Directions:

- Complete the exercises below and either write up or type up your solutions. Solutions must be submitted as PDF or Word documents, uploaded to the appropriate assignment area on Blackboard.
- If you choose to submit handwritten work, it must be neat and legible; if you do your handwritten work on paper, it must be **scanned to a PDF file** and submitted to Blackboard. Instructions and practice for scanning work to PDFs is given in the Startup Assignment. **Do not just take a picture**, **and do not submit a graphics file (JPG**, **PNG**, **etc.)** — such submissions will not be graded.
- Your work will be graded using the EMPX rubric and evaluated **not just on the basis of a right or wrong answer**, **but on the quality**, **completeness**, **and clarity of your work**. Therefore you need to show all work and explain your reasoning on each item.
- Every item must have a good-faith effort at a complete and correct response. If any item is left blank, or shows minimal effort (such as answering "I don't know"), or is significantly incomplete, the entire assignment will be graded "X" (Not Assessable) and you will have to spend a token to revise it.

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1. Let $[a]_n$ be the congruence class of a under integer congruence modulo n. For example

$$[8]_3 = \{\ldots, -4, -1, 2, 5, 8, 11, 14, \ldots\}$$

(We usually drop the subscript if the context makes it clear we are only working with a single value of n, but in this exercise we'll be using several different values of n.) Label each of the following statements with **True** if the statement is true; or **False** if the statement is false. Explain your reasoning for each one, being as specific and clear as possible.

   (a) $572 \in [11]_{17}$
   (b) $[11]_3 \in [8]_3$
   (c) $-37 \in [7]_{10}$
   (d) $[7]_4 = [7]_8$
   (e) $[5]_7 \subseteq [10]_{14}$
   (f) $[3]_8 \subseteq [3]_4$
   (g) $[3]_4 \subseteq [3]_8$
   (h) $10 \subseteq [20]_5$

2. Go to this website and spin the wheel to get a random two-digit integer. (Note that some of the integers on the wheel may be single-digit integers; if you get one of those as a result of the spin, throw it out and spin again.) Call your two-digit integer n. Do the following:

   (a) Pick five elements from $\mathbb{Z}_n$ and state their *additive* inverses. These **must** be elements of $\mathbb{Z}_n$. For example, in $\mathbb{Z}_5$ the additive inverse of $[3]$ is not "$-[3]$" or "$[-3]$"; the correct additive inverse must be one of $[0], [1], [2], [3]$, of $[4]$ because those are the elements of $\mathbb{Z}_5$. (The answer is $[2]$. Ask yourself: Why?)
   (b) Find *three* elements of $\mathbb{Z}_n$ that are zero divisors. For each one $[x]$, find a nonzero $[y]$ such that $[x][y] = [0]$.
   (c) List *all* the zero divisors of $\mathbb{Z}_n$ and explain your reasoning. (Don't do it using brute-force methods! Do we have a theorem that would be useful?)

(d) Find *three* elements in $\mathbb{Z}_n$ that are units. For each one, find its *multiplicative* inverse. Remember, like additive inverses, these must actually be elements of $\mathbb{Z}_n$. For example in $\mathbb{Z}_5$, the *multiplicative* inverse of $[4]$ is not "$[1/4]$" because there is no such thing as $[1/4]$. The answer must be one of $[0]$, $[1]$, $[2]$, $[3]$, of $[4]$ because those are the elements of $\mathbb{Z}_5$. (The answer is $[4]$. Ask yourself: Why?)

(e) List *all* the units of $\mathbb{Z}_n$ and explain your reasoning. (Don't do it using brute-force methods! Do we have a theorem that would be useful?) You do not have to find all their multiplicative inverses (but it's good practice to do a few of them on your own).

3. Spin the wheel again and repeat the previous item for a new two-digit n. If you somehow get the same integer as before, or a single-digit integer, throw it out and spin again.

4. Go to: `https://flipgrid.com/d0550a14`. Give a 3-minute-or-less reply to these prompts:

- What's something you learned this week in MTH 350 that really stands out for you?
- What did you do this week that was helpful in your learning?
- What would you like to do differently next week?