Proposition. For each real number x, $x(1-x) \leq \frac{1}{4}$.

Proof. A proof by contradiction will be used. So we assume the proposition is false. This means that there exists a real number x such that $x(1-x) > \frac{1}{4}$. If we multiply both sides of this inequality by 4, we obtain $4x(1-x) > 1$. However, if we let $x = 3$, we then see that

$$\begin{aligned} 4x(1-x) &> 1 \\ 4 \cdot 3(1-3) &> 1 \\ -12 &> 1 \end{aligned}$$

The last inequality is clearly a contradiction and so we have proved the proposition. ■