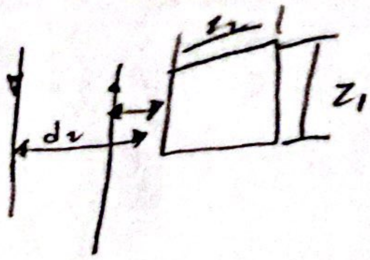
9.1



在x处的磁感应强度的大小为

$$B=\frac{\mu_0 I}{2\pi(x-d_2+d_1)}-\frac{\mu_0 I}{2\pi x}$$

B的方向垂直纸面向里

$$d\Phi=B\cdot dS=B\cdot dS=\left[\frac{\mu_0 I}{2\pi(x-d_2+d_1)}-\frac{\mu_0 I}{2\pi x}\right]l_1 dx$$

矩形线圈的磁通量为 $\Phi=\int_{d_2}^{d_2+l_2}\left[\frac{\mu_0 I}{2\pi(x-d_2+d_1)}-\frac{\mu_0 I}{2\pi x}\right]l_1 dx$

$$=\frac{\mu_0 I l_1}{2\pi}\left(\ln\frac{d_1+l_2}{d_1}-\ln\frac{d_2+l_2}{d_2}\right)$$

$$=\frac{\mu_0 I l_1}{2\pi}\ln\frac{(d_1+l_2)d_2}{(d_2+l_2)d_1}$$

矩形线圈中的感生电动势为

$$\varepsilon=-\frac{d\Phi}{dt}=-\frac{\mu_0 l_1}{2\pi}\ln\frac{(d_1+l_2)d_2}{(d_2+l_2)d_1}\frac{d}{dt}(I_0\sin\omega t)$$

$$=-\frac{\mu_0 I_0\omega l_1}{2\pi}\ln\frac{(d_1+l_2)d_2}{(d_2+l_2)d_1}\cos\omega t$$

9.5

10cm, 30°角, $v=15m/s$, $B=25\times10^{-2}T$

取电动势的假定方向沿导线 $P\to M\to N$

$$\varepsilon_{PN}=\varepsilon_{PM}+\varepsilon_{MN}$$

其中 $\varepsilon_{PM}=(v\times B)\cdot l_{PM}=vBl_{PM}\cos\pi=-vBl_{PM}$

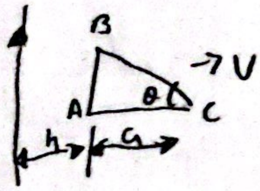
$\varepsilon_{MN}=(v\times B)\cdot l_{MN}=vBl_{MN}\cos150^\circ=-vBl_{MN}\cos30^\circ$ 因 $l_{PM}=l_{MN}$

所以 $\varepsilon_{PN}=\varepsilon_{PM}+\varepsilon_{MN}=-vBl_{PM}(1+\cos30^\circ)=-7.0\times10^{-3}V$ 式中负号表明导线上的动生电动势方向与假设方向相反，沿 $N\to M\to P$

P,N两端间的电势差为 $U_{PN}=V_P-V_N=-\varepsilon_{PN}=7.0\times10^{-3}V$

运动导线上P端的电势高。

9.6 恒定电流I的磁感应强度大小



$$B=\frac{\mu_0 I}{2\pi x}$$

动生电动势 $\varepsilon_{AB}=(v\times B)\cdot l_{AB}=vBl_{AB}=\frac{\mu_0 I l_{AB} v}{2\pi b}=\frac{\mu_0 I a v}{2\pi b}\tan\theta$

式中利用了几何关系 $l_{AB}=a\tan\theta$，ε_{AB}的方向由A指向B

取微元 dl_{BC}，有

$$d\varepsilon_{BC}=(v\times B)\cdot dl_{BC}=vBdl_{BC}\cos\left(\frac{\pi}{2}+\theta\right)=-vBdl_{BC}\sin\theta$$

$$=-vB\tan\theta dx=-\frac{\mu_0 I v}{2\pi}\tan\theta\frac{dx}{x}$$

斜边 l_{BC} 的动生电动势

$$\varepsilon_{BC}=\int d\varepsilon_{BC}=-\frac{\mu_0 I v}{2\pi}\tan\theta\int_b^{a+b}\frac{dx}{x}=-\frac{\mu_0 I v}{2\pi}\ln\frac{a+b}{b}\tan\theta$$

横边 l_{AC} 在运动不切割磁感线，总电动势

$$\varepsilon=\varepsilon_{AB}+\varepsilon_{BC}+\varepsilon_{AC}=\frac{\mu_0 I v}{2\pi}\left(\frac{a}{b}-\ln\frac{a+b}{b}\right)\tan\theta$$

通过x处dx长狭条面元的磁通量

$$d\phi=B\cdot dS=\frac{\mu_0 I}{2\pi x}y\,dx=\frac{\mu_0 I}{2\pi x}\tan\theta(a+x'-x)dx$$

t磁通量

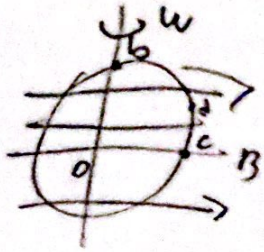
$$\phi=\int d\phi=\int_{x'}^{a+x'}\frac{\mu_0 I}{2\pi x}\tan\theta(a+x'-x)dx=\frac{\mu_0 I}{2\pi}\tan\theta\left[(a+x')\ln\frac{a+x'}{x'}-a\right]$$

总电动势为

$$\varepsilon=-\frac{d\phi}{dt}=-\frac{\mu_0 I}{2\pi}\tan\theta\frac{d}{dt}\left[(a+x')\ln\frac{a+x'}{x'}-a\right]=\frac{\mu_0 I v}{2\pi}\tan\theta\left(\frac{a}{x'}-\ln\frac{a+x'}{x'}\right)$$

$x'=b$时 $\varepsilon=\frac{\mu_0 I v}{2\pi}\left(\frac{a}{b}-\ln\frac{a+b}{b}\right)\tan\theta$

9.8



取环内感应电流为 b→d→c

$$\Phi = B\cdot S = BS\cos\phi = Ba^2\pi\cos\omega t$$

圆环中的电动势 $\varepsilon = -\frac{d\phi}{dt} = Ba^2\pi\omega\sin\omega t$

圆环中的电流为 $I = \frac{\varepsilon}{R} = \frac{Ba^2\pi\omega}{R}\sin\omega t$

$\omega t = \frac{\pi}{2}$ 时 $\varepsilon = \varepsilon_{max} = Ba^2\pi\omega$, $I = I_{max} = \frac{\varepsilon_{max}}{R} = \frac{Ba^2\pi\omega}{R}$

感应电流方向取线元 dl $d\varepsilon = (v\times B)\cdot dl = vBdl\cos(\frac{\pi}{2}-\theta) = vBdl\sin\theta$

式中 $v = r\omega = a\omega\sin\theta$, $dl = a\,d\theta$ 有

$$d\varepsilon = a^2\omega B\sin^2\theta\, d\theta$$

圆环上 b,c 段的电动势为

$$\varepsilon_{bc} = \int d\varepsilon = a^2\omega B\int_0^{\frac{\pi}{2}}\sin^2\theta\, d\theta = \frac{Ba^2\pi\omega}{4}$$

dc 段电动势 $\varepsilon_{dc} = a^2\omega B\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\sin^2\theta\, d\theta = Ba^2\omega(\frac{\pi}{8}+\frac{1}{4})$

bd . . . . $\varepsilon_{bd} = a^2\omega B\int_0^{\frac{\pi}{4}}\sin^2\theta\, d\theta = Ba^2\omega(\frac{\pi}{8}-\frac{1}{4})$

$V_b + \varepsilon_{bc} - IR_{bc} = V_c$ $V_{bc} = V_b - V_c = IR_{bc} - \varepsilon_{bc} = 0$ (电阻为 $\frac{\pi}{4}$,

dc 电阻为 $\frac{\pi}{8}$ $V_{dc} = V_d - V_c = IR_{dc} - \varepsilon_{dc} = -\frac{1}{4}Ba^2\omega$

bd . . . . . . $V_{bd} = V_b - V_d = IR_{bd} - \varepsilon_{bd} = \frac{1}{4}Ba^2\omega$

b点的电势高。