

Class Work: Proof by Contradiction

This is a FULL-TIME activity worth 10 points. These are all proofs; you may use know-show tables to set up your arguments, but please develop a formal, paragraph-style proof to turn in.

Below are four statements that might be suited for a proof by contradiction.

- For each of the four, state clearly all the assumptions you would make at the beginning of a proof by contradiction. Write this as an actual sentence that you would use in the opening statement of a proof, for example "For a contradiction, we will assume that..."
- Choose TWO of these four and write a complete proof by contradiction.

1. For every real number θ , if $0 < \theta < \frac{\pi}{2}$, then $(\sin \theta + \cos \theta) > 1$.
2. For all real numbers a and b , if $a \neq 0$ and $b \neq 0$, then $\sqrt{a^2 + b^2} \neq 0$.
3. There is no integer x such that $x^3 - 4x^2 = 7$.
4. For all real numbers a and b , if $a > 0$ and $b > 0$ then

$$\frac{2}{a} + \frac{2}{b} \neq \frac{4}{a+b}$$

If you finish all of these...

...then go back and write a complete proof of one of the other statements in the list above.

Successful completion of each additional problem will add 1 point to both your Class Work score and the overall Class Work total. However, you must attain a score of at least 8/10 on the main Class Work to receive these points. (That is, you've completed all the problems and they are mostly correct.)

Specifications

Please hand in a clean copy of your work by the end of your class period. This copy should be written up on paper neatly and in an organized way. All members of the group working on problems should add their names to the paper. Do not include the paper with the problems on it. All groups are expected to submit their work by the end of class. I have the right to grant extensions if the majority of groups are working productively but still not completing the problems on time. *However:* Don't expect extensions. Work as if the end of class were a hard deadline.