GUIDED PRACTICE: Proof Part 8

Instructions: This is the final lesson in our unit on proof, and it is focused on **evaluation of a written proof**. Below are three propositions with proposed proofs. However, *the proposition may or may not be true*; and even if the proposition is true, *the proof may be incorrect* due to logical, semantic, syntactic, or computational error; and even if the proof is free of errors, *it may not be well-written*.

For each of these propositions and their proposed proofs, do the following:

- Determine if the proposition is true (using methods we discussed in Part 1 of this unit). If the proposition is false, then the proposed proof is, of course, incorrect. In that situation, find the error in the proof and then provide a counterexample showing that the proposition is false.
- If the proposition is true, look at the proposed proof and identify the method (direct proof, proof by contrapositive, proof by contradiction, proof by induction) being used. Then, having analyzed the method being used, read through the proof carefully and determine if the proof is correct — meaning, it's free of any fatal errors in logic, syntax, semantics, or computation. (Note that you have to know the proof method before you can do this.)
- Finally, if the proposition is true and the proof is correct, decide if the proof is well written — meaning it's free of any significant errors of any sort in logic, syntax, semantics, or computation. If the proof is not well written, suggest some improvements that would make it better.

Do your work at this form: http://bit.ly/1WlipkH

We will discuss the results in class and analyze proofs written by other MTH 225 students. Note that a large portion of the Proofs bundle assessments at Level 2 will consist of this sort of activity.

Sample #1 (Note: The notation x|y means "x divides y".)

Proposition. For all integers a, b, and c, if $a \mid (bc)$, then $a \mid b$ or $a \mid c$.

Proof. We assume that a, b, and c are integers and that a divides bc. So, there exists an integer k such that $bc = ka$. We now factor k as $k = mn$, where m and n are integers. We then see that

$$bc = mna.$$

This means that $b = ma$ or $c = na$ and hence, $a \mid b$ or $a \mid c$. ■

Sample #2

For each natural number n with $n \geq 2$, $2^n > 1 + n$.

Proof. We let k be a natural number and assume that $2^k > 1 + k$. Multiplying both sides of this inequality by 2, we see that $2^{k+1} > 2 + 2k$. However, $2 + 2k > 2 + k$ and, hence,

$$2^{k+1} > 1 + (k + 1).$$

By mathematical induction, we conclude that $2^n > 1 + n$. ■

Sample #3

Proposition. For each real number x, if x is irrational and m is an integer, then mx is irrational.

Proof. We assume that x is a real number and is irrational. This means that for all integers a and b with $b \neq 0$, $x \neq \dfrac{a}{b}$. Hence, we may conclude that $mx \neq \dfrac{ma}{b}$ and, therefore, mx is irrational. ■