MTH 225: Checkpoint 7

This Checkpoint contains new problems for Core skills C1-C6, and for Supplemental skills S1-S8.

- Do *each* problem on a *separate* page. Do not put more than one Skill on a page.
- T**urn in your work on the Core Skills before leaving the class session**. Please ensure your name is on each page.
- Turn in your work on the **Supplemental** Skills by converting your work to a PDF file and uploading to the appropriate assignment area on Blackboard, before **12:00pm noon ET on Thursday, March 13**.
- You may use a simple graphing or scientific calculator; a calculator app on a smartphone is acceptable but your phone must be in airplane mode.
- No collaboration is allowed with any person or software.
- Be sure to read the Success Criteria below each problem to know what is required for "Success".

Skill C1

> I can identify the hypothesis and conclusion of a conditional statement and state its converse, contrapositive, inverse, and negation.

Consider the following conditional statement:

"If it is raining outside, then I will bring an umbrella."

Clearly state the following:

1. The hypothesis of the statement
2. The conclusion of the statement
3. The converse of the statement
4. The contrapositive of the statement
5. The inverse of the statement
6. The negation of the statement (Do not just put "Not" or "It is not the case that..." or a similar phrase in front of the statement; rewrite it as a new proposition)

Success criteria: All six items are correct and clearly stated.

Skill C2

> Given the statement of a proposition to be proven with mathematical induction, state the complete induction proof framework.

Consider the following proposition, and suppose we want to prove it with mathematical induction:

For every integer $n \geq 5$, $4n < 2^n$.

1. Write out the predicate involved in this statement: $P(n)$ says...

2. What value of n corresponds to the base case?
3. Prove that the base case holds, by direct computation or demonstration.
4. Clearly state the inductive hypothesis without using the notation "$P(n)$".
5. Clearly state what needs to be proven in the inductive step. Do not use the notation "$P(n)$" . You do NOT have to actually prove this statement; just state it.

Success criteria: The predicate is correctly identified and has no quantifiers. The base case is correctly stated and has a complete and correct proof. The inductive hypothesis and proof step are correctly stated in specific terms.

Skill C3

> I can use set-theoretic notation correctly and convert a set from set-builder notation to roster notation and vice versa.

1. Below are several statements about sets and elements. Label each one as **TRUE** or **FALSE**.

   a) $2 \subseteq \{0, 1, 2, 3, 4\}$

   b) $25 \in \{n^2 : n \in \{0, 1, 2, 3, 4, \ldots\}\}$

   c) $-1 \in \mathbb{Z}$

   d) $\mathbb{N} \subseteq \mathbb{Z}$

   e) $\emptyset \subseteq \mathbb{N}$

2. Here are three sets written in set-builder notation. For each, if the set is written using correct set syntax, rewrite it using roster notation. If the set is written using incorrect syntax, write **INCORRECT SYNTAX**.

   a) $\{n/2 : n \in \{3, 4, 5\}\}$

   b) $\{n \in \{8, 9, 10, 11\} : n^2 \leq 100\}$

   c) $\{x \in \{1, 2, 3, \ldots 10\} : x \text{ is even}\}$

Success criteria: All items involving element, subset, and equality determinations are correct. All sets using incorrect set-builder syntax are correctly identified. Up to two simple mistakes are allowed in rewriting sets into roster notation.

Skill C4

> I can find the intersection, union, difference, symmetric difference, power set, cardinality, cartesian product, and complement of sets.

Let $A = \{1, 3, 5\}, B = \{2, 3\}, C = \{0, 5\}$, and $D = \{7\}$. Find each of the following. You do not need to show intermediate steps, but this is often helpful for you (and for the person grading your work).

1. $A \cap B$
2. $A \cup B$
3. $B \setminus A$
4. $A \cup (B \cap C)$
5. $(A \cup B) \cap C$

6. $|A \times C|$

Success criteria: All responses are correct up to two simple errors.

Skill C5

> I can apply the Additive and Multiplicative Principles and the Principle of Inclusion/Exclusion to formulate and solve basic combinatorics problems.

1. A state in the USA has license plates that are made up of three letters followed by three more characters, each of which can be either a letter (A..Z) or a single-digit number (0..9). Examples include "YYZ 123" and "ABC Z82". How many license plates are possible in this system?
2. Using the system of license plates in item 1, how many license plates are there that either start with a vowel (A, E, I, O, or U) or end in an even number (0, 2, 4, 6, or 8)?

Success criteria: On each part, the setup and subsequent work is shown; and the answers are correct with up to two simple errors allowed.

Skill C6

> I can compute factorials and binomial coefficients, and apply these concepts to solve basic combinatorics problems involving permutations, selections, and distributions.

1. Compute the values of each of the following, and show your steps:

   (a) $9!$

   (b) $\binom{199}{197}$
2. How many subsets of the set $\{0, 1, 2, 3, 4, 5\}$ have exactly three (3) elements? An example would be $\{1, 3, 4\}$.

Success criteria: The factorial and binomial coefficient are correct. In the counting problems, the setup and intermediate work is shown, and the answers are correct with up to two simple errors allowed.

Skill S1

> I can convert a positive integer from base 10 to base 2, 8, and 16 and vice versa and represent a negative integer in base 2 using two's complement notation.

1. Convert $(11B)_{16}$ to base 10.
2. Convert $(1600)_{10}$ to base 16.

3. Convert $(88)_{10}$ base 2.
4. The binary form of the base 10 integer 34 is `0010 0010`. Find the binary form of -34 using two's complement notation. Show all steps and assume an 8-bit system.

Success criteria: All, or all but one, of the base conversions must be correct, and conversions from base 10 must involve the base conversion algorithm. The computation of the negative integer using two's complement must be correct and show all the intermediate steps.

Skill S2

> I can add, subtract, multiply, and divide positive integers in base 2.

1. Compute `111001` + `100111` without first converting to base 10.
2. Compute `111001` - `100111` without first converting to base 10.
3. Compute `111001` $\times$ `11` without first converting to base 10.
4. Compute `111001` $\div$ `10` without first converting to base 10.

Success criteria: At least three of the four answers must be correct with intermediate steps shown.

Skill S3

> I can construct truth tables for propositions involving two or three atomic propositions and use truth tables to determine if two propositions are logically equivalent.

1. Construct a truth table for $(p \vee (\neg q)) \rightarrow r\,.$.
2. Construct truth tables for $\neg(p \wedge q)$ and for $(\neg p) \vee q$. Based on the truth tables, are these two statements logically equivalent?

Success criteria: All truth tables have the correct number of rows with no duplicated rows. All intermediate columns are shown. No more than three total errors are permitted. (If you make a mistake in an intermediate column but the rest of the row is correct given that mistake, then the mistake only counts once.) The answer about whether the last two statements are logically equivalent is clearly indicated and is consistent with the truth table results.

Skill S4

> I can determine the truth value of a predicate at a specific input, the truth value of a quantified predicate, and the negation of a quantified predicate.

1. Let P and Q be these two predicates. Assume the domain of each is the set of all natural numbers: $0, 1, 2, 3, 4, \ldots$
   - $P(x)$: $x^2 = 100$
   - $Q(x)$: x^2 ends in a "3"   (Example of ending in a 3: the number 19995473)

   For each of the following, state whether the expression is **TRUE** or whether it is **FALSE**.

(a) $P(10)$

(b) $Q(13)$

(c) $\forall x P(x)$

(d) $\exists x(\neg Q(x))$

2. Consider the statement: *Every problem on this quiz has the same answer.* State the negation of this statement, without merely putting the word "not" or the phrase "it is not the case that" in front of the statement; that is, phrase the negation in terms of another quantified statement.

Success criteria: At least three of the answers in the first part are correct; and the negation in the second part is correct and clearly stated. (No intermediate work is required.)

Skill S5

> I can determine elements of a recursively-defined sequence using a recurrence relation.

Consider the sequence of integers a_n given by the following recursive definition: $a_0 = 2$, $a_1 = 3$,and for $n \geq 2$ the terms are given by the recurrence relation $a_n = 2a_{n-1} + a_{n-2}$. Find the values of a_2 through a_6 and show your work on each step. (Note, you are already given a_0 and a_1.)

Success criteria: The first three terms to be computed are correct and the intermediate steps are clearly shown. Of the remaining terms all of them, or all but one, are correct.

Skill S6

> I can derive a recurrence relation for a recursive visual pattern, number sequence, or other sequence of objects.

Consider the following visual pattern:

Let $R(n)$ be the number of blocks in each step of the pattern, with $n = 1$ representing the very first figure in the pattern.

1. State the value of $R(1)$. $R(2)$. and $R(3)$. .
2. Give a correct recurrence relation for $R(n)$, the number of objects in step n, and explain your reasoning clearly but briefly using the visual pattern only. (That is, do not simply find $R(1)$, $R(2)$, and $R(3)$ by counting blocks and then guessing at a numerical relationship – **your reasoning must be based on the visual pattern**.)

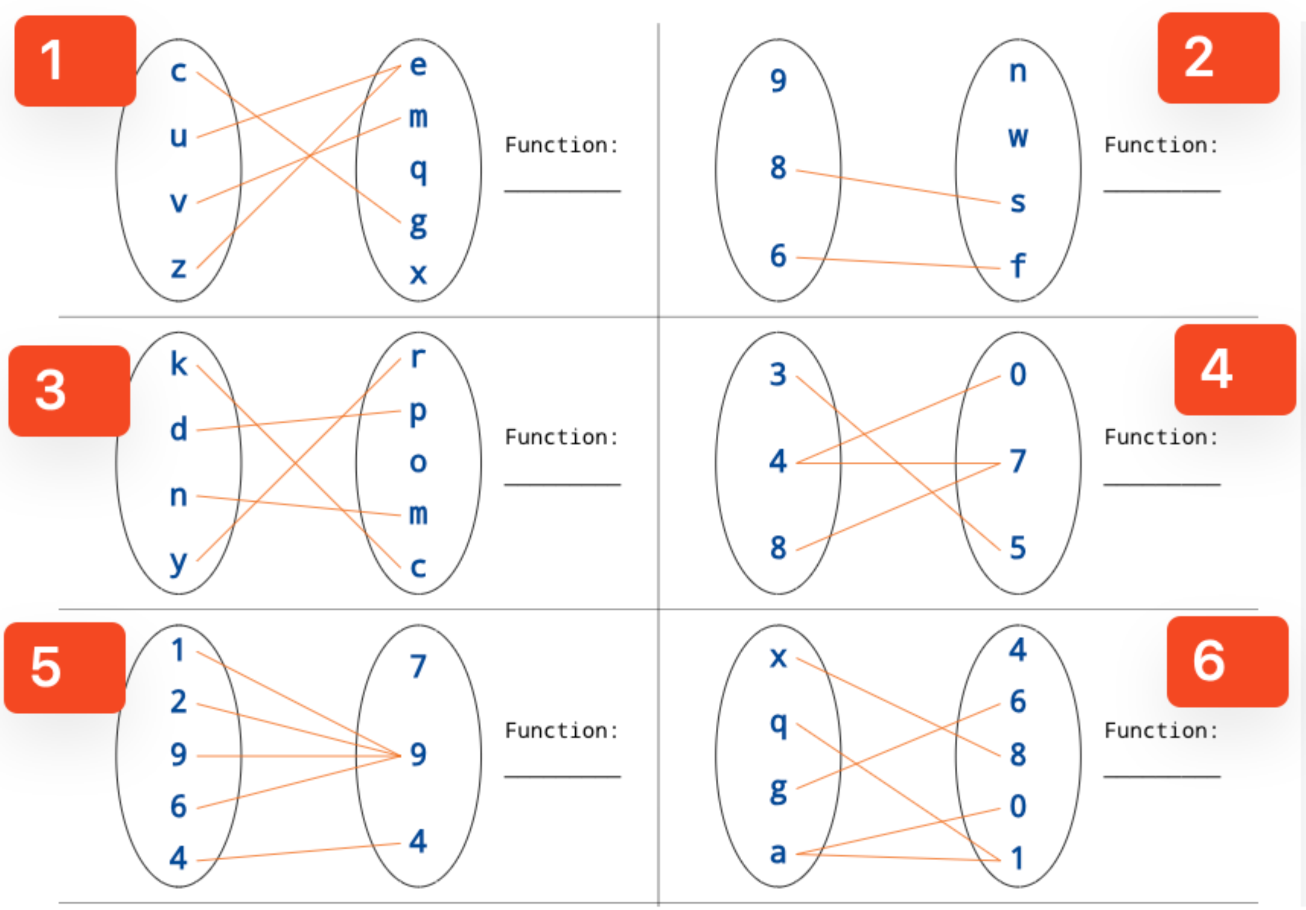
Success criteria: The values of R are stated correctly in the first part. The recurrence relation is a recurrence relation and not a closed formula; and it is both correct and clearly explained in a way that does not merely use the list of numbers $R(1), R(2), R(3)$.

Skill S7

> I can determine if a mapping is a function; identify the domain, range, and codomain of a function; and determine the image of a specific input in one function or a composition of functions.

Below are six mappings. For each one:

- State whether the mapping is a function or whether it is not a function.
- If the mapping is a function, state its domain, range, and codomain. Phrase each one as a set using correct notation and syntax and make sure to label which one is which.



Success criteria: All mappings are correctly labeled as "function" or "not a function". All mappings that are functions have the domain, codomain, and range listed with up to two misakes allowed. You do not need to justify your reasoning (although having reasoning present might help you).

Skill S8

> I can determine if a function is injective, surjective, or bijective.

For each of the functions below, state whether the function is *injective but not surjective*, *surjective but not injective*, *neither injective nor surjective*, or *bjective*. (Note that every function must fall into exactly one of these categories.) If a function **DOES HAVE** a property, you **DO NOT** have to explain why; just state it. (For example "F is injective".) If a function **DOES NOT HAVE** a property (for example if the function is not surjective) then **give a specific, concrete example where the property fails**.

1. The function $f : \mathbb{R} \to \mathbb{R}$ given by the formula $f(x) = 3x + 2$. (Reminder, $\mathbb{R}$ is the set of all real numbers.)
2. The function $g : \mathbb{N} \to \mathbb{N}$ given by the formula $g(n) = 3n + 2$. (Reminder, $\mathbb{N} = \{0, 1, 2, 3, 4, \ldots\}$)
3. The function $h : \{1, 2, 3, 4, 5\} \to \{a, b, c\}$ given as follows: $h(1) = b$, $h(2) = b$, $h(3) = b$, $h(4) = b$, and $h(5) = c$.

Success criteria: At least two of the three functions are correctly labeled. And, all instances of failure to have a property are accompanied by a correct, specific, concrete example that shows the property fails. **General explanations for why a property fails are not acceptable; you must give a concrete, specific counterexample.**