Class Work: Proof using Cases

This is a FULL-TIME activity worth 10 points.

1. Consider the following proposition:

   For all integers a, b and d with $d \neq 0$, if d divides a or d divides b, then d divides ab.

   (a) Notice the hypothesis to this proposition is a disjunction. How could you use this fact to set up a direct proof of the proposition that involves two cases? What would those two cases be?

   (b) In the first of those two cases, what are you assuming? And what do you want to show?

   - **Assuming**:
   - **Proving**:

   (c) Make a "forward" step by taking the assumption from Case 1 and rewriting it using a definition.

   (d) Make a "backward" step by taking the conclusion of Case 1 and rewriting it using a definition.

   (e) In the space below, sketch out a proof of Case 1. ("Sketching" a proof means writing some kind of a rough — but reasonably complete and correct — version of the proof. That could be a know-show table or a paragraph-style proof that is not completely up to the writing standards yet.)

   (f) Now move on to Case 2. What will you assume? What to you want to prove?

   - **Assuming**:
   - **Proving**:

(g) Make a "forward" step by taking the assumption from Case 2 and rewriting it using a definition.

(h) Make a "backward" step by taking the conclusion of Case 2 and rewriting it using a definition.

(i) In the space below, sketch out a proof of Case 2.

(j) Now in the space below, write a complete, formal paragraph proof of the entire proposition.

2. Consider the following proposition:

   For each integer n, if n is odd, then $8|(n^2 - 1)$.

   (a) This is a conditional statement, so you have three basic methods for proving it: direct proof, contraposition, or contradiction. Which one seems most likely to work? Why?

   (b) If you used a direct proof on this statement, what would you assume? And what do you want to prove?

   - **Assuming**:
   - **Proving**:

   (c) Make a "forward" step by rewriting your assumption using a definition.

   (d) Make a "backward" step by rewriting your conclusion using a definition.

   (e) You've assumed something now about n and want to show something about $n^2 - 1$. Make a substitution based on what you know at this point and do some basic algebra to get an expression for $n^2 - 1$:

   $$n^2 - 1 =$$

   (f) Can you tell just from this expression that $n^2 - 1$ is divisible by 8? Why or why not?

   (g) Think of two cases you could introduce at this point that might help the proof move forward.

   - Case 1:
   - Case 2:

   (h) Sketch a proof of Case 1 below.

(i) Sketch a proof of Case 2 below.

(j) Now write up a formal proof for the whole proposition.