Class Work: More Methods of Proof

This is a FULL-TIME activity worth 10 points. These are all proofs; you may use know-show tables to set up your arguments, but please develop a formal, paragraph-style proof to turn in.

1. Prove that for every positive real number x, if x is irrational, then $\sqrt{x}$ is irrational. (*Hint*: What's the contrapositive of this statement? Also, make sure you know the precise definition of a rational number before you start proving anything.)

2. Suppose we have a right triangle whose hypotenuse has length c, and the lengths of the other sides are a and b. Prove that this right triangle is isosceles if and only if its area equals $\frac{1}{4}c^2$.

If you finish all of these...

- Prove that for all $p, q \in \mathbb{Q}$ with $p < q$, there exists an $x \in \mathbb{Q}$ such that $p < x < q$. (In English, this would say that between any two distinct rational numbers is another rational number.)

- Let a, b be natural numbers such that $a^2 = b^3$. Prove that if $4|b$, then $8|a$.

Successful completion of each additional problem will add 1 point to both your Class Work score and the overall Class Work total. However, you must attain a score of at least 8/10 on the main Class Work to receive these points. (That is, you've completed all the problems and they are mostly correct.)

Specifications

Please hand in a clean copy of your work by the end of your class period. This copy should be written up on paper neatly and in an organized way. All members of the group working on problems should add their names to the paper. Do not include the paper with the problems on it. All groups are expected to submit their work by the end of class. I have the right to grant extensions if the majority of groups are working productively but still not completing the problems on time. *However*: Don't expect extensions. Work as if the end of class were a hard deadline.