Skill 1: (CORE) I can outline a proof by mathematical induction.

Consider the following proposition: For every integer $n \geq 1$, $10^{2n-1} + 1$ is divisible by 11. (Note, this is 10 raised to the $2n$-1 power, plus 1. The exponent on 10 is $2n$-1; the additional +1 is not in the exponent.)

1. State the value of n that corresponds to the base case, then prove that the base case holds.
2. Clearly state the inductive hypothesis. Your answer should be phrased as a complete sentence. (No explanation is required here; simply state the inductive hypothesis.)
3. Clearly state what you would need to prove, after assuming the inductive hypothesis. Your answer should be phrased as a complete sentence. (You do not need to give a completed proof the statement; simply state what you would need to prove.)

Skill 2: (CORE) I can outline a proof using direct, contrapositive, and indirect approaches.

Consider the following proposition: Suppose that G is a graph. **If the degree of each vertex of G is even, then G has an Euler circuit.**

1. Clearly state what you would assume and what you would need to prove, if you were to prove this statement with a *direct proof*. (No further explanation is necessary.)
2. Clearly state what you would assume and what you would need to prove, if you were to prove this statement with a *proof by contrapositive*. (No further explanation is necessary.)
3. Clearly state all assumptions you would make, if you were to prove this statement with a *proof by contradiction* (also known as an *indirect proof*). (No further explanation is necessary.)

You are not being asked to give completed proofs here. Simply state the frameworks for each type of proof.

Skill 3 (CORE) I can represent a graph in different ways, determine information (degree, degree sequence, paths of given length, etc.) about a graph using different representations, and give examples of graphs with specified properties.

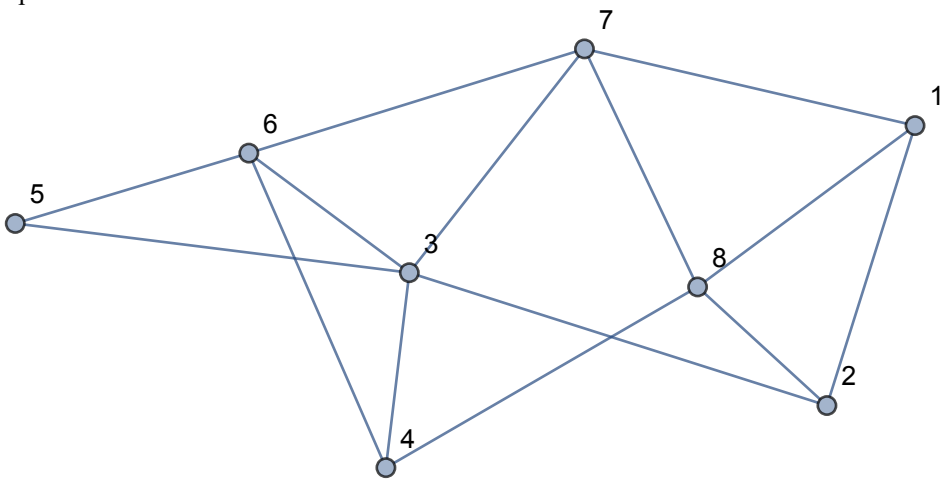
Consider the graph G given by this adjacency matrix:

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \end{bmatrix}$$

1. Assume that the vertices are labeled *a, b, c, d, e, f* and the rows and columns of this matrix correspond to that ordering (so the first row and column represent vertex *a*, the second represent vertex *b* and so on). State the Python dictionary for this graph.
2. State the degree of each vertex. You don't need to explain your answers here, just make sure they are right.
3. Give the number of edges in the graph and show your work or explain your reasoning.
4. Give an example of a cycle of length 4 in this graph. If no such cycle exists, say so and explain your reasoning.

Skill 4: I can determine whether a graph has an Euler trail or Euler circuit, and whether a graph has a Hamiltonian path or circuit.

Consider the graph *G* shown below:



1. Determine whether this graph has an Euler trail, and explain how you know.
2. Determine whether this graph has an Euler circuit, and explain how you know.
3. Determine whether this graph has a Hamilton path, and explain how you know.
4. Determine whether this graph has a Hamilton circuit, and explain how you know.