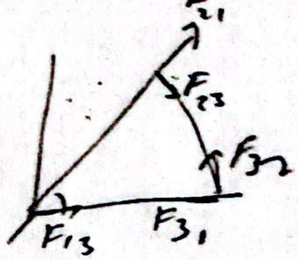
7.2 $q_1=1.0\times10^{-6}C$ $q_2=3.0\times10^{-6}C$ $q_3=-1.0\times10^{-6}C$ 点电荷

(1) 哪一个点电荷所受的力最大

库仑定律 $\frac{1}{4\pi\varepsilon}\frac{|q_3q_2|}{l^2}=|F_{23}|=|F_{32}|=|F_{12}|=|F_{21}|$

$$=\frac{1.0\times10^{-6}\times3.0\times10^{-6}}{4\pi\times8.85\times10^{-12}\cdot0.02^2}=67.4N$$



$$|F_{13}|=|F_{31}|=\frac{|q_1q_3|}{4\pi\varepsilon l^2}=22.5N$$

F_1 F_2 F_3 q_1 受合力为

$$F_1=(F_{13}-F_{12}\cos\alpha)i-F_{12}\sin\alpha j=-(11.2i+58.4j)N$$

$$F_1=\sqrt{(F_{13}-F_{12}\cos\alpha)^2+(F_{12}\sin\alpha)^2}=57.5N$$

F_1 与 Ox 的夹角 $\theta_1=\arctan\frac{F_y}{F_x}=\arctan\left(\frac{F_{12}\sin\alpha}{F_{13}-F_{12}\cos\alpha}\right)$

$$=\pi+\arctan\left(\frac{58.4}{11.2}\right)=259.1^\circ$$

q_2 受合力为 $F_2=2F_{21}\cos\alpha i=67.4iN$

q_3 受合力

$$F_3=-(F_{31}+F_{32}\cos\alpha)i+F_{32}\sin\alpha j=(-56.2i+58.4j)N$$

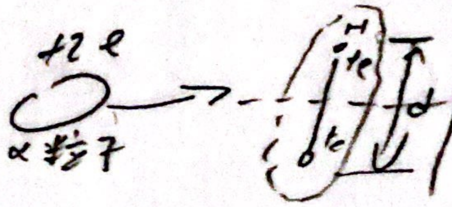
$$F_3=\sqrt{(F_{31}+F_{32}\cos\alpha)^2+(F_{32}\sin\alpha)^2}=81.0N$$

与夹角 $\theta_3=\arctan\frac{F_{3y}}{F_{3x}}=\arctan\left(\frac{F_{32}\sin\alpha}{F_{31}+F_{32}\cos\alpha}\right)=\pi-\arctan\frac{58.4}{56.2}$

q_3 受合力最大 $=133.9^\circ$

(2) q_2 受力也就是 67.4N，沿 Ox 轴正向。

7.5



α粒子在何处受到的力最大

$$F = F_x = 2F\cos\theta = 2\frac{2e^2}{4\pi\varepsilon r^2}\frac{x}{r} = \frac{xe^2}{\pi\varepsilon r^3}$$

x<0时 α粒子向x轴负方向，x>0时α粒子向x正方向

$$r=\sqrt{x^2+(\frac{d}{2})^2} \Rightarrow F = \frac{8e^2}{\pi\varepsilon}\frac{x}{(4x^2+d^2)^{\frac{3}{2}}}$$

∴α粒子受力最大的位置为 $x=\pm\frac{d}{2\sqrt{2}}$

7.6 A点 $q_1=1.8\times10^{-9}C$ B点 $q_2=-4.8\times10^{-9}C$ $BC=0.04m$, $AC=0.03m$

(1) C处的电场强度

$$E_1=\frac{1}{4\pi\varepsilon}\frac{q_1}{r^2}(-j)=-1.8\times10^4 j\ V/m$$

$$E_2=\frac{1}{4\pi\varepsilon}\frac{q_2}{r_2^2}i=2.7\times10^4 i\ V/m$$

那么 $E_C=E_1+E_2=\frac{1}{4\pi\varepsilon}\frac{q_2}{r_2^2}i-\frac{1}{4\pi\varepsilon}\frac{q_1}{r^2}j$

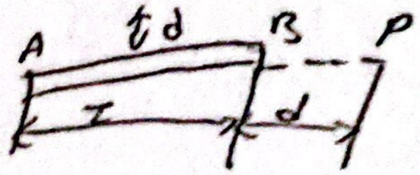
E_C的大小为

$$E_C=\sqrt{(-1.8\times10^4)^2+(2.7\times10^4)^2}=3.24\times10^4\ V/m$$

夹角(与x)

$$\theta=\arctan\frac{E_1}{E_2}=\arctan(-\frac{2}{3})=-33.7°$$

7.8 AB的长 L，电荷线密度为 λ

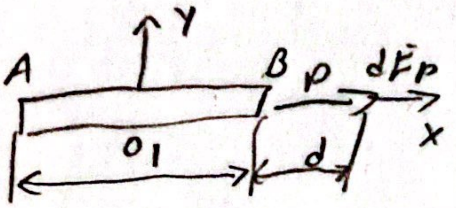


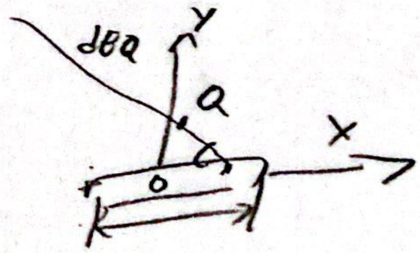
(1) P处电场强度

$$dE_p = \frac{1}{4\pi\varepsilon}\frac{dq}{r^2}\,i = \frac{1}{4\pi\varepsilon}\frac{\lambda dx}{(\frac{L}{2}+d-x)^2}\,i$$

我们能知道的是沿x正方向

$$E_p = \int dE_p = \frac{\lambda}{4\pi\varepsilon}\int_{-\frac{L}{2}}^{\frac{L}{2}}\frac{dx}{(\frac{L}{2}+d-x)^2}\,i = \frac{\lambda L}{4\pi\varepsilon(d+L)d}\,i$$





(2) Q点处电场强度

$$dE_Q = \frac{1}{4\pi\varepsilon}\frac{dq}{r^2}e_r$$

各dq在Q点的电场强度 dE_Q 也相对y轴分布

$$dE_Q = dE_Q\sin\alpha = \frac{1}{4\pi\varepsilon}\frac{dq}{r^2}\sin\alpha,\quad \sin\alpha = \frac{d}{r_Q}$$

积分

$$E_Q = \int dE_Q = \frac{\lambda d}{4\pi\varepsilon}\int_{-\frac{L}{2}}^{\frac{L}{2}}\frac{dx}{(d^2+x^2)^{\frac{3}{2}}}$$

$$= \frac{\lambda d}{4\pi\varepsilon}\frac{x}{d^2\sqrt{d^2+x^2}}\Bigg|_{-\frac{L}{2}}^{\frac{L}{2}} = \frac{\lambda L}{4\pi\varepsilon d}\frac{1}{\sqrt{d^2+(\frac{L}{2})^2}}$$

$$= E_Q = \frac{\lambda L}{4\pi\varepsilon d\sqrt{d^2+(\frac{L}{2})^2}}\,j$$