Class Work: Relations

This is a full-time activity worth 10 points.

Problem of the Day

1. Let $\mathbb{Z}_9 = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$. Define the relation $\sim$ on $\mathbb{Z}_9$ as follows: For all $a, b \in \mathbb{Z}_9$, $a \sim b$ if and only if $a^2 \equiv b^2 \pmod 9$. It can be proven (although you do not have to do this right now) that this is an equivalence relation on $\mathbb{Z}_9$.

   (a) Fill in the following table:

x	0	1	2	3	4	5	6	7	8
$x^2 \pmod 9$									

   (b) List all the elements of $[0]$. Hint: There are three of them.

   (c) Find all the distinct equivalence classes under this relation and list their contents in roster form. Hint: There are four classes, and they are not all the same size.

2. Define the relation $\sim$ on $\mathbb{R}$ as follows: For $x, y \in \mathbb{R}$, $x \sim y$ if and only if $xy \geq 0$.

   R - {0}

   (a) Prove that this relation is reflexive.

   (b) Prove that this relation is symmetric.

   (c) Prove that this relation is transitive.

   (d) You've now proven that this relation is an equivalence relation, so we can talk about its equivalence classes. How many distinct equivalence classes does this relation have? Prove that your answer is right and list all the distinct classes.

Parameters

If your group finishes your work, please hand it in at the end of class. If all groups finish by the end of class, we will take time to debrief the solutions to one or more of these.