Class Work: Injective functions
This is a full-time activity worth 10 points.

Problems of the Day

For each of the functions below, decide whether the function is an injection or not. If not, give an example that shows this. If so, then give a proof.

1. $A : \mathbb{Z}_6 \to \mathbb{Z}_6$ given by $A(x) = x^2 + 4 \pmod 6$

2. $B : \mathbb{Z}_6 \to \mathbb{Z}_6$ given by $B(x) = x^3 + 4 \pmod 6$

3. $C : \mathbb{N} \to \mathbb{N}$ given by $C(n) =$ the sum of all the positive divisors of n. (Examples: $C(6) = 1 + 2 + 3 + 6 = 12$, $C(10) = 18$)

4. $D : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z} \times \mathbb{Z}$ given by $D(a, b) = (2a, a + b)$.

5. trans $: \mathcal{M}_2(\mathbb{R}) \to \mathcal{M}_2(\mathbb{R})$ given by

$$\text{trans}\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$$

Recall that $\mathcal{M}_2(\mathbb{R})$ is the set of all 2×2 matrices with real number entries. Also, if two matrices are equal, it means all their corresponding entries are equal.

Parameters

If your group finishes your work, please hand it in at the end of class. If all groups finish by the end of class, we will take time to debrief the solutions to one or more of these.