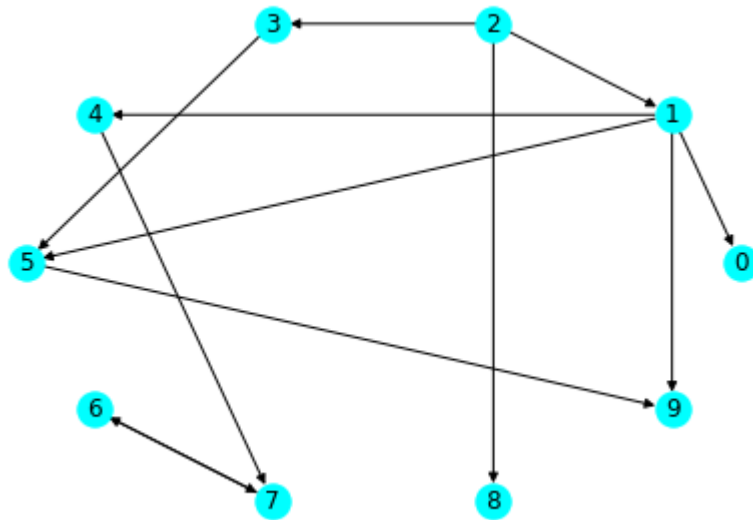


**DR.1: I can determine information about a directed graph and its individual vertices and edges using different representations.**

Using the directed graph below: Label the in- and out-degrees of each vertex. Then write this digraph as an edge list and as an adjacency matrix.



Additional question: What if there were self-loops?

**DR.2: I can give examples of relations on a set that have combinations of the properties of reflexivity, symmetry, antisymmetry, and transitivity.**

**DR.3: I can determine if a relation is an equivalence relation; I can determine the equivalence class of an element under an equivalence relation and determine whether two elements belong to the same equivalence class.**

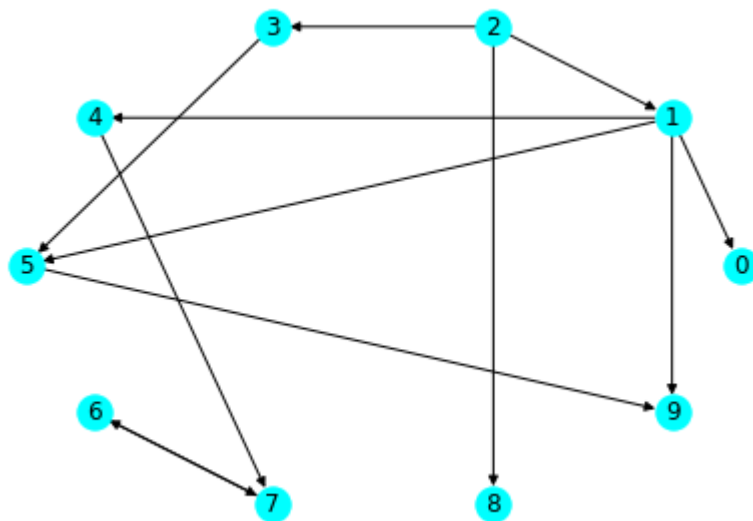
- There are  $2^4 = 16$  ways for a relation to have the possible combinations of the reflexive, symmetric, antisymmetric, and transitive properties. Give an example of each, using the set  $\{0, 1, 2, 3\}$ . (Yes, they are all possible.)
- Decide if each of the following relations on  $\mathbb{N} = \{0, 1, 2, 3, \dots\}$  is an equivalence relation. If it is, pick a number and write out all the elements in its equivalence class.
  - $a \sim b$  if and only if  $a$  divides  $b$
  - $a \sim b$  if and only if  $a$  and  $b$  have the same number of 1 bits in their binary representation
  - $a \sim b$  if and only if  $ab \leq 10$
  - $a \sim b$  if and only if  $a^2 = b^2$

**DR.4: I can find the nth order composition of a relation with itself.**

**DR.5: I can sketch the transitive closure of a relation as a directed graph.**

Given the relation  $r$  below:

- Draw, then write both the edge list and adjacency matrix, for  $r^2$ .
- Draw, then write both the edge list and adjacency matrix, for  $r^3$ .
- Draw the directed graph for its transitive closure.



**DR.6: I can determine when a set with a relation is a partially ordered set; I can draw the Hasse diagram of a poset and identify maximal/minimal elements and/or greatest/least elements, if they exist.**

Generate a random list of integers between 1 and 100 using this website: <https://www.random.org/integers/>

Put a relation on this list by saying  $a \sim b$  if  $a$  divides  $b$ .

The set of numbers with this relation is a partially ordered set. Draw its Hasse diagram, then identify the maximal, minimal, greatest, and least elements if they exist.