Directions:

- Complete the exercises below and either write up or type up your solutions. Solutions must be submitted as PDF or Word documents, uploaded to the appropriate assignment area on Blackboard.
- If you choose to submit handwritten work, it must be neat and legible; if you do your handwritten work on paper, it must be **scanned to a PDF file** and submitted to Blackboard. Instructions and practice for scanning work to PDFs is given in the Startup Assignment. **Do not just take a picture**, **and do not submit a graphics file (JPG**, **PNG**, **etc.)** — such submissions will not be graded.
- Your work will be graded using the EMPX rubric and evaluated **not just on the basis of a right or wrong answer**, **but on the quality**, **completeness**, **and clarity of your work**. Therefore you need to show all work and explain your reasoning on each item.
- Every item must have a good-faith effort at a complete and correct response. If any item is left blank, or shows minimal effort (such as answering "I don't know"), or is significantly incomplete, the entire assignment will be graded "X" (Not Assessable) and you will have to spend a token to revise it.

---

1. In Module 1 we looked closely at the Axioms of Integer Arithmetic, revolving around the operations of integer addition and multiplication. These are the only two operations defined in the axioms; on page 5, *subtraction* is defined in terms of addition and additive inverses, but *division* never makes an appearance at all! Let's try to understand why that is the case.
   (a) Are the integers closed under division? That is, for all integers a and b, is a/b also an integer? If so, explain how you know; if not, give a counterexample.
   (b) Are the *natural numbers* (that is, just the positive integers $1, 2, 3, \ldots$) closed under division? If so, explain how you know; if not, give a counterexample.
   (c) Why didn't we discuss division of integers in Module 1 alongside the Axioms of Arithmetic?

2. An important idea we encountered in our work this week is that **every mathematical claim we make needs to be backed up by either an Axiom**, **or a theorem that we prove from the Axioms**. If something is not an Axiom, it needs to be proven. This can be very tricky when trying to prove "facts" that we have taken on faith our entire lives. Below is a result that you've used countless times ("a positive times a negative is a negative") but in all likelihood never *proven*. The proof is given in a line-by-line format, but none of the steps are justified. Provide a justification for each line. The justifications must come from the Arithmetic Axioms or the Ordering Axioms, a result that is proven in the textbook, or a result that we proved in our course work. *We're not allowed to say "obviously" or refer back to what we learned in high school!*

   Theorem: For all integers x and y, if $x > 0$ and $y < 0$, then $xy < 0$.
   Proof: Let x and y be integers such that $x > 0$ and $y < 0$.

   (a) Because $y < 0$, we have $-y > 0$.

   (b) Therefore $x(-y) > 0$.

   (c) Therefore $-(xy) > 0$.

   (d) Therefore $-(-(xy)) < 0$.

   (e) Therefore $xy < 0$, which is what we wanted to prove.

   (Side note: Once we provide the justifications for this, it's a theorem and we are allowed to use it to prove other things.)

3. On each Weekly Practice, you'll be asked to reflect on your work for the week and share what you've learned. We'll typically do this by video, using the app **Flipgrid**. Go here: `https://flipgrid.com/cef1218e` and sign in with your GVSU email credentials, watch the video of me (Talbert) that's posted there, then give a 3-minute (or less) video response to the three prompts that are given.