大学物理作业21

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13-14 X射线的光子能量为0.60 MeV，在康普顿散射后波长改变了20%，求反冲电子获得的能量和动量

解：

$\varepsilon_0 = h\nu_0 = \frac{hc}{\lambda_0}$ $\lambda = \lambda_0 + \Delta\lambda = 1.2\lambda_0$

$\varepsilon_\varphi = h\nu = \frac{hc}{\lambda} = \frac{hc}{1.2\lambda_0} = \frac{\varepsilon_0}{1.2}$

能量守恒

$\varepsilon_0 = \varepsilon_\varphi + \varepsilon_e$ 反冲电子能量 $\varepsilon_e = \varepsilon_0 - \varepsilon_\varphi = (1-\frac{1}{1.2})\varepsilon_0$

$\Rightarrow (1-\frac{1}{1.2}) \times 0.60\,\mathrm{MeV} \Rightarrow \underline{0.10\,\mathrm{MeV}}$

2) $\Delta\lambda = \lambda - \lambda_0 = 0.2\lambda_0 = \frac{2h}{m_{e0}c}\sin^2\frac{\varphi}{2} = 2\lambda_c \sin^2\frac{\varphi}{2}$（波长改变量）

解得散射角 $\varphi = 2\arcsin\sqrt{\frac{0.2\lambda_0}{2\lambda_c}} = 2\arcsin\sqrt{\frac{0.2hc}{2\lambda_c\varepsilon_0}}$

$\Rightarrow 2\arcsin\sqrt{8.5\times10^{-2}} = 0.59\,\mathrm{rad} = 33.96°$

电子的康普顿波长 $\lambda_c = \frac{h}{m_{e0}} = 2.43\times10^{-12}\,m$

反冲电子的动量 P_e 可由动量守恒定律求得，由

$0 = P_\varphi \sin\varphi - P_e \sin\theta$

$P_e^2 = P_e^2 + P_0^2 - 2P_0P_\varphi\cos\varphi$

两式解得 $p_e = \sqrt{p_\varphi^2 + p_0^2 - 2p_0 p_\varphi \cos\varphi}$

$= \frac{h}{\lambda\lambda_0}\sqrt{\lambda^2+\lambda_0^2-2\lambda\lambda_0\cos\varphi} = \frac{h}{1.2\lambda_0}\sqrt{1.2^2+1-2\times1.2\cos\varphi}$

$= \frac{\varepsilon_0}{1.2c}\sqrt{2.44-2.4\cos\varphi} = 1.78\times10^{-22}\ \mathrm{kg\cdot m/s}$

和 $\theta = \arcsin\left(\frac{p_\varphi}{p_e}\sin\varphi\right) = \arcsin\left(\frac{\varepsilon_0}{1.2cp}\sin\varphi\right)$

$= \arcsin 0.83 = 0.99\ \mathrm{rad} = 56.57^\circ$

$p = \sqrt{(h/\lambda_0)^2 + (h/\lambda)^2}$ # $\tan\varphi = (h/\lambda)/(h/\lambda) = \lambda_0/\lambda$

$\theta = \arctan\frac{0.20}{0.22} = 42.3^\circ$

13-16 $\lambda_1 = 400nm$ $\lambda_2 = 0.104nm$，在 $\theta = \frac{\pi}{2}$ 的方向上观察散射光

解：在散射角 $\theta = \frac{\pi}{2}$ 的方向上

$$\Delta\lambda = \lambda_c = 2.43 \times 10^{-3} nm$$

电子获得的动能为 $\Delta\varepsilon = \varepsilon_0 - \varepsilon_\varphi$

$$\Delta\varepsilon = \frac{hc}{\lambda_0} - \frac{hc}{\lambda} = hc\frac{\lambda - \lambda_0}{\lambda\lambda_0} = hc\frac{\Delta\lambda}{(\lambda_0 + \Delta\lambda)\lambda_0}$$

(1)
$$\frac{\Delta\lambda/\lambda_1}{\Delta\lambda/\lambda_2} = \frac{\lambda_2}{\lambda_1} = \frac{0.04}{400} = 1 \times 10^{-4}$$

电子获得的动能之比为

$$\frac{\Delta\varepsilon_1}{\Delta\varepsilon_2} = \frac{(\lambda_2 + \Delta\lambda)\lambda_2}{(\lambda_1 + \Delta\lambda)\lambda_1} \approx \left(\frac{\lambda_2}{\lambda_1}\right)^2 = 1 \times 10^{-8}$$

(2) 由以上计算可知，用X射线与自由电子碰撞，波长的相对改变量和电子获得的动能，均远大于可见光与自由电子碰撞的结果。所以，欲获得明显的康普顿效应，入射光应选择短波长的X射线。