Conceptual Questions: Exam 1 Review

MTH 210: Communicating in Mathematics
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Which of the following are you allowed to use during Exam 1?

(a) A pen

(b) A 3×5 notecard with notes on it

(c) An iPod touch calculator app

(d) All of the above

(e) Jut (a) and (b)

(f) None of the above

Which of the following are statements?

(a) $3^2 + 4^2 = 5^2$.

(b) $a^2 + b^2 = c^2$.

(c) If $x^2 = 4$, then $x = 2$.

(d) Every square is a rectangle.

(e) All of the above

(f) Three of the above (specify)

(g) Two of the above (specify)

Consider the statement

If $8 < 5$*, then* $a + 2 = 10$*.*

This statement is true

(a) Always

(b) Sometimes (depends on a)

(c) Never

Every conditional statement $P \to Q$ is logically equivalent to

(a) Its converse

(b) Its contrapositive

(c) Its negation

(d) Its equivalent biconditional statement $P \leftrightarrow Q$

(e) More than one of the above (be ready to specify)

The negation of $P \to Q$ is

(a) $\neg P \to Q$

(b) $P \to \neg Q$

(c) $P \vee \neg Q$

(d) $\neg P \wedge Q$

(e) None of the above

Let $A = \{2, 4, 6, 8\}$ and $B = \{x \in \mathbb{N} \mid x^2 < 100\}$. Then

(a) $A \subseteq B$

(b) $B \subseteq A$

(c) $A = B$

(d) All of the above

(e) None of the above

Consider the predicate "n is prime and congruent to 0 (mod 4)". The truth set of this statement is

(a) $\mathbb{Z}$

(b) The set of all prime numbers

(c) The set of all natural numbers divisible by 4

(d) $\{2\}$

(e) $\emptyset$ (the empty set)

Consider the statement:

> *There exists no prime number that is congruent to* 0 *modulo* 4.

The negation of this statement would say

(a) There exists a prime number that is congruent to 0 modulo 4.

(b) There exists a prime number that is not congruent to 0 modulo 4.

(c) Every prime number is congruent to 0 modulo 4.

(d) Every prime number fails to be congruent to 0 modulo 4.

What is the smallest nonnegative integer that is congruent to 1179 modulo 10?

(a) 0

(b) 1

(c) 9

(d) 79

(e) This number does not exist

Suppose $a \equiv 0 \pmod{12}$. Then

(a) a divides 12

(b) a is even

(c) a is divisible by 6

(d) All of the above

(e) Just (b) and (c)

Consider the statement:

If $a \equiv 0 \pmod{12}$, *then* $6|a$.

If you tried to prove this using proof by contraposition, you would assume

(a) $a \equiv 0 \pmod{12}$

(b) $a \not\equiv 0 \pmod{12}$

(c) $6|a$

(d) $6 \not| a$

(e) $a \equiv 0 \pmod{12}$ and $6 \not| a$

Consider the statement:

If $a \equiv 5 \pmod{12}$*, then* $a^2 \equiv 1 \pmod{12}$*.*

If you wanted to use cases in a direct proof of this statement, which of the following sets of cases are valid?

(a) Two cases: $a \geq 0$ and $a < 0$

(b) Two cases: a prime and a not prime

(c) Three cases: $a < 0$, $0 \leq a < 11$, and $a \geq 12$

(d) All of the above

For each of the following proofs, read carefully and then vote:

1. The proposition is false (and therefore the “proof” cannot be correct)
2. The proposition is true, but the proof is wrong
3. The proposition is true and the proof is right, but the proof needs improvement
4. The proposition is true and the proof needs no correction

Proposition. If m is an even integer, then $(5m + 4)$ is an even integer.

Proof. We see that $5m + 4 = 10n + 4 = 2\,(5n + 2)$. Therefore, $(5m + 4)$ is an even integer. $\blacksquare$

Proposition. For each real number x, $x(1-x) \le \dfrac{1}{4}$.

Proof. A proof by contradiction will be used. So we assume the proposition is false. This means that there exists a real number x such that $x(1-x) > \dfrac{1}{4}$. If we multiply both sides of this inequality by 4, we obtain $4x(1-x) > 1$. However, if we let $x = 3$, we then see that

$$\begin{aligned} 4x(1-x) &> 1 \\ 4 \cdot 3(1-3) &> 1 \\ -12 &> 1 \end{aligned}$$

The last inequality is clearly a contradiction and so we have proved the proposition. ■

Proposition. For all integers a, b, and c, if $a \mid (bc)$, then $a \mid b$ or $a \mid c$.

Proof. We assume that a, b, and c are integers and that a divides bc. So, there exists an integer k such that $bc = ka$. We now factor k as $k = mn$, where m and n are integers. We then see that

$$bc = mna.$$

This means that $b = ma$ or $c = na$ and hence, $a \mid b$ or $a \mid c$. ■