Module 4B: Predicates and quantifiers

MTH 225
30 Sept 2020

Agenda

- Review of Daily Prep activity + Q/A time
- Activity:
- Wrap up with ungraded quiz + feedback time

The statement "n is a prime number" is

A true proposition

A false proposition

An underdetermined proposition

The predicate "$n^2 + 1$ is odd" (where n is an integer) is

Never true, no matter what n is

Sometimes true, but it depends on what n is

Always true, no matter what n is

Tc 0

The statement "$\forall n(n^2 + 1$ is positive)" (where n is an integer) is

A true proposition

A false proposition

An underdetermined proposition

Which of the following are TRUE statements, assuming the domain of discourse is the set of all integers? Select all that apply.

- $\exists n(n^2 + 1 \text{ is even})$
- $\forall n(n^2 + 1 \text{ is even})$
- $\exists n(n + m \text{ is even})$
- $\exists n(n^2 + 1 \text{ is positive})$
- $\forall n(n^2 + 1 \text{ is positive})$

Tc 0

Q&A time

Predicates with two variables

c(x,y) = x is the capital of y

Domain of x: Set of all cities in the world
Domain of y: Set of all countries in the world

c(Washington, USA) = TRUE
c(Sao Paolo, Brazil) = FALSE

∀x(c(x,y)) is __________

UNDERTERMINED because y is still a free variable
Need TWO quantifiers to “bind” the variables

c(x,y) = x is the capital of y

Domain of x: Set of all cities in the world
Domain of y: Set of all countries in the world

Combinations of two quantifiers

- FALSE **∀x∀y(c(x,y))**: Every city is the capital of every country.
- TRUE **∃x∃y(c(x,y))**: There is a city and a country, such that the city is the capital of that country.
- FALSE **∀x∃y(c(x,y))**: Every city is the capital of at least one country.
- FALSE **∃x∀y(c(x,y))**: A city exists that is the capital of every country.
- TRUE **∀y∃x(c(x,y))**: Every country has at least one capital.

Let p(x,y) = "x is a parent of y". The domain of both variables is the set of all people. Which of the following are TRUE propositions? Select all that apply.

- $\forall x \forall y (p(x,y))$
- $\forall x \exists y (p(x,y))$
- $\exists y \forall x (p(x,y))$
- $\exists x \exists y (p(x,y))$
- $\exists x \forall y (p(x,y))$
- $\forall y \exists x (p(x,y))$

To 0

Consider the universally quantified statement, "All GVSU mathematics professors are geniuses." The *negation* of this statement would say

- At least one GVSU math professor is not a genius.
- None of the GVSU mathematics professors are geniuses.
- All of the GVSU mathematics professors are non-geniuses.
- None of the geniuses at GVSU are mathematics professors.

$$\neg(\forall x P(x)) \equiv \exists x(\neg P(x))$$

$$\neg(\exists x P(x)) \equiv \forall x(\neg P(x))$$

Let p(x,y) = "x is older than y" where the domain for x,y is the set of all people. Consider the statement $\forall x \exists y (p(x,y))$. The *negation* would be

- $\forall x \exists y (\neg p(x,y))$
- $\exists x \forall y (\neg p(x,y))$
- $\forall x \forall y (\neg p(x,y))$
- $\exists x \exists y (\neg p(x,y))$

To 0

Quick review

Let $d(a, b)$ be the statement "a divides b", and the domain of discourse is the set of all positive integers. Which of the following are true?

- $d(10, 2)$
- $d(3, 3)$
- $d(8, 32)$
- $d(4, x)$

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Let $d(a, b)$ be the statement "a divides b". Which of the following are true?

- $\forall a(d(1, a))$
- $\exists b(d(5, b))$
- $\forall b(d(b, 5))$
- $\forall a \forall b(d(a, b))$
- $\exists a \forall b(d(a, b))$

Tc 0

Let $d(a,b)$ be the statement "a divides b". Consider the statement "Every positive integer can be divided by at least one positive integer." In symbols, this would look like

- $\exists a \exists b (d(a,b))$
- $\forall a \exists b (d(a,b))$
- $\forall b \exists a (d,(a,b))$
- $\forall a \forall b (d(a,b))$

Tc 0

Consider the statement "Every positive integer can be divided by at least one positive integer." The negation of this statement would say

- No positive integers can be divided by any other positive integer.
- There is a positive integer that cannot be divided by any positive integer.
- Every positive integer can be divided by every positive integer.
- There is no such thing as a positive integer.

To 0