Class Work: Relations

This is a full-time activity worth 10 points.

Problems of the Day

1. Let $A = \{a, b, c\}$ and let $R = \{(a, a), (a, c), (b, b), (b, c), (c, a), (c, b)\}$. This makes R a relation on A. Are the following statements true, or are they false? State your answer and explain each one.
   (a) For each $x \in A$, $(x, x) \in R$.
   (b) For every $x \in A$, if $(x, y) \in R$, then $(y, x) \in R$.
   (c) For every $x \in A$, if $(x, y) \in R$ and $(y, z) \in R$, then $(x, z) \in R$.
   (d) R is a function from A into A.
2. Draw the directed graph for the relation R in problem 1. Does the directed graph help you answer the questions more easily?
3. Let $U = \{x, y, z\}$ and recall that $\mathcal{P}(U)$ is the power set of U, that is, the set whose elements are all the subsets of U. Define a relation R on $\mathcal{P}(U)$ by:
$$R = \{(S, T) \in \mathcal{P}(U) \times \mathcal{P}(U) \mid S \subseteq T\}$$
   (a) List the elements of $\mathcal{P}(U)$. There should be eight of these.
   (b) Draw the directed graph for the relation R. Note that the number of vertices in this graph should equal the number you found in part (a).
   (c) Is it the case that for each $S \in \mathcal{P}(U)$, $(S, S) \in R$? Why or why not?
   (d) Is it the case that for each $S, T \in \mathcal{P}(U)$, if $(S, T) \in R$, then $(T, S) \in R$? Why or why not?
   (e) Is it the case that for each $S, T, V \in \mathcal{P}(U)$, if $(S, T) \in R$ and $(T, V) \in R$, then $(T, V) \in R$? Why or why not?

Parameters

If your group finishes your work, please hand it in at the end of class. If all groups finish by the end of class, we will take time to debrief the solutions to one or more of these.