Directions:

- Complete the exercises below and either write up or type up your solutions. Solutions must be submitted as PDF or Word documents, uploaded to the appropriate assignment area on Blackboard.
- If you choose to submit handwritten work, it must be neat and legible; if you do your handwritten work on paper, it must be **scanned to a PDF file** and submitted to Blackboard. Instructions and practice for scanning work to PDFs is given in the Startup Assignment. **Do not just take a picture**, **and do not submit a graphics file (JPG**, **PNG**, **etc.)** — such submissions will not be graded.
- Your work will be graded using the EMPX rubric and evaluated **not just on the basis of a right or wrong answer**, **but on the quality**, **completeness**, **and clarity of your work**. Therefore you need to show all work and explain your reasoning on each item.
- Every item must have a good-faith effort at a complete and correct response. If any item is left blank, or shows minimal effort (such as answering "I don't know"), or is significantly incomplete, the entire assignment will be graded "X" (Not Assessable) and you will have to spend a token to revise it.

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1. Here's a proof that if R and S are rings, 0_R and 0_S are the additive identities in those rings, and $f : R \longrightarrow S$ is an isomorphism, then $f(0_R) = 0_S$. That is, isomorphisms of rings must map the additive identity elements onto each other. All the steps are given but none of them are explained. Fill in the reasoning for each step.

   First of all, we know that $f(0_R) \in S$. (No need to explain that step.) Let $y \in S$. Consider the sum $y + f(0_R)$.

   (a) There exists some point $x \in R$ such that $f(x) = y$.

   (b) Therefore $y + f(0_R) = f(x) + f(0_R)$.

   (c) This equals $f(x + 0_R)$.

   (d) This equals $f(x)$.

   (e) Therefore $y + f(0_R) = y$.

   (f) Therefore $f(0_R) = 0_S$.

2. Suppose R and S are rings and $f : R \longrightarrow S$ is an isomorphism. Suppose further than for each $r \in R$, r is its own additive inverse. Prove every element of S its own additive inverse in S. (The previous exercise might be useful.) Then give a mathematical explanation why the following statement is true: If R and S are rings and every element of R is its own additive inverse, but not every element of S is its own additive inverse, then R and S are *not* isomorphic.

3. In earlier Weekly Practices, we introduced the set $R = \{a, b, c, d, e, f, g, h\}$, and defined "addition" (+) and

“multiplication” $(\cdot)$ on R by the following tables:

$+$	a	b	c	d	e	f	g	h
a	a	b	c	d	e	f	g	h
b	b	a	d	c	f	e	h	g
c	c	d	a	b	g	h	e	f
d	d	c	b	a	h	g	f	e
e	e	f	g	h	a	b	c	d
f	f	e	h	g	b	a	d	c
g	g	h	e	f	c	d	a	b
h	h	g	f	e	d	c	b	a

$\cdot$	a	b	c	d	e	f	g	h
a	a	a	a	a	a	a	a	a
b	a	b	c	d	e	f	g	h
c	a	c	e	g	d	b	h	f
d	a	d	g	f	h	e	b	c
e	a	e	d	h	g	c	f	b
f	a	f	b	e	c	h	d	g
g	a	g	h	b	f	d	c	e
h	a	h	f	c	b	g	e	d

This ring has 8 elements in it, obviously. So does $\mathbb{Z}_8$. Is this ring isomorphic to $\mathbb{Z}_8$? If you think so, then prove it by setting up a function from this ring to $\mathbb{Z}_8$ that satisfies the definition of an isomorphism. If you think not, then give a mathematical explanation for why not. A “mathematical explanation” will use proofs or proven results plus logic, not just “look at a table”.