北京航空航天大学
BEIHANG UNIVERSITY

飞行力学
Flight Mechanics

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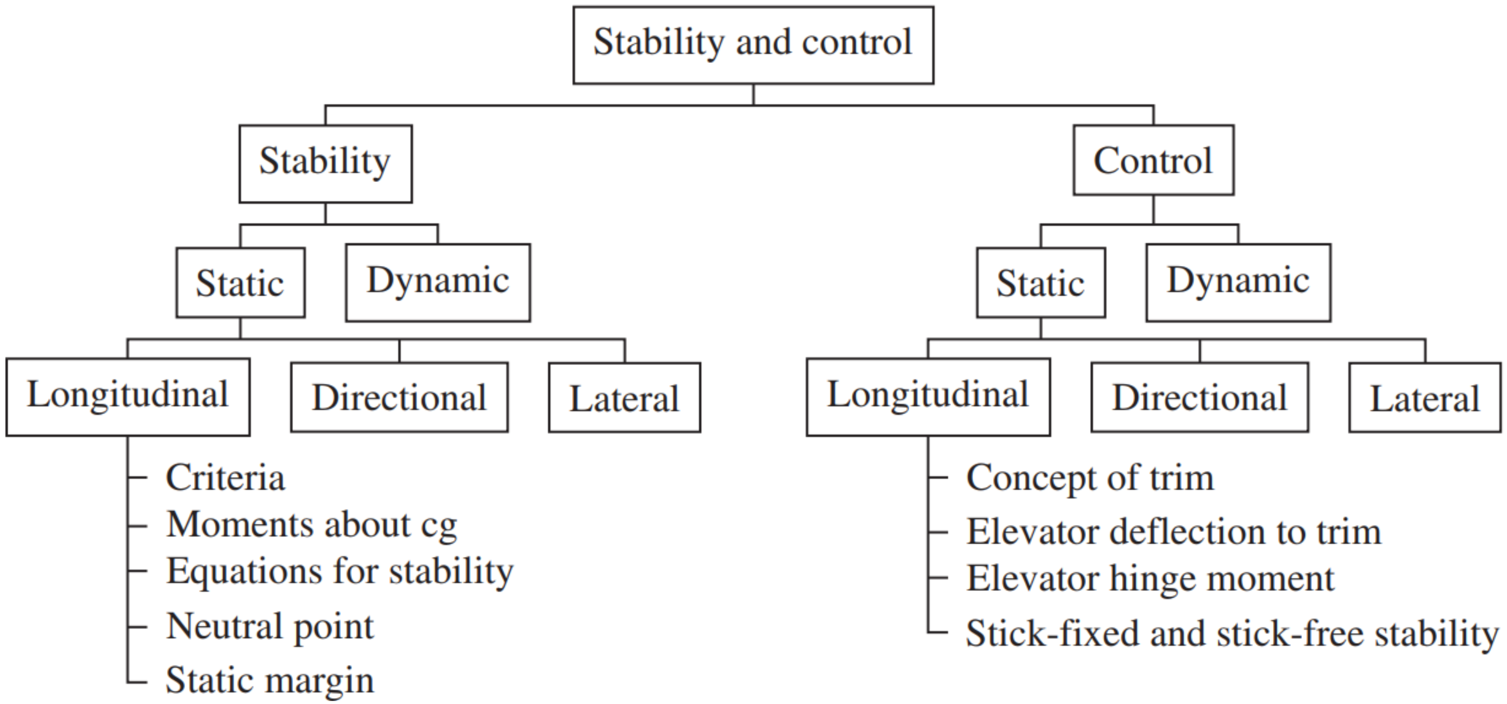
Contents

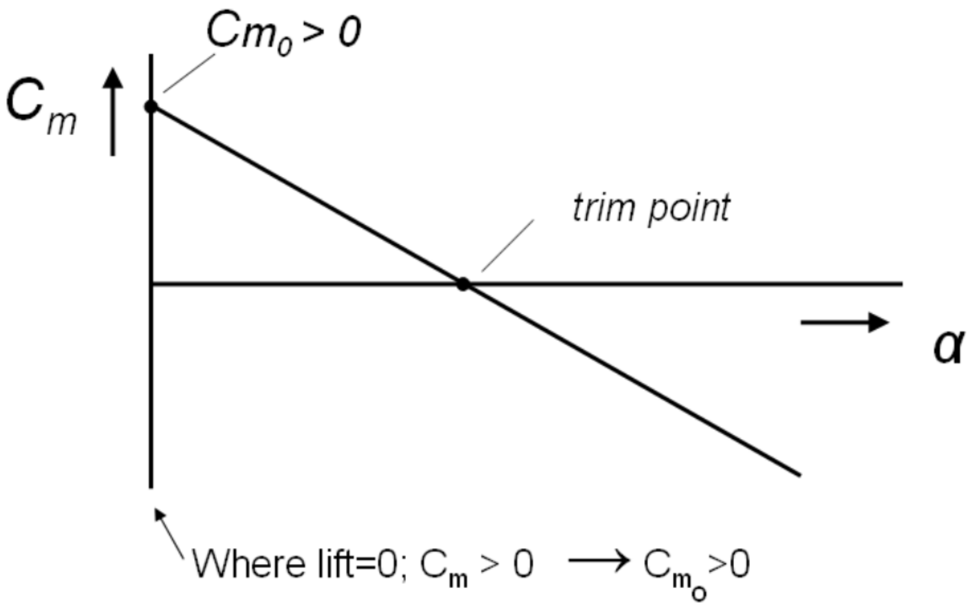
Introduction



Road map for Stability and Control.

Review - stability

Criteria for longitudinal static stability



1) $C_{M,0}$ must be positive

2) $\partial C_{M,cg} / \partial \alpha_a$ must be negative

Introduction

Questions

- How to arrange the wing and tail to obtain stability ?
- Condition 1 is easy to check, what about condition 2 ?

1) $C_{M,0}$ must be positive

2) $\partial C_{M,cg} / \partial \alpha_a$ must be negative

Introduction

Airfoil nomenclature

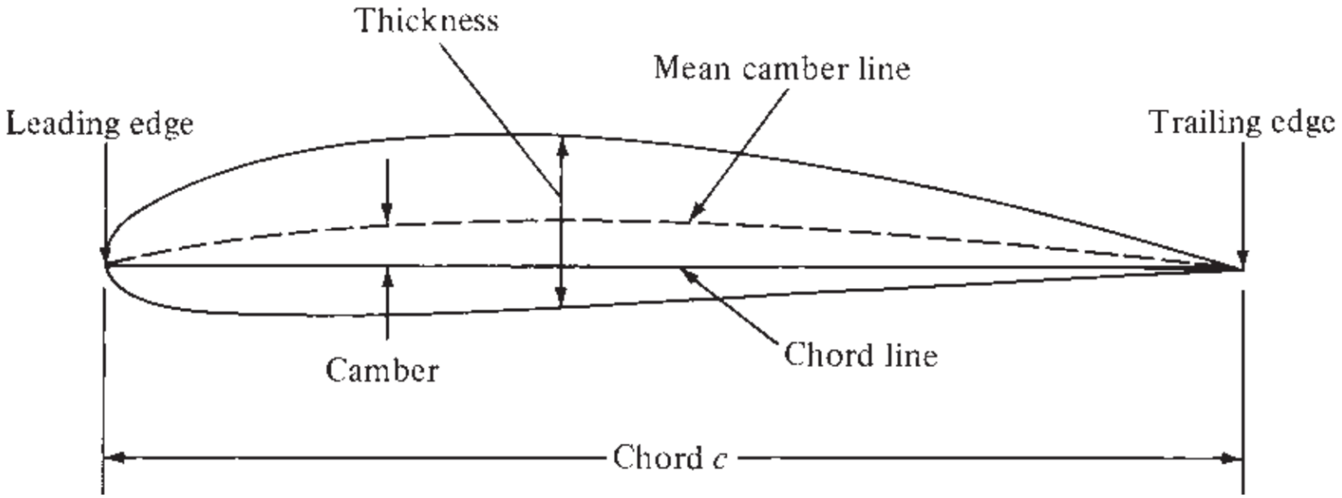


Figure 5.3 Airfoil nomenclature. The shape shown here is a NACA 4415 airfoil.

Introduction

Some definitions

Mean camber line （中弧线）： the locus of points halfway between the upper and lower surfaces, as measured perpendicular to the mean camber line itself.

Leading and trailing edges （前缘和尾缘）： the most forward and rearward points of the mean camber line.

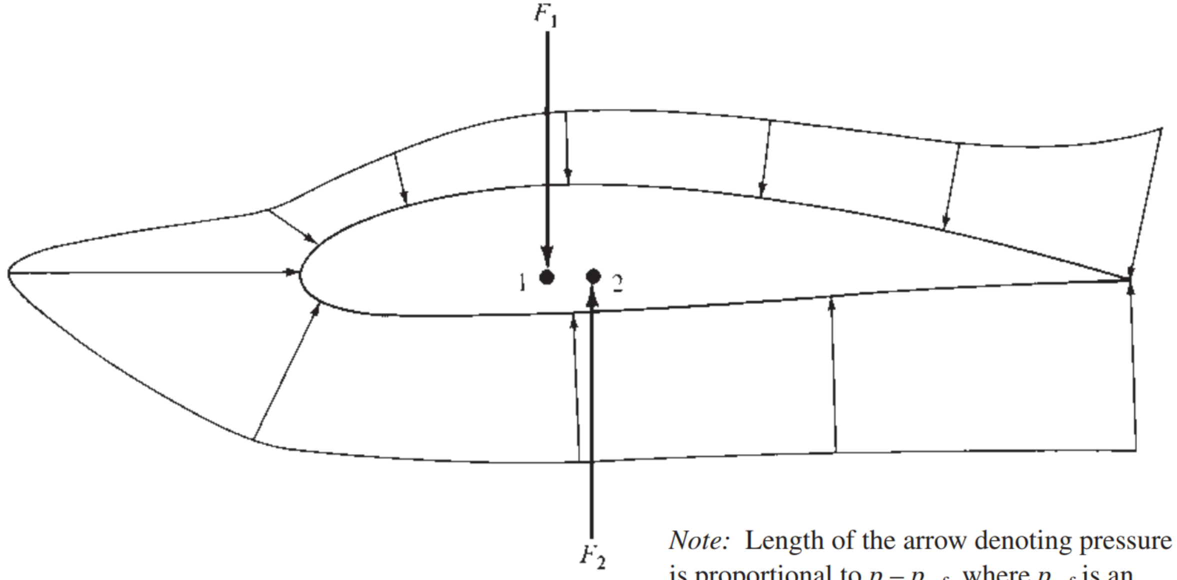
Chord line（弦线）: the straight line connecting the leading and trailing edges.

Chord（弦长）: the precise distance from the leading to the trailing edge measured along the chord line, given by the symbol c.

Moments about cg

The aerodynamic moment

$M_{ac,w}$: moment of wing about aerodynamic center



Note: Length of the arrow denoting pressure is proportional to $p - p_{\text{ref}}$, where p_{ref} is an arbitrary reference pressure slightly less than the minimum pressure on the airfoil.

Figure 5.5 The physical origin of moments on an airfoil.

Moments about cg

The aerodynamic moment

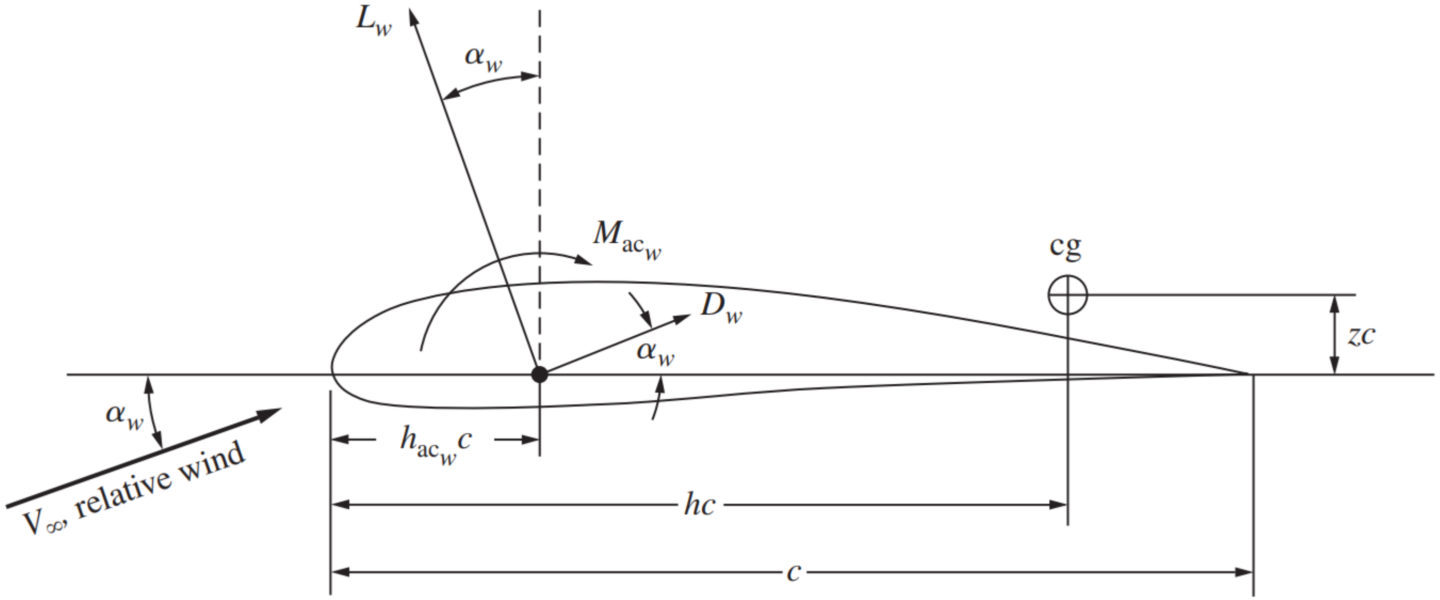


Figure 7.19 Airfoil nomenclature and geometry.

Moments about cg

Summary of the moment and forces

- $M_{ac,w}$
- Lift force L
- Drag force D
- Weight W

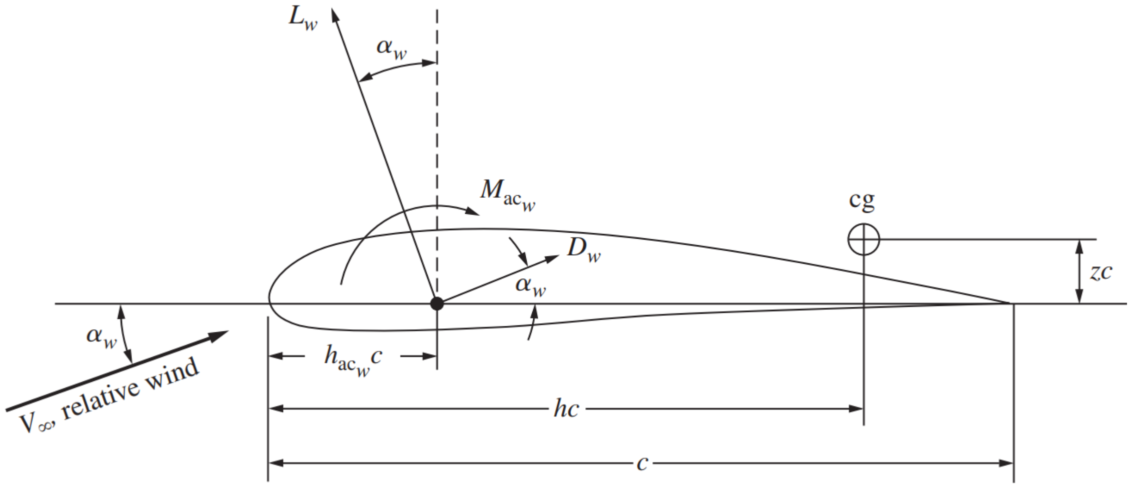


Figure 7.19 Airfoil nomenclature and geometry.

Moments about cg

The moments about the center of gravity

- $M_{ac,w}$
- $L_w \rightarrow L_w \cos\alpha_w (hc - h_{ac,w}c) + L_w \sin\alpha_w \, zc$
- $D_w \rightarrow D_w \sin(hc - h_{ac,w}c) - D_w \cos\alpha_w \, zc$
- $W \rightarrow 0$

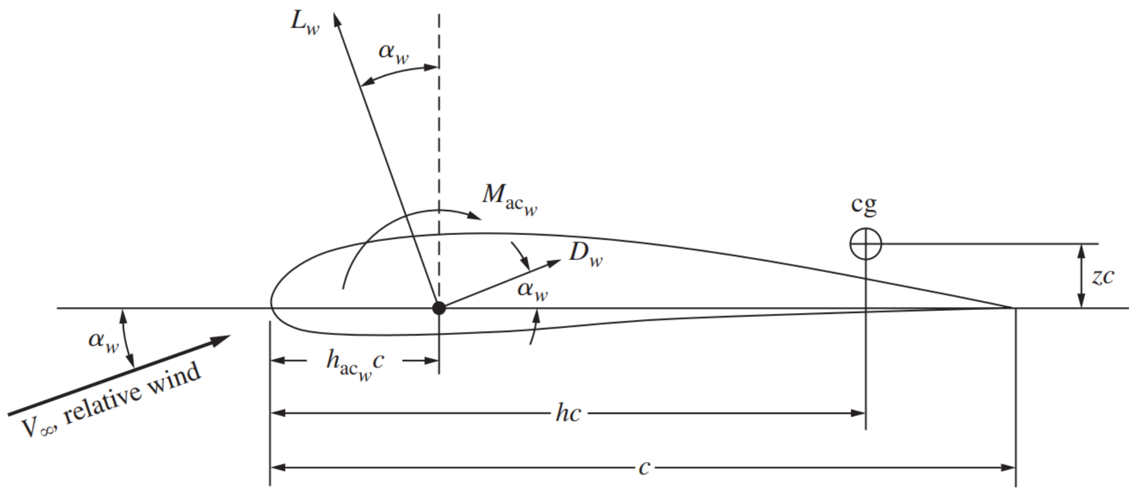


Figure 7.19 Airfoil nomenclature and geometry.

Moments about cg

The moments about the center of gravity

$$M_{cg,w} = M_{ac,w} + L_w \cos\alpha_w (hc - h_{ac,w}c) + D_w \sin(h - h_{ac,w}c) + L_w \sin\alpha_w \, zc - D_w \cos\alpha_w \, zc$$

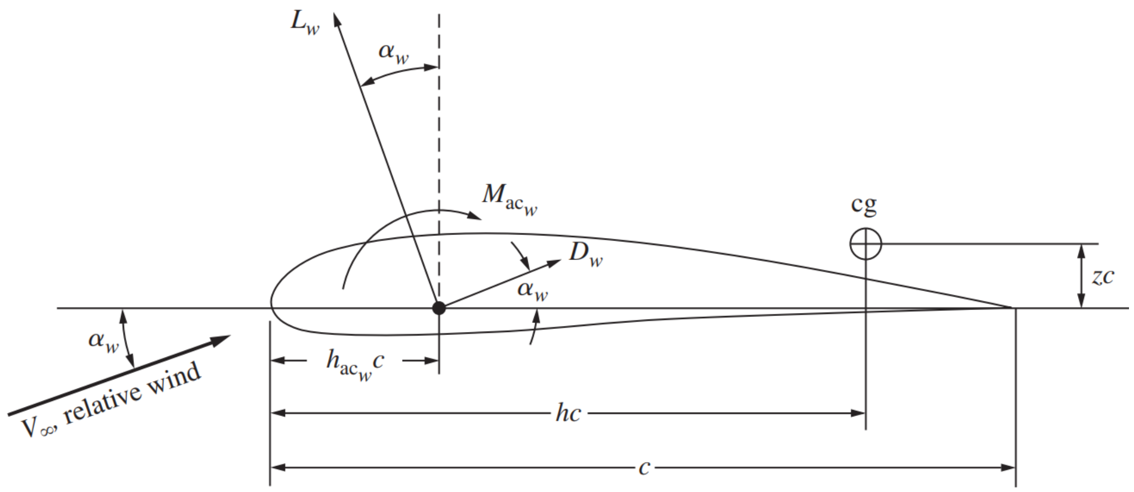


Figure 7.19 Airfoil nomenclature and geometry.

Moments about cg

Assumption 1: α_w is small

$$\Rightarrow \cos\alpha_w \approx 1, \sin\alpha_w \approx \alpha_w$$

$$\Rightarrow M_{cg,w} = M_{ac,w} + (L_w + D_w\alpha_w)\left(h - h_{ac,w}\right)\mathrm{c} + (L_w\alpha_w - D_w)zc$$

Moments about cg

Nondimensionalize by $q_\infty S c$

$$\boxed{M_{cg,w} = M_{ac,w} + (L_w + D_w\alpha_w)(h - h_{ac,w})\mathrm{c} + (L_w\alpha_w - D_w)zc}$$

$$\Rightarrow C_{M,cgw} = C_{M,acw} + (C_{L,w} + C_{D,w}\alpha_w)(h - h_{ac,w}) + (C_{L,w}\alpha_w - C_{D,w})z$$

Moments about cg

Assumption 2: $z \approx 0$

$$\boxed{C_{M,cgw} = C_{M,acw} + \left(C_{L,w} + C_{D,w}\alpha_w\right)\left(h - h_{ac,w}\right) + \left(C_{L,w}\alpha_w - C_{D,w}\right)z}$$

$$\Rightarrow C_{M,cgw} = C_{M,acw} + \left(C_{L,w} + C_{D,w}\alpha_w\right)\left(h - h_{ac,w}\right)$$

Since α_w: is small, the expression can be further simplified to

$$\Rightarrow C_{M,cgw} = C_{M,acw} + C_{L,w}\left(h - h_{ac,w}\right)$$

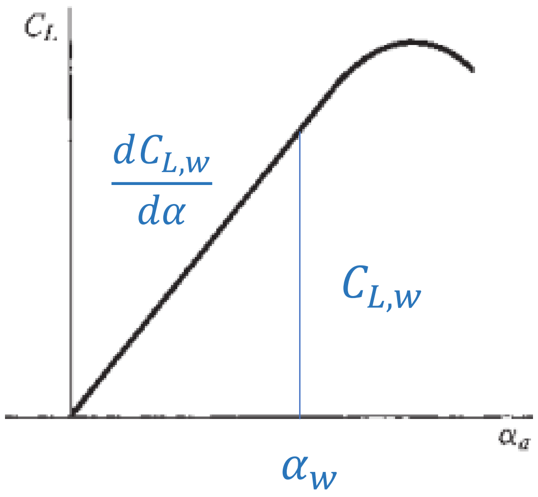
Moments about cg

Assume: $a_w = dC_{L,w}/d\alpha$

$$\boxed{C_{M,cgw} = C_{M,acw} + C_{L,w}(h - h_{ac,w})}$$

$$\Rightarrow C_{L,w} = \frac{dC_{L,w}}{d\alpha}\alpha_w = a_w\ \alpha_w$$

$$\Rightarrow C_{M,cgw} = C_{M,acw} + a_w\ \alpha_w(h - h_{ac,w})$$



Moments about cg

Take the fuselage into consideration

$$C_{M,cg-wb} = C_{M,ac-wb} + C_{L,wb}(h - h_{ac,wb})$$

$$C_{M,cg-wb} = C_{M,ac-wb} + a_{wb}\ \alpha_{wb}(h - h_{ac,wb})$$

Practice

Example 7.3

A wing–body model is tested in a subsonic wind tunnel. The lift is found to be zero at a geometric angle of attack $\alpha = -1.5°$. At $\alpha = 5°$ the lift coefficient is measured as 0.52. Also, at $\alpha = 1.0°$ and 7.88°, the moment coefficients about the center of gravity are measured as –0.01 and 0.05, respectively. The center of gravity is located at $0.35c$. Calculate the location of the aerodynamic center and the value of $C_{M,\mathrm{ac}_{\mathrm{wb}}}$.

Practice

Example 7.3

A wing–body model is tested in a subsonic wind tunnel. The lift is found to be zero at a geometric angle of attack $\alpha = -1.5°$. At $\alpha = 5°$ the lift coefficient is measured as 0.52. Also, at $\alpha = 1.0°$ and 7.88°, the moment coefficients about the center of gravity are measured as –0.01 and 0.05, respectively. The center of gravity is located at $0.35c$. Calculate the location of the aerodynamic center and the value of $C_{M,\mathrm{ac}_{\mathrm{wb}}}$.

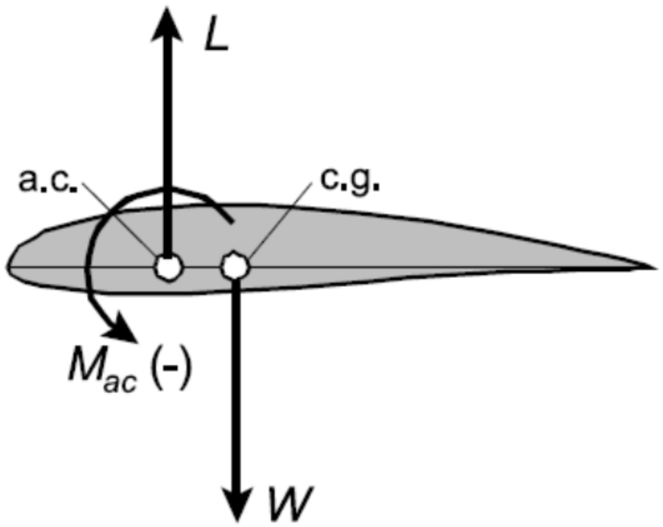
$$C_{M,cg-wb} = C_{M,ac-wb} + a_{wb}\,\alpha_{wb}\left(h - h_{ac,wb}\right)$$

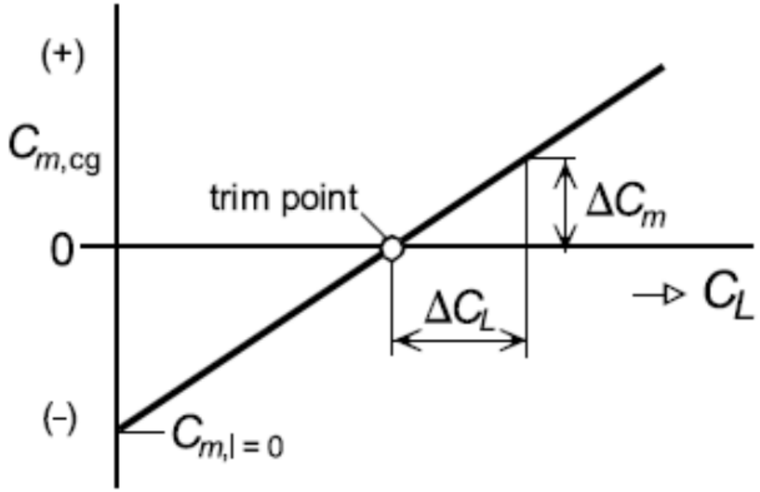
$$\boxed{C_{M,ac-wb} = -0.032}$$

Problems…

X $C_{M,0}$ must be positive

X $\partial C_{M,0} / \partial \alpha_a$ must be negative



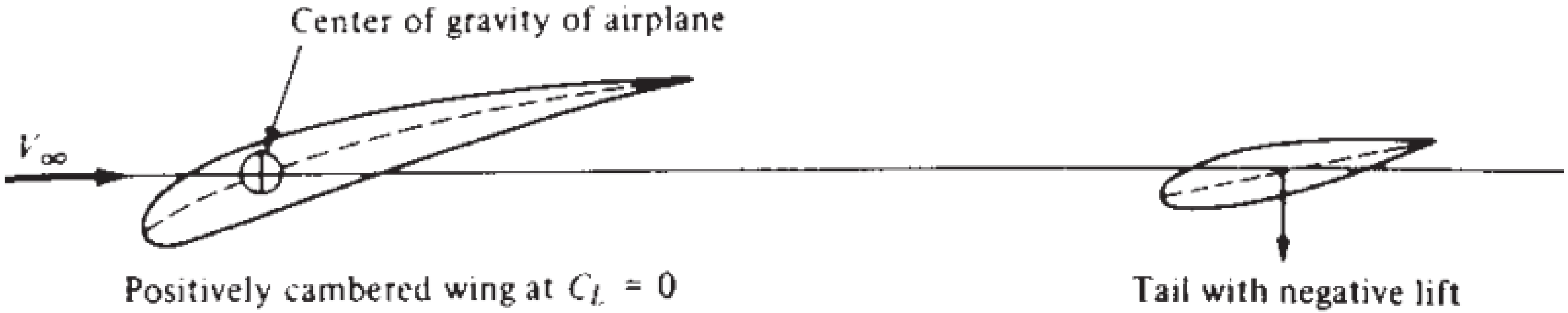


(b) statically unstable

Unfortunately, wing alone is unstable.

Contribution of tail

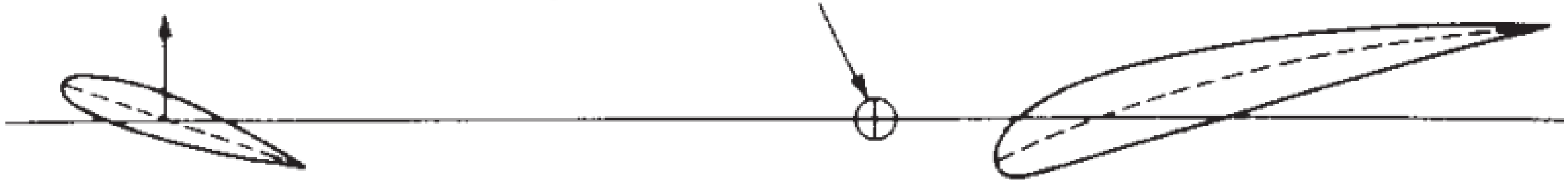
Horizontal stabilizer



a) Conventional wing-tail combination

Contribution of tail

Horizontal stabilizer



b) Canard wing-tail combination.

Contribution of tail

Figure 5.44 Wing-tip vortices made visible by smoke ejected at the wing tips of a Boeing 727 test airplane.
(Source: *NASA*.)

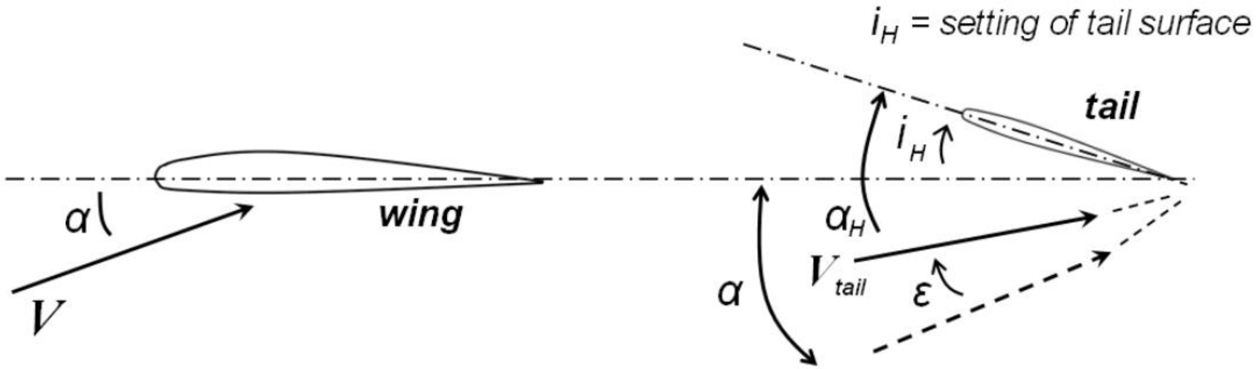
Contribution of tail

The effects of
airflow downwash ?

Figure 5.44 Wing-tip vortices made visible by smoke ejected at the wing tips of a Boeing 727 test airplane.
(Source: *NASA*.)

Contribution of tail

The effects of downwash



$$\alpha_H = \alpha - \varepsilon - i_H$$

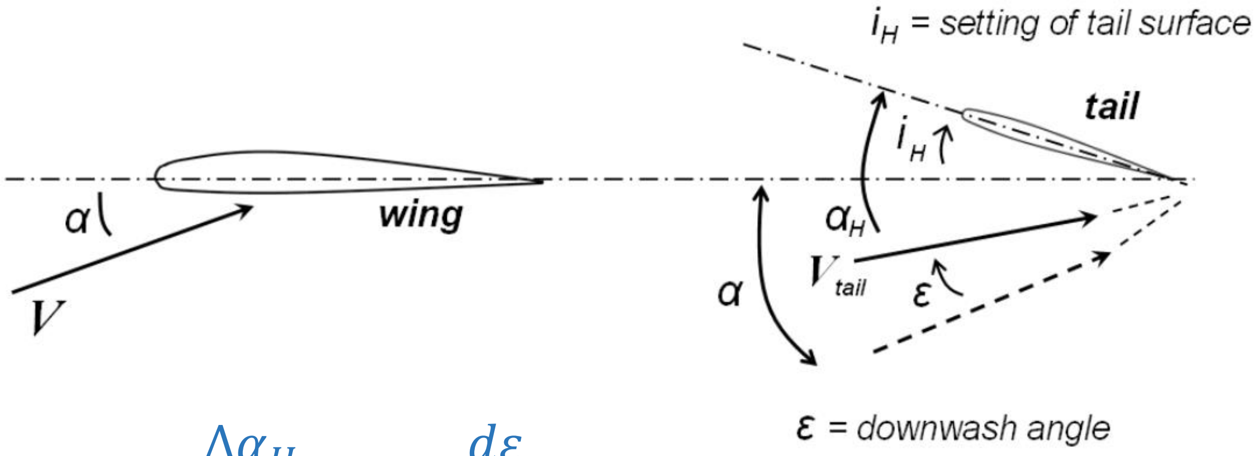
ε = downwash angle

Airflow direction: $\alpha \to \alpha - \varepsilon$

Airflow magnitude: $V \to V_{tail}$

Contribution of tail

The effects of downwash

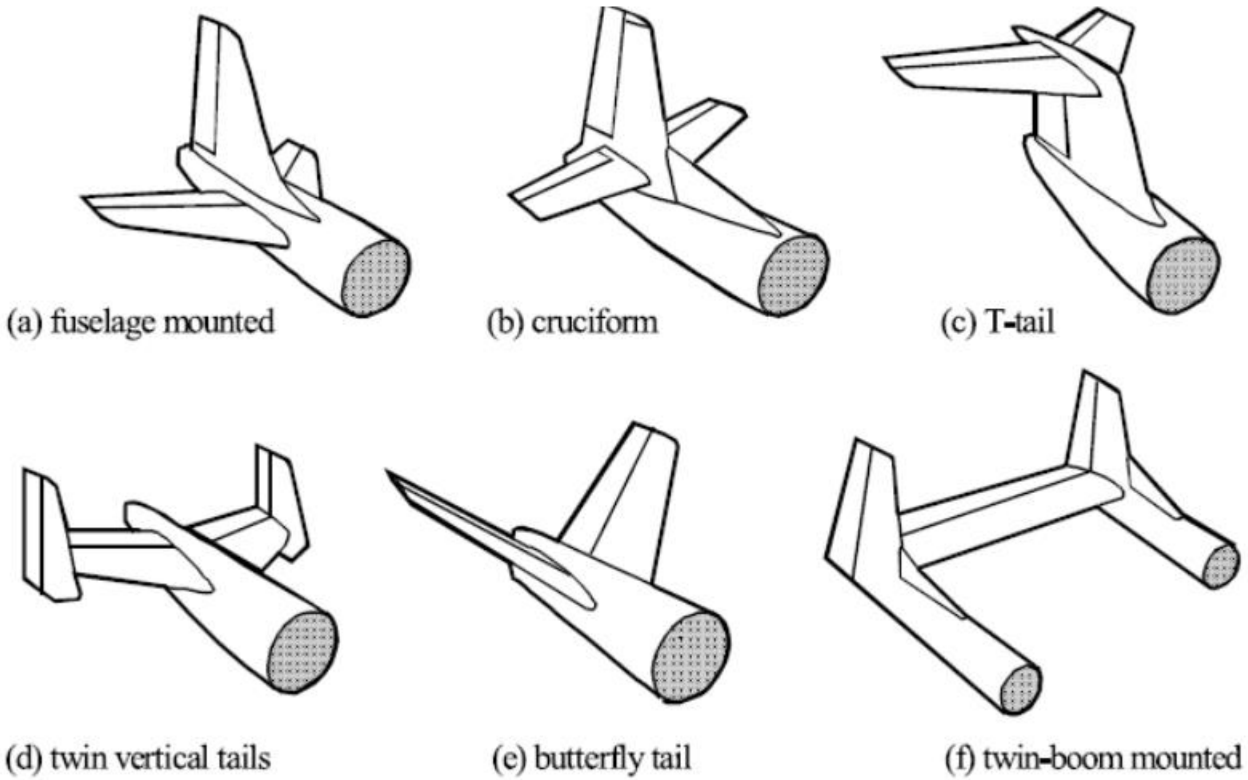


$$\frac{\Delta\alpha_H}{\Delta\alpha} = 1 - \frac{d\varepsilon}{d\alpha}$$

Typical value for $d\varepsilon/d\alpha$ is 0.01 (depend on the tail configuration).

Contribution of tail

Typical tail configurations



Contribution of tail

Wing-tail combination

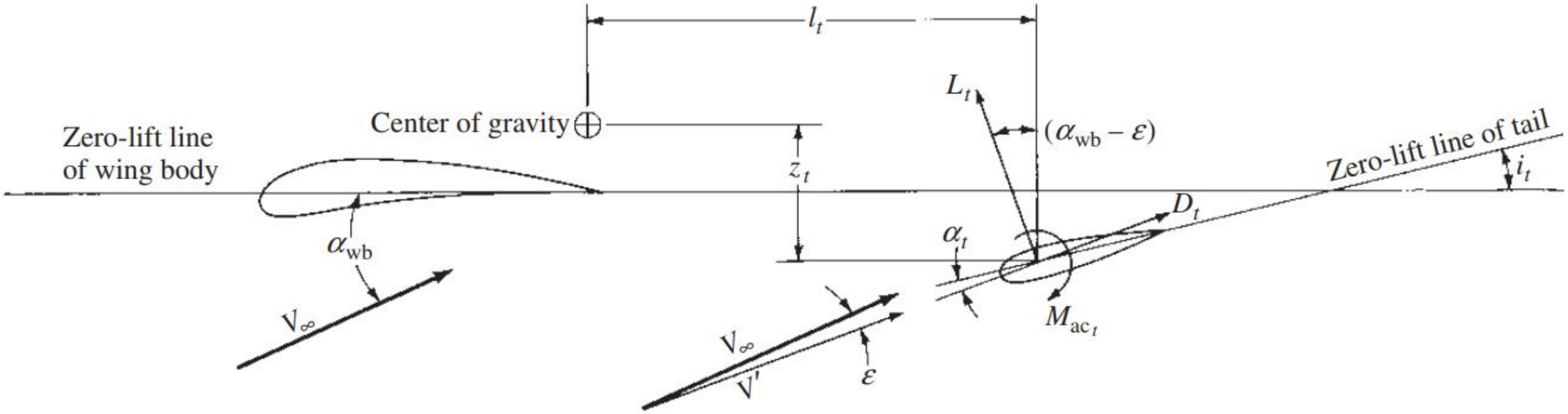
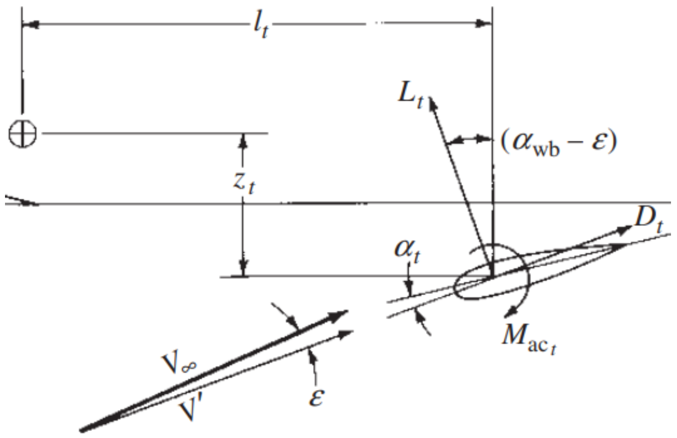


Figure 7.21 Geometry of wing–tail combination.

What is the total pitching moment about the center of gravity of the wing-tail combination?

Contribution of tail

The sum of tail moments



$$M_{cg_t} = -l_t[L_t \cos(\alpha_{wb} - \varepsilon) + D_t \sin(\alpha_{wb} - \varepsilon)] + z_t L_t \sin(\alpha_{wb} - \varepsilon) - z_t D_t \cos(\alpha_{wb} - \varepsilon) + M_{ac_t} \qquad (7.11)$$

The lift force is perpendicular to the airspeed direction (no relate to the i_t)

Contribution of tail

Approximations

1. $z_t \ll l_t$. Vertical distance to c.g. is far smaller than horizontal
2. $D_t \ll L_t$. Drag force is far smaller than lift force
3. The angle $\alpha_{\mathrm{wb}} - \varepsilon$ is small; hence $\sin(\alpha_{\mathrm{wb}} - \varepsilon) \approx 0$ and $\cos(\alpha_{\mathrm{wb}} - \varepsilon) \approx 1$.
4. M_{ac_t} is small in magnitude.

$$\Rightarrow \quad M_{\mathrm{cg}_t} = -l_t L_t$$

Contribution of tail

Define tail lift coefficient

$$C_{L,t} = \frac{L_t}{q_\infty S_t}$$

$$\Rightarrow \quad M_{\mathrm{cg}_t} = -l_t q_\infty S_t C_{L,t}$$

$$\Rightarrow \quad \frac{M_{\mathrm{cg}_t}}{q_\infty S c} \equiv C_{M,\mathrm{cg}_t} = -\frac{l_t S_t}{cS} C_{L,t}$$

Contribution of tail

Define tail lift coefficient

$$C_{L,t} = \frac{L_t}{q_\infty \boxed{S_t}}$$

Tail planform area

$$\Rightarrow \quad M_{cg_t} = -l_t q_\infty S_t C_{L,t}$$

Tail volume ratio V_H

$$\Rightarrow \quad \frac{M_{cg_t}}{q_\infty S c} \equiv C_{M,cg_t} = -\boxed{\frac{l_t S_t}{cS}} C_{L,t}$$

Contribution of tail

$$\frac{M_{\mathrm{cg}_t}}{q_\infty S c} \equiv C_{M,\mathrm{cg}_t} = -\frac{l_t S_t}{cS} C_{L,t}$$

Given $$C_{M,\mathrm{cg}_t} = -V_H C_{L,t}$$

$$\varepsilon = \varepsilon_0 + \frac{\partial \varepsilon}{\partial \alpha}\alpha_{\mathrm{wb}}$$

Obtained by the experiment

The lift coefficient of tail $$C_{L,t} = a_t \alpha_t = a_t(\alpha_{\mathrm{wb}} - i_t - \boxed{\varepsilon})$$

$\partial C_{L,t} / \partial \alpha_t$ lift slope of the tail

$$\Rightarrow \quad C_{L,t} = a_t \alpha_{\mathrm{wb}}\left(1 - \frac{\partial \varepsilon}{\partial \alpha}\right) - \boxed{a_t}(i_t + \varepsilon_0)$$

$$\Rightarrow \quad C_{M,\mathrm{cg}_t} = -a_t V_H \alpha_{\mathrm{wb}}\left(1 - \frac{\partial \varepsilon}{\partial \alpha}\right) + a_t V_H(\varepsilon_0 + i_t)$$

The total pitching moment

Define total pitching moment M_{cg}

$$C_{M,\,\mathrm{cg}} = C_{M,\,\mathrm{cg}_{\mathrm{wb}}} + C_{M,\,\mathrm{cg}_t}$$

$$C_{M,ac-wb} + C_{L,wb}\left(h - h_{ac,wb}\right) \qquad -V_H C_{L,t}$$

$$\boxed{C_{M,\,\mathrm{cg}} = C_{M,\,\mathrm{ac}_{\mathrm{wb}}} + a_{\mathrm{wb}}\alpha_{\mathrm{wb}}\left[h - h_{\mathrm{ac}_{\mathrm{wb}}} - V_H \frac{a_t}{a_{\mathrm{wb}}}\left(1 - \frac{\partial\varepsilon}{\partial\alpha}\right)\right] + V_H a_t (i_t + \varepsilon_0)}$$

Practice

Example 7.4

Consider the wing–body model in Example 7.3. The area and chord of the wing are 0.1 m^2 and 0.1 m, respectively. Now assume that a horizontal tail is added to this model. The distance from the airplane's center of gravity to the tail's aerodynamic center is 0.17 m; the tail area is 0.02 m^2; the tail-setting angle is 2.7°; the tail lift slope is 0.1 per degree; and from experimental measurement, $\varepsilon_0 = 0$ and $\partial\varepsilon/\partial\alpha = 0.35$. If $\alpha = 7.88°$, calculate $C_{M,\mathrm{cg}}$ for the airplane model.

Longitudinal static stability

Criteria for static stability

1) $C_{M,0}$ must be positive

2) $\partial C_{M,0} / \partial \alpha_a$ must be negative

$$C_{M,0} \equiv (C_{M,\text{cg}})_{L=0} = C_{M,\text{ac}_\text{wb}} + V_H a_t (i_t + \varepsilon_0)$$

$$\frac{\partial C_{M,\text{cg}}}{\partial \alpha_a} = a\left[h - h_{\text{ac}_\text{wb}} - V_H \frac{a_t}{a}\left(1 - \frac{\partial \varepsilon}{\partial \alpha}\right)\right]$$

Any conclusions?

Longitudinal static stability

Conclusions

- i_t should be a positive quantity
- A larger tail ($S_t \uparrow$) will contribute to static stability
- A longer distance between tail and wing ($l_t \uparrow$) will contribute to stability
- Center of gravity that is just after the wing or even before the wing contributes to stability ($h - h_{ac,wb} \leq 0$, i.e., forward cg $\Rightarrow$ more stable)

$$C_{M,0} \equiv (C_{M,\mathrm{cg}})_{L=0} = C_{M,\mathrm{ac_{wb}}} + V_H a_t (i_t + \varepsilon_0)$$

$$\frac{\partial C_{M,\mathrm{cg}}}{\partial \alpha_a} = a\left[h - h_{\mathrm{ac_{wb}}} - \boxed{V_H}\,\frac{a_t}{a}\left(1 - \frac{\partial \varepsilon}{\partial \alpha}\right)\right]$$

$$\frac{l_t S_t}{cS}$$

Practice

Example 7.5

Consider the wing–body–tail wind tunnel model of Example 7.4 . Does this model have longitudinal static stability and balance?

$$\frac{\partial C_{M,\mathrm{cg}}}{\partial \alpha_a} = a\left[h - h_{\mathrm{ac_{wb}}} - V_H \frac{a_t}{a}\left(1 - \frac{\partial \varepsilon}{\partial \alpha}\right)\right]$$

Neutral point

Definition

$$\frac{\partial C_{M,\text{cg}}}{\partial \alpha_a} = a\left[h - h_{\text{ac}_{\text{wb}}} - V_H \frac{a_t}{a}\left(1 - \frac{\partial \varepsilon}{\partial \alpha}\right)\right]$$

Neutral point (中性点): the value of h when $\partial C_{M,cg}/\partial \alpha_a = 0$, denoted by h_n.

$$\Rightarrow \quad h_n = h_{\text{ac}_{\text{wb}}} + V_H \frac{a_t}{a}\left(1 - \frac{\partial \varepsilon}{\partial \alpha}\right)$$

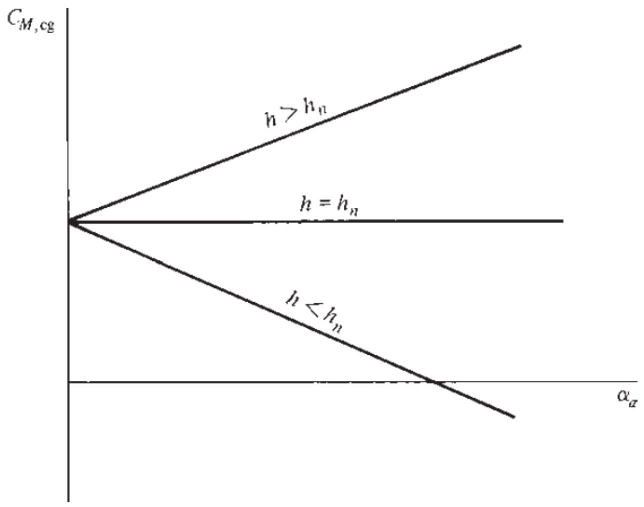
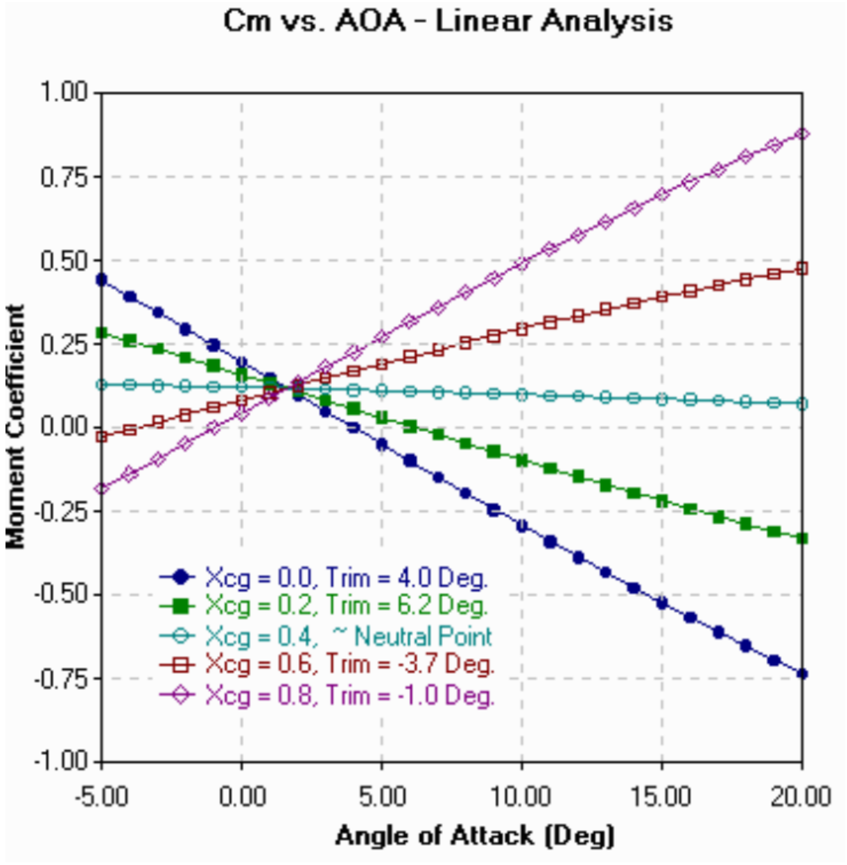


Figure 7.23 Effect of the location of the center of gravity, relative to the neutral point, on static stability.

Neutral point

Stability analysis



Estimate neutral point:

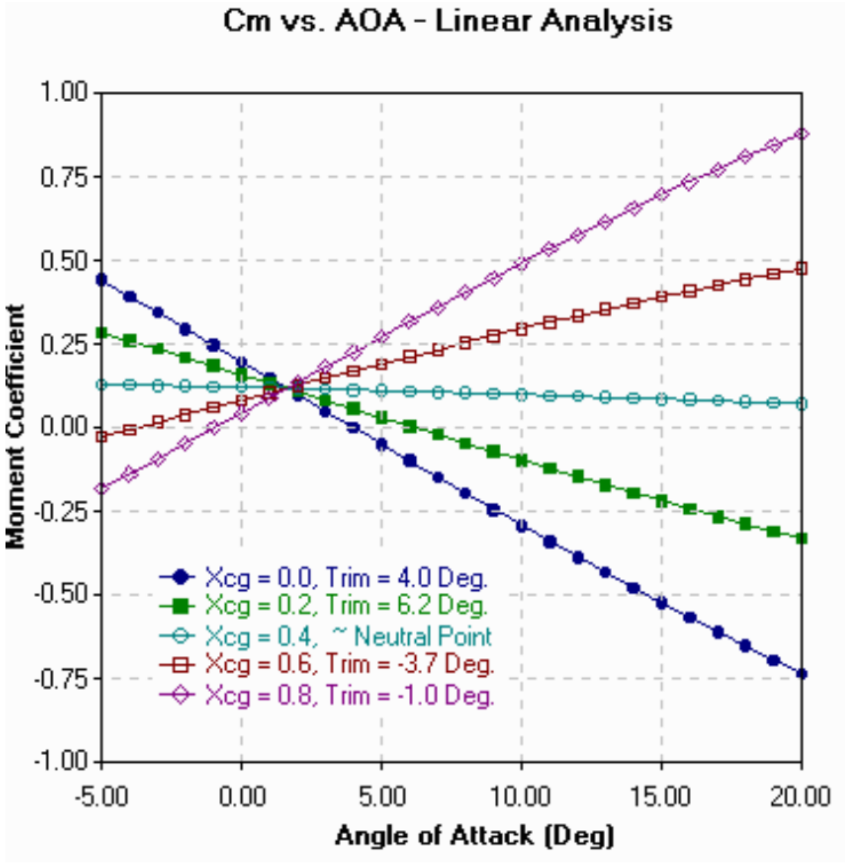
more or less than 0.4 ?

Neutral point

Key positions for longitudinal stability

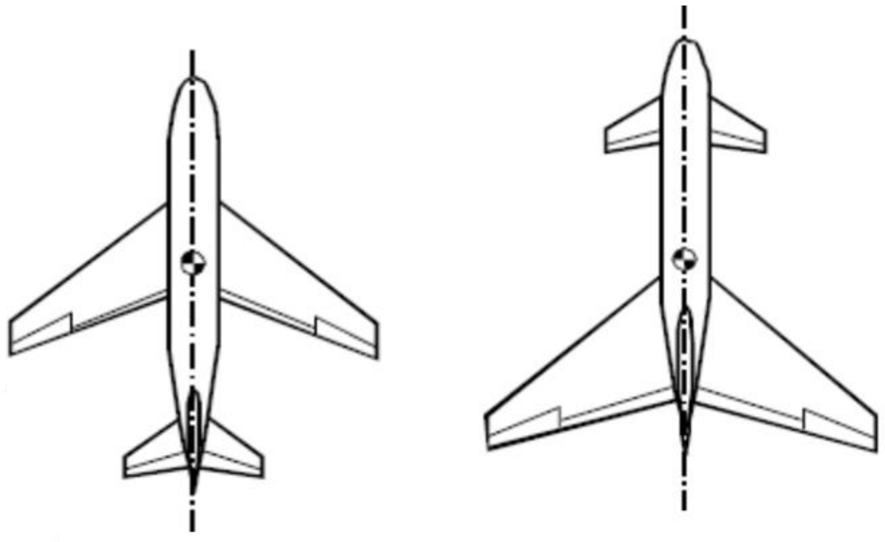
- Position of tail surface
- Position of center of gravity

For longitudinal static stability, the position of the center of gravity must always be forward of the neutral point.



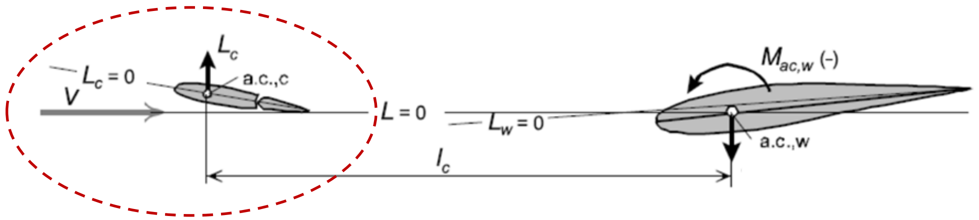
Neutral point

How about a canard (鸭翼) ?



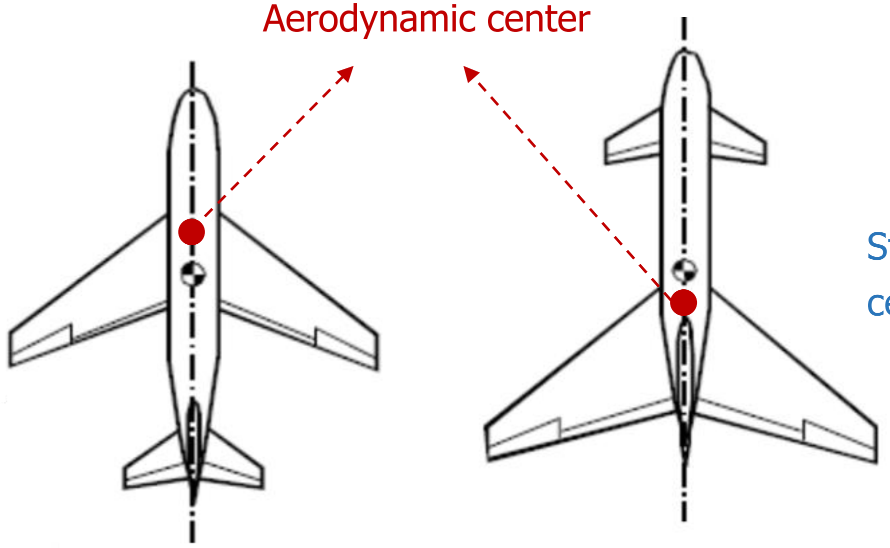
Neutral point

How about a canard (鸭翼)



$C_{M,0} > 0$

Aerodynamic center



Statically stable canard, by moving the center of gravity ahead of wing

Neutral point

How about a canard (鸭翼)

比亚乔P180（空中法拉利）

Static Margin

Definition

$$h_n = h_{\mathrm{ac_{wb}}} + V_H \frac{a_t}{a}\left(1 - \frac{\partial \varepsilon}{\partial \alpha}\right)$$

$$\frac{\partial C_{M,\mathrm{cg}}}{\partial \alpha_a} = a\left[h - h_{\mathrm{ac_{wb}}} - V_H \frac{a_t}{a}\left(1 - \frac{\partial \varepsilon}{\partial \alpha}\right)\right]$$

$$\Rightarrow \quad h_{\mathrm{ac_{wb}}} = h_n - V_H \frac{a_t}{a}\left(1 - \frac{\partial \varepsilon}{\partial \alpha}\right)$$

$$\Rightarrow \quad \frac{\partial C_{M,cg}}{\partial \alpha_a} \approx a(h - h_n)$$

Static Margin

Definition

$$\frac{\partial C_{M,cg}}{\partial \alpha_a} \approx a(h - h_n) = -a(h_n - h)$$

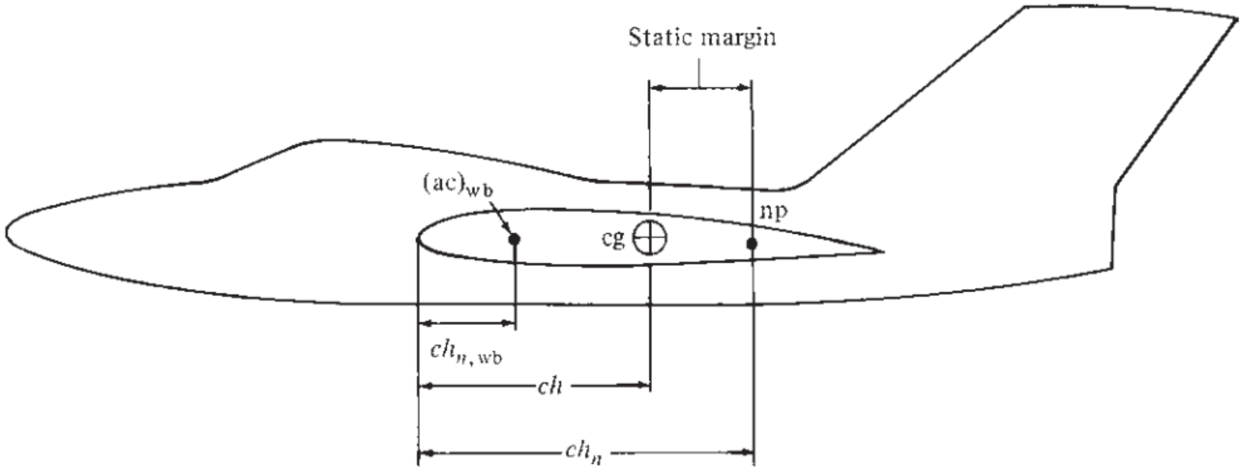
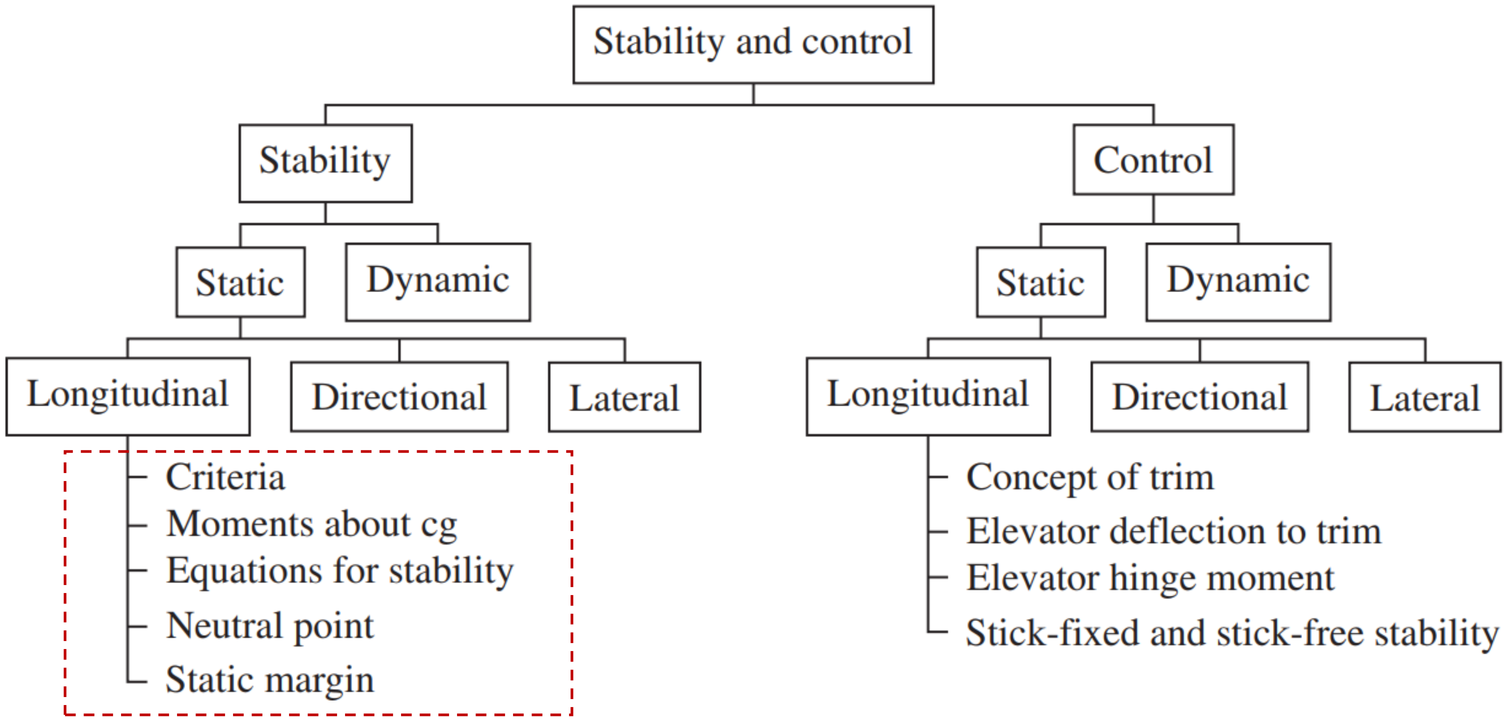


Figure 7.24 Illustration of the static margin.

Summary

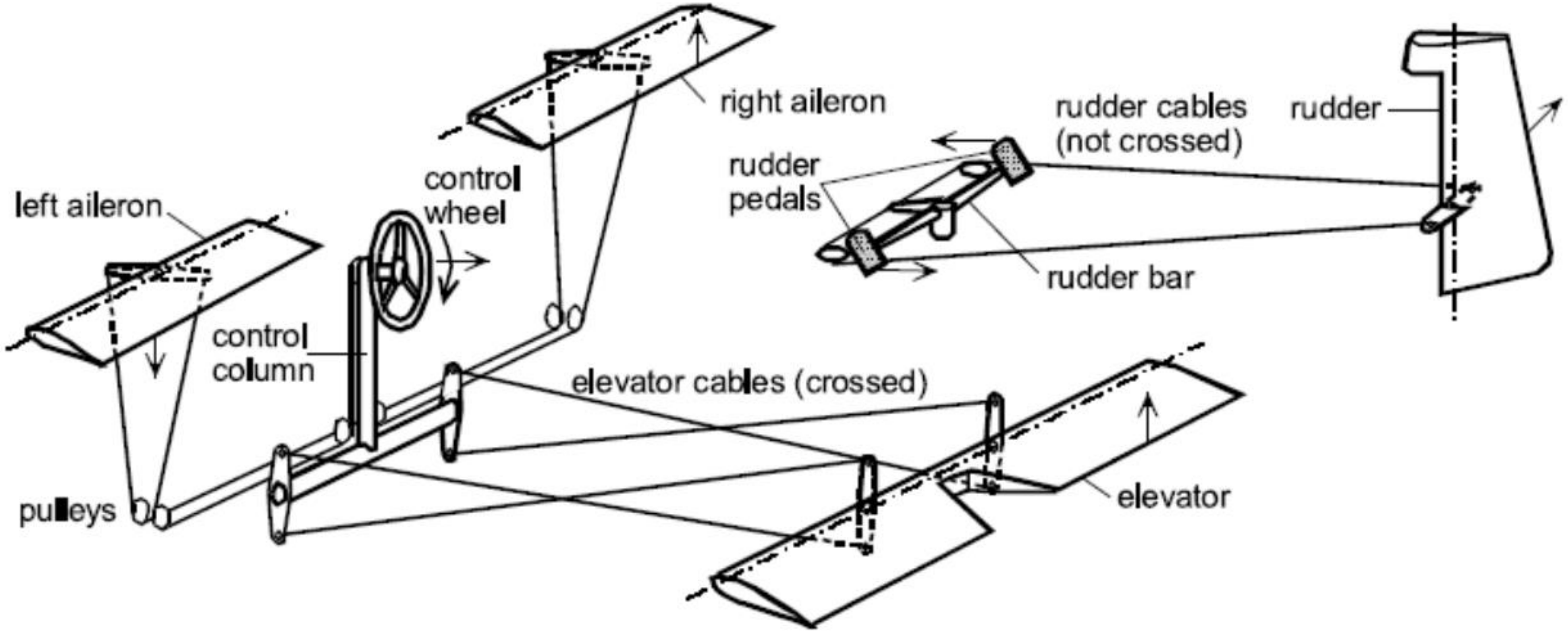
Stability



Static longitudinal control

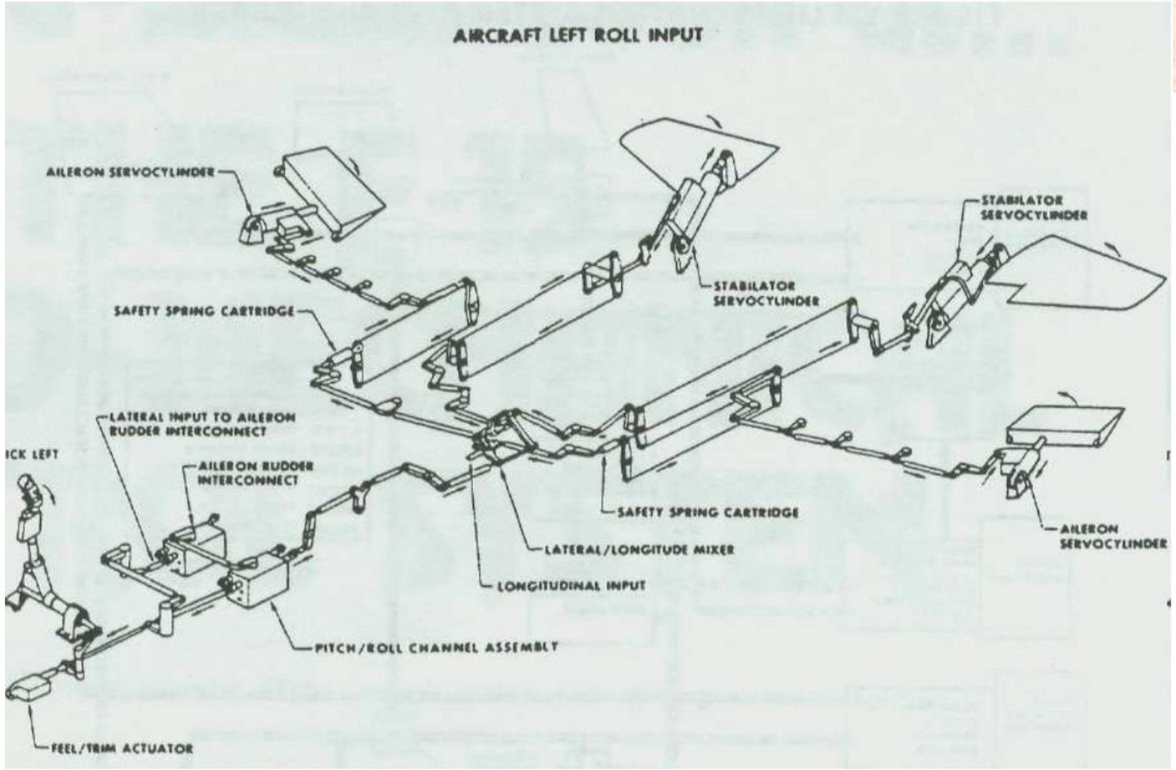
Mechanical Flight Control System (FCS)

Early pure mechanical system



Static longitudinal control

Electrical FCS



Static longitudinal control

Fly by wire FCS