7.9 半径为R

$$dE_{0x} = dE_0\cos\theta = \frac{1}{4\pi\varepsilon}\frac{dq}{R^2}\cos\theta$$

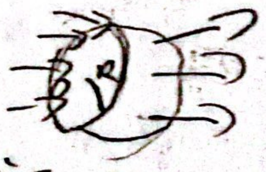
将 $dq=\lambda dl$, $dl=Rd\theta$, $\lambda=\frac{Q}{\pi R}$

$$dE = \frac{Q}{4\pi^2\varepsilon R^2}\cos\theta d\theta$$

积分 $E_0=\int dE_0 = \frac{Q}{4\pi^2\varepsilon R^2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos\theta d\theta = \frac{Q}{2\pi^2\varepsilon R^2}$

即 $E=\frac{Q}{2\pi^2\varepsilon R^2}i$

7.14



(1) 分别通过闭合半球面底面为和球面的电场强度通量

$\Phi = E\cdot S = ES\cos\pi = -\pi R^2 E$ 半球底面为S_1

$\Phi_2 = \int_{S_2} E\cdot dS = \int_{S_2} E d\cos\theta = E\int_{S_2} d\cos\theta = \theta R^2 E$

(2) 半球面内的总电荷量

$\oint_S E\cdot dS = \Phi_1+\Phi_2 = 0$ 闭合半球面所围电荷的代数和为零

7.18 电荷体密度为 $\rho = kr$

高斯定理有 $\oint_S E\cdot dS=\oint_S E\,dS = E\oint_S dS = E\cdot 4\pi r^2 = \frac{1}{\varepsilon}\int_V \rho dV$

离球心 r 处的电场强度为 $E=\frac{1}{4\pi\varepsilon r^2}\int_V \rho dV = \frac{q}{4\pi\varepsilon r^2}$

$q=\int_V \rho dV$

半径 $<R$ 的高斯球面包围的电荷量

$$\int_V \rho dV = \int_0^r kr4\pi r^2 dr = 4\pi k\int_0^r r^3 dr = \pi k r^4$$

得球体内的电场强度为 $E=\frac{kr^2}{4\varepsilon}$

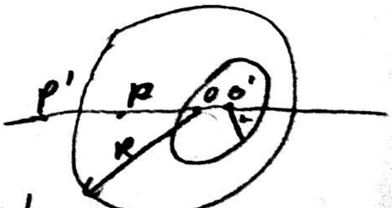
半径 $>R$

$$\int_V \rho dV = \int_0^R kr4\pi r^2 dr = 4\pi K\int_0^R r^3 dr = \pi k R^4$$

得球体外的电场强度

$$E=\frac{kR^4}{4\varepsilon r^2}$$

7.22



$$E=E_1+E_2$$

利用高斯定理 $E_1=\begin{cases}\frac{\rho r}{3\varepsilon}e & r<a\\ \frac{\rho a^3}{3\varepsilon r^2}e & r>a\end{cases}$ $E=\begin{cases}-\frac{\rho r'}{3\varepsilon}e' & r'<b\\ -\frac{\rho b^3}{3\varepsilon r'^2}e' & r'>b\end{cases}$

在O点 $r=0,\ r'=ce'=-ce$ $E_1=0$ 所以O点电场强度 $E_O=E_{2O}=-\frac{\rho c}{3\varepsilon}e'=\frac{\rho c}{3\varepsilon}e$

在O'点 $E_{O'}=E_{1O'}=\frac{\rho c}{3\varepsilon}e=E_O$

P'点为于两个球外 $E_{P'}=E_{1P'}+E_{2P'}=\frac{\rho a^3}{3\varepsilon r^2}e-\frac{\rho b^3}{3\varepsilon r'^2}e'=\frac{\rho}{3\varepsilon}\left(\frac{a^3}{r_P^2}-\frac{b^3}{r'^2_P}\right)e$

P点为于大球内小球外电场强度

$$E_P=E_{1P}+E_{2P}=\frac{\rho r}{3\varepsilon}-\frac{\rho b^3}{3\varepsilon r'^2}e'=\frac{\rho}{3\varepsilon}\left(r_P-\frac{b^3}{r'^2}\right)e$$