MTH 210: Communicating in Mathematics
Proof Portfolio Problems 6 and 7

Problem 6
Choose either Problem 6A or Problem 6B to do.

Problem 6A: Make a conjecture about a formula for the product

$$\left(1-\frac{1}{4}\right)\cdot\left(1-\frac{1}{9}\right)\cdot\left(1-\frac{1}{16}\right)\cdot\cdots\cdot\left(1-\frac{1}{n^2}\right)$$

where n is a natural number. Then, state your conjecture as a proposition and prove it.

Problem 6B: Make a conjecture about a formula for the product

$$\left(1+\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1+\frac{1}{4}\right)\cdots\left(1+\frac{(-1)^n}{n}\right)$$

where n is a natural number. Then, state your conjecture as a proposition and prove it.

Problem 7
Choose ONE of the following problems to do.

All of the problems in this group involve the Fibonacci sequence, defined by

$$f_1 = 1 \quad f_2 = 1 \quad f_n = f_{n-1} + f_{n-2} \text{ if } n \geq 3$$

Problem 7A: Let $\alpha = \dfrac{1+\sqrt{5}}{2}$ and $\beta = \dfrac{1-\sqrt{5}}{2}$. Show that for all natural numbers n,

$$f_n = \frac{\alpha^n - \beta^n}{\alpha - \beta}$$

Problem 7B: Prove that for all natural numbers n, f_{5n} is a multiple of 5.

Problem 7C: (For those with some familiarity with matrix multiplication.) Prove that for all natural numbers $n \geq 2$,

$$\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^n = \begin{bmatrix} f_{n+1} & f_n \\ f_n & f_{n-1} \end{bmatrix}$$

Matrix exponentiation A^n is defined as $\underbrace{A\cdot\cdots\cdot A}_{n\text{ times}}$.