北京航空航天大學
BEIHANG UNIVERSITY

HT: Convection

L8: Introduction to convection heat transfer

Learning Objectives:

- **What is convection heat transfer.**
- **The governing equation of convection HT**
- **The boundary layer and equations**
- **Analogy theory & similarity principle & dimensional analysis**

§ 5-1 introduction to convection heat transfer

1.Definiton and Features

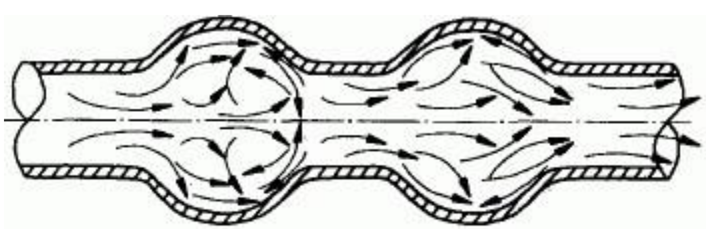
★ **Convection heat transfer：** **the heat transfer process occurs between a fluid in motion and a bounding surface when the two are at different temperatures.**

Different with heat convection：

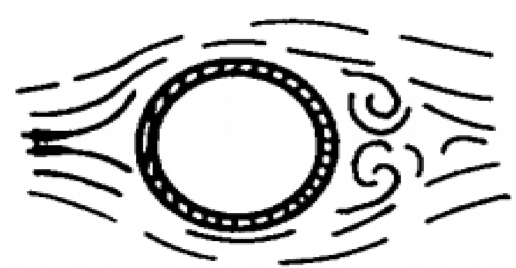
a heat conduction and convection both exist

b the flow and the surface must contact; the flow motion must occurs and there must be have temperature difference

c A boundary layer exists nearby the surface



Forced Convection HT in a tube



Natural Convection HT around a tube

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2.Newton’s cooling equation

$$\Phi = hA(t_w - t_f)[W] \quad q = \Phi/A = h(t_w - t_f)[W/m^2]$$

$\boldsymbol{\Phi}$ — heat rate [W]， the heat transfer per unit of time

$\boldsymbol{q}$ — heat flux $\left[\mathrm{W/m^2}\right]$

$\boldsymbol{A}$ — the contact area $\left[\mathrm{m^2}\right]$

t_w — the temperature of the surface of the solid $\left[{}^\circ C\right]$

t_∞ — the temperature of the fluid $\left[{}^\circ C\right]$

h — Convection heat transfer coefficient $\left[\mathrm{W/(m^2 \cdot K)}\right]$

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3. The convection heat transfer coefficient

$$h = \Phi/\left(A\left(t_w - t_f\right)\right)\,[W/(m^2 \bullet K)] \qquad \left[\mathrm{W/(m^2 \cdot {}^\circ C)}\right]$$

to represent the heat transfer per unit of time and area, when the temperature difference between fluid and solid surface at 1 Celsius degree

Methods to investigate the *h*:

（1） analytical method

（2） experimental method

（3） analogy theory method

（4） Numerical simulation method

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3. The convection heat transfer coefficient

$$h = \Phi / \left(A \left(t_w - t_f \right) \right) [W/(m^2 \bullet K)] \quad \left[\mathrm{W/(m^2 \cdot {}^\circ C)} \right]$$

to represent the heat transfer per unit of time and area, when the temperature difference between fluid and solid surface at 1 Celsius degree

Who can influence the *h*： flow velocity、physical properties、surface structure and roughness, etc。

How to determine the ***h*** and how to intensify the ***h*** is the key issues of convection heat transfer.

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4.The effect factors

those that influence fluid flow (motion) and heat conduction

Can be classified as 5 aspects：

(1)The origin of flow.

(2)Flow state.

(3)Whether phase change exist;

(4)The structural characteristics of the surface;

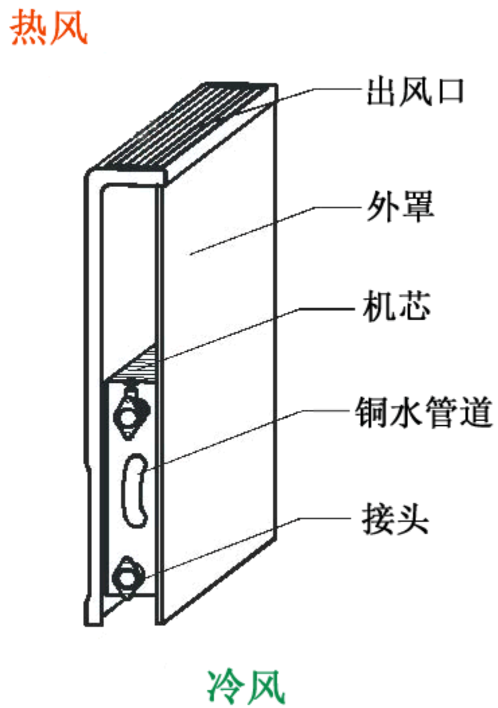
(5)Thermal properties of the fluid。

(1) The origin of the flow

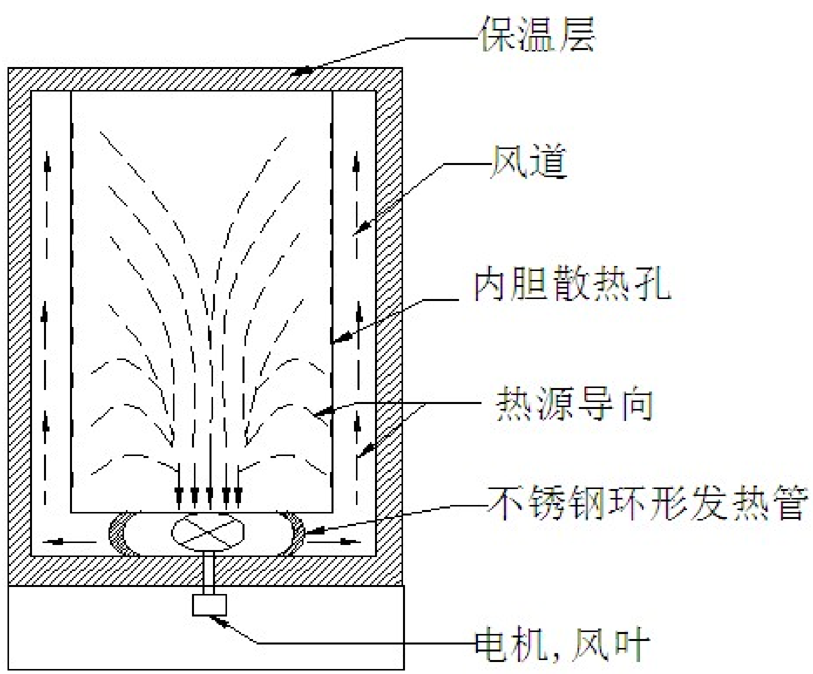
$$h_{forced} > h_{natural}$$

Natural convection： driven by the density differences due to the temperature distribution

Forced convection： due to the external force



Natural convection



Forced convection

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(2) Phase change

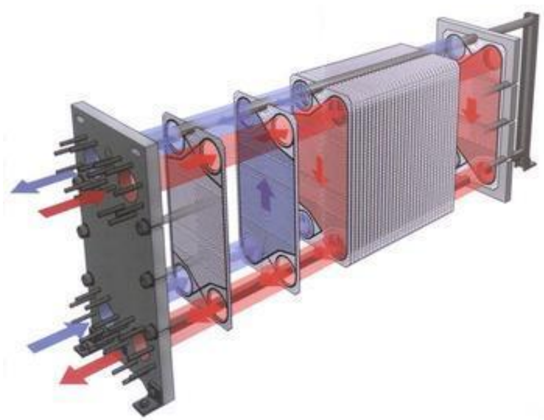
$h_{phase\ change} > h_{without\ phase\ change}$

单相传热：（**Single phase heat transfer**）

Sensible heat

相变传热：（**Phase change**）**:**condensation、boiling、Sublimation、melt, etc.

Latent heat



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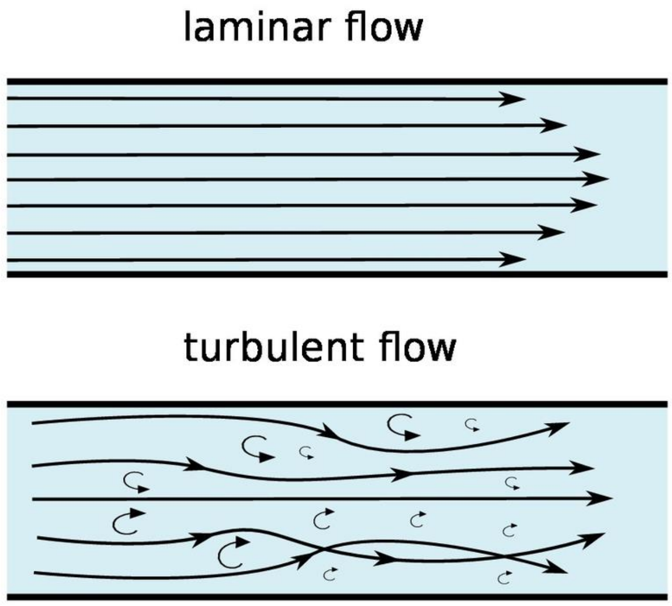
(3) Flow state

$$h_{turbulence} > h_{laminar}$$

层流（Laminar flow）： develops an insulating blanket (thermal boundary) around the channel wall and restricts heat transfer

湍流（紊流，**Turbulent flow**）： develops no insulating blanket due to the perturbations. Heat is transferred very rapidly.



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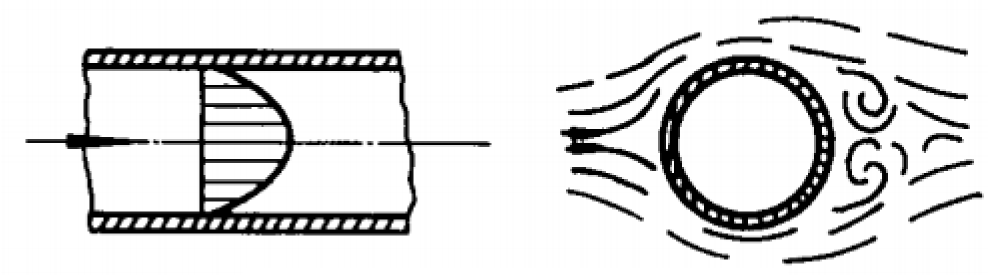
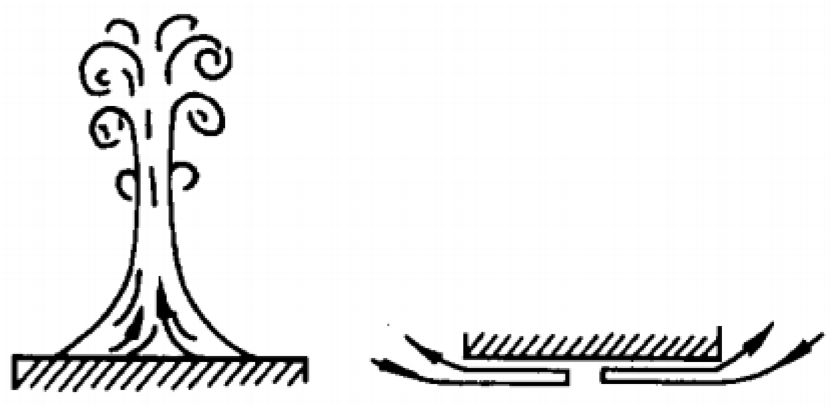
(4) Structural properties of the surface：

- ► Shape and size
- ► How to contact
- ► roughness



Internal or external



Upward or downward

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(5) Physical properties:

Thermal conductivity $\lambda\,[\mathrm{W/(m\cdot{}^\circ C)}]$

Heat capacity $c\,[\mathrm{J/(kg\cdot{}^\circ C)}]$

kinematic viscosity $\nu = \eta/\rho \quad [\mathrm{m^2/s}]$

density $\rho\,[\mathrm{kg/m^3}]$

viscosity $\eta \quad [\mathrm{N\cdot s/m^2}]$

cubic expansion coefficient $\alpha \quad [1/\mathrm{K}]$

$$\alpha = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_p = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_p$$

$\lambda \uparrow \Rightarrow h \uparrow$ *(low thermal resistance)*

ρ、$c \uparrow \Rightarrow h \uparrow$ *(more heat could transfer)*

$\eta \uparrow \Rightarrow h \downarrow$ *(negtive to flow behavior)*

$\alpha \uparrow \Rightarrow$ *(increase the natural convenction)*

Therefore:

$$h = f(\vec{v},\, t_w,\, t_f,\, \lambda,\, c_p,\, \rho,\, \alpha,\, \eta,\, l)$$

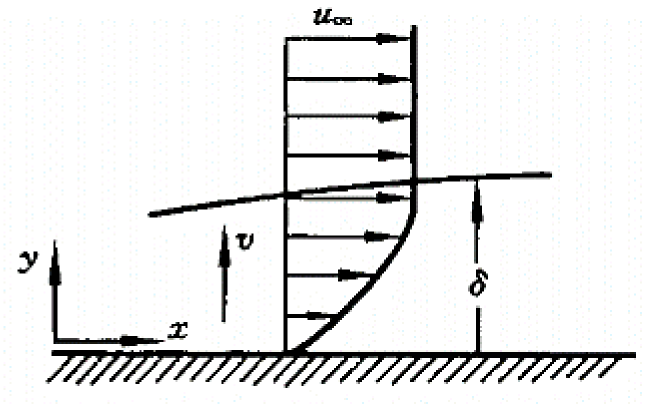
§ 5-1 introduction to convection heat transfer

5.classification

- 对流换热
  - 无相变换热
    - 受迫对流换热
      - 内部流动
        - 圆管内受迫流动
        - 非圆形管内受迫流动
      - 外部流动
        - 外掠平板
        - 外掠单管
        - 外掠管束（光管；翅片管）
    - 自然对流换热
      - 无限空间
        - 竖壁；竖管
        - 横管
        - 水平壁（上表面和下表面）
      - 有限空间——夹层空间
    - 混合对流换热（受迫对流与自然对流并存）
  - 相变换热
    - 凝结换热
      - 垂直壁凝结换热
      - 水平单圆管及管束外凝结换热
      - 管内凝结换热
    - 沸腾换热
      - 大空间沸腾换热
      - 管内沸腾换热（横管，竖管）

§ 5-1 introduction to convection heat transfer

6.How to calculate the *h*



When the viscous fluid flows on the wall, due to the viscosity, the flow velocity decreases with the distance from the wall reduces and stagnates at the wall.(y=0, u=0).

No flip boundary condition

Within such boundary layers, the heat only can be transfer by conduction

According to Fourier's law: $q_{w,x} = -\lambda \left(\frac{\partial t}{\partial y} \right)_{w,x}$ $\left[\mathrm{W/m^2} \right]$

$\lambda - thermal\ conductivity\ of\ the\ fluid [\mathrm{W/(m \cdot ℃)}]$
$(\partial t / \partial y)_{w,x} — temperature\ gradient\ of\ the\ fluid\ at\ (x,0)$

§ 5-1 introduction to convection heat transfer

$$q_{w,x} = -\lambda \left(\frac{\partial t}{\partial y} \right)_{w,x}$$

According to the **Newtonian cooling equation**:

$$q_{w,x} = h_x (t_w \text{-} t_\infty) \quad \left[\mathrm{W/m^2} \right]$$

h_x— *the local heat transfer coefficients at x* [W/（m^2 · °C)]

therefore:

$$h_x = -\frac{\lambda}{t_w - t_\infty} \left(\frac{\partial t}{\partial y} \right)_{w,x} \qquad \left[\mathrm{W/(m^2 \cdot {}^\circ C)} \right]$$

Convective heat transfer differential equation

§ 5-1 introduction to convection heat transfer

$$h_x = -\frac{\lambda}{t_w - t_\infty}\left(\frac{\partial t}{\partial y}\right)_{w,x}$$

h_x depends on thermal conductivity of **fluid**、 temperature difference and temperature gradient near the wall。

Where is the influence of velocity?

How to determine the temperature gradient?

The velocity and temperature field are determined by：

Mass conservation equation, momentum conservation equation, energy conservation equation

§ 5-2 the governing equation of convection

1.Energy conservation equation

In our lecture, only two dimensional heat transfer is considered

assume:

a) continuity hypothesis

b) incompressible flow

Newtonian flow $\tau = \eta \frac{\partial u}{\partial y}$

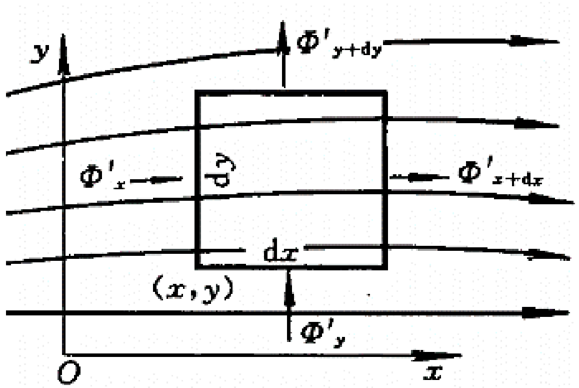
c) constant thermal properties

§ 5-2 the governing equation of convection

The energy conservation in element:

[heat increased by conduction] + [heat increased by convection] +[inner heat source energy] = [thermodynamic energy increase] + [work to external]

$$Q = \Delta E + W$$



Q — $Q_{conduction} + Q_{convection} + Q_{inner\ source}$

ΔE — $\Delta U_{thermodynamic} + \Delta U_{kinetic}$

W — due to thebody force and the gravity

assume:

(1) no external work → **W＝0**

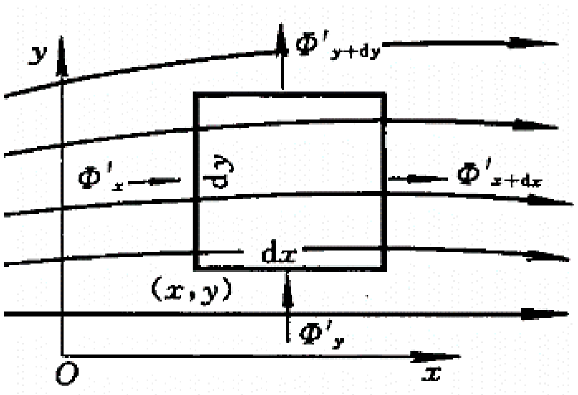
(2) no inner heat source → $Q_{\text{inner heat source}}=0$

(3) The heat of dissipation from viscous dissipation is negligible

§ 5-2 the governing equation of convection

$$Q_{conduction} + Q_{convection} = \Delta U_{thermodynamic}$$

$$Q_{conduction} = \lambda \frac{\partial^2 t}{\partial x^2} dxdy + \lambda \frac{\partial^2 t}{\partial y^2} dxdy$$



The heat increased by convection along *x* direction:

$$Q_x^{''} - Q_{x+dx}^{''} = Q_x^{''} - \left(Q_x^{''} + \frac{\partial Q_x^{''}}{\partial x} dx \right) = -\frac{\partial Q_x^{''}}{\partial x} dx = -\rho c_p \frac{\partial (ut)}{\partial x} dxdy$$

The heat increased by convection along *y* direction:

$$Q_y^{''} - Q_{y+dy}^{''} = Q_y^{''} - \left(Q_y^{''} + \frac{\partial Q_y^{''}}{\partial y} dy \right) = -\frac{\partial Q_y^{''}}{\partial y} dy = -\rho c_p \frac{\partial (vt)}{\partial y} dydx$$

§ 5-2 the governing equation of convection

$$Q_{conduction} = \lambda \frac{\partial^2 t}{\partial x^2} dxdy + \lambda \frac{\partial^2 t}{\partial y^2} dxdy$$

$$\begin{aligned} Q_{convection} &= -\rho c_p \frac{\partial (ut)}{\partial x} dxdy - \rho c_p \frac{\partial (vt)}{\partial y} dxdy \\ &= -\rho c_p \left[u \frac{\partial t}{\partial x} + v \frac{\partial t}{\partial y} + t \frac{\partial u}{\partial x} \right. \\ &= -\rho c_p \left[u \frac{\partial t}{\partial x} + v \frac{\partial t}{\partial y} \right] dxdy \end{aligned}$$

$$\Delta U = \rho c_p dxdy \frac{\partial t}{\partial \tau} d\tau$$

Energy conservation equation

$$\frac{\lambda}{\rho c_p} \left[\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} \right] = u \frac{\partial t}{\partial x} + v \frac{\partial t}{\partial y} + \frac{\partial t}{\partial \tau}$$

§ 5-2 the governing equation of convection

(2) Mass conservation equation

M **mass flow rate [kg/s]**

The mass flows in left interface along the *x* direction

$$M_x = \rho u dy$$

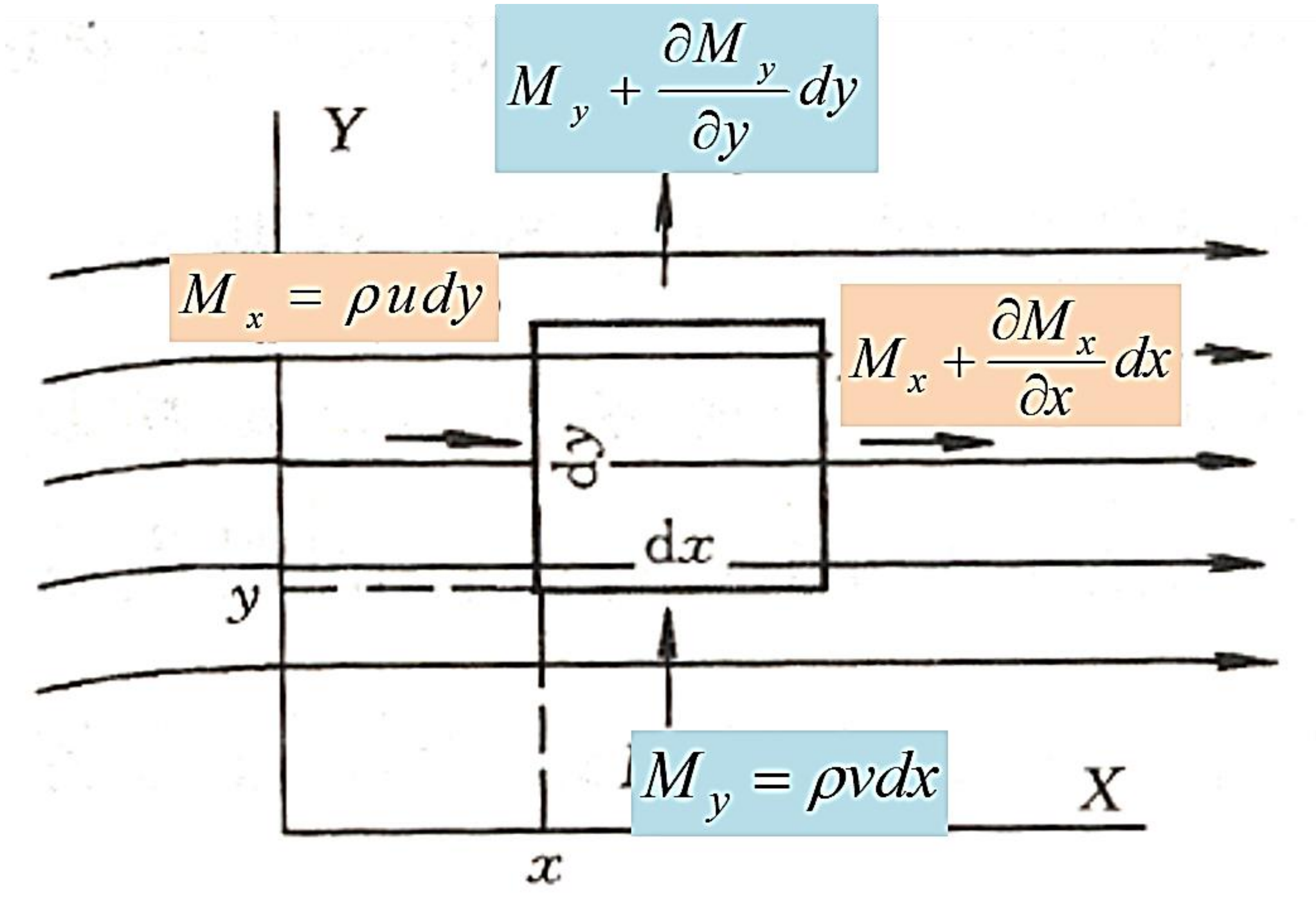
The mass flows in right interface along the *x* direction

$$M_{x+dx} = M_x + \frac{\partial M_x}{\partial x} dx$$

The mass variation along *x* direction:

$$M_x - M_{x+dx} = -\frac{\partial M_x}{\partial x} dx = -\frac{\partial (\rho u)}{\partial x} dxdy$$

§ 5-2 the governing equation of convection



流体微元质量守恒示意图

§ 5-2 the governing equation of convection

The mass variation along *y* direction:

$$M_y - M_{y+dy} = -\frac{\partial M_y}{\partial y}dy = -\frac{\partial(\rho v)}{\partial y}dxdy$$

The mass variation within the element:

$$\frac{\partial(\rho dxdy)}{\partial \tau} = \frac{\partial \rho}{\partial \tau}dxdy$$

Mass conservation within the element: **(per unit of time)**

Flows in – Flows out = mass variation

$$-\frac{\partial(\rho u)}{\partial x}dxdy - \frac{\partial(\rho v)}{\partial y}dxdy = \frac{\partial \rho}{\partial \tau}dxdy$$

§ 5-2 the governing equation of convection

$$-\frac{\partial(\rho u)}{\partial x}dxdy-\frac{\partial(\rho v)}{\partial y}dxdy=\frac{\partial\rho}{\partial\tau}dxdy$$

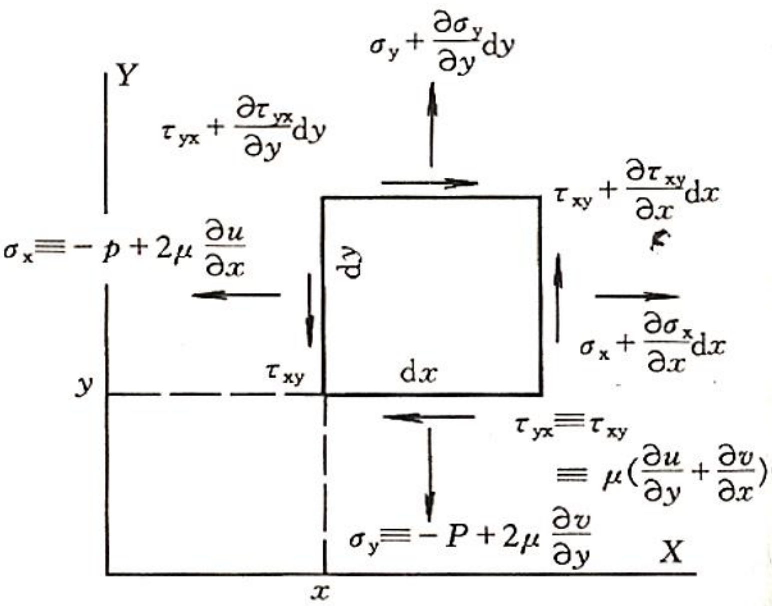
Two dimensional continuity equation:

$$\frac{\partial\rho}{\partial\tau}+\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}=0$$

Steady-state and constant properties：

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

(3)Momentum conservation equation



动量守恒方程推导中的微元体

Force = m ×a（*F=ma*）

Force：body force、surface force

body: **gravity、centrifugal force**

surface： **pressure, viscous stress**

§ 5-2 the governing equation of convection

动量微分方程 — Navier-Stokes方程（N-S方程）

$$\rho\left(\frac{\partial u}{\partial \tau}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=F_x-\frac{\partial p}{\partial x}+\eta\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)$$

$$\rho\left(\frac{\partial v}{\partial \tau}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=F_y-\frac{\partial p}{\partial y}+\eta\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)$$

(1) (2) (3) (4)

(1)— inertial force；(2) — body force；(3) — surface force；(4) — viscous force

steady： $$\frac{\partial u}{\partial \tau}=0; \quad \frac{\partial v}{\partial \tau}=0$$

Only gravity： $$F_x=\rho g_x; \quad F_y=\rho g_y$$

§ 5-2 the governing equation of convection

Governing equation of convection heat transfer:(constant、no inner heat source、2D、incompressible fluid)

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

$$\rho\left(\frac{\partial u}{\partial \tau}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=F_x-\frac{\partial p}{\partial x}+\eta\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)$$

$$\rho\left(\frac{\partial v}{\partial \tau}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=F_y-\frac{\partial p}{\partial y}+\eta\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)$$

$$\rho c_p\left(\frac{\partial t}{\partial \tau}+u\frac{\partial t}{\partial x}+v\frac{\partial t}{\partial y}\right)=\lambda\left(\frac{\partial^2 t}{\partial x^2}+\lambda\frac{\partial^2 t}{\partial y^2}\right)$$

§ 5-2 the governing equation of convection

4 equaitons，4 variables. Get velovity and temperature distribution and conduct Newton cooling equation:

$$h_x = -\frac{\lambda}{\Delta t}\left(\frac{\partial t}{\partial y}\right)_{w,x}$$

We can calculate the local h_x

§ 5-2 the governing equation of convection

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\rho\ (\frac{\partial u}{\partial \tau} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}) = F_x - \frac{\partial p}{\partial x} + \eta(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2})$$

$$\rho\ (\frac{\partial v}{\partial \tau} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}) = F_y - \frac{\partial p}{\partial y} + \eta(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2})$$

$$\rho c_p\left(\frac{\partial t}{\partial \tau} + u\frac{\partial t}{\partial x} + v\frac{\partial t}{\partial y}\right) = \lambda\left(\frac{\partial^2 t}{\partial x^2} + \lambda\frac{\partial^2 t}{\partial y^2}\right)$$

$$h_x = -\frac{\lambda}{\Delta t}\left(\frac{\partial t}{\partial y}\right)_{w,x}$$

§ 5-2 the governing equation of convection

1、geometrical conditions

Indicate the size and shape…

如：平壁或圆筒壁；厚度、直径等。

2、Physical properties

Indicates the parameters like specific heat, thermal conductivity and dense…说明导热体的物理特征

3、time conditions　说明在时间上导热过程进行的特点

- **Steady-state— independent with time**
- For transient state:

时间条件又称为**初始条件**　（Initial conditions）

§ 5-2 the governing equation of convection

4) Boundary conditions

a fixed temperature

given the temperature of the interface (fluid and solid)

b fixed heat flux

given the heat flux of the interface (fluid and solid)