MTH 210: Communicating in Mathematics
Proof Portfolio Problems 9 and 10

Problem 9

Problem 9: Let C be the set of all real functions (that is, the set of all functions that accept real numbers as input and produce real numbers as output) that are continuous on the closed interval $[0, 1]$. Define the function $A : C \to \mathbb{R}$ as follows: For each $f \in C$,

$$A(f) = \int_0^1 f(x)\, dx$$

Prove or disprove that A is a bijection. If A is not a bijection, check to see if it is either an injection or surjection. If it is one of these, prove it. Otherwise, give counterexamples for both properties.

Be sure you understand the function A before embarking on a proof. Play around with it and calculate several outputs first. Those are not to be submitted with your draft but are an essential step in moving towards a formal writeup.

Problem 10

Let $g : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ be defined by

$$g(a, b) = 3a + 2b \quad \forall a, b \in \mathbb{Z}$$

Prove or disprove that g is a bijection. If g is not a bijection, check to see if it is either an injection or surjection. If it is one of these, prove it. Otherwise, give counterexamples for both properties.