MTH 225 Final Exam Fall 2023 -- Part "A"

Instructions

- Please do all work on this handout. Do not turn in any extra paper.
- You are allowed to use a basic hand calculator on this portion of the exam but no notes and no other forms of technology without prior approval.
- Unless explicitly stated otherwise, you are being graded primarily on your reasoning and only partially on the correctness of your answers. Be sure to explain everything unless the problem says no explanation is needed.

Section 1: Multiple Choice (30 points)

On each item below, circle the ONE answer that you think is most correct. Correct responses earn 2 points each; incorrect responses earn 0 points each. You do not need to explain your reasoning on this section.

1. If an integer represented in binary (base 2) has a `0` in the rightmost bit, then

   (a) The integer is even

   (b) The integer is odd

   (c) The integer is a multiple of 4

   (d) The integer is a prime number

   (e) None of these

2. The number of rows (excluding the header row) in a truth table for the proposition $(p \wedge q) \to r$ is

   (a) 4

   (b) 6

   (c) 8

   (d) 9

   (e) None of these

3. The converse of the statement "I will eat turkey if it's Thanksgiving" is

   (a) If it's Thanksgiving then I will eat turkey

   (b) If I eat turkey, then it's Thanksgiving

   (c) If I don't eat turkey then it's not Thanksgiving

   (d) It's Thanksgiving but I did not eat turkey

(e) None of these

4. The negation of the statement "All electric cars are expensive" is

   (a) All electric cars are inexpensive

   (b) No electric cars are expensive

   (c) Some electric cars are expensive

   (d) Some electric cars are inexpensive

   (e) None of these

5. A conditional statement and its contrapositive are

   (a) Exactly the same sentence

   (b) Logically equivalent but not exactly the same sentence

   (c) Negations of each other

   (d) Sometimes logically equivalent but not always

   (e) None of these

6. Consider the predicate "$x \,\%\, 5 = 3$" where % is the modulus function. The domain of the predicate is the set of all natural numbers. The truth set of this predicate is

   (a) $\{0,1,2,3, \dots\}$

   (b) $\{3,6,9,12, \dots\}$

   (c) $\{5,10,15,20, \dots\}$

   (d) $\{3,13,23,33, \dots\}$

   (e) None of these

7. The set of all integers $\{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$ is represented by the symbol

   (a) $\mathbb{I}$

   (b) $\mathbb{N}$

   (c) $\mathbb{R}$

   (d) $\mathbb{Z}$

   (e) None of these

8. The cardinality of the set $\{x \in \{1,2,3, \ldots ,20\} :\ x \,\%\, 4 = 0\}$ is

(a) 5

(b) 20

(c) Infinite

(d) Does not apply since this is incorrect set syntax

(e) None of these

9. Which of the following are true statements? (Select the ONE answer that is most correct.)

(a) $\emptyset \in \{-1,0,5,7\}$

(b) $\emptyset \subseteq \{-1,0,5,7\}$

(c) $\emptyset \subseteq \emptyset$

(d) All of these

(e) Both (b) and (c) but not (a)

10. The set $\{x \,\%\, 4 :\ x \in \mathbb{N}\}$ is...

(a) Equal to $\{0,1,2,3\}$

(b) Equal to $\{0, 0.25, 0.75, 1, 1.25, 1.5, 1.75, 2, \ldots\}$

(c) Equal to {True,False}

(d) Written in correct syntax but not equal to any of the above

(e) Not written in correct set syntax

11. Consider the function $f: \mathbb{Z} \to \mathbb{N}$ given by the formula $f(a) = a^2$. The range of this function is

(a) $\mathbb{Z}$

(b) $\mathbb{N}$

(c) $\{0,1,4,9,16,25, \ldots\}$

(d) $\{1,2,4,8,16,32, \ldots\}$

(e) None of these

12. The function $f: \mathbb{R} \to \mathbb{R}$ given by the formula $f(x) = 3x + 2$ is

(a) Injective but not surjective

(b) Surjective but not injective

(c) Neither injective nor surjective

(d) Bijective

(e) None of these

13. The number of subsets of $\{A, B, C, D, E, F\}$ that contain the letter "A" and which have four elements in all is

(a) 15

(b) 20

(c) 120

(d) 216

(e) None of these

14. The number of bitstrings that have length 10 and which contain exactly 3 “1” bits, is equal to

(a) The number of bitstrings that have length 10 and which contain exactly 7 “1” bits

(b) The number of 3-element subsets of $\{0,1,2, \dots ,9\}$

(c) The number of bitstrings that have length 9 and have exactly 3 “1” bits plus the number of bitstrings of length 9 that have exactly 2 “1” bits

(d) All of these

(e) Just (a) and (b)

15. The first two terms of a sequence are 2 and 10. If the sequence were *geometric*, the third term would be

(a) 12

(b) 20

(c) 1024

(d) It's not possible to tell what the third term would be

(e) It *is* possible to tell what the third term would be but it's not any of the above

Section 2: Computation (36 points)

This section asks you to perform some basic computational tasks. Do each one and show all intermediate steps. Each one is worth 6 points.

1. Compute the value of 10!.

2. Compute the value of $\binom{50}{45}$. Give an exact value, not a decimal or scientific notation approximation.

3. A sequence is defined recursively by $a_0 = 3$, $a_1 = 5$, and for $n > 1$, $a_n = 2a_{n-1} + a_{n-2}$. Compute a_5.

4. Write the converse and contrapositive of the statement "If the weather is snowy, then I will wear gloves." Be sure to label which is which.

5. Write a truth table for the statement $(p \vee q) \rightarrow r$. Show all intermediate columns.

6. If $A = \{1,3,5\}$, $B = \{2,3\}$, and $C = \{0,5\}$, find $(A \cup B) \cap C$.

Section 3: Problem Solving (16 points)

Pick **EXACTLY ONE** of the following problems and give a complete, correct, and convincing solution. **DO NOT SUBMIT WORK ON MORE THAN ONE** or this section will be graded 0/16. If needed, use a separate page and attach it to the rest of the exam.

1. How many 8-bit strings are there that:

   (a) Start (on the left) with the string `101`?

   (b) Have weight 5?

   (c) Either start (on the left) with `101` or end (on the right) with `11`?

   (d) Have weight 5 and either start (on the left) with `101` or end (on the right) with `11`?

2. Willy Wonka gives everyone who visits his factory 7 pieces of candy to take home. He never gives a person 2 or more pieces of the same type of candy. If Mr. Wonka has 25 different types of candy, in how many different ways could Mr. Wonka give a visitor his candy?

3. Mike owns 7 different mathematics books and 5 different computer science books and wish to fill 5 positions on a shelf. If the first 3 positions are to be occupied by math books and the last 2 by computer science books, in how many ways can this be done?

4. Prove using mathematical induction that for all integers $n \geq 1$, $1 + 2 + 4 + 8 + \cdots + 2^n = 2^{n+1} - 1$.

5. Prove using mathematical induction that for all integers $n \geq 0$, a set with n elements has 2^n subsets.

Section 4: 18 free points for answering a few questions about the class

I'm turning around in 3 weeks to teach MTH 225 again, so I want to know what worked for you and what could be improved. Your email should have a course announcement in it right now with a link to a Google Form that asks some questions about how the class went.

Between now and noon tomorrow (December 14), go to the link in your email and on the course announcement and fill out the survey. Doing so will earn you the last 18 points on this final exam. Thank you in advance for your feedback.