System Dynamics and Vibrations

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Chapter 2: Concepts from vibrations
Exercises

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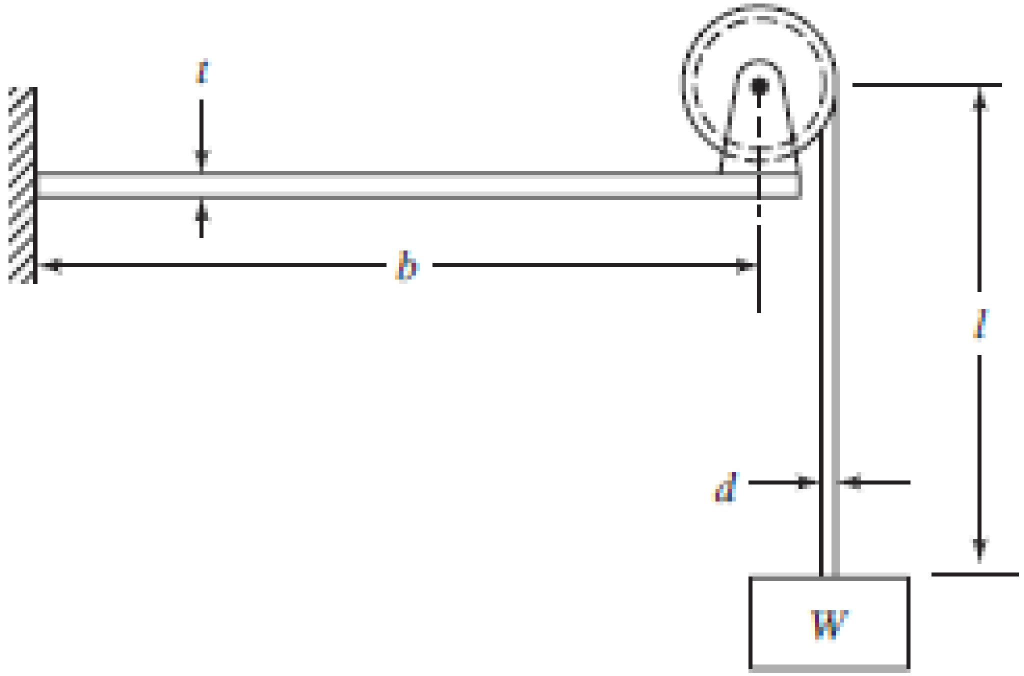
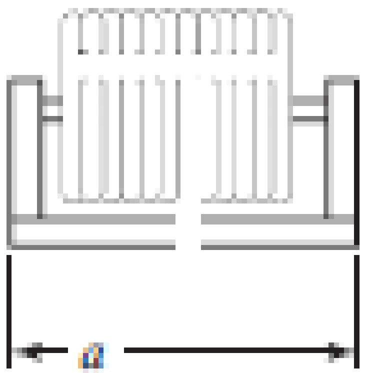
Exercise 1

- A hoisting drum, carrying a steel wire rope, is mounted at the end of a cantilever beam as shown in the figure. Determine the equivalent spring constant of the system when the suspended length of the wire rope is *l*. Assume that the net cross-sectional diameter of the wire rope is *d* and the Young's modulus of the beam and the wire rope is E.





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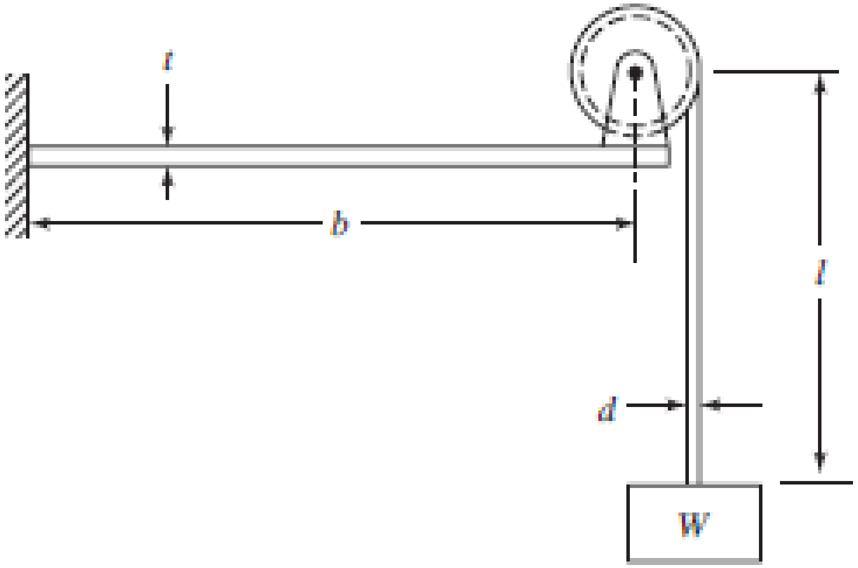

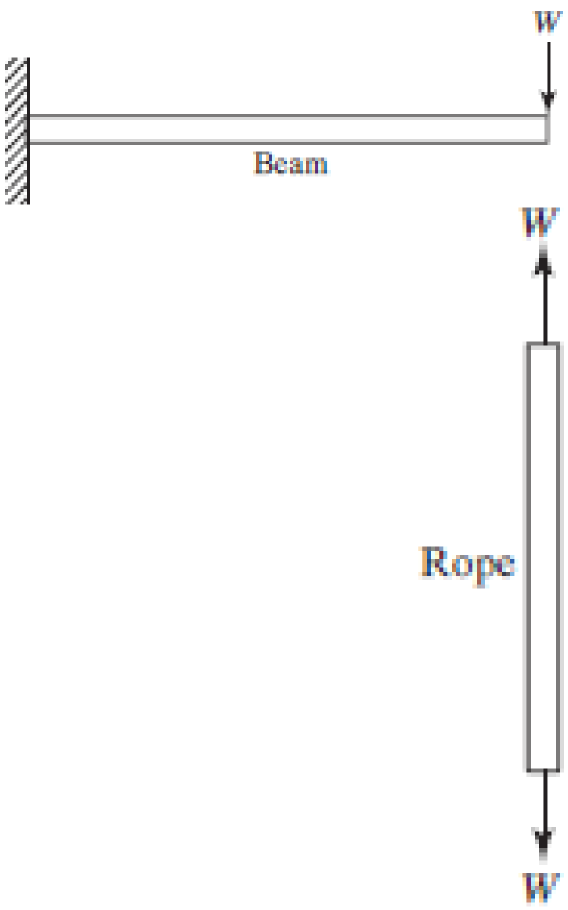

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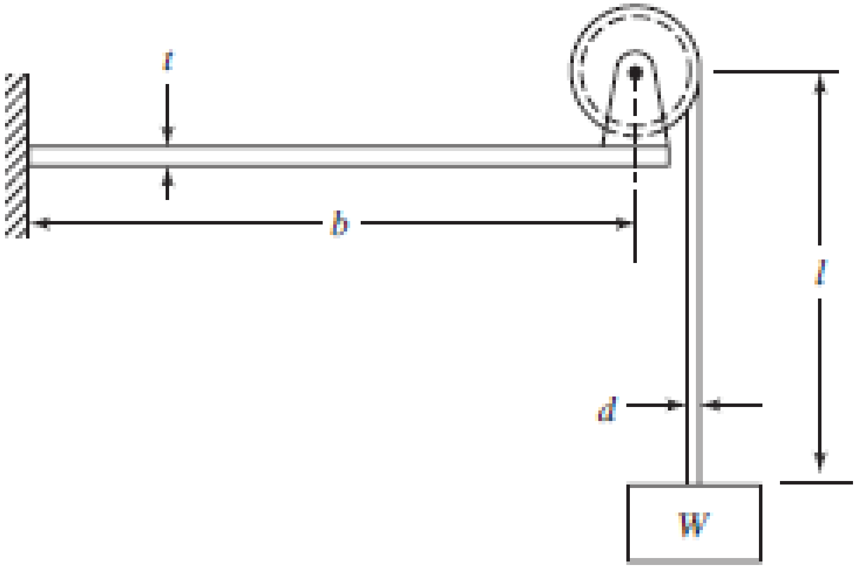

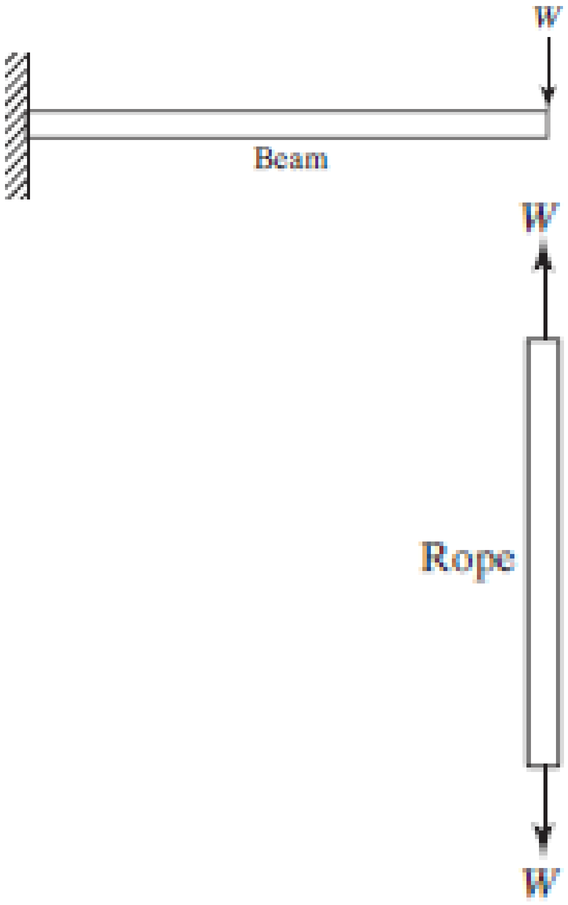

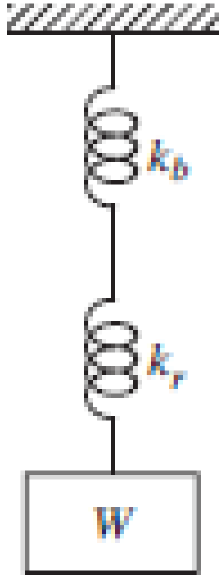

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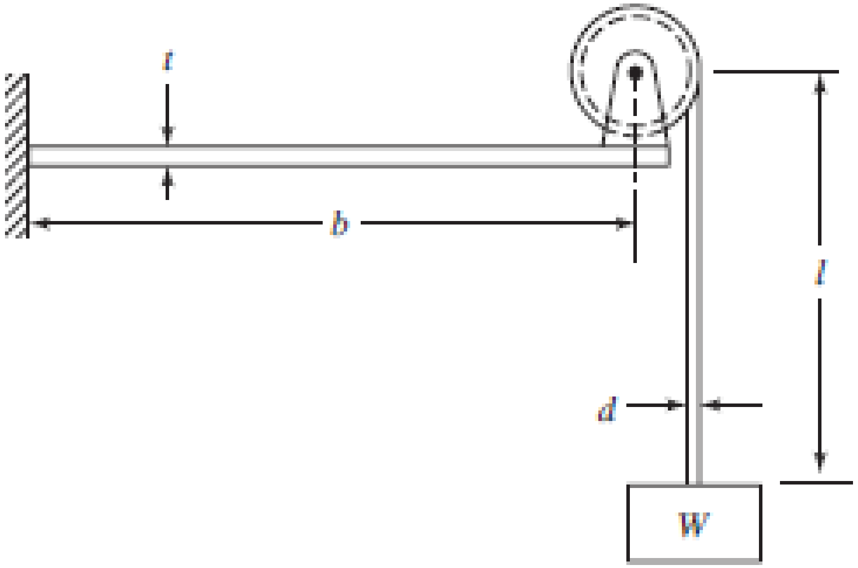
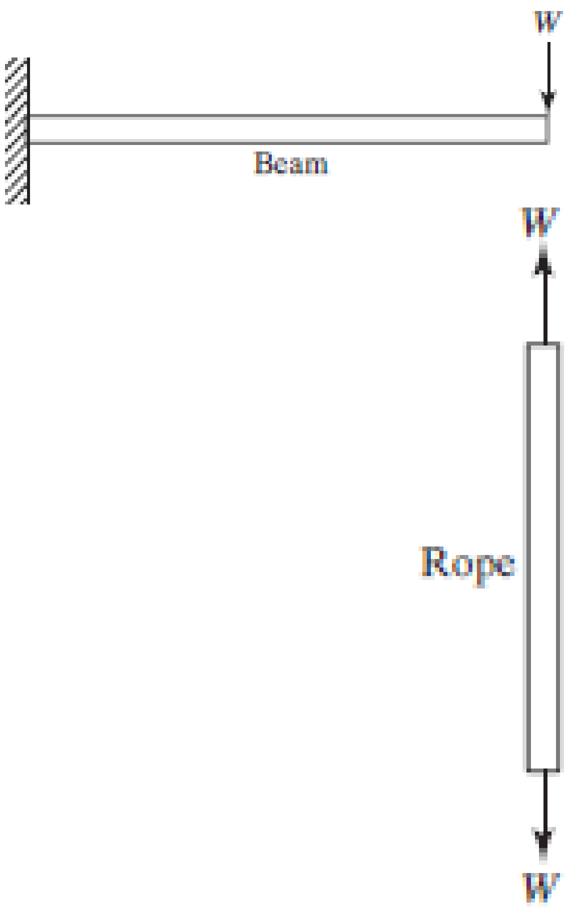
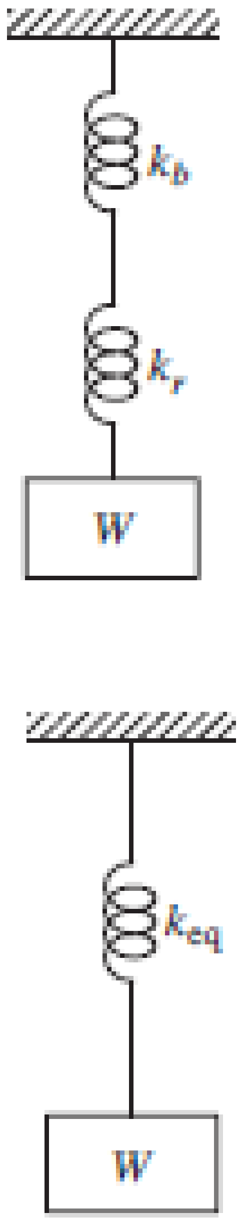






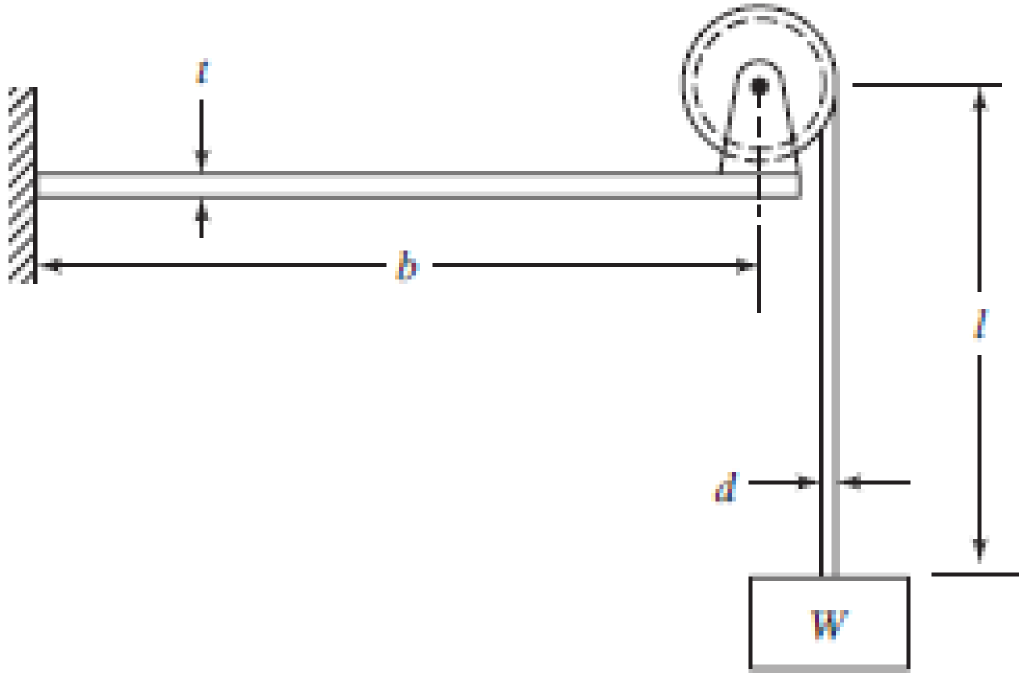
Exercise 1

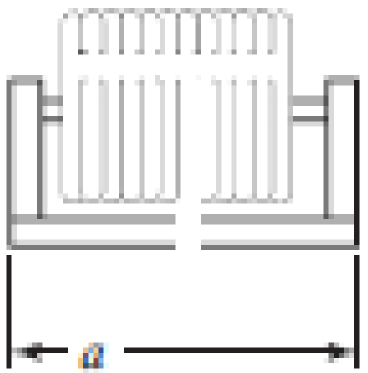
$$k_b = \frac{3EI}{b^3} = \frac{3E}{b^3}\left(\frac{1}{12}at^3\right) = \frac{Eat^3}{4b^3}$$

$$k_r = \frac{AE}{l} = \frac{\pi d^2 E}{4l}$$

$$\frac{1}{k_{eq}} = \frac{1}{k_b} + \frac{1}{k_r} = \frac{Eat^3}{4b^3} + \frac{\pi d^2 E}{4l}$$

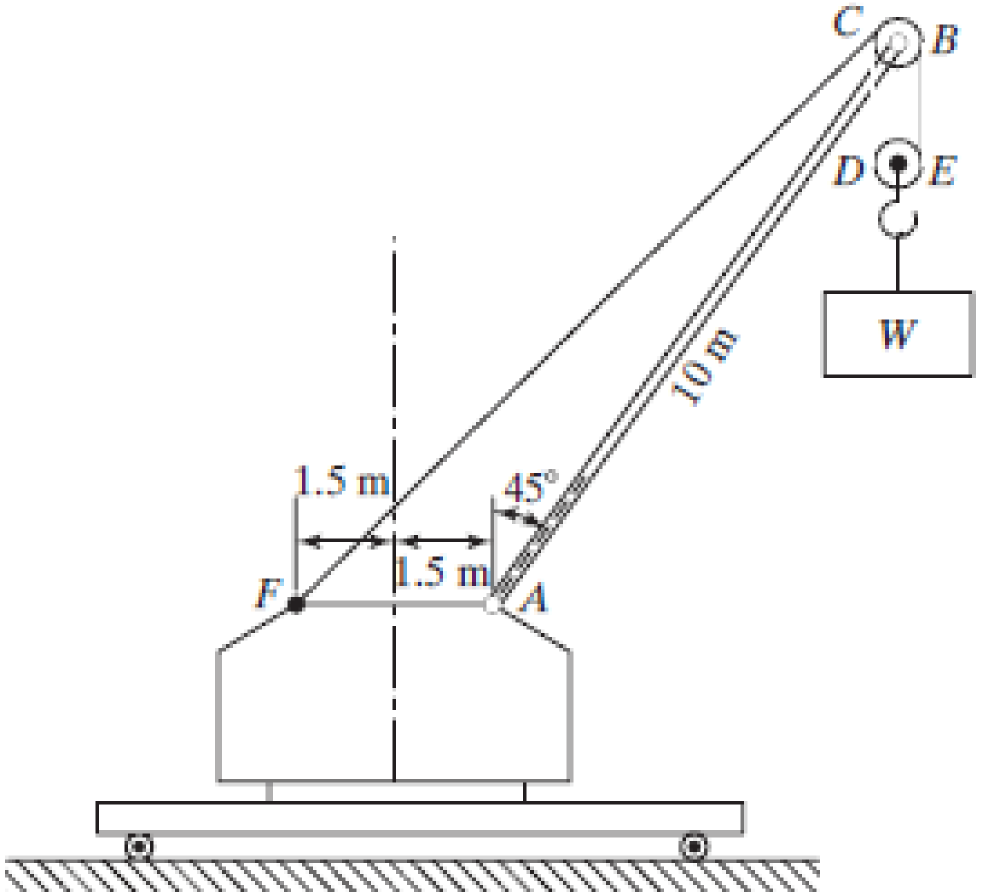
$$k_{eq} = \frac{E}{4}\left(\frac{\pi a t^3 d^2}{\pi d^2 b^3 + l a t^3}\right)$$





Exercise 2

- The boom AB of the crane shown in the figure is a uniform steel bar of length 10 m and area of cross section 2500 mm^2. A weight W is suspended while the crane is stationary. The cable CDEBF is made of steel and has a cross-sectional area of 100 mm^2 Neglecting the effect of the cable CDEB, find the equivalent spring constant of the system in the vertical direction.



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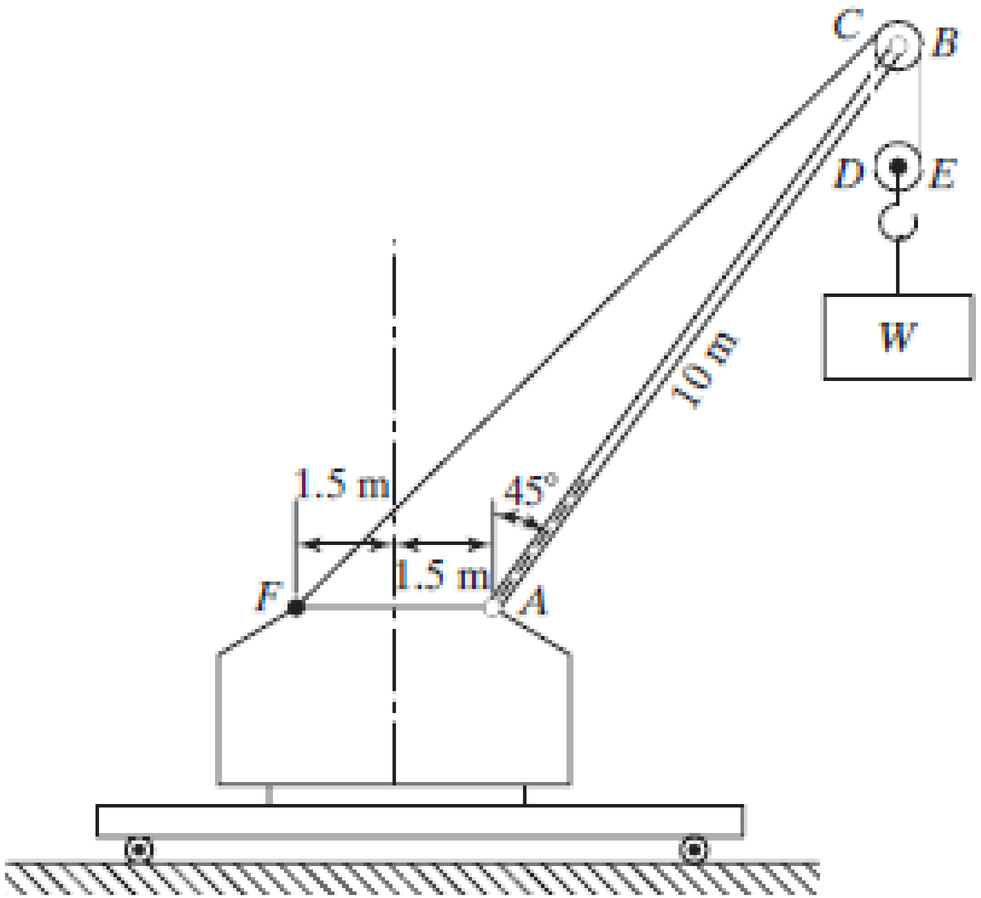
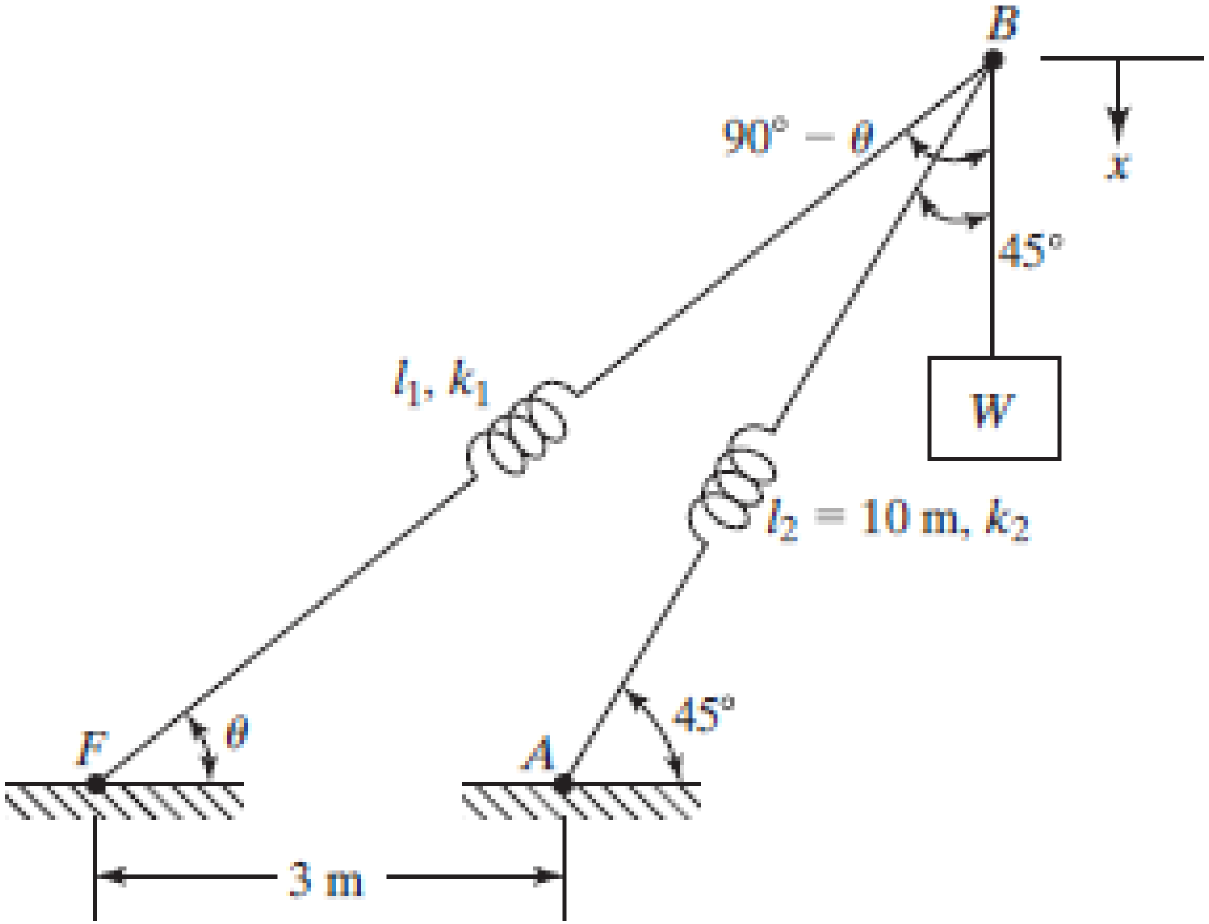
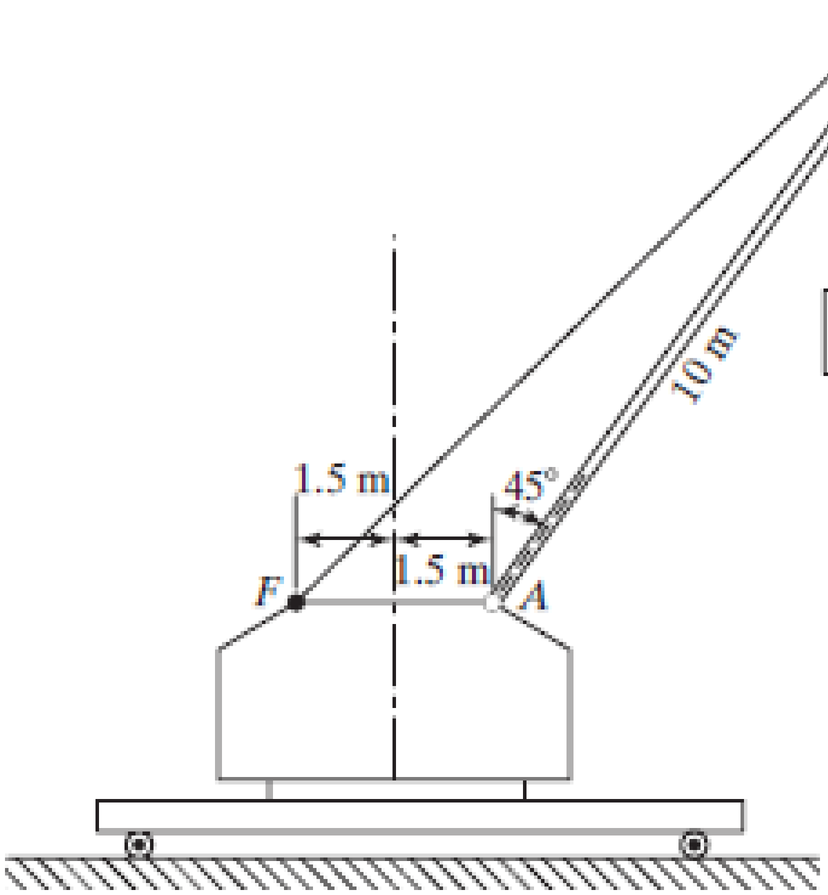
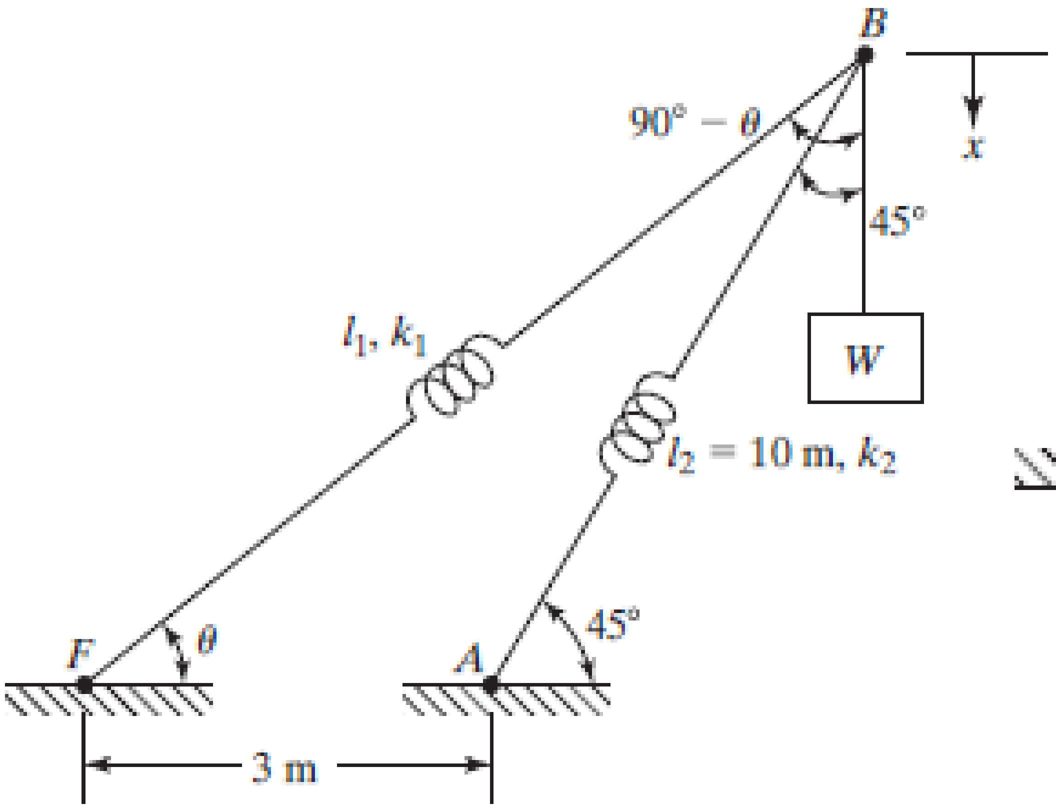
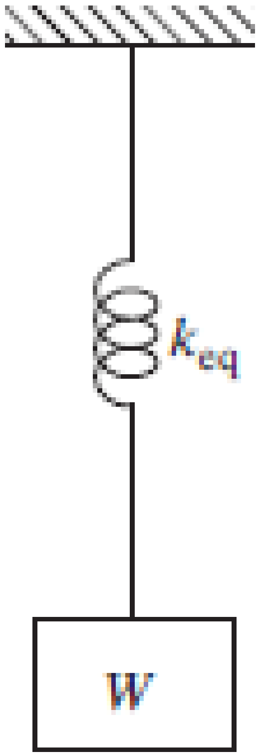




Exercise 2







Exercise 2

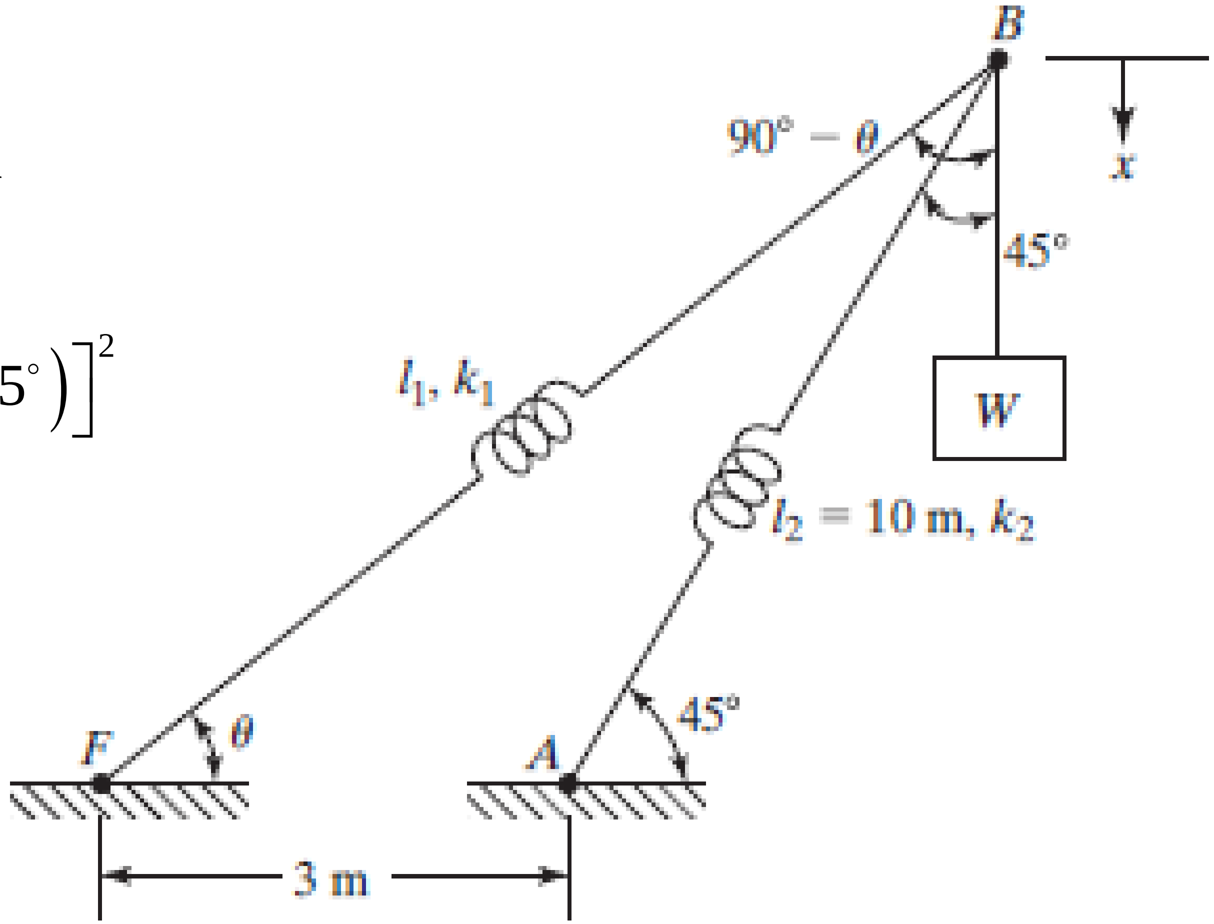
$$l_1^2 = 3^2 + 10^2 - 2 \cdot 3 \cdot 10 \cos 135^\circ \Rightarrow l_1 = 12.3055\text{m}$$

$$l_1^2 + 3^2 - 2 l_1 \cdot 3 \cos\theta = 10^2 \Rightarrow \theta = 35.0736^\circ$$

$$V = \frac{1}{2} k_1 \left[x \cos\left(90^\circ - \theta\right)\right]^2 + \frac{1}{2} k_2 \left[x \cos\left(90^\circ - 45^\circ\right)\right]^2$$

$$k_1 = \frac{A_1 E_1}{l_1} = 1.6822 \cdot 10^6 \text{ N/m}$$

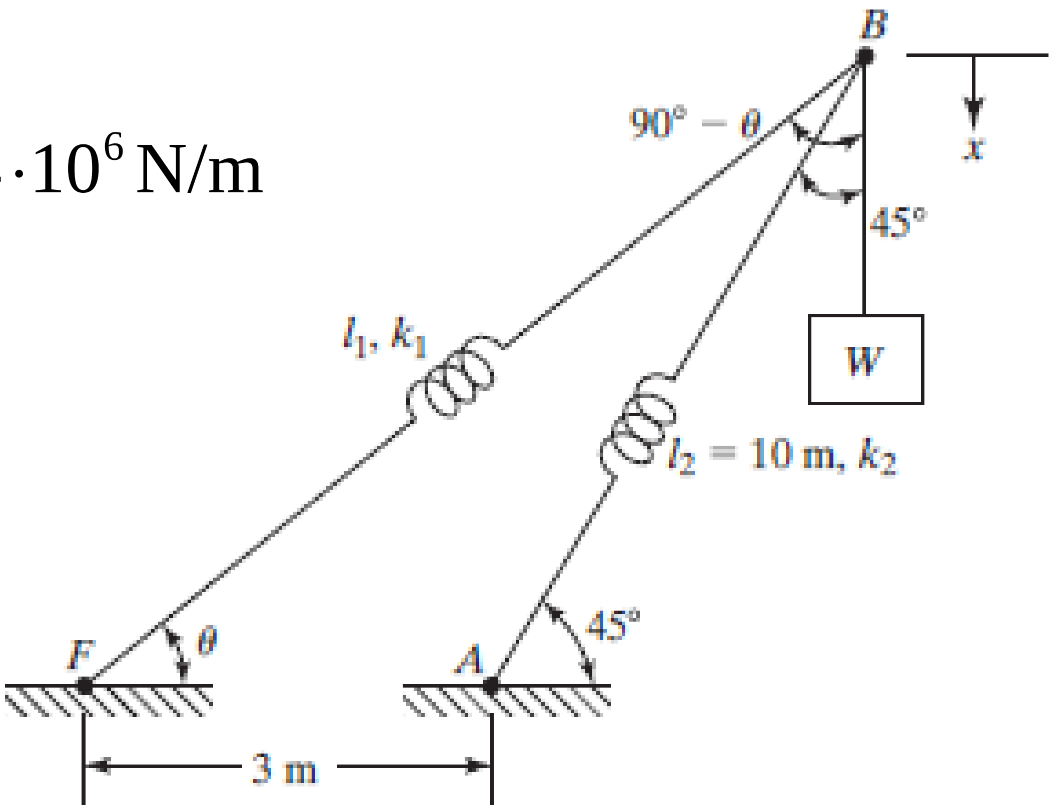
$$k_2 = \frac{A_2 E_2}{l_2} = 5.1750 \cdot 10^7 \text{ N/m}$$

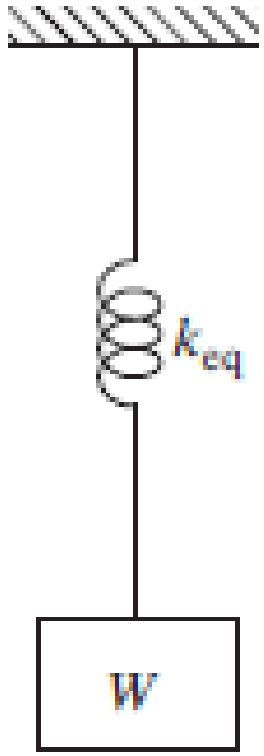


Exercise 2

$$\mathrm{V}_{eq} = \frac{1}{2} k_{eq} x^2$$

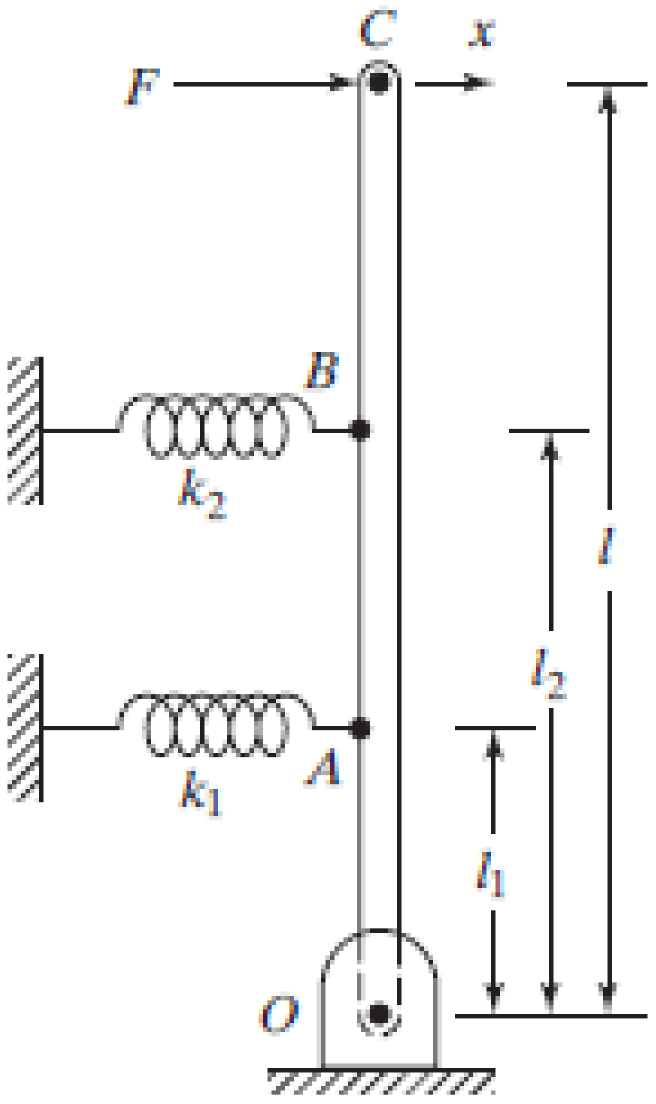
$$V = \mathrm{V}_{eq} \Rightarrow k_{eq} = 26.4304 \cdot 10^6 \,\mathrm{N/m}$$



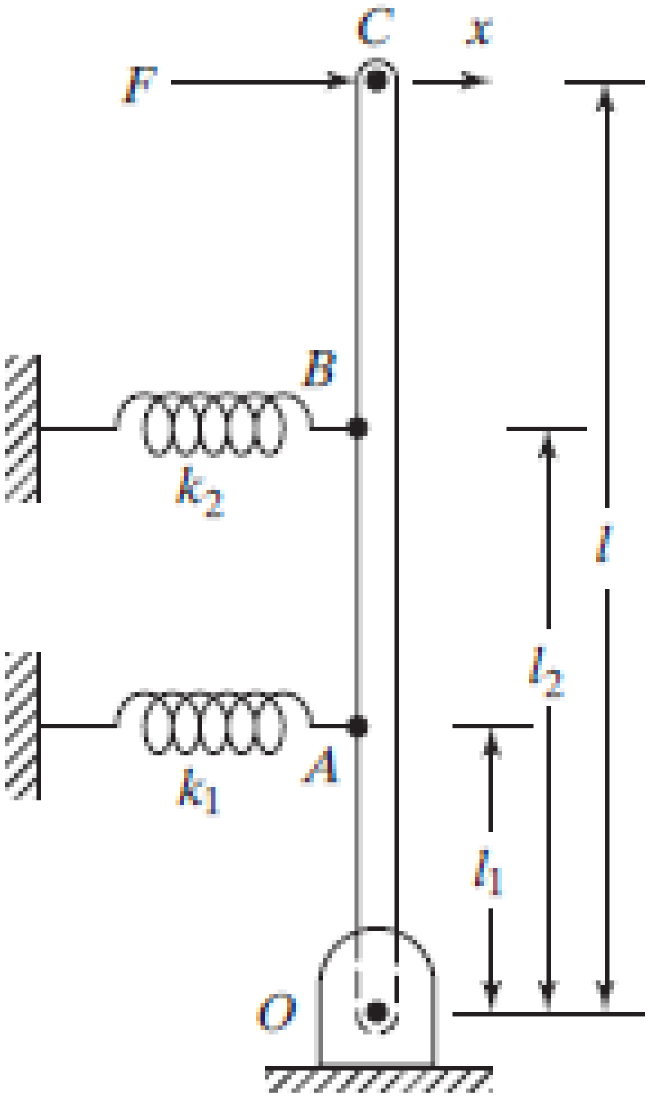


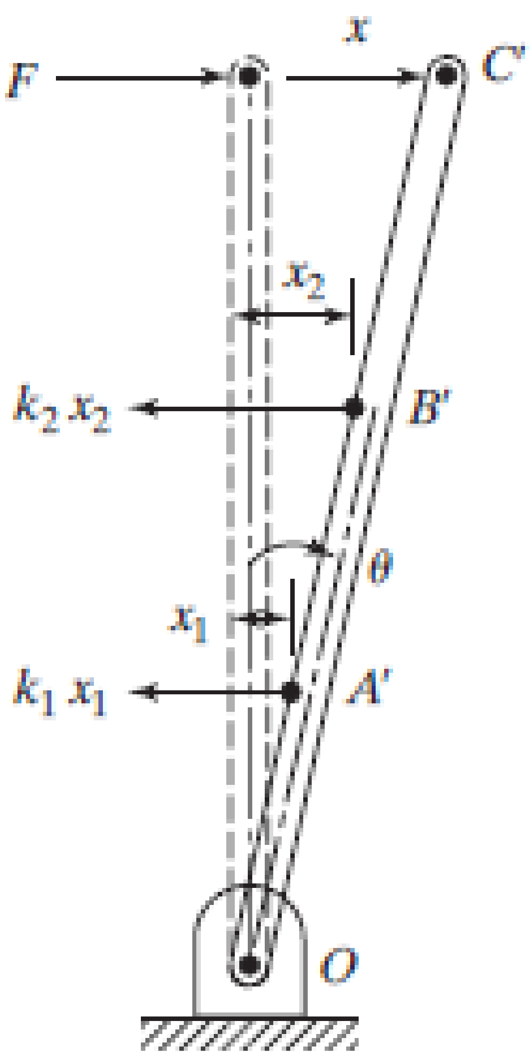
Exercise 3

- A hinged rigid bar of length *l* is connected by two springs of stiffnesses k_1 and k_2 and and is subjected to a force *F* as shown in the figure. Assuming that the angular displacement of the bar (θ) is small, find the equivalent spring constant of the system that relates the applied force *F* to the resulting displacement *x*.



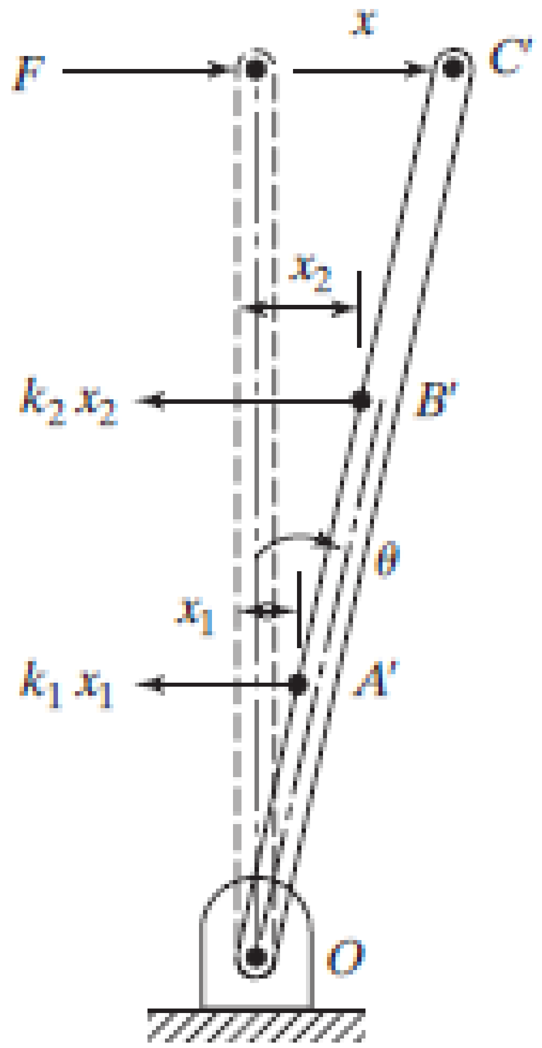
Exercise 3





Exercise 3

$$
\left.
\begin{aligned}
&k_1 x_1 + k_2 x_2 = F \\
&x_1 = l_1 \theta \\
&x_2 = l_2 \theta \\
&x = l\theta
\end{aligned}
\right\}
F = k_1 \left(\frac{x_1 l_1}{l} \right) + k_2 \left(\frac{x_2 l_2}{l} \right)
$$

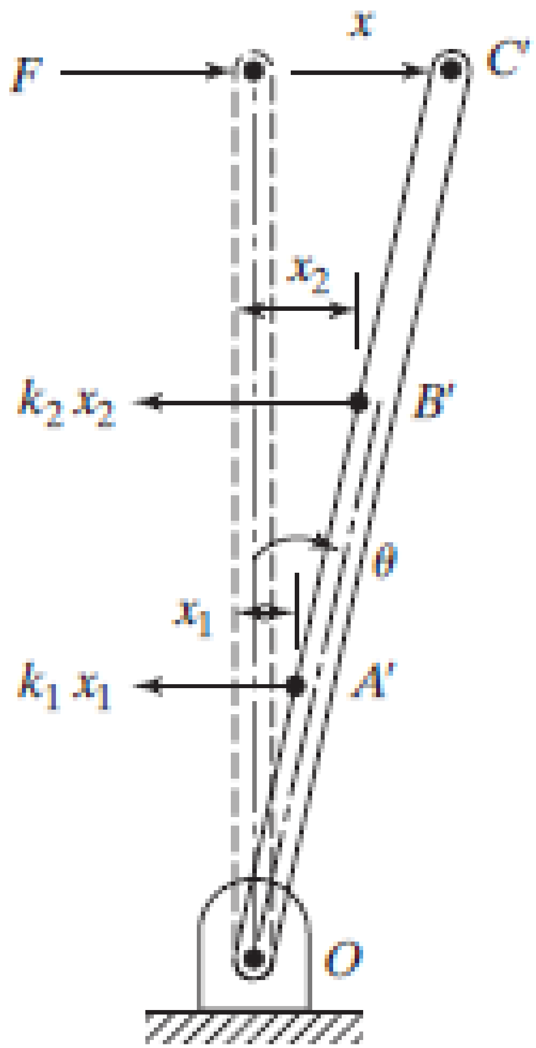


Exercise 3

$$F = k_1\left(\frac{x_1 l_1}{l}\right) + k_2\left(\frac{x_2 l_2}{l}\right)$$

$$F = k_{eq} x$$

$$\Rightarrow k_{eq} = k_1\left(\frac{l_1}{l}\right)^2 + k_2\left(\frac{l_2}{l}\right)^2$$

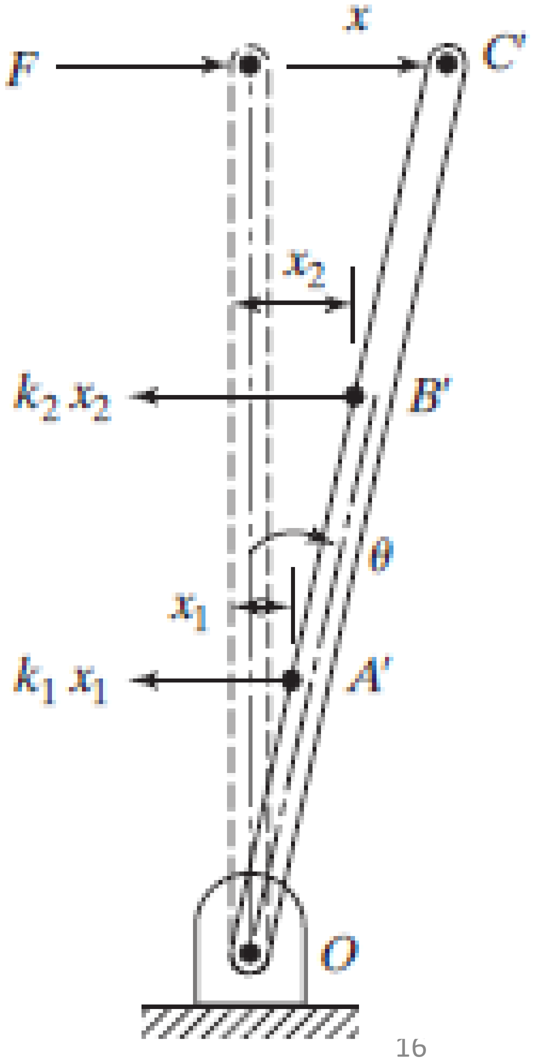


Exercise 3

- Another method:

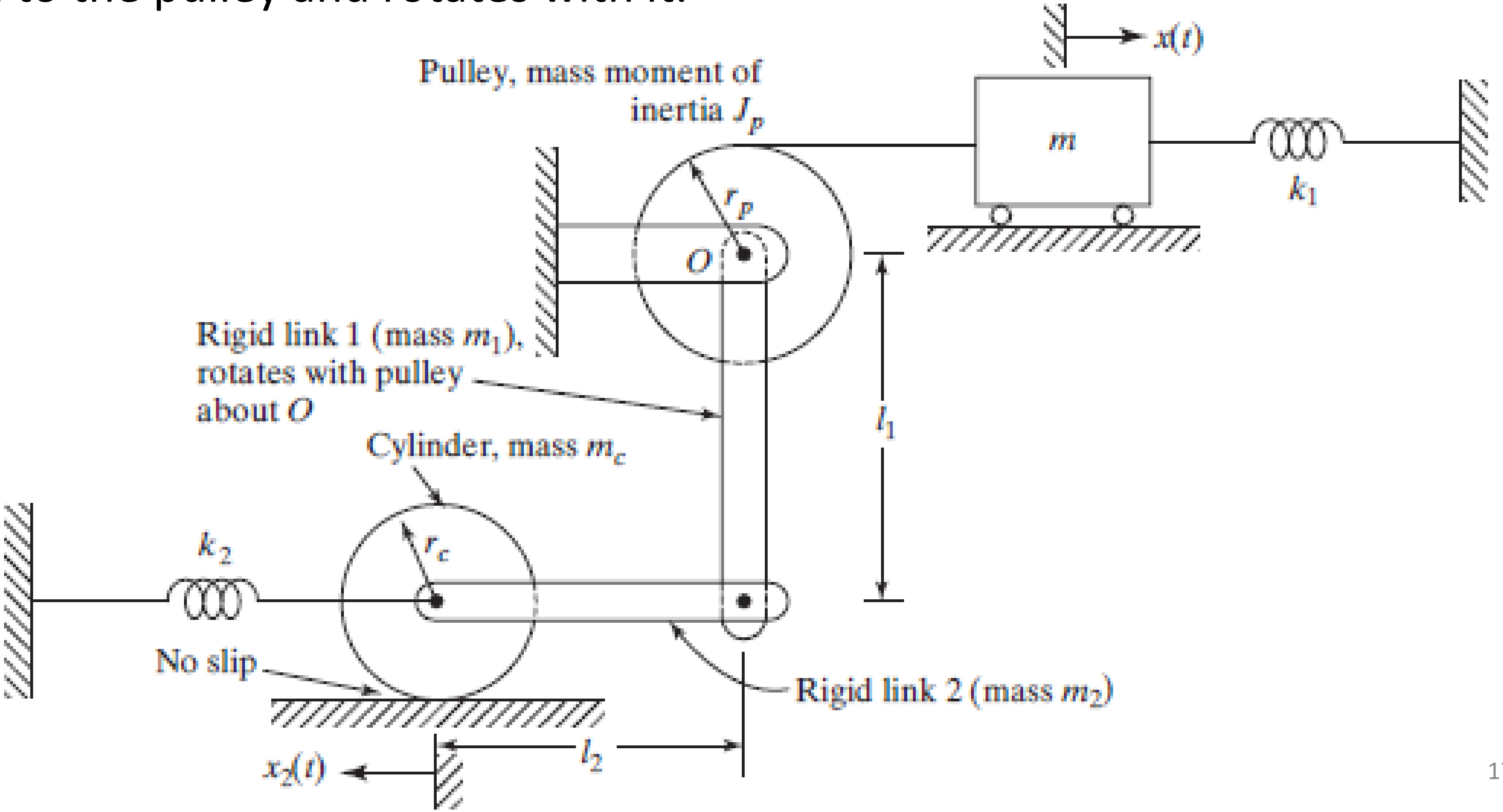
Work done by F = Potential energy of springs

$$\frac{1}{2}Fx = \frac{1}{2}k_1 x_1^2 + \frac{1}{2}k_2 x_2^2$$



Exercise 4

- Find the equivalent mass of the system shown in the figure, where the rigid link 1 is attached to the pulley and rotates with it.



Exercise 4

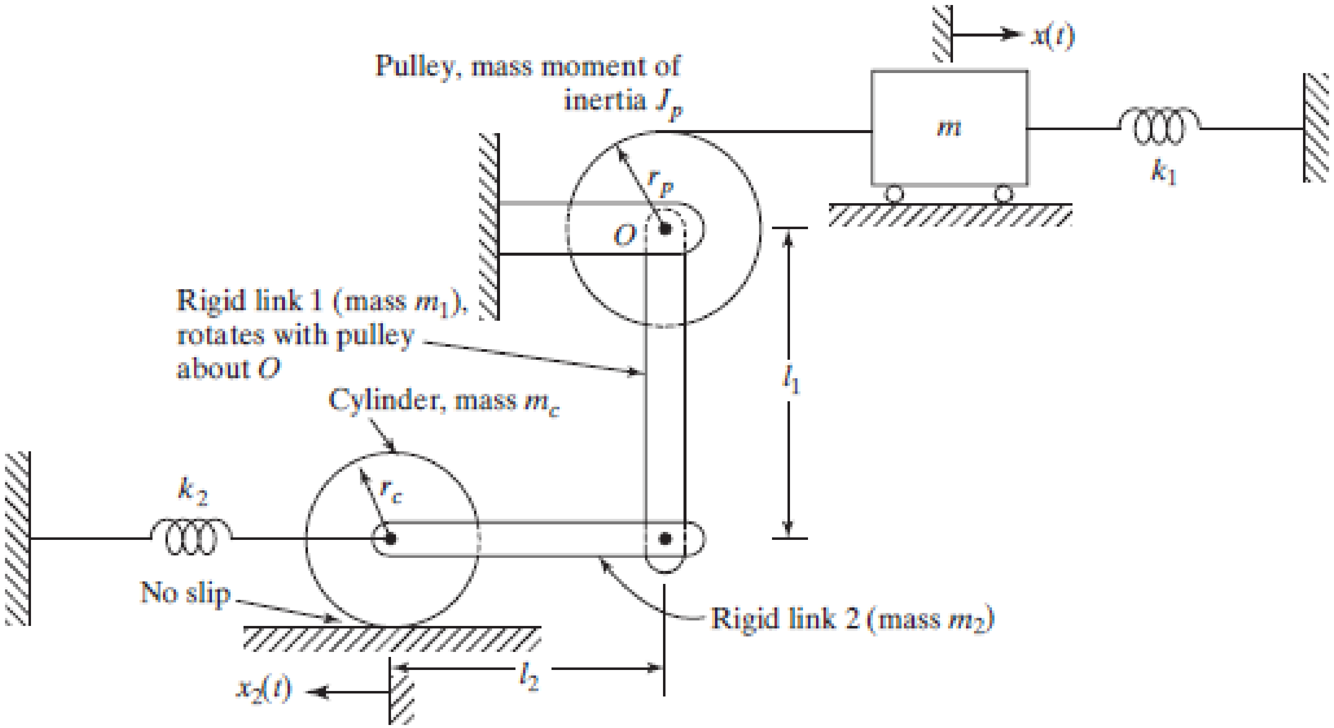
- Assumption: small displacements: the equivalent mass can be determined using the equivalence of the kinetic energies of the two systems

Motion:

$$\theta_p = \theta_1 = {}^{x}\!/_{r_p}$$

$$x_2 = \theta_p l_1 = {}^{xl_1}\!/_{r_p}$$

$$\theta_c = {}^{x_2}\!/_{r_c} = {}^{xl_1}\!/_{r_p r_c}$$



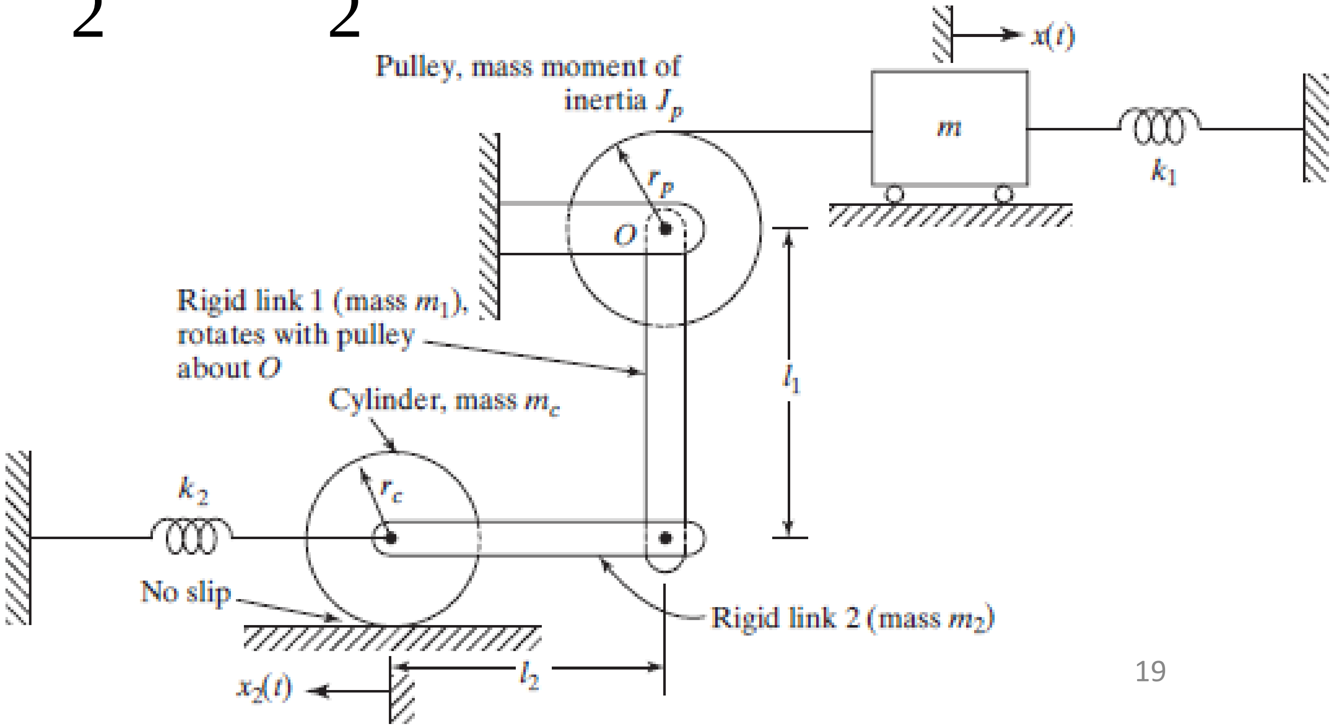
Exercise 4

- Assumption: small displacements: the equivalent mass can be determined using the equivalence of the kinetic energies of the two systems

$$T = \frac{1}{2} m\dot{x}^2 + \frac{1}{2} J_p \dot{\theta}_p^2 + \frac{1}{2} J_1 \dot{\theta}_1^2 + \frac{1}{2} m_2 \dot{x}_2^2 + \frac{1}{2} J_c \dot{\theta}_c^2 + \frac{1}{2} m_c \dot{x}_2^2$$

$$J_c = \frac{1}{2} m_c r_c^2$$

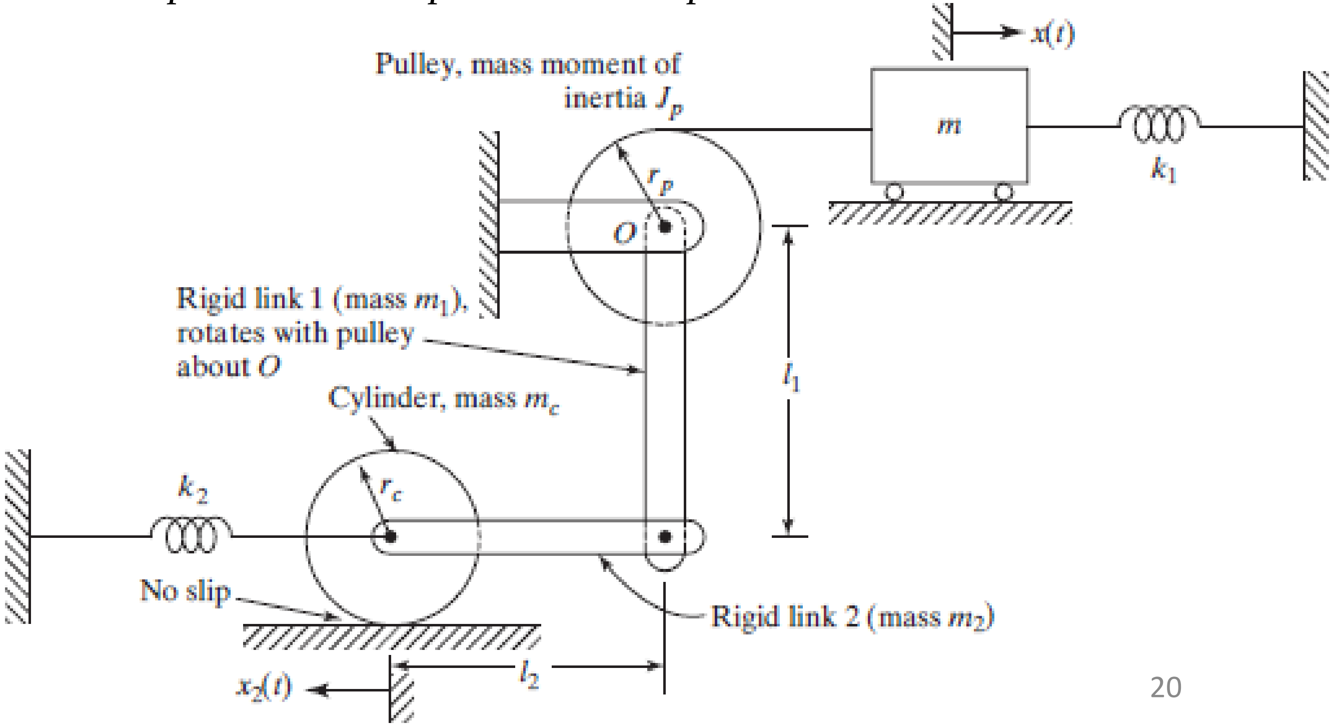
$$J_1 = \frac{1}{3} m_1 l_1^2$$



Exercise 4

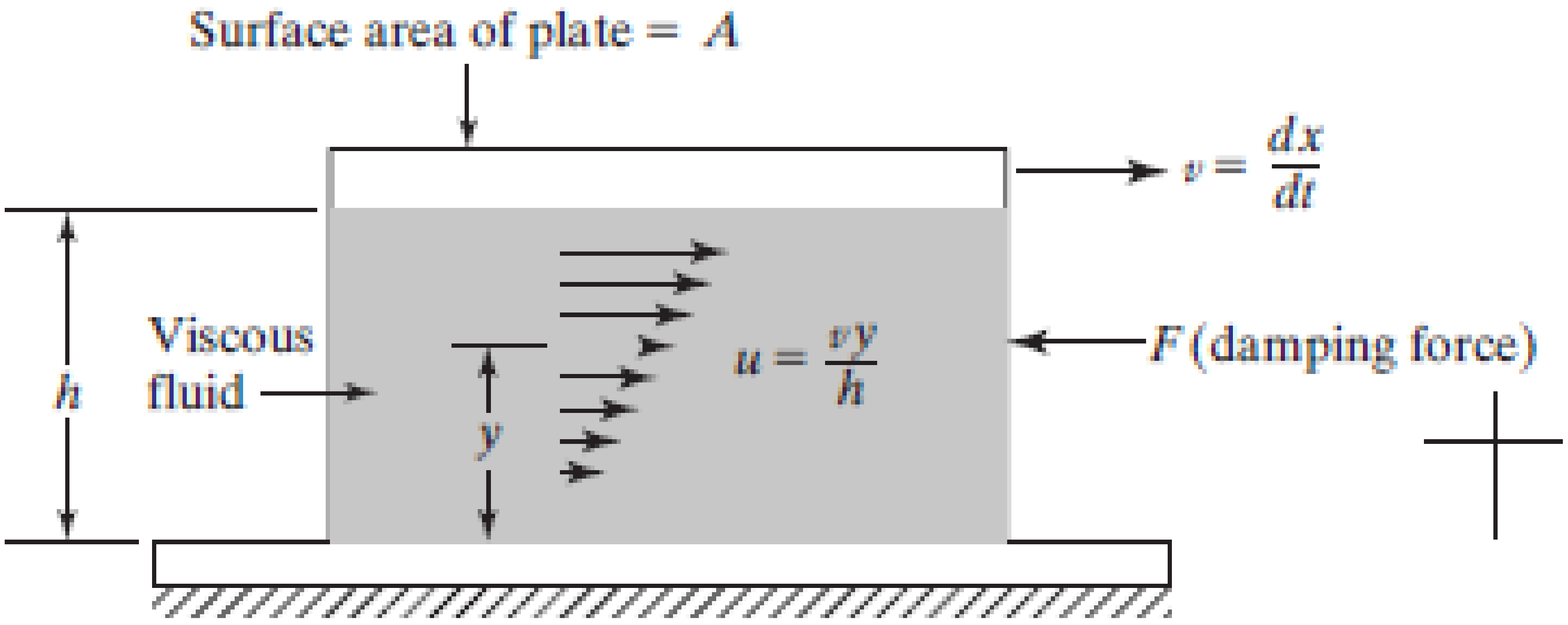
- Assumption: small displacements: the equivalent mass can be determined using the equivalence of the kinetic energies of the two systems

$$T = \frac{1}{2} m_{eq} \dot{x}^2 \Rightarrow m_{eq} = m + \frac{J_p}{r_p^2} + \frac{1}{3}\frac{m_1 l_1^2}{r_p^2} + \frac{m_2 l_1^2}{r_p^2} + \frac{1}{2}\frac{m_c l_1^2}{r_p^2} + m_c \frac{l_1^2}{r_p^2}$$



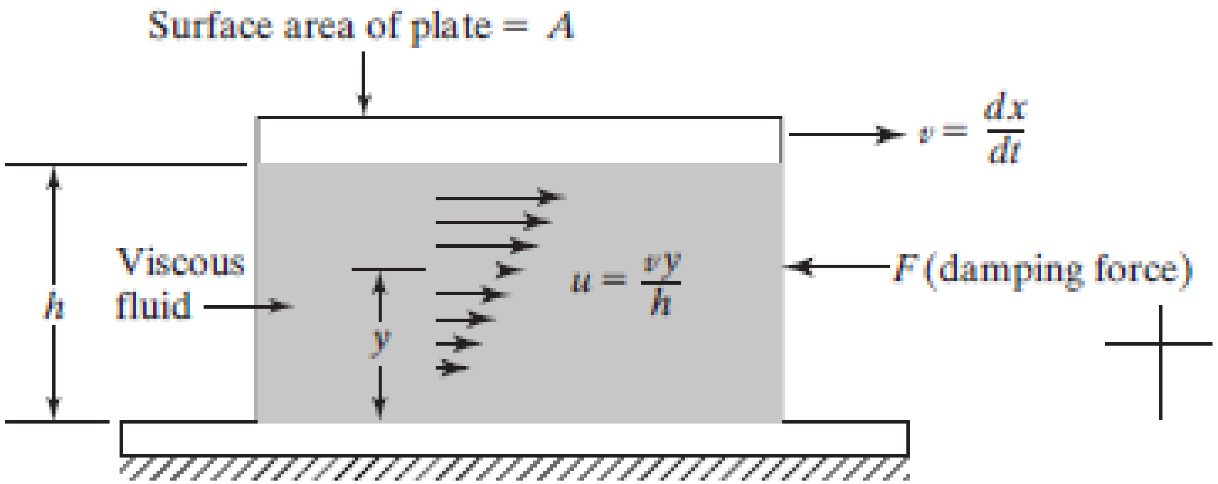
Exercise 5

- Consider two parallel plates separated by a distance *h*, with a fluid of viscosity between the plates. Derive an expression for the damping constant when one plate moves with a velocity *v* relative to the other as shown in the figure.



Exercise 5

- According to Newton s law of viscous flow, the shear stress developed in the fluid layer at a distance *y* from the fixed plate is given by:



$$\tau = \mu \frac{du}{dy}$$

$$\left.\begin{aligned} F &= \tau A = \frac{\mu A v}{h} \\ F &= cv \end{aligned}\right\} \quad c = \frac{\mu A}{h}$$