北京航空航天大學
BEIHANG UNIVERSITY

飞行力学
Flight Mechanics

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Chapter 2

- Static performance

  Horizontal flight,

  climbing and descending flight,

  Range and endurance.

- Dynamic performance

  Takeoff,

  Landing,

  ……

Contents

- Introduction climbing and descending flight
- Equations of straight and symmetric flight
- Climbing flight
- Gliding flight
- Example

Questions

Aircraft performance

How fast can an aircraft climb?

How steep can an aircraft climb?

J-20 Climb

Equation of Motion

General equation for symmetric flight

$$\parallel V:\ T\cos\alpha_T - D - W\sin\gamma = \frac{W}{g}\frac{dV}{dt}$$

$$\perp V:\ L - W\cos\gamma + T\sin\alpha_T = \frac{W}{g}V\frac{d\gamma}{dt}$$

Equation of Motion

Steady straight-line, symmetric flight

$\mathrm{d}V/dt = 0, \mathrm{d}\gamma/dt = 0$, $\beta = 0$, $\mathrm{C} = 0$, $(\alpha + \varphi) \approx 0$

$$T = D + W \sin \gamma$$

$$L = W \cos \gamma$$

Equation of Motion

Assumptions

γ is small. Available thrust T_a is used for climbing.

$$T = D + W\sin\gamma$$

$$L = W\cos\gamma$$

$$T_a \approx T_R + W\sin\gamma$$

$$L \approx W$$

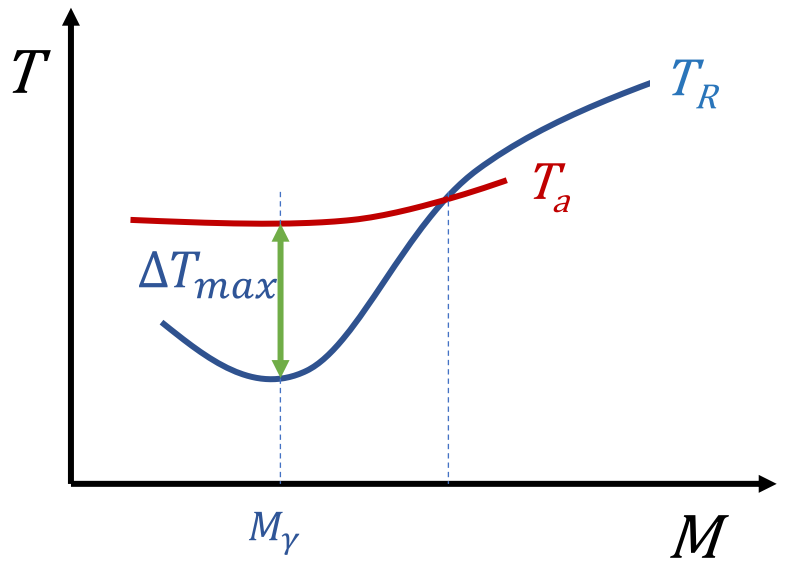
Equation of Motion

The maximum flight-path angle γ_{max}

$$\gamma = \arcsin\frac{\Delta T}{W}$$

$$\gamma_{\max} = \arcsin\frac{\Delta T_{max}}{W}$$

$$\gamma_{\max} \Leftrightarrow \Delta T_{max} \Leftrightarrow M_{\gamma}$$



Steepest climb speed（最陡上升速度）: $V_{\gamma} = cM_{\gamma}$

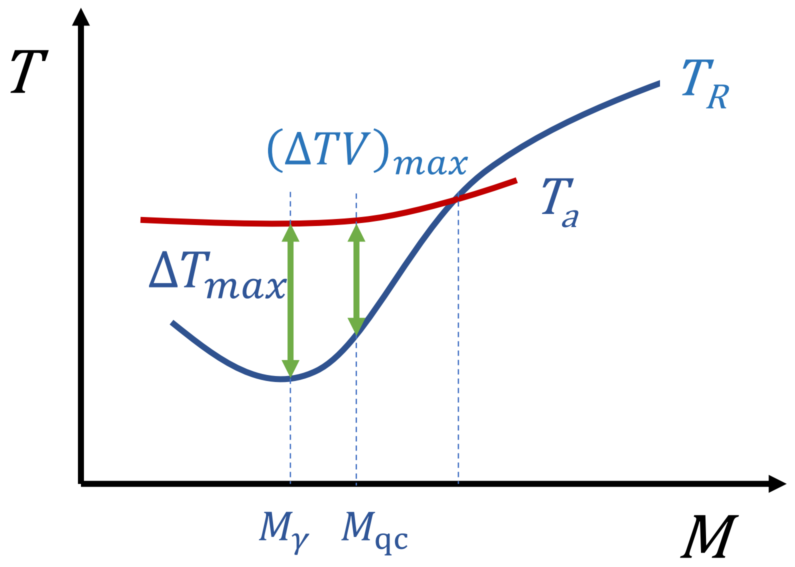
Climbing and Descending

Rate of climb（上升率） RC

ΔTV: Excess power（剩余功率）

$$RC = \frac{dH}{dt} = V\sin\gamma = \frac{\Delta TV}{W}$$

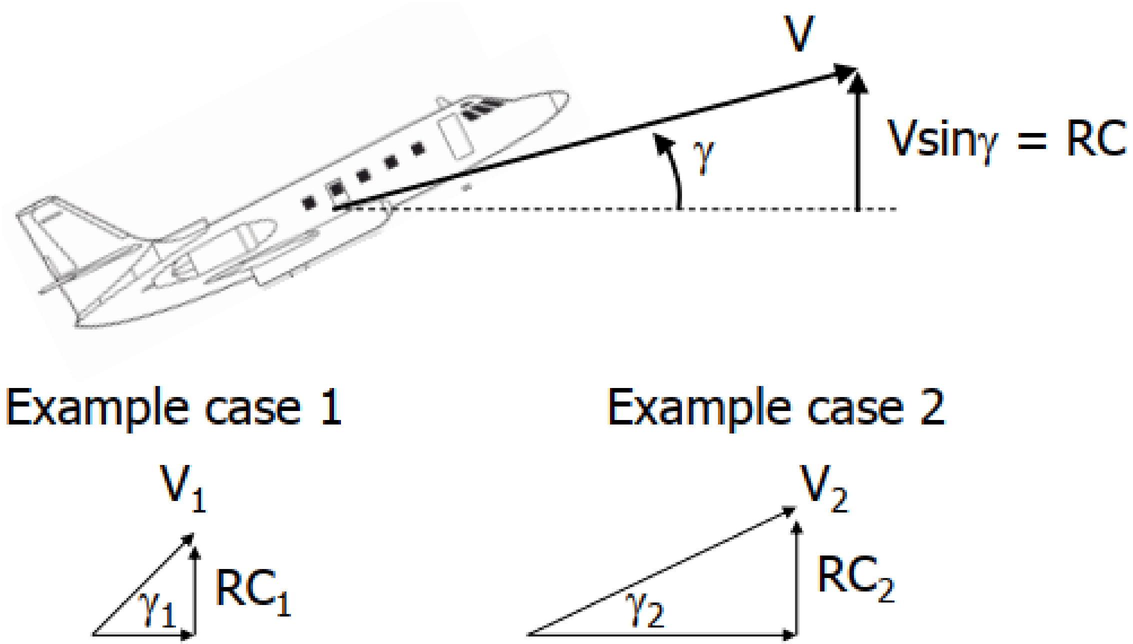
$$RC_{\max} = \frac{(\Delta TV)_{max}}{W}$$



Quick climb speed（快升速度）: $V_{qc} = cM_{qc}$

Climbing and Descending

Rate of climb and flight-path angle



$$\gamma_1 > \gamma_1$$

$$RC_1 < RC_2$$

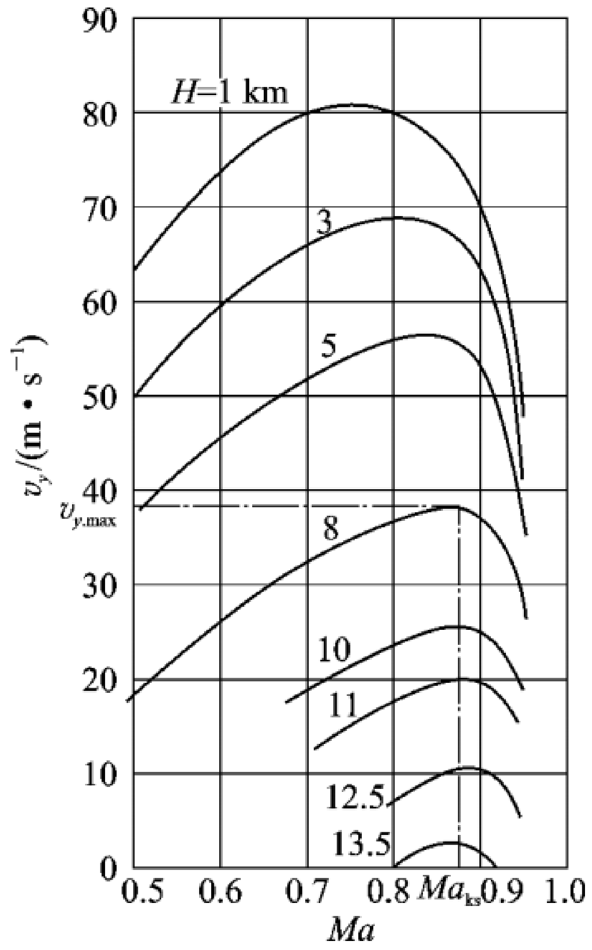
Climbing and Descending

Calculate the rate of climb RC

Example in the Page 43 of Textbook.

1) Given a Ma number ($V = c \cdot Ma$)
2) Calculate excess Thrust ΔT
3) Calculate $\sin \gamma = \Delta T / W$
4) Calculate RC = $\Delta T V / W$

RC as a function of Ma and H →



As H increases, $RC_{\max}$ decreases and V_{qc} increases.

Climbing and Descending

Theoretical static ceiling（理论静升限）： $H_{max,a}$

Explanation in the Page 45 of Textbook.

$$H_{max,a} \Leftrightarrow RC_{\max} = 0$$

1) The height $\mathrm{H}_{\mathrm{max,a}}$ can’t reached by steady straight-line flight
2) The aircraft is hard to maintain stable at $\mathrm{H}_{\mathrm{max,a}}$

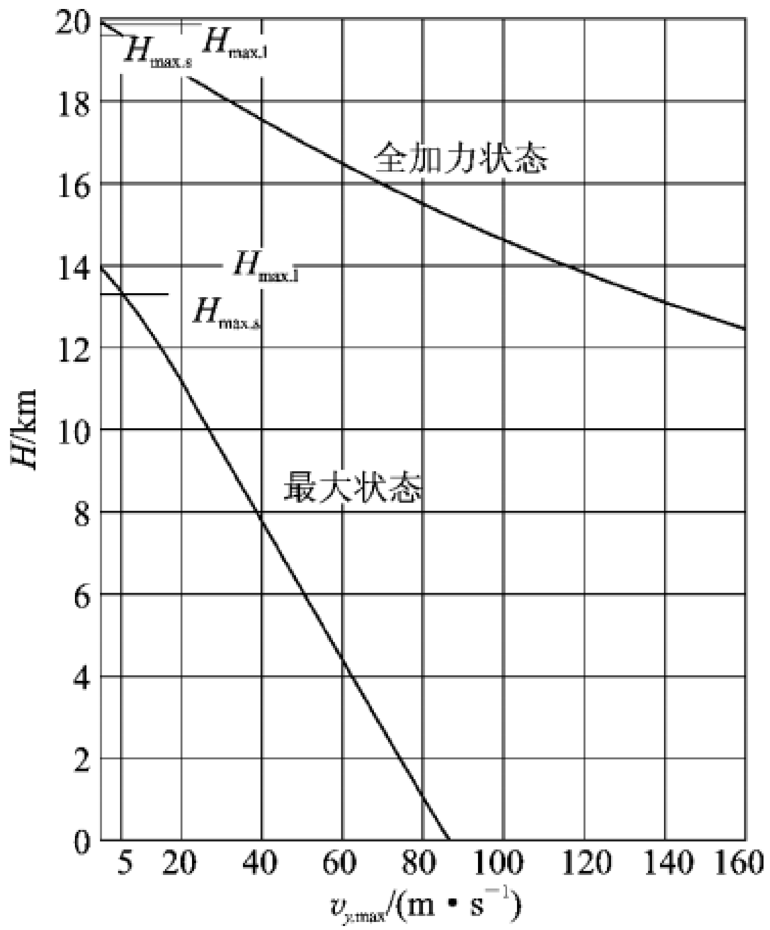
Climbing and Descending

Practical static ceiling（实用静升限）

$H_{max,s}$ is defined as the height when:

1) $RC_{\max} = 5$m/s for supersonic aircraft;

2) $RC_{\max} = 0.5$m/s for subsonic aircraft.

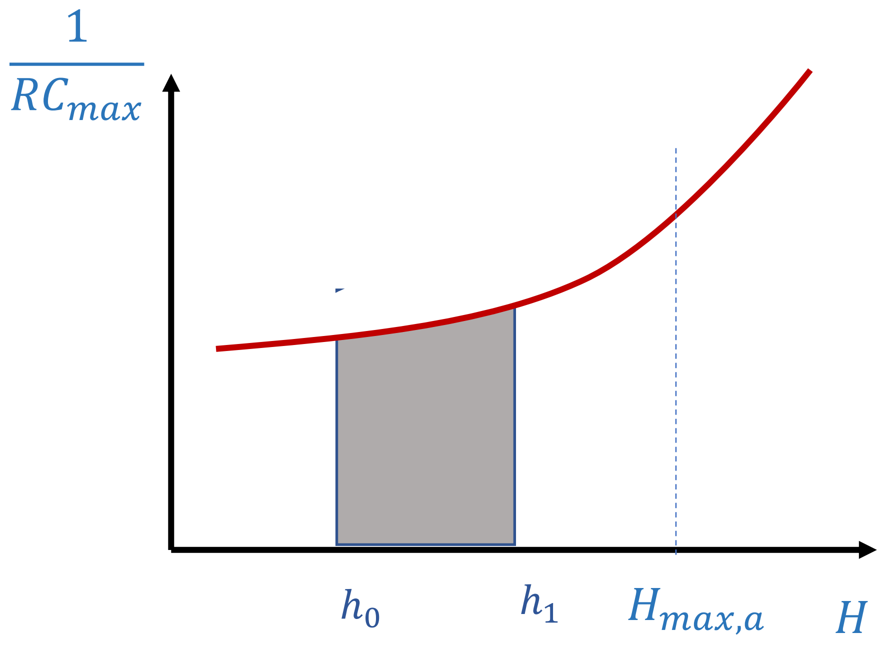


Climbing and Descending

The minimum climb time

$$dt = \frac{dH}{RC}$$

$$t_{c,min} = \int_{h0}^{h1} \frac{dH}{RC_{max}}$$



RC_{max} corresponding to V_{qc}（快升速度）.

Climbing and Descending

Horizontal distance during climb

$$R_c = \int_{h0}^{h1} \cot\gamma \, dH$$

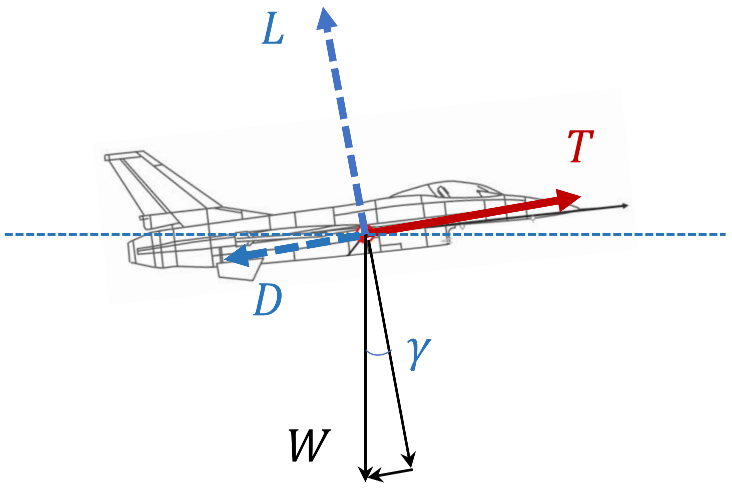
$$\gamma = \arcsin\frac{\Delta T}{W}$$

Climbing and Descending

Unsteady climb

$$\parallel V:\ T - D - W\sin\gamma = \frac{W}{g}\frac{dV}{dt}$$

$$\perp V:\ L = W$$



When is γ small, $\mathrm{W}\cos\gamma \approx W$. However, since W is typically far larger than T or D, thus $\mathrm{W}\sin\gamma$ cannot be ignored!

Climbing and Descending

Rate of climb for unsteady climb

Steady rate of climb

$$RC = \frac{dH}{dt} = \frac{\Delta TV}{W}\frac{1}{1+\frac{1}{2g}\frac{dV^2}{dH}} = RC^*\chi$$

$$\chi = \frac{1}{1+\frac{1}{2g}\frac{dV^2}{dH}} = \frac{1}{1+\frac{V}{g}\frac{dV}{dH}} \longrightarrow$$

Correction factor

Climbing and Descending

Rate of climb for unsteady climb

$$RC = \frac{dH}{dt} = \frac{\Delta TV}{W} \frac{1}{1 + \frac{1}{2g}\frac{dV^2}{dH}}$$

$$\Rightarrow \frac{P_a - P_R}{W} = \frac{dH}{dt} + \frac{1}{2g}\frac{dV^2}{dt}$$

= Potential energy + kinetic energy increases

Climbing and Descending

Maximum Rate of climb

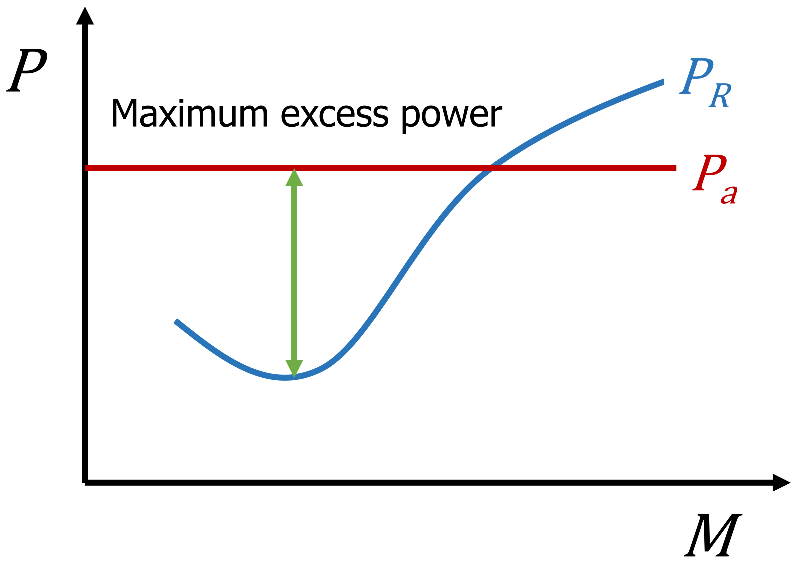
$$\frac{P_a - P_R}{W} = \frac{dH}{dt} + \frac{1}{2g}\cancelto{0}{\frac{dV^2}{dt}}$$

All the excess power is used for climb

$$\Rightarrow \frac{P_a - P_R}{W} = \frac{dH}{dt} = RC$$

Climbing and Descending

Assumption:



$$\frac{P_a - P_R}{W} = RC$$

$$RC_{max} \Leftrightarrow P_{R,min}$$

$$P_{R,min} \Rightarrow \left(\frac{C_L^3}{C_D^2}\right)_{\max} \Rightarrow C_L = \sqrt{3C_{D0}\pi\lambda_e}$$

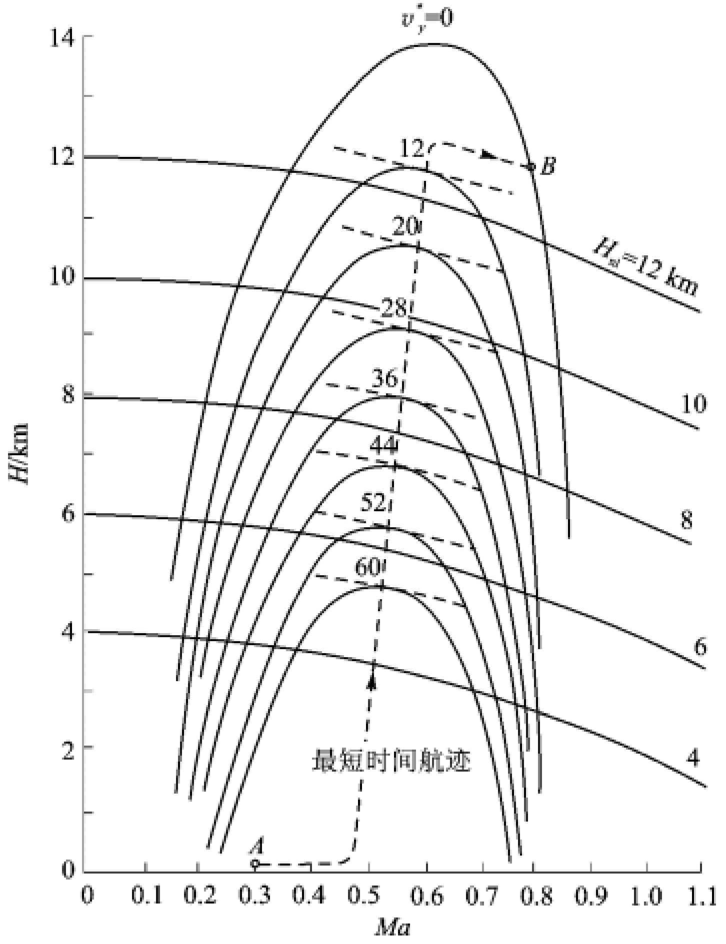
P_a is independent of speed (Propeller engine)

Climbing and Descending

Climb time and the fast climb path

$$t_c = \int_{h0}^{h1} \frac{dH}{RC} = \int_{h0}^{h1} \frac{dH_e}{RC_{max}^*}$$

Page 48 of Textbook



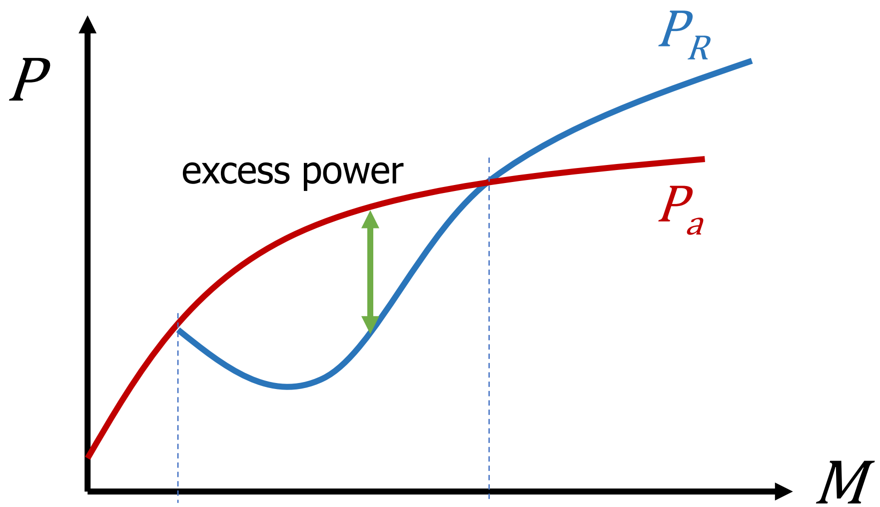
Summary

Climbing performance

$$\frac{P_a - P_R}{W} = V \sin\gamma + \frac{1}{2g}\frac{dV^2}{dt}$$

$$L = W$$

= (potential energy + kinetic energy increases)



Example

Climbing performance of the Beach King Air

Two engine propeller aircraft

$C_D = C_{D0} + kC_L^2$

$C_{D0} = 0.02$,

$k = 0.04$,

$W = 60$ [kN],

$S = 28.2$ [m^2] .

Maximum power available (741 kW) can be assumed independent of airspeed. The aircraft is performing a steady symmetrical climb

Question: What is the maximum rate of climb of this aircraft at sea level ($\rho = 1.225$ [kg/m^3]) and what is the corresponding airspeed

Climbing and Descending

Steady straight-line gliding, symmetric flight

$\mathrm{d}V/dt = 0, \mathrm{d}\gamma/dt = 0, T = 0$, $\beta = 0$, $\mathrm{C} = 0$, $(\alpha + \varphi) \approx 0$

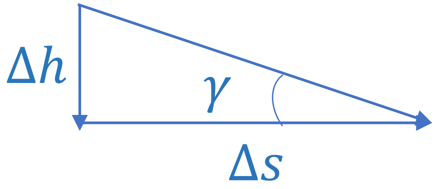
$$\boxed{\begin{aligned} \mathrm{D} &= -W \sin\gamma \\ L &= W \cos\gamma \end{aligned}}$$

Climbing and Descending

Glide ratio (滑翔比)

Glide ratio is defined as the ratio of the distance forwards to downwards during the descending period.

$$\varepsilon = \frac{\Delta s}{\Delta h}$$



The angle of descend

$$\gamma = \tan^{-1}\frac{D}{L} = \tan^{-1}\frac{C_D}{C_L} = \tan^{-1}\frac{1}{K}$$

Glide ratio equals to lift-to-drag ratio K for steady straight-line flight.

Climbing and Descending

Glide ratio（滑翔比）

Glide ratio is an important indicator for aerodynamic efficiency. The glide ratio for typical aircrafts are:

Aircraft	Glide ratio
Cessna 172	10 : 1
Airbus 320	17 : 1
Glider (ASW 22)	60 : 1

Cessna 172


Airbus 320


ASW 22

Climbing and Descending

Gliding flight discussion

$$\frac{P_a - P_R}{W} = \frac{dH}{dt} + \frac{1}{2g}\frac{dV^2}{dt}$$

Assume $P_a = 0$ $\Rightarrow$ $$-P_R = W\frac{dH}{dt} + \frac{W}{2g}\frac{dV^2}{dt}$$

Climbing and Descending

Gliding flight discussion

$$\boxed{-P_R = \mathrm{W}\frac{dH}{dt} + \frac{W}{2g}\frac{dV^2}{dt}}$$

1) Gliding at horizonal plane (dH/dt=0):

$$-P_R = \frac{W}{2g}\frac{dV^2}{dt} \Longrightarrow \text{Aircraft decelerates}$$

2) Gliding at a constant speed:

$$-P_R = \mathrm{W}\frac{dH}{dt} = WV\sin\gamma$$

Climbing and Descending

Best gliding performance

Option 1: RC_{min} (Long time)

$$-P_R = WV\sin\gamma \Rightarrow RC_{min}\ at\ \ P_{R,min}$$

Option 2: γ_{min} (Long distance)

$$-DV = WV\sin\gamma \Rightarrow \gamma_{min}\ at\ \ D_{min}$$

Conclusion:

1. To glide as **far** as possible, one must glide at the condition for minimum drag (e.g. useful for engine failure)

2. To glide as **long** (time wise) as possible, one must glide at the condition for minimum power required (e.g. useful for glider)

Climbing and Descending

Gliding as long as possible

$$-P_R = WV\sin\gamma \Rightarrow |RC| = \frac{P_R}{W} \qquad P_R = W\sqrt{\frac{W}{S}\frac{2}{\rho}\frac{C_D^2}{C_L^3}}$$

Minimum rate of descend at $P_{R,min}$:

$$P_{R,min} \Rightarrow \left(\frac{C_L^3}{C_D^2}\right)_{\max} \Rightarrow C_L = \sqrt{3C_{D0}\pi\lambda_e}$$

Climbing and Descending

Gliding as far as possible ($\gamma \rightarrow \gamma_{min}$)

$$\boxed{\begin{aligned} \mathrm{D} &= -W\sin\gamma \\ L &= W\cos\gamma \end{aligned}}$$

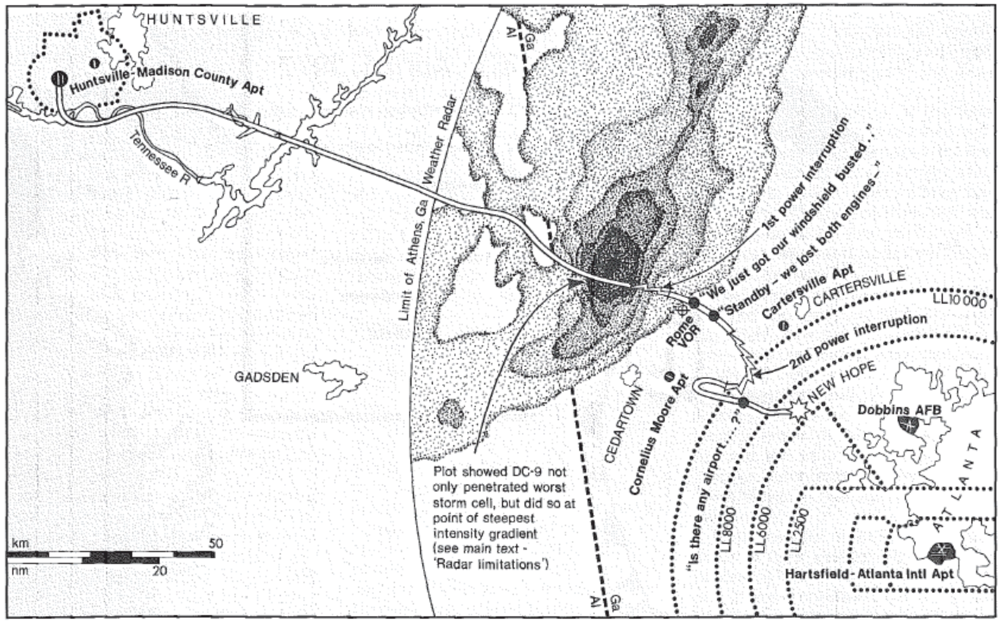
$$\gamma_{min} = \tan^{-1}\frac{1}{K_{max}}$$

$$K_{max} = \left(\frac{C_L}{C_D}\right)_{max} \Rightarrow C_L = \sqrt{C_{D0}\pi\lambda_e}$$

Question: Does aircraft weight influence the glide angle ?

Climbing and Descending

Story: An aircraft lost both engines



Larger scale map of northern Alabama and Georgia, showing track flown by N1335U from Huntsville to accident site, as determined by investigators from radar plots and eyewitness sightings. Some published accounts of this accident have speculated that the seemingly inexplicable turn back towards the west might have been the result of the crew's sighting of Cornelius Moore Airport through breaks in the rain and cloud as they descended. Loss of visual contact, or a sudden realisation of the airport's unsuitability have similarly been held as the reason for the further course reversal back towards the southeast. The second interruption to the DC-9's electrical power at this time, however, obliterated any evidence there might have been on the CVR to support this theory. The stippling shows areas of storm activity recorded by National Weather Service radar at the time of the aircraft's total loss of engine power, its density indicating the estimated intensity of precipitation. (Matthew Tesch, with acknowledgement to NTSB)

Climbing and Descending

Story: An aircraft lost both engines

Question: Could the aircraft have made it to Dobbins Air Force Base if the pilots had decided to glide there?

W = 90,000 lb (= 400500 N)

S = 1001 sq ft (=93 m^2)

b = 93.3 ft (=28.4 m)

C_{D0} = 0.02

e = 0.85

Distance to Dobbins AFB: 20 nautical miles (=36 km)

Altitude: 7000 ft (=2134 m)