1. 阻抗的串联

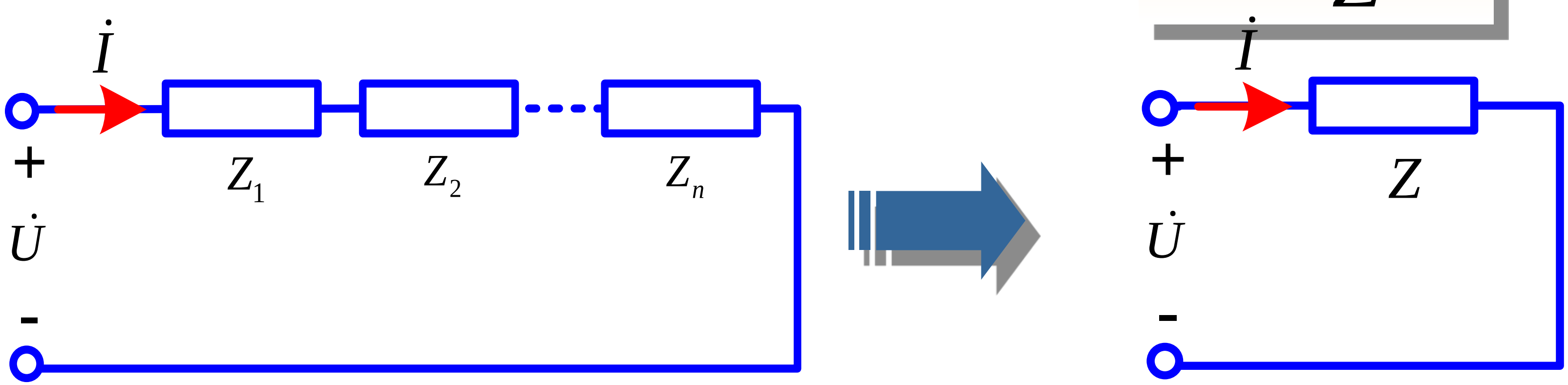
串联：瞬时值方程 $\sum u = 0$　　相量形式方程 $\sum \dot{U} = 0$

$$\dot{U} = \dot{U}_1 + \dot{U}_2 + \ldots + \dot{U}_n = Z_1 \cdot \dot{I} + Z_2 \cdot \dot{I} + \ldots + Z_n \cdot \dot{I}$$

$$\dot{U} = Z\dot{I}$$

$$\therefore Z = Z_1 + Z_2 + \ldots + Z_n$$

$$\dot{U}_i = \frac{Z_i}{Z}\dot{U}$$



2. 阻抗的并联

并联：瞬时值方程 $\sum i = 0$ 　　相量形式方程 $\sum \dot{I} = 0$

$$\dot{I} = \dot{I}_1 + \dot{I}_2 + \ldots + \dot{I}_n = Y_1 \cdot \dot{U} + Y_2 \cdot \dot{U} + \ldots + Y_n \cdot \dot{U}$$

$$\dot{I} = Y\dot{U}$$

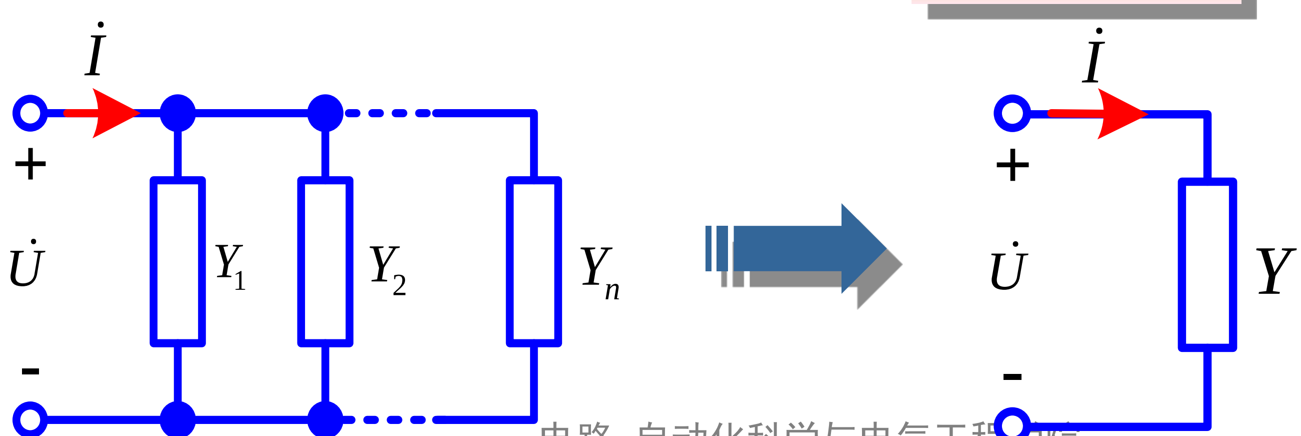
$$\therefore Y = Y_1 + Y_2 + \ldots + Y_n$$

分流公式

$$\dot{I}_i = \frac{Y_i}{Y}\dot{I}$$

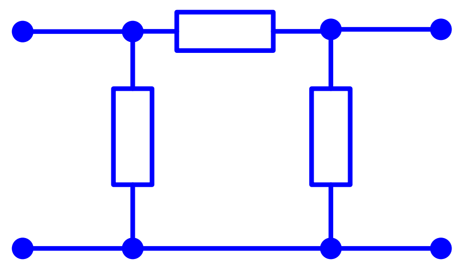
两个阻抗Z_1、Z_2的并联等效阻抗为：

$$Z = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

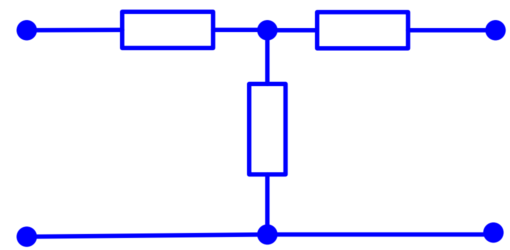


9.2 阻抗（导纳）的联接

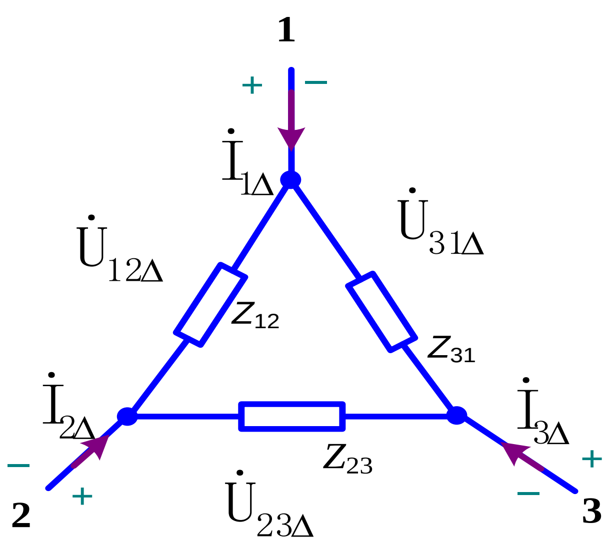
3. 阻抗的Y-△联接

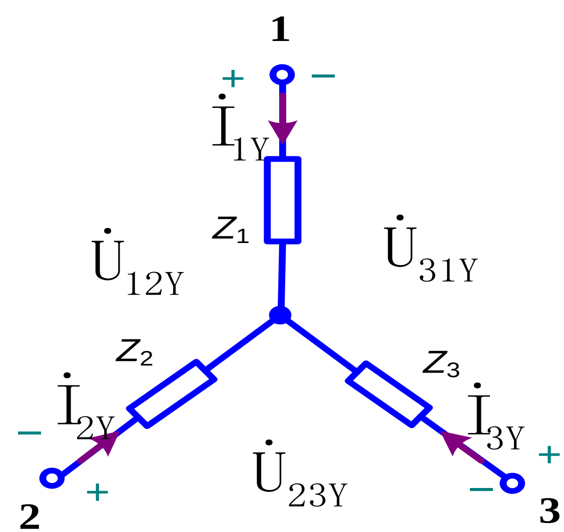


π 型电路 (Δ 型)

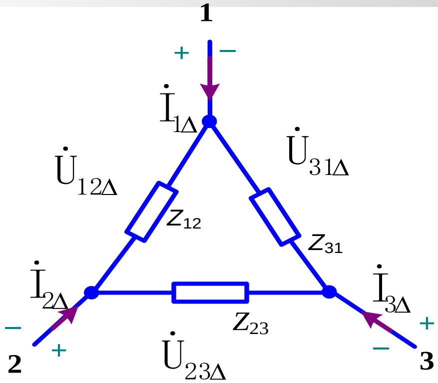


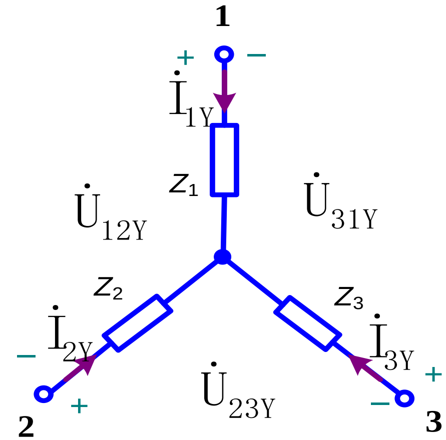
T 型电路 (Y、星型)





9.2 阻抗（导纳）的联接

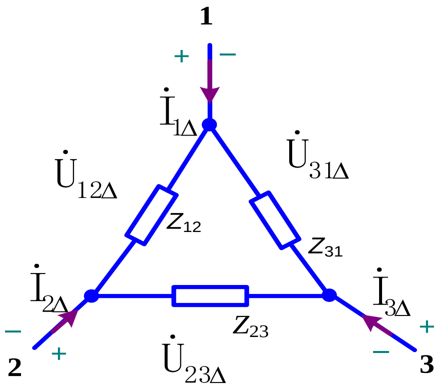


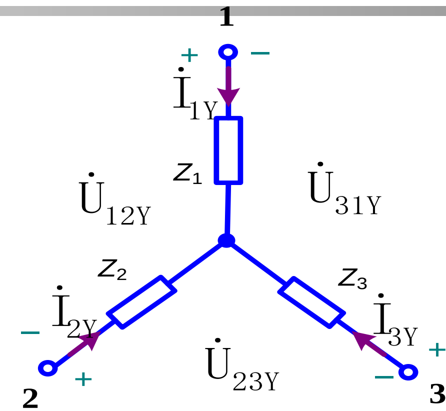


$$
\left.
\begin{aligned}
\dot{I}_{1\mathrm{Y}} &= \frac{\dot{U}_{12\mathrm{Y}} Z_3 - \dot{U}_{31\mathrm{Y}} Z_2}{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1} \\
\dot{I}_{2\mathrm{Y}} &= \frac{\dot{U}_{23\mathrm{Y}} Z_1 - \dot{U}_{12\mathrm{Y}} Z_3}{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1} \\
\dot{I}_{3\mathrm{Y}} &= \frac{\dot{U}_{31\mathrm{Y}} Z_2 - \dot{U}_{23\mathrm{Y}} Z_1}{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}
\end{aligned}
\right\}
\qquad
\left\{
\begin{aligned}
\dot{I}_{1\Delta} &= \frac{\dot{U}_{12\Delta}}{Z_{12}} - \frac{\dot{U}_{31\Delta}}{Z_{31}} \\
\dot{I}_{2\Delta} &= \frac{\dot{U}_{23\Delta}}{Z_{23}} - \frac{\dot{U}_{12\Delta}}{Z_{12}} \\
\dot{I}_{3\Delta} &= \frac{\dot{U}_{31\Delta}}{Z_{31}} - \frac{\dot{U}_{23\Delta}}{Z_{23}}
\end{aligned}
\right.
$$

根据端电压相等则电流相等的等效条件，得Y型→Δ型的变换条件

9.2 阻抗（导纳）的联接





得Y型→Δ型的变换条件：

$$Y_{12}=\frac{Y_1Y_2}{Y_1+Y_2+Y_3}$$

$$Y_{23}=\frac{Y_2Y_3}{Y_1+Y_2+Y_3}$$

$$Y_{31}=\frac{Y_3Y_1}{Y_1+Y_2+Y_3}$$

Δ型→ Y型的变换条件：

$$Z_1=\frac{Z_{12}Z_{31}}{Z_{12}+Z_{23}+Z_{31}}$$

$$Z_2=\frac{Z_{23}Z_{12}}{Z_{12}+Z_{23}+Z_{31}}$$

$$Z_3=\frac{Z_{31}Z_{23}}{Z_{12}+Z_{23}+Z_{31}}$$

$$Y_\Delta=\frac{\text{Y相邻导纳乘积}}{\sum Y_Y}$$

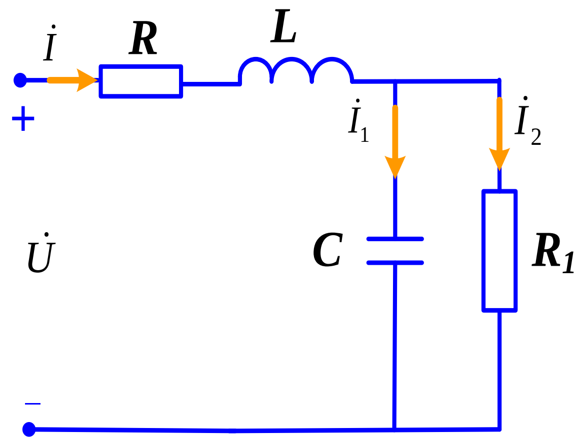
动化

$$Z_Y=\frac{\Delta\text{相邻阻抗乘积}}{\sum Z_\Delta}$$

【例】 已知：$\omega = 1000 rad/s, U = 100V, R = 10\Omega, R_1 = 50\Omega,$

$L = 20mH, C = 10\mu F$

求：各支路电流。



解 **设** $\dot{U} = 100\angle 0^\circ \mathrm{V}$

$$Z = R + \mathrm{j}\omega L + \frac{R_1(-\mathrm{j}\dfrac{1}{\omega C})}{R_1 - \mathrm{j}\dfrac{1}{\omega C}} = 50\angle 0^\circ \Omega$$

$$\dot{I} = \frac{\dot{U}}{Z} = \frac{100\angle 0^\circ}{50\angle 0^\circ} = 2\angle 0^\circ \mathrm{A}$$

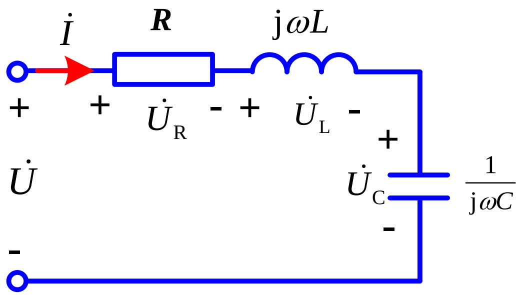
$$\dot{I}_2 = \frac{-\mathrm{j}\dfrac{1}{\omega C}}{R_1 - \mathrm{j}\dfrac{1}{\omega C}}\dot{I} = 1.79\angle -26.6^\circ \mathrm{A}$$

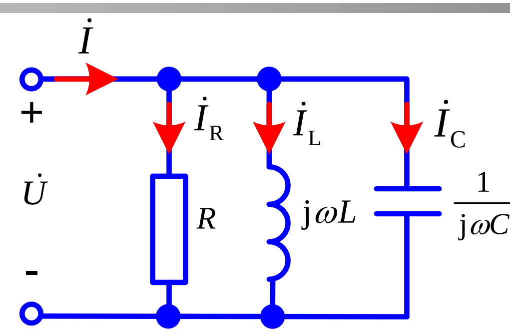
$$\dot{I}_1 = \frac{R_1}{R_1 - \mathrm{j}\dfrac{1}{\omega C}}\dot{I} = 0.894\angle 63.4^\circ \mathrm{A}$$

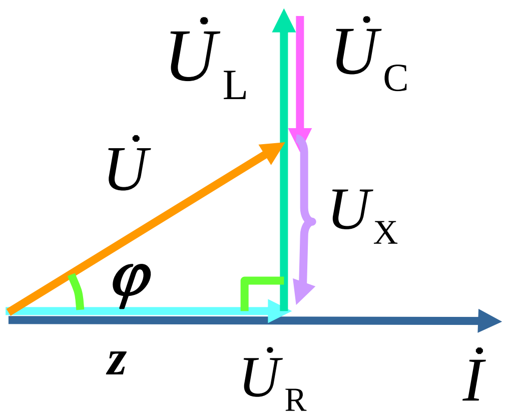
9.3 电路的相量图

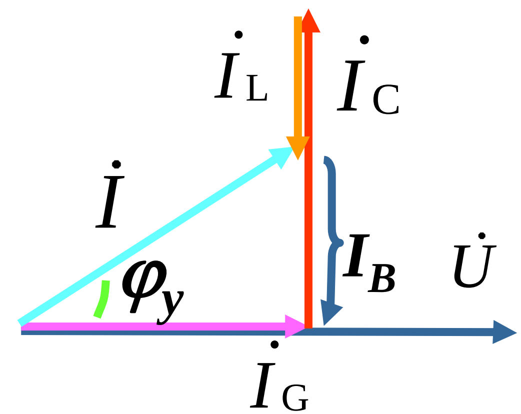
- **相量图**
  - 描述电路中电流、电压相量关系的图叫相量图
  - 利用相量图分析求解
- **相量图画法**
  - 任取一相量作为参考相量（相位为零），不需画出复平面的实轴、虚轴；
  - 具有一般性，参考相量的初相可以不为零，其它相量与参考相量之间的相位关系为相对关系；
  - 选择距离电源最远端的一个复杂环节，串联则以电流作为参考相量；并联则以电压作为参考相量；
  - 相量间首尾相连以反映KCL与KVL，不要画成放射状。

9.3 电路的相量图







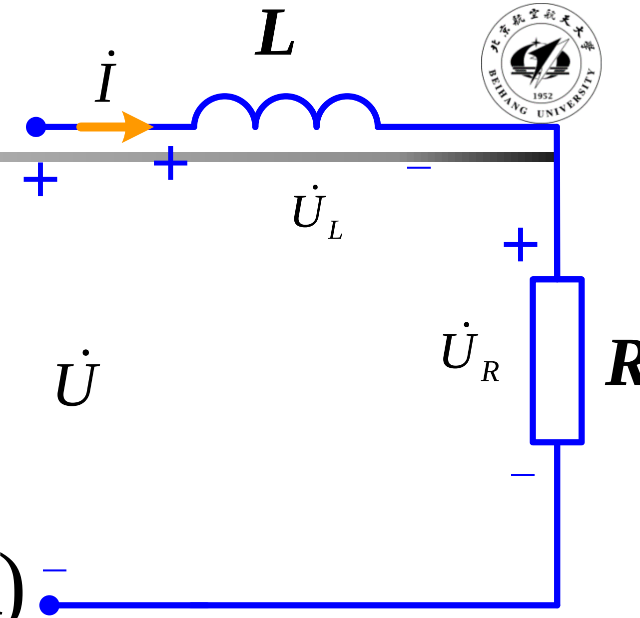


$$U=\sqrt{U_R^2+U_X^2}=\sqrt{U_R^2+\left(U_L-U_C\right)^2}$$

$$I=\sqrt{I_G^2+I_B^2}=\sqrt{I_G^2+(I_L-I_C)^2}$$

电流参考⟶VCR⟶KVL　　电压参考⟶ VCR ⟶KCL

【例】



$R=800\Omega, U=220V, f=50HZ, U_R=110V,$

求：元件L参数值。

解 （一）解析法

设 $\dot{I}=I\angle 0^\circ$ $\quad I=\dfrac{U_R}{R}=0.1375(A)$

$$\dot{U}_{\mathrm{R}}=\dot{I}R=110\angle 0^\circ(V)$$

$$\dot{U}_{\mathrm{L}}=j\omega L\times\dot{I}=0.1375\omega L\angle 90^\circ(V)$$

$$\dot{U}=\dot{U}_{\mathrm{R}}+\dot{U}_{\mathrm{L}}=\sqrt{110^2+(0.1375\omega L)^2}\angle\arctan(\frac{0.1375\omega L}{110})(V)$$

$$U=220=\sqrt{110^2+(0.1375\omega L)^2} \Rightarrow L=4.4\mathrm{H}$$

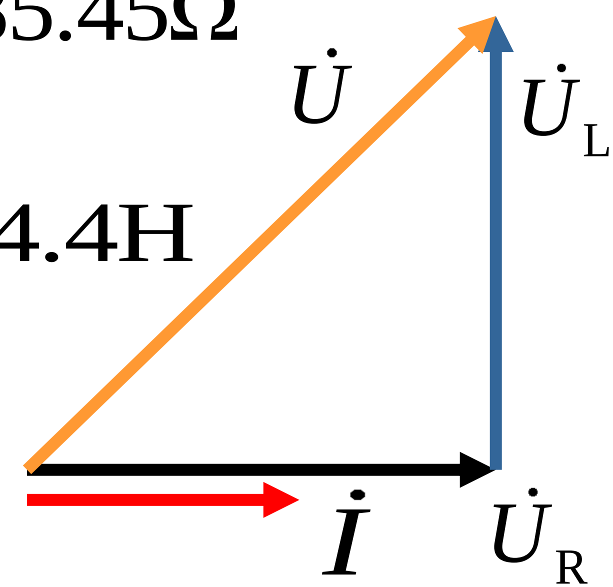
（二）相量图法

$$U_L = \sqrt{U^2 - U_R^2} = \sqrt{220^2 - 110^2} = 190.5\text{V}$$

$$I = \frac{U_R}{R} = \frac{110}{800} = 0.1375\text{A}$$

$$X_L = \omega L = \frac{U_L}{I} = \frac{190.5}{0.1375} = 1385.45\Omega$$

$$L = \frac{X_L}{\omega} = \frac{X_L}{2\pi f} = \frac{1385.45}{2\pi \times 50} = 4.4\text{H}$$

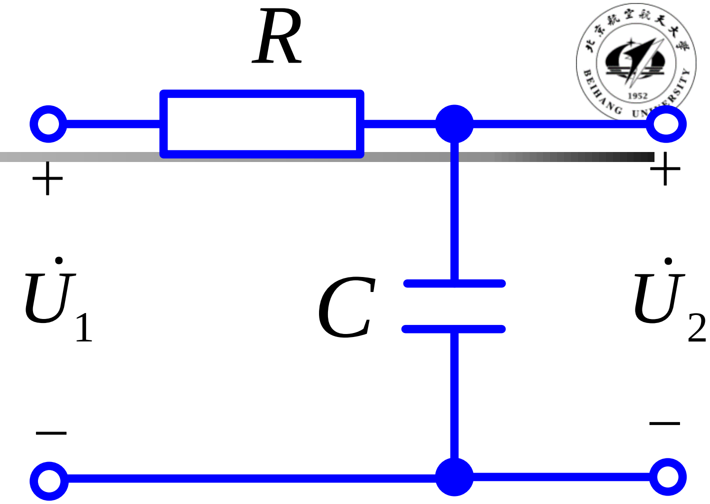


相量图是很好的工具!

相量图法

【例】已知：R=1kΩ f=5kHz

若使 $\dot{U}_2$ 滞后于 $\dot{U}_1$ 30°应取 C 为何值。



解　法1：解析法

$$\dot{U}_2=\frac{-\mathrm{j}\dfrac{1}{\omega C}}{R-\mathrm{j}\dfrac{1}{\omega C}}\dot{U}_1$$

$$=\frac{-\mathrm{j}\dfrac{1}{\omega C}(R+\mathrm{j}\dfrac{1}{\omega C})}{R^2+(\dfrac{1}{\omega C})^2}\dot{U}_1=\frac{\dfrac{1}{\omega C}}{R^2+(\dfrac{1}{\omega C})^2}(\frac{1}{\omega C}-\mathrm{j}R)\dot{U}_1$$

$$\dot{U}_2=\frac{\frac{1}{\omega C}}{R^2+(\frac{1}{\omega C})^2}(\frac{1}{\omega C}-\mathrm{j}R)\dot{U}_1$$

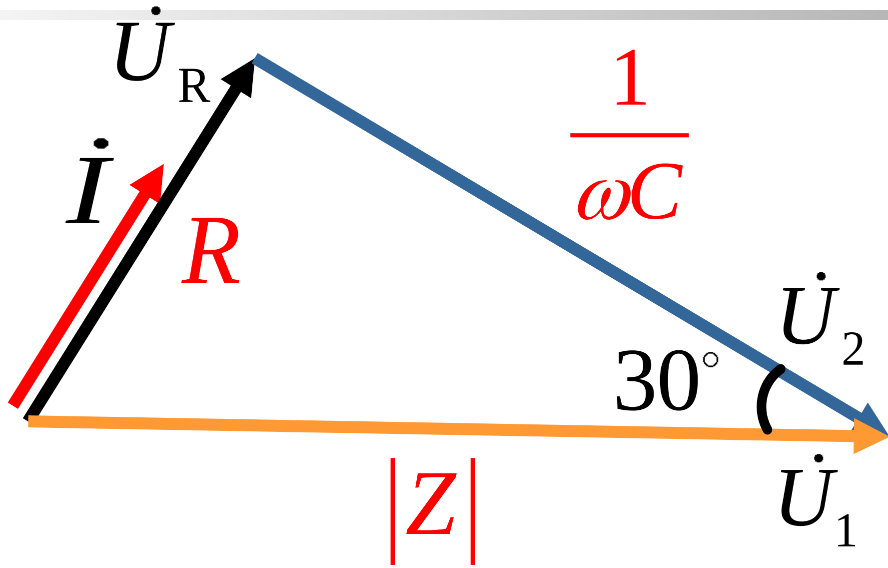
$$=\dot{U}_1\frac{\frac{1}{\omega C}\sqrt{R^2+(\frac{1}{\omega C})^2}}{R^2+(\frac{1}{\omega C})^2}\angle\mathrm{arctg}\frac{-R}{\frac{1}{\omega C}}$$

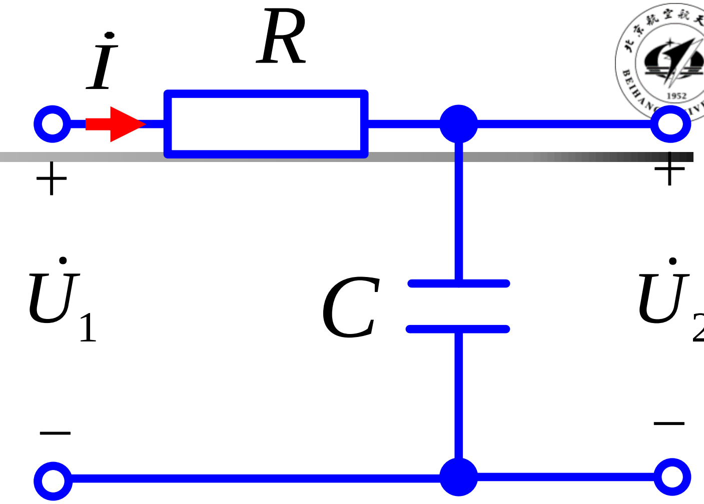
$$\mathrm{arctg}(-\omega CR)=-30^\circ$$

$$\mathrm{tg}(-30^\circ)=-\omega C\mathrm{R}=-\frac{\sqrt{3}}{3}$$

$$C=\frac{\frac{\sqrt{3}}{3}}{2\pi fR}=0.0184\mu\mathrm{F}$$

法2相量图法





$$\mathrm{tg}30^\circ = \frac{R}{\frac{1}{\omega C}} = \omega CR = 2\pi fCR$$

$$C = \frac{\mathrm{tg}\,30^\circ}{2\pi fR} = \frac{\frac{\sqrt{3}}{3}}{2\pi \times 5 \times 10^3 \times 10^3} = 0.0184\mu\mathrm{F}$$

作业

【9-3】

【9-4】

【9-5】

【9-6】

【9-7】

【9-8】