Topic

- ☐ **Chapter 1: Introduction to Writing Proofs in Mathematics**
  - ☐ **1.1: Statements and conditional statements**
    - ☐ Tell the difference between a mathematical statement (proposition) and a sentence that is not a mathematical statement.
    - ☐ Explain what it means for a mathematical statement to be "true" or "false".
    - ☐ Decide whether a statement is true or false using various exploration techniques.
    - ☐ Identify a statement as a conditional statement, and identify its hypothesis and conclusion.
    - ☐ Construct the truth table for a simple conditional statement and state the conditions under which a conditional statement is true.
    - ☐ Find a counterexample for a false conditional statement.
    - ☐ Draw conclusions from conditional statements, given information about the smaller statements involved.
    - ☐ Define the sets of real numbers, rational numbers, irrational numbers, natural numbers, and integers and determine whether these sets are closed under various arithmetic operations.
  - ☐ **1.2: Constructing direct proofs**
    - ☐ Construct examples and non-examples of a definition. (Even/odd)
    - ☐ Construct a "know-show" table for a direct proof by use of both forward questions and backward questions.
    - ☐ Convert a know-show table into a verbal proof (using the standards for writing outlined in the syllabus).
    - ☐ Explain what a mathematical proof is, in general.
- ☐ **Chapter 2: Logical Reasoning**
  - ☐ **2.1: Statements and Logical Operators**
    - ☐ Identify the conjunction, disjunction, negation, and implication operators and use their notation correctly.
    - ☐ Phrase the negation of a statement in a way that does not use the word "not".
    - ☐ Construct truth tables for conjunctions, disjunctions, negations, and implications.
    - ☐ Phrase a conditional statement in several different English formats.
    - ☐ Construct truth tables for compound statements involving three or more variables.
    - ☐ Identify a biconditional statement; construct a truth table and phrase it in several different English formats.
    - ☐ Define the terms "tautology" and "contradiction" and use truth tables to determine if a statement is a tautology or a contradiction.
  - ☐ **2.2: Logically Equivalent Statements**
    - ☐ Use truth tables to determine if two statements are logically equivalent.
    - ☐ State the converse and contrapositive of a conditional statement.
    - ☐ State DeMorgan's Laws and apply them to simplify a statement involving negations and either conjunctions or disjunctions.
    - ☐ Derive, state and use the results of Theorem 2.6.
    - ☐ Form the negation of a conditional statement.
    - ☐ Use previously-established logical equivalencies to determine if two statements are logically equivalent.
    - ☐ Derive, state, and use the results of Theorem 2.8 (Important Logical Equivalencies).
  - ☐ **2.3: Open Sentences and Sets**
    - ☐ Define the term "set" and write the contents of a set using the roster method.
    - ☐ Use correct notation for the sets real numbers, rational numbers, integers, and natural numbers.
    - ☐ Use correct notation to denote that an object is an element of a set (or not an element of a set).
    - ☐ Define what it means for two sets to be equal.
    - ☐ Define what it means for one set to be a subset of another, and use correct notation to denote this relationship.
    - ☐ Define the term "open sentence" (a.k.a. "predicate" or "propositional function") and distinguish between an open sentence and a logical statement.
    - ☐ Find values of variables that make an open sentence true or false; then determine the truth set of that open sentence.
    - ☐ Write the contents of a set using set builder notation.
    - ☐ Define the term "empty set" and use correct notation to denote this set.
  - ☐ **2.4: Quantifiers and Negations**
    - ☐ Define the terms "universal quantifier" and "existential quantifier"; identify these quantifiers in practice and use correct notation to denote them.
    - ☐ Determine whether a quantified statement is true or false.
    - ☐ Phrase both universally and existentially quantified statements in several different English formats.
    - ☐ Find a counterexample for a (false) universally quantified statement.
    - ☐ Form the negation of universally quantified and existentially quantified statements; write these negations in forms that do not use the negation symbol.
    - ☐ Rewrite a conditional statement as a universally quantified statement.
    - ☐ Construct examples and non-examples of definitions that involve quantified statements.
    - ☐ Phrase doubly-quantified statements in English, and determine if such statements are true or false.
    - ☐ Form the negation of a doubly-quantified statement.
- ☐ **Chapter 3: Constructing and Writing Proofs in Mathematics**
  - ☐ **3.1: Direct Proofs**
    - ☐ State and instantiate the definitions of "divides", "divisor", "factor", and "multiple" and use correct notation to work with these concepts.
    - ☐ Explain what a "proof" is and what it takes for a proof to be correct.
    - ☐ Explain the concepts of "undefined terms" and "axioms" and give examples of these in real life.

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- ☐ Explain the terms "conjecture", "theorem", "proposition", "lemma", and "corollary".
- ☐ Construct a know-show table for a conditional statement using a combination of forward and backward steps.
- ☐ Convert a completed know-show table into a well-written English paragraph proof that adheres to MTH 210 writing guidelines.
- ☐ State what it means for two integers to be "congruent modulo n".
- ☐ Determine the truth value of propositions involving divisibility, integer congruence, and other basic arithmetic concepts. If a proposition is true, construct a correct proof. If not, demonstrate a working counterexample.

- ☐ **3.2: More Methods of Proof**
  - ☐ Write the contrapositive of a conditional statement (possibly complex).
  - ☐ Construct a proof for a conditional statement by proving the contrapositive.
  - ☐ Construct a proof for a biconditional statement by proving two conditional statements.
  - ☐ Explain the concept of a "constructive proof" and write a correct constructive proof for a proposition when it makes sense to do so.
  - ☐ Explain the difference between a constructive and non-constructive proof.
- ☐ **3.3: Proof by Contradiction**
  - ☐ Explain the basic strategy behind a proof by contradiction and why such a proof should be convincing.
  - ☐ Correctly formulate all the assumptions made at the beginning of a proof by contradiction, and then correctly state what you want to show having made those assumptions.
  - ☐ Explain some situations in which a proof by contradiction might be an appropriate strategy for proof.
  - ☐ Construct a correct proof by contradiction for a proposition when it makes sense to do so, including: proofs of the nonexistence of an object, proofs involving rational and irrational numbers, and so on.
  - ☐ Prove that sqrt(2) is irrational.
- ☐ **3.4: Using Cases in Proofs**
  - ☐ Explain the basic concept and strategy behind a proof using cases.
  - ☐ Give examples of situations in which using cases in a proof might be appropriate, and give the cases appropriate for each such situation.
  - ☐ Construct correct proofs using cases for a proposition when it makes sense to do so, including: proofs involving even/odd integers, proofs involving rational/irrational numbers, and proofs involving absolute values.