Proposition. For all real numbers x and y, if x is irrational and y is rational, then $x + y$ is irrational.

Proof. We will use a proof by contradiction. So we assume that the proposition is false, which means that there exist real numbers x and y where $x \notin \mathbb{Q}$, $y \in \mathbb{Q}$, and $x + y \in \mathbb{Q}$. Since the rational numbers are closed under subtraction and $x + y$ and y are rational, we see that

$$(x + y) - y \in \mathbb{Q}.$$

However, $(x + y) - y = x$, and hence we can conclude that $x \in \mathbb{Q}$. This is a contradiction to the assumption that $x \notin \mathbb{Q}$. Therefore, the proposition is not false, and we have proven that for all real numbers x and y, if x is irrational and y is rational, then $x + y$ is irrational. ■