Directions:

- Do only the problems that you need to take, and feel ready to take. If you have already earned Fluency on a Learning Target, do not attempt a problem for that Target! You can skip a Target if you need more time to practice with it, and take it on the next round.
- Do not put any work on this form; do all your work on separate pages and create one page per Learning Target.
- Clearly indicate which Learning Target you are attempting at the beginning of its solution.
- **No internet-connected technology is allowed**, **including smartphones**, **tablets**, **or laptops**. Handheld calculators, including graphing calculators, are OK as long as they do not connect to the internet.
- The criteria for "success" is shown on each problem. Please note, some problems may *require* work or an explanation to be shown while others may not require it. On those that don't require work or explanations, you may still show work or explanations if you want, and if there is an error that can be traced to a "simple" mistake in your work, this will be taken into consideration when grading.

Learning Target L.1 (CORE): *I can identify the parts of a conditional statement and write the negation, converse, and contrapositive of a conditional statement.*

Do the following for each of the conditional statements below:

- State the hypothesis and the conclusion, and clearly label each.
- Write the negation (without just putting "not" in front of the statement).
- Write the converse.
- Write the contrapositive.

Success means: There is no more than two "simple" errors in the answers and no more than one (1) non-simple error.

1. $R \to S$
2. If it snows, the temperature is below freezing.

Learning Target L.2: *I can construct truth tables for propositions involving two or three variables and use truth tables to determine if two propositions are logically equivalent.*

Do each of the following.

Success means: There is no more than one error in the final column of the results, and the statement about logical equivalence is made and is consistent with the truth tables.

1. Use truth tables to determine whether the two statements $A \wedge (\neg B)$ and $(\neg A) \vee B$ are logically equivalent.
2. Make a truth table for the statement $(P \to Q) \wedge (Q \to R)$. Include columns for all intermediate steps.

Learning Target L.3: *I can identify the truth value of a predicate, determine whether a quantified predicate is true or false, and state the negation of a quantified statement.*

Let P and Q be the following predicates where the domain of each is the set of all positive integers (that is, $\{1, 2, 3, \dots\}$).

- $P(x) : x$ is a multiple of 8
- $Q(x) : x^2 > 10$

Do each of the following.

Success means: The negation is stated correctly and there is no more than one (1) error in the rest.

1. State the truth values of each of the following: $P(43)$, $P(11)$, $Q(1)$, $Q(5)$.
2. State the truth values of each of the following and give a one-sentence explanation for each. (Answers without explanations are not Satisfactory.)
   (a) $\forall x P(x)$
   (b) $\exists x P(x)$
   (c) $\forall x Q(x)$
3. State the negation of the statement, "Every faculty senate meeting runs over time" without merely putting the word "not", "It is not the case that", etc. on the statement.

Learning Target SF.1 (CORE): *I can represent a set in roster notation and set-builder notation; determine if an object is an element of a set; and determine set relationships (equality, subset).*

Do each of the following.

Success means: There is no more than two "simple" errors in the answers and no more than one (1) non-simple error.

1. Restate each of the following sets using roster notation:
   (a) $\{x \in \{1, 2, \dots, 100\} \ : \ x \ \% \ 10 = 0\}$
   (b) $\{a \ \% \ 2 \ : \ a \in \mathbb{N}\}$
2. Write the set $S = \{3, 13, 23, 33, 43, \dots\}$ using correct set-builder notation. There is more than one way to do it; but your answer must be correct and use correct notation and syntax. You may *not* use $\{x : x \in S\}$ as an answer.
3. Mark each of the following as TRUE or FALSE.
   (a) $\mathbb{N} \subseteq \mathbb{R}$
   (b) $\mathbb{R} \subseteq \mathbb{Z}$
   (c) $\emptyset \subseteq \mathbb{N}$
   (d) $3 \in \mathbb{N}$
   (e) $1 \in \mathbb{R}$
   (f) $\{1, 2, 3, 4\} \subseteq \{2, 3, 4\}$

Learning Target SF.2: *I can perform operations on sets (intersection, union, complement, Cartesian product), determine the cardinality of a set, and write the power set of a finite set.*

Let $A = \{0, 1, 2, 3, 4, 5\}$, $B = \{2, 4, 6, 8\}$, and $C = \{4, 8\}$. The universal set for these is $U = \{0, 1, 2, \dots, 10\}$. Find each of the following. You do not need to show work, but do show it if it helps you; and your answers must be correct. **Use correct set notation on each answer.**

Success means: The final two items are correct; and in the rest, there is no more than two "simple" errors in the answers and no more than one (1) non-simple error.

1. $A \cap C$
2. $A \cup B$
3. $A \setminus C$

4. $\overline{B}$
5. $(A \cup B) \cap C$
6. $|B \times C|$
7. $\mathcal{P}(C)$

Learning Target SF.3 (CORE): *I can determine whether or not a given relation is a function; determine the domain, range, and codomain of a function.*

Below are three mappings from $\{1,2,3,4,5\}$ to $\{x,y,z,t\}$. For each one, state whether the mapping is a function. **If the mapping is not a function, give a SPECIFIC explanation why**. Otherwise if the mapping is a function, state the domain, range, and codomain; you do not need to explain your reasoning if the mapping is a function but your answers must be correct.

Success means: All answers are correct and all explanations are clear and correct.

1. The mapping f defined by $f(1) = y$, $f(2) = z$, $f(2) = t$, $f(3) = z$, $f(4) = y$, $f(5) = x$
2. The mapping g defined by this table:

Input	1	2	3	4	5
Output	x	y	t	x	

3. The mapping h given by this matrix: $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ z & t & x & z & t \end{pmatrix}$

Learning Target SF.4: *I can determine whether a function is injective, surjective, or bijective.*

Below are three functions. **For each, state whether the function is injective, then state whether it is surjective, then state whether it is bijective. If a function fails to have one or more of these properties, give a SPECIFIC EXAMPLE that shows why.** Otherwise you do not need to explain your reasoning unless it helps you; but your answers must be correct.

Success means: Each function is correctly identified as injective or not, surjective or not, and bijective or not; and each failure of a property is explained with a specific counterexample.

1. $f : \{1,2,3,4,5\} \to \{1,2,3,4\}$ given by $f(a) = (a \,\%\, 4) + 1$
2. $h : \mathbb{Z} \to \mathbb{Z}$ defined by $h(n) = n^2 + 1$
3. $h : \mathbb{N} \to \mathbb{N}$ defined by $h(n) = n + 1$

Learning Target C.1 (CORE): *I can use the additive and multiplicative principles and the Principle of Inclusion and Exclusion to formulate and solve counting problems.*

Show your work or explain your reasoning on each of these, and clearly indicate your answer. Answers without explanations, or vice versa, are not considered successful demonstrations of skill.

Success means: Both answers are correct (although up to two simple errors are allowed) and justified with a brief and clear explanation.

1. How many seven-digit numbers (in regular base 10) are there that either start or end with an even number?
2. A professor has three different shirts, four different pairs of pants, and two different pairs of shoes. How many outfits can the professor make from these items?

Learning Target C.2 (CORE): *I can calculate a binomial coefficient and correctly apply the binomial coefficient to formulate and solve counting problems.*

Show your work or explain your reasoning on each of these, and clearly indicate your answer. Answers without explanations, or vice versa, are not considered successful demonstrations of skill.

Success means: All answers are correct (although up to two simple errors are allowed) and justified with a brief and clear explanation.

1. Compute the numerical value of each of the following, and justify your reasoning with a formula or explanation.
   (a) $\binom{12}{8}$
   (b) $\binom{120}{120}$
   (c) $\binom{120}{119}$
2. How many ways can you select six cards from a deck of 12 cards? (Assume all the cards are different.)

Learning Target C.3: *I can count the number of permutations of a group of objects and the number of k-permutations from a set of n objects.*

Show your work or explain your reasoning on each of these, and clearly indicate your answer. Answers without explanations, or vice versa, are not considered successful demonstrations of skill.

Success means: Both answers are correct (although up to two simple errors are allowed) and justified with a brief and clear explanation.

1. How many ways can you give six cards from a deck of 12 cards to six different people? (Assume all the cards are different.)
2. License plates in a given state have three letters following by three digits. There are 26 letters in the alphabet. How many license plates are possible?

Learning Target C.4: *I can use the "stars and bars" method to count the number of ways to distribute objects among a group.*

Show your work or explain your reasoning on each of these, and clearly indicate your answer. Answers without explanations, or vice versa, are not considered successful demonstrations of skill.

Success means: Both answers are correct (although up to two simple errors are allowed) and justified with a brief and clear explanation.

1. How many ways are there to give 10 identical books to three children?
2. How many ways are there to give 10 identical books to three children if we want each child to get at least two copies of the book?

Learning Target RI.1 (CORE): *I can generate several values in a sequence defined using a closed-form expression or using recursion.*

List the first six (6) terms of each of the following sequences. You do not need to show your work, but your answers must be correct.

Success means: At least three of the four sequences have all six terms correctly listed.

1. $a_n = 2(3^n) + 3,$ where $n = 1, 2, 3, \ldots$
2. $b_n = 3 - 2n$ where $n = 0, 1, 2, \ldots$
3. $c_0 = 2,$ and $c_n = 2c_{n-1} + 3n$ if $n > 0$

4. $d_0 = 1, d_1 = 2$ and $d_n = 7d_{n-1} - 10d_{n-2}$ if $n > 0$

Learning Target RI.2: *I can use sigma notation to rewrite a sum and determine the sum of an expression given in sigma notation.*

Either compute the value of the sum given in sigma notation, or write sigma notation that correctly represents the sum. You do not need to show your work, but your answers must be correct.

Success means: At least three of the four answers are completely correct.

1. Compute both sums below:
   (a) $\sum_{n=1}^{4}(2+3n)$
   (b) $\sum_{n=2}^{4}(2(3)^n)$
2. For each sum below, write the sum correctly using sigma notation:
   (a) $2+4+6+8+10+\cdots+100$
   (b) $2+6+18+54+162$

Learning Target RI.3: *I can find closed-form and recursive expressions for arithmetic and geometric sequences.*

For each sequence below, find *both* a closed formula *and* a complete recursive definition for the sequence. You do not need to show your work, but your results must be correct.

Success means: At least three of the four answers are completely correct.

1. $2, 6, 18, 54, 162, \ldots$
2. $2, 6, 10, 14, 18, \ldots$
3. $2, 5, 8, 11, 14, \ldots$
4. $1, 0.1, 0.01, 0.001, 0.0001, \ldots$

Learning Target RI.4: *I can determine a recurrence relation for a given recursive sequence and check whether a proposed solution to a recurrence relation is valid.*

Consider the recurrence relation given by $a_0 = 1$ and for $n > 0$, $a_n = 1 + 2a_{n-1}$. Determine whether the function $f(n) = 2^{n+1} - 1$ is or is not a solution to this recurrence relation. If it is not a solution, give an instance of a sequence term where the function differs from the recurrence relation. If it *is* a solution, show all steps that prove this (as in the examples done on video and in class).

Success means: If the function is not a solution, a specific counterexample is correctly and clearly stated. If the function is a solution, all steps in the checking process are shown and correct, with up to two (2) simple errors allowed.

Learning Target RI.5: *I can solve a second-order linear homogeneous recurrence relation using the characteristic root method.*

Solve the recurrence relation given by $a_0 = 2, a_1 = 1$, and $a_n = 7a_{n-1} - 10a_{n-2}$ for $n > 1$.

Success means: A correct and complete function is given, and all significant work is shown clearly. Up to two (2) simple errors are allowed but the solution still must result in a complete function (i.e. you can't stop mid-solution if you make an error).