Class Work: Introduction to Functions
This is a full-time activity worth 10 points.

Problems of the Day

1. Let $d : \mathbb{N} \to \mathbb{N}$ be the function defined as follows: For each $n \in \mathbb{N}$, $d(n)$ is the number of natural number divisors of n. For example, $d(6) = 4$ because 6 has 4 natural number divisors (1, 2, 3, and 6). Similarly, $d(81) = 5$ because 81 has 5 natural number divisors (what are they?).

   (a) Calculate $d(k)$ for each natural number k from 1 through 12.

   (b) Find all the preimages of the number 1. (Review the definition of "preimage" if needed.)

   (c) Find all the preimages of the number 2.

   (d) Is the following statement true, or false? Justify your conclusion:

   If $m, n \in \mathbb{Z}$ and $m \neq n$, then $d(m) \neq d(n)$.

   (e) Calculate $d(2^k)$ for $k = 1, 2, 3, 4, 5, 6$. Based on your results, make a conjecture for a formula for $d(2^k)$ where k is a nonnegative integer.

   (f) Is the following statement true, or false? Justify your conclusion:

   For each $n \in \mathbb{N}$, there exists a natural number m such that $d(m) = n$.

   If this statement is true, what does it mean about the relationship between the codomain of d and the range of d?

2. Now define a function S that accepts a natural number as input and produces the set of its natural number divisors as output. For example, $S(6) = \{1, 2, 3, 6\}$ and $S(81) = \{1, 3, 9, 27, 81\}$.

   (a) What is the domain of this function? What is the codomain of this function? (Careful with the second question.)

   (b) Determine $S(k)$ for $k = 1, 2, \ldots, 10$ and then for three other values of k.

   (c) Is the following statement true, or false? Justify your conclusion:

   If $m, n \in \mathbb{Z}$ and $m \neq n$, then $S(m) \neq S(n)$.

   (d) Is the range of S equal to the codomain? Why or why not?

Parameters

If your group finishes your work, please hand it in at the end of class. If all groups finish by the end of class, we will take time to debrief the solutions to one or more of these.