Assignment #3

Robert Smith

October 8, 2013

1. Consider an ARMA(1,1) model with $\phi = 0.5$ and $\theta = -0.45$ with $\mu = 3$ and $\sim N(0, 1)$ errors.

```
library(TSA)
library(xtable)
library(ggplot2)
library(gridExtra)

ARMA11 <- function(n, phi, theta) {
  # modify theta for Cryer definition
  theta <- theta * -1
  return(
    data.frame(
      var_phi = ((1 - phi**2) / n) * (((1 - phi * theta) / (phi - theta))**2),
      var_theta = ((1 - theta**2) / n) * (((1 - phi * theta) / (phi - theta))**2),
      corr = sqrt((1 - phi**2) * (1 - theta**2)) / (1 - phi * theta)
    ))
}

print(
  xtable(
    ARMA11(n = 100, phi = 0.5, theta = -0.45),
    digits = 6,
    caption = "Question 1-a"
  ), type = "latex",
  caption.placement = "top")
```

Table 1: Question 1-a

	var_phi	var_theta	corr
1	1.801875	1.915994	0.997917

```r
print(
  xtable(
    ARMA11(n = 300, phi = 0.5, theta = -0.45),
    digits = 6,
    caption = "Question 1-b"
  ), type = "latex",
  caption.placement = "top")
```

Table 2: Question 1-b

	var_phi	var_theta	corr
1	0.600625	0.638665	0.997917

```r
ARIMA.MC <- function(reps, sample.size, phi, theta, mean, err = list(mean = 0, sd = 1)) {
  # correcting to Cryer theta notation
  theta <- theta * -1
  # chained output initializers
  mle <- vars <- matrix(0, ncol = 3, nrow = reps)

  for (i in 1:reps) {
    sim <- arima.sim(
      n = sample.size,
      list(ar = phi, ma = theta),
      innov = rnorm(sample.size, err$mean, err$sd)
    ) + mean

    est <- arima(sim, order = c(1, 0, 1))
    mle[i,] <- est$coef
    vars[i,] <- diag(est$var.coef)
  }

  return(as.data.frame(cbind(mle,vars)))
}

Q1C.100 <- ARIMA.MC(reps = 1000, sample.size = 100, phi = 0.5, theta = -0.45,
                    mean = 3, err = list(mean = 0, sd = 1) )

MLE <- as.data.frame(Q1C.100[,1])
VARS <- as.data.frame(Q1C.100[,4])
names(MLE) <- names(VARS) <- "data"

plot1 <- ggplot(MLE, aes(x = data, y = ..density..)) +
  geom_histogram(binwidth = sum(abs(range(MLE$data)))/50 ) +
```

```r
  geom_density(color = "red", fill = NA) +
  ggtitle(paste(expression(phi), expression(mu)))

plot2 <- ggplot(VARS, aes(x = data, y = ..density..)) +
  geom_histogram(binwidth = sum(abs(range(VARS$data)))/50 ) +
  geom_density(color = "red", fill = NA) +
  ggtitle(paste(expression(phi), expression(sigma)))

MLE <- as.numeric(MLE[[1]]); VARS <- as.numeric(VARS[[1]])
print(
  xtable(
    data.frame(
      phi.mean = mean(MLE),
      phi.sd = sd(MLE),
      phi.err.mean = sqrt(mean(VARS))
      ), digits = 6
    ), type = "latex"
  )
```

	phi.mean	phi.sd	phi.err.mean
1	0.468353	0.116474	0.116200

```r
MLE <- as.data.frame(Q1C.100[,2])
VARS <- as.data.frame(Q1C.100[,5])
names(MLE) <- names(VARS) <- "data"

plot3 <- ggplot(MLE, aes(x = data, y = ..density..)) +
  geom_histogram(binwidth = sum(abs(range(MLE$data)))/50 ) +
  geom_density(color = "red", fill = NA) +
  ggtitle(paste(expression(theta), expression(mu)))

plot4 <- ggplot(VARS, aes(x = data, y = ..density..)) +
  geom_histogram(binwidth = sum(abs(range(VARS$data)))/50 ) +
  geom_density(color = "red", fill = NA) +
  ggtitle(paste(expression(theta), expression(sigma)))

MLE <- as.numeric(MLE[[1]]); VARS <- as.numeric(VARS[[1]])
print(
  xtable(
    data.frame(
      theta.mean = mean(MLE),
      theta.sd = sd(MLE),
      theta.err.mean = mean(VARS)
```

```
      ), digits = 6
    ), type = "latex"
  )
```

	theta.mean	theta.sd	theta.err.mean
1	0.468788	0.124102	0.014154

```
MLE <- as.data.frame(Q1C.100[,3])
VARS <- as.data.frame(Q1C.100[,6])
names(MLE) <- names(VARS) <- "data"

plot5 <- ggplot(MLE, aes(x = data, y = ..density..)) +
  geom_histogram(binwidth = sum(abs(range(MLE$data)))/50 ) +
  geom_density(color = "red", fill = NA) +
  ggtitle(paste(expression(mu), expression(mu)))

plot6 <- ggplot(VARS, aes(x = data, y = ..density..)) +
  geom_histogram(binwidth = sum(abs(range(VARS$data)))/50 ) +
  geom_density(color = "red", fill = NA) +
  ggtitle(paste(expression(mu), expression(sigma)))

MLE <- as.numeric(MLE[[1]]); VARS <- as.numeric(VARS[[1]])
print(
  xtable(
    data.frame(
      mean.mean = mean(MLE),
      mean.sd = sd(MLE),
      mean.err.mean = mean(VARS)
      ), digits = 6
    ), type = "latex"
  )
```

	mean.mean	mean.sd	mean.err.mean
1	3.004916	0.285230	0.080186

```
grid.arrange(plot1, plot2, plot3, plot4, plot5, plot6, ncol=2)

Warning: position_stack requires constant width: output may be incorrect
```

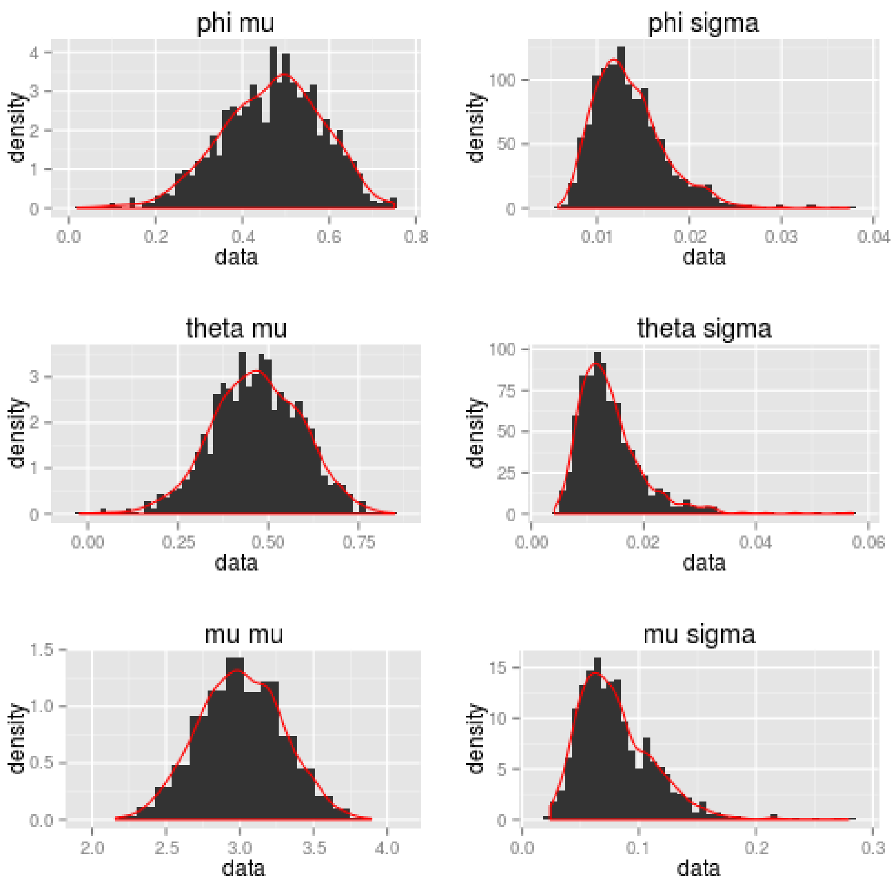

Figure 1: plot of chunk Q1C_100

```r
Q1C.300 <- ARIMA.MC(reps = 1000, sample.size = 300, phi = 0.5, theta = -0.45,
                    mean = 3, err = list(mean = 0, sd = 1) )

MLE <- as.data.frame(Q1C.300[,1])
VARS <- as.data.frame(Q1C.300[,4])
names(MLE) <- names(VARS) <- "data"

plot1 <- ggplot(MLE, aes(x = data, y = ..density..)) +
  geom_histogram(binwidth = sum(abs(range(MLE$data)))/50 ) +
  geom_density(color = "red", fill = NA) +
  ggtitle(paste(expression(phi), expression(mu)))

plot2 <- ggplot(VARS, aes(x = data, y = ..density..)) +
  geom_histogram(binwidth = sum(abs(range(VARS$data)))/50 ) +
  geom_density(color = "red", fill = NA) +
  ggtitle(paste(expression(phi), expression(sigma)))

MLE <- as.numeric(MLE[[1]]); VARS <- as.numeric(VARS[[1]])
print(
  xtable(
    data.frame(
      phi.mean = mean(MLE),
      phi.sd = sd(MLE),
      phi.err.mean = sqrt(mean(VARS))
      ), digits = 6
    ), type = "latex"
  )
```

	phi.mean	phi.sd	phi.err.mean
1	0.488480	0.068250	0.065454

```r
MLE <- as.data.frame(Q1C.300[,2])
VARS <- as.data.frame(Q1C.300[,5])
names(MLE) <- names(VARS) <- "data"

plot3 <- ggplot(MLE, aes(x = data, y = ..density..)) +
  geom_histogram(binwidth = sum(abs(range(MLE$data)))/50 ) +
  geom_density(color = "red", fill = NA) +
  ggtitle(paste(expression(theta), expression(mu)))

plot4 <- ggplot(VARS, aes(x = data, y = ..density..)) +
```

```
  geom_histogram(binwidth = sum(abs(range(VARS$data)))/50 ) +
  geom_density(color = "red", fill = NA) +
  ggtitle(paste(expression(theta), expression(sigma)))

MLE <- as.numeric(MLE[[1]]); VARS <- as.numeric(VARS[[1]])
print(
  xtable(
    data.frame(
      theta.mean = mean(MLE),
      theta.sd = sd(MLE),
      theta.err.mean = mean(VARS)
      ), digits = 6
    ), type = "latex"
  )
```

	theta.mean	theta.sd	theta.err.mean
1	0.454661	0.070568	0.004526

```
MLE <- as.data.frame(Q1C.300[,3])
VARS <- as.data.frame(Q1C.300[,6])
names(MLE) <- names(VARS) <- "data"

plot5 <- ggplot(MLE, aes(x = data, y = ..density..)) +
  geom_histogram(binwidth = sum(abs(range(MLE$data)))/50 ) +
  geom_density(color = "red", fill = NA) +
  ggtitle(paste(expression(mu), expression(mu)))

plot6 <- ggplot(VARS, aes(x = data, y = ..density..)) +
  geom_histogram(binwidth = sum(abs(range(VARS$data)))/50 ) +
  geom_density(color = "red", fill = NA) +
  ggtitle(paste(expression(mu), expression(sigma)))

MLE <- as.numeric(MLE[[1]]); VARS <- as.numeric(VARS[[1]])
print(
  xtable(
    data.frame(
      mean.mean = mean(MLE),
      mean.sd = sd(MLE),
      mean.err.mean = mean(VARS)
      ), digits = 6
    ), type = "latex"
  )
```

	mean.mean	mean.sd	mean.err.mean
1	3.000948	0.163556	0.027519

```
grid.arrange(plot1, plot2, plot3, plot4, plot5, plot6, ncol=2)
```

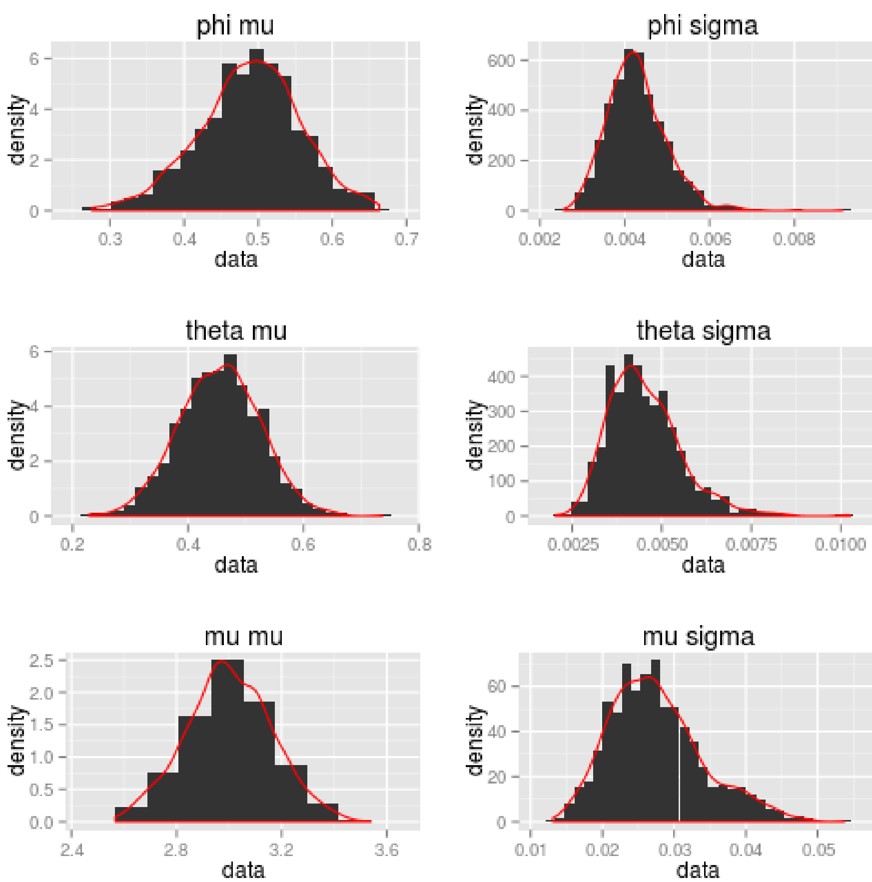

Figure 2: plot of chunk Q1C_300

The monte carlo simuation indicates that $\phi \approx 0.5$ and $\theta \approx 0.5$, but $\sigma_\phi > \sigma_\theta$

The simulation indicates that as N approaches infinity that the MLE will converge towards the theoretical value.

2. Consider an ARMA(1,1) with $\phi = 0.5$ and $\theta = 0.45$ with $\mu = 3$ and $\sim N(0, 1)$ errors.

```
print(
  xtable(
    ARMA11(n = 1000, phi = 0.5, theta = 0.45),
    digits = 6,
    caption = "Question 1-a"
  ), type = "latex",
  caption.placement = "top")
```

Table 3: Question 1-a

	var_phi	var_theta	corr
1	0.001247	0.001326	0.631335

```
print(
  xtable(
    ARMA11(n = 3000, phi = 0.5, theta = 0.45),
    digits = 6,
    caption = "Question 1-b"
  ), type = "latex",
  caption.placement = "top")
```

Table 4: Question 1-b

	var_phi	var_theta	corr
1	0.000416	0.000442	0.631335

```
# 1000 reps - Monte Carlo
Q2C.1000 <- ARIMA.MC(reps = 1000, sample.size = 100, phi = 0.5, theta = -0.45,
                    mean = 3, err = list(mean = 0, sd = 1) )

MLE <- as.data.frame(Q2C.1000[,1])
VARS <- as.data.frame(Q2C.1000[,4])
names(MLE) <- names(VARS) <- "data"

plot1 <- ggplot(MLE, aes(x = data, y = ..density..)) +
  geom_histogram(binwidth = sum(abs(range(MLE$data)))/50 ) +
  geom_density(color = "red", fill = NA) +
```

```r
  ggtitle(paste(expression(phi), expression(mu)))

plot2 <- ggplot(VARS, aes(x = data, y = ..density..)) +
  geom_histogram(binwidth = sum(abs(range(VARS$data)))/50 ) +
  geom_density(color = "red", fill = NA) +
  ggtitle(paste(expression(phi), expression(sigma)))

MLE <- as.numeric(MLE[[1]]); VARS <- as.numeric(VARS[[1]])
print(
  xtable(
    data.frame(
      phi.mean = mean(MLE),
      phi.sd = sd(MLE),
      phi.err.mean = sqrt(mean(VARS))
      ), digits = 6
    ), type = "latex"
  )
```

	phi.mean	phi.sd	phi.err.mean
1	0.465387	0.121720	0.116419

```r
MLE <- as.data.frame(Q2C.1000[,2])
VARS <- as.data.frame(Q2C.1000[,5])
names(MLE) <- names(VARS) <- "data"

plot3 <- ggplot(MLE, aes(x = data, y = ..density..)) +
  geom_histogram(binwidth = sum(abs(range(MLE$data)))/50 ) +
  geom_density(color = "red", fill = NA) +
  ggtitle(paste(expression(theta), expression(mu)))

plot4 <- ggplot(VARS, aes(x = data, y = ..density..)) +
  geom_histogram(binwidth = sum(abs(range(VARS$data)))/50 ) +
  geom_density(color = "red", fill = NA) +
  ggtitle(paste(expression(theta), expression(sigma)))

MLE <- as.numeric(MLE[[1]]); VARS <- as.numeric(VARS[[1]])
print(
  xtable(
    data.frame(
      theta.mean = mean(MLE),
      theta.sd = sd(MLE),
      theta.err.mean = mean(VARS)
      ), digits = 6
```

```
    ), type = "latex"
  )
```

	theta.mean	theta.sd	theta.err.mean
1	0.469115	0.124380	0.014054

```
MLE <- as.data.frame(Q2C.1000[,3])
VARS <- as.data.frame(Q2C.1000[,6])
names(MLE) <- names(VARS) <- "data"

plot5 <- ggplot(MLE, aes(x = data, y = ..density..)) +
  geom_histogram(binwidth = sum(abs(range(MLE$data)))/50 ) +
  geom_density(color = "red", fill = NA) +
  ggtitle(paste(expression(mu), expression(mu)))

plot6 <- ggplot(VARS, aes(x = data, y = ..density..)) +
  geom_histogram(binwidth = sum(abs(range(VARS$data)))/50 ) +
  geom_density(color = "red", fill = NA) +
  ggtitle(paste(expression(mu), expression(sigma)))

MLE <- as.numeric(MLE[[1]]); VARS <- as.numeric(VARS[[1]])
print(
  xtable(
    data.frame(
      mean.mean = mean(MLE),
      mean.sd = sd(MLE),
      mean.err.mean = mean(VARS)
      ), digits = 6
    ), type = "latex"
  )
```

	mean.mean	mean.sd	mean.err.mean
1	2.988990	0.294159	0.079409

```
grid.arrange(plot1, plot2, plot3, plot4, plot5, plot6, ncol=2)
```

```
## 3000 reps - Monte Carlo
Q2C.3000 <- ARIMA.MC(reps = 3000, sample.size = 100, phi = 0.5, theta = -0.45,
```

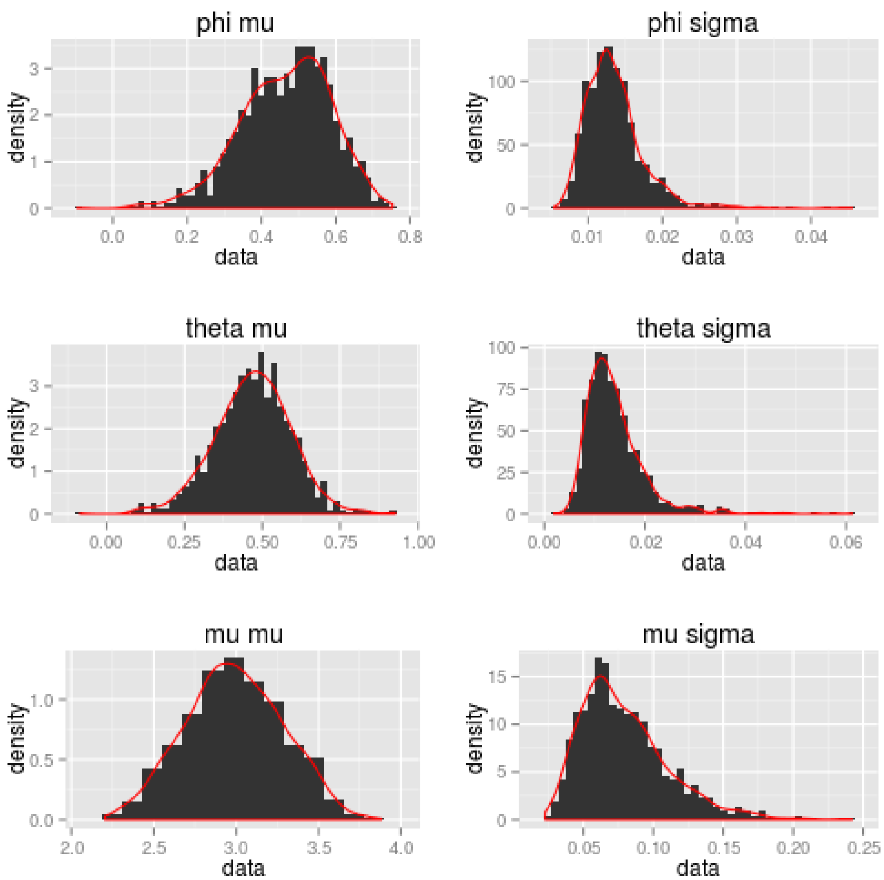

Figure 3: plot of chunk Q2C_1000

```r
                    mean = 3, err = list(mean = 0, sd = 1) )

MLE <- as.data.frame(Q2C.3000[,1])
VARS <- as.data.frame(Q2C.3000[,4])
names(MLE) <- names(VARS) <- "data"

plot1 <- ggplot(MLE, aes(x = data, y = ..density..)) +
  geom_histogram(binwidth = sum(abs(range(MLE$data)))/50 ) +
  geom_density(color = "red", fill = NA) +
  ggtitle(paste(expression(phi), expression(mu)))

plot2 <- ggplot(VARS, aes(x = data, y = ..density..)) +
  geom_histogram(binwidth = sum(abs(range(VARS$data)))/50 ) +
  geom_density(color = "red", fill = NA) +
  ggtitle(paste(expression(phi), expression(sigma)))

MLE <- as.numeric(MLE[[1]]); VARS <- as.numeric(VARS[[1]])
print(
  xtable(
    data.frame(
      phi.mean = mean(MLE),
      phi.sd = sd(MLE),
      phi.err.mean = sqrt(mean(VARS))
      ), digits = 6
    ), type = "latex"
  )
```

	phi.mean	phi.sd	phi.err.mean
1	0.466501	0.117781	0.116903

```r
MLE <- as.data.frame(Q2C.3000[,2])
VARS <- as.data.frame(Q2C.3000[,5])
names(MLE) <- names(VARS) <- "data"

plot3 <- ggplot(MLE, aes(x = data, y = ..density..)) +
  geom_histogram(binwidth = sum(abs(range(MLE$data)))/50 ) +
  geom_density(color = "red", fill = NA) +
  ggtitle(paste(expression(theta), expression(mu)))

plot4 <- ggplot(VARS, aes(x = data, y = ..density..)) +
  geom_histogram(binwidth = sum(abs(range(VARS$data)))/50 ) +
  geom_density(color = "red", fill = NA) +
  ggtitle(paste(expression(theta), expression(sigma)))
```

```r
MLE <- as.numeric(MLE[[1]]); VARS <- as.numeric(VARS[[1]])
print(
  xtable(
    data.frame(
      theta.mean = mean(MLE),
      theta.sd = sd(MLE),
      theta.err.mean = mean(VARS)
      ), digits = 6
    ), type = "latex"
  )
```

	theta.mean	theta.sd	theta.err.mean
1	0.466570	0.119078	0.014295

```r
MLE <- as.data.frame(Q2C.3000[,3])
VARS <- as.data.frame(Q2C.3000[,6])
names(MLE) <- names(VARS) <- "data"

plot5 <- ggplot(MLE, aes(x = data, y = ..density..)) +
  geom_histogram(binwidth = sum(abs(range(MLE$data)))/50 ) +
  geom_density(color = "red", fill = NA) +
  ggtitle(paste(expression(mu), expression(mu)))

plot6 <- ggplot(VARS, aes(x = data, y = ..density..)) +
  geom_histogram(binwidth = sum(abs(range(VARS$data)))/50 ) +
  geom_density(color = "red", fill = NA) +
  ggtitle(paste(expression(mu), expression(sigma)))

MLE <- as.numeric(MLE[[1]]); VARS <- as.numeric(VARS[[1]])
print(
  xtable(
    data.frame(
      mean.mean = mean(MLE),
      mean.sd = sd(MLE),
      mean.err.mean = mean(VARS)
      ), digits = 6
    ), type = "latex"
  )
```

	mean.mean	mean.sd	mean.err.mean
1	3.007895	0.283936	0.079979

```
grid.arrange(plot1, plot2, plot3, plot4, plot5, plot6, ncol=2)
```

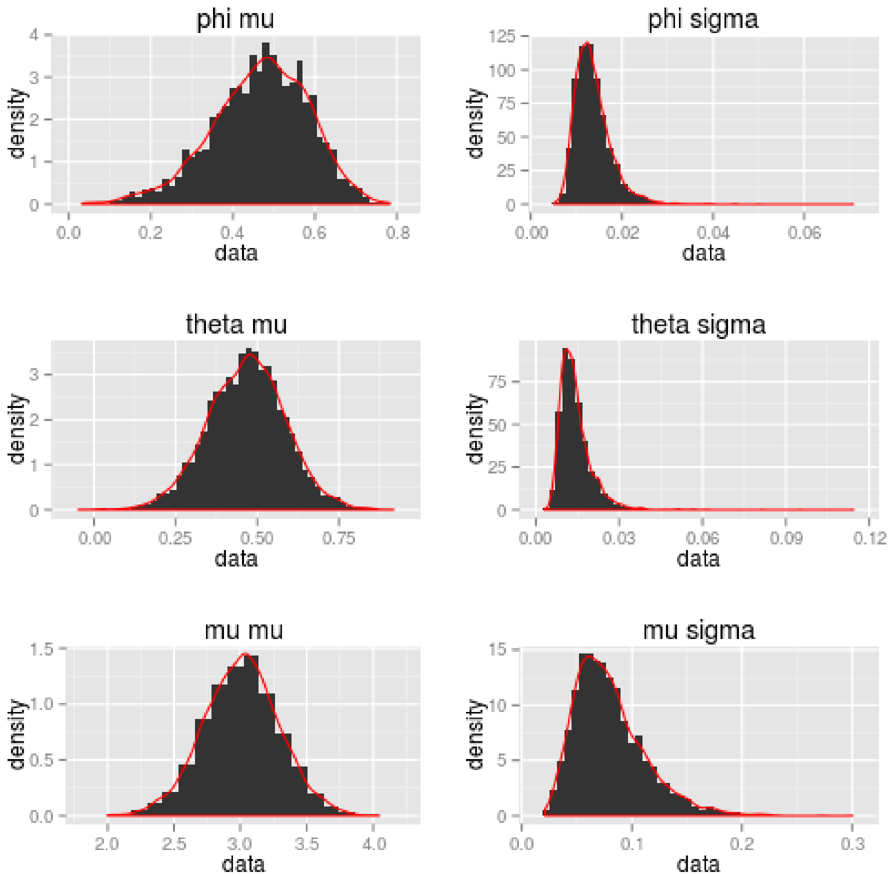

Figure 4: plot of chunk Q2C_3000

Given the samples with $n = 1000$ and $n = 3000$ Monte Carlo simulation repetitions, we see that the mean values for $\hat{\phi}$ and $\hat{\theta}$ have essentially converged to their theoretical values. Also, when we compare these runs to the previous runs at $n = 1000$ and $n = 3000$ we can see that the error values appear closer to the theoretical distribution for χ^2.

As with before, as n approaches infinity for the Monte Carlo simulation, the simulation results become closer to the theoretical expected values.

3. Plot histogram of $\hat{\phi}$ and $\hat{\theta}$ from Problem 1 and for $n = 1000$, $n = 3000$ from problem 2.

Please see plots included above.