Assignment #3

Robert Smith

October 8, 2013

1. Consider an ARMA(1,1) model with $\phi = 0.5$ and $\theta = -0.45$ with $\mu = 3$ and $\sim N(0,1)$ errors.

```
library(TSA)
library(xtable)
library(ggplot2)
library(gridExtra)

ARMA11 <- function(n, phi, theta) {
    # modify theta for Cryer definition
    theta <- theta * -1
    return(data.frame(var_phi = ((1 - phi^2)/n) * (((1 - phi * theta)/(phi -
        theta))^2), var_theta = ((1 - theta^2)/n) * (((1 - phi * theta)/(phi -
        theta))^2), corr = sqrt((1 - phi^2) * (1 - theta^2))/(1 - phi * theta)))
}

print(xtable(ARMA11(n = 100, phi = 0.5, theta = -0.45), digits = 6, caption = "Question 1-a'
    type = "latex", caption.placement = "top")
```

Table 1: Question 1-a

	var_phi	var_theta	corr
1	1.801875	1.915994	0.997917

```
print(xtable(ARMA11(n = 300, phi = 0.5, theta = -0.45), digits = 6, caption = "Question 1-b'
    type = "latex", caption.placement = "top")
```

```
ARIMA.MC <- function(reps, sample.size, phi, theta, mean, err = list(mean = 0,
```

Table 2: Question 1-b

	var_phi	var_theta	corr
1	0.600625	0.638665	0.997917

```
    sd = 1)) {
    # correcting to Cryer theta notation
    theta <- theta * -1
    # chained output initializers
    mle <- vars <- matrix(0, ncol = 3, nrow = reps)

    for (i in 1:reps) {
        sim <- arima.sim(n = sample.size, list(ar = phi, ma = theta), innov = rnorm(sample.s
            err$mean, err$sd)) + mean

        est <- arima(sim, order = c(1, 0, 1))
        mle[i, ] <- est$coef
        vars[i, ] <- diag(est$var.coef)
    }

    return(as.data.frame(cbind(mle, vars)))
}

Q1C.100 <- ARIMA.MC(reps = 1000, sample.size = 100, phi = 0.5, theta = -0.45,
    mean = 3, err = list(mean = 0, sd = 1))

MLE <- as.data.frame(Q1C.100[, 1])
VARS <- as.data.frame(Q1C.100[, 4])
names(MLE) <- names(VARS) <- "data"

plot1 <- ggplot(MLE, aes(x = data, y = ..density..)) + geom_histogram(binwidth = sum(abs(ran
    geom_density(color = "red", fill = NA) + ggtitle(paste(expression(phi),
    expression(mu)))

plot2 <- ggplot(VARS, aes(x = data, y = ..density..)) + geom_histogram(binwidth = sum(abs(ra
    geom_density(color = "red", fill = NA) + ggtitle(paste(expression(phi),
    expression(sigma)))

MLE <- as.numeric(MLE[[1]])
VARS <- as.numeric(VARS[[1]])
print(xtable(data.frame(phi.mean = mean(MLE), phi.sd = sd(MLE), phi.err.mean = sqrt(mean(VAR
    digits = 6), type = "latex")



MLE <- as.data.frame(Q1C.100[, 2])
```

	phi.mean	phi.sd	phi.err.mean
1	0.468580	0.117678	0.117099

```
VARS <- as.data.frame(Q1C.100[, 5])
names(MLE) <- names(VARS) <- "data"

plot3 <- ggplot(MLE, aes(x = data, y = ..density..)) + geom_histogram(binwidth = sum(abs(ran
    geom_density(color = "red", fill = NA) + ggtitle(paste(expression(theta),
    expression(mu)))

plot4 <- ggplot(VARS, aes(x = data, y = ..density..)) + geom_histogram(binwidth = sum(abs(ra
    geom_density(color = "red", fill = NA) + ggtitle(paste(expression(theta),
    expression(sigma)))

MLE <- as.numeric(MLE[[1]])
VARS <- as.numeric(VARS[[1]])
print(xtable(data.frame(theta.mean = mean(MLE), theta.sd = sd(MLE), theta.err.mean = mean(VA
    digits = 6), type = "latex")
```

	theta.mean	theta.sd	theta.err.mean
1	0.461219	0.126833	0.014582

```
MLE <- as.data.frame(Q1C.100[, 3])
VARS <- as.data.frame(Q1C.100[, 6])
names(MLE) <- names(VARS) <- "data"

plot5 <- ggplot(MLE, aes(x = data, y = ..density..)) + geom_histogram(binwidth = sum(abs(ran
    geom_density(color = "red", fill = NA) + ggtitle(paste(expression(mu), expression(mu)))

plot6 <- ggplot(VARS, aes(x = data, y = ..density..)) + geom_histogram(binwidth = sum(abs(ra
    geom_density(color = "red", fill = NA) + ggtitle(paste(expression(mu), expression(sigma)

MLE <- as.numeric(MLE[[1]])
VARS <- as.numeric(VARS[[1]])
print(xtable(data.frame(mean.mean = mean(MLE), mean.sd = sd(MLE), mean.err.mean = mean(VARS)
    digits = 6), type = "latex")
```

	mean.mean	mean.sd	mean.err.mean
1	2.999603	0.293162	0.079693

```
grid.arrange(plot1, plot2, plot3, plot4, plot5, plot6, ncol = 2)
```

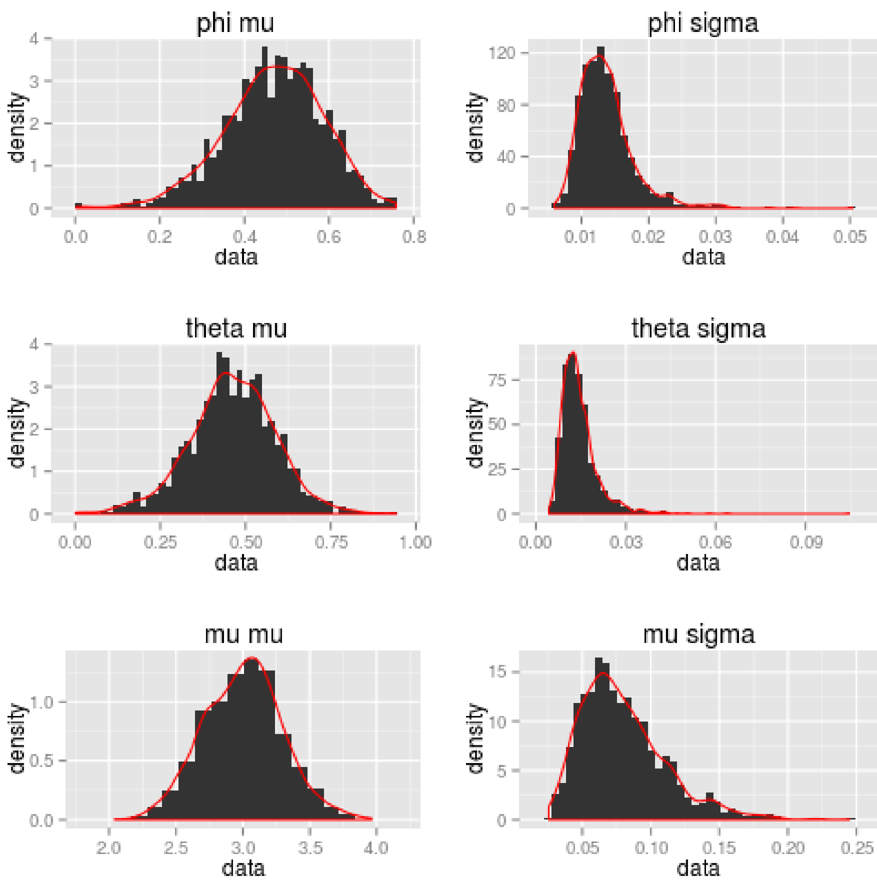

Figure 1: plot of chunk Q1C_100

```
Q1C.300 <- ARIMA.MC(reps = 1000, sample.size = 300, phi = 0.5, theta = -0.45,
    mean = 3, err = list(mean = 0, sd = 1))

MLE <- as.data.frame(Q1C.300[, 1])
VARS <- as.data.frame(Q1C.300[, 4])
names(MLE) <- names(VARS) <- "data"

plot1 <- ggplot(MLE, aes(x = data, y = ..density..)) + geom_histogram(binwidth = sum(abs(ran
    geom_density(color = "red", fill = NA) + ggtitle(paste(expression(phi),
    expression(mu)))

plot2 <- ggplot(VARS, aes(x = data, y = ..density..)) + geom_histogram(binwidth = sum(abs(ra
    geom_density(color = "red", fill = NA) + ggtitle(paste(expression(phi),
    expression(sigma)))

MLE <- as.numeric(MLE[[1]])
VARS <- as.numeric(VARS[[1]])
print(xtable(data.frame(phi.mean = mean(MLE), phi.sd = sd(MLE), phi.err.mean = sqrt(mean(VAR
    digits = 6), type = "latex")
```

	phi.mean	phi.sd	phi.err.mean
1	0.486549	0.063630	0.065725

```
MLE <- as.data.frame(Q1C.300[, 2])
VARS <- as.data.frame(Q1C.300[, 5])
names(MLE) <- names(VARS) <- "data"

plot3 <- ggplot(MLE, aes(x = data, y = ..density..)) + geom_histogram(binwidth = sum(abs(ran
    geom_density(color = "red", fill = NA) + ggtitle(paste(expression(theta),
    expression(mu)))

plot4 <- ggplot(VARS, aes(x = data, y = ..density..)) + geom_histogram(binwidth = sum(abs(ra
    geom_density(color = "red", fill = NA) + ggtitle(paste(expression(theta),
    expression(sigma)))

MLE <- as.numeric(MLE[[1]])
VARS <- as.numeric(VARS[[1]])
print(xtable(data.frame(theta.mean = mean(MLE), theta.sd = sd(MLE), theta.err.mean = mean(VA
    digits = 6), type = "latex")
```

	theta.mean	theta.sd	theta.err.mean
1	0.457854	0.065185	0.004577

```
MLE <- as.data.frame(Q1C.300[, 3])
VARS <- as.data.frame(Q1C.300[, 6])
names(MLE) <- names(VARS) <- "data"

plot5 <- ggplot(MLE, aes(x = data, y = ..density..)) + geom_histogram(binwidth = sum(abs(ran
    geom_density(color = "red", fill = NA) + ggtitle(paste(expression(mu), expression(mu)))

plot6 <- ggplot(VARS, aes(x = data, y = ..density..)) + geom_histogram(binwidth = sum(abs(ra
    geom_density(color = "red", fill = NA) + ggtitle(paste(expression(mu), expression(sigma)

MLE <- as.numeric(MLE[[1]])
VARS <- as.numeric(VARS[[1]])
print(xtable(data.frame(mean.mean = mean(MLE), mean.sd = sd(MLE), mean.err.mean = mean(VARS)
    digits = 6), type = "latex")
```

	mean.mean	mean.sd	mean.err.mean
1	2.998598	0.167201	0.027257

```
grid.arrange(plot1, plot2, plot3, plot4, plot5, plot6, ncol = 2)
```

The monte carlo simuation indicates that $\phi \approx 0.5$ and $\theta \approx 0.5$, but $\sigma_\phi > \sigma_\theta$

The simulation indicates that as N approaches infinity that the MLE will converge towards the theoretical value.

2. Consider an ARMA(1,1) with $\phi = 0.5$ and $\theta = 0.45$ with $\mu = 3$ and $\sim N(0,1)$ errors.

```
print(xtable(ARMA11(n = 1000, phi = 0.5, theta = 0.45), digits = 6, caption = "Question 1-a"
    type = "latex", caption.placement = "top")
```

```
print(xtable(ARMA11(n = 3000, phi = 0.5, theta = 0.45), digits = 6, caption = "Question 1-b"
    type = "latex", caption.placement = "top")
```

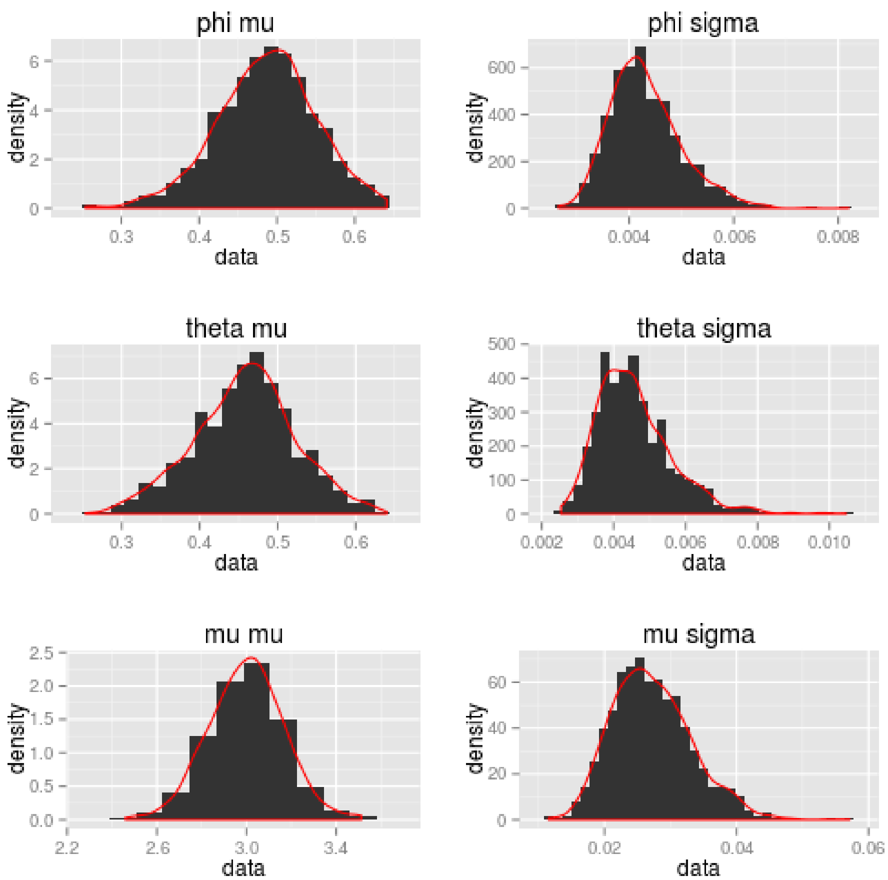

Figure 2: plot of chunk Q1C_300

Table 3: Question 1-a

	var_phi	var_theta	corr
1	0.001247	0.001326	0.631335

Table 4: Question 1-b

	var_phi	var_theta	corr
1	0.000416	0.000442	0.631335

```
# 1000 reps - Monte Carlo
Q2C.1000 <- ARIMA.MC(reps = 1000, sample.size = 100, phi = 0.5, theta = -0.45,
    mean = 3, err = list(mean = 0, sd = 1))

MLE <- as.data.frame(Q2C.1000[, 1])
VARS <- as.data.frame(Q2C.1000[, 4])
names(MLE) <- names(VARS) <- "data"

plot1 <- ggplot(MLE, aes(x = data, y = ..density..)) + geom_histogram(binwidth = sum(abs(ran
    geom_density(color = "red", fill = NA) + ggtitle(paste(expression(phi),
    expression(mu)))

plot2 <- ggplot(VARS, aes(x = data, y = ..density..)) + geom_histogram(binwidth = sum(abs(ra
    geom_density(color = "red", fill = NA) + ggtitle(paste(expression(phi),
    expression(sigma)))

MLE <- as.numeric(MLE[[1]])
VARS <- as.numeric(VARS[[1]])
print(xtable(data.frame(phi.mean = mean(MLE), phi.sd = sd(MLE), phi.err.mean = sqrt(mean(VAR
    digits = 6), type = "latex")
```

	phi.mean	phi.sd	phi.err.mean
1	0.464903	0.111855	0.116878

```
MLE <- as.data.frame(Q2C.1000[, 2])
VARS <- as.data.frame(Q2C.1000[, 5])
names(MLE) <- names(VARS) <- "data"

plot3 <- ggplot(MLE, aes(x = data, y = ..density..)) + geom_histogram(binwidth = sum(abs(ran
    geom_density(color = "red", fill = NA) + ggtitle(paste(expression(theta),
    expression(mu)))
```

```r
plot4 <- ggplot(VARS, aes(x = data, y = ..density..)) + geom_histogram(binwidth = sum(abs(ra
    geom_density(color = "red", fill = NA) + ggtitle(paste(expression(theta),
    expression(sigma)))

MLE <- as.numeric(MLE[[1]])
VARS <- as.numeric(VARS[[1]])
print(xtable(data.frame(theta.mean = mean(MLE), theta.sd = sd(MLE), theta.err.mean = mean(VA
    digits = 6), type = "latex")
```

	theta.mean	theta.sd	theta.err.mean
1	0.472167	0.115269	0.014172

```r
MLE <- as.data.frame(Q2C.1000[, 3])
VARS <- as.data.frame(Q2C.1000[, 6])
names(MLE) <- names(VARS) <- "data"

plot5 <- ggplot(MLE, aes(x = data, y = ..density..)) + geom_histogram(binwidth = sum(abs(ran
    geom_density(color = "red", fill = NA) + ggtitle(paste(expression(mu), expression(mu)))

plot6 <- ggplot(VARS, aes(x = data, y = ..density..)) + geom_histogram(binwidth = sum(abs(ra
    geom_density(color = "red", fill = NA) + ggtitle(paste(expression(mu), expression(sigma)

MLE <- as.numeric(MLE[[1]])
VARS <- as.numeric(VARS[[1]])
print(xtable(data.frame(mean.mean = mean(MLE), mean.sd = sd(MLE), mean.err.mean = mean(VARS)
    digits = 6), type = "latex")
```

	mean.mean	mean.sd	mean.err.mean
1	3.009976	0.297299	0.079493

```r
grid.arrange(plot1, plot2, plot3, plot4, plot5, plot6, ncol = 2)
```

```r
## 3000 reps - Monte Carlo
Q2C.3000 <- ARIMA.MC(reps = 3000, sample.size = 100, phi = 0.5, theta = -0.45,
    mean = 3, err = list(mean = 0, sd = 1))

MLE <- as.data.frame(Q2C.3000[, 1])
VARS <- as.data.frame(Q2C.3000[, 4])
```

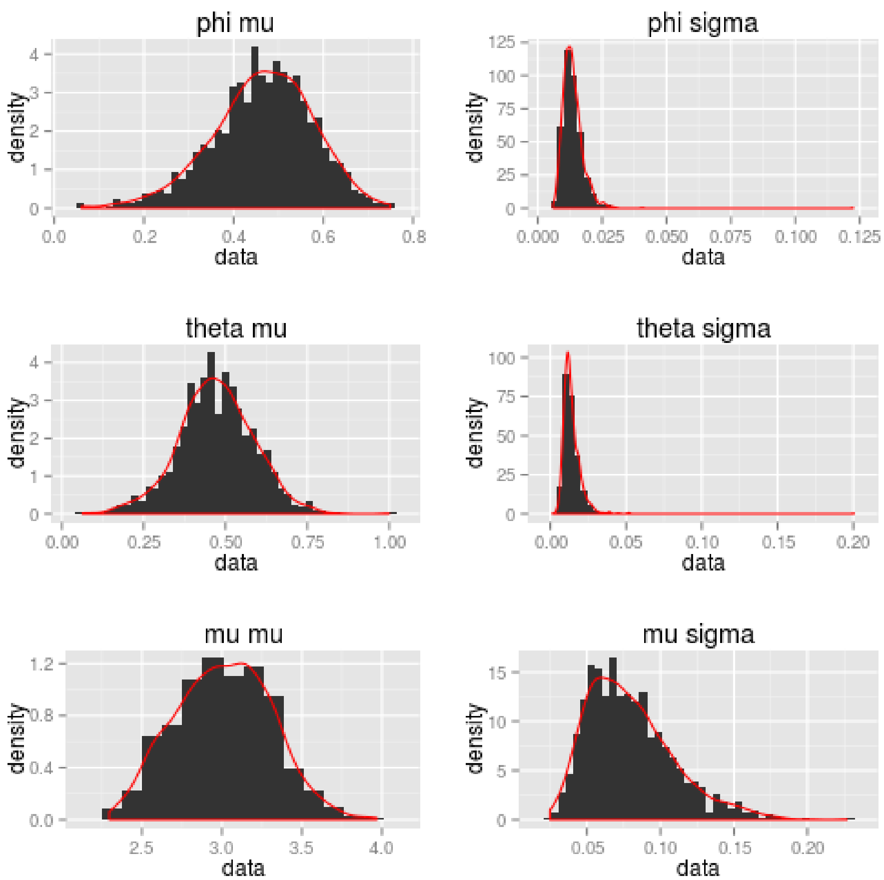

Figure 3: plot of chunk Q2C_1000

```r
names(MLE) <- names(VARS) <- "data"

plot1 <- ggplot(MLE, aes(x = data, y = ..density..)) + geom_histogram(binwidth = sum(abs(ran
    geom_density(color = "red", fill = NA) + ggtitle(paste(expression(phi),
    expression(mu)))

plot2 <- ggplot(VARS, aes(x = data, y = ..density..)) + geom_histogram(binwidth = sum(abs(ra
    geom_density(color = "red", fill = NA) + ggtitle(paste(expression(phi),
    expression(sigma)))

MLE <- as.numeric(MLE[[1]])
VARS <- as.numeric(VARS[[1]])
print(xtable(data.frame(phi.mean = mean(MLE), phi.sd = sd(MLE), phi.err.mean = sqrt(mean(VAR
    digits = 6), type = "latex")
```

	phi.mean	phi.sd	phi.err.mean
1	0.467349	0.118512	0.116943

```r
MLE <- as.data.frame(Q2C.3000[, 2])
VARS <- as.data.frame(Q2C.3000[, 5])
names(MLE) <- names(VARS) <- "data"

plot3 <- ggplot(MLE, aes(x = data, y = ..density..)) + geom_histogram(binwidth = sum(abs(ran
    geom_density(color = "red", fill = NA) + ggtitle(paste(expression(theta),
    expression(mu)))

plot4 <- ggplot(VARS, aes(x = data, y = ..density..)) + geom_histogram(binwidth = sum(abs(ra
    geom_density(color = "red", fill = NA) + ggtitle(paste(expression(theta),
    expression(sigma)))

MLE <- as.numeric(MLE[[1]])
VARS <- as.numeric(VARS[[1]])
print(xtable(data.frame(theta.mean = mean(MLE), theta.sd = sd(MLE), theta.err.mean = mean(VA
    digits = 6), type = "latex")
```

	theta.mean	theta.sd	theta.err.mean
1	0.466804	0.125334	0.014504

```r
MLE <- as.data.frame(Q2C.3000[, 3])
VARS <- as.data.frame(Q2C.3000[, 6])
names(MLE) <- names(VARS) <- "data"
```

```r
plot5 <- ggplot(MLE, aes(x = data, y = ..density..)) + geom_histogram(binwidth = sum(abs(ran
    geom_density(color = "red", fill = NA) + ggtitle(paste(expression(mu), expression(mu)))

plot6 <- ggplot(VARS, aes(x = data, y = ..density..)) + geom_histogram(binwidth = sum(abs(ra
    geom_density(color = "red", fill = NA) + ggtitle(paste(expression(mu), expression(sigma)

MLE <- as.numeric(MLE[[1]])
VARS <- as.numeric(VARS[[1]])
print(xtable(data.frame(mean.mean = mean(MLE), mean.sd = sd(MLE), mean.err.mean = mean(VARS)
    digits = 6), type = "latex")
```

	mean.mean	mean.sd	mean.err.mean
1	3.002306	0.285497	0.079412

```r
grid.arrange(plot1, plot2, plot3, plot4, plot5, plot6, ncol = 2)
```

Given the samples with $n = 1000$ and $n = 3000$ Monte Carlo simulation repetitions, we see that the mean values for $\hat{\phi}$ and $\hat{\theta}$ have essentially converged to their theoretical values. Also, when we compare these runs to the previous runs at $n = 1000$ and $n = 3000$ we can see that the error values appear closer to the theoretical distribution for χ^2.

As with before, as n approaches infinity for the Monte Carlo simulation, the simulation results become closer to the theoretical expected values.

3. Plot histogram of $\hat{\phi}$ and $\hat{\theta}$ from Problem 1 and for $n = 1000$, $n = 3000$ from problem 2.

Please see plots included above.

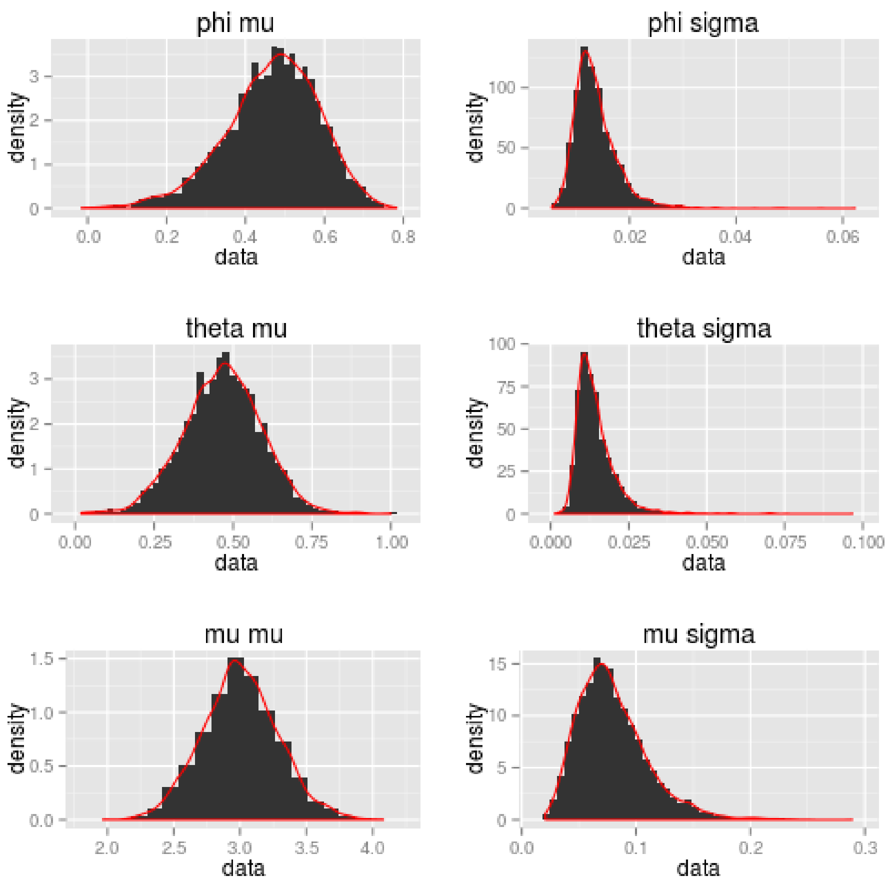

Figure 4: plot of chunk Q2C_3000