

**Class Work: Quantifiers and Negations**

1. Each of the following universally quantified statements is false. Give a specific counterexample to show this, and then write the negation of the statement in English (without using any symbols for quantifiers).

(a)  $(\forall m \in \mathbb{Z}) \left( \frac{m}{3} \in \mathbb{Z} \right)$

(b)  $(\forall a \in \mathbb{Z}) (\sqrt{a^2} = a)$

2. For each of the following statements, write the statement as an English sentence that does not use quantifier symbols. Then write the negation of the statement that does use quantifier symbols. Finally, write the negation in English without quantifier symbols.

(a)  $(\exists x \in \mathbb{Q}) (x > \sqrt{2})$

(b)  $(\forall x \in \mathbb{Z}) (x \text{ is even or } x \text{ is odd})$

(c)  $(\forall x \in \mathbb{Z}) (\text{If } x^2 \text{ is odd, then } x \text{ is odd})$

3. Let  $\mathbb{Z}^*$  be the set of all nonzero integers. Here's a statement with two quantifiers:

For each  $x \in \mathbb{Z}^*$ , there exists a  $y \in \mathbb{Z}^*$  such that  $xy = 1$ .

(a) Is this statement true or false? Why?

(b) Write the negation of this statement in English with no quantifier symbols.

**Specifications**

Please hand in a clean copy of your work by the end of your class period. This copy should be written up on paper neatly and in an organized way. All members of the group working on problems should add their names to the paper. Do not include the paper with the problems on it. All groups are expected to submit their work by the end of class. I have the right to grant extensions if the majority of groups are working productively but still not completing the problems on time. *However:* Don't expect extensions. Work as if the end of class were a hard deadline.