M.rref(pivots=False)
```

Getting inverses from SymPy directly

In [ ]:
```
A = randMatrix(4,4,-20,20)
A
```

In [ ]:
```
A.inv()
```

In [ ]:
```
# What if A is non-invertible
A = Matrix(2,2,[1,-1,2,-2])
A.inv()
```

Activity

Work in groups on today's activity, Part 2.

Skill Quiz 3

Find it on Blackboard > Skill Quizzes.

Next time

- The Invertible Matrix Theorem
- Determinants
- How inverses and determinants connect to linear independence, span, etc.