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4.1

解：H_{20}模式的~~等~~横向电场、横向磁场

$$\dot{E}_y = A\sin\left(\frac{2\pi}{a}x\right)\cdot Be^{-j\beta z} = e_y(x)\cdot \dot{U}(z)$$

$$\dot{H}_x = C\sin\left(\frac{2\pi}{a}x\right)\cdot De^{-j\beta z} = h_x(x)\cdot \dot{I}(z)$$

满足归一化条件：

$$\frac{1}{2}\int_S (\vec{e_T}\times\vec{h_T})\cdot\hat{i}_z\, ds = 1$$

$$AC\int_0^b dy\int_0^a \sin^2\frac{2\pi}{a}x\, dx = -1.$$

$\therefore\quad AC = -\frac{2}{ab}.$

$$\frac{E_y}{H_x} = -\eta_{TE20} = \frac{AB}{CD}$$

$$\frac{B}{D} = \frac{\dot{U}(z)}{\dot{I}(z)} = Z_0$$

$\therefore\quad \frac{A}{C} = -\frac{\eta_{TE20}}{Z_0}$

取 $Z_0 = \eta_{TE20}$. $\frac{A}{C} = -1.$

解得
$$\begin{cases} A = \sqrt{\frac{2}{ab}} \\ B = -\sqrt{\frac{2}{ab}}. \end{cases}$$

矢量模式函数.

$$\begin{cases} \vec{e_T} = \hat{i}_y\sqrt{\frac{2}{ab}}\sin\left(\frac{2\pi}{a}x\right) \\ \vec{h_T} = -\hat{i}_x\sqrt{\frac{2}{ab}}\sin\left(\frac{2\pi}{a}x\right). \end{cases}$$

$$\begin{cases} \dot{U}(z) = Be^{-j\beta z} \\ \dot{I}(z) = \frac{B}{\eta_{TE20}}e^{-j\beta z}. \end{cases}$$

4-3.

解：TE_{10} 模式.

$$k_c = \sqrt{\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2}.$$

$$\lambda_c = \frac{2\pi}{k_c} = 2a = 6.97\,cm.$$

$z<0$ 时.

$$\lambda_0 = \frac{c}{f} = 6.67\,cm.$$

$$G = \sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2} = 0.2902.$$

$$\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 377\,\Omega.$$

$z>0$ 时

$$\lambda_1 = \frac{c}{\sqrt{\varepsilon_r}f} = 4.17\,cm$$

$$\eta_1 = \sqrt{\frac{\mu_0}{\varepsilon_0\varepsilon_r}} = 235.6\,\Omega.$$

~~取 Z_0~~ $$G_1 = \sqrt{1-\left(\frac{\lambda_1}{\lambda_c}\right)^2} = 0.8013.$$

取 等效特性阻抗. $Z_0 = \eta_w$

$$Z_{0a} = \frac{\eta_0}{G_0} = 589\,\Omega$$

$$Z_{0d} = \frac{\eta_1}{G_1} = 133.3\,\Omega$$

$$\Gamma = \frac{Z_{0d}-Z_{0a}}{Z_{0d}+Z_{0a}} \approx -0.63.$$