北京航空航天大學
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飞行力学
Flight Mechanics

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Chapter 2

- Static performance

  Horizontal flight,

  climbing and descending flight,

  Range and endurance.

- Dynamic performance

  Takeoff,

  Landing,

  ……

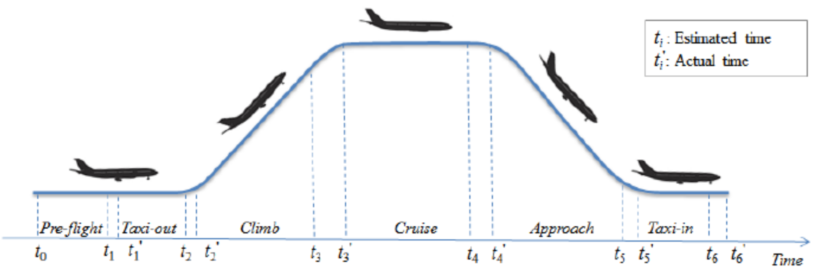
Contents

- Introduction
- Equation of motion
- Take-off distance and time
- Summary
- Examples
- Landing

Introduction

Question

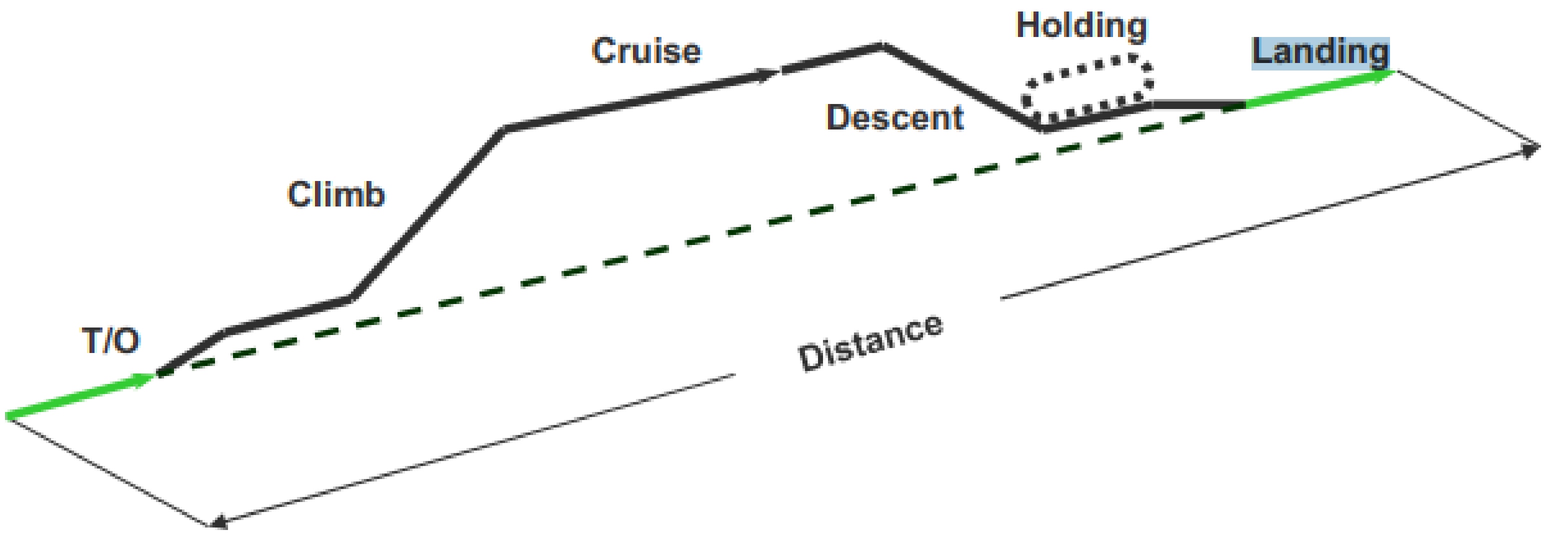
- What are the major phases of flight?
- Which is the most dangerous phase?



Introduction

Phases of flight – civil aviation

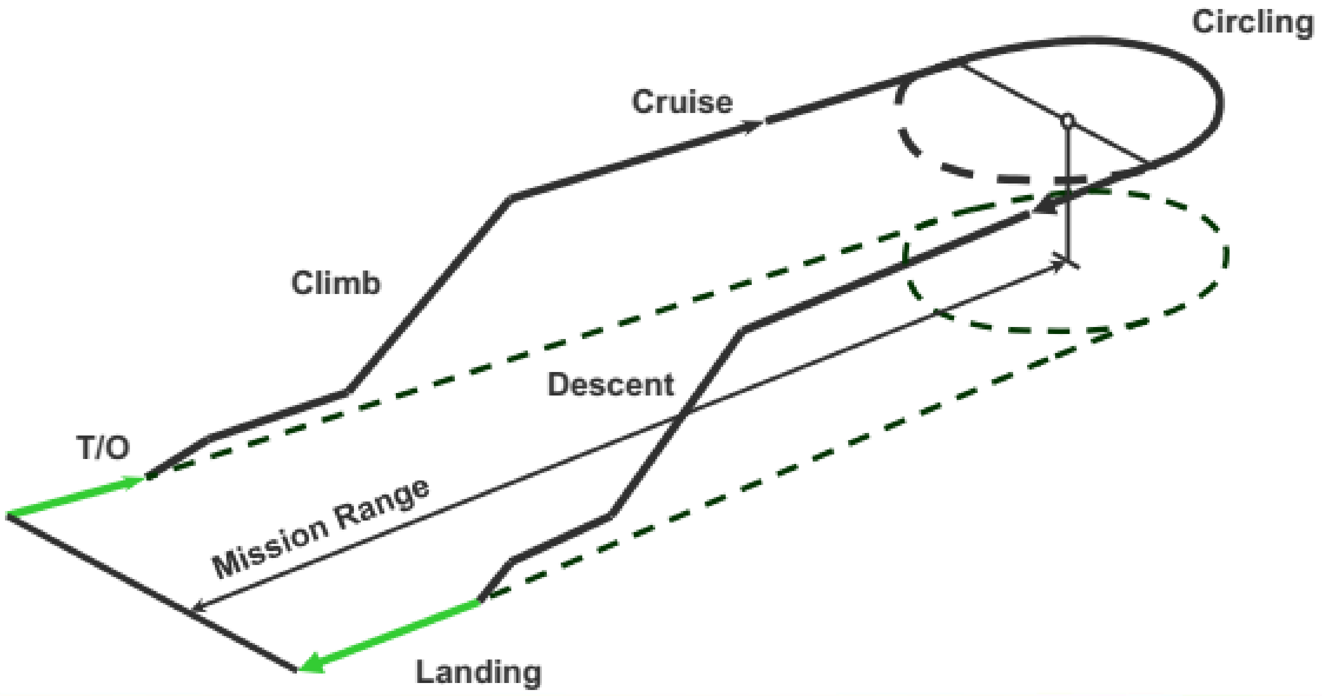
- Taxi
- Take-off, T/O
- Initial climb
- Departure/climb
- Cruise
- Descend/Arrival
- Holding
- Approach
- Landing
- Taxi



Introduction

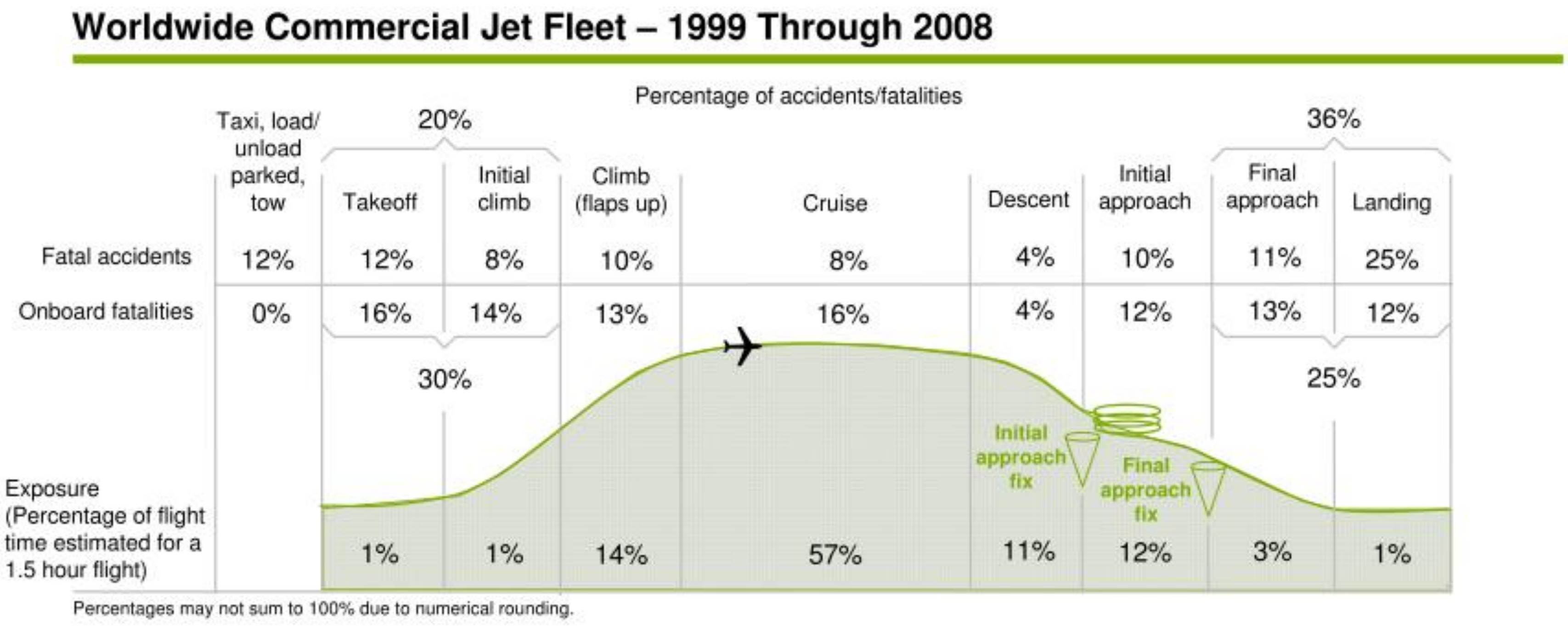
Phases of flight – exploration

- Taxi
- Take-off, T/O
- Initial climb
- Departure/climb
- Cruise
- Military Section (Circling, …)
- Cruise
- Descent/Arrival
- Approach
- Landing



Introduction

Fatal accidents and onboard fatalities by phase of flight



STATISTICAL SUMMARY OF COMMERCIAL JET AIRPLANE ACCIDENTS Worldwide Operations | 1959 – 2008

Introduction

Some definitions

Take-off（起飞）：The take-off can be defined as the maneuver by which the airplane is accelerated from rest on the runway to the climb out speed V_H over the screen height.

Landing（降落）：The landing is the maneuver by which the airplane is brought from a steady approach speed V_A over a 15 m obstacle at the runway threshold to standstill at the runway.

Screen height（安全高度）: Screen height is defined in as an imaginary obstacle that the aircraft can safely fly through on takeoff or landing. In China, the screen height is 15 meters for civil transports and 25 meters for military airplanes.

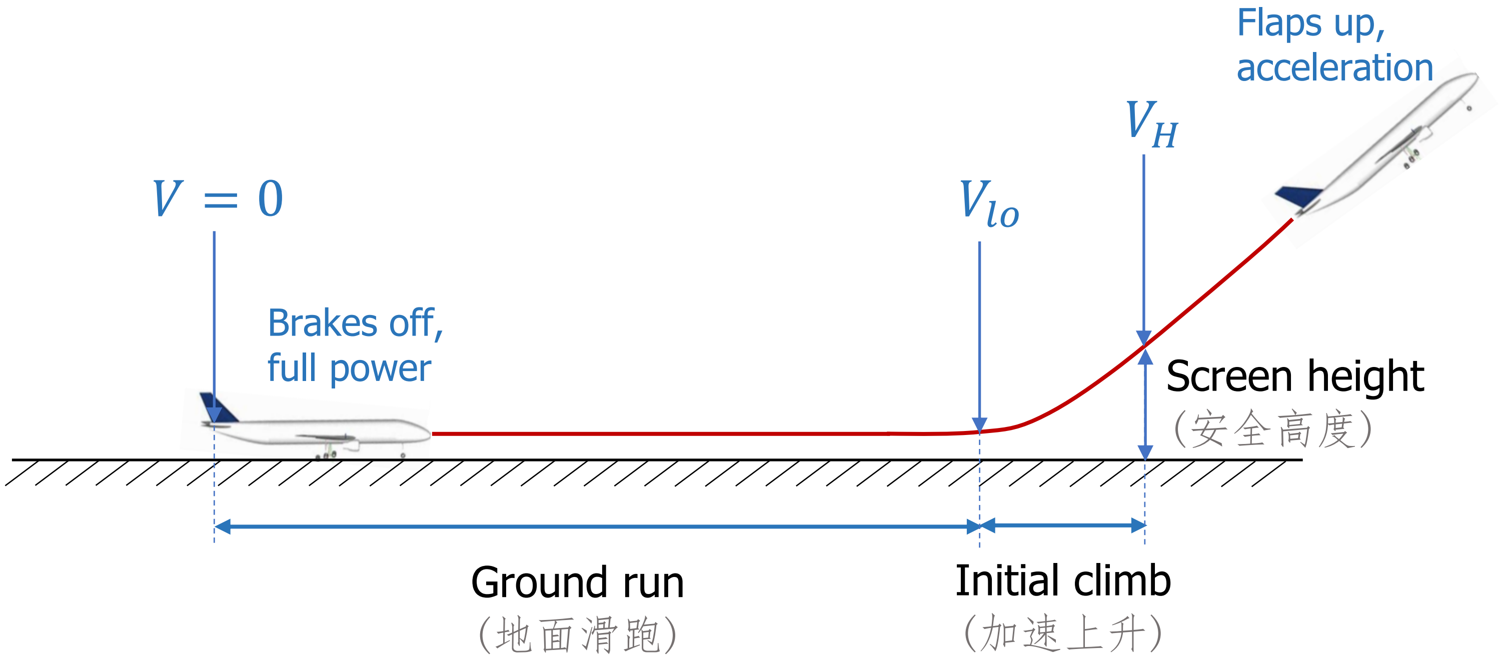
Introduction

Key features of take-off

- Unsteady motion with rapid velocity change.
- The aircraft is subjected to the support and frictional forces of the ground against the wheels.
- The aircraft will experience structure changes such as lowering the landing gear, extending the flaps and other lift-increase devices, opening the speed brakes, etc.
- The ground effects should be taken into consideration.

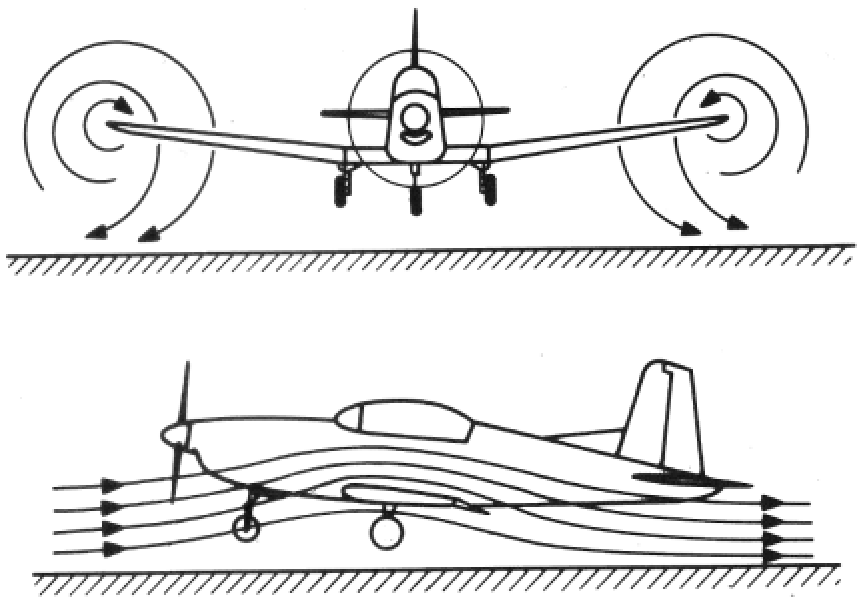
Introduction

Take-off maneuver



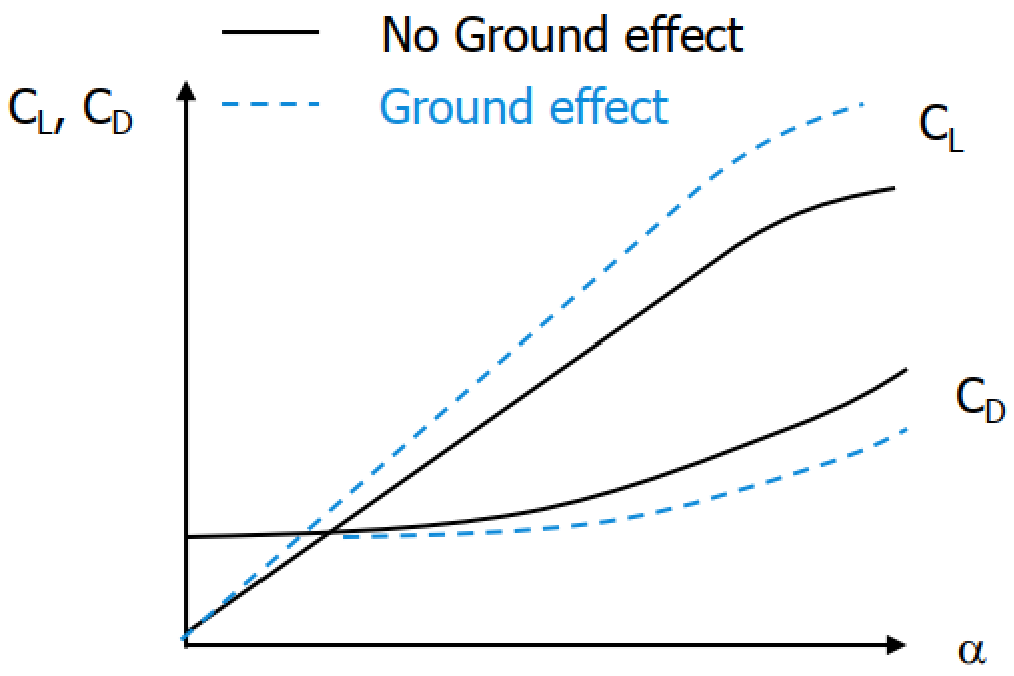
Introduction

Ground effects（地面效应）



Introduction

Ground effects (地面效应)



Contents

- Introduction
- **Equation of motion**
- Take-off distance and time
- Summary
- Examples
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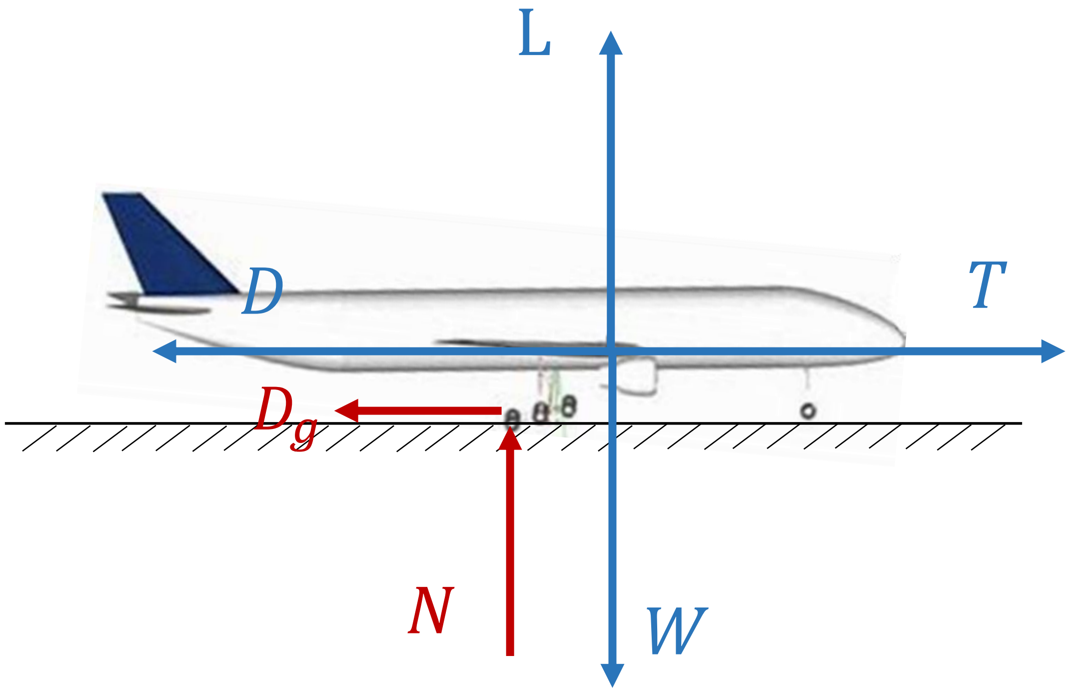
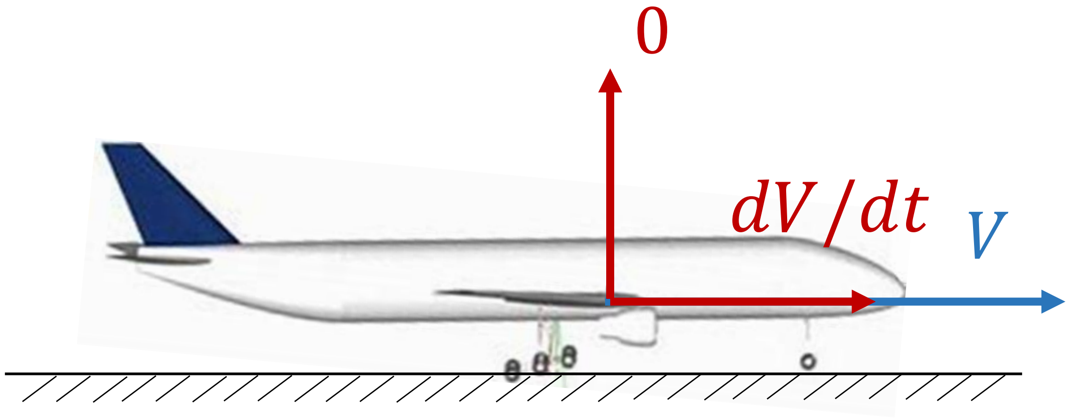
Equation of motion

Ground run – Free Body Diagram



Equation of motion

Ground run – Kinetic Diagram



There is no vertical acceleration during the ground phase
(aircraft travels along a straight line)

Equation of motion

- Parallel to the airspeed vector

$$\parallel V: \frac{W}{g}\frac{dV}{dt} = T - D - D_g$$

- Perpendicular to the airspeed

$$\perp V: N = W - L$$

- Friction of wheels $D_g = \mu N$

$$\boxed{\begin{aligned} \frac{W}{g}\frac{dV}{dt} &= T - D - \mu(W - L) \\ N &= W - L \end{aligned}}$$

Equation of motion

Ground run – Kinetic equation

- Horizontal runway
- No wind
- Thrust vector parallel to airspeed

$$\boxed{\frac{ds}{dt} = V}$$

Equation of motion

Ground run（地面滑跑段）

The average acceleration for ground run

$$\bar{a} = \frac{g}{W}\left(\bar{T} - \bar{D} - \bar{D}_g\right)$$

Equation of motion

Ground run – distance and time

$$S_1 = \frac{V_{lo}^2}{2\bar{a}} = \frac{1}{2g}\frac{V_{lo}^2}{\bar{T}/W - \mu}$$

$$t_1 = \frac{V_{lo}}{\bar{a}} = \frac{1}{g}\frac{V_{lo}}{\bar{T}/W - \mu}$$

Equation of motion

Ground run – distance and time

$$S_1 = \frac{V_{lo}^2}{2\bar{a}} = \frac{1}{2g}\frac{V_{lo}^2}{\bar{T}/W - \mu}$$

$$t_1 = \frac{V_{lo}}{\bar{a}} = \frac{1}{g}\frac{V_{lo}}{\bar{T}/W - \mu}$$

Question: How to reduce S and t ?

Equation of motion

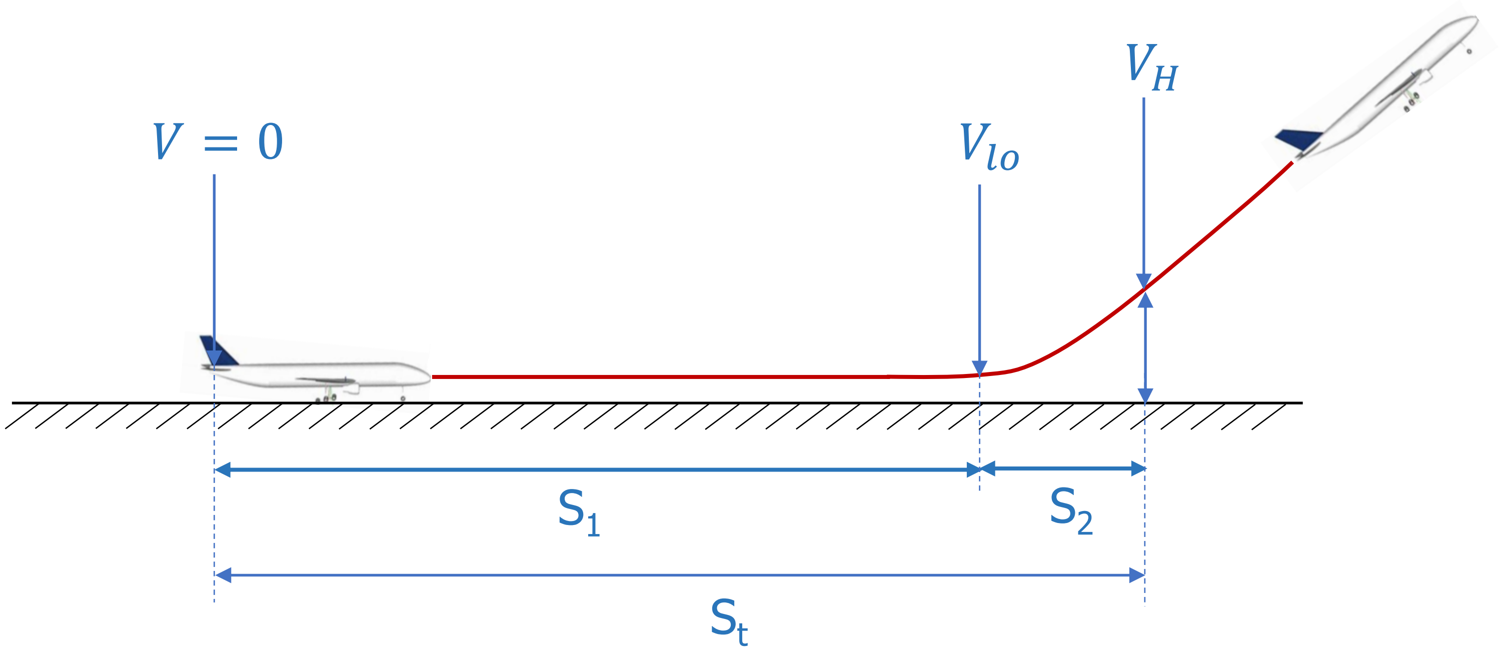
Ground run – the lift-off speed （离地速度）

$$V_{lo} = \sqrt{\frac{2W}{\rho S C_{L,lo}}}$$

Assumptions ?

Equation of motion

The initial climb（加速上升段）



Equation of motion

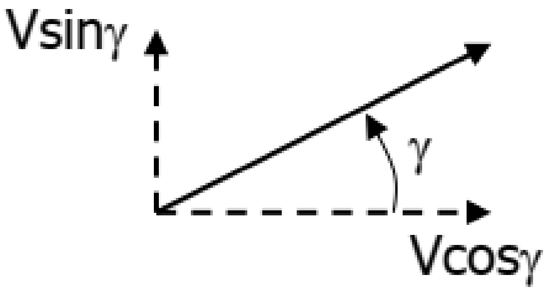
The initial climb（加速上升段）

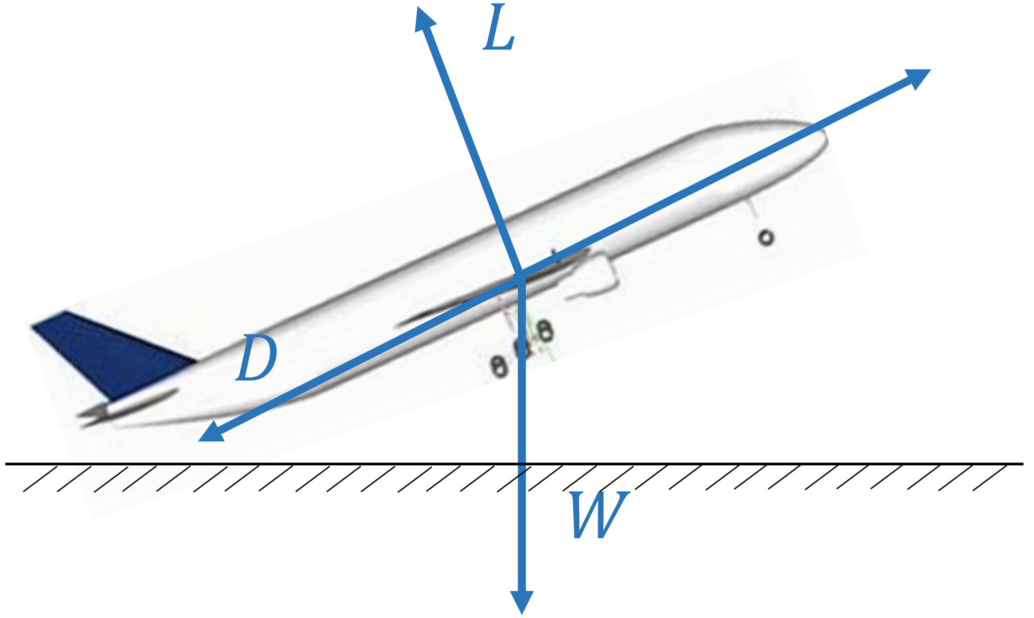
$$\frac{W}{g}\frac{dV}{dt}=T-D-W\sin\gamma$$

$$\frac{W}{g}V\frac{d\gamma}{dt}=L-W\cos\gamma$$

$$\frac{dH}{dt}=V\sin\gamma$$

$$\frac{ds}{dt}=V\cos\gamma$$





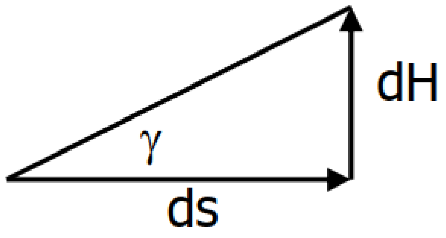
Equation of motion

The initial climb（加速上升段）

$$\frac{W}{g}\frac{dV}{ds}V = T - D - W\sin\gamma$$

$$\frac{W}{g}VdV = (T - D)ds - W\sin\gamma ds$$

$$\frac{W}{g}VdV = (T - D)ds - Wdh$$



$$\sin\gamma \approx \tan\gamma = \frac{dh}{ds}$$

$$\sin\gamma ds = dh$$

$$\frac{W}{g}\int_{V_{lo}}^{V_H} VdV = \int_{S_1}^{S_t}(T - D)ds - \int_0^H Wdh$$

Equation of motion

The initial climb – Assumptions

$$\frac{W}{g}\int_{V_{lo}}^{V_H} V dV = \int_{S_{lo}}^{S_{to}} (T-D)ds - \int_0^H W dh$$

- Assume constant (average) T-D

$$T - D = \bar{T} - \bar{D}$$

$$\frac{W}{2g}\left(V_H^2 - V_{lo}^2\right) = \bar{T} - \bar{D}(S_{to} - S_{lo}) - WH$$

- Assume steady climb at screen height

$$\frac{\bar{T} - \bar{D}}{W} = \sin\gamma_H \qquad \frac{V_H^2 - V_{lo}^2}{2g} = \sin\gamma_H(S_{to} - S_{lo}) - H$$

Equation of motion

The initial climb – distance and time

Distance and time for initial climb

$$S_2 = \frac{1}{\sin\gamma_H}\left(\frac{V_H^2 - V_{lo}^2}{2g} + H\right)$$

$$t_2 = \frac{S_2}{V_{av}} = \frac{S_2}{(V_H + V_{lo})/2}$$

Summary

The take-off distance

$$S_1 = \frac{V_{lo}^2}{2\bar{a}} = \frac{1}{2g}\frac{V_{lo}^2}{\bar{T}/W - \mu}$$

$$S_2 = \frac{1}{\sin\gamma_H}\left(\frac{V_H^2 - V_{lo}^2}{2g} + H\right)$$

$$S_t = S_1 + S_2$$

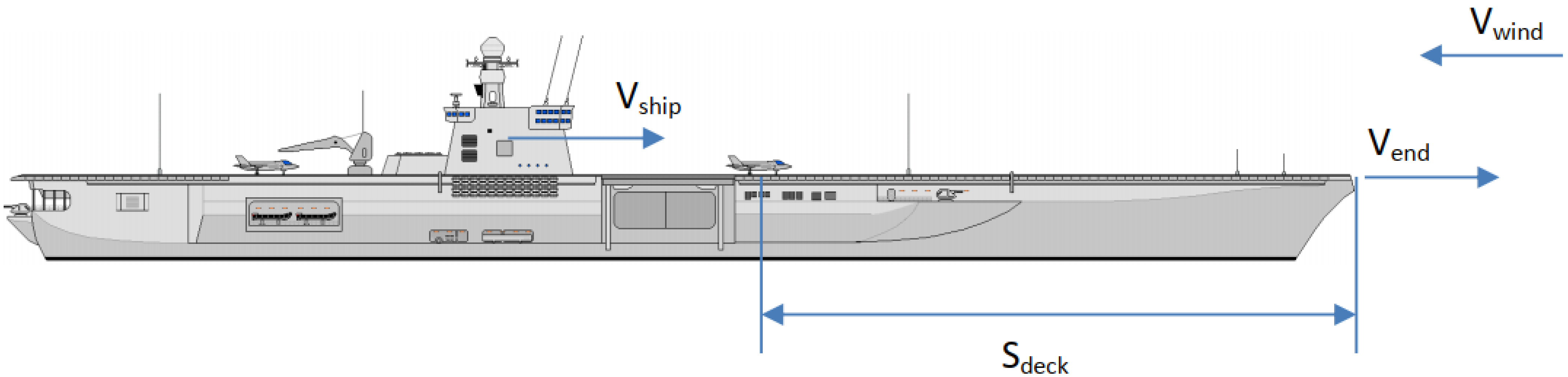
Example - 1

Textbook, Page 74

$$\left.\begin{aligned} W &= 87280\ N \\ S &= 28\ m^2 \\ C_{L.lo} &= 0.75 \\ \mu &= 0.0335 \end{aligned}\right\} \qquad \begin{aligned} S_t &= ? \\ t_t &= ? \end{aligned}$$

Example - 2

Aircraft carriers make use of catapult systems to launch aircraft from the limited distance available on the deck. During the launch, maximum thrust is also applied by the aircraft. In general, the ship will have a forward speed into the direction of the wind (as indicated in the picture), to improve the take-off performance.



Example - 2

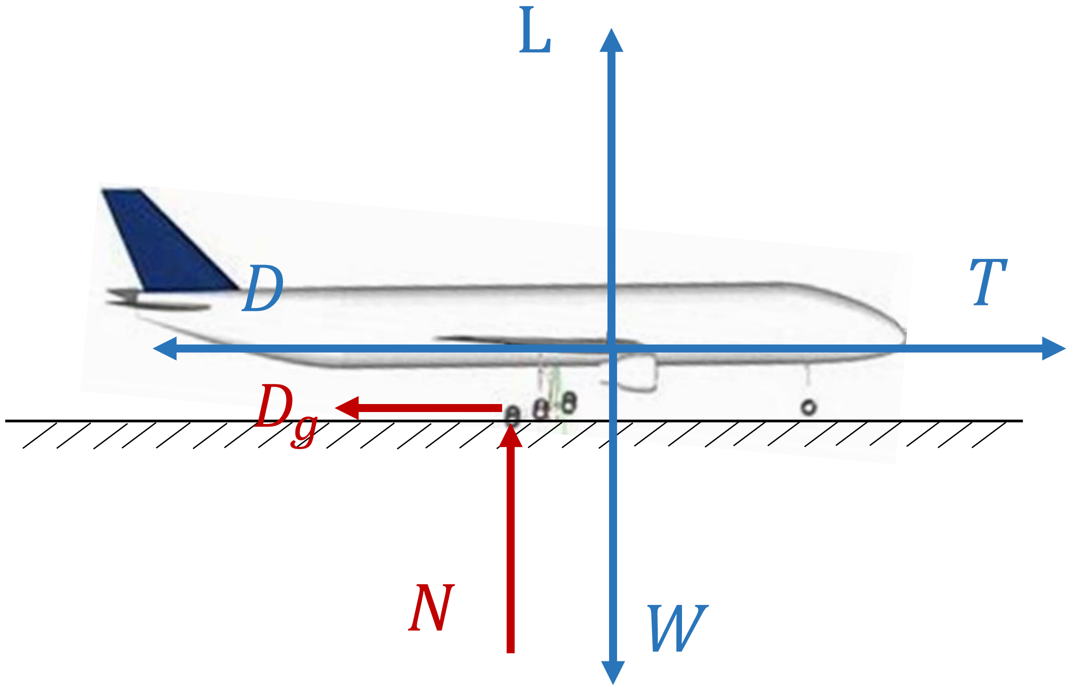
a) Draw a clear free body diagram (FBD) and kinetic diagram (KD) in which all the relevant forces, accelerations, angles and velocities are indicated.

b) Derive the equations of motion for the aircraft during the acceleration over the ship deck

c) Derive an expression for the ground run distance s_{deck} in terms of a mean acceleration and the speed at the moment the aircraft leaves the deck (V_{end}). Clearly indicate if the velocity in the equation is expressed relative to the air or relative to the ship.

Equation of motion

Landing（降落） – the free body diagram



$$\frac{W}{g}\frac{dV}{dt} = T - D - \mu(W - L)$$

$$N = W - L$$

Equation of motion

Landing – the ground run

$$S_3 = K_{av}\left(\frac{V_H^2 - V_{td}^2}{2g} + H\right)$$

Error in the textbook page 76, Eq. (2.72)

$$t_3 = \frac{S_3}{V_{av}} = \frac{S_2}{(V_H + V_{td})/2}$$

Equation of motion

Landing – initial climb

$$S_4 = \frac{1}{2g}\int_0^{V_{td}} \frac{dV^2}{\mu + \frac{\rho V^2 S}{2W}(C_D - \mu C_L)}$$

Error in the textbook page 77, Eq. (2.76)

$$S_4 = -\frac{V_{td}^2}{2\bar{a}} = \frac{V_{td}^2}{g\left(\mu + \frac{1}{K_{td}}\right)}$$

K_{td} is the lift-to-drag ratio at touch down

$$t_4 = -\frac{V_{td}}{\bar{a}} = \frac{2V_{td}}{g\left(\mu + \frac{1}{K_{td}}\right)}$$

Equation of motion

Landing – touch down speed

$$V_{td} = K_1 \sqrt{\frac{2W}{\rho S C_{L,td}}}$$

K_1 is the modified coefficient. $C_{L,td}$ is the touch down lift coefficient

Summary

The landing distance

$$S_3 = K_{av}\left(\frac{V_H^2 - V_{td}^2}{2g} + H\right)$$

$$S_4 = -\frac{V_{td}^2}{2\bar{a}} = \frac{V_{td}^2}{g\left(\mu + \frac{1}{K_{td}}\right)}$$

$$S_t = S_3 + S_4$$

Example - 3

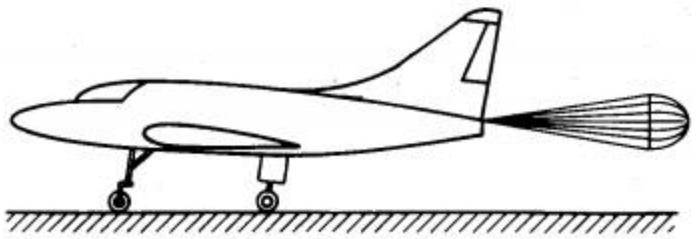
Textbook, Page 77

$$\left.\begin{aligned} W &= 70710\ N \\ S &= 28\ m^2 \\ K_1 &= 0.95 \\ \mu &= 0.3 \end{aligned}\right\} \quad \begin{aligned} S_t &= ? \\ t_t &= ? \end{aligned}$$

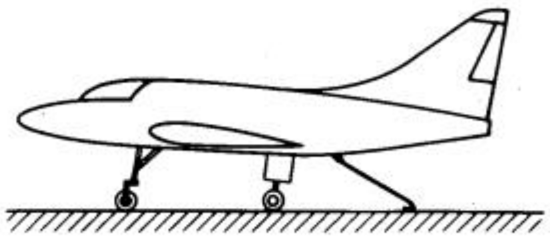
Special devices

Spoilers, thrust reversers, etc.

a) drag parachute



b) arrester hook



Special devices

Speed brakes and spoilers