Directions:

- Complete the exercises below and either write up or type up your solutions. Solutions must be submitted as PDF or Word documents, uploaded to the appropriate assignment area on Blackboard.
- If you choose to submit handwritten work, it must be neat and legible; if you do your handwritten work on paper, it must be **scanned to a PDF file** and submitted to Blackboard. Instructions and practice for scanning work to PDFs is given in the Startup Assignment. **Do not just take a picture**, **and do not submit a graphics file (JPG**, **PNG**, **etc.)** — such submissions will not be graded.
- Your work will be graded using the EMPX rubric and evaluated **not just on the basis of a right or wrong answer**, **but on the quality**, **completeness**, **and clarity of your work**. Therefore you need to show all work and explain your reasoning on each item.
- Every item must have a good-faith effort at a complete and correct response. If any item is left blank, or shows minimal effort (such as answering "I don't know"), or is significantly incomplete, the entire assignment will be graded "X" (Not Assessable) and you will have to spend a token to revise it.

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1. Last week we introduced the set $R = \{a, b, c, d, e, f, g, h\}$, and defined "addition" (+) and "multiplication" $(\cdot)$ on R by the following tables:

$+$	a	b	c	d	e	f	g	h
a	a	b	c	d	e	f	g	h
b	b	a	d	c	f	e	h	g
c	c	d	a	b	g	h	e	f
d	d	c	b	a	h	g	f	e
e	e	f	g	h	a	b	c	d
f	f	e	h	g	b	a	d	c
g	g	h	e	f	c	d	a	b
h	h	g	f	e	d	c	b	a

$\cdot$	a	b	c	d	e	f	g	h
a	a	a	a	a	a	a	a	a
b	a	b	c	d	e	f	g	h
c	a	c	e	g	d	b	h	f
d	a	d	g	f	h	e	b	c
e	a	e	d	h	g	c	f	b
f	a	f	b	e	c	h	d	g
g	a	g	h	b	f	d	c	e
h	a	h	f	c	b	g	e	d

   Your work in Weekly Practice 9 showed that this set with these operations forms a ring. In fact it can be shown (by brute force calculation, so we won't do that here) that $x \cdot y = y \cdot x$ for all $x, y \in R$, making R a *commutative ring.*

   (a) Show that R is a commutative ring with identity, by stating what the identity element 1_R is and proving that it has the property that defines an identity. Remember, simply stating this property does not constitute an explanation; give specific reasoning as to why the property is satisfied.

   (b) Is R an integral domain? If you think so, explain why. If you think not, give a specific counterexample. Remember, simply stating the definition of "integral domain" does not constitute an explanation; give specific reasoning as to why the definition is satisfied.

   (c) Is R a field? If you think so, explain why. If you think not, give a specific counterexample. Remember, simply stating the definition of "field" does not constitute an explanation; give specific reasoning as to why the definition is satisfied.

2. Here's another ring defined abstractly. Let S be the six-element set $S = \{r, s, t, u, v, w\}$ and define + and $\cdot$ using these tables:

$+$	r	s	t	u	v	w
r	r	s	t	u	v	w
s	s	t	u	v	w	r
t	t	u	v	w	r	s
u	u	v	w	r	s	t
v	v	w	r	s	t	u
w	w	r	s	t	u	v

$\cdot$	r	s	t	u	v	w
r	r	r	r	r	r	r
s	r	s	t	u	v	w
t	r	t	v	r	t	v
u	r	u	r	u	r	u
v	r	v	t	r	v	t
w	r	w	v	u	t	s

As with R, it can be shown (hint: this is good practice) that S forms a commutative ring under these operations. With this knowledge, repeat the three items from the previous question, using S: Prove that S has an identity element; prove or disprove that S is an integral domain; prove or disprove that S is a field.

3. Look back at the ring S with six elements. For each element $x \in S$, compute the following, if possible. If it's not possible, explain why. Double check your answers to ensure they make semantic sense; in particular each answer must be either r, s, t, u, v, or w — nothing else makes sense. **Show all your steps unless there is a pattern that you notice that will make some calculations unnecessary.** In the latter case, state that pattern and use it to arrive at your answer.

   (a) $2x$
   (b) $3x$
   (c) $6x$
   (d) $-x$
   (e) $-5x$
   (f) x^2
   (g) x^3
   (h) x^{-1}
   (i) x^{-2}