MTH 210 Learning Objectives

Topic

- ☐ **Chapter 1: Introduction to Writing Proofs in Mathematics**
    - ☐ **1.1: Statements and conditional statements**
        - ☐ Tell the difference between a mathematical statement (proposition) and a sentence that is not a mathematical statement.
        - ☐ Explain what it means for a mathematical statement to be "true" or "false".
        - ☐ Decide whether a statement is true or false using various exploration techniques.
        - ☐ Identify a statement as a conditional statement, and identify its hypothesis and conclusion.
        - ☐ Construct the truth table for a simple conditional statement and state the conditions under which a conditional statement is true.
        - ☐ Find a counterexample for a false conditional statement.
        - ☐ Draw conclusions from conditional statements, given information about the smaller statements involved.
        - ☐ Define the sets of real numbers, rational numbers, irrational numbers, natural numbers, and integers and determine whether these sets are closed under various arithmetic operations.
    - ☐ **1.2: Constructing direct proofs**
        - ☐ Construct examples and non-examples of a definition. (Even/odd)
        - ☐ Construct a "know-show" table for a direct proof by use of both forward questions and backward questions.
        - ☐ Convert a know-show table into a verbal proof (using the standards for writing outlined in the syllabus).
        - ☐ Explain what a mathematical proof is, in general.
- ☐ **Chapter 2: Logical Reasoning**
    - ☐ **2.1: Statements and Logical Operators**
        - ☐ Identify the conjunction, disjunction, negation, and implication operators and use their notation correctly.
        - ☐ Phrase the negation of a statement in a way that does not use the word "not".
        - ☐ Construct truth tables for conjunctions, disjunctions, negations, and implications.
        - ☐ Phrase a conditional statement in several different English formats.
        - ☐ Construct truth tables for compound statements involving three or more variables.
        - ☐ Identify a biconditional statement; construct a truth table and phrase it in several different English formats.
        - ☐ Define the terms "tautology" and "contradiction" and use truth tables to determine if a statement is a tautology or a contradiction.
    - ☐ **2.2: Logically Equivalent Statements**
        - ☐ Use truth tables to determine if two statements are logically equivalent.
        - ☐ State the converse and contrapositive of a conditional statement.
        - ☐ State DeMorgan's Laws and apply them to simplify a statement involving negations and either conjunctions or disjunctions.
        - ☐ Derive, state and use the results of Theorem 2.6.
        - ☐ Form the negation of a conditional statement.
        - ☐ Use previously-established logical equivalencies to determine if two statements are logically equivalent.
        - ☐ Derive, state, and use the results of Theorem 2.8 (Important Logical Equivalencies).
    - ☐ **2.3: Open Sentences and Sets**
        - ☐ Define the term "set" and write the contents of a set using the roster method.
        - ☐ Use correct notation for the sets real numbers, rational numbers, integers, and natural numbers.
        - ☐ Use correct notation to denote that an object is an element of a set (or not an element of a set).
        - ☐ Define what it means for two sets to be equal.
        - ☐ Define what it means for one set to be a subset of another, and use correct notation to denote this relationship.
        - ☐ Define the term "open sentence" (a.k.a. "predicate" or "propositional function") and distinguish between an open sentence and a logical statement.
        - ☐ Find values of variables that make an open sentence true or false; then determine the truth set of that open sentence.
        - ☐ Write the contents of a set using set builder notation.

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- ☐ Define the term "empty set" and use correct notation to denote this set.
- ☐ **2.4: Quantifiers and Negations**
    - ☐ Define the terms "universal quantifier" and "existential quantifier"; identify these quantifiers in practice and use correct notation to denote them.
    - ☐ Determine whether a quantified statement is true or false.
    - ☐ Phrase both universally and existentially quantified statements in several different English formats.
    - ☐ Find a counterexample for a (false) universally quantified statement.
    - ☐ Form the negation of universally quantified and existentially quantified statements; write these negations in forms that do not use the negation symbol.
    - ☐ Rewrite a conditional statement as a universally quantified statement.
    - ☐ Construct examples and non-examples of definitions that involve quantified statements.
    - ☐ Phrase doubly-quantified statements in English, and determine if such statements are true or false.
    - ☐ Form the negation of a doubly-quantified statement.
- ☐ **Chapter 3: Constructing and Writing Proofs in Mathematics**
  - ☐ **3.1: Direct Proofs**
    - ☐ State and instantiate the definitions of "divides", "divisor", "factor", and "multiple" and use correct notation to work with these concepts.
    - ☐ Explain what a "proof" is and what it takes for a proof to be correct.
    - ☐ Explain the concepts of "undefined terms" and "axioms" and give examples of these in real life.
    - ☐ Explain the terms "conjecture", "theorem", "proposition", "lemma", and "corollary".
    - ☐ Construct a know-show table for a conditional statement using a combination of forward and backward steps.
    - ☐ Convert a completed know-show table into a well-written English paragraph proof that adheres to MTH 210 writing guidelines.
    - ☐ State what it means for two integers to be "congruent modulo n".
    - ☐ Determine the truth value of propositions involving divisibility, integer congruence, and other basic arithmetic concepts. If a proposition is true, construct a correct proof. If not, demonstrate a working counterexample.
  - ☐ **3.2: More Methods of Proof**
    - ☐ Write the contrapositive of a conditional statement (possibly complex).
    - ☐ Construct a proof for a conditional statement by proving the contrapositive.
    - ☐ Construct a proof for a biconditional statement by proving two conditional statements.
    - ☐ Explain the concept of a "constructive proof" and write a correct constructive proof for a proposition when it makes sense to do so.
    - ☐ Explain the difference between a constructive and non-constructive proof.
  - ☐ **3.3: Proof by Contradiction**
    - ☐ Explain the basic strategy behind a proof by contradiction and why such a proof should be convincing.
    - ☐ Correctly formulate all the assumptions made at the beginning of a proof by contradiction, and then correctly state what you want to show having made those assumptions.
    - ☐ Explain some situations in which a proof by contradiction might be an appropriate strategy for proof.
    - ☐ Construct a correct proof by contradiction for a proposition when it makes sense to do so, including: proofs of the nonexistence of an object, proofs involving rational and irrational numbers, and so on.
    - ☐ Prove that sqrt(2) is irrational.
  - ☐ **3.4: Using Cases in Proofs**
    - ☐ Explain the basic concept and strategy behind a proof using cases.
    - ☐ Give examples of situations in which using cases in a proof might be appropriate, and give the cases appropriate for each such situation.
    - ☐ Construct correct proofs using cases for a proposition when it makes sense to do so, including: proofs involving even/odd integers, proofs involving rational/irrational numbers, and proofs involving absolute values.
  - ☐ **3.5: The Division Algorithm and Congruence**
    - ☐ State the Division Algorithm and use it to write an integer as a combination of a quotient and remainder.

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- ☐ Construct a correct proof using cases where the cases are created by the remainder in the Division Algorithm.
- ☐ Derive, state, and use the results of Theorems 3.28 and 3.30 (Properties of Congruence Modulo n).
- ☐ Construct a correct proof using cases where the cases are created by congruence modulo n.

- ☐ **Chapter 4: Mathematical Induction**
  - ☐ **4.1: The Principle of Mathematical Induction**
    - ☐ State and instantiate the definition of "inductive set".
    - ☐ State the Principle of Mathematical Induction and explain why it is true.
    - ☐ Explain the basic strategy behind a proof by induction and why such a proof should be convincing.
    - ☐ State the basis step in a proof by induction.
    - ☐ State the inductive step and the inductive hypothesis in a proof by induction.
    - ☐ Construct a complete and correct proof using mathematical induction.
  - ☐ **4.2: Other Forms of Mathematical Induction**
    - ☐ State and instantiate the definitions of n! (n factorial), "prime number", and "composite number".
    - ☐ State the Extended Principle of Mathematical Induction; explain how it is different from the Principle of Mathematical Induction and why we might want to use it.
    - ☐ Construct a complete and correct proof using the Extended Principle of Mathematical Induction.
    - ☐ State the Second Principle of Mathematical Induction; explain how it is different from the Principle of Mathematical Induction and why we might want to use it.
    - ☐ Construct a complete and correct proof using the Second Principle of Mathematical Induction.
  - ☐ **4.3: Induction and Recursion**
    - ☐ List elements of a sequence whose terms are defined recursively (including the integers in the Fibonacci sequence).
    - ☐ Construct proofs using mathematical induction for propositions involving recursively-defined sequences.
- ☐ **Chapter 5: Set Theory**
  - ☐ **5.1: Sets and Operations on Sets**
    - ☐ State and instantiate the definitions of "intersection", "union", "set difference", "complement", "proper subset"
    - ☐ Illustrate the concepts of intersection, union, difference, and complement using a Venn diagram.
    - ☐ Phrase a statement about proper subsets into a quantified statement and vice-versa.
    - ☐ State and use the result of Theorem 5.2 (test for set equality).
    - ☐ Construct the power set of a finite set.
    - ☐ State and instantiate the definition of "cardinality" of a finite set.
  - ☐ **5.2: Proving Set Relationships**
    - ☐ Use the "choose an element" method to prove that one set is a subset of another.
    - ☐ Use the "choose an element" method to prove that one set is equal to another.
    - ☐ State and instantiate the definition of "disjoint" sets.
    - ☐ Prove that two sets are disjoint.
  - ☐ **5.3: Properties of Set Operations**
    - ☐ Derive, state, and use the results of Theorem 5.18 (Algebra of Set Operations).
    - ☐ Derive, state, and use the results of Theorem 5.20.
    - ☐ Prove that three or more statements are equivalent.
  - ☐ **5.4: Cartesian Products**
    - ☐ State and instantiate the definitions of "ordered pair", "coordinates", and the "Cartesian product" of two sets.
    - ☐ Visualize Cartesian products of sets of real numbers in the plane.
    - ☐ Derive, state, and use the results of Theorem 5.25.
  - ☐ ***5.5: Indexed Families of Sets***
    - ☐ *State and instantiate the definition of the union and intersection of an indexed family of sets.*
    - ☐ *Derive, state, and use the results of Theorem 5.30.*
- ☐ **Chapter 6: Functions**
  - ☐ **6.1: Introduction to Functions**

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- ☐ State and instantiate the definitions of "function", "domain", "codomain", "range", "image" (of a point and of a function), and "preimage" (of a point), and use correct mathematical notation when working with functions.
- ☐ Exhibit fluency with working with functions whose domains or codomains are not sets of real numbers and which are not given by closed-form formulas (e.g. the birthday function, the sum of divisors function).
- ☐ Represent a function between finite sets using an arrow diagram.

- ☐ **6.2: More about Functions**
  - ☐ State what it means for two functions to be equal, and either prove or disprove that two functions are equal.
  - ☐ State and instantiate the "identity function" on a set.
  - ☐ Work with functions involving congruences, mathematical processes, matrices, and sequences.
  - ☐ Work with functions having more than one input.
- ☐ **6.3: Injections, Surjections, and Bijections**
  - ☐ State and instantiate the definitions of "injection", "surjection", and "bijection".
  - ☐ Determine whether a given function is an injection, surjection, or bijection.
  - ☐ Give examples of functions that are injections but not surjections; surjections but not injections; neither injections nor surjections; or bijections.
- ☐ **6.4: Composition of Functions**
  - ☐ State and instantiate the definition of the "composition" of two functions.
  - ☐ Derive, state, and use the results of Theorem 6.20.
- ☐ **6.5: Inverse Functions**
  - ☐ Represent a function as a set of ordered pairs.
  - ☐ State and instantiate the definition of the "inverse" of a function.
  - ☐ Derive, state, and use the result of Theorem 6.25 (conditions under which f^{-1} is a function).
  - ☐ Explain the notation for inverse functions and use this notation correctly.
  - ☐ Derive, state, and use the result of Corollary 6.28 (compositions of inverse functions).
- ☐ **6.6: Functions Acting on Sets**
  - ☐ State and instantiate the definitions of the "image" and "preimage" of a set under a function.
  - ☐ Derive, state, and use the results of Theorem 6.34 (images of unions and intersections) and 6.35 (preimages of unions and intersections).

- ☐ **Chapter 7: Equivalence Relations**
  - ☐ **7.1: Relations**
    - ☐ State and instantiate the definition of a "relation" from one set to another (or on a set by itself).
    - ☐ Determine whether a set of ordered pairs forms a relation. If so, find the domain and the range.
    - ☐ Work with standard mathematical relations given in Table 7.1.
    - ☐ Draw the directed graph for a relation.
  - ☐ **7.2: Equivalence Relations**
    - ☐ Determine whether a relation is symmetric, reflexive, and/or transitive. Give examples of relations which have exactly zero, one, two, or three of these three properties.
    - ☐ State and instantiate the definition of "equivalence relation" on a set, and use correct mathematical notation when working with equivalence relations.
    - ☐ Give examples of standard mathematical equivalence relations, including equality, congruence modulo n, and equivalence of rational numbers.
  - ☐ **7.3: Equivalence Classes**
    - ☐ Determine the equivalence class of a point under a given equivalence relation.
    - ☐ Derive, state, and use the results of Theorem 7.14 (properties of equivalence classes).
    - ☐ State and instantiate the definition of a "partition" of a set.
    - ☐ Derive, state, and use the results of Theorem 7.18 (equivalence classes yield partitions and vice versa) .
  - ☐ **7.4: Modular Arithmetic**
    - ☐ State and instantiate the definition of the "set of integers modulo n" (Z_n).
    - ☐ Derive, state, and use the results of Theorems 3.28 and Corollary 7.19 (properties of modular arithmetic).
    - ☐ Carry out addition and multiplication modulo n.
    - ☐ Prove the validity of divisibility tests for 3 and 9.