Peer instruction in linear algebra

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MAA Session on Innovative and Effective Ways to Teach Linear Algebra, II
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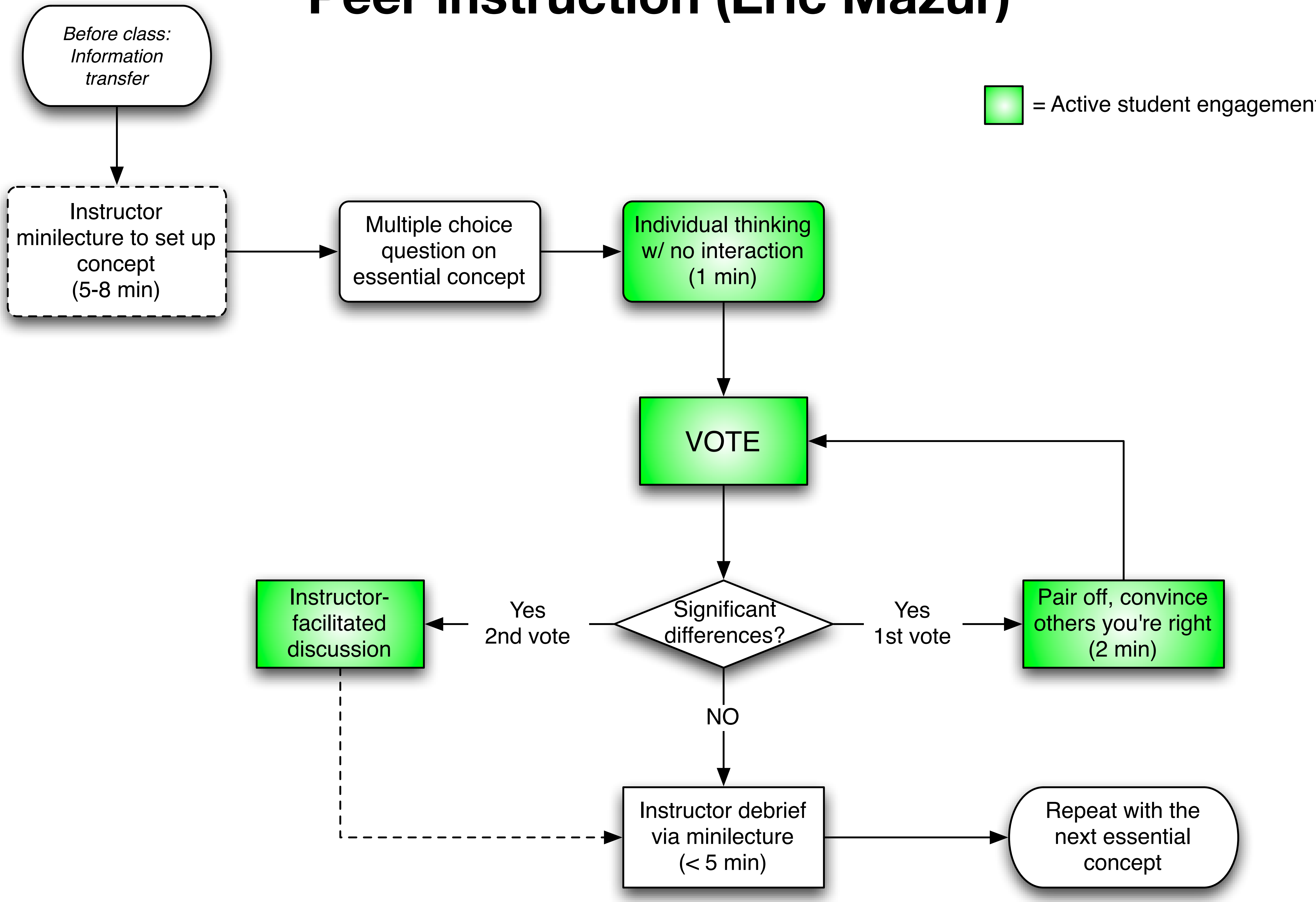
Overview

- What are some learning goals for students in linear algebra that can be problematic?
- What is peer instruction and how can it address those problems?
- Mini-case study: A peer-instruction focused Linear Algebra class

Problematic learning goals in linear algebra

Goal	Problematic because…
Use the language of linear algebra fluently and correctly	Focus on computation rather than on semantics/syntax
Draw connections between concepts (within/between courses)	Focus on math as set of discrete, unconnected “tools”
Communicate one’s reasoning clearly	Focus on right answers vs. coherent processes
Map basic content knowledge onto new problems	Focus on getting the right answer rather than identifying unifying principles

Peer instruction (Eric Mazur)



Redesigning Linear Algebra at GVSU

- MTH 227: 3-credit intro to Linear Algebra
- Received grant from Faculty Teaching and Learning Center to purchase software licenses and tablet devices
- Redesigned course structure around peer instruction with a semi-flipped class design, for Winter 2013

Peer instruction in action

Using Learning Catalytics (learningcatalytics.com)

Round 1

1. multiple choice

How many solutions does the following linear system have?

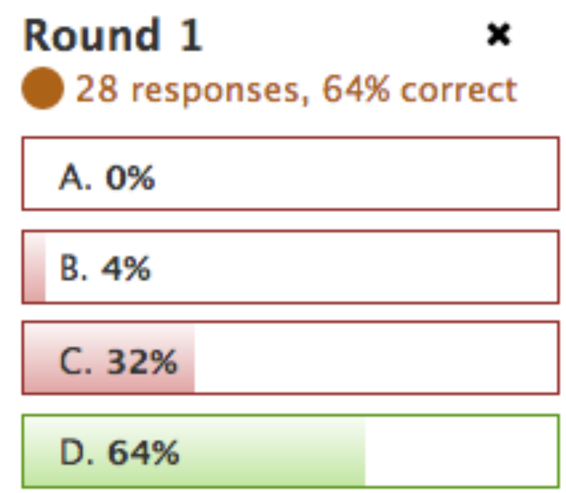
$$\begin{aligned} x_1 + x_2 &= 1 \\ 2x_1 + 2x_2 &= 2 \end{aligned}$$

A. None

B. Exactly 1

C. A finite number of solutions, but more than just 1

D. An infinite number of solutions



Round 2

1. multiple choice

How many solutions does the following linear system have?

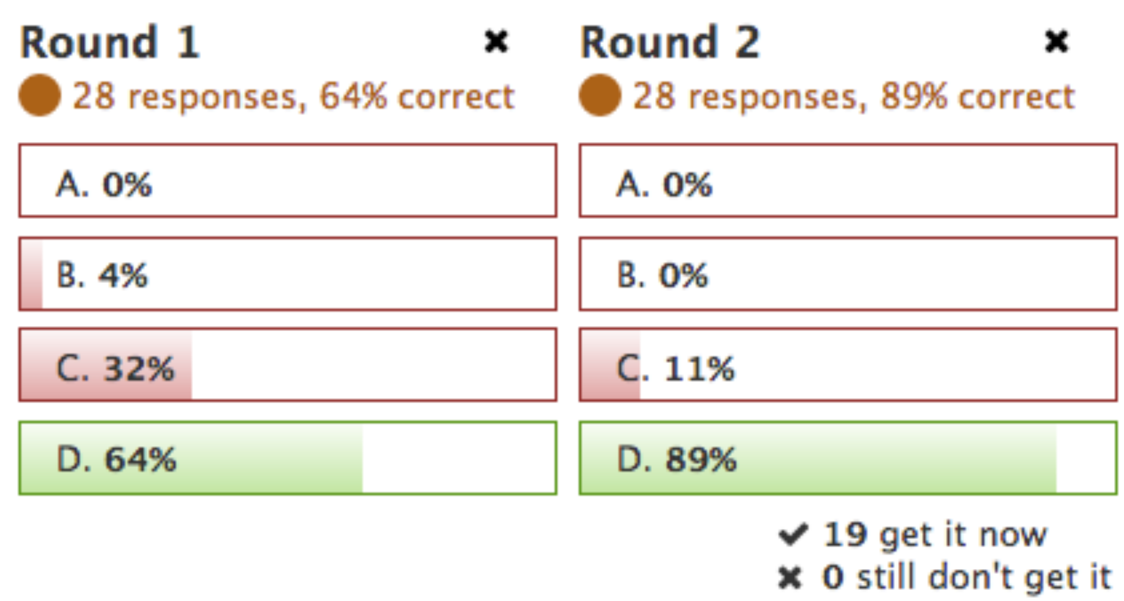
$$\begin{aligned} x_1 + x_2 &= 1 \\ 2x_1 + 2x_2 &= 2 \end{aligned}$$

A. None

B. Exactly 1

C. A finite number of solutions, but more than just 1

D. An infinite number of solutions



Using PI to probe understanding of span

1. many choice

Suppose that the vector $\mathbf{b}$ is in the set spanned by $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_p$. What does this mean?

A. $\mathbf{b}$ is a scalar multiple of one of the vectors $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_p$

B. $\mathbf{b}$ is a linear combination of $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_p$

C. The system with augmented matrix $[\mathbf{v}_1\ \mathbf{v}_2\ \cdots\ \mathbf{v}_p\ \mathbf{b}]$ is consistent

D. The system with augmented matrix $[\mathbf{v}_1\ \mathbf{v}_2\ \cdots\ \mathbf{v}_p\ \mathbf{b}]$ is inconsistent

Round 1 ✖

26 responses, 58% correct

A. 27%

B. 88%

C. 77%

D. 0%

✔ 2 get it now

✖ 0 still don't get it

2. multiple choice

Let $\mathbf{v} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$. Then $\mathrm{Span}\{\mathbf{v}\}$ equals

A. The empty set

B. The set containing only $\mathbf{v}$

C. The line y = 2x in $\mathbb{R}^2$

D. All of $\mathbb{R}^2$

Round 1 ✖

28 responses, 21% correct

A. 0%

B. 57%

C. 21%

D. 21%

When things go backwards

5. multiple choice

Suppose S is a set of five vectors in $\mathbb{R}^3$. Then

A. S must be linearly independent

B. S might be linearly independent, but it might also be linearly dependent depending on what the vectors are

C. S must be linearly dependent

Round 1

26 responses, 50% correct

A. 4%

B. 46%

C. 50%

5. multiple choice

Suppose S is a set of five vectors in $\mathbb{R}^3$. Then

A. S must be linearly independent

B. S might be linearly independent, but it might also be linearly dependent depending on what the vectors are

C. S must be linearly dependent

Round 1

26 responses, 50% correct

A. 4%

B. 46%

C. 50%

Round 2

25 responses, 32% correct

A. 0%

B. 68%

C. 32%

Using PI to clear up concepts and terminology

2. multiple choice

Consider the transformation T defined by the formula:

$$T(\mathbf{x}) = \begin{bmatrix} 4 & -5 & -5 & -3 & 3 & -4 \\ 4 & 1 & -5 & 1 & -3 & -2 \\ -4 & -4 & -2 & -3 & 1 & -2 \end{bmatrix} \mathbf{x}$$

What are the domain and codomain of T, respectively?

A. Domain is $\mathbf{R}$, codomain is $\mathbf{R}$

B. Domain is $\mathbf{R}^3$, codomain is $\mathbf{R}^3$

C. Domain is $\mathbf{R}^3$, codomain is $\mathbf{R}^6$

D. Domain is $\mathbf{R}^6$, codomain is $\mathbf{R}^3$

E. Domain is $\mathbf{R}^6$, codomain is $\mathbf{R}^6$

F. None of the above

Round 1 ✖

26 responses, 46% correct

A. 0%

B. 23%

C. 27%

D. 46%

E. 4%

F. 0%

Round 2 ✖

26 responses, 69% correct

A. 0%

B. 19%

C. 8%

D. 69%

E. 4%

F. 0%

✔ 3 get it now

✖ 0 still don't get it

6. many choice

Suppose A is a 5×5 matrix and $\det(A) = 3$. Which of the following can we conclude from this information alone?

A. The columns of A form a basis for $\mathbb{R}^5$

B. The columns of A are linearly independent

C. The rank of A equals 3

D. The null space of A equals the set $\{0\}$

Round 1 ✖

24 responses, 38% correct

A. 63%

B. 67%

C. 25%

D. 54%

Round 2 ✖

23 responses, 52% correct

A. 91%

B. 96%

C. 17%

D. 74%

Using PI to make conceptual connections

6. multiple choice

Consider the matrix transformation $V : \mathbb{R}^3 \to \mathbb{R}^2$ given by:

$$V(\mathbf{x}) = \begin{bmatrix} 2 & 4 & 8 \\ 0 & 3 & -1 \end{bmatrix} \mathbf{x}$$

How many vectors in $\mathbb{R}^3$ are mapped to the zero vector in $\mathbb{R}^2$? That is, how many vectors $\mathbf{x}$ in $\mathbb{R}^3$ satsify $V(\mathbf{x}) = \mathbf{0}$?

A. None

B. One

C. More than one but finitely many

D. Infinitely many

Round 1

26 responses, 50% correct

- A. 0%
- B. 31%
- C. 19%
- D. 50%

Round 2

26 responses, 69% correct

- A. 0%
- B. 19%
- C. 12%
- D. 69%

2. multiple choice

Let $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ be a set of vectors such that none of the vectors is a scalar multiple of any of the other vectors. Then the set is

A. Linearly dependent

B. Linearly independent

C. (Not enough information)

Round 1

26 responses, 54% correct

- A. 4%
- B. 42%
- C. 54%

Round 2

27 responses, 81% correct

- A. 0%
- B. 19%
- C. 81%

✔ 5 get it now

✖ 0 still don't get it

Benefits of using PI in linear algebra

- Students receive instruction from a multiplicity of channels (peers + instructor)
- Class time focuses on concepts, connections, and reasoning rather than only computation
- Instructor receives real-time formative assessment, can engage in “agile teaching” and make decisions based on actual knowledge of student learning
- Students get immediate feedback on their learning

Thank you.

Software-based classroom response platforms:
Learning Catalytics (http://learningcatalytics.com)
Top Hat (http://tophat.com)

talbertr@gvsu.edu
@RobertTalbert on Twitter
http://google.com/+RobertTalbert

http://chronicle.com/blognetwork/castingoutnines

http://proftalbert.com