Class Work: Mathematical Induction, day 2

Problems of the Day

1. Write a complete proof of one of the five propositions you worked with on Friday (your choice). If you finished this on Friday, just make sure you have a clean writeup to turn in today.

   Proposition 1: For each natural number n,

   $$2 + 5 + 8 + \cdots + (3n - 1) = \frac{n(3n + 1)}{2}$$

   Proposition 2: For each natural number n, $4^n \equiv 1 \pmod 3$.

   Proposition 3: For each natural number n, $x - y$ divides $x^n - y^n$.

   Proposition 4: For each natural number n, 3 divides $n^3 + 23n$.

   Proposition 5: Suppose we draw n straight lines in the plane (which may cross but not lie coincident with each other), dividing it into a number of regions. Then for every natural number n, we may color each region either red or blue in such a way that no two neighboring regions have the same color.

2. Below are some problems to consider that involve creating a conjecture and then proving it using mathematical induction. Choose one to do in your groups.

   (a) Calculate the values of

   $$\frac{1}{2} + \frac{1}{4} \qquad \frac{1}{2} + \frac{1}{4} + \frac{1}{8} \qquad \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}$$

   Based on your results, make a conjecture about the value of the sum

   $$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots + \frac{1}{2^n}$$

   for $n \in \mathbb{N}$. Then prove that conjecture using mathematical induction. What is the base case? What is the inductive hypothesis? What are you going to try to prove after you assume the inductive hypothesis?

   (b) Calculate the values of

   $$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} \qquad \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} \qquad \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \frac{1}{4 \cdot 5}$$

Proposition 0: (*Example*) For each natural number n,

$$1 + 2 + 3 + \cdots + n = \frac{n(n + 1)}{2}$$

- For the base case, I will prove: 1 equals $\frac{1(1+1)}{2}$ (showing left side of the equation equals the right side when $n = 1$)

- For the inductive step, I will **assume**: That for some $k \in \mathbb{N}$,

$$1 + 2 + 3 + \cdots + k = \frac{k(k + 1)}{2}$$

- For the inductive step, I will **prove**:

$$1 + 2 + 3 + \cdots + (k+1) = \frac{(k+1)(k+1+1)}{2}$$

Proposition 1: For each natural number n,

$$2 + 5 + 8 + \cdots + (3n-1) = \frac{n(3n+1)}{2}$$

- For the base case, I will prove:

- For the inductive step, I will **assume**:

- For the inductive step, I will **prove**:

Proposition 2: For each natural number n, $4^n \equiv 1 \pmod 3$.

- For the base case, I will prove:

- For the inductive step, I will **assume**:

- For the inductive step, I will **prove**:

Proposition 3: For each natural number n, $x - y$ divides $x^n - y^n$.

- For the base case, I will prove:

- For the inductive step, I will **assume**:

- For the inductive step, I will **prove**:

Proposition 4: For each natural number n, 3 divides $n^3 + 23n$.

- For the base case, I will prove:

- For the inductive step, I will **assume**:

- For the inductive step, I will **prove**:

Proposition 5: Suppose we draw n straight lines in the plane (which may cross but not lie coincident with each other), dividing it into a number of regions. Then for every natural number n, we may color each region either red or blue in such a way that no two neighboring regions have the same color.

- For the base case, I will prove:

- For the inductive step, I will **assume**:

- For the inductive step, I will **prove**:

Part 2: Group work (2 points)

After time is called, hand in your individual work to the instructor. Then get together with your group and check each others' work. Then, **make an attempt at a proof of one of the propositions labelled 1–5 above (your choice)**. *You do not have to complete a proof today*, but you do need to make a good-faith effort to begin a proof using the setup that you worked out in Part 1. If you make it through the end of the proof, you'll be given some bonus credit for today. Hand in a neat sketch of your work in progress at the end of the session.