

Class Work: Using the Division Algorithm in a Proof

This is a FULL-TIME activity worth 10 points.

Problem of the Day

Today you'll prove the proposition:

Proposition 1. *The number $\sqrt{3}$ is irrational.*

You'll be proving this proposition in two stages, the first of which draws upon what you've learned about the Division Algorithm and its use in setting up cases.

1. Before proving Proposition 1, let's consider the following lemma¹:

Lemma 1. *For every integer a , if a^2 is divisible by 3, then a is divisible by 3.*

- (a) What does the contrapositive of Lemma 1 say?
 - (b) Let's try to prove Lemma 1 by proving its contrapositive. What are you going to assume, and what are you going to prove?
 - We will assume:
 - We will try to prove:
- Remember your eventual proof of this Lemma must not treat the statement to be proven as a fact. Don't assume what you are trying to prove is true.**
- (c) What are the possible remainders you could get if you take a and divide it by 3?
 - (d) Which, if any, of those remainders are ruled out in the contrapositive of Lemma 1?
 - (e) Set up two cases for proving Lemma 1.
 - **Case 1:**
 - **Case 2:**
 - (f) On the very last page, there is space for a formal proof of Lemma 1. Sketch out your proof on a separate page, and then designate one person to write the proof up formally on the last page.

Continued →

¹A "lemma" is a mini-theorem whose main purpose is to provide justification for a step in a larger proof.

2. Now that we've proven Lemma 1, we're ready to try a proof of the main Proposition.

(a) What overall strategy (direct proof, contraposition, contradiction, etc.) are you going to use?

(b) What are you going to assume, and what are you going to try to show?

(c) Use the space below or a blank page to sketch out your proof. The proof here might be very similar to the proof that $\sqrt{2}$ is irrational but you will need to use Lemma 1. Think about how Lemma 1 will play a role. Then write your formal proof in the final page.

Extra Problem

If your group completes the Problem of the Day successfully with time remaining, try this:

Proposition 2. *The number $\sqrt{5}$ is irrational.*

What's a lemma you might need for this proof? Make sure you prove that as part of your work.

Can this method of proof be extended to prove that $\sqrt{6}$ is irrational? Why or why not?

Successful completion of each additional problem will add 1 point to both your Class Work score and the overall Class Work total. However, you must attain a score of at least 8/10 on the main Class Work to receive these points. (That is, you've completed all the problems and they are mostly correct.)

Futher practice

For more practice, please try the following exercises from §3.5, pages 153—157:

- 6, 7, 11, 12, 17, 18.

These are all optional. But if you submit writeups of one or more of these that represent good-faith efforts to complete the problem correctly, I will give you feedback on whatever you submit.

Writeup for Class Work on §3.5 (October 2)

Lemma 1: For all integers a , if 3 divides a^2 , then 3 divides a .

Proof:

Proposition 1: The number $\sqrt{3}$ is irrational.

Proof: