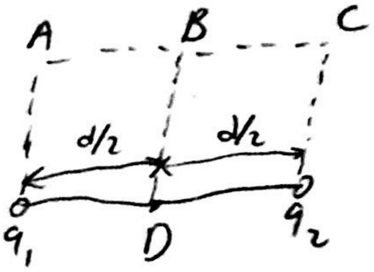
7.24

$r = 6\,\text{cm}$ $q_1 = 3\times10^{-8}\,\text{C}$

$d = 8\,\text{cm}$ $q_2 = -3\times10^{-8}\,\text{C}$



(1) 将电荷量为$2\times10^{-9}\,\text{C}$的点电荷从A点移到B点，电场力做功多少？

$$V_A = \frac{q_1}{4\pi\varepsilon_0 r} + \frac{q_2}{4\pi\varepsilon_0\sqrt{r^2+d^2}} = 1.8\times10^3\,\text{V}$$

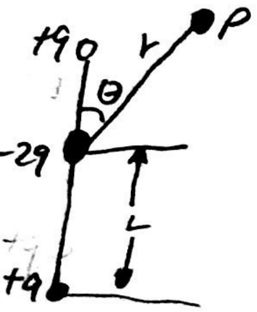
$$V_B = \frac{q_1}{4\pi\varepsilon\sqrt{r^2+\frac{d^2}{4}}} + \frac{q_2}{4\pi\varepsilon\sqrt{r^2+\frac{d^2}{4}}} = 0$$

$$V_C = \frac{q_1}{4\pi\varepsilon\sqrt{r^2+d^2}} + \frac{q_2}{4\pi\varepsilon r} = -1.8\times10^3\,\text{V} \qquad V_D = \frac{q_1}{4\pi\varepsilon(\frac{d}{2})} + \frac{q_2}{4\pi\varepsilon(\frac{d}{2})} = 0$$

$$A_{AB} = -q_0(V_B - V_A) = q_0 V_A = \boxed{3.6\times10^{-6}\,\text{J}} = (2\times10^{-9}\cdot 1.8\times10^3)$$

(2) $$A_{CD} = -q_0(V_D - V_C) = q_0 V_C = 2\times10^{-9}\cdot -1.8\times10^3 = \boxed{-3.6\times10^{-6}\,\text{J}}$$

7.25 $r \gg L$



$$V_P = \frac{1}{4\pi\varepsilon}\left(\frac{q}{r_1} - \frac{2q}{r} + \frac{q}{r_2}\right)$$

$r \gg l$ 时有 $r_1 \approx r - a$，$r_2 \approx r + a$，$a \approx l\cos\theta + a$

代入V_P式

$$V_P = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_1} - \frac{2}{r} + \frac{1}{r_2}\right) \approx \frac{q}{4\pi\varepsilon}\left(\frac{1}{(r-a)} - \frac{2}{r} + \frac{1}{(r+a)}\right)$$

$$= \frac{1}{4\pi\varepsilon}\frac{2qa^2}{r(r^2-a^2)} \approx \frac{ql^2}{2\pi\varepsilon r^3}\cos^2\theta$$

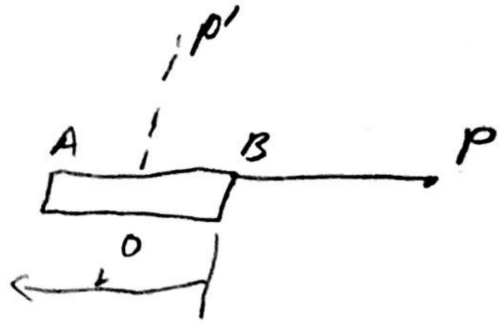
利用电场强度与电势梯度的关系 $E_r = \frac{dU}{dr} = \frac{3ql^2}{2\pi\varepsilon r^4}\cos^2\theta$，

$$E_\theta = -\frac{1}{r}\frac{dU}{d\theta} = \frac{ql^2}{2\pi\varepsilon r^4}\sin 2\theta$$

电场强度大小为 $E = \sqrt{E_r^2 + E_\theta^2} = \frac{ql^2}{2\pi\varepsilon r^4}\sqrt{9\cos^4\theta + \sin^2 2\theta}$

7.29

①



在直杆上x处取电荷微元

$dq=\lambda dx \quad \lambda=q/L$

$$dV_P=\frac{dq}{4\pi\varepsilon r_P}=\frac{q\,dx}{4\pi\varepsilon L\left(\frac{L}{2}+r-x\right)}$$

对上式积分，得直杆上所有电荷在P点的电势

$$V_P=\int dV=\frac{q}{4\pi\varepsilon L}\int_{-\frac{L}{2}}^{\frac{L}{2}}\frac{dx}{\left(\frac{L}{2}+r-x\right)}=\frac{q}{4\pi\varepsilon L}\ln\left(1+\frac{L}{r}\right)$$

杆的延长线上任意点P的电势 $V_P=V(r)$。利用电场强度与电势梯度的关系

$$E_x=-\frac{dV}{dr}$$

所以 $E_x=\frac{-q}{4\pi\varepsilon L}\frac{d}{dr}\left[\ln\left(\frac{r+L}{r}\right)\right]=\frac{q}{4\pi\varepsilon L}\left(\frac{1}{r}-\frac{1}{r+L}\right)=\frac{q}{4\pi\varepsilon r(r+L)}$

Ex沿x轴正向；在A端外侧各点处则沿x轴负向

(2) $dV_{P'}=\frac{dq}{4\pi\varepsilon_0 r_{P'}}=\frac{q\,dx}{4\pi\varepsilon L\sqrt{x^2+r^2}}$ P'点对直杆对称

$$V_{P'}=\int dV_{P'}=\frac{2q}{4\pi\varepsilon L}\int_0^{\frac{L}{2}}\frac{dx}{\sqrt{x^2+r^2}}=\frac{q}{2\pi\varepsilon L}\ln\frac{L+\sqrt{L^2+4r^2}}{2r}$$

可求得垂直平分线上的电场强度

$$E_y=-\frac{dV}{dr}=\frac{-q}{2\pi\varepsilon L}\frac{d}{dr}\left[\ln\frac{L+\sqrt{L^2+4r^2}}{2r}\right]$$

$$=\frac{q}{2\pi\varepsilon L}\left[\frac{1}{r}-\frac{4r}{\sqrt{L^2+4r^2}\left(L+\sqrt{L^2+4r^2}\right)}\right]=\frac{q}{2\pi\varepsilon r\sqrt{L^2+4r^2}}$$

当 $r\gg L$ 时，以上计算得到的 E_x 和 E_y，都将趋于点电荷的电场强度表达式

7.32

$R=8cm$ 的圆盘，面密度 $\sigma=2\times10^{-5}C/m^2$ 的电荷

(1)

$$dq=\sigma dS=\sigma 2\pi r dr$$

该微元圆环在轴线上 x 处的电势为

$$dV=\frac{dq}{4\pi\varepsilon\sqrt{x^2+r^2}}=\frac{\sigma r dr}{2\varepsilon_0\sqrt{x^2+r^2}}$$

圆盘轴线上 x 处的电势为

$$V=\int dV=\int_0^R\frac{\sigma r dr}{2\varepsilon\sqrt{x^2+r^2}}=\frac{\sigma}{2\varepsilon}(\sqrt{R^2+x^2}-x)$$

(2) 轴线上的电场强度分布为

$$E=-\frac{dV}{dx}i=\frac{\sigma}{2\varepsilon}\left(1-\frac{x}{\sqrt{R^2+x^2}}\right)i$$

(3) 代入数据 得 $x=6cm$ 处

$$V=4.52\times10^4V \quad E=4.52\times10^5 iV/m$$