Class Work: Inverse functions
This is a full-time activity worth 10 points.

Problems of the Day

1. Let A and B be nonempty sets and let $f : A \to B$ be a bijection. Prove that for every $y \in B$, $(f \circ f^{-1})(b) = b$.

2. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = e^x$.

    (a) Explain why the inverse of this function is not a function.

    (b) Change the codomain of f so that the inverse f^{-1} will now be a function. What is the new codomain and why is f^{-1} now a function?

    (c) Is f^{-1} injective? Surjective? (Hint: You should not need to prove anything in this problem. Look back over your known theorem results.)

    (d) What's the more common name for f^{-1}, the inverse function for $f(x) = e^x$? What are its domain and codomain?

    (e) Explain how problem 1 in this activity proves the identities:

    $$e^{\ln x} = x, \ \forall x > 0 \qquad \text{and} \qquad \ln(e^x) = x, \ \forall x \in \mathbb{R}$$

Parameters

If your group finishes your work, please hand it in at the end of class. If all groups finish by the end of class, we will take time to debrief the solutions to one or more of these.