Directions:

- Do only the problems that you need to take, and feel ready to take.
- If you have already earned Fluency on a Learning Target, do not attempt a problem for that Target.
- **Do all your work either handwritten on paper, or typed into a file. Convert each Learning Target into a separate PDF file when done, and then upload to the Learning Target's assignment area on Blackboard.**
- **You may use technology to check your work, but your solutions must not involve technology. The success criteria for many of the Learning Targets on this quiz have been changed to require the showing of work, so read each of those carefully.**
- **Use of the textbook and your notes is allowed**, but no other sources of information or help are allowed. And again, many problems have enhanced requirements for showing work and explaining reasoning.
- All solutions must be uploaded to Blackboard **no later than 11:59pm Eastern on Monday, December 6. This deadline may not be extended with a token.** Solutions submitted after that deadline will not be accepted.
- If you encounter technology issues when submitting your work, **follow the protocols for dealing with those issues found in the "Technology" section of the Syllabus, page 9. Late work due to technology issues will not be accepted if you have not followed those procedures.**

Learning Target CA.1 (CORE): *I can represent an integer in base 2, 8, 10, and 16 and represent a negative integer in base 2 using two's complement notation.*

Perform each of the following actions.

1. Given 77 in decimal, convert to binary and hexadecimal.
2. Given 277 in octal, convert to decimal.
3. Given `11010101` in binary, convert to decimal.
4. Write the base 2 representation of -80 using two's complement notation. The base 2 representation of +80 is `01010000`. (Assume 8-bit representation.)

Success means: All answers are correct other than a maximum of two (2) "simple" errors and all work is shown or clear explanations given for how the answers were obtained.

Learning Target L.1 (CORE): *I can identify the parts of a conditional statement and write the negation, converse, and contrapositive of a conditional statement.*

Consider the conditional statement: *If I am not vaccinated, I will be disenrolled from my Winter semester courses.*

- State the hypothesis and the conclusion, and clearly label each.
- Write the negation (without just putting "not" in front of the statement).
- Write the converse.
- Write the contrapositive.

Success means: All answers are correct.

Learning Target L.2: *I can construct truth tables for propositions involving two or three variables and use truth tables to determine if two propositions are logically equivalent.*

Do each of the following.

1. Use truth tables to determine whether the two statements $(\neg A) \wedge (\neg B)$ and $(\neg A) \vee B$ are logically equivalent.
2. Make a truth table for the statement $(P \vee Q) \wedge (Q \to R)$. Include columns for all intermediate steps.

Success means: There is no more than one error in the final column of the results, and the statement about logical equivalence is made and is consistent with the truth tables.

Learning Target L.3: *I can identify the truth value of a predicate, determine whether a quantified predicate is true or false, and state the negation of a quantified statement.*

Let P and Q be the following predicates where the domain of each is the set of all positive integers (that is, $\{1, 2, 3, \ldots\}$).

- $P(x) : x$ is odd
- $Q(x) : x^2$ is a multiple of 5

Do each of the following.

1. State the truth values of each of the following: $P(100)$, $P(110)$, $Q(1)$, $Q(5)$.
2. State the truth values of each of the following and give a one-sentence explanation for each. (Answers without explanations are not Satisfactory.)
   (a) $\exists x P(x)$
   (b) $\exists x Q(x)$
   (c) $\forall x Q(x)$
3. State the negation of the statement, "Every rectangle has opposite sides parallel" without merely putting the word "not", "It is not the case that", etc. on the statement.

Success means: The negation is stated correctly and there is no more than one (1) error in the rest.

Learning Target SF.1 (CORE): *I can represent a set in roster notation and set-builder notation; determine if an object is an element of a set; and determine set relationships (equality, subset).*

Do each of the following.

1. Restate each of the following sets using roster notation:
   (a) $\{\lfloor x/3 \rfloor \ : \ x \in \{0, 1, 2, 3, 4, 5, 6\}\}$ where $\lfloor x \rfloor$ is the "floor" function.
   (b) $\{x \in \mathbb{N} \ : \ x + 5 = 0\}$
2. Write the set $S = \{1, 3, 9, 81, 243, \ldots\}$ using correct set-builder notation. There is more than one way to do it; but your answer must be correct and use correct notation and syntax. You may *not* use $\{x : x \in S\}$ as an answer.
3. Mark each of the following as TRUE or FALSE.
   (a) $\mathbb{N} \subseteq \mathbb{Z}$
   (b) $\mathbb{R} \subseteq \mathbb{Z}$
   (c) $\emptyset \subseteq \{1, 2, 3\}$
   (d) $0 \in \mathbb{N}$
   (e) $0 \in \mathbb{R}$
   (f) $\{1, 2, 3, 4\} \subseteq \{2, 3, 4\}$

Success means: All parts of items 1 and 2 are correct except for a maximum of two (2) simple errors; and no more than one incorrect response in item 3.

Learning Target SF.2: *I can perform operations on sets (intersection, union, complement, Cartesian product), determine the cardinality of a set, and write the power set of a finite set.*

Let $X = \{a, b, c, d\}$, $Y = \{d, e, f, g\}$, and $Z = \{a, z\}$. The universal set for these is $U = \{a, b, c, d, \ldots, y, z\}$. Find each of the following. You do not need to show work, but do show it if it helps you; and your answers must be correct. **Use correct set notation on each answer.**

1. $Z \cup Y$
2. $X \cap Y$
3. $Z \setminus X$
4. $X \triangle Y$
5. $(X \cup Y) \cap Z$
6. $|X \times Y|$
7. $\mathcal{P}(Z)$

Success means: The final two items are correct; and all other answers are correct except for a maximum of two (2) simple errors.

Learning Target SF.3 (CORE): *I can determine whether or not a given relation is a function; determine the domain, range, and codomain of a function.*

Below are three mappings. For each one, state whether the mapping is a function. If a mapping is not a function, give a specific and clearly-stated explanation. Otherwise if a mapping is a function, state the domain, codomain, and range (no explanation needed).

1. $f : \{1, 2, 3\} \to \mathbb{N}$ given by $f(1) = 0$, $f(2) = 10$, $f(3) = 100$
2. The mapping $g : \{1, 2, 3, 4\} \to \{x, y, z, t\}$ given by this matrix: $\begin{pmatrix} 1 & 2 & 3 & 4 \\ z & t & x & \text{(nothing)} \end{pmatrix}$
3. The mapping h from the set of all possible English words to the set of letters in the alphabet $\{A, B, C, D, \ldots, Z\}$ given by the following rule: Take a word, and map it to the first letter of the word. (Ignore lower case letters — just pretend everything is capitalized.)

Success means: All answers are correct and all explanations are clear and correct.

Learning Target SF.4: *I can determine whether a function is injective, surjective, or bijective.*

Below are three functions. **For each, state whether the function is injective, then state whether it is surjective, then state whether it is bijective.** So you should be stating three things for each function. If a function fails to have one or more of these properties, give a SPECIFIC EXAMPLE that shows why.

1. $f : \{1, 2, 3, 4, 5\} \to \{0, 1, 2, 3, 4\}$ given by $f(a) = \lfloor a/2 \rfloor + 1$ where $\lfloor x \rfloor$ is the floor function.
2. g is the function from the set of all 4-bit binary strings to the set $\{0, 1, 2, 3, 4\}$, defined by the following rule: Given a binary string for input, add up the bits (in base 10). For example $g(1011) = 3$.
3. $h : \mathbb{Z} \to \mathbb{Z}$ defined by $h(n) = n + 1$

Success means: Each function is correctly identified as injective or not, surjective or not, and bijective or not; and each failure of a property is explained with a specific counterexample.

Learning Target SF.5: *I can evaluate special computer science functions: floor, ceiling, factorial, DIV, and MOD (%).*

Notation reminder: $\lceil x \rceil$ is the ceiling of x; $\lfloor x \rfloor$ is the floor of x. Also $DIV(a, b)$ means the same as the Python expression `a // b`.

State the values of the following. **Give a brief explanation for each of these**.

1. $\lfloor 1.1 \rfloor$
2. $\lfloor -1.1 \rfloor$
3. $\lceil 1.1 \rceil$
4. $\lceil -1.1 \rceil$
5. 6!
6. 0!
7. $DIV(50, 20)$
8. $50 \,\%\, 7$
9. $6 \,\%\, 7$

Success means: All of the answers are correct (no mistakes allowed on this one!) and each item has a brief explanation attached.

Learning Target C.1 (CORE): *I can use the additive and multiplicative principles and the Principle of Inclusion and Exclusion to formulate and solve counting problems.*

Solve each problem below. Show your work or explain your reasoning on each of these, and clearly indicate the numerical answer on each (don't just plug into a formula and leave it).

1. How many 16-bit binary strings have either `10` in the leftmost two bits, or `0` in the rightmost bit?
2. Your wardrobe consists of 5 shirts, 3 pairs of pants, and 17 bow ties. How many different outfits can you make (out of these articles of clothing)?

Success means: Both answers are correct (although up to two simple errors are allowed) and justified with a brief and clear explanation. **Answers without a brief and clear verbal explanation are not considered successful demonstrations**.

Learning Target C.2 (CORE): *I can calculate a binomial coefficient and correctly apply the binomial coefficient to formulate and solve counting problems.*

Solve each problem below. Show your work or explain your reasoning on each of these, and clearly indicate the numerical answer on each (don't just plug into a formula and leave it).

1. You have 17 bow ties in your wardrobe. These are all distinct (no two are alike). You're going on a trip and need to pack three of them. How many different choices are possible?
2. How many ways are there to flip a (fair) coin 10 times and get exactly 7 "heads"?

Success means: All answers are correct except for a maximum of two (2) simple errors, and each answer justified with a brief and clear explanation. **Answers without explanations, or vice versa, are not considered successful demonstrations of skill**.

Learning Target C.3: *I can count the number of permutations of a group of objects and the number of k-permutations from a set of n objects.*

Solve each problem below. Show your work or explain your reasoning on each of these, and clearly indicate the numerical answer on each (don't just plug into a formula and leave it).

1. How many ways are there to award first, second, and third place to a group of runners in a race, if there are 50 runners?
2. In this race, there is a group of 11 runners who are on a team. The team wants to line up for a group photo. How many ways are there to arrange the team if they are lined up in a single row?

Success means: All answers are correct except for a maximum of two (2) simple errors, and each answer justified with a brief and clear explanation. **Answers without explanations, or vice versa, are not considered successful demonstrations of skill.**

Learning Target C.4: *I can use the "stars and bars" method to count the number of ways to distribute objects among a group.*

Solve each problem below. Show your work or explain your reasoning on each of these, and clearly indicate the numerical answer on each (don't just plug into a formula and leave it).

1. How many natural number solutions are there to the equation $x + y + z = 20$?
2. How many solutions are there to the equation $x + y + z = 20$, if the value of each variable must be greater than or equal to 3?

Success means: All answers are correct except for a maximum of two (2) simple errors, and each answer justified with a brief and clear explanation. **Answers without explanations, or vice versa, are not considered successful demonstrations of skill.**

Learning Target RI.1 (CORE): *I can generate several values in a sequence defined using a closed-form expression or using recursion.*

List the first six (6) terms of each of the following sequences. **Show your work on at least two calculations (not including initial conditions) for each sequence.**

1. $a_n = 2n + 10,$ where $n = 1, 2, 3, \ldots$
2. $b_n = 2(0.1)^n$ where $n = 0, 1, 2, \ldots$
3. $c_0 = 1,$ and $c_n = c_{n-1} + n^2$ if $n > 0$
4. $d_0 = 2, d_1 = 3$ and $d_n = d_{n-1} + 3d_{n-2}$ if $n > 1$

Success means: At least three of the four sequences have all six terms correctly listed and work is shown on at least two calculations (not including initial conditions) for each sequence.

Learning Target RI.2: *I can use sigma notation to rewrite a sum and determine the sum of an expression given in sigma notation.*

Either compute the value of the sum given in sigma notation, or write sigma notation that correctly represents the sum.

1. Compute both sums below. **Show all your work on both.**
   (a) $\displaystyle\sum_{n=1}^{6}(2 + 4n)$
   (b) $\displaystyle\sum_{n=2}^{3}(2(10)^n)$
2. For each sum below, write the sum correctly using sigma notation:
   (a) $1 + 5 + 9 + 13 + \cdots + 397$
   (b) $4 + 6 + 9 + 13.5 + 20.25 + 30.375$

Success means: At least three of the four answers are completely correct, and all work is shown on item 1.

Learning Target RI.3: *I can find closed-form and recursive expressions for arithmetic and geometric sequences.*

For each sequence below, find *both* a closed formula *and* a complete recursive definition for the sequence. You do not need to show your work, but your results must be correct.

1. $4,\ 6,\ 9,\ 13.5,\ 20.25,\ \ldots$
2. $1, 5, 9, 13, 17, \ldots$
3. $2, 5, 8, 11, 14, \ldots$
4. $4, 8, 16, 32, 64, 128, 256, \ldots$

Success means: At least three of the four answers are completely correct.

Learning Target RI.4: *I can determine a recurrence relation for a given recursive sequence and check whether a proposed solution to a recurrence relation is valid.*

Consider the recurrence relation given by $a_0 = 4$, $a_1 = 1$ and for $n > 1$, $a_n = 2a_{n-1} - a_{n-2}$. Determine whether the function $f(n) = 4 - 3n$ is or is not a solution to this recurrence relation. If it is not a solution, give an instance of a sequence term where the function differs from the recurrence relation. If it *is* a solution, show all steps that prove this (as in the examples done on video and in class).

Success means: If the function is not a solution, a specific counterexample is correctly and clearly stated. If the function is a solution, all steps in the checking process are shown and correct.

Learning Target RI.5: *I can solve a second-order linear homogeneous recurrence relation using the characteristic root method.*

Solve the recurrence relation given by $a_0 = 3, a_1 = -3$, and $a_n = -6a_{n-1} - 9a_{n-2}$ for $n > 1$.

Success means: A correct and complete function is given, and all significant work is shown clearly. Up to two (2) simple errors are allowed but the solution still must result in a complete function (i.e. you can't stop mid-solution if you make an error).

Learning Target RI.6 (CORE): *Given a statement to be proven by mathematical induction, I can state and prove the base case, state the inductive hypothesis, and outline the proof.*

Consider the statement: For any positive integer n,

$$1 + 3 + 5 + 7 + \cdots + (2n - 1) = n^2$$

Write out a complete, correct framework for a proof by mathematical induction for this statement.

Success means:

1. The base case has been clearly and correctly stated; and a complete, correct, and clearly-written argument has been given that proves the base case.
2. The induction hypothesis is clearly and correctly stated.
3. Following the induction hypothesis, there is a clear and correct statement of what needs to be proven; and there is at least one suggestion given for the first step in a proof that is reasonably likely to be useful in proving the statement.