Name: ____________________________

Instructions: Welcome to Exam 2. You may use the 3×5 notecard with notes from Exam 1, a new 3×5 notecard made specifically for this exam, and a calculator. You may NOT use any device that can communicate with another device. The backs of each page are blank; use them if needed. On all questions other than multiple choice, give complete and correct solutions; answers without accompanying work will be given no credit.

The test will end at the normal ending time of your class (10:50am for Section 01 and 11:50am for Section 02). No extensions or extra time will be given unless you have received prior permission from the instructor.

Items 1—10 are multiple choice questions that address a variety of learning objectives. Please circle the ONE response you believe is most correct. You do not need to justify your answer.

1. (2 points) If an integer k is congruent to $5 \pmod 7$, it means that
   (a) The quotient obtained when dividing k by 7 is 5
   (b) The quotient obtained when dividing k by 5 is 7
   (c) The remainder obtained when dividing k by 7 is 5
   (d) The remainder obtained when dividing k by 5 is 7
   (e) The remainder obtained when dividing 7 by 5 is k

2. (2 points) The main difference between the Principle of Mathematical Induction (which we can abbreviate "PMI") and the Extended Principle of Mathematical Induction ("EPMI") is
   (a) The EPMI uses a different base case than the PMI
   (b) The EPMI allows induction over the set of real numbers
   (c) The EPMI assumes a larger induction hypothesis than the PMI
   (d) The EPMI does not use a base case
   (e) The EPMI does not use an induction hypothesis at all

3. (2 points) In the proof that 4 divides $5^n - 1$, what would be the inductive hypothesis?
   (a) Assume that 4 divides $5^1 - 1$
   (b) Assume that 4 divides $5^k - 1$ for *all* $k \in \mathbb{N}$.
   (c) Assume that 4 divides $5^k - 1$ for *some* $k \in \mathbb{N}$.
   (d) Assume that if 4 divides $5^k - 1$ for some $k \in \mathbb{N}$, then 4 divides $5^{k+1} - 1$ for all $k \in \mathbb{N}$.
   (e) Assume that if 4 divides $5^1 - 1$, then 4 divides $5^k - 1$ for all $k \in \mathbb{N}$.

4. (2 points) Suppose A and B are sets such that card$(A) = 5$ and card$(B) = 3$. Then card$(A \times B)$
   (a) Equals 8
   (b) Equals 15
   (c) Equals 125
   (d) Equals 243
   (e) Is infinite

5. (2 points) Let A and B be sets. If $(a, b) \notin A \times B$, it means that
   (a) $a \notin A$ and $b \notin B$
   (b) $a \notin A$ or $b \notin B$
   (c) $a \in A$ and $b \notin B$
   (d) $a \notin A$ and $b \in B$
   (e) None of the above

Continued →

6. (2 points) Suppose T_n is the sequence defined recursively by $T_n = \frac{1}{2}T_{n-1}$ for all $n \geq 2$. Then T_{10}
   (a) Equals 0
   (b) Equals $1/2048$
   (c) Equals $1/1024$
   (d) Equals $1/512$
   (e) Is impossible to determine based on this information alone

7. (2 points) Let A and B be sets. If $x \in (A \cap B)^c$, then
   (a) $x \notin A$ and $x \notin B$
   (b) $x \notin A$ or $x \notin B$
   (c) $x \in A - B$
   (d) $x \in B - A$
   (e) None of the above

8. (2 points) Which of the following must be explicitly specified when defining a function f?
   (a) The domain of f
   (b) The range of f
   (c) The codomain of f
   (d) All of the above
   (e) Just (a) and (c)

9. (2 points) Let $f : \mathbb{N} \to \mathbb{R}$ be the function defined by $f(x) = \sqrt{x}$. Then the set of preimages of the number 4 is
   (a) $\emptyset$
   (b) $\{-2\}$
   (c) $\{2\}$
   (d) $\{16\}$
   (e) $\{-2, 2\}$

10. (2 points) Suppose f and g are functions whose domain is the set $\mathbb{R}$ of real numbers and that $f(x) = g(x)$ for all x. Then
    (a) The range of f equals the range of g
    (b) The codomain of f equals the codomain of g
    (c) $f = g$ as functions
    (d) All of the above
    (e) None of the above

Continued →

11. Suppose that $U = \mathbb{N}$ and define the following sets:

- $A = \{x \in \mathbb{N} \mid x \geq 7\}$
- $B = \{x \in \mathbb{N} \mid x \text{ is odd}\}$
- $C = \{x \in \mathbb{N} \mid x \text{ is a multiple of } 3\}$
- $D = \{x \in \mathbb{N} \mid x \text{ is even}\}$

Use the roster method to list the elements of the following sets. You do not need to explain your reasoning.

(a) (4 points) $A^c \cap B^c$

(b) (4 points) $(A \cup B) \cap C$

(c) (6 points) $(A - D) \cup (B - D)$

(d) (6 points) $B \times \{x, y\}$

Continued →

12. Define the function $F : \mathbb{N} \to \mathbb{Z}$ by defining $F(n)$ to be the n^{th} Fibonacci number.

(a) (4 points) State the domain and codomain of this function. (Be sure to label which is which.)

(b) (6 points) State the images of the numbers $1, 2, 3, \ldots, 10$. (This is ten items to compute and state.) You do not need to show your work.

(c) (8 points) Is the range of this function equal to its codomain? Explain.

(d) (8 points) Consider the function $G : \mathbb{Z} \to \mathbb{R}$ given by

$$G(n) = \frac{\left(\frac{1}{2}(1+\sqrt{5})\right)^n - \left(\frac{1}{2}(1-\sqrt{5})\right)^n}{\sqrt{5}}$$

It can be verified that for all natural numbers n, $G(n)$ is the n^{th} Fibonacci number (see Portfolio Problem 7a). Does this mean $F = G$ as functions? Explain.

Continued →

13. (12 points) Choose EXACTLY ONE of the following true statements and give a formal proof.

    (a) For each natural number n, 3 divides $n^3 + 23n$.

    (b) For each natural number n, $4^n \equiv 1 \pmod 3$.

    (c) For all integers $n > 4$, $n^2 < 2^n$.

Continued →

14. (12 points) Choose EXACTLY ONE of the following true statements and give a formal proof. In all of these, assume the sets A, B, and C are subsets of some universal set U.

   (a) Prove that the sets $A \cap B$ and $A - B$ are disjoint.

   (b) Prove that $A = (A - B) \cup (A \cap B)$.

   (c) Prove that $A \times (B - C) = (A \times B) - (A \times C)$.

Continued →

The following items are for reflection and application. They are to be completed outside of class and returned to me by email (**talbertr@gvsu.edu**) before **noon on Tuesday, November 6**. Please use as your subject line:

[Lastname] [section] final item #exam2

where [Lastname] is replaced by your last name and [section] is replaced with either 01 or 02. The hashtag (#exam2) is important for filing purposes, so please remember to include it. **No late submissions will be accepted.** Also please type your responses directly into the body of your email — **do not send email attachments this time**.

15. (6 points) Please take some time to reflect on the Exam you just took, and respond to these questions. Please be substantive and specific. You CAN lose points on these questions if you aren't.

    - What item on this exam shows your best work, and why do you think so? Note that "your best work" may not be the problem you feel most sure about — it might be a problem you are not completely sure of, but which shows your best effort or which shows your attainment of the learning objectives most clearly.
    - What are some concepts and learning objectives from the exam that you still need to work on? Why do you think so?
    - How do you see the content from this Exam (Division Algorithm, induction, sets, functions) connecting back to the content from the first exam? Give specific examples, and more than just one or two of them.

16. (4 points) Give me a status report on your experience in the class so far. Address the questions:

    - What's working particularly well for you in terms of your learning? In particular, if something has been improved since the last time you were asked this question on Exam 1, please point it out.
    - As we move into the last 4–5 weeks of the semester, what's something that we can do that will help your learning and attainment of the course objectives?