# MTH 325 Fall 2024 – Exam 2, Skills 5—8

<u>Instructions</u>: This packet contains problems for the newly acquired Skills 5—8. **If you need to retake Skills 1, 2, 3 or 4 you can find those printed separately**. Complete your work in the spaces provided below each set of Skill questions. Reminder: unless otherwise stated, you are required to provide not only correct answers but clear explanations for each response. When you are done, place your name at the top of each paper you turn in, then turn in all the skills you are attempting (including the earlier skills you are reattempting).

A solution guide for this Exam will be posted by early next week; watch your announcements.

Skill 5: I can use a greedy algorithm to find a vertex coloring for a graph, and I can determine a graph's chromatic number.

Consider the graph below:
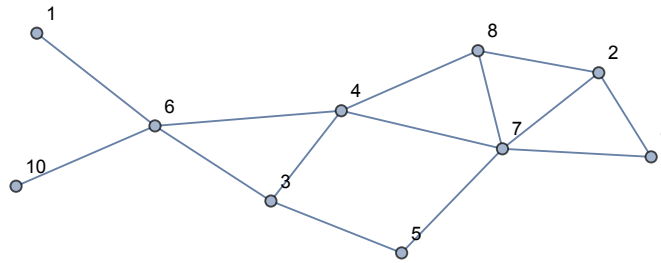
1. Use a greedy algorithm to construct a proper vertex coloring for this graph. For the initial ordering of the vertices, use the degree of the vertices from highest degree to lowest, and use numerical ordering in the case of a tie. Your work should consist of a list of vertices in the order in which they are considered; and the color assigned to each one, given as a non-negative integer.
2. State the chromatic number of this graph and explain your reasoning.

Given the two graphs below, state whether they are isomorphic. If they are isomorphic, give an explicit isomorphism between the two and explain why your mapping is really an isomorphism. If they are not isomorphic, give an explicit isomorphism invariant property that one has but the other does not have.

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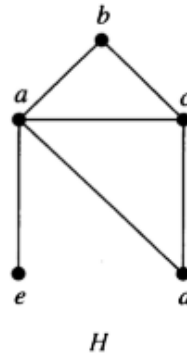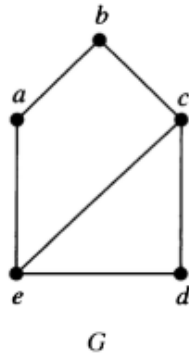
Skill 7 (CORE): I can determine whether a graph is a tree and state information about it.

1. Draw (or give a Python dictionary for) a graph that has degree sequence 2, 2, 2, 1, 1 that is *not* a tree. If no such example is possible, explain why.
2. How many vertices does a tree with 100 edges have? State your answer; if there is not enough information to arrive at an answer, say so. No explanation needed either way.
3. Consider the rooted tree shown below, with vertex 6 as the root. State the following. (No explanation needed but be sure to label your answers.)
   a. The children of vertex 9
   b. The parent(s) of vertex 3
   c. The height of the tree
   d. The leaves of the tree
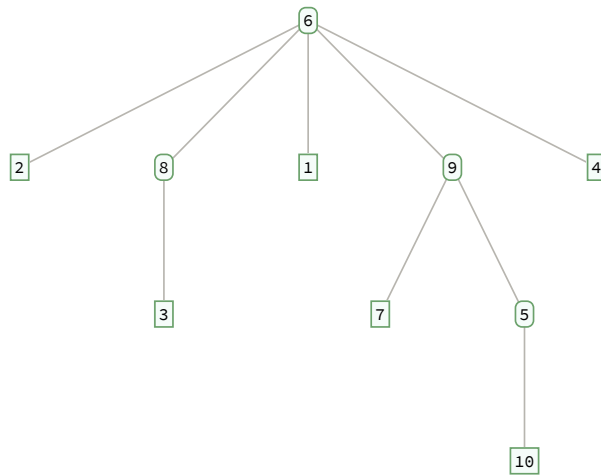
Skill 8: I can use Prim's Algorithm and Kruskal's Algorithm to construct a minimum spanning tree for a weighted graph.

Consider the weighted graph below:
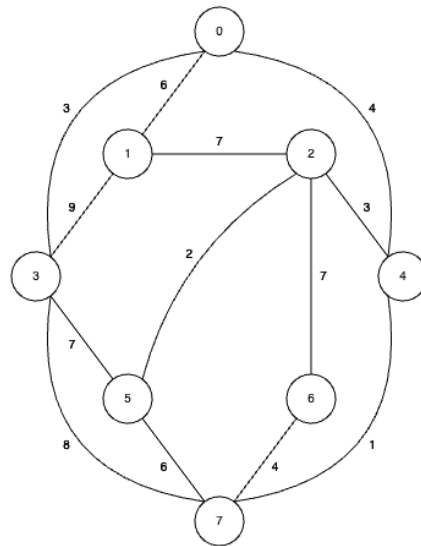
1. Use Prim's algorithm, with initial vertex 3, to construct a minimum spanning tree for this graph. Your work should consist of a list of edges in the tree, given in the order in which they are added.
2. Use Kruskal's algorithm to construct a minimum spanning tree for this graph. Your work should consist of a list of edges in the tree, given in the order in which they are added.