Directions:

- Complete the exercises below and either write up or type up your solutions. Solutions must be submitted as PDF or Word documents, uploaded to the appropriate assignment area on Blackboard.
- If you choose to submit handwritten work, it must be neat and legible; if you do your handwritten work on paper, it must be **scanned to a PDF file** and submitted to Blackboard. Instructions and practice for scanning work to PDFs is given in the Startup Assignment. **Do not just take a picture**, **and do not submit a graphics file (JPG, PNG, etc.)** — such submissions will not be graded.
- Your work will be graded using the EMPX rubric and evaluated **not just on the basis of a right or wrong answer**, **but on the quality**, **completeness**, **and clarity of your work**. Therefore you need to show all work and explain your reasoning on each item.
- Every item must have a good-faith effort at a complete and correct response. If any item is left blank, or shows minimal effort (such as answering "I don't know"), or is significantly incomplete, the entire assignment will be graded "X" (Not Assessable) and you will have to spend a token to revise it.

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1. A lot of people are annoyed that 1 is not considered a prime number. It's understandable, but there's a very good reason. If 1 were to be considered a prime number, how would that affect the conclusions of the Fundamental Theorem of Arithmetic? Give specific examples to support your answer.

2. You might think it would be easy to go to a computer and ask it to generate a list of the first n prime numbers, but in fact it's not obvious how this would work, and not clear that it's easy to do. (Try doing it yourself for $n = 20$ without looking them up!) One famous *algorithm* for generating a list of the first n primes is called the **Sieve of Eratosthenes**. A "sieve" is the thing you use in the kitchen to strain your pasta when you cook it; similarly, a sieve in this context will take a list of numbers and "strain out" the composite ones, leaving only the primes. This video shows you how it works: `https://www.youtube.com/watch?v=V08g_lkKj6Q`. Yes, this video pushes the boundaries on cheesiness.

   (a) In the video, why did the "professor" stop when he got to 10? The answer is not "because all the remaining uncrossed-out numbers are prime". This is true, but the professor didn't look through the list and check each one to see if it's prime; he stopped because he knew without checking that they were all prime. How would he know? (Hint: Does this have something to do with the fact that we were looking at the list from 1 to 100?)

   (b) Use the Sieve of Eratosthenes method to generate a list of all the primes between 1 and 200. Start by making a list or grid of all numbers from 1 to 200; then show your work on the method. Be especially careful to state clearly where you will stop, and why.

3. Prime numbers are at the heart of some of the most famous open problems in all of mathematics. One of those is **Goldbach's Conjecture**, which states:

   > Every even integer greater than 2 can be written as the sum of two (not necessarily distinct) prime numbers.

   Another is the **Twin Prime Conjecture**:

   > There are infinitely many "twin primes", i.e. prime numbers that differ by 2.

   As of today, neither of these conjectures has been either proven or disproven; they are open problems, although there have been some close calls.[1]

[1] Fun fact: The mathematician mentioned in this article, Prof. Richard Arenstorf, was my (Talbert's) professor for Complex Analysis during my Ph.D. work at Vanderbilt.

(a) Click here to choose a random 4-digit even number. (This just generates a 4-digit integer; if your first one is odd, refresh the page to try again.) Then take your number and write it as the sum of two prime numbers. Explain how you went about doing it. (There's no single right way.)

(b) Can you find an even integer that can be written as the sum of two primes *in more than one way*? (I.e. is the sum unique, like the factorization that the Fundamental Theorem of Arithmetic gives?) If so, find the *smallest* integer.

(c) The smallest pair of twin primes is $(3, 5)$. The next smallest is $(5, 7)$. Find the next eight.

4. We're back to Flipgrid this week: `https://flipgrid.com/d0550a14`. Give a 3-minute-or-less reply to these prompts:

- What's something you learned this week in MTH 350 that really stands out for you?
- What did you do this week that was helpful in your learning?
- What would you like to do differently next week?