Weekly Practice 6 key

1. (a) $f = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 5 & 4 & 3 & 2 & 1 & 0 \end{pmatrix}$; range $= S$; bijective

   (b) $g = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 0 & 1 & 2 & 0 \end{pmatrix}$; range $= \{0, 1, 2\}$; neither injective nor surjective

   (c) $h = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 3 & 3 & 3 & 3 & 4 & 5 \end{pmatrix}$; range $= \{3, 4, 5\}$; neither injective nor surjective

   (d) $j = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 0 & 1 & 1 & 1 & 2 & 2 \end{pmatrix}$; range $= \{0, 1, 2\}$; neither injective nor surjective

   (e) $k = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 0 & 1 & 2 & 3 & 0 & 1 \end{pmatrix}$; range $= \{0, 1, 2, 3\}$; neither injective nor surjective

   (f) $n = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 0 & 1 & 4 & 3 & 4 & 1 \end{pmatrix}$; range $= \{0, 1, 3, 4\}$; neither injective nor surjective

2. Note: Where examples exist at all, there are many possible examples. Only a few are given here.

   (a) No examples of any of the first two combinations are possible. If $f : A \to B$ is not surjective, then there would have to be an element of B that has nothing mapping to it, leaving only four points for the domain to map to. But there are five points in the domain, so at least one pair would have to map to the same output. Similarly if f is not injective, then there are two inputs that map to the same output, leaving a maximum of four possible distinct outputs, meaning the function cannot be surjective.

   An example of a function that is neither injective nor surjective would be the function that maps all points in the domain to a.

   (b) There is no injective function from $\mathbb{N}$ to $\{1, 2, 3, 4, 5\}$ in the first place since $\mathbb{N}$ is infinite but the codomain is finite.

   A surjective function that is not injective would be $f(n) = n \,\%\, 6$.

   A function that is neither injective nor surjective would be $f(n) = 1$ (everything maps to 1).

   (c) An injective function $\mathbb{Z} \to \mathbb{R}$ would be $f(n) = n$.

   There is no surjective function $\mathbb{Z} \to \mathbb{R}$. A proof is beyond the scope of the course (although you can read it here: `http://mathonline.wikidot.com/the-set-of-real-numbers-is-uncountable`) , but basically it's because $\mathbb{Z}$ is a "smaller size of infinite" than $\mathbb{R}$.

   Similar to the other parts, to get a function that's neither injective nor surjective, you can usually just map everything to one point, for example $f(n) = 0$.