MTH 225 Learning Target Quiz 4 Fall 2021

Directions:

- Do only the problems that you need to take, and feel ready to take. If you have already earned Fluency on a Learning Target, do not attempt a problem for that Target! You can skip a Target if you need more time to practice with it, and take it on the next round.
- Do not put any work on this form; do all your work on separate pages and create one page per Learning Target.
- Clearly indicate which Learning Target you are attempting at the beginning of its solution.
- **No internet-connected technology is allowed, including smartphones, tablets, or laptops**. Handheld calculators, including graphing calculators, are OK as long as they do not connect to the internet.
- **Unless explicitly stated otherwise, you must show your work or explain your reasoning clearly on each item of each problem you do**. Responses that consist of only answers with no work shown, or where the work is insufficient or difficult to read, or which have significant gaps or omissions (including parts left blank) will not constitute a successful attempt.

Learning Target CA.1 (CORE): *I can represent an integer in base 2, 8, 10, and 16 and represent a negative integer in base 2 using two's complement notation.*

Show all work or explain your reasoning on each of the following.

1. Given 42 in decimal, convert to binary and hexadecimal.
2. Given `47A` in hexadecimal, convert to decimal.
3. Given `11011011` in binary, convert to decimal.
4. Write the base 2 representation of $-45 usingtwo's complementnotation. The base 2 representation of +45$ is `00101101`. (Assume 8-bit representation.)

Learning Target CA.2: *I can perform addition, subtraction, multiplication, and division in binary.*

Perform all of the following computations in binary, without changing to base 10. Show all work or explain your reasoning.

1. `10001000 + 01101011`
2. `10001000 - 01101011`
3. `11010` $\times$ `10`
4. `11011100` $\div$ `11` (That is, 11011100 divided by 11)

Learning Target L.1 (CORE): *I can identify the parts of a conditional statement and write the negation, converse, and contrapositive of a conditional statement.*

Do the following for each of the conditional statements below:

- State the hypothesis and the conclusion, and clearly label each.
- Write the negation (without just putting "not" in front of the statement).
- Write the converse.
- Write the contrapositive.

1. $A \to B$
2. If a student is enrolled in MTH 203, the student passed MTH 201.

Learning Target L.2: *I can construct truth tables for propositions involving two or three variables and use truth tables to determine if two propositions are logically equivalent.*

1. Use truth tables to determine whether the two statements $\neg(P \to Q)$ and $P \wedge (\neg Q)$ are logically equivalent.
2. Make a truth table for the statement $P \wedge (Q \vee R)$. Include columns for all intermediate steps.

Learning Target L.3: *I can identify the truth value of a predicate, determine whether a quantified predicate is true or false, and state the negation of a quantified statement.*

Let P and Q be the following predicates where the domain of each is the set of all positive integers (that is, $\{1, 2, 3, \dots\}$).

- $P(x)$: x is even
- $Q(x)$: $x/2$ is an integer

1. State the truth values of each of the following: $P(2)$, $P(7)$, $Q(9)$, $Q(15)$.
2. State the truth values of each of the following and give a one-sentence explanation for each. (Answers without explanations are not Satisfactory.)
   (a) $\forall x P(x)$
   (b) $\exists x P(x)$
   (c) $\exists x Q(x)$
3. State the negation of the statement, "Every numbered street in Allendale has a number divisible by 4" without merely putting the word "not", "It is not the case that", etc. on the statement.

Learning Target SF.1 (CORE): *I can represent a set in roster notation and set-builder notation; determine if an object is an element of a set; and determine set relationships (equality, subset).*

1. Restate each of the following sets using roster notation:
   (a) $\{x \in \mathbb{N} : x \% 5 = 0\}$
   (b) $\{x \% 5 : x \in \mathbb{N}\}$
   (c) $\{x \in \{1, 2, 3, \dots, 10\} : x^3 \leq 5\}$
2. Write the set $S = \{1, 3, 5, 7, 9, \dots\}$ using correct set-builder notation. There is more than one way to do it; but your answer must be correct and use correct notation and syntax. You may *not* use $\{x : x \in S\}$ as an answer.
3. Mark each of the following as TRUE or FALSE.
   (a) $0 \in \mathbb{N}$
   (b) $0 \in \mathbb{R}$
   (c) $0 \in \mathbb{Z}$
   (d) $0 \in \emptyset$
   (e) $\mathbb{Z} \subseteq \mathbb{R}$
   (f) $\emptyset \subseteq \mathbb{R}$

Learning Target SF.2: *I can perform operations on sets (intersection, union, complement, Cartesian product), determine the cardinality of a set, and write the power set of a finite set.*

Let $A = \{1,3,5,7,9\}$, $B = \{1,2,4,8\}$, and $C = \{5\}$. The universal set for these is $U = \{0,1,2,\dots,10\}$. Find each of the following. You do not need to show work, but do show it if it helps you; and your answers must be correct. **Use correct set notation on each answer**.

1. $A \cap B$
2. $A \cup C$
3. $C \setminus B$
4. $\overline{B}$
5. $A \triangle B$
6. $A \cap (\overline{B} \cup C)$
7. $|\mathcal{P}(\mathcal{B})|$

Learning Target SF.3 (CORE): *I can determine whether or not a given relation is a function; determine the domain, range, and codomain of a function.*

Below are three mappings from $\{1,2,3,4\}$ to $\{x,y,z,t\}$. For each one, state whether the mapping is a function. **If the mapping is not a function, explain why**. Otherwise if the mapping is a function, state the domain, range, and codomain; you do not need to explain your reasoning if the mapping is a function but your answers must be correct.

1. The mapping f defined by $f(1) = t$, $f(2) = x$, $f(3) = z$, $f(4) = y$
2. The mapping g defined by this table:

Input	1	2	3	4
Output	x	t	z	t

3. The mapping h given by this matrix: $\begin{pmatrix} 1 & 2 & 3 & 4 \\ z & t & x & \text{(nothing)} \end{pmatrix}$

Learning Target SF.4: *I can determine whether a function is injective, surjective, or bijective.*

Below are three functions. For each, state whether the function is injective, surjective, and/or bijective. **If a function fails to have one or more of these properties, explain why**. Otherwise you do not need to explain your reasoning unless it helps you; but your answers must be correct.

1. $f : \{1,2,3,4\} \to \{x,y,z,t\}$ given by $f(1) = t$, $f(2) = z$, $f(3) = x$, $f(4) = y$
2. $g : \{1,2,3,4\} \to \{x,y,z,t\}$ given by $g(1) = z$, $g(2) = z$, $g(3) = x$, $g(4) = y$
3. $h : \mathbb{N} \to \mathbb{Z}$ defined by $h(n) = 2^n$

Learning Target SF.5: *I can evaluate special computer science functions: floor, ceiling, factorial, DIV, and MOD (%).*

Notation reminder: $\lceil x \rceil$ is the ceiling of x; $\lfloor x \rfloor$ is the floor of x. Also $DIV(a,b)$ means the same as the Python expression `a // b`.

State the values of the following. You do not need to give steps or reasoning; you are allowed two incorrect responses.

1. $\lfloor 1.1 \rfloor$
2. $\lfloor -1.1 \rfloor$

3. $\lceil 1.1 \rceil$
4. $\lceil -1.1 \rceil$
5. $6!$
6. $0!$
7. $DIV(50, 20)$
8. $50 \% 7$
9. $6 \% 7$

Learning Target C.1 (CORE): *I can use the additive and multiplicative principles and the Principle of Inclusion and Exclusion to formulate and solve counting problems.*

Show your work or explain your reasoning on each of these, and clearly indicate your answer. Answers without explanations, or vice versa, are not considered successful demonstrations of skill.

1. How many 3-digit octal (base 8) numbers are possible?
2. How many 4-bit binary strings either start with a "1" or end in a "1"?

Learning Target C.2 (CORE): *I can calculate a binomial coefficient and correctly apply the binomial coefficient to formulate and solve counting problems.*

Show your work or explain your reasoning on each of these, and clearly indicate your answer. Answers without explanations, or vice versa, are not considered successful demonstrations of skill.

1. (a) $\binom{10}{6}$
   (b) $\binom{30}{29}$
   (c) $\binom{100}{0}$
2. A shelf in a bookstore has 20 different books on it. How many ways are there to select 5 books from that shelf to give away?

Learning Target C.3: *I can count the number of permutations of a group of objects and the number of k-permutations from a set of n objects.*

Show your work or explain your reasoning on each of these, and clearly indicate your answer. Answers without explanations, or vice versa, are not considered successful demonstrations of skill.

1. $P(22, 8)$
2. MTH 225 is having a t-shirt giveaway, where there are five different shirts to give out (blue, red, green, yellow, and purple). In a class of 24 students, how many ways are there to give away those shirts? (Assume no student gets more than one shirt.)

Learning Target C.4: *I can use the "stars and bars" method to count the number of ways to distribute objects among a group.*

Show your work or explain your reasoning on each of these, and clearly indicate your answer. Answers without explanations, or vice versa, are not considered successful demonstrations of skill.

1. How many ways are there to distribute five *identical* t-shirts to a class of 24 students? Assume that it's possible for a student to get more than one shirt.
2. How many natural number solutions are there to the equation $x + y + z = 10$? Remember $0 \in \mathbb{N}$.