

# 第11章 电路的频率响应

## 本章重点

**\*网络函数的概念，幅频特性和相频特性的意义；**

**\* $RLC$ 串联的谐振分析；**

**$RLC$ 串联电路的频率响应及 $Q$ 值、带宽、通频带；**

**\* $RLC$ 并联电路的谐振分析；**

**$RC$ 低通滤波器、 $RC$ 高通滤波器。**

# 11.1 网络函数

单一正弦激励下，电路稳态响应的都为同频率的正弦量。

## 1. 定义

电路的工作状态随频率而变化的现象，称为频率特性（频率响应）

响应与激励之间的函数关系称为 **网络函数**

$$H(j\omega) \Rightarrow H(j\omega) = \frac{\dot{R}_k(j\omega)}{\dot{E}_{Sj}(j\omega)}$$

$\dot{R}_k(j\omega)$   $\Rightarrow$  端口 $k$ 的正弦稳态响应相量  $\dot{I}_k(j\omega)$  或  $\dot{U}_k(j\omega)$

$\dot{E}_{Sj}(j\omega)$   $\Rightarrow$  端口 $j$ 处的输入变量（正弦激励） $\dot{I}_{Sj}(j\omega)$  或  $\dot{U}_{Sj}(j\omega)$

**网络函数与激励无关，是系统参数和结构决定的。**

**网络函数是一个复数。**  $H(j\omega) = |H(j\omega)| \angle \varphi(j\omega)$

## 2. 网络函数是一个复数

$$H(j\omega) = \frac{\dot{R}_k(j\omega)}{\dot{E}_{sj}(j\omega)} = |H(j\omega)| \angle \varphi(j\omega)$$

**幅频特性：正弦量有效值或振幅值之比**

**相频特性：正弦量初相位之差**

} **频响特性**

**已知网络函数，已知激励，可以求响应。**

## 3. 网络函数的物理意义

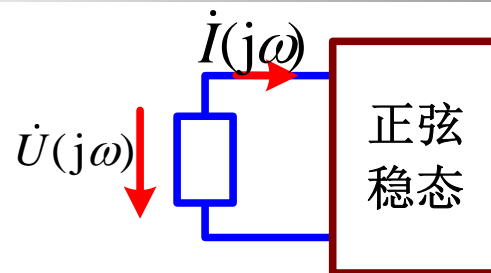
**网络函数有多种类型**

**当激励和响应属于同一对端子时，称为驱动点函数。**

**当激励和响应不属于同一对端子时，称为转移函数。**

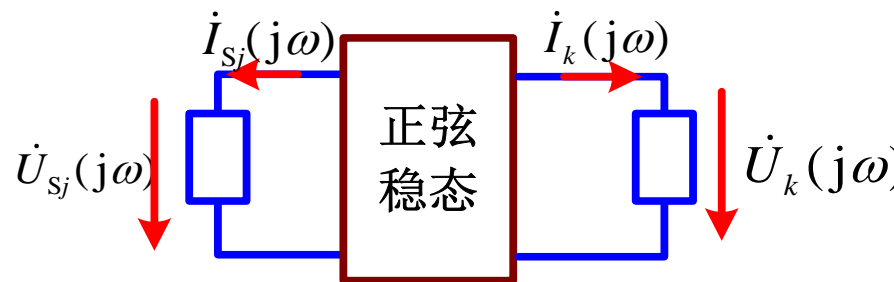
# 驱动点函数

**驱动点阻抗**  $Z = H(j\omega) = \frac{\dot{U}(j\omega)}{\dot{I}(j\omega)}$   
**驱动点导纳**  $Y = H(j\omega) = \frac{\dot{I}(j\omega)}{\dot{U}(j\omega)}$



# 转移函数

**转移阻抗**  $H(j\omega) = \frac{\dot{U}_k(j\omega)}{\dot{I}_{sj}(j\omega)}$   
**转移导纳**  $H(j\omega) = \frac{\dot{I}_k(j\omega)}{\dot{U}_{sj}(j\omega)}$   
**转移电压比**  $H(j\omega) = \frac{\dot{U}_k(j\omega)}{\dot{U}_{sj}(j\omega)}$   
**转移电流比**  $H(j\omega) = \frac{\dot{I}_k(j\omega)}{\dot{I}_{sj}(j\omega)}$



# 【例】

求电路的网络函数  $\frac{\dot{I}_2}{\dot{U}_s}, \frac{\dot{U}_L}{\dot{U}_s}$ 。

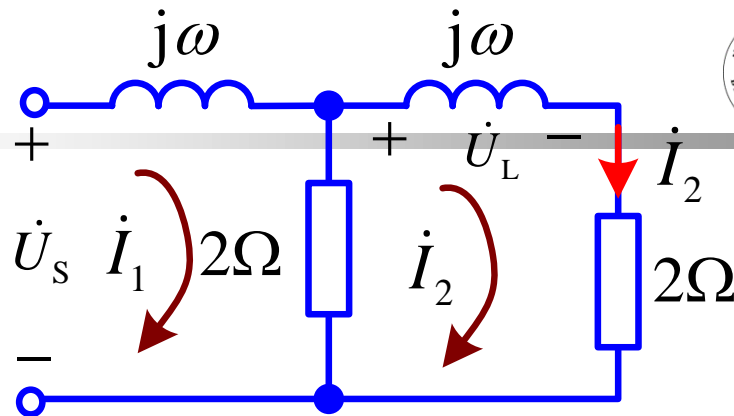
解

$$\begin{cases} (2 + j\omega) \dot{I}_1 - 2\dot{I}_2 = \dot{U}_s \\ -2\dot{I}_1 + (4 + j\omega) \dot{I}_2 = 0 \end{cases}$$

$$\dot{I}_2 = \frac{2\dot{U}_s}{4 + (j\omega)^2 + j6\omega}$$

$$\frac{\dot{I}_2}{\dot{U}_s} = \frac{2}{(4 - \omega^2) + j6\omega}$$

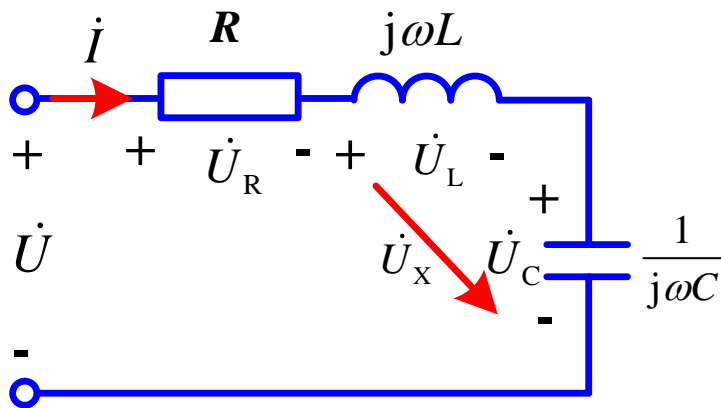
转移导纳



$$\frac{\dot{U}_L}{\dot{U}_s} = \frac{j2\omega}{(4 - \omega^2) + j6\omega}$$

转移电压比

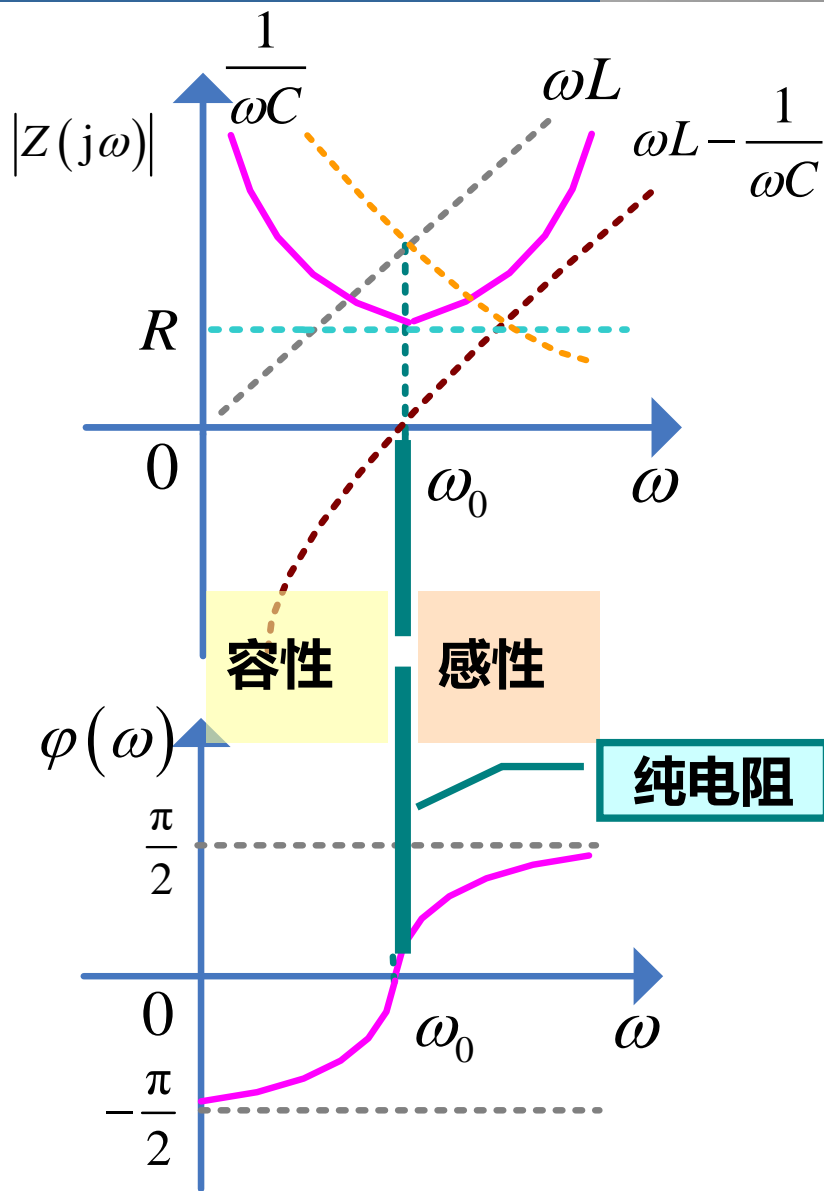
# 11.2 RLC串联电路的谐振



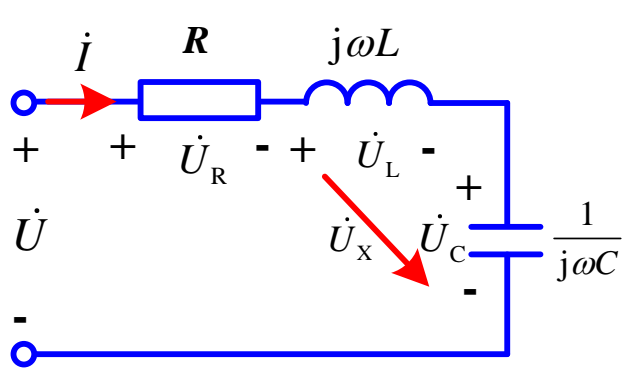
**驱动点阻抗**  $Z = R + j\left(\omega L - \frac{1}{\omega C}\right)$

$$|Z(j\omega)| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

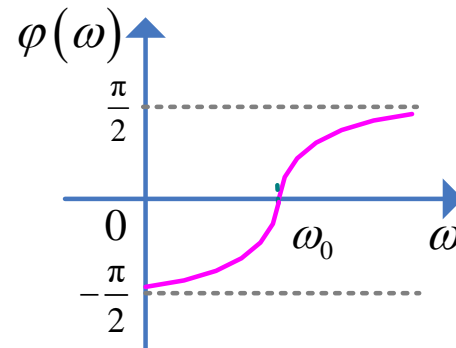
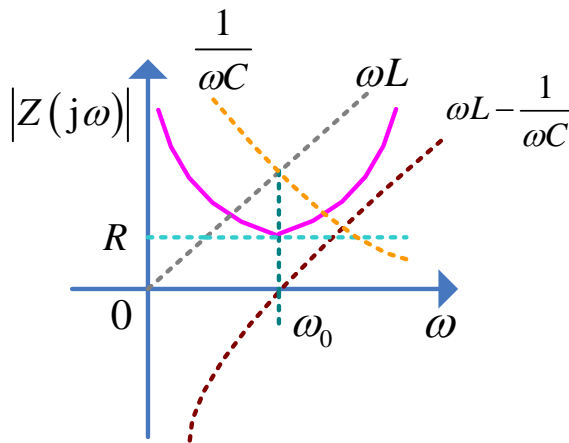
$$\varphi(j\omega) = \arctan\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)$$



## 11.2 RLC串联电路的谐振



$$Z = R + j\left(\omega L - \frac{1}{\omega C}\right)$$



### 1. 定义

端口电压、电流出现同相位的现象时，称电路发生了谐振；对于RLC串联电路，则称为串联谐振。

### 2. 条件

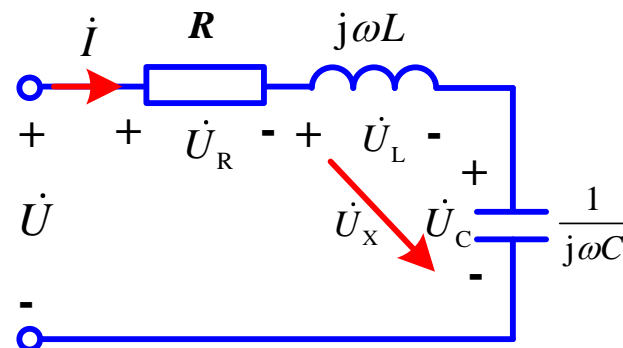
$$\text{Im}[Z(j\omega)] = 0$$

# 11.2 RLC串联电路的谐振

## 2. 条件 $\text{Im}[Z(j\omega)] = 0$

\*由谐振条件得串联电路实现谐振的**方式**为：

- (1)  $L$ 、 $C$  不变，改变  $\omega$  达到谐振。
- (2) 电源频率不变，改变  $L$  或  $C$  ( 常改变  $C$  ) 达到谐振。



谐振角频率

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

谐振频率



### 3. 特点 (阻抗、电流、电压、功率)

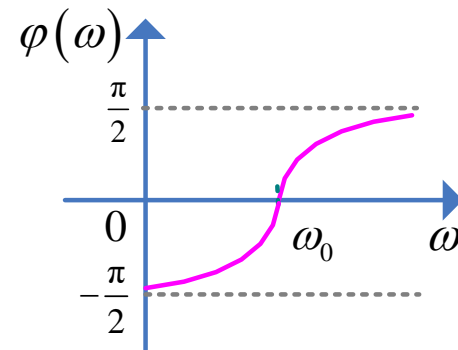
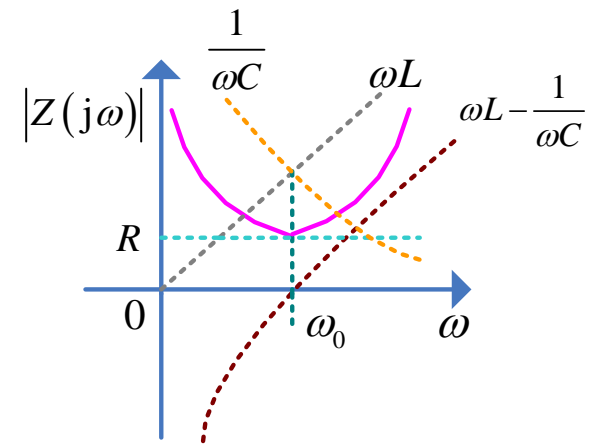
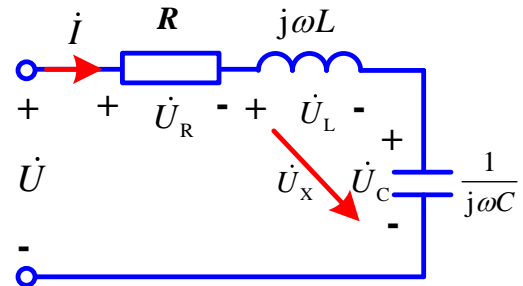
$$Z(j\omega_0) = R + j\left(\omega_0 L - \frac{1}{\omega_0 C}\right) = R$$

(1)  $\dot{U}$ 、 $\dot{I}$  同相  $\varphi = 0^\circ$

$$Z = R \quad |Z| = |Z|_{\min} = R$$

(2) 若  $U$  一定

$$I = \frac{U}{|Z|} = \frac{U}{R} = I_{\max}$$



$$(3) \quad \dot{U}_R = R\dot{I} = R\frac{\dot{U}}{R} = \dot{U}$$

$$\dot{U}_X = \dot{U}_L + \dot{U}_C = 0$$

➡ **电压谐振 L串C部分视作短路**

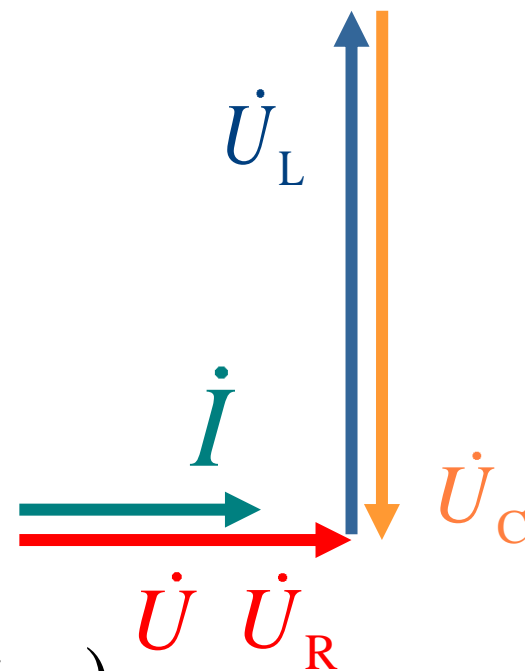
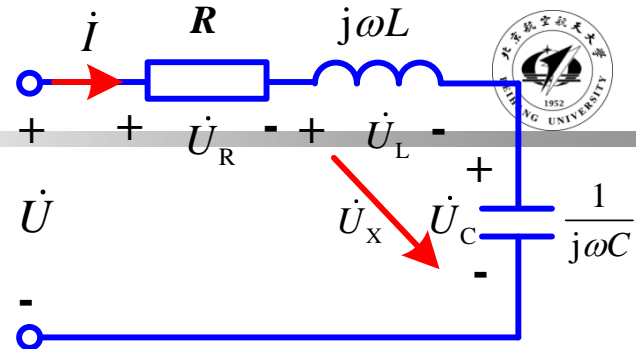
$$U_L = U_C$$

$$(4) \quad P = UI = RI^2 = \frac{U^2}{R}$$

$$Q = Q_L + Q_C = 0$$

$$W_{\text{存储}}(j\omega_0) = W_L(j\omega_0) + W_C(j\omega_0)$$

$$W_{\text{存储}}(j\omega_0) = \frac{1}{2}LI_m^2(j\omega_0) = \frac{1}{2}CU_{Cm}^2(j\omega_0)$$



## 4. 品质因数 $Q$

**谐振时**  $U_C = U_L = \omega_0 L I = \omega_0 L \frac{U}{R} = \frac{\omega_0 L}{R} U$

**定义**  $Q = \frac{U_L(\omega_0)}{U} \Rightarrow Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 C R} = \frac{1}{R} \sqrt{\frac{L}{C}}$

$Q = \frac{Q_L(j\omega_0)}{P(j\omega_0)} = \frac{|Q_C(j\omega_0)|}{P(j\omega_0)} \leftarrow Q = \frac{\omega_0 L I^2(j\omega_0)}{R I^2(j\omega_0)}$

$$W(j\omega_0) = \frac{1}{2} L I_m^2(j\omega_0) = \frac{1}{2} C U_{Cm}^2(j\omega_0) = C Q^2 U_s^2(j\omega_0)$$

**!  $Q$ 综合反映了电路三个参数对谐振状态的影响!**

# 11.3 RLC串联电路的频率特性

**驱动点阻抗**  $Z = H(j\omega) = \frac{\dot{U}(j\omega)}{\dot{I}(j\omega)}$

**转移电压比**  $H(j\omega) = \frac{\dot{U}_R(j\omega)}{\dot{U}_s(j\omega)}$   $H(j\omega) = \frac{\dot{U}_L(j\omega)}{\dot{U}_s(j\omega)}$   $H(j\omega) = \frac{\dot{U}_C(j\omega)}{\dot{U}_s(j\omega)}$

## (1) 电阻电压频率特性

$$H_R(j\omega) = \frac{\dot{U}_R(j\omega)}{\dot{U}_s(j\omega)} = \frac{R}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

$$H_R(j\omega) = \frac{1}{1 + j\left(\frac{\omega L}{R} - \frac{1}{R\omega C}\right)} = \frac{1}{1 + j\left(\frac{\omega}{\omega_0} \frac{\omega_0 L}{R} - \frac{1}{R\omega_0 C} \frac{\omega_0}{\omega}\right)}$$

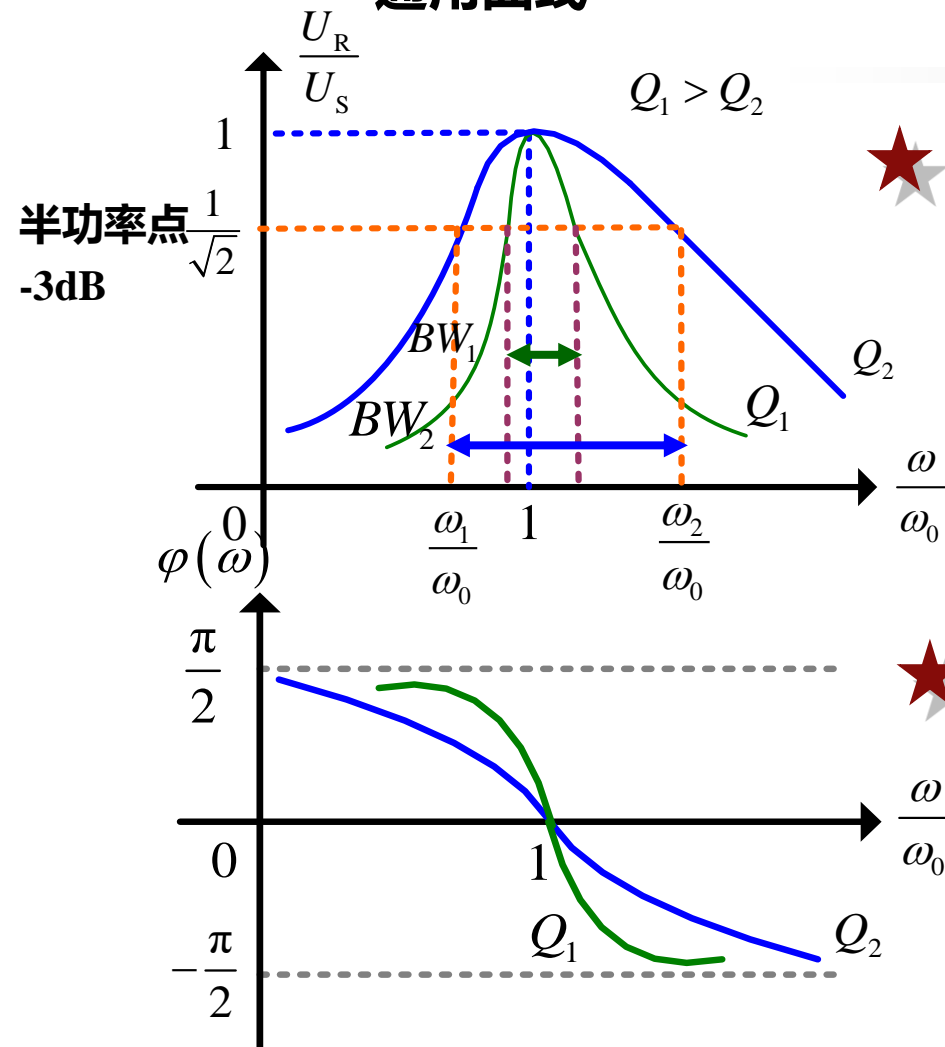
$$Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 C R} \quad \eta = \frac{\omega}{\omega_0}$$

$$H(j\eta) = \frac{1}{1 + jQ\left(\eta - \frac{1}{\eta}\right)}$$



$$\begin{cases} |H(j\eta)| = \frac{1}{\sqrt{1 + Q^2\left(\eta - \frac{1}{\eta}\right)^2}} \\ \varphi(j\eta) = -\arg \tan \left[ Q\left(\eta - \frac{1}{\eta}\right) \right] \end{cases}$$

## 通用曲线



$$|H(j\eta)| = \frac{1}{\sqrt{1 + Q^2 \left( \eta - \frac{1}{\eta} \right)^2}}$$

$$\varphi(j\eta) = -\arg \tan \left[ Q \left( \eta - \frac{1}{\eta} \right) \right]$$

频率选择性:  $Q$  越大越好

抑制能力:  $Q$  越大抑制能力越强

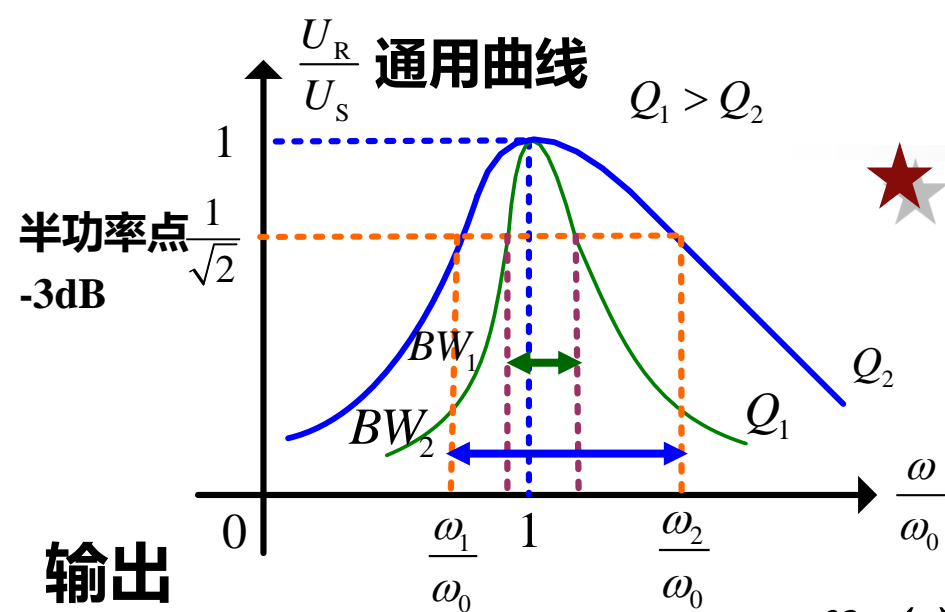
通频带 (通带)

## 品质因数试验方法

$$Q = \frac{\omega_0}{\omega_2 - \omega_1} = \frac{f_0}{f_2 - f_1} = \frac{1}{\eta_2 - \eta_1} = \frac{\omega_0}{BW}$$

!  $Q$ 反映电路选择性的好坏!

通频带带宽



$$|H(j\eta)| = \frac{1}{\sqrt{1 + Q^2 \left( \eta - \frac{1}{\eta} \right)^2}}$$

$$\varphi(j\eta) = -\arg \tan \left[ Q \left( \eta - \frac{1}{\eta} \right) \right]$$

输出  
幅值  
指标

$$|H| \geq \frac{1}{\sqrt{2}}$$

$$|H| = \frac{1}{\sqrt{2}}$$

$$\eta_1 \leftrightarrow \omega_1 \leftrightarrow f_1$$

$$\eta_2 \leftrightarrow \omega_2 \leftrightarrow f_2$$

截止频率 转折频率

$$\eta_1 = -\frac{1}{2Q} + \sqrt{\left(\frac{1}{2Q}\right)^2 + 1}$$

$$\eta_2 = \frac{1}{2Q} + \sqrt{\left(\frac{1}{2Q}\right)^2 + 1}$$



通频带 (通带)

$$\omega_1 \leq \omega \leq \omega_2 \quad f_1 \leq f \leq f_2$$

带宽  $BW = \begin{cases} \omega_2 - \omega_1 \\ f_2 - f_1 \\ \eta_2 - \eta_1 \end{cases}$

$$Q = \frac{\omega_0}{BW} \sim \frac{f_0}{BW} \sim \frac{1}{BW}$$

波特图

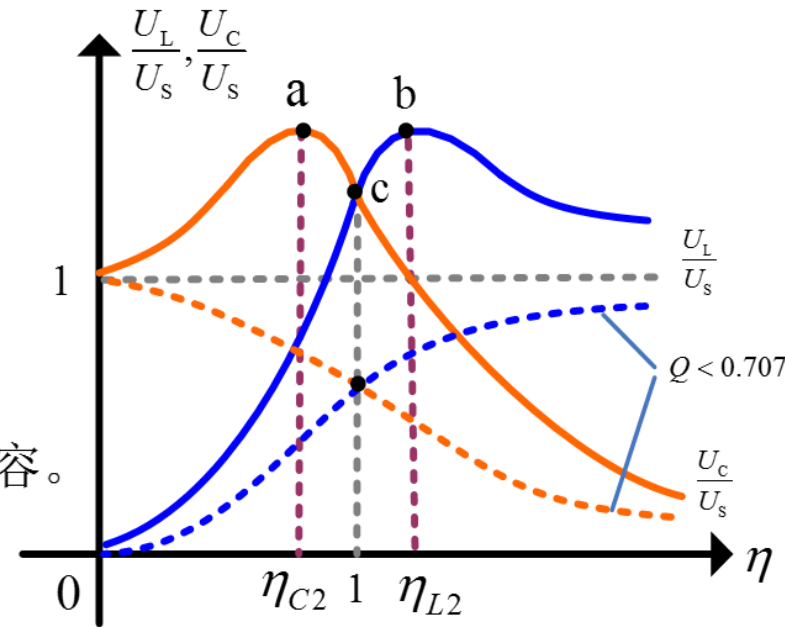
$$\begin{cases} H_{dB} = 20 \lg |H(j\omega)| \\ \lg \omega \end{cases} \quad \begin{cases} \varphi(j\omega) \\ \lg \omega \end{cases}$$

## (2) 电感电压和电容电压频率特性

$$H_c(j\eta) = \frac{\dot{U}_c(j\eta)}{\dot{U}_s(j1)} = \frac{-jQ}{\eta + jQ(\eta^2 - 1)}$$

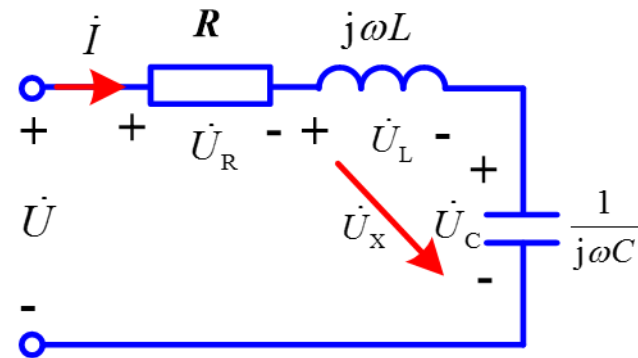
### 低通频率特性

若  $Q \gg 1$ , 则  $U_c \gg U_s$ , 可应用于无线电系统;  
而电力系统应避免谐振, 以免  $U_c$  过大, 击毁电容。



$$H_L(j\eta) = \frac{\dot{U}_L(j\eta)}{\dot{U}_s(j1)} = \frac{jQ}{\frac{1}{\eta} + jQ\left(1 - \frac{1}{\eta^2}\right)}$$

### 高通频率特性

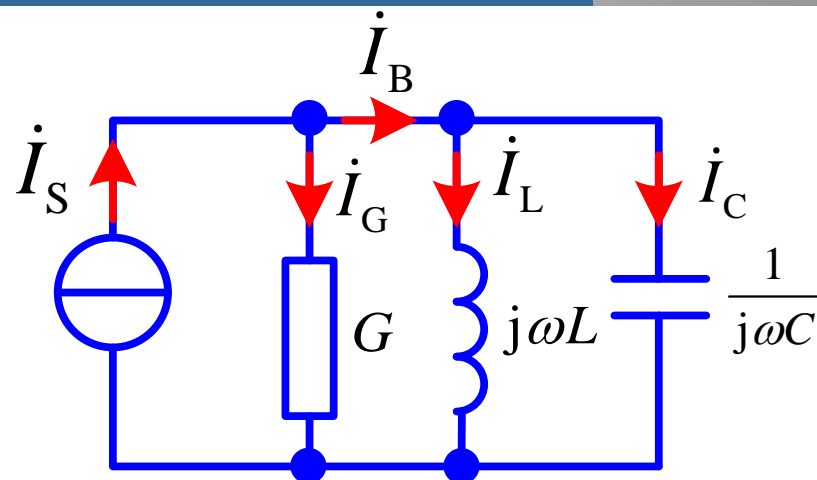


## 11.4 RLC并联谐振电路

$$Y = G + j\left(\omega C - \frac{1}{\omega L}\right)$$

### 1. 定义

端口电压、电流出现同相位的现象时，称电路发生了谐振；对于RLC并联电路，则称为并联谐振。



### 2. 条件

$$\text{Im}[Y(j\omega)] = 0$$

谐振角频率

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

谐振频率



### 3. 特点 (导纳、电压、电流、功率)

(1)  $\dot{U}$ 、 $\dot{I}$  同相,  $\varphi = 0^\circ$

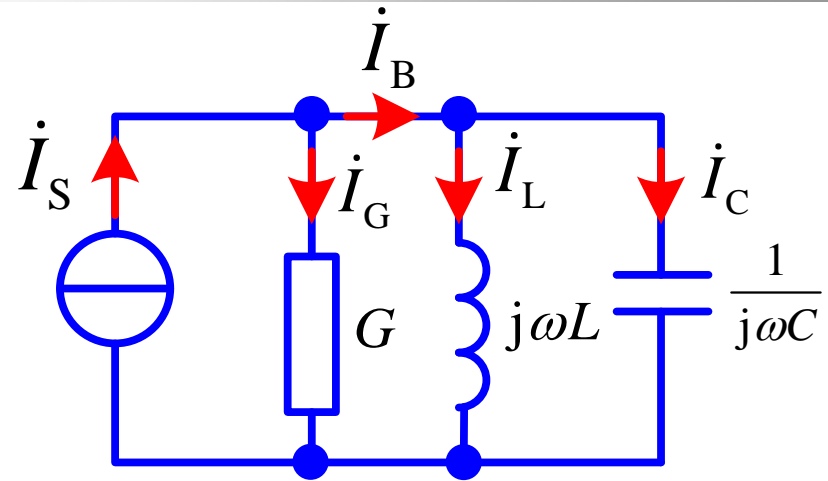
$$Y = G$$

$$|Y| = |Y|_{\min} = G$$

$$|Z| = \frac{1}{|Y|} = |Z|_{\max}$$

(2) 若  $I$  一定

$$U = I|Z| = IR = U_{\max}$$

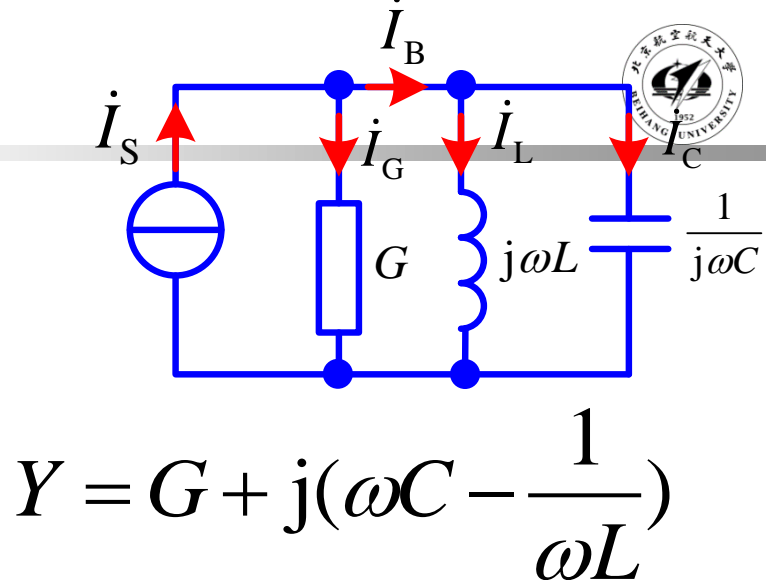


$$Y = G + j(\omega C - \frac{1}{\omega L})$$

$$(3) \quad \dot{I}_B = \dot{I}_L + \dot{I}_C = 0$$

$$I_L = I_C$$

$$I_S = I_G$$



**电流谐振 L并C视作开路**

$$(4) \quad P = UI = RI^2 = \frac{U^2}{R}$$

$$Q = Q_L + Q_C = 0$$

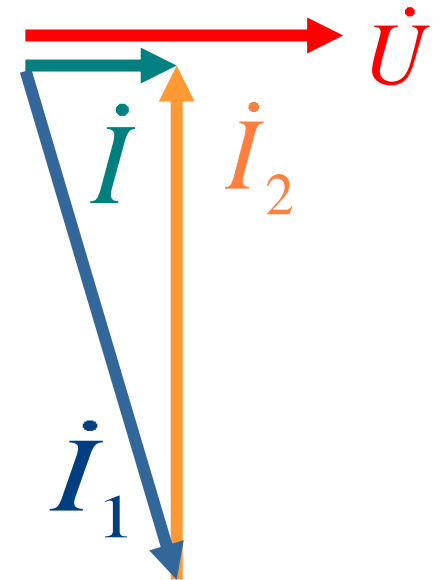
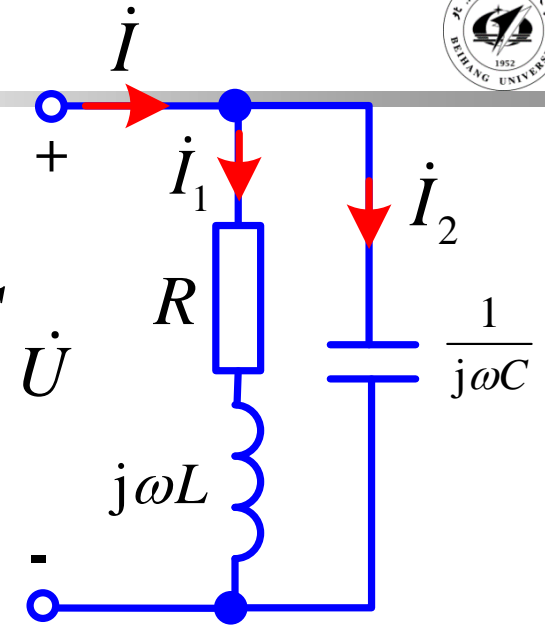
$$Y = \frac{1}{R + j\omega L} + j\omega C$$

$$= \frac{R}{R^2 + (\omega L)^2} - j \frac{\omega L}{R^2 + (\omega L)^2} + j\omega C$$

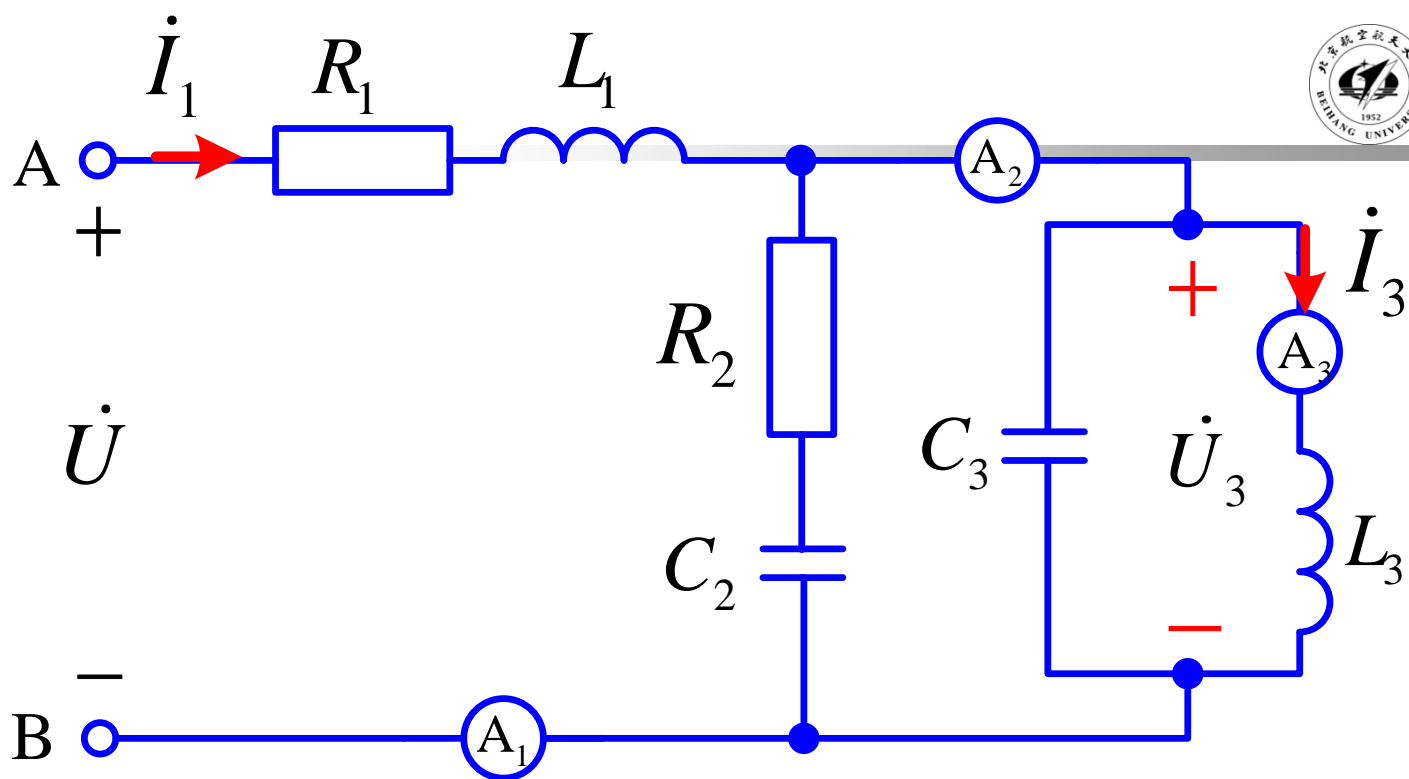
$$\text{Im}[Y] = 0$$

$$\omega C - \frac{\omega L}{R^2 + (\omega L)^2} = 0$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \sqrt{1 - \frac{CR^2}{L}}$$



# 【例】



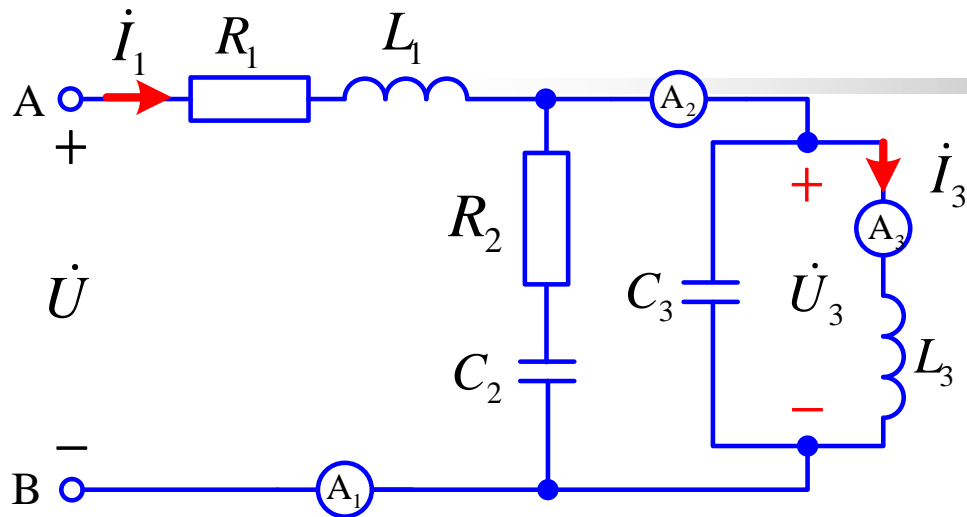
**已知：**  $R_1 = 50\Omega$   $R_2 = 50\Omega$   $L_1 = 200\text{mH}$   $C_2 = 5\mu\text{F}$

$L_3 = 100\text{mH}$   $C_3 = 10\mu\text{F}$   $U = 200\text{V}$

**电流表A2指示为零，所有电流表内阻忽略不计**

**求：(1) A1, A3的读数**

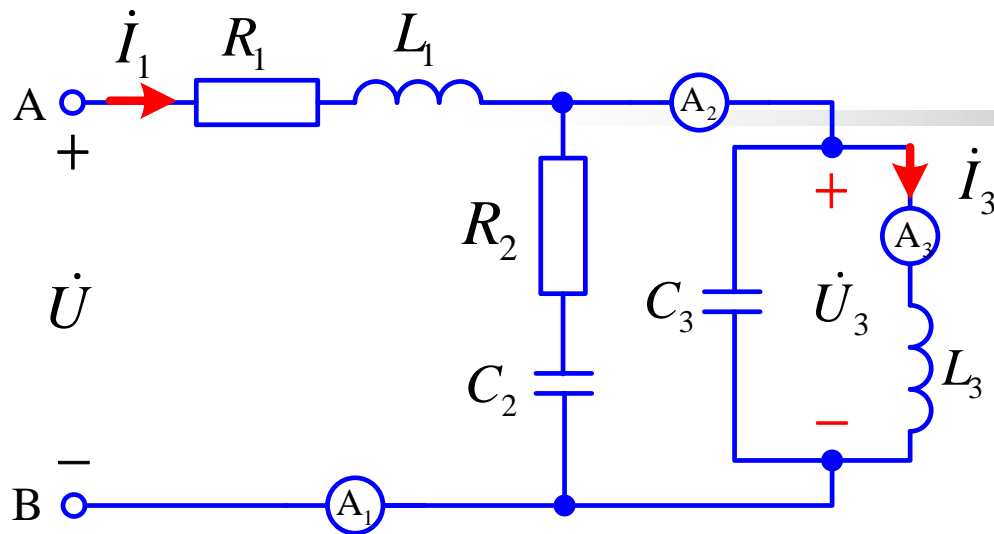
**(2) 输入AB端的功率和功率因数。**



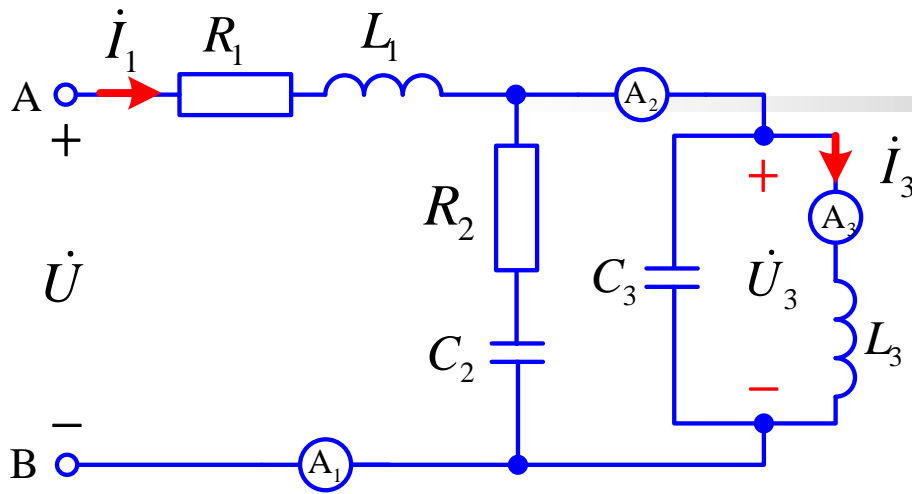
解

$$(1) \quad \omega C_3 = \frac{1}{\omega L_3}$$

$$\omega = \frac{1}{\sqrt{L_3 C_3}} = \frac{1}{\sqrt{100 \times 10^{-3} \times 10 \times 10^{-6}}} = 10^3 \text{ rad/s}$$



$$\begin{aligned}
 Z &= (R_1 + R_2) + j \left( \omega L_1 - \frac{1}{\omega C_2} \right) \\
 &= 50 + 50 + j \left( 10^3 \times 200 \times 10^{-3} - \frac{1}{10^3 \times 5 \times 10^{-6}} \right) \\
 &= 100 \angle 0^\circ \Omega \\
 I_1 &= \frac{U}{|Z|} = \frac{200}{100} = 2 \text{ A}
 \end{aligned}$$



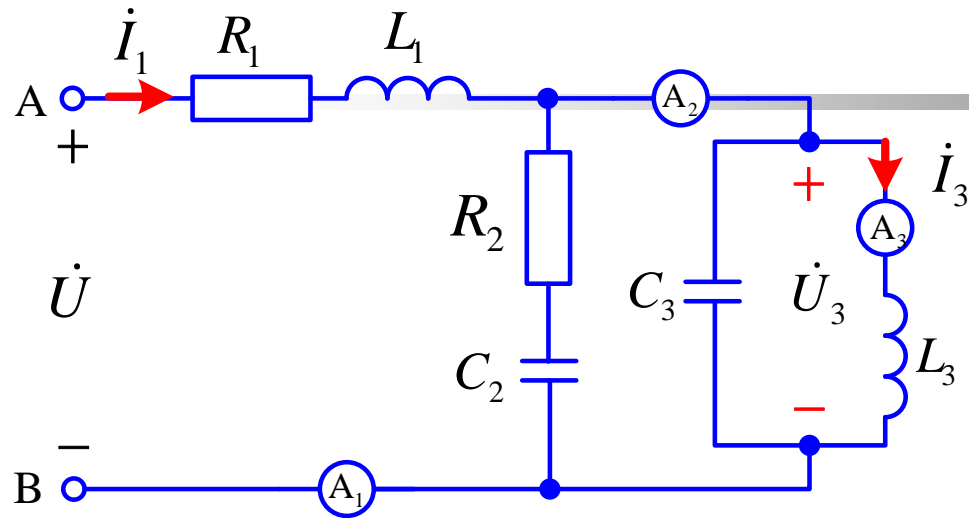
$$U_3 = I_1 |Z_2| = I_1 \sqrt{R_2^2 + \left( \frac{1}{\omega C_2} \right)^2} = 412.3 \text{ V}$$

$$I_3 = \frac{U_3}{\omega L_3} = \frac{412.3}{10^3 \times 100 \times 10^{-3}} = 4.12 \text{ A}$$

(2)

$$\varphi = 0^\circ$$

$$\cos \varphi = 1$$



$$P = UI_1 \cos \varphi = 200 \times 2 = 400 \text{ W}$$

$$\text{或 } P = I_1^2 (R_1 + R_2) = 2^2 \times (50 + 50) = 400 \text{ W}$$



## 【例】

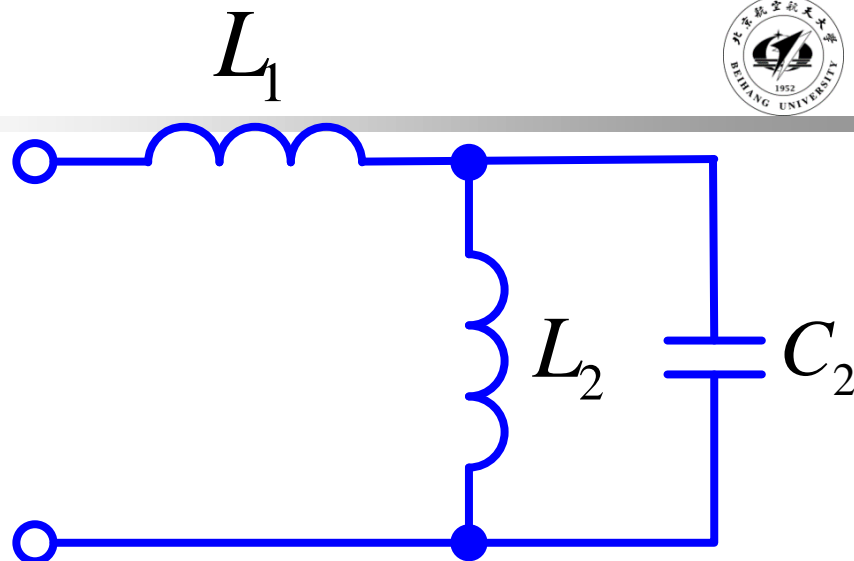
分析图示电路的串并联谐振。

解

既有串联谐振，  
也有并联谐振。

先求  $Z, I_m[Z] = 0$ , 串联谐振

再求  $Y, I_m[Y] = 0$ , 并联谐振



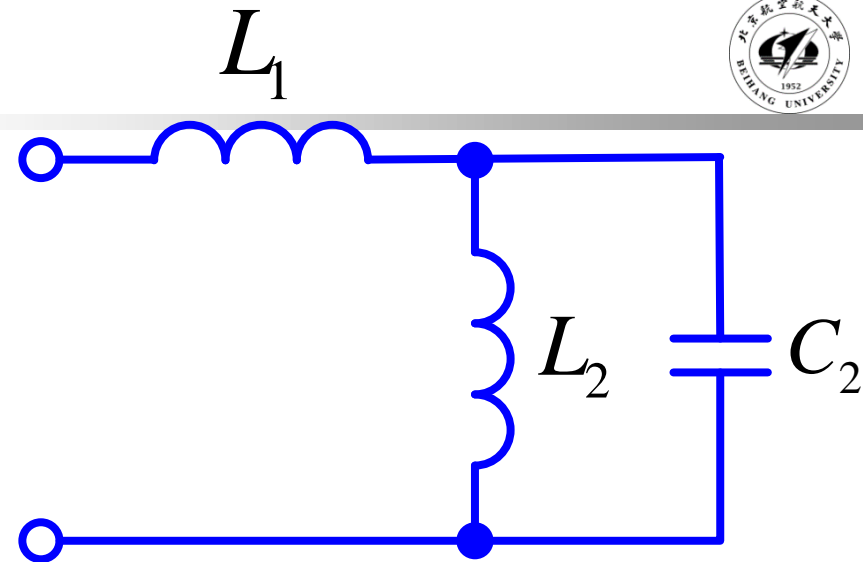
$$Z = j\omega L_1 + \frac{j\omega L_2 \left( -j \frac{1}{\omega C_2} \right)}{j\omega L_2 - j \frac{1}{\omega C_2}} = j \frac{\omega^3 L_1 L_2 C_2 - \omega (L_1 + L_2)}{\omega^2 L_2 C_2 - 1}$$

**串联谐振**  $I_m [Z] = 0$

$$\omega_{01} = \sqrt{\frac{L_1 + L_2}{L_1 L_2 C_2}}$$

**并联谐振**  $I_m [Y] = 0$

$$\omega_{02} = \sqrt{\frac{1}{L_2 C_2}}$$



**谐振特点：** 电流电压同相；  
LC间能量交换，与外部没有能量交换

**低频段，并联环节呈感性，整个电路呈感性；**

**$\omega$  上升，达到  $\omega_{02}$ ，发生并联谐振；**

**$\omega$  继续上升，并联环节呈容性，当  $\omega = \omega_{01}$  时，发生串联谐振。**

## 【练习】

分析图示电路的串并联谐振。

解

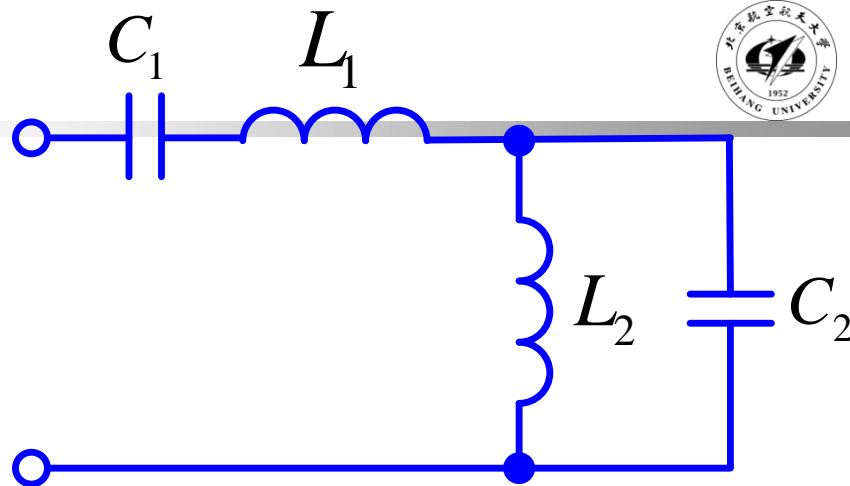
既有串联谐振，  
也有并联谐振。

求  $Z, I_m[Z] = 0$ , 串联谐振

求  $Y, I_m[Y] = 0$ , 并联谐振

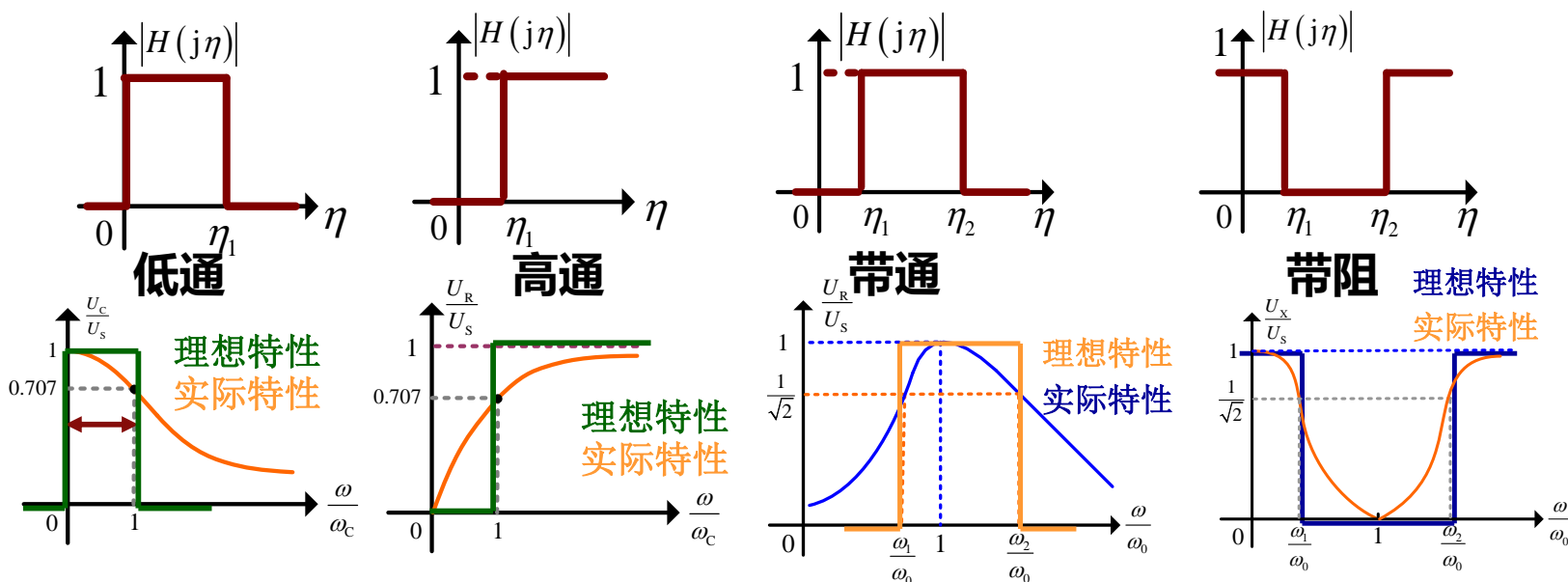
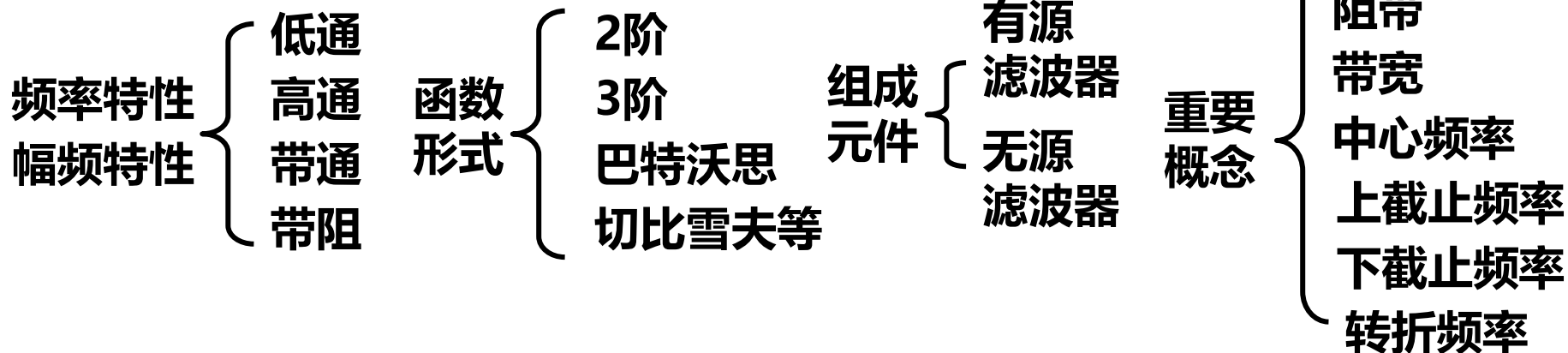
$C_2, L_2$  并联, 并联谐振

$C_1, L_1$  串联,  $C_2, L_2$  并联, 整个电路串联谐振



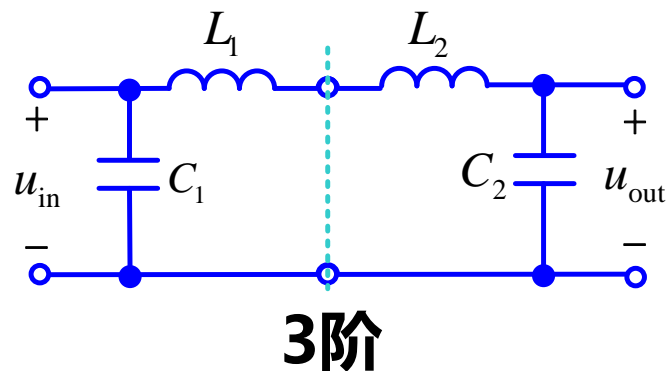
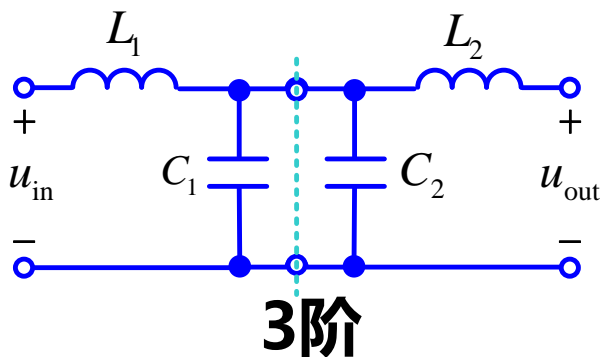
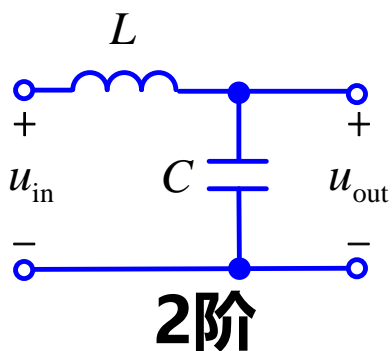
# 11.5 滤波器简介

滤波器是具有频率选择作用的网络

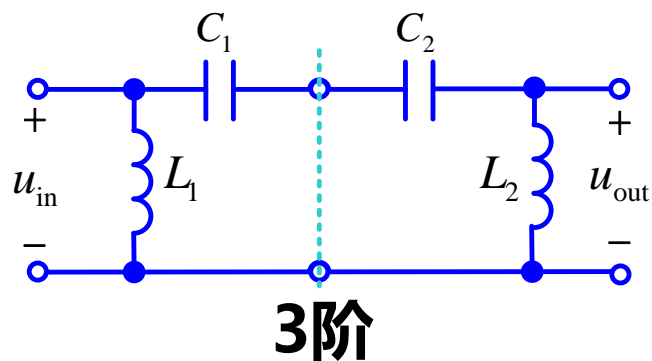
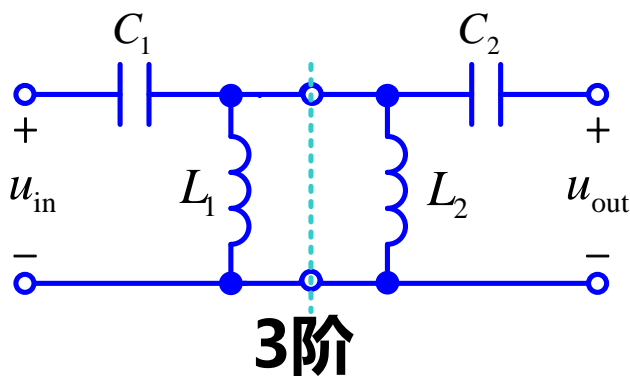
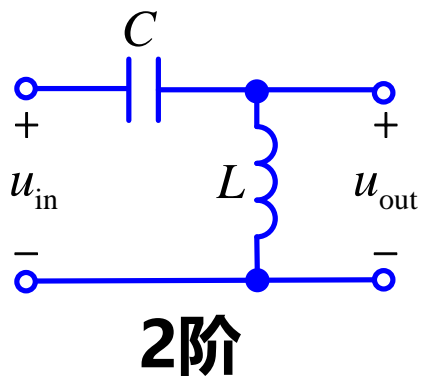


# 11.5 滤波器简介

## 低通滤波器 Low Pass Filter (LPF)



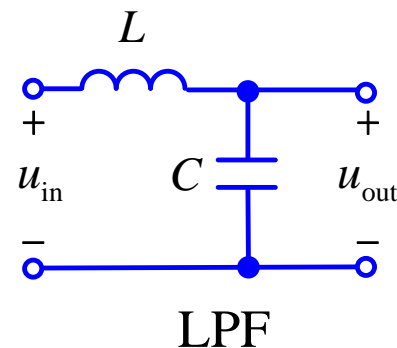
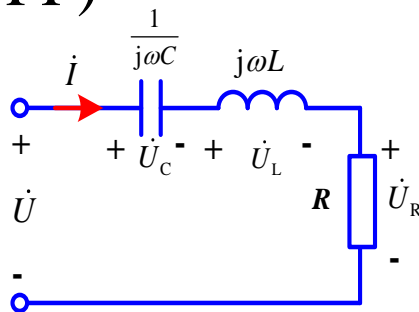
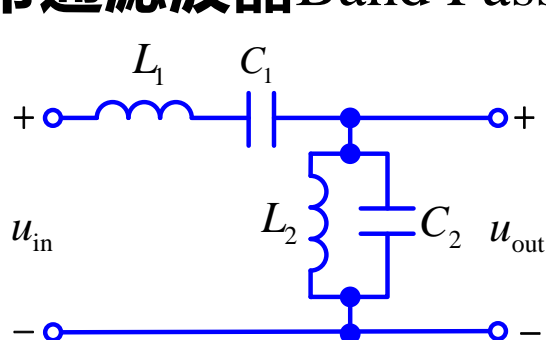
## 高通滤波器 High Pass Filter (HPF)



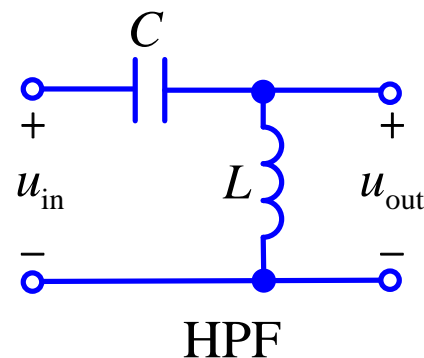
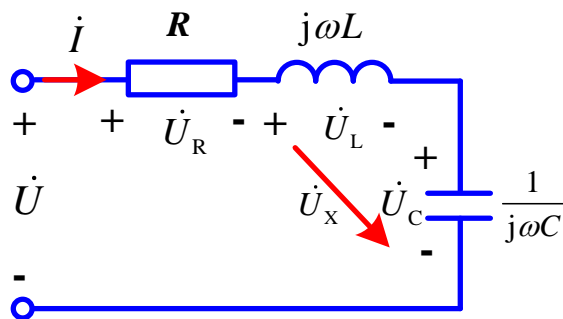
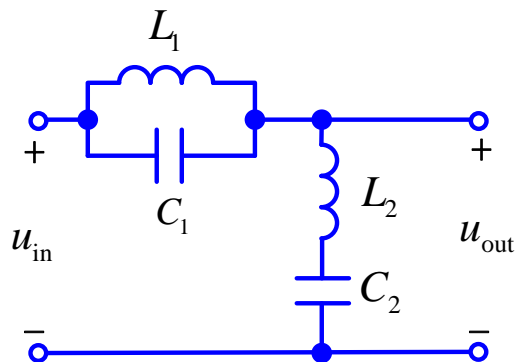
# 11.5 滤波器简介 有源滤波器 无源滤波器



## 带通滤波器 Band Pass Filter(BPF)



## 带阻滤波器 Band Reject Filter(BRF)



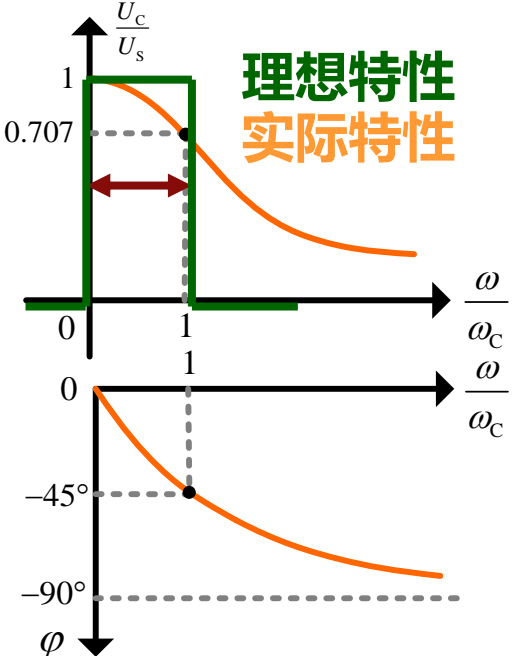
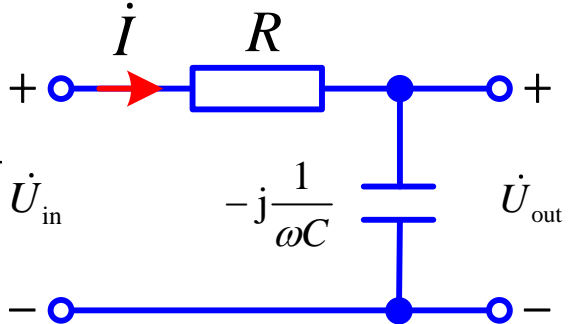
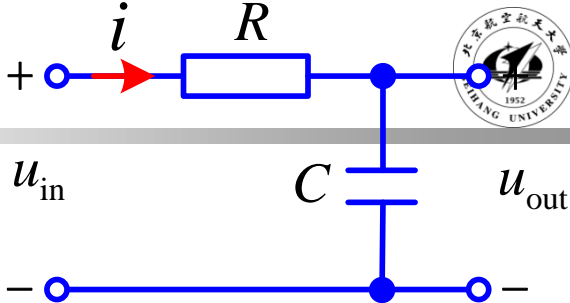
# (1) RC低通滤波器

$$\dot{U}_{out} = \frac{-j\frac{1}{\omega C}}{R - j\frac{1}{\omega C}} \dot{U}_{in} = \frac{1 - jR\omega C}{(R\omega C)^2 + 1} \dot{U}_{in}$$

$$U_{out} = \frac{1}{\sqrt{(R\omega C)^2 + 1}} U_{in} \quad |H(j\omega)| = \frac{U_{out}}{U_{in}} = \frac{1}{\sqrt{(R\omega C)^2 + 1}}$$

$$\omega_c = \frac{1}{\tau} = \frac{1}{RC} \quad |H(j\omega)| = \frac{1}{\sqrt{\left(\frac{\omega}{\omega_c}\right)^2 + 1}}$$

是一个截止频率为 $\omega_c$ 低通滤波器。



## (2) RC高通滤波器

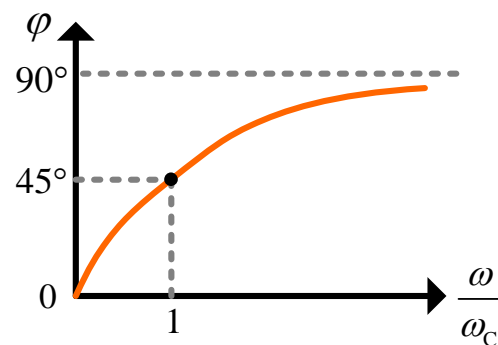
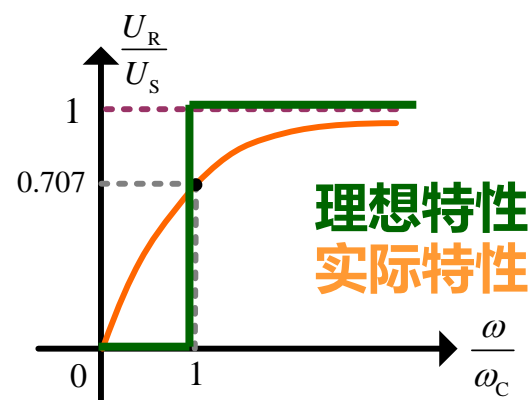
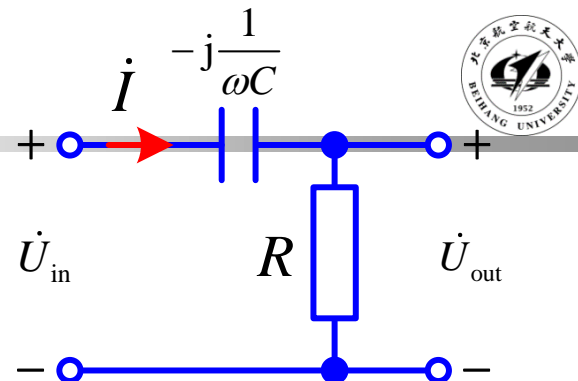
$$\dot{U}_{\text{out}} = \frac{R}{R - j\frac{1}{\omega C}} \dot{U}_{\text{in}} = \frac{R\omega C(R\omega C + j)}{(R\omega C)^2 + 1} \dot{U}_{\text{in}}$$

$$U_{\text{out}} = \frac{R\omega C}{\sqrt{(R\omega C)^2 + 1}} U_{\text{in}} = \frac{1}{\sqrt{1 + \left(\frac{1}{R\omega C}\right)^2}} U_{\text{in}}$$

$$|H(j\omega)| = \frac{U_{\text{out}}}{U_{\text{in}}} = \frac{1}{\sqrt{1 + \left(\frac{1}{R\omega C}\right)^2}}$$

$$\omega_c = \frac{1}{\tau} = \frac{1}{RC} \quad |H(j\omega)| = \frac{1}{\sqrt{\left(\frac{\omega_c}{\omega}\right)^2 + 1}}$$

是一个截止频率为 $\omega_c$ 高通滤波器。

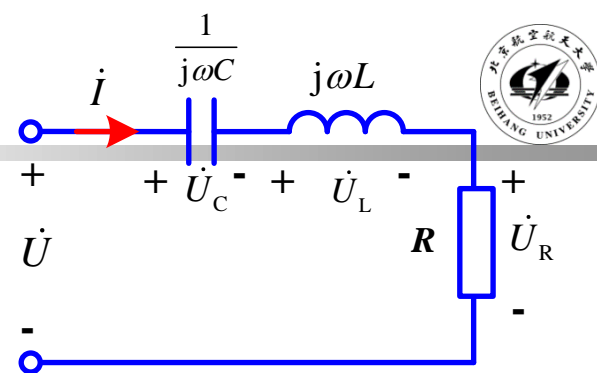
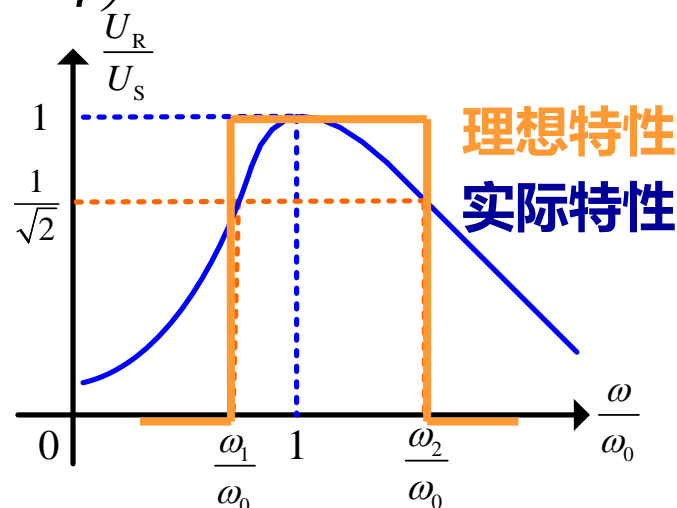
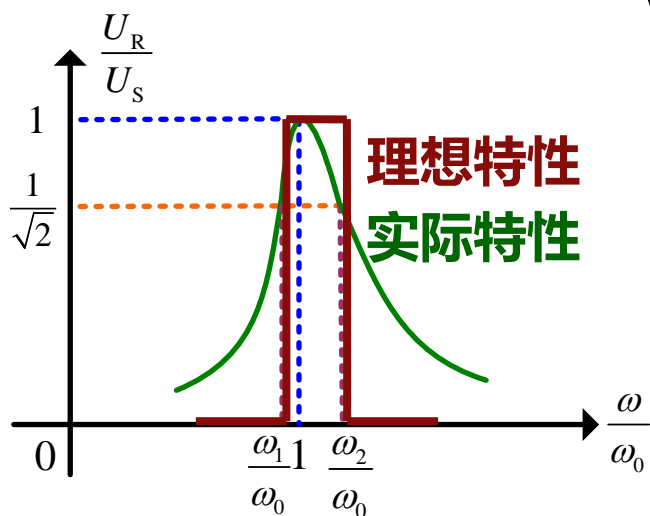




### (3) $RLC$ 带通滤波器

$$H_R(j\omega) = \frac{\dot{U}_R(j\omega)}{\dot{U}_S(j\omega)} = \frac{R}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

$$\eta = \frac{\omega}{\omega_0} \quad H(j\eta) = \frac{1}{1 + jQ\left(\eta - \frac{1}{\eta}\right)}$$



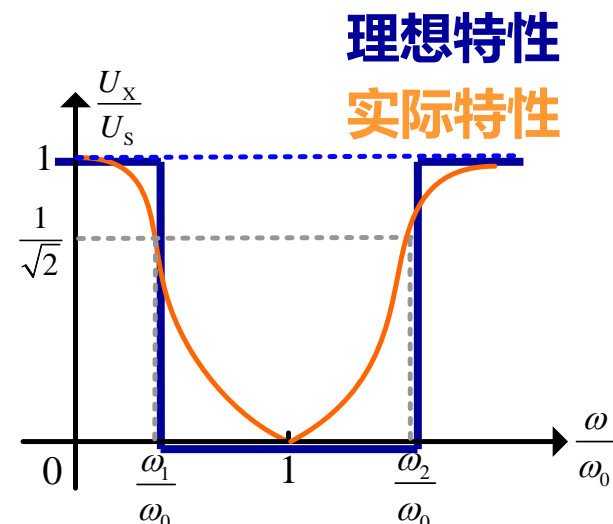
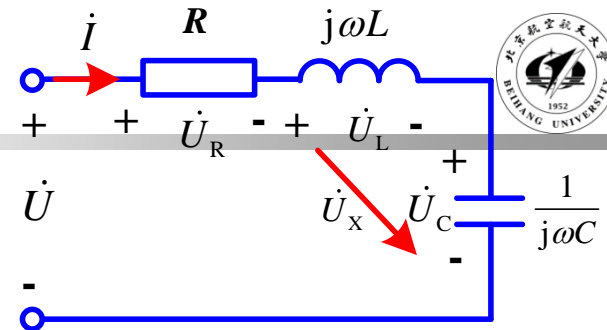
通频带为  $\omega_1, \omega_2$  之间，带通滤波器。

## (4) $RLC$ 带阻滤波器

$$H_X(j\omega) = \frac{\dot{U}_X(j\omega)}{\dot{U}_S(j\omega)} = \frac{j\left(\omega L - \frac{1}{\omega C}\right)}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

$$\eta = \frac{\omega}{\omega_0} \quad H(j\eta) = \frac{jQ\left(\eta - \frac{1}{\eta}\right)}{1 + jQ\left(\eta - \frac{1}{\eta}\right)}$$

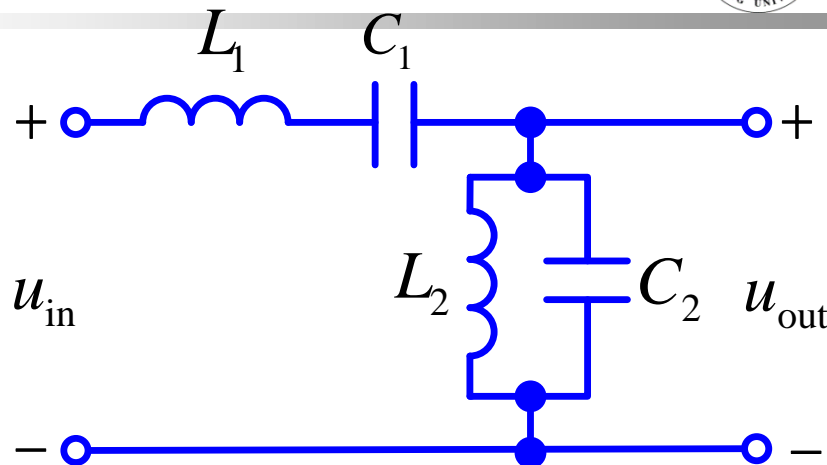
阻频带为  $\omega_1, \omega_2$  之间，带阻滤波器。



# 思考:

$u_{in}$  中有多个频率谐波信号,

希望  $u_{out}$  中只有一种  
频率信号无衰减, 分  
析电路工作状态。



$L_1 C_1$  串联谐振  
 $L_2 C_2$  并联谐振

其谐振频率是输出信号频率。