System Dynamics and Vibrations

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Chapter 2: Concepts from vibrations
Part 1

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Contents

- Introduction
- Modeling of mechanical systems
- System differential equations of motion
- Linearization about equilibrium points
- System and response characteristics. The superposition principle.

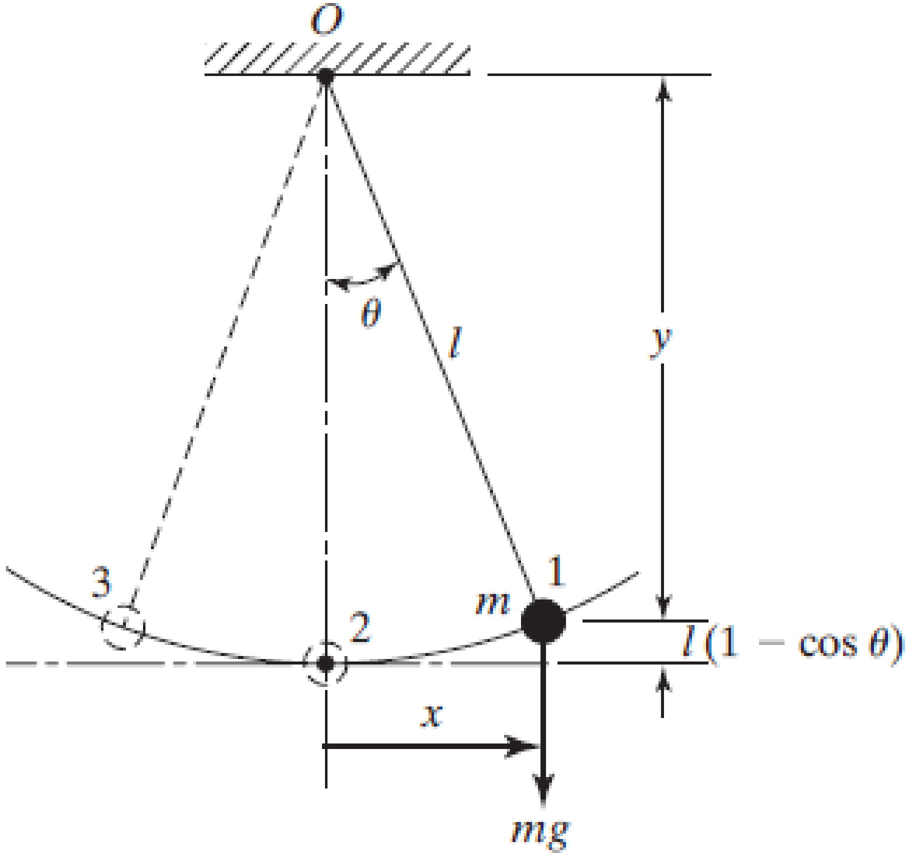
Introduction

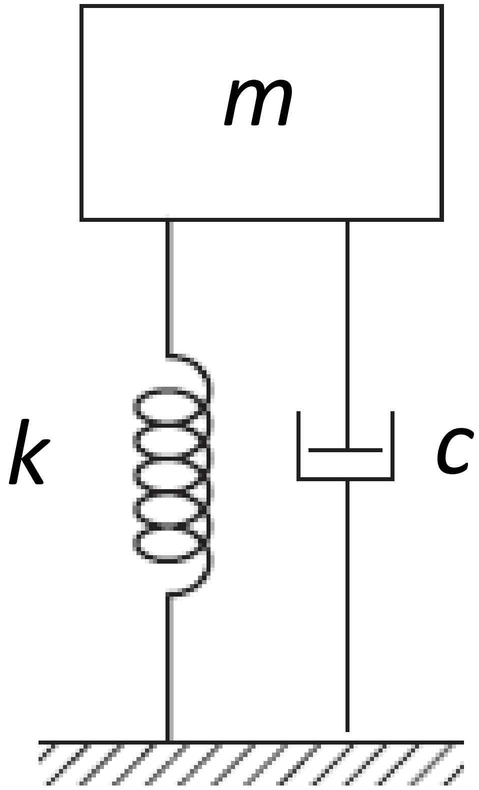
- Vibrations are present in many human activities:
  - Noise: speaking, musical instruments, ... ➔ fluid-structure interaction
  - Engineering problems: design of machines, foundations, structures, engines, turbines, control systems, ...
  - Problem: the natural frequency of vibration of a machine or structure coincides with the frequency of the external excitation ➔ resonase ➔ excessive deflections and failure
  - Solution: design, analysis and testing
  - Beneficial applications: washing machines, elecric toothbrushes, clocks, compactors, finishing manufacturing processes ...

Introduction

- Basic concepts:
  - **Vibration**: any motion that repeats itself after an interval of time (**oscillation**)
  - The theory of vibration deals with the study of oscillatory motions of bodies and the forces associated with them
  - A vibratory system, in general, includes a means for storing potential energy (**spring** or elasticity), a means for storing kinetic energy (**mass** or inertia), and a means by which energy is gradually lost (**damper**).
  - The vibration of a system involves the transfer of its **potential energy** to kinetic energy and of **kinetic energy** to potential energy, alternately. If the system is damped, some energy is dissipated in each cycle of vibration and must be replaced by an external source if a state of steady vibration is to be maintained.

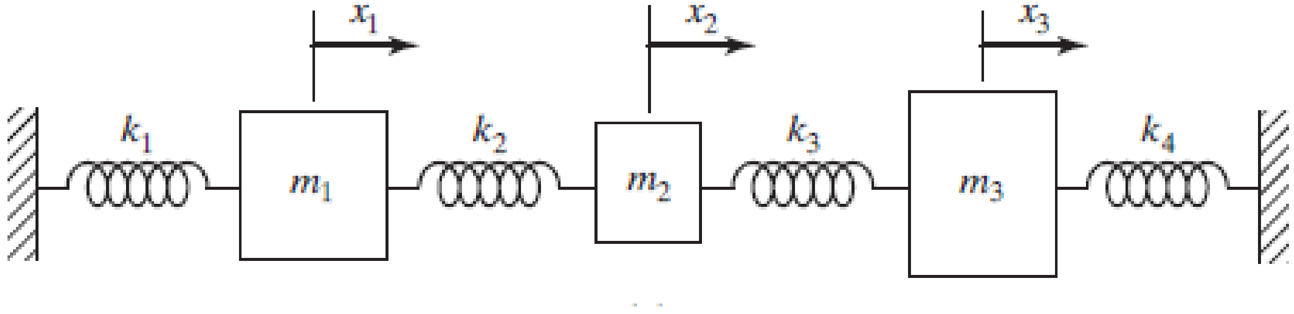
Introduction

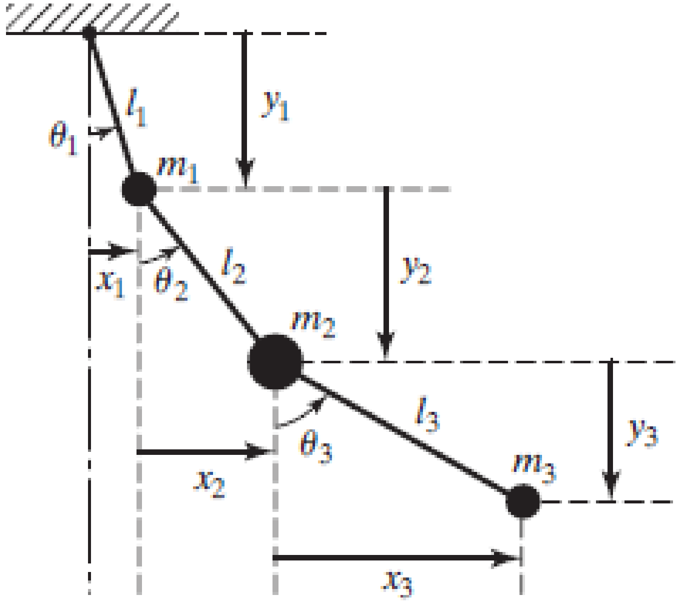




Introduction

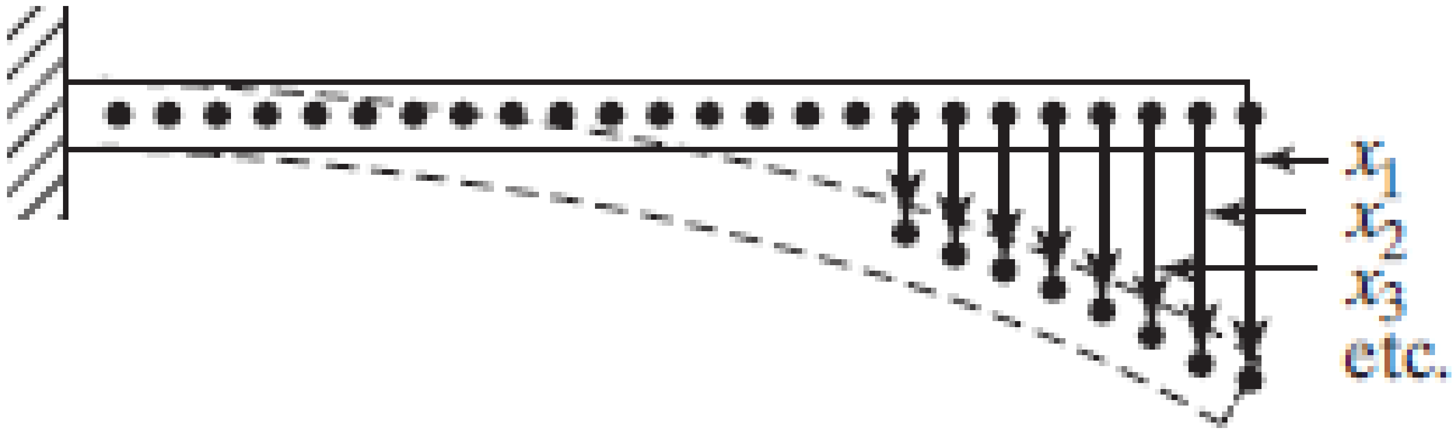
- Basic concepts:
  - Degrees of freedom: The minimum number of independent coordinates required to determine completely the positions of all parts of a system at any instant of time





Introduction

- Basic concepts:
  - Discrete / continuous system: continuous elastic members, have an infinite number of degrees of freedom



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- **Modeling of mechanical systems**
- System differential equations of motion
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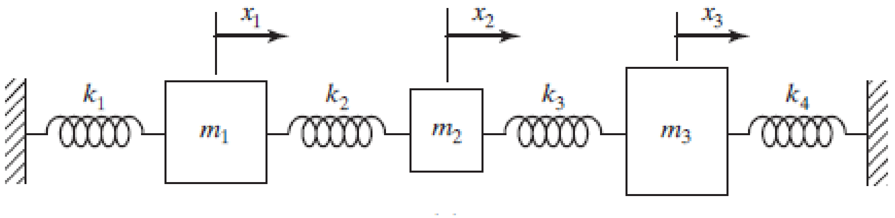
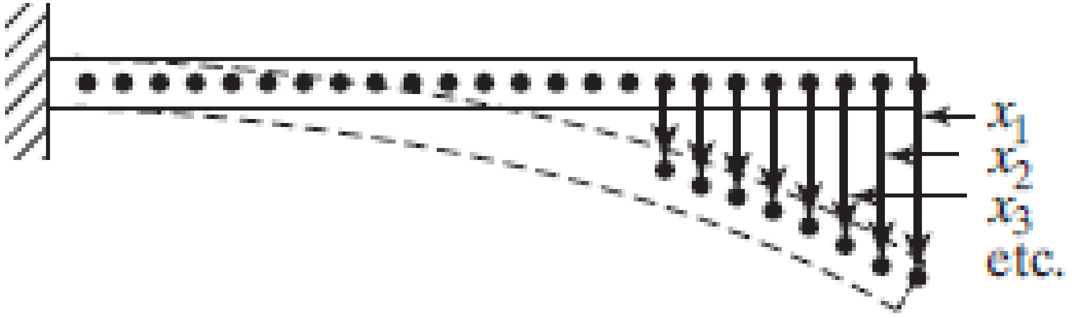
Modeling of mechanical systems

- Physical systems are complex ➔ an exact description is not feasible
- In many cases is not even necessary
- Models represent only an approximation of actual physical systems
- Models retain all the essential dynamic characteristics of the system ➔ the behaviour predicted by the model must match the observed behaviour of the actual system

Modeling of mechanical systems

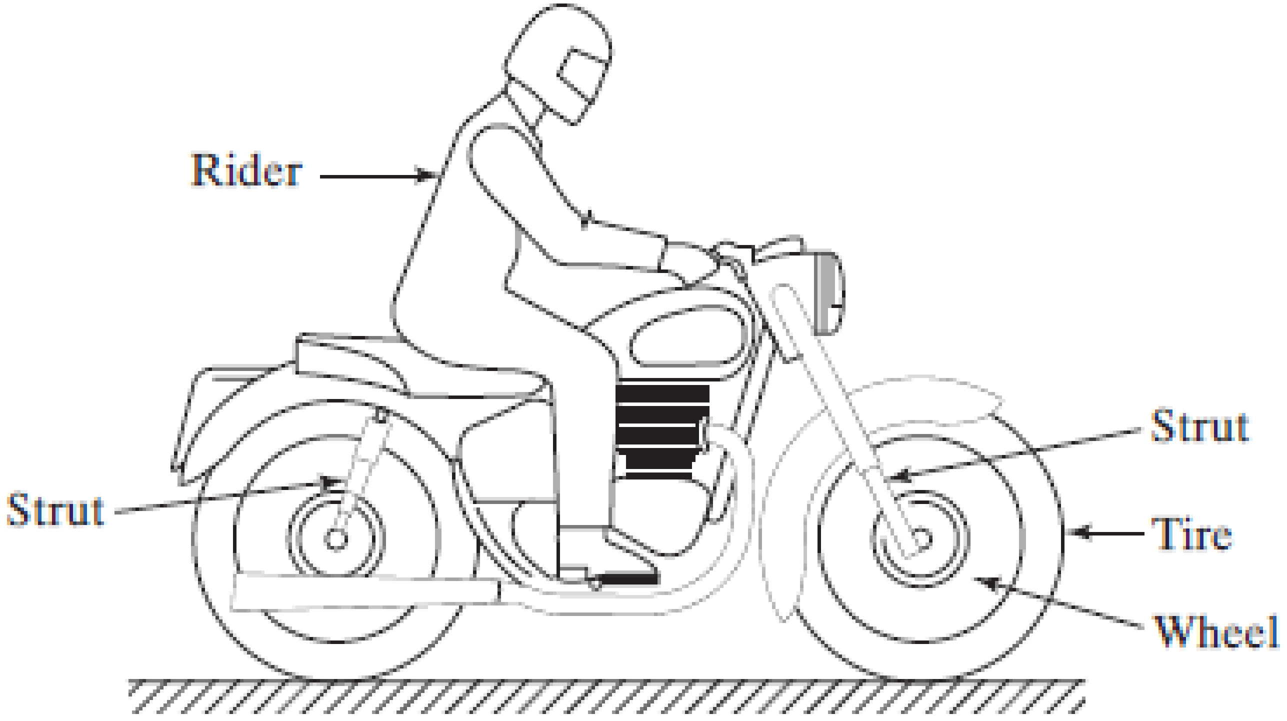
- Models of vibrating mechanical systems
  - Lumped-parameter (discrete)
  - Distributed-parameter
  - Combination of both





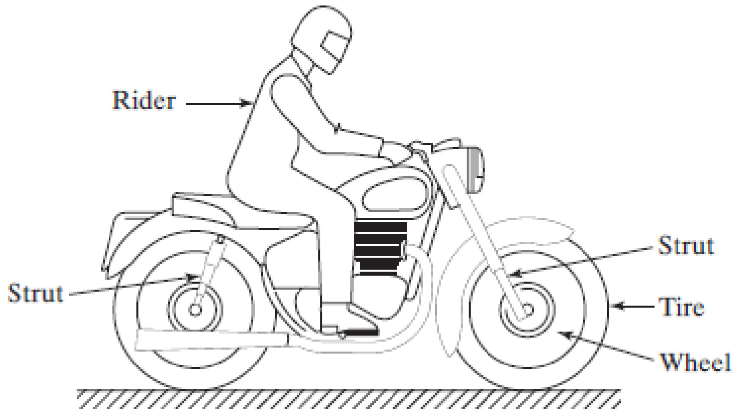
Modeling of mechanical systems

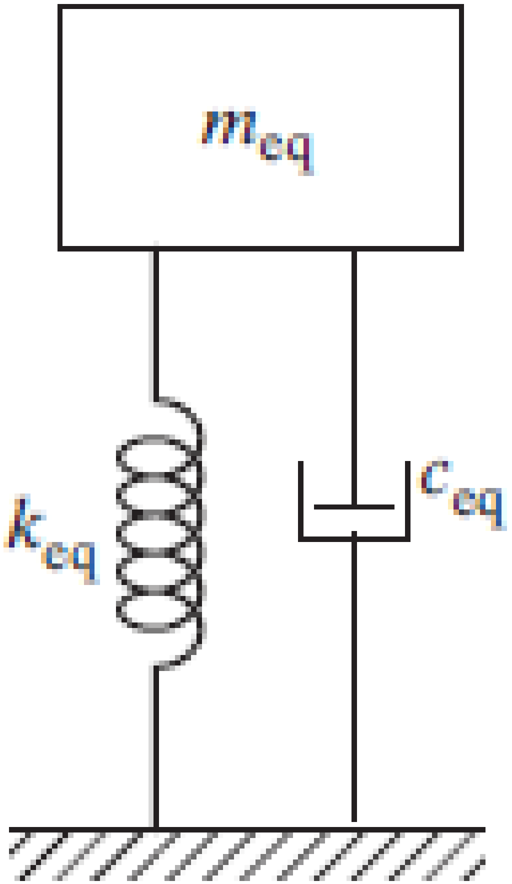
- Examples: motorcycle



Modeling of mechanical systems

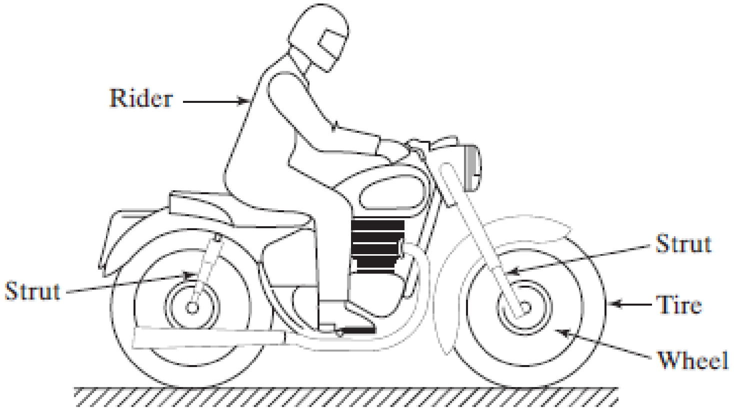
- Examples

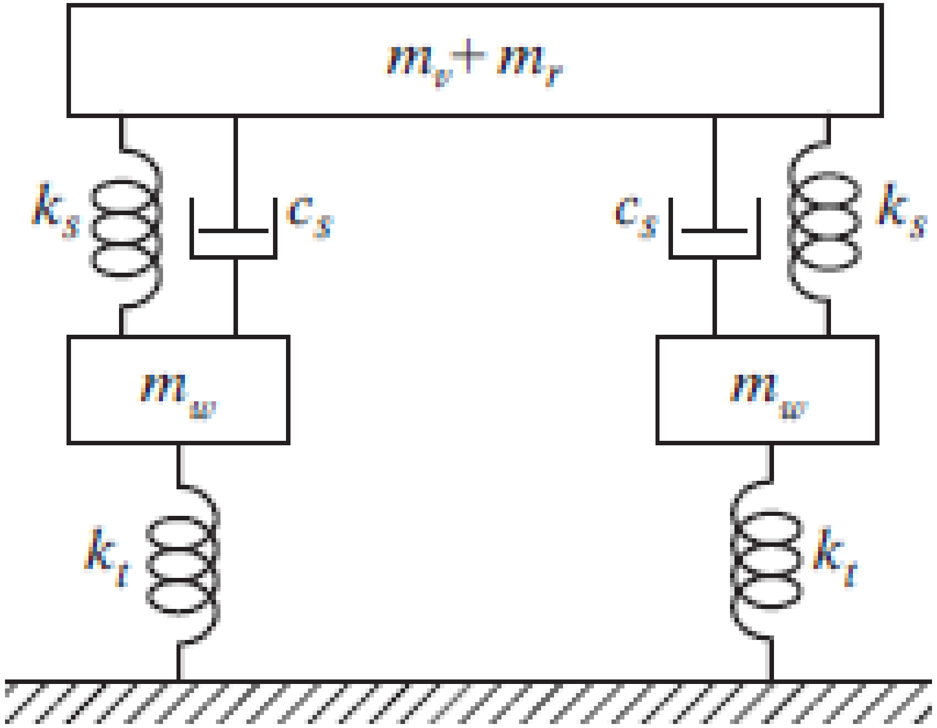




Modeling of mechanical systems

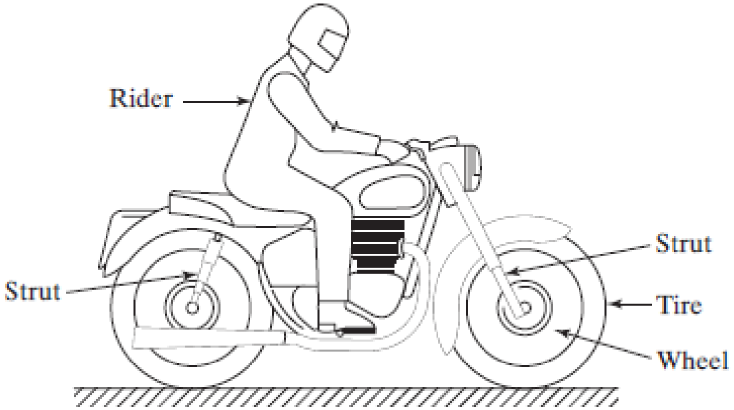
- Examples

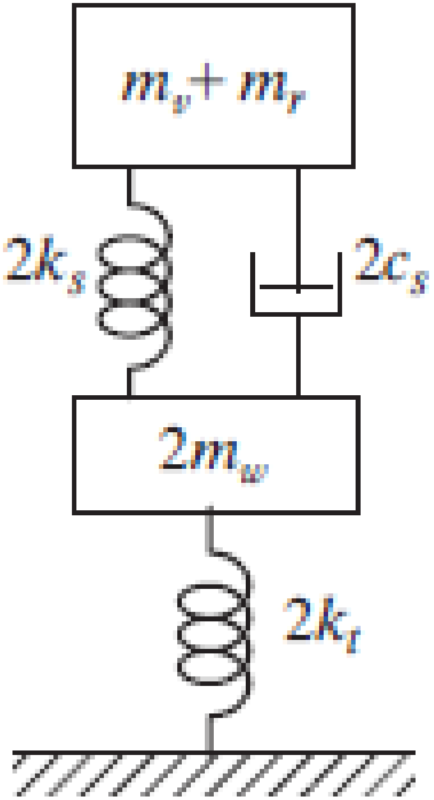




Modeling of mechanical systems

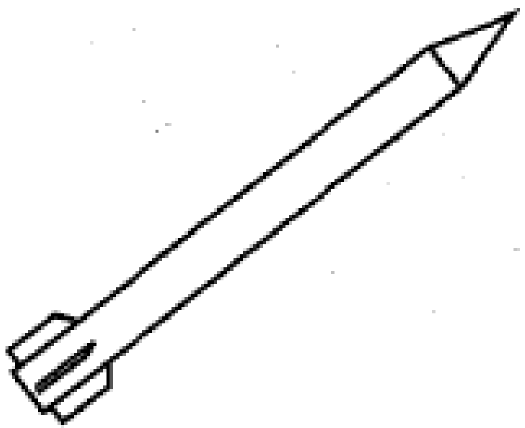
- Examples



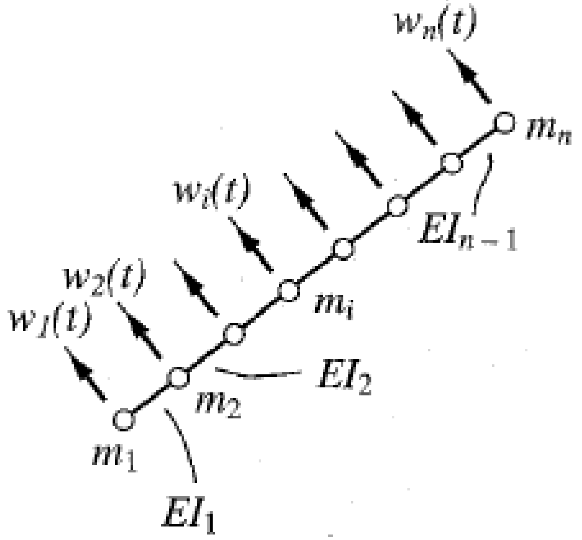


Modeling of mechanical systems

- Examples: missile / rocket

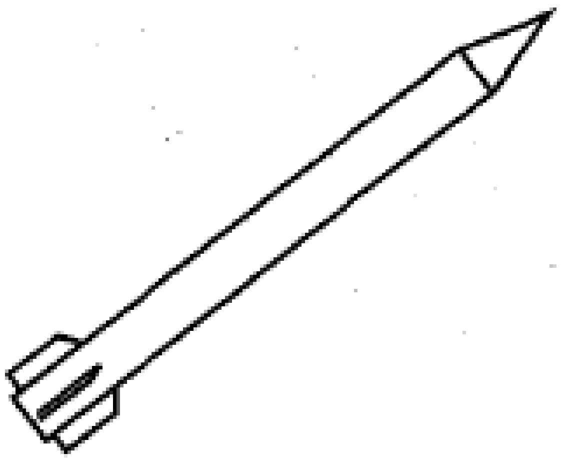


lumped-parameter model (discrete)

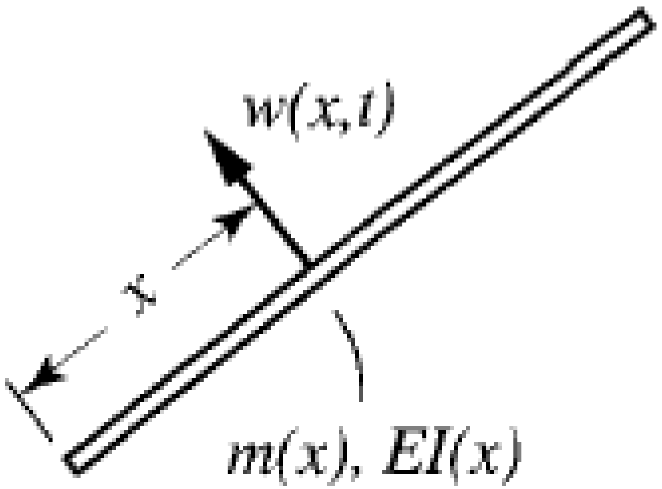


Modeling of mechanical systems

- Examples: missile / rocket

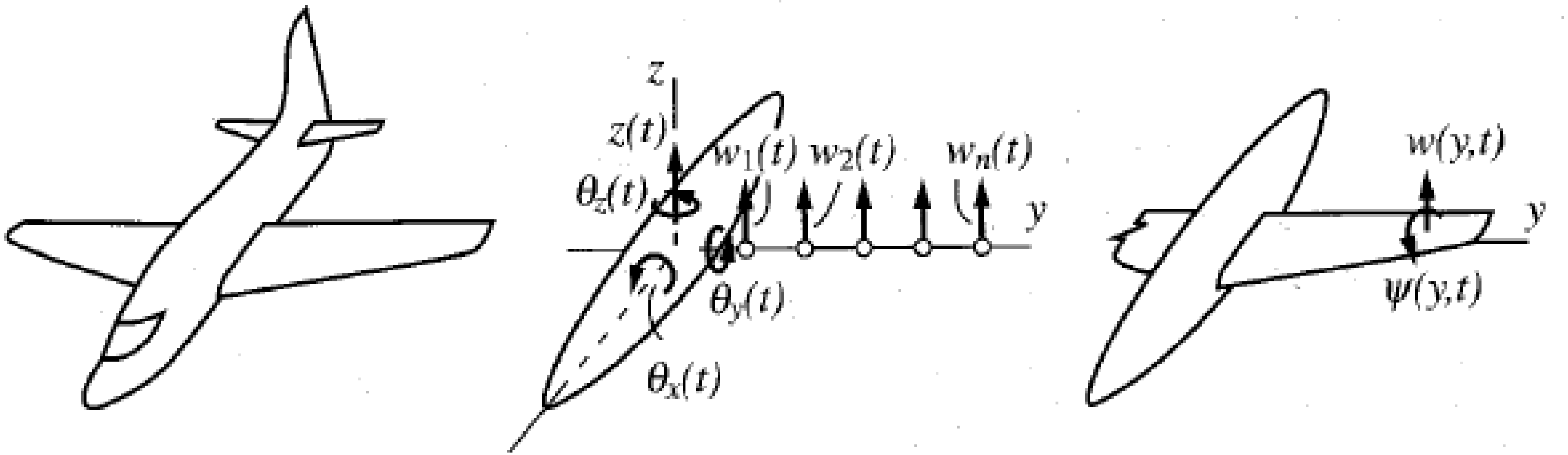


continuous



Modeling of mechanical systems

- Examples: aircraft wing



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- **System differential equations of motion**
- Linearization about equilibrium points
- System and response characteristics. The superposition principle.

System differential equations of motion

- The response of a system subjected to excitations depends on:
  - The nature of the excitation
  - The system characteristics
- Excitations:
  - Initial excitations
  - Applied forces / moments

System differential equations of motion

- Initial excitations:
  - Initial displacements
  - Initial velocities
  - Both
- Initial excitations: the effect is to impart energy to the system:
  - Potential energy (initial displacements)
  - Kinetic energy (initial velocities)
- Free vibration (free response): no further external factors affecting the system

System differential equations of motion

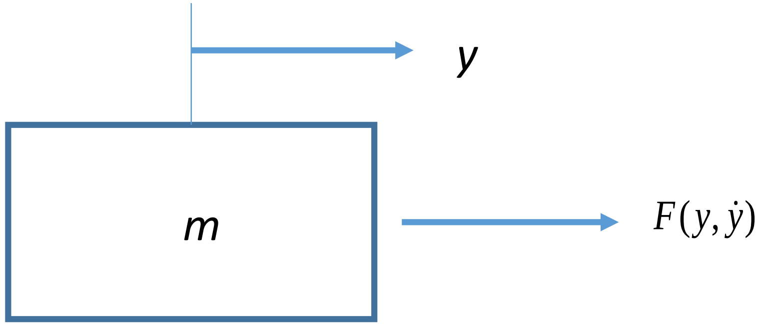
- Applied forces / moments ➔ forced vibration / response
- The response depends on the type of applied (external) forces / moments

System differential equations of motion

- System characteristics: internal characteristics of the individual components and the manner in which these components are arranged
- Need to be reproduced in the model ➔ differential equations of motion
- Determine the system response to a given excitation
- Examples: components mass, moments of inertia, stiffness, damping, etc.

System differential equations of motion

- Let’s consider a single-degree-of-freedom system:



- The model is described by the generic differential equation of motion:

$$m\ddot{y} = F(y, \dot{y})$$

System differential equations of motion

$$m\ddot{y} = F(y, \dot{y})$$

- *m* is the mass
- *F* is in general a nonlinear function of the displacement and velocity
- General solutions to the equation are not possible
- We are interested in special solutions, to understand the system behaviour

System differential equations of motion

$$m\ddot{y} = F(y, \dot{y})$$

- Special solution:

$$y = y_e = \text{constant}$$
$$\dot{y} = \ddot{y} = 0$$

- These constant solutions represent equilibrium points, obtained from:

$$m\ddot{y} = 0 = F(y, \dot{y}) = F(y_e, 0) \Rightarrow F(y_e, 0) = 0$$

System differential equations of motion

- Equilibrium equation: $$F(y_e, 0) = 0$$
- The solution depends on the type of function $F(y_e, 0)$
  - If *F* is a polynomial: as many solutions as the degree of the polynomial
  - If *F* is linear: just one solution
  - If *F* is a trascendental function: potentially an infinite number of solutions
- Physically there is only a finite number of equilibrium points
- If $y_e = 0$ is a solution ➔ trivial solution

System differential equations of motion

- How the system behaves when disturbed from equilibrium?:
  - The system returns to the same equilibrium point ➔ <u>asymptotically stable</u>
  - The system oscillates about the same equlibrium point (without any secular trend) ➔ <u>stable</u>
  - The system moves away from the equlibrium point ➔ <u>unstable</u>

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Linearization about equilibrium points

$$m\ddot{y} = F(y, \dot{y})$$

- Let's consider a solution having the form:

$$y(t) = y_e + x(t)$$

- being *x(t)* a relatively small displacement from equilibrium
- then:

$$\dot{y}(t) = \dot{x}(t)$$

$$\ddot{y}(t) = \ddot{x}(t)$$

Linearization about equilibrium points

- Expandig $F(y,\dot{y})$ in a Taylor series about an equlibrium point y_e:

$$F(y,\dot{y}) = F(y_e,0) + \left.\frac{\partial F(y,\dot{y})}{\partial y}\right|_{\substack{y=y_e\\ \dot{y}=0}} x + \left.\frac{\partial F(y,\dot{y})}{\partial \dot{y}}\right|_{\substack{y=y_e\\ \dot{y}=0}} \dot{x} + O(x^2)$$

$$m\ddot{y} = F(y,\dot{y}) \Rightarrow \left\{ \begin{aligned} \frac{1}{m}\left.\frac{\partial F(y,\dot{y})}{\partial y}\right|_{\substack{y=y_e\\ \dot{y}=0}} &= -b \\ \frac{1}{m}\left.\frac{\partial F(y,\dot{y})}{\partial \dot{y}}\right|_{\substack{y=y_e\\ \dot{y}=0}} &= -a \end{aligned} \right\} \Rightarrow m\ddot{x} + a\dot{x} + bx = 0$$

Linearization about equilibrium points

- We have assumed that displacements from equilibrium are sufficiently small that the nonlinear terms can be ignored

$$m\ddot{y} = F(y, \dot{y}) \quad \Rightarrow \quad \ddot{x} + a\dot{x} + bx = 0$$

➔*linearized equation of motion about equilibrium*

(small motions assumption)

- The motion characteristics in the neighborhoud of equilibrium depend on parameters *a*, *b*

Linearization about equilibrium points

$$\ddot{x} + a\dot{x} + bx = 0$$

- Linear equation with constant coefficients:

$$x(t) = Ae^{st}$$

A: amplitude

s: constant exponent

- Combining

$$\left.\begin{aligned} m\ddot{x} + a\dot{x} + bx = 0 \\ x(t) = Ae^{st} \end{aligned}\right\} \quad s^2 + as + b = 0$$

Linearization about equilibrium points

$$s^2 + as + b = 0$$

➔ Characteristic equation (algebraic equation)

- The roots are:

$$\begin{matrix} s_1 \\ s_2 \end{matrix} = -\frac{a}{2} \pm \sqrt{\left(\frac{a}{2}\right)^2 - b}$$

- So the solution to $m\ddot{x} + a\dot{x} + bx = 0$ is:

$$x(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

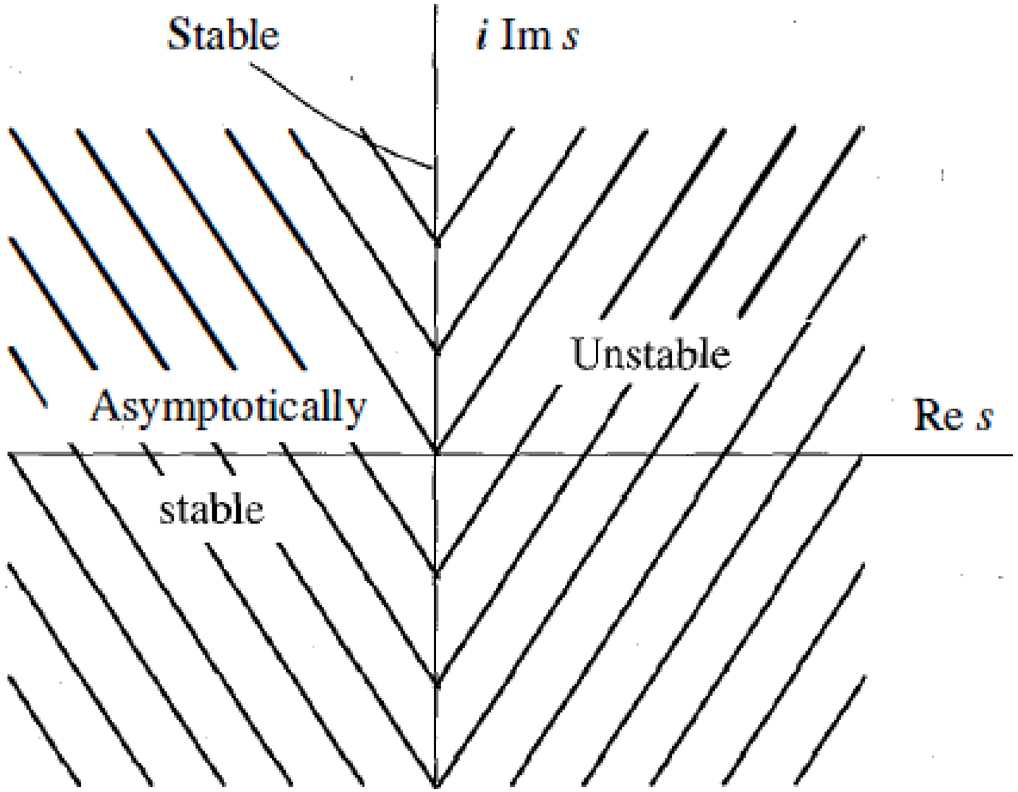
Linearization about equilibrium points

$$x(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

- The nature of the motion (around equilibrium points) depends on the values of the roots *s* (complex numbers, in general):
  - In all cases in which s_1 and s_2 are both real and negative or complex conjugates with negative real part the motion in the neighborhood of an equilibrium point is asymptotically stable
  - In all cases in which s_1 and s_2 are pure imaginary the motion is merely stable
  - If either s_1 or s_2 is real and positive, or both s_1 and s_2 are real and positive, or s_1 and s_2 are complex conjugates with positive real part, the motion is unstable

Linearization about equilibrium points

$$x(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

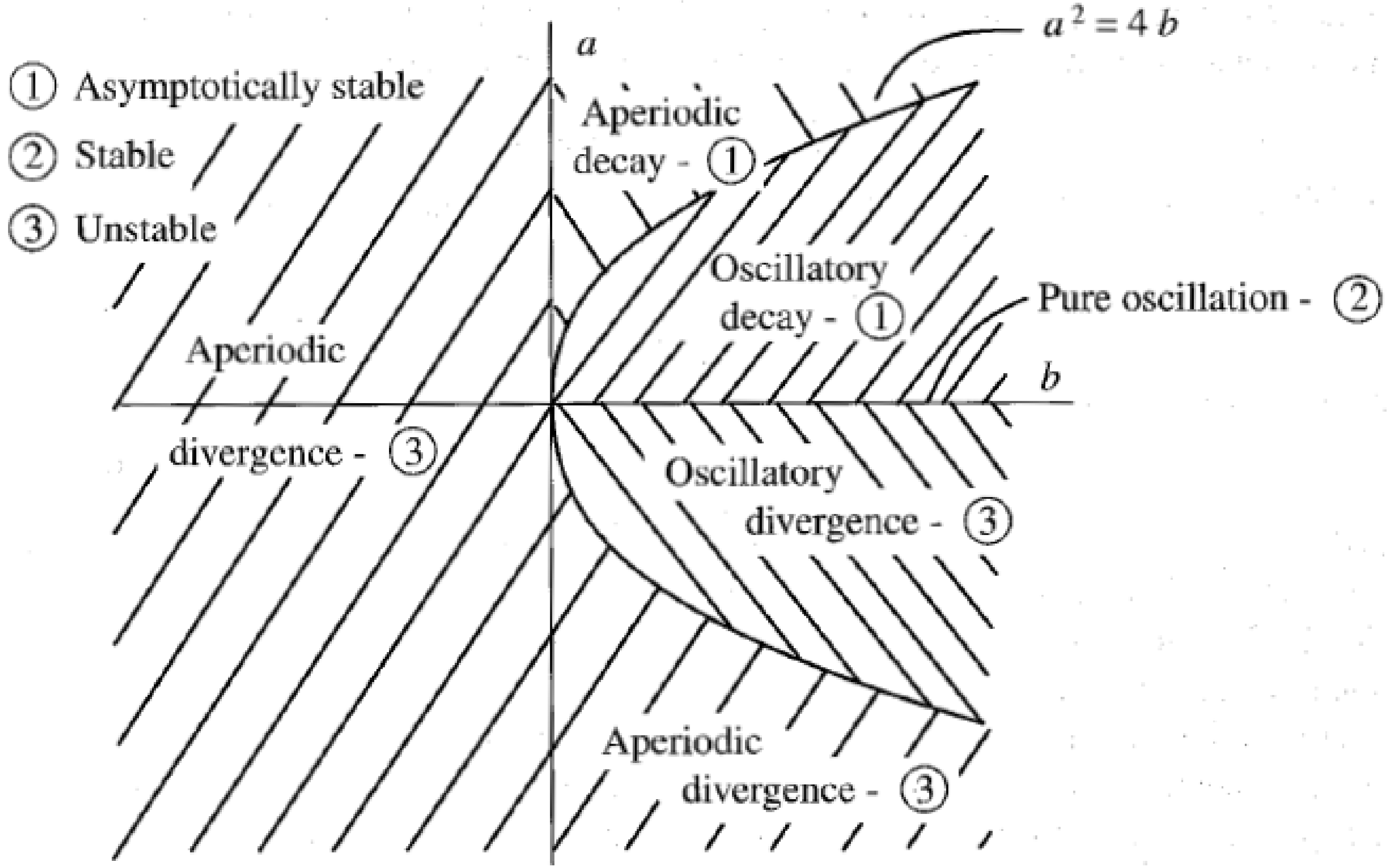


Linearization about equilibrium points

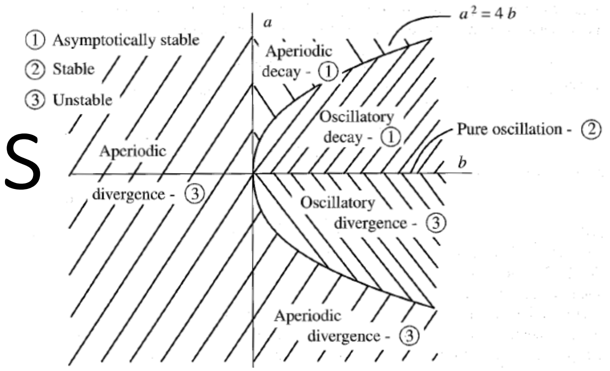
$$s^2 + as + b = 0$$

$$\begin{matrix} s_1 \\ s_2 \end{matrix} = -\frac{a}{2} \pm \sqrt{\left(\frac{a}{2}\right)^2 - b}$$

$$x(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

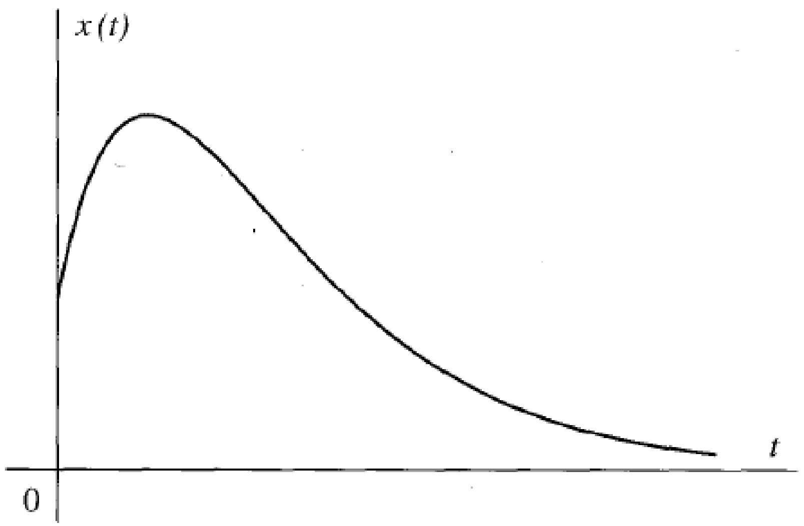


Linearization about equilibrium points

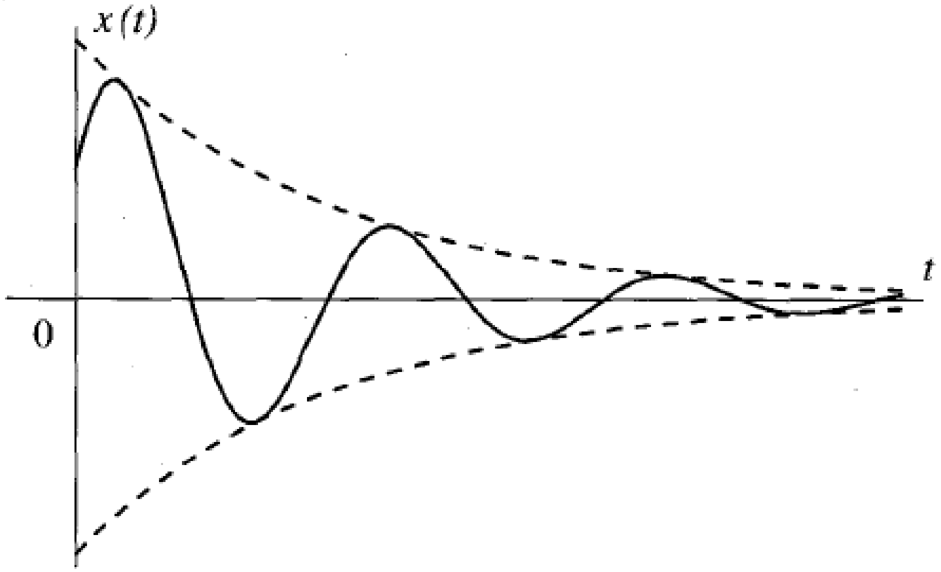


1. Asymptotically stable solution (a>0, b>0)

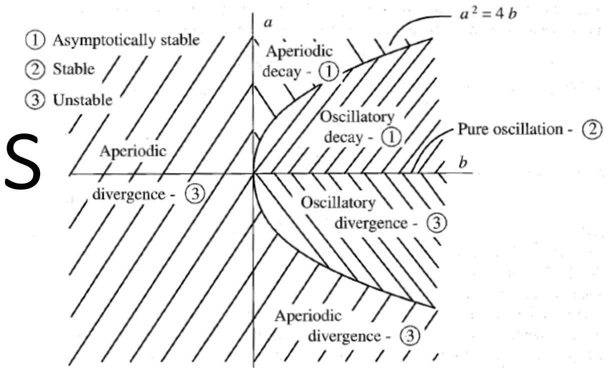
Aperiodically decay



Decaying oscillation

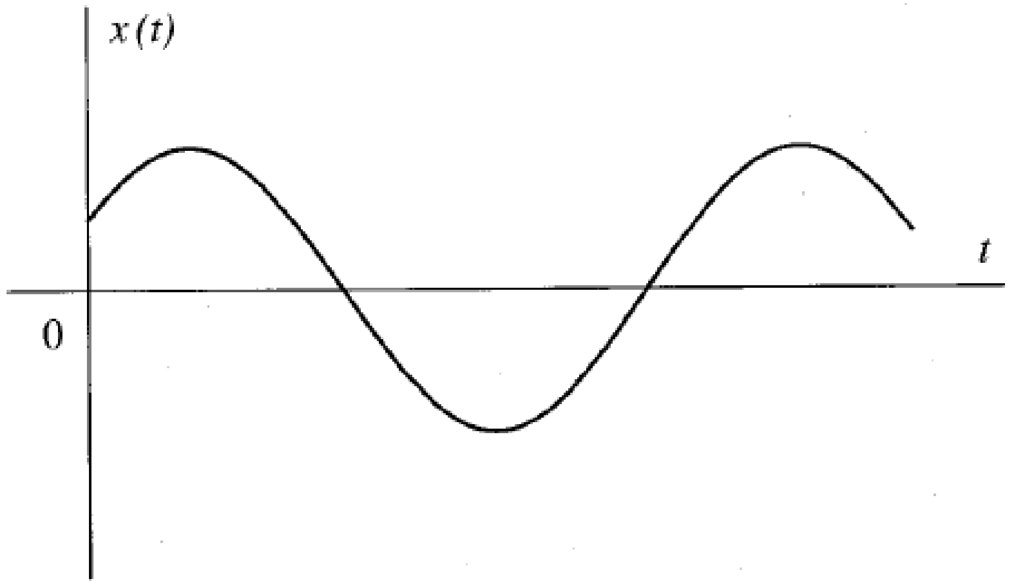


Linearization about equilibrium points

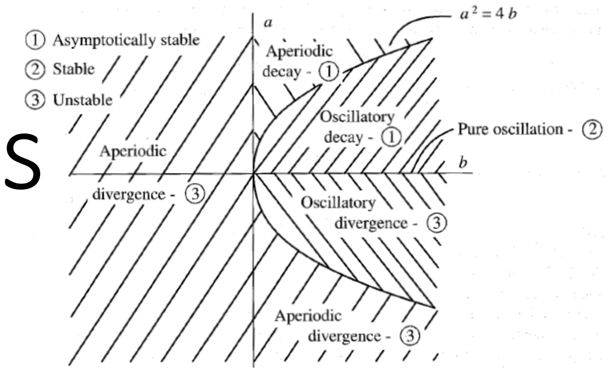


2. Stable motion (a=0, b>0)

Harmonic oscillation

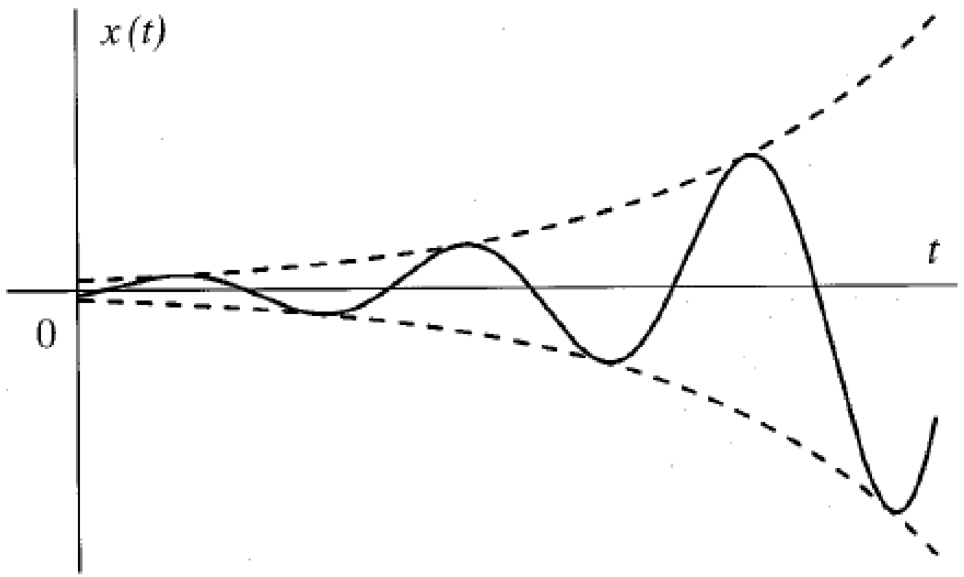


Linearization about equilibrium points



3. Unstable motion (b<0, b>0 & a<0)

Diverging oscillation



Aperiodically diverging motion