

MTH 225: Checkpoint 9

Learning Targets on this Checkpoint: 2, 3, 6, 7, 8, 9, 12, 13.

All eight of these are Core (in fact it's the entire Core collection) and so **all work is due in class**.

Instructions:

- Do not attempt any problem for which one or both of the following is true;
 - You **aren't ready to take it** because you need more practice; or
 - You **don't need to take it** because you are already at Level 2 (or at Level 1 and you don't intend to go to Level 2).
- **A basic scientific or graphing calculator is the only technology allowed.** If you are using a smartphone app as your calculator, your phone *must* be switched to airplane mode. The use of any other technology, or phone apps with internet enabled, will be treated as academic dishonesty.
- **You are not allowed any notes** on Checkpoints.
- Make sure to **put your name on each page you are handing in**.
- Note that each problem has *Success Criteria* listed. These are the standards for what constitutes a "successful" demonstration of skill. Make sure to read through those criteria and double check your work against them before turning in your work.

Learning Target 2

(CORE) I can add, subtract, multiply, and divide positive integers in base 2.

1. Add the base-2 integers 11110011 and 10000011 . **Show your work and circle your answer.**
2. Subtract the base-2 integers 11110011 and 10000011 . **Show your work and circle your answer.**
3. Multiply the base-2 integers 1011 and 101 . **Show your work and circle your answer.**
4. Divide the base-2 integer 110110 by 101 . **Show your work and circle your answers.**

Success criteria: All four answers are correct, and the work leading to each answer is clear and legible. **All arithmetic must be done entirely in base 2 without converting to base 10 first.** Up to two "simple errors" (as described in the *Standards for Student Work* document) are allowed.

Learning Target 3

(CORE) I can identify the hypothesis and conclusion of a conditional statement and state its converse, contrapositive, inverse, and negation.

Consider the statement: ***If the store is closed, I will come back tomorrow.*** State, in clear English:

1. The hypothesis of the statement
2. The conclusion of the statement
3. The converse of the statement
4. The contrapositive of the statement
5. The inverse of the statement
6. The negation of the statement (Do not just put "not" or "it is not the case that" in front of the statement)

Success criteria: All six statements are given correctly. (There is no "work" required on this one.)

Learning Target 6

(CORE) I can determine elements of a recursively-defined sequence using a recurrence relation and derive a recurrence relation for a recursively-defined sequence.

1. Consider the sequence of numbers a_n given by the recursive formula $a_0 = 2$, $a_1 = 1$, and for $n > 1$, $a_n = a_{n-1} + a_{n-2}$. Find the values of a_2 , a_3 , a_4 and a_5 . Show your work on each calculation.
2. Consider the sequence of numbers: 3, 8, 13, 18, 23, . . . If $a_0 = 3$, give a recurrence relation that generates the rest of the sequence. (Do *not* give a closed formula, but a recursive definition.)

Success criteria: On the first part, the recursive formula must be used correctly with the correct base cases, and the mathematical work leading to the answers must be shown. On the second part, the recurrence relation must be recursive (not a closed formula) and correctly generate the terms of the sequence. No "work" is required on the second part. No more than two simple errors are allowed total.

Learning Target 7

(CORE) Given a statement to prove using mathematical induction, I can state the framework of a proof. (Identify the predicate, identify and prove the base case, state the inductive hypothesis, and state what needs to be proven in the inductive step)

Suppose we want to prove the following statement using mathematical induction:

For every integer $n \geq 1$, $1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$.

1. Write out the predicate involved in this statement: $P(n)$ says...
2. What value of n corresponds to the base case?
3. Prove that the base case holds, by direct computation or demonstration.
4. Clearly state the inductive hypothesis without using the notation " $P(n)$ ".
5. Clearly state what needs to be proven in the inductive step. Do not use the notation " $P(n)$ ". You do NOT have to actually prove this statement; just state it.

Success criteria: The predicate is correctly identified and does not include a quantifier. The correct value of n for the base case is identified. The proof of the base case is clearly written and is correct. The inductive hypothesis is correctly and clearly stated and does not include " $P(n)$ ". The statement to be proven in the inductive step is correctly and clearly stated and does not include " $P(n)$ ".

Learning Target 8

(CORE) I can convert a set given in roster notation to set-builder notation and vice versa; and determine if an object is an element of the set.

- Here are three sets written in set-builder notation. For each, if the set is written using correct set syntax, rewrite it using roster notation. If the set is written using incorrect syntax, write **INCORRECT SYNTAX**. See the success criteria below for what constitutes correct roster notation.
 - $\{x \in \{10, 11, 12, \dots, 30\} : \text{The digits of } x \text{ add up to } 5\}$
 - $\{x \% 2 : x \in \mathbb{N}\}$
 - $\{x \in \{0, 1, 2, 3, \dots, 10\} : x \% 4\}$
- Rewrite the set $S = \{x, y, z\}$ using set-builder notation. There is more than one way to do this correctly; your answer must equal S and must use correct set notation and syntax. You may not use " $\{x : x \in S\}$ " as an answer.

Success criteria: On the first part, all sets with incorrect syntax must be indicated; sets with correct syntax must:

- Have ALL its elements listed if there are 4 or fewer elements;
- Have AT LEAST FOUR elements listed if there are 5 or more elements;
- If the set is infinite, the roster notation must reflect this and not give a finite roster like $\{1, 2, 3, 4\}$
- If the set is finite, the roster notation must reflect this and not give an infinite roster like $\{1, 2, 3, 4, \dots\}$
- Have no more than two errors in the elements

On the second part, the set must equal S and the syntax of the set notation must be correct and rephrase the set in a meaningful way.

Learning Target 9

(CORE) I can find the intersection, union, difference, symmetric difference, power set, cardinality, cartesian product, and complement of sets.

Let $A = \{3, 4, 5, 6\}$, $B = \{2, 4, 6, 8\}$, and $C = \{0, 5\}$ and suppose the universal set is $U = \{0, 1, 2, \dots, 10\}$. Find each of the following. You do not need to show intermediate steps, but this is often helpful for you (and for the person grading your work).

- $A \cap C$
- $B \cup C$
- $B \setminus A$
- \overline{A}
- $B \setminus (A \cap C)$
- $((A \cap B) \cup (C \cap D)) \cap \emptyset$
- $\mathcal{P}(C)$
- $|B \times C|$

Success criteria: The final two items are correct; and at least 5 of the first 6 are correct.

NEW Learning Target 12

(CORE) I can apply the Additive and Multiplicative Principles and the Principle of Inclusion/Exclusion to formulate and solve basic combinatorics problems.

Suppose you need to create a 4-character password using the 26 lowercase letters of the English alphabet (a-z) and the 10 digits (0-9). Repeated letters are OK, as are repeated digits.

Find the following and show your steps:

1. How many possible password combinations are there?
2. How many possible password combinations are there that end in either a 0 or a 5?
3. How many possible password combinations are there that end in a 0 or start with the letter "o"?

Success criteria: All three items use a correct method for counting; correct computations are set up and shown clearly; and all three answers are correct except for no more than two simple errors.



Learning Target 13

(CORE) I can compute factorials and binomial coefficients, and apply these concepts to solve basic combinatorics problems. (permutations, selections, distributions)

1. Compute the values of each of the following, and show your steps:

(a) $6!$

(b) $\binom{11}{6}$

2. Find the following and show your steps:

(a) Suppose you have 5 different books, and you want to arrange them on a bookshelf in a specific order. How many different ways can you arrange the books?

(b) Suppose you have 10 different books and you want to put four of them on a bookshelf in a specific order. How many different ways can you arrange the books?

Success criteria: Both parts of the first item are correct except for no more than one simple error. On the second part, a correct method for counting is used on each, correct computations are set up and shown clearly, and the answers are correct except for no more than one simple error.