

Class Work: Direct Proof

This is a FULL-TIME activity worth 10 points. These are all proofs; you may use know-show tables to set up your arguments, but please develop a formal, paragraph-style proof to turn in.

1. Let a be an integer and let $n \in \mathbb{N}$.
 - (a) Prove that if $a \equiv 0 \pmod{n}$, then $n|a$.
 - (b) Prove that if $n|a$, then $a \equiv 0 \pmod{n}$.
2. Prove that for every integer a , if $a \equiv 3 \pmod{8}$, then $a^2 \equiv 1 \pmod{8}$. Is the converse of this statement also true?

If you finish all of these...

Prove or disprove the following statements:

- For all integers a and b , if $ab \equiv 0 \pmod{6}$ then either $a \equiv 0 \pmod{6}$ or $b \equiv 0 \pmod{6}$.
- For all real numbers x and y , we have $xy \leq \left(\frac{x+y}{2}\right)^2$. If you think this statement is true, beware of giving a “backwards” proof that begins with the inequality you are trying to prove.

Successful completion of this additional problem will add 2 points to both your Class Work score and the overall Class Work total. However, you must attain a score of at least 8/10 on the main Class Work to receive these points. (That is, you’ve completed all the problems and they are mostly correct.)

Specifications

Please hand in a clean copy of your work by the end of your class period. This copy should be written up on paper neatly and in an organized way. All members of the group working on problems should add their names to the paper. Do not include the paper with the problems on it. All groups are expected to submit their work by the end of class. I have the right to grant extensions if the majority of groups are working productively but still not completing the problems on time. *However:* Don’t expect extensions. Work as if the end of class were a hard deadline.