# Summary and review of counting techniques

MTH 225 -- Module 9B 6 November 2020

#### What we've learned about counting

- There are different techniques for different situations
- Very different situations can look incredibly similar -- careful thought needed!
- Techniques: Multiplicative Principle; Additive Principle/Principle of Inclusion and Exclusion; Binomial Coefficients; Combinations and Permutations; Stars and Bars
- Many counting situations mix and match these rules --- usually not just one tool for the job

In each of these, choose the method that is most likely to be used.

A university committee needs to choose someone from the Math Department -- either a faculty member or a math major -- to be on it. There are 30 faculty available and 300 math majors (and no overlap). We want to count the number of possible choices.

Multiplicative Rule

Additive Rule/PIE

Binomial coefficient

Permutation/combination



The committee decides to select just from among the math faculty, but it also decides it needs two faculty members.

To count: The number of ways to select two faculty members.

Multiplicative Rule

Additive Rule/PIE

Binomial coefficient

Permutation/combination



One of the math majors, angry at having been left off the committee, prints up 50 flyers airing his grievances and wants to distribute them to his MTH 201 class (28 students). To count: The number of ways to distribute the flyers.

Multiplicative Rule

Additive Rule/PIE

Binomial coefficient

Permutation/combination



The math majors, agitated by the flyers, decide to protest. So they organize into a group, and need to pick a president, vice president, and secretary. To count: The number of ways to select these positions from the 300 majors.

Multiplicative Rule

Additive Rule/PIE

Binomial coefficient

Permutation/combination



## Counting cards

### A standard deck of cards has 52 distinct cards in. Shuffling the deck reorders the cards. How many different ways are there to shuffle the deck?

$$1+2+3+\cdots+52=1378$$

$$\binom{52}{5} = 2598960$$

$$P(52,5) = 311875200$$

$$52! \approx 8.07 \times 10^{67}$$

None of the above



The age of the known universe is estimated at 4.32 x 10<sup>17</sup> seconds

If you counted one possible card shuffle every nanosecond, it would take  $10^{58}$  seconds to count them all –  $10^{41}$  times the age of the known universe

https://www.physicsoftheuniverse.com/numbers.html

When you shuffle a deck of cards, the result has very likely never been created before.

#### How many different ways are there to deal a 5-card hand from a standard 52-card deck?

$$1+2+3+\cdots+52=1378$$

$$\binom{52}{5} = 2598960$$

$$P(52,5) = 311875200$$

None of the above



# A group of 4 friends has gotten together to play poker. Poker hands are typically 5 cards each. The dealer shuffles the deck and deals out four 5-card hands. How many different outcomes are there?

#### Join by Web



- 1 Go to PollEv.com
- 2 Enter TALBERT

#### Join by Text



- 1 Text TALBERT to 37607
- 2 Text in your message



#### Summary

- The counting techniques we've learned are tools, each of which is specialized for a particular situation
- Most situations call for a combination of tools.
- But careful examination of the problem first, before applying the tool, can guide our choices and how we apply them.
- So we should do that.