Welcome to Mathematics 331, Fall 2016
Algebraic Structures

Course Orientation and Syllabus

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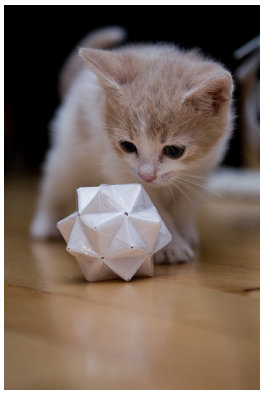

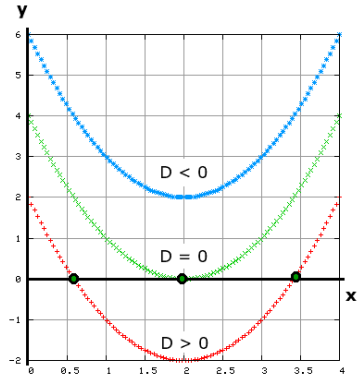

1 Welcome!

[1]

Welcome to Algebraic Structures! I am excited to be working with you because I *looooooooove* this course. Here are two ways of describing what we will be studying in this course. First, the short version: Symmetry.

Now the long version: You all learned about the quadratic formula in high school: The solution to $ax^2+bx+c=0$ is

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}.$$

The quadratic formula has been known for at least 3000 years (this is not a typo). In fact, there are similar (and more complicated) versions for cubic equations (ax^3+bx^2+cx+d) and quartic equations ($ax^4+bx^3+cx^2+dx+e$). These two formulas have been known since roughly 1500 A.D., and people started immediately looking for a solution to a quintic equation ($ax^5+bx^4+cx^3+dx^2+ex+f$).

People had trouble finding the solution to the quintic, though. They tried hard for about 250 or 300 years, so it isn't like they were slacking. But Paolo Ruffini and Niels Henrik Abel demonstrated in about 1800 A.D. that no matter how long you look, no matter how hard you look, no matter how smart you are, it is quite literally impossible to find an analogous formula for the quintic.

This should be a little weird—how do you show that something can't exist? The answer is 'from the material we learn in this class.' (Unfortunately, we won't be able to get to the quintic until Spring semester).

2 Office Hours and Contacting Bret

Office hours are a place to get help—they are designed for you to use. There is a sign-up sheet for office hours on Canvas (or you can go to `bretbenesh.youcanmeet.me` directly). I usually post new office hours every Wednesday for the following week. If none of the times listed work for you and you need to meet with me immediately, please email me with times you are available, and I will respond with a time that works for both of us. Additionally, you can drop by my office if you are near. In this case, I am not guaranteed to be free to meet, but I often will be.

There is free drop-in help at the Math Skills Center on both campuses (in the basement of the HAB at CSB and in PEngel at SJU). This is a particularly good resource for the Gateway Exams.
Email is by far the best way to reach Bret if you need to contact me.

There is a Canvas page for this course. Seek it out; read it often. Like every day after class.

3 Textbook and Reading

None required. I am supplying a draft of Thomas Q. Sibley's upcoming textbook on Canvas for reference. Please look at it as you see fit.

[1] I thank Kate Owens and Robert Talbert for much of the structure of this syllabus.

4 Learning Objectives

Algebraic Structures is designed to both be a key part of your liberal arts education and help you progress as a mathematician. Math 331 will help move you from 'pre-rigorous' thinking, which is fuzzy and informal, to 'rigorous' thinking, which is formal and precise (there is a 'post-rigorous' way of thinking, too, but that does not typically come until graduate school). The ability to think rigorously is an important aspect of critical thinking.

As such, here is what you should expect to learn from this class (h/t to Joshua Bowman, Theron Hitchman, and Jeremy Kun for helping me put this into words).

1. You will be able to write clearly, concisely, and precisely—this is a writing-intensive course. This involves
   (a) Writing in prose.
   (b) Using proper mathematical style.
   (c) Using proper mathematical notation.
   (d) Using precise language.
   (e) Writing enough to communicate what you need to, but no more.
2. You will develop a healthy respect for the importance of definitions. This involves
   (a) Internalizing definitions so that you truly understand them.
   (b) Stating definitions and explain notation.
   (c) Identifying whether or not an object meets the conditions of a given definition.
   (d) Giving examples that both satisfy and do not satisfy a given definition.
   (e) Testing an unfamiliar definition using examples.
   (f) Creating your own definitions.
3. You will understand the structure and importance of theorems. This involves
   (a) Stating theorems accurately, paying particularly close assumptions to their assumptions and conclusions.
   (b) Determining where the conditions of a theorem hold in a particular situation.
   (c) Recognizing logically-equivalent forms of a theorem.
   (d) Formulating and testing conjectures.
4. You will understand the structure and importance of proofs. Proofs are a form of persuasive writing that are logical, comprehensible, and explanatory. This involves
   (a) Evaluating existing proofs.
   (b) Creating your own proofs.
5. You will be able to abstract appropriately. This involves
   (a) Being comfortable with a minimal amount of details.
   (b) Being able to apply a theorem to a particular case.
   (c) Generalize based on common properties of specific examples.

To achieve these goals, we will be studying seven main Topics.

1. Groups (and their elements)
2. Subgroups
3. Quotient Groups (and cosets)
4. Isomorphisms
5. Homomorphisms
6. Permutation Groups
7. Rings (and integral domains, fields, and ideals)

Note that we will not spend much time in class talking about permutations, but rather will treat it as an application of the ideas from Groups and Subgroups.

5 How to study

Neuroscience is teaching us that there are good ways to study and bad ways to study. This is overly simplistic, but still useful: you should only study if you are either (1) writing something down or (2) debating with someone. Here are some specific examples of how to study and how not to study. These DO's and DON'Ts are not specific to mathematics—they appear to work in all subjects.

DO: Regularly quiz yourself. Make up a problem, see if you can solve it, and figure out what you did right and what you did wrong. Do this for old material as well as new material. (The science seems to be suggesting that testing is the best way of learning material).

DO: Read the assigned reading from the textbook once.

DO: While reading, try to solve examples before looking at the answers.

DO: Write up a short paragraph summarizing the main point of a chapter. Just make sure it is in your own words—don't copy or paraphrase from any sort of chapter summary that the textbook author has written.

DO: Write up a short paragraph that summarizes what you learned during a class period.

DO: Describe to a friend what you have learned in class.

DO: Listen to a friend describe what you have learned in class. Be skeptical, and ask a lot of questions.

DO: Look through the textbook to find answers to very specific questions ("What is the definition of continuity?" "How do you take the derivative of an exponential function?").

DON'T: Re-read the textbook beyond the first time.

DON'T: Re-read your class notes.

DON'T: Do nothing.

The first two DON'Ts are to prevent you from wasting time; re-reading the textbook and your classnotes is time intensive, yet does not help you learn much. Worse yet, it makes you more confident that you understand the material without actually helping you understand it (the book seems more *familiar*, which makes it seem like you are learning. But you aren't). Also, I apologize for the double negative in the last DON'T.

One final note: I speak from experience when I say that the DON'Ts are much more pleasant to do. It is much more pleasant and much less confusing to re-read the textbook than to create and take a quiz. In fact, most people feel that they actually learn more from re-reading the text as compared to creating and taking a quiz. But this is a false confidence—studies show that the people who quizzes themselves actually did better on later quizzes (even though they felt like they didn't learn the material as well).

6 Cooperative Learning

Several of the policies of this course have been set up to promote *cooperative learning* (as opposed to competitive or individual learning). This may differ from what you are used to, so I will provide a brief explanation as to why these policies are in place.

1. The research from the past 115 years overwhelmingly shows that students learn more in cooperative environments than individual environments, which seem to be the norm at most colleges. Cooperative learning does even better when compared to competitive rather than individual learning.

2. The research shows that the vast majority of students who have had true cooperative learning experiences overwhelmingly prefer those experiences to individual or competitive experiences.

3. Cooperative learning is in keeping with the Benedictine values of this school. "We seek the common good...We integrate a commitment to the common good with respect for the individual...We call the community together for counsel to make decisions."

Your teams are the main people you will be "cooperating" with.

7 Expectations for Interacting in Class

Mathematics is done in a social context, and there are certain expectations in how people interact with each other. Here is a list of how you are expected to behave in this class.

1. It is your responsibility to listen to what your classmates are saying.
2. It is your responsibility to share ideas that you think might be interesting.
3. It is your responsibility to speak up if you do not understand someone else's ideas. A good way of doing this is by asking the person to rephrase what they said.
4. It is your responsibility to speak up if you disagree with someone's ideas. Just do so respectfully.
5. Generally speaking, treat people with respect. If you do this, you will be fine.

Everyone in this class is important, and I will not hesitate to ask individuals who act disrespectfully to leave class immediately (Note: this almost never happens—Bennies and Johnnies tend to be very respectful toward each others in class).

8 Assignments

All assignments will be graded according to a PASS/NOTYET scale. That is, there will be no points. There will be no partial credit, either, as you will be given chances to resubmit work instead. The criteria to receive a PASS will be spellout out for each assignment (see the document on Canvas), and these criteria are called *Specifications*.

8.1 Daily Homework

At the end of most class periods, you will be assigned Daily Homework that is due the class period that immediately follows. This work will be collected at the end of each class period. A grade of PASS will be given to everyone who made an honest effort at answering the questions (regardless of whether they are correct); NOTYET will be given to people who did not complete the assignment or did not put honest effort into thinking about the assignment. Check the Canvas page after each class period to see what the Daily Homework is for the following class period.

As part of the Daily Homework, you will be required to write two short papers related to self-regulated learning. These will be graded on completion only. The goals of these papers are to help you become better students in this class and your other classes. Additionally, you will be asked to write a short reflection on a paper describing the advantages of being a mathematics major.

The purpose of Daily Homework is to help prepare you for that day's class. Because the class experience cannot be made up, a Daily Homework score of NOTYET will not be able to be improved.

8.2 Definition Checks

You will be given 'pop quizzes' regularly (usually once or twice each week) on the statements of definitions and theorems. You do not need to write the definition/theorem statement down verbatim, but you must capture every key idea contained in the statement. A list of definitions and theorems that you need to know will be on the Canvas page.

Your definitions (and statements of theorems) will be graded as either PASS or NOTYET according to whether it captures all of the important ideas of the statement in a clear way. You will have more than one opportunity to get a PASS for each definition/theorem statement, although everything will be better if you get PASS's as soon as possible.

The purpose of the Definition Checks is to give you an incentive to internatize definitions and theorem statements as soon as possible, since internalizing definitions and theorem statements will help you in every aspect of the course.

8.3 Problem Sets

You will have Problem Sets based on the seven main topics of the course:

1. Groups (and their elements)
2. Subgroups
3. Quotient Groups (and cosets)

4. Isomorphisms
5. Homomorphisms
6. Permutation Groups
7. Rings (and integral domains, fields, and ideals)

The Problem Sets should be hand-written (and legible), and care should be taken to make sure that you meet all of the Specifications. You may submit up to two Problem Sets per week for evaluation. This could be two first attempts, two revisions, one revision and one first attempt, one first attempt, one revision, or no submissions, although you will rarely want to submit nothing.

Every *problem* in a Problem Set will be given one of the following grades (the entire Problem Set will be given a grade of PASS or NOYET).

Grade	Description
E	Excellent or Exemplary. The work has essentially no errors. The work is clearly and concisely communicated. The work is justified in sufficient detail. The work may be, in some sense, beautiful.
M	Meets expectations for the assignment (but is not "excellent"). The work is complete and reasonably well-communicated. It is clear that the student understands the material. The explanations are adequate, with at most minor gaps or omissions.
R	Revisions needed. There is a significant gap or omission, or there is evidence of a misconception or misunderstanding of the problem.
N	Not assessable. There are major errors or omissions.

Each problem is broken up into three parts.

8.3.1 Team Problems

There will several problems of each Problem Set that you will do as a Team. You should meet together, work to understand the problem, and discuss how to write up the solution. You should then separate and each write up your own version of the solution. Then come back together and critique each of the other write-ups, ensuring that all of the specifications are met.

You will want each of your team member's proofs to be as good as possible because these will be graded as a team. On the day the Team Problems are due, I will randomly chose one of your team member's write-ups to grade. Everyone in the team who submitted a Team Problem Set that day will receive the same grade, which is based on the randomly chosen write-up (if a team member does not submit

Each problem will be graded according to the usual specificiations, and you (and everyone else in your Team) will receive a PASS for the Team Problem Set of a given topic if you complete (with a grade of E or M) the number of problems specified at the top of the Team Problem Set page (which will be fewer than the total number of problems).

If you do not receive a PASS on your initial attempt, you may resubmit your work as either a Team or as an Individual. If you resubmit as a Team, you will receive a PASS if you meet the criteria in the above paragraph; if you resubmit as an Individual, you will receive a PASS for the Team Problem Set if you receive an E or M on *all* of the problems (so it is easier to receive a PASS on the entire assignment if you do it as a Team).

In the event that one of your Team members does not do the Team Problem Set (or does not do it to the standards of the rest of the Team), the Team must notify me *before* I randomly select the paper. That way, I can remove that Team member from consideration so that the other Team Members are not harmed by their work (that member will have to later submit all of the problems as an Individual, which has a higher bar to receive a PASS).

8.3.2 Individual Problems

There will be Individual Problems in each Topic's Problem Set, which is split into two parts: Standard Problems and Advanced Problems. You will write up one of these as an Individual, with the Standard Problems being easier than the Advanced Problems. It is your choice as to which type you do, although Advanced Problems will help you receive a higher grade than Standard Problems will. You will need to receive an E or M on all problems to get a PASS on this portion of the Problem Set for the Topic.

If you receive a PASS on the Standard Problem Set and later wish to attempt the Advanced Problem Set for a Topic, you may do so. However, you are limited in the number of Problem Sets you may submit in a week, and this will count toward your quota.

Once you have received a PASS on both the Team Problems and either the Standard or Advanced Problems for a Topic, you do not need to do any more work with that Topic's Problem Set.

The purpose of the Problem Sets is to give you an opportunity to help both you and me understand how well you have achieved the course's learning goals.

8.4 Quizzes

You will have Quizzes based on the seven main topics of the course:

1. Groups (and their elements)
2. Subgroups
3. Quotient Groups (and cosets)
4. Isomorphisms
5. Homomorphisms
6. Permutation Groups
7. Rings (and integral domains, fields, and ideals)

We will have a Quiz roughly every other week. The Quiz will be broken into up to seven sections based on the above Topics. Each Topic Section will be graded according to the same set of Specifications as the Problem Sets. Each Quiz Topic will be graded either PASS or NOTYET, with each Quiz problem being graded E, M, R, or N as with the Problem Sets.

For each Topic, you will aim to receive a PASS on at least one Quiz. To do this, you will need to get a an E or M on every question in that Topic Section, in which case you will be given a PASS. Additionally, you will need to Recertify on that Topic by getting an E or M on one problem of that Topic during the last week of class or Final Exam Week; you will have a PASS for that Topic after Recertifying. The Final Exam time (Thursday, December 15th from 1:15–3:15 pm in our usual classroom) will be used exclusively to work on getting a PASS on Quiz Topics and Recertifying (you can simultaneously get a PASS and Recertify on a single Quiz at the end of the semester).

The purpose of the Quizzes is to give you an opportunity to help both you and me understand how well you have achieved the course's learning goals.

9 Grading

There will be five components to your semester grade:

- Daily Homework
- Definition Checks
- Problem Sets
- Quizzes
- Recertifications

10 Semester Grades

Below are what you need to do to get each grade in this class—your semester grade will be the highest grade that applies to you. Every column save for the first and last describe the number of items PASSed for that assignment, and 'PS' is short for 'Problem Set.'

11 Academic Honesty

I expect you to abide by the CSBSJU Plagiarism policy, found in the link below.

`http://www.csbsju.edu/academics/2016-2017-catalog/academic-policies-and-regulations/rights/academic-misconduct`

Because we work together so much in this class, I will give you some examples of acceptable and unacceptable behaviors.

Grade	Daily Homework	Definitions	Team PS	Individual PSTs	Advanced PSTs	QTs	Recertified QTs
A	90%	100%	7	7	5	7	7
AB	90%	100%	7	7	4	7	7
B	90%	90%	7	7	2	7	7
BC	90%	90%	7	7	1	7	6
C	80%	80%	7	6	0	6	5
CD	80%	80%	7	5	0	5	3
D	60%	60%	4	3	0	0	0

Table 1: PSTs denotes Problem Set Topics and QTs denotes Quiz Topics. Recall that there are 7 Topics total.

Assignment	Collaboration Allowed	Unacceptable Behavior
Daily Homework	Unlimited, but you must do the write-up yourself. Note that relying too much on other people will hurt you elsewhere.	Outright copying.
Definition Check	None	Consulting with other people, your computer, your notes, or a text.
Team Problem Sets	You may (*should*) discuss any aspect of the problem with your Team. You may speak with people from other Teams.	Copying another's written work.
Standard and Advanced Problem Sets	You may discuss very general ideas with other people, but not details.	Showing your written work with others. Looking at another's written work. Looking for the answer online or in a textbook. Discussing the details of problems.
Quizzes	None	Consulting with other people, your computer, your calculator, your notes, or a text.

I will act in full accordance with the official policy if there is a violation of school policy—do not expect leniency.

12 Disabilities Requiring Accommodation

If you require accommodations or assistance for a documented disability, please contact Bret as soon as possible. It is best to do this well in advance of the first exam so that we can make whatever arrangements are needed.

13 Careers for Math Majors

There is a meeting on careers for mathematics majors on Tuesday, October 18th at 7 pm in Gorecki 204A. Please attend if you are interested.