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飞行力学
Flight Mechanics

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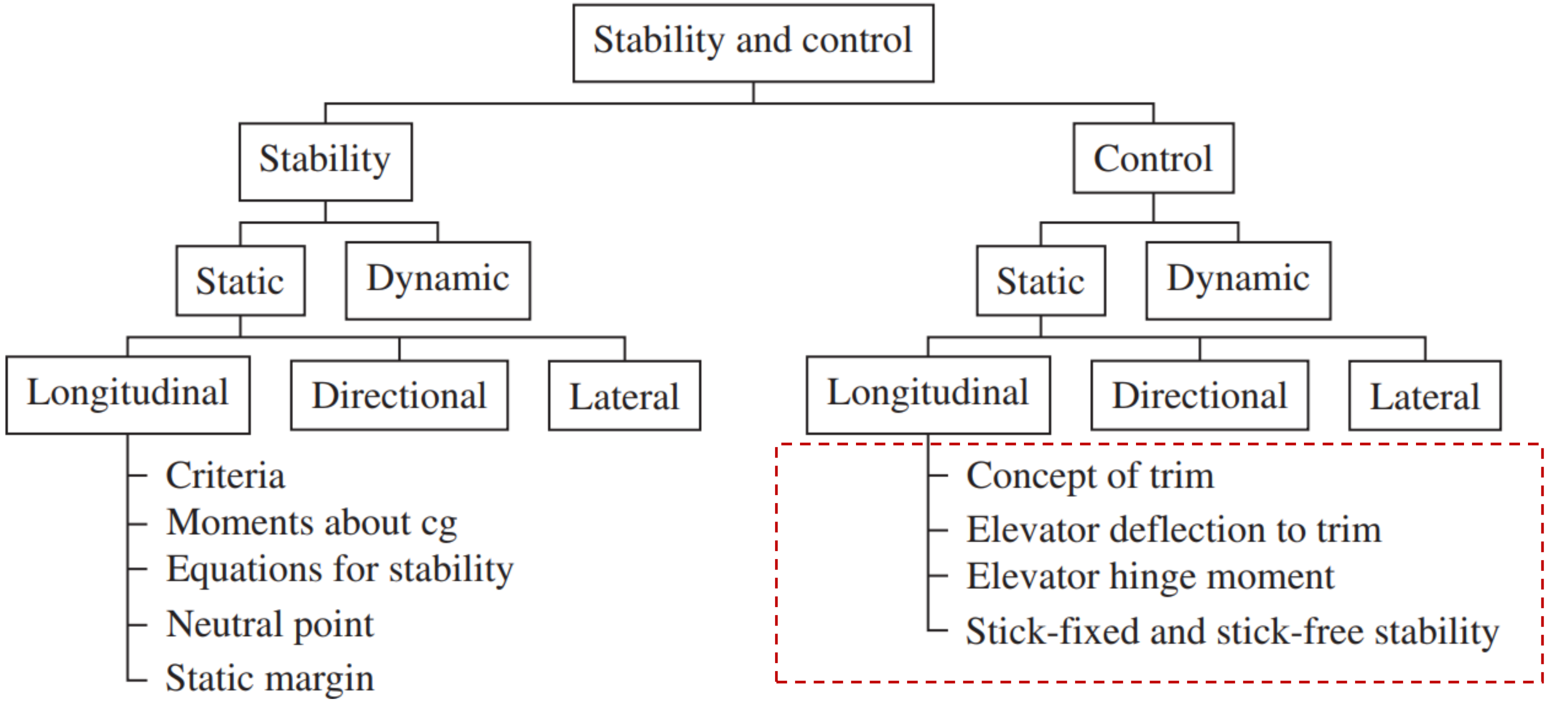
Contents

- Concept of trim
- Elevator angle to trim
- Elevator hinge moment
- Stick-free longitudinal static stability
- Directional static stability
- Lateral static stability

Introduction



Road map for Stability and Control.

Introduction

Questions

- What are the important things the pilot need to bear in mind?
- Why flight control is necessary?

Introduction

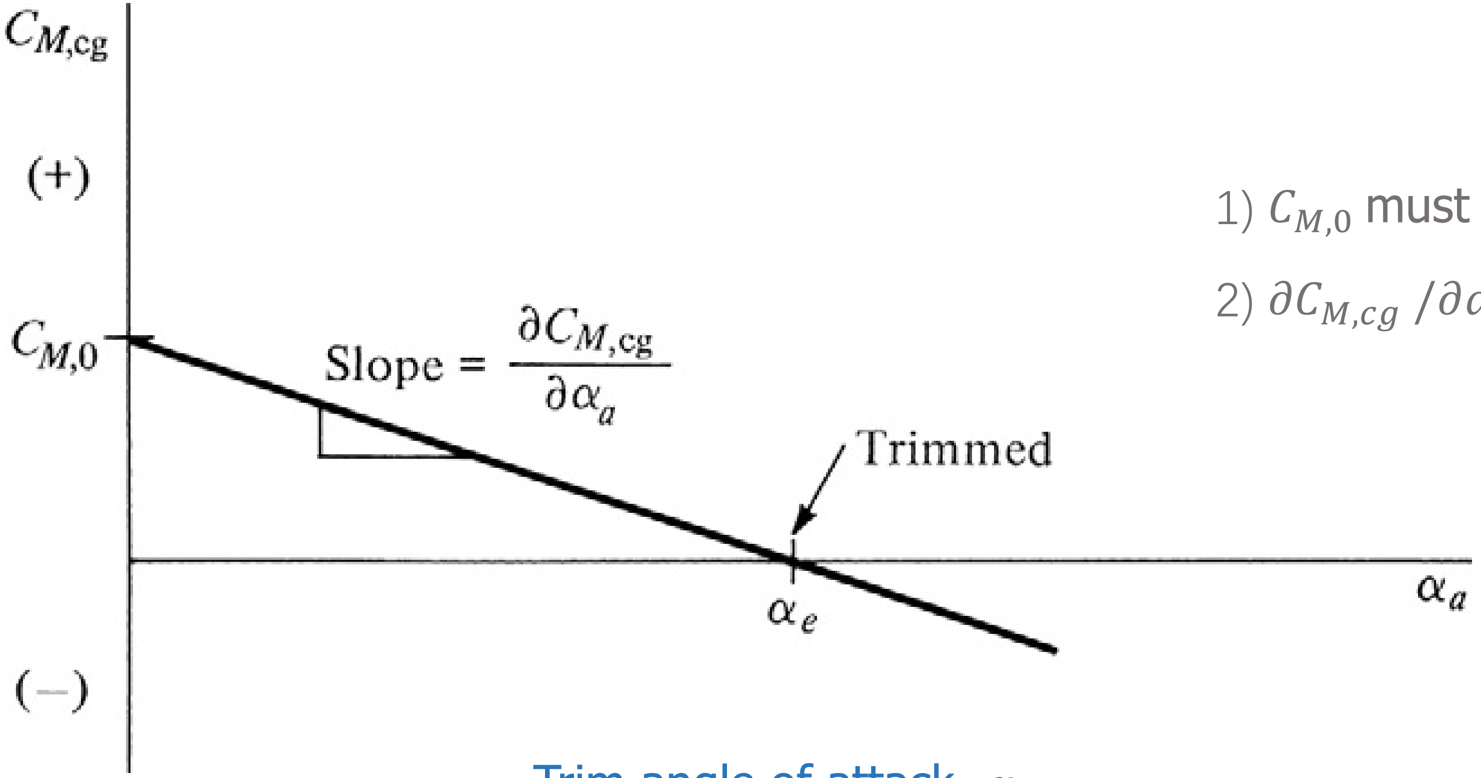
Two different philosophies for stability

Inherent stability → statically stable aircraft → aircraft tends to right itself without pilot → Hard to maneuver

Discard Inherent stability → statically unstable aircraft → Pilot control → High maneuverability

If you were the engineer and pilot during the first decade of powered flight, what will you choose ?

Concept of trim



Trim angle of attack, α_e

1) $C_{M,0}$ must be positive

2) $\partial C_{M,cg} / \partial \alpha_a$ must be negative

Concept of trim

Problem:

$$V_{\text{trim}} = \sqrt{\frac{2W}{\rho_\infty S C_{L_{\text{trim}}}}}$$

$$L = W$$

$$C_L \tfrac{1}{2} \rho V^2 S = W$$

$$V = \sqrt{\frac{W}{S}\frac{2}{\rho}\frac{1}{C_L}}$$

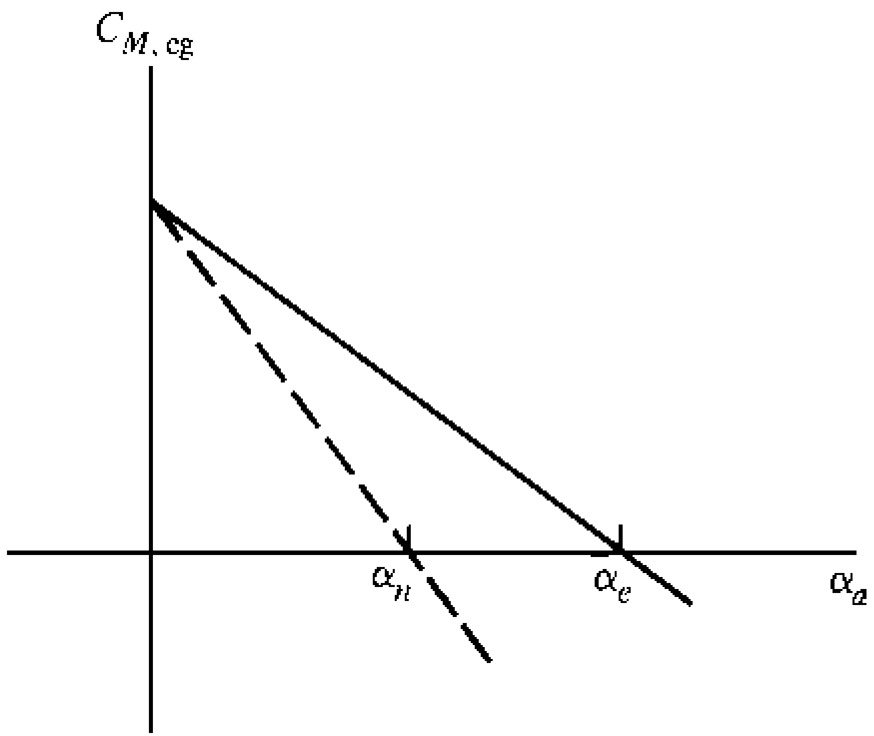
$$V_{\min} = \sqrt{\frac{W}{S}\frac{2}{\rho}\frac{1}{C_{L_{\max}}}}$$

(From lecture 4)

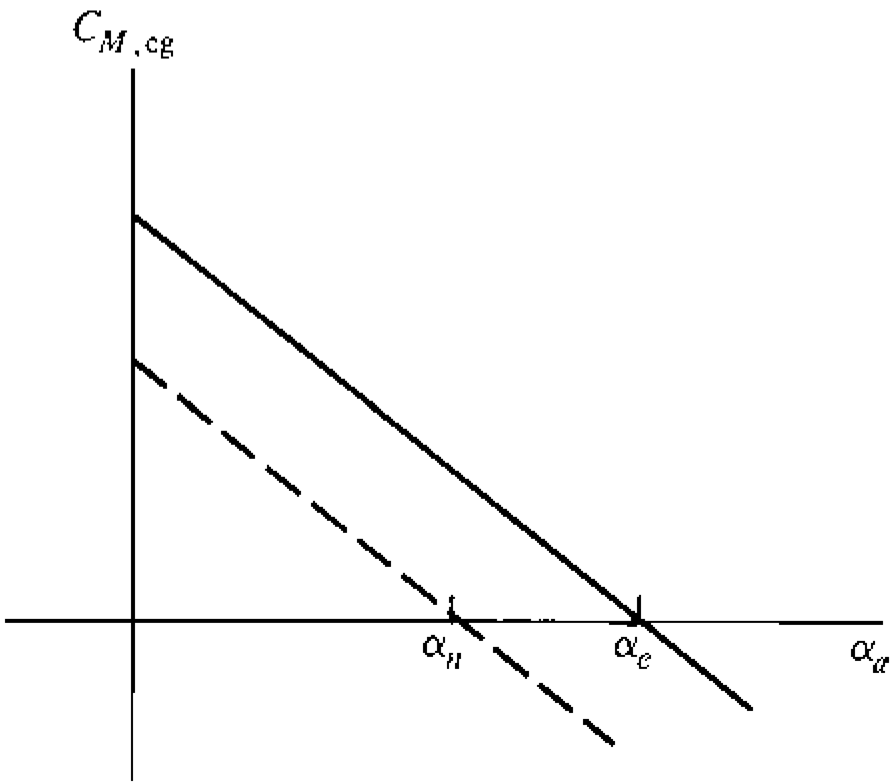
If V increase, C_L has to decrease to balance the weight, corresponding to a smaller α. Then, the airplane is no longer in equilibrium.

Concept of trim

Solution



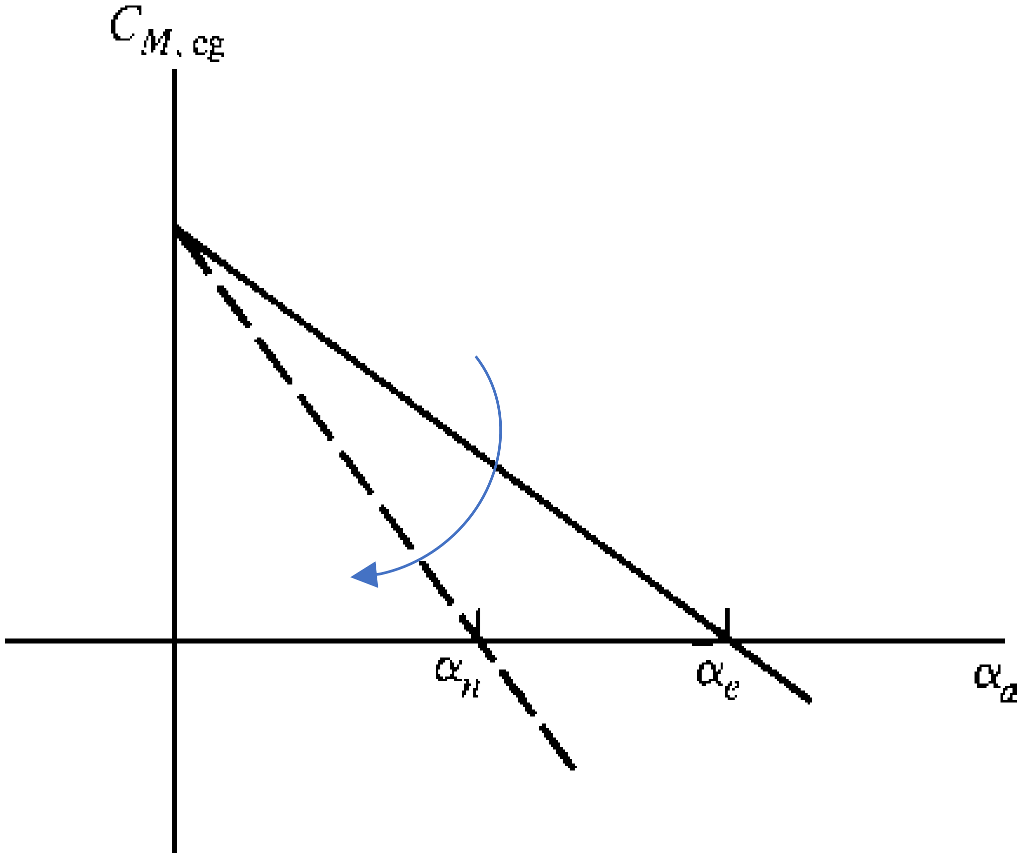
Change the slope of $C_{M,cg}$



Change the $C_{M,0}$

Concept of trim

Solution



Concept of trim

Change the slope of $C_{M,cg}$

$$\frac{\partial C_{M,cg}}{\partial \alpha_a} \approx a(h - h_n) = -a(h_n - h)$$

(From lecture 11)

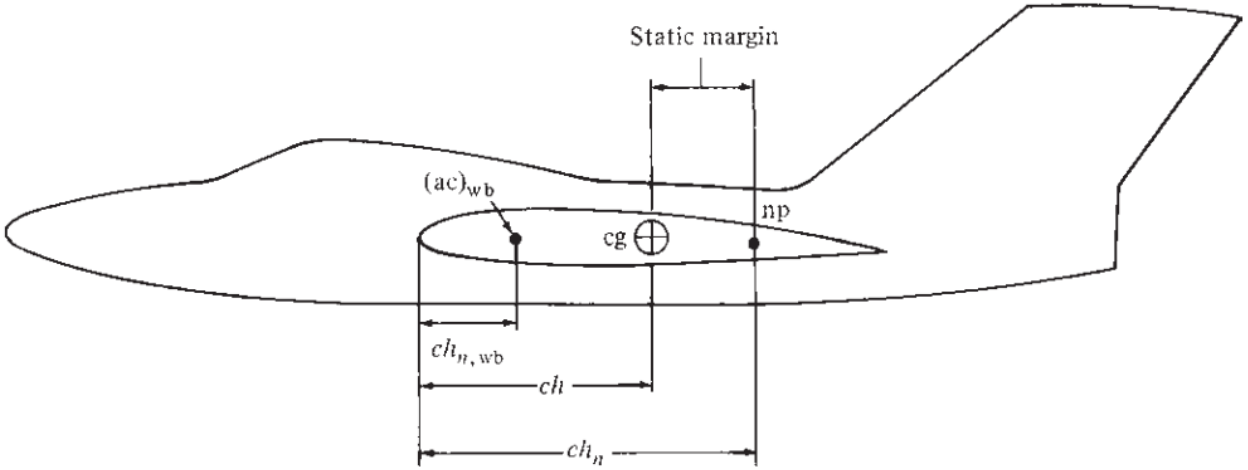


Figure 7.24 Illustration of the static margin.

The slope can be changed by shifting the center of gravity $h - h_n$

Concept of trim

The total moment coefficient $C_{M,cg}$

Moment of wing-body about the a.c.

Wing-body lift coefficient

Tail volume ratio $\frac{l_t S_t}{cS}$

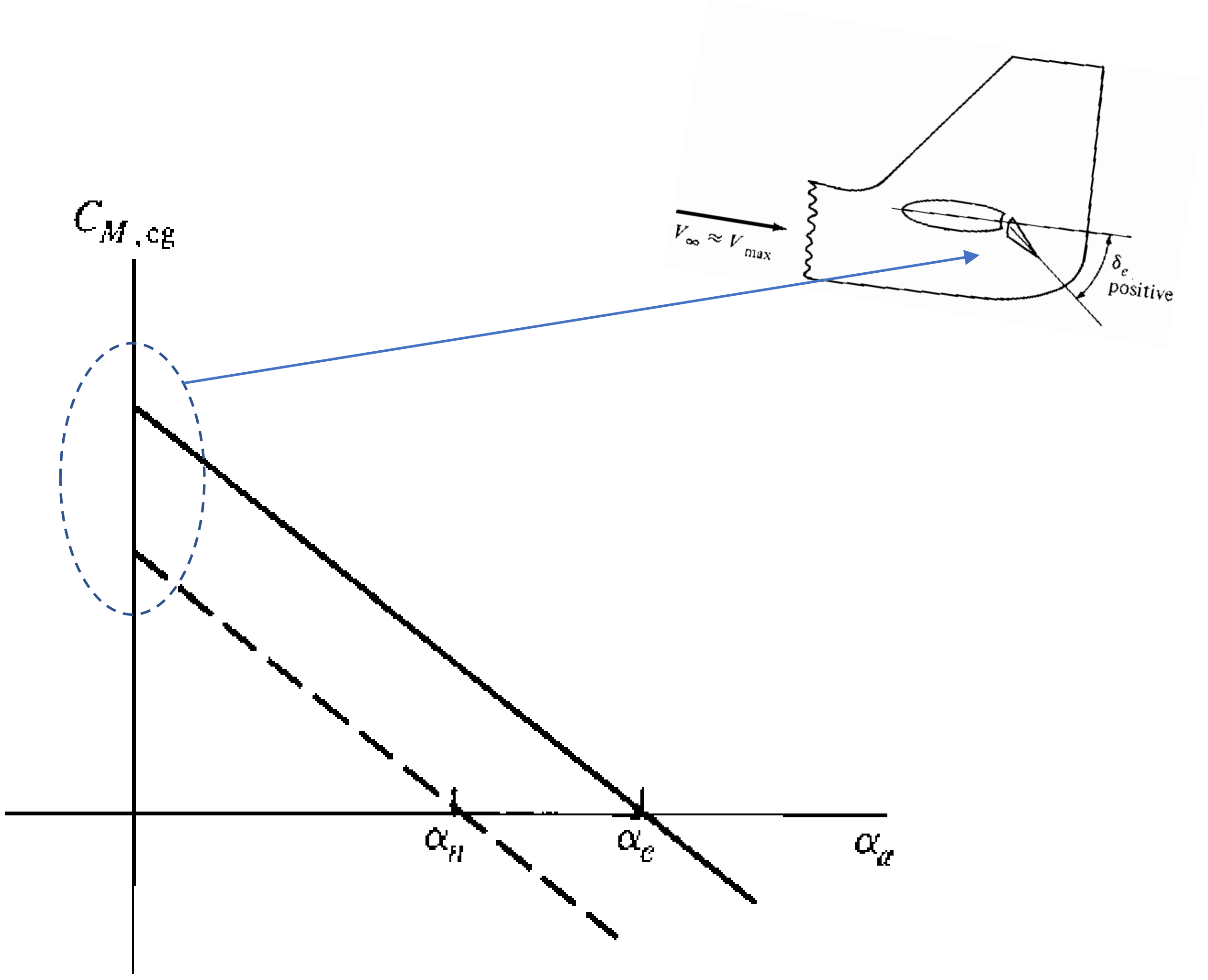
$$C_{M,\mathrm{cg}} = C_{M,\mathrm{ac}_{\mathrm{wb}}} + \boxed{C_{L_{\mathrm{wb}}}}\,\underline{(h - h_{\mathrm{ac}_{\mathrm{wb}}})} - \boxed{V_H}\,\underline{C_{L,t}}$$

Relative position of center of gravity

The tail lift coefficient

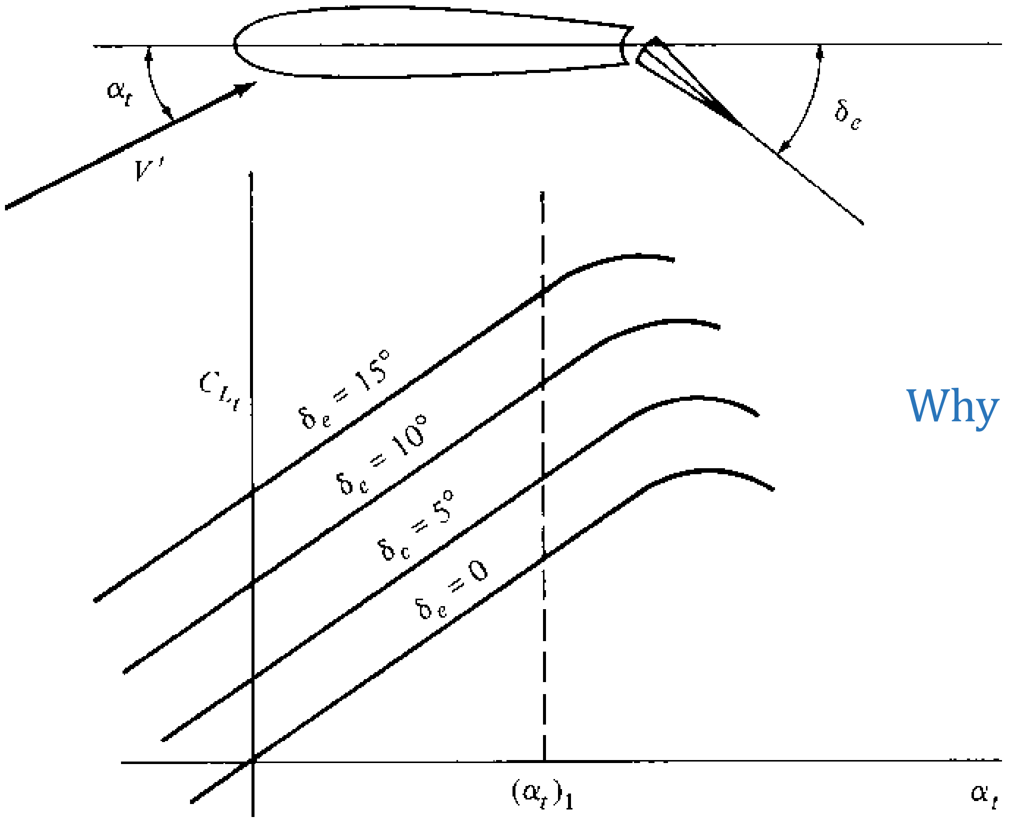
Concept of trim

Change $C_{M,0}$



Concept of trim

Tail deflection angle δ_e

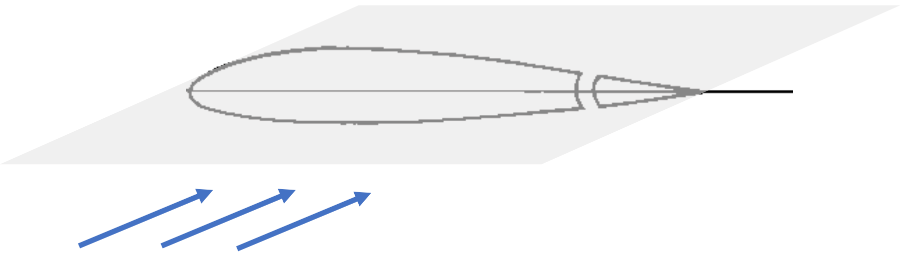


Why $C_{L,t}$ increases as δ_e?

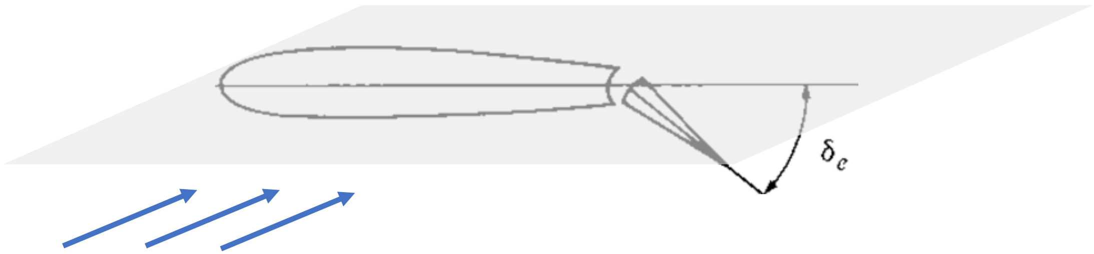
Concept of trim

$C_{L,t}$ as a function of δ_e

$$L_t = C_{L,t}\frac{1}{2}\rho V^2 S_t$$



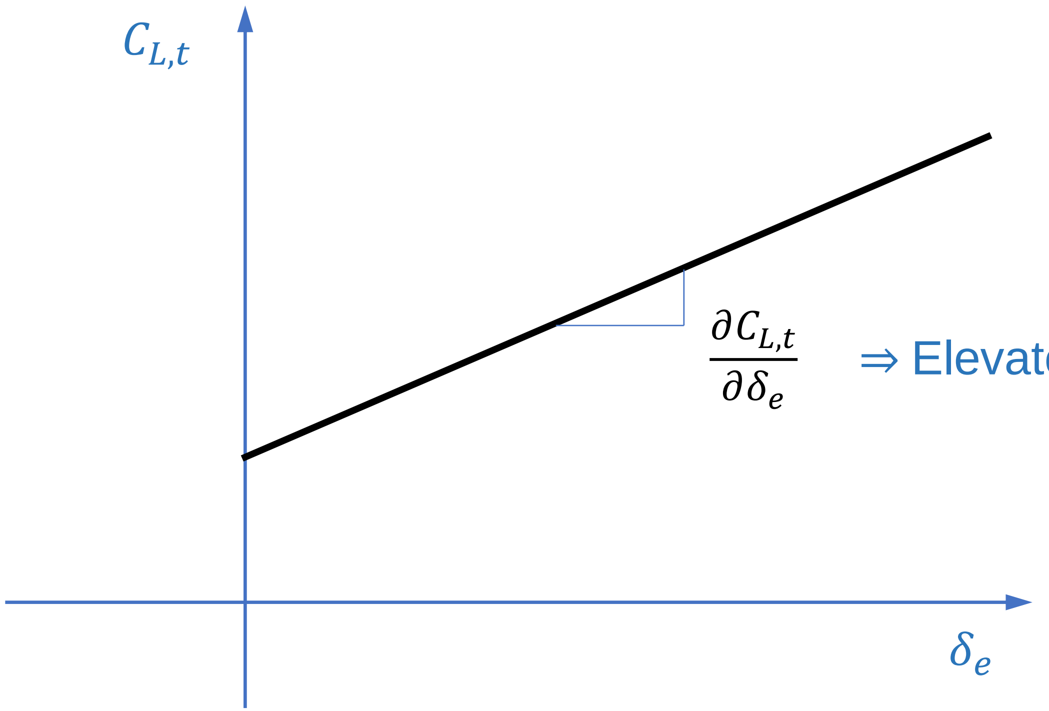
Airflow V



Airflow V

Concept of trim

$C_{L,t}$ as a function of δ_e



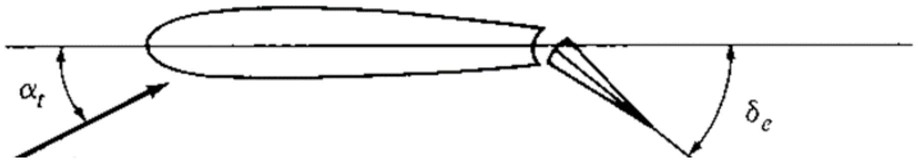
$\Rightarrow$ Elevator control effectiveness

Concept of trim

Lift coefficient of the tail

$$C_{L,t} = \frac{\partial C_{L,t}}{\partial \alpha_t}\alpha_t + \frac{\partial C_{L,t}}{\partial \delta_e}\delta_e$$

$$= a_t \alpha_t + \frac{\partial C_{L,t}}{\partial \delta_e}\delta_e$$



$C_{L,t} = f(\alpha_t, \delta_e)$

$$\Rightarrow C_{M,cg} = C_{M,ac_{wg}} + C_{L_{wb}}(h - h_{ac_{wb}}) - V_H\left(a_t\alpha_t + \frac{\partial C_{L,t}}{\partial \delta_e}\delta_e\right)$$

Concept of trim

Effect of δ_e on $C_{M,cg}$

$$\boxed{C_{M,cg} = C_{M,ac_{wg}} + C_{L_{wb}}\left(h - h_{ac_{wb}}\right) - V_H\left(a_t\alpha_t + \frac{\partial C_{L,t}}{\partial \delta_e}\delta_e\right)}$$

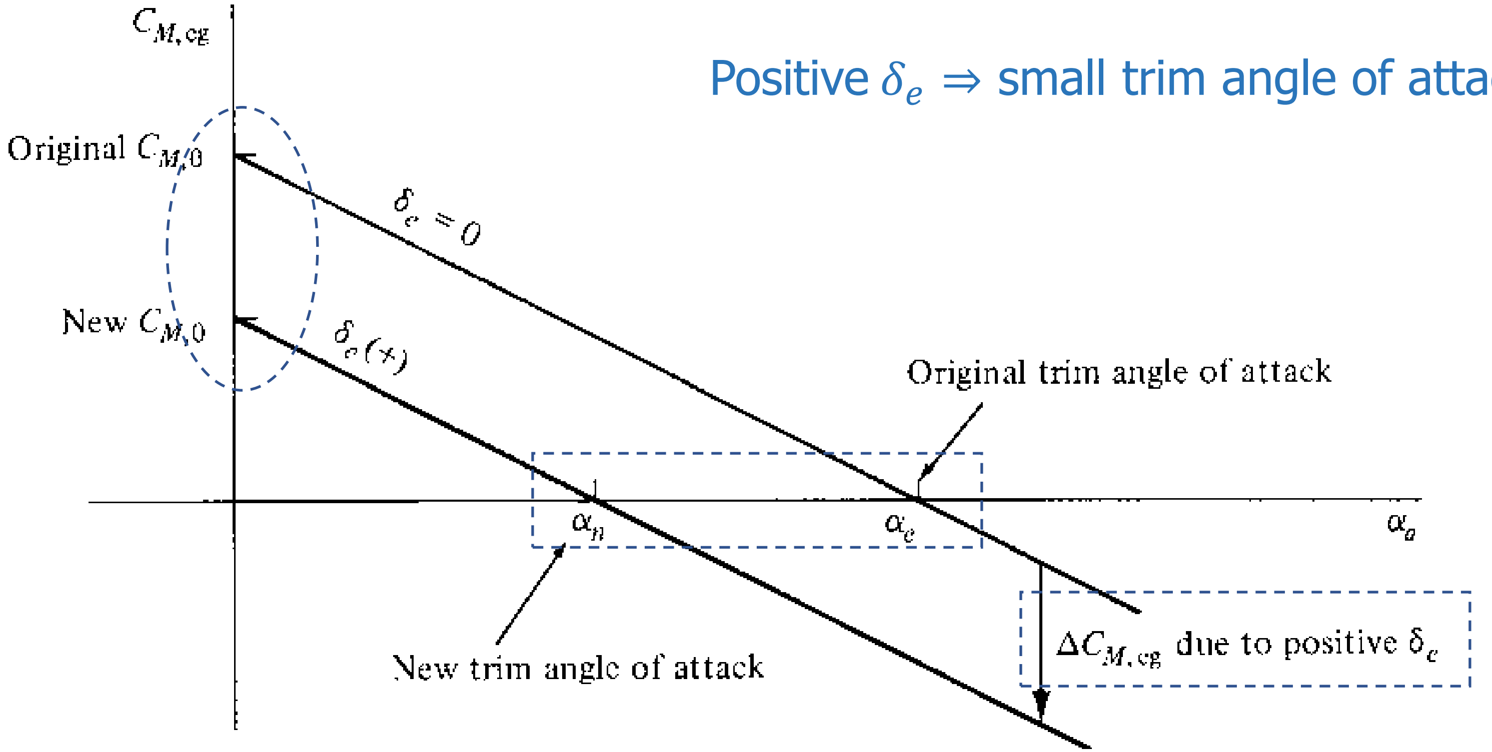
$$\Rightarrow \frac{\partial C_{M,cg}}{\partial \delta_e} = -V_H\frac{\partial C_{L,t}}{\partial \delta_e}$$

Increment in $C_{M,cg}$ due to a given elevator deflection δ_e

$$\Rightarrow \Delta C_{M,cg} = -V_H\frac{\partial C_{L,t}}{\partial \delta_e}$$

Concept of trim

Effect of δ_e on $C_{M,cg}$



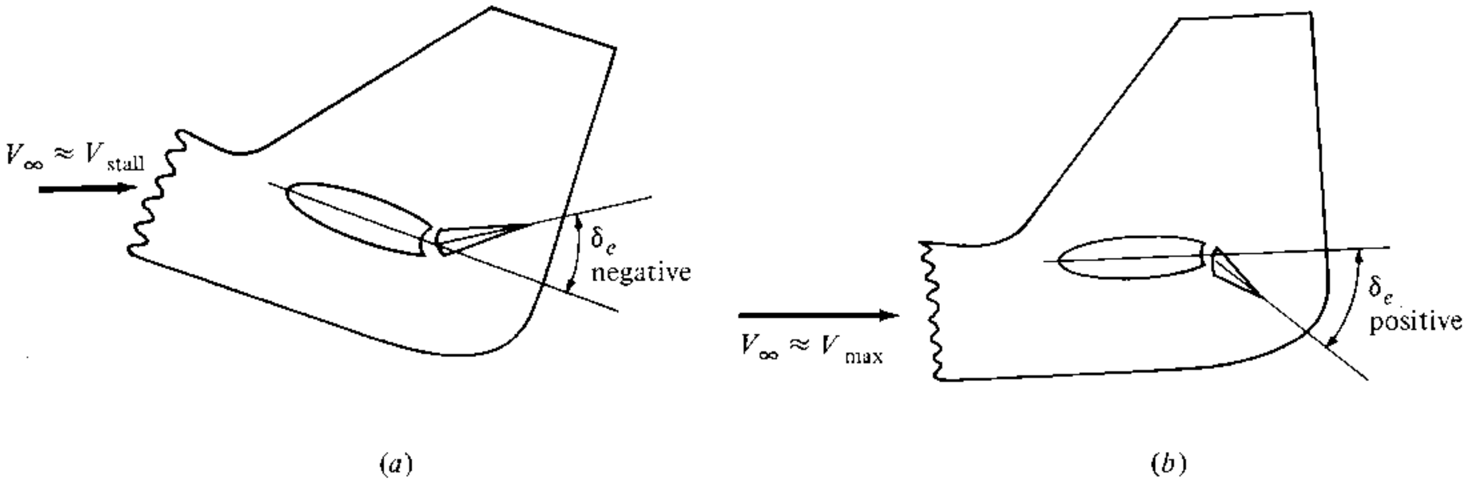
Positive δ_e $\Rightarrow$ small trim angle of attack

Concept of trim

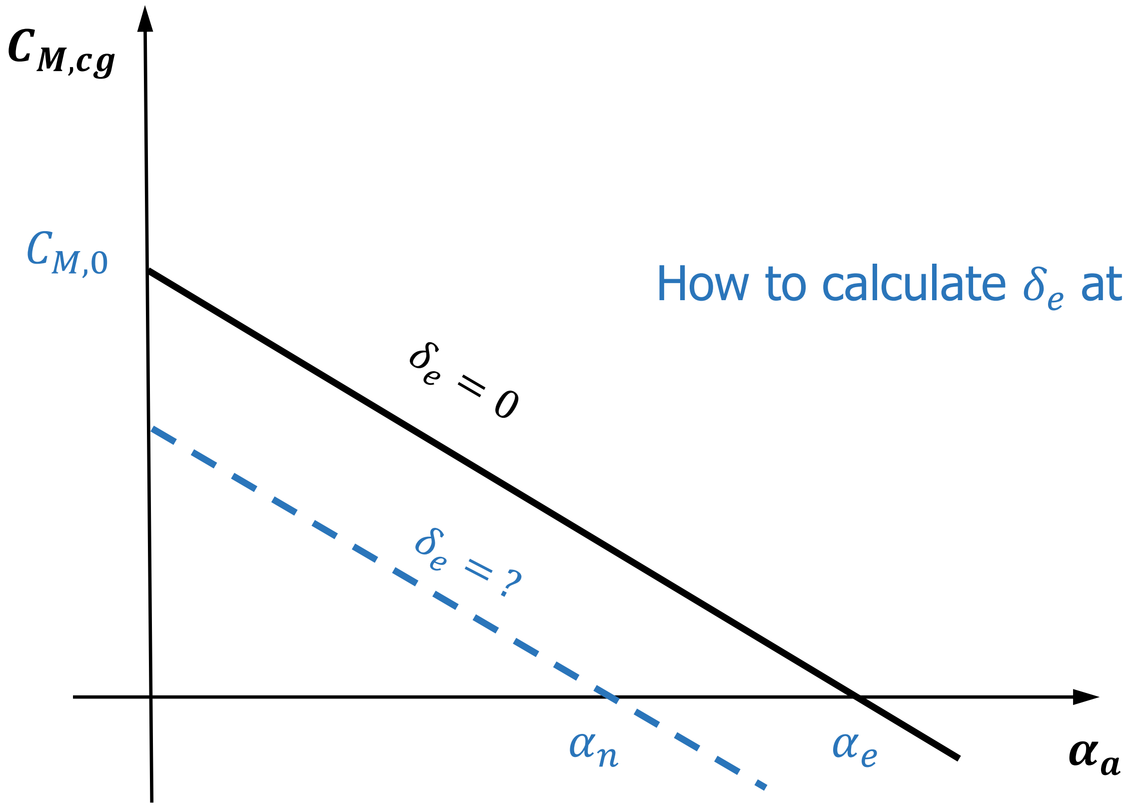
$$V_{trim} = \sqrt{\frac{2W}{\rho_\infty S C_{L_{trim}}}}$$

Summary

- Positive $\delta_e \Rightarrow$ small $\alpha_e \Rightarrow$ small $L_{trim} \Rightarrow$ high V_{trim}
- Negative $\delta_e \Rightarrow$ large $\alpha_e \Rightarrow$ large $L_{trim} \Rightarrow$ low V_{trim}



Elevator angle to trim



How to calculate δ_e at the new trim angle?

Elevator angle to trim

$$C_{M,cg} = C_{M,0} + \frac{\partial C_{M,cg}}{\partial \alpha_a}\alpha_a$$

Calculate δ_e at α_n

$$C_{M,cg} = C_{M,0} + \frac{\partial C_{M,cg}}{\partial \alpha_a}\alpha_a + \Delta C_{M,cg}$$

Increment in $C_{M,cg}$ due to a given elevator deflection δ_e

$$C_{M,cg} = C_{M,0} + \frac{\partial C_{M,cg}}{\partial \alpha_a}\alpha_a - V_H \frac{\partial C_{L,t}}{\partial \delta_e}\delta_e$$

Elevator angle to trim

Calculate δ_e at α_n

$$0 = C_{M,0} + \frac{\partial C_{M,cg}}{\partial \alpha_a}\alpha_n - V_H \frac{\partial C_{L,t}}{\partial \delta_e}\delta_e$$

Solve for the tail deflection angle δ_{trim}

$$\Rightarrow \delta_{trim} = \frac{C_{M,0} + \left(\partial C_{M,cg}/\partial \alpha_a\right)\alpha_n}{V_H\left(\partial C_{L,t}/\partial \delta_e\right)}$$

Where V_H is known design parameter, $C_{M,0}$, $\left(\partial C_{M,cg}/\partial \alpha_a\right)$ and $\left(\partial C_{L,t}/\partial \delta_e\right)$ are usually obtained from wind tunnel or flight data. (A good example for solving real-world problems!)

Practice – example 7.8

Consider a full-size airplane with the same aerodynamic and design characteristics as the wind tunnel model of Examples 7.3 to 7.7. The airplane has a wing area of 19 m^2, a weight of 2.27×10^4 N, and an elevator control effectiveness of 0.04. Calculate the elevator deflection angle necessary to trim the airplane at a velocity of 61 m/s at sea level.

J.D. Anderson, 7th edition, page 627

Given parameters:

$C_{M,0} = 0.06$ (from Example 7.5)

$\dfrac{C_{M,cg}}{\partial \alpha_a} = -0.0133$ (from Example 7.5)

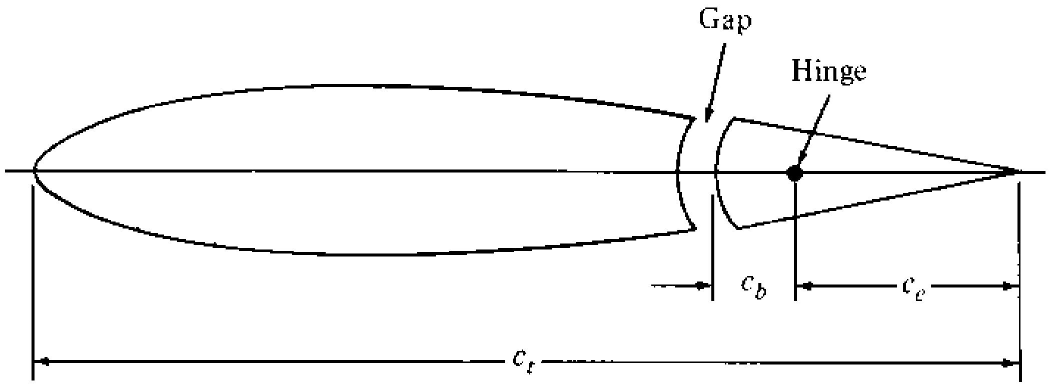
$\alpha_n = 6.5°$ (this is the α_a calculated previously)

$V_H = 0.34$ (from Example 7.4)

$\dfrac{\partial C_{L,t}}{\partial \delta_e} = 0.04$ (given in the preceding information)

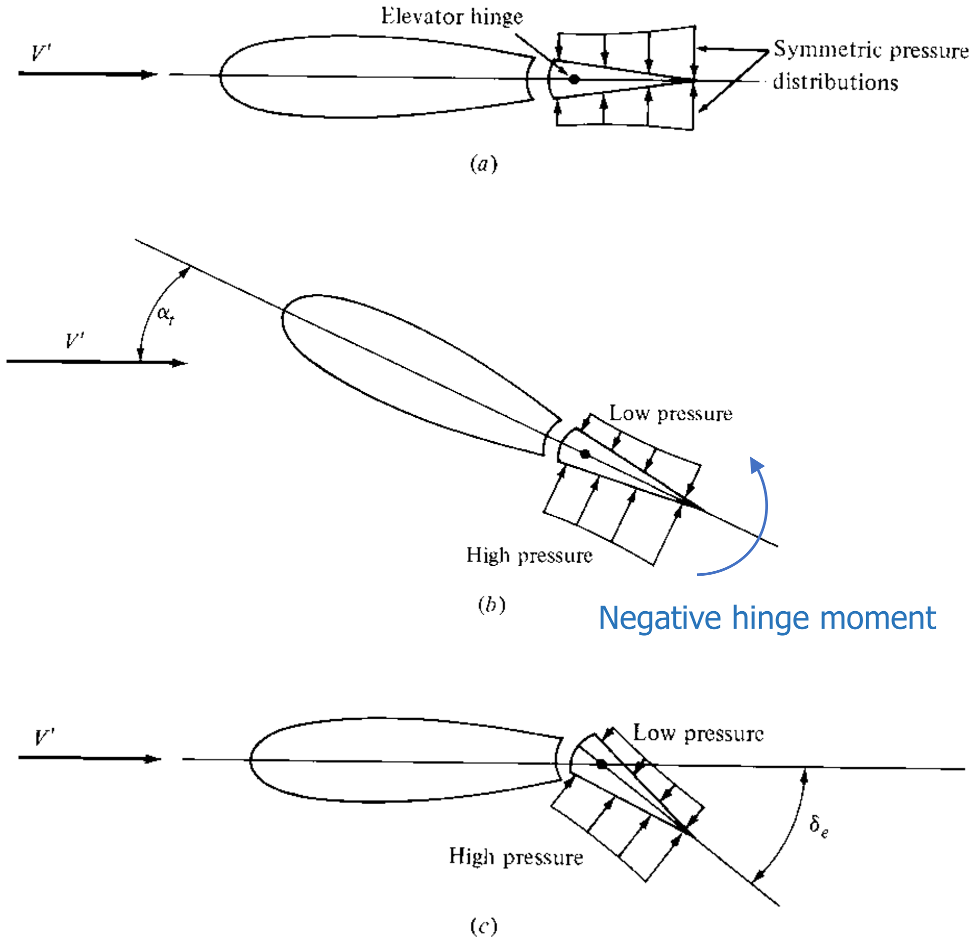
Elevator hinge moment

An elevator with hinge



$$C_{h_e} = \frac{H_e}{\frac{1}{2}\rho_\infty V_\infty^2 S_e c_e}$$

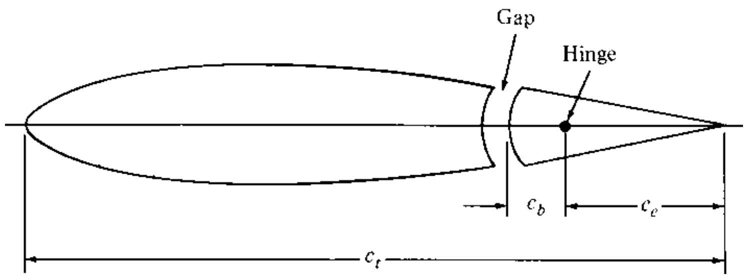
?



Elevator hinge moment

Calculation of hinge moment coefficient

$$C_{h_e} = \frac{\partial C_{h_e}}{\partial \alpha_t}\alpha_t + \frac{\partial C_{h_e}}{\partial \delta_e}\delta_e$$



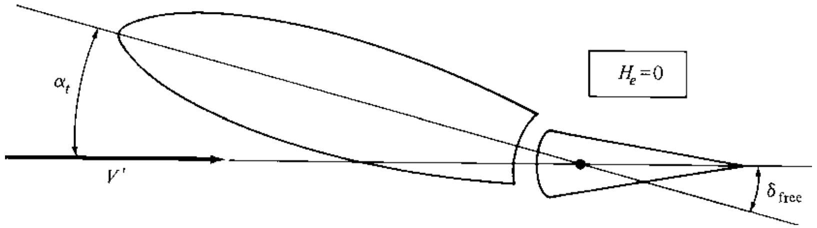
$C_{he} = f(\alpha_t, \delta_e)$

Where $(\partial C_{he}/\partial \alpha_t)$ and $(\partial C_{he}/\partial \delta_e)$ are approximately constant. Its value can be obtained empirically from wind tunnel experiments

Stick-free stability

Deflection angle at “Stick-free”

$$C_{he} = 0 = \frac{\partial C_{he}}{\partial \alpha_t}\alpha_t + \frac{\partial C_{he}}{\partial \delta_e}\delta_{free}$$



$$\Rightarrow \delta_{free} = -\frac{\partial C_{he}/\partial \alpha_t}{\partial C_{he}/\partial \delta_e}\alpha_t$$

Both partial derivatives are usually negative, thus a positive α_t yields a negative δ_{free}

Stick-free stability

$\delta_e \rightarrow \delta_{free}$

$$C_{L,t} = a_t\alpha_t + \frac{\partial C_{L,t}}{\partial \delta_e}\delta_e$$

$$C'_{L,t} = a_t\alpha_t + \frac{\partial C_{L,t}}{\partial \delta_e}\delta_{\text{free}}$$

$$C'_{L,t} = a_t\alpha_t - \frac{\partial C_{L,t}}{\partial \delta_e}\frac{\partial C_{h_e}/\partial \alpha_t}{\partial C_{h_e}/\partial \delta_e}\alpha_t$$

Define F is free elevator factor:

$$F = 1 - \frac{1}{a_t}\frac{\partial C_{L,t}}{\partial \delta_e}\frac{\partial C_{h_e}/\partial \alpha_t}{\partial C_{h_e}/\partial \delta_e}$$

$$\Rightarrow C'_{L,t} = a_t\alpha_t F$$

Stick-free stability

$C_{M,cg}$ for a free elevator

$$\boxed{C_{M,\mathrm{cg}} = C_{M,\mathrm{ac}_{\mathrm{wb}}} + C_{L_{\mathrm{wb}}}(h - h_{\mathrm{ac}_{\mathrm{wb}}}) - V_H C_{L,t}}$$

$$C'_{M,\mathrm{cg}} = C_{M,\mathrm{ac}_{\mathrm{wb}}} + C_{L_{\mathrm{wb}}}(h - h_{\mathrm{ac}_{\mathrm{wb}}}) - V_H \boxed{C'_{L,t}}$$

$$\boxed{C'_{L,t} = a_t \alpha_t F}$$

$$\Rightarrow \quad C'_{M,\mathrm{cg}} = C_{M,\mathrm{ac}_{\mathrm{wb}}} + C_{L_{\mathrm{wb}}}(h - h_{\mathrm{ac}_{\mathrm{wb}}}) - V_H a_t \alpha_t F$$

Another example for solving problems you don’t fully understand.

Stick-free stability

Apply the same analyses as stick-fixed stability

$$\boxed{\begin{aligned} C'_{M,0} &= C_{M,\mathrm{ac_{wb}}} + FV_H a_t (i_t + \varepsilon_0) \\ h'_n &= h_{\mathrm{ac_{wb}}} + FV_H \frac{a_t}{a}\left(1 - \frac{\partial \varepsilon}{\partial \alpha}\right) \\ \frac{\partial C'_{M,\mathrm{cg}}}{\partial \alpha} &= -a(h'_n - h) \end{aligned}}$$

(7.51)

(7.52)

(7.53)

Where $h'_n - h$ is the stick-free static margin

Practice – example 7.9

Consider the airplane of Example 7.8. Its elevator hinge moment derivatives are $\partial C_{h_e}/\partial \alpha_t = -0.008$ and $\partial C_{h_e}/\partial \delta_e = -0.013$. Assess the *stick-free* static stability of this airplane.

J.D. Anderson, 7th edition, page 634

Directional Static Stability

Effect of the vertical stabilizer

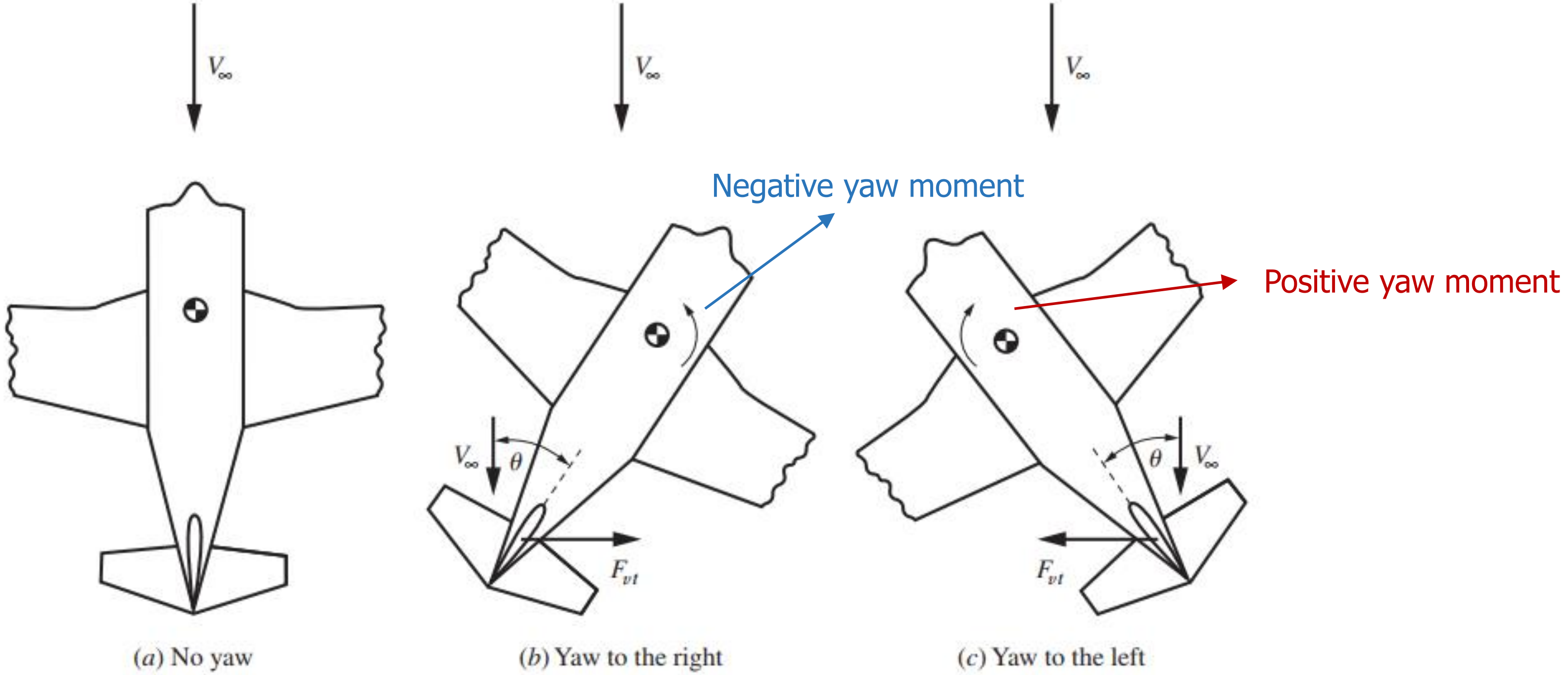


Figure 7.38 Effect of the vertical stabilizer on directional stability.

Directional Static Stability

Moment of vertical tail

$$M_{cg,vt} = F_{vt} l_{vt}$$

Vertical tail volume ratio:

$$V_{vt} = \frac{l_{vt} S_{vt}}{bS}$$

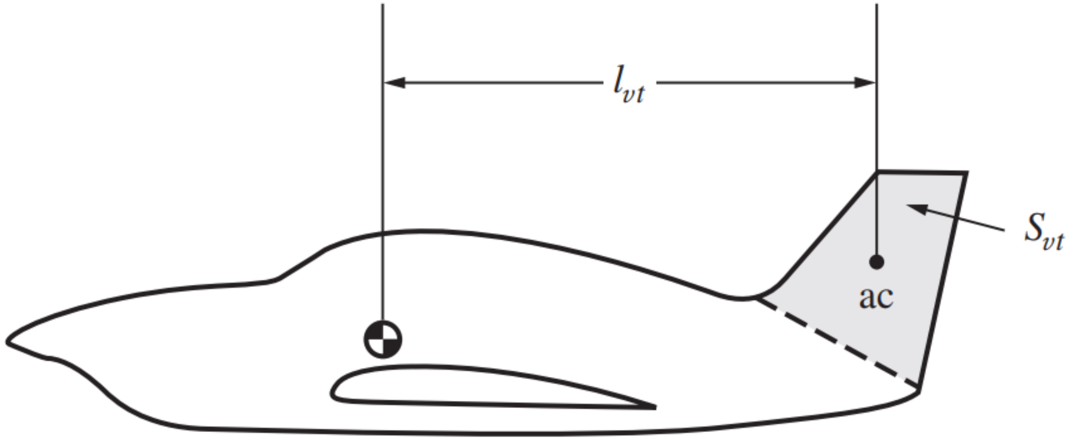


Figure 7.39 Moment arm of the vertical tail.

Lateral Static Stability

Dihedral:

Upward angle of wings

Why the wings are designed upward?

Lateral Static Stability

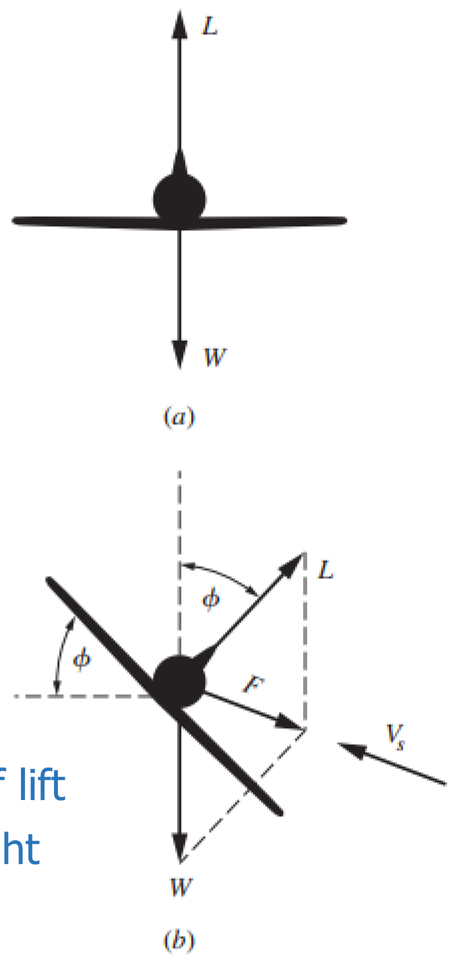


Figure 7.40 Generation of slideslip.

Horizontal component of lift causes aircraft to slip right

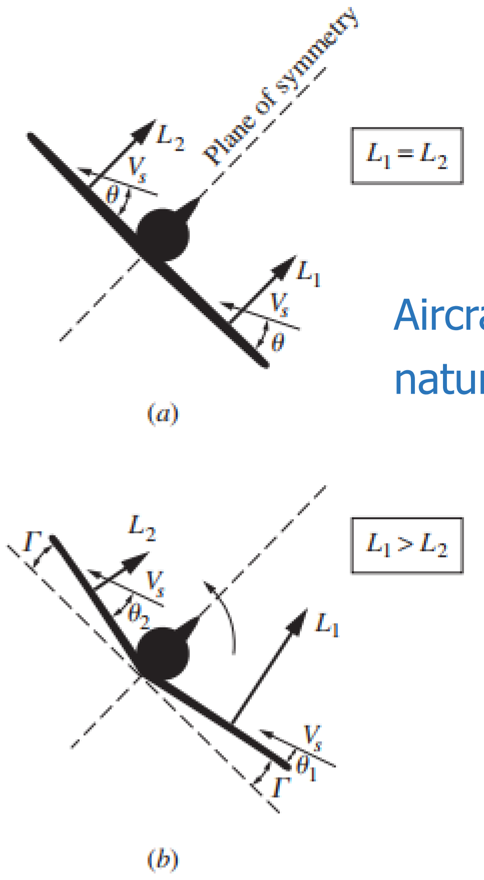


Figure 7.41 Effect of dihedral.

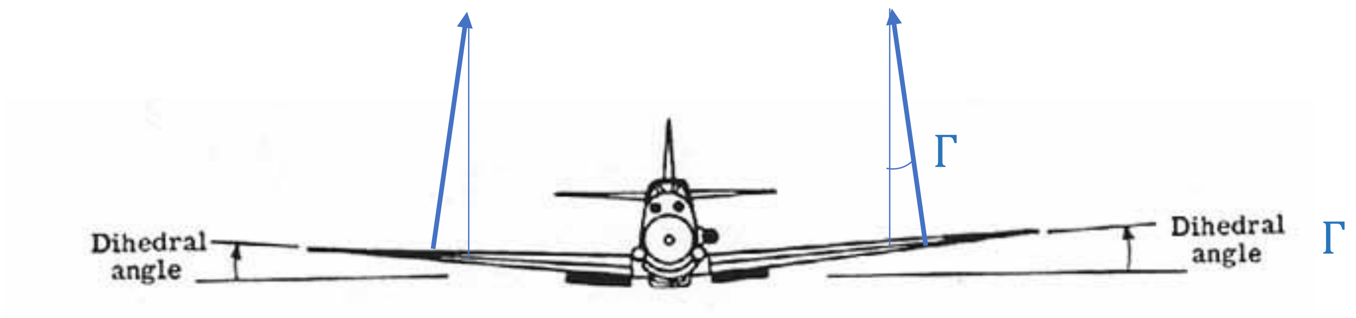
Aircraft without dihedral won't naturally return to wing-level!

Lateral Static Stability

Cost of dihedral ?

Lateral Static Stability

Cost of dihedral ?



1) Actual Lift force $L' = L\cos\Gamma$ is smaller
2) Reduce the roll rate, low maneuverability

Lateral Static Stability

An example of anhedral wing

Why Y-20 use anhedral wing?

Lateral Static Stability

Dihedral vs Anhedral Wings

Definition:

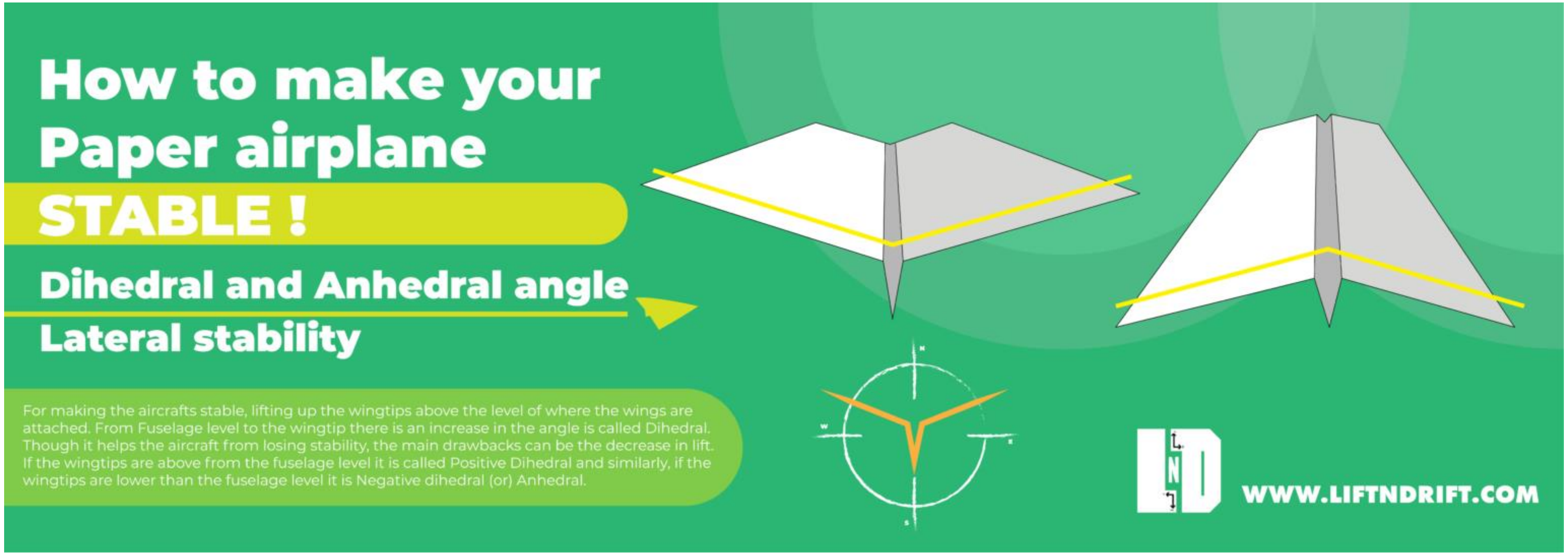
- **Dihedral wing** (上反翼) is where the wing slopes in a positive (upward) degree in relation to the wing base;
- **Anhedral wing** (下反翼) has a negative (downward) slope from the wing base.

Application:

- **Dihedral wing:** most passenger airliner, civil aviation aircraft;
- **Anhedral wing:** high performance military jets, heavy lifting high-wing cargo planes.

Lateral Static Stability

Paper airplane experiment (纸飞机实验)



Summary

Stability and control

Statically stable → Low requirements for control → Pilot can fly hands off

Statically unstable → High requirements for control → Pilots must concentrate on fly

With the help of modern autopilot system, more aircraft can be designed unstable.

Summary

Page 644 – 645 of Textbook

$$C_{M,0} = C_{M,\mathrm{ac}_{\mathrm{wb}}} + V_H a_t (i_t + \varepsilon_0)$$

$$\frac{\partial C_{M,\mathrm{cg}}}{\partial \alpha_a} = a\left[h - h_{\mathrm{ac}_{\mathrm{wb}}} - V_H \frac{a_t}{a}\left(1 - \frac{\partial \varepsilon}{\partial \alpha}\right)\right]$$

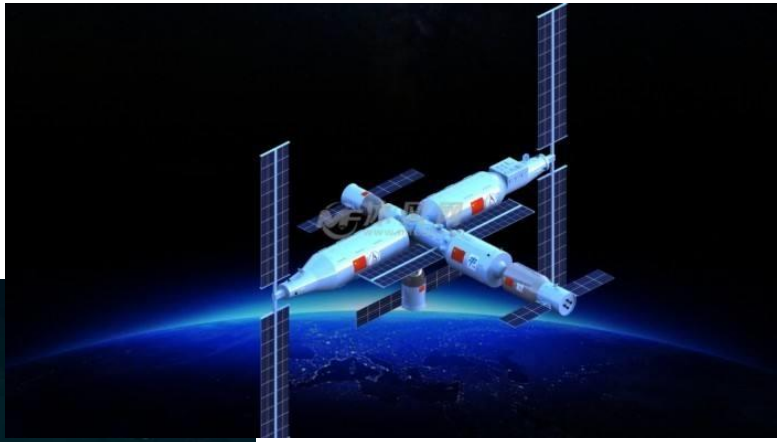
$$C_{M,\mathrm{cg}} = C_{M,\mathrm{ac}_{\mathrm{wb}}} + C_{L,\mathrm{wb}}(h - h_{\mathrm{ac}}) - V_H\left(a_t \alpha_t + \frac{\partial C_{L,t}}{\partial \delta_e}\delta_e\right)$$

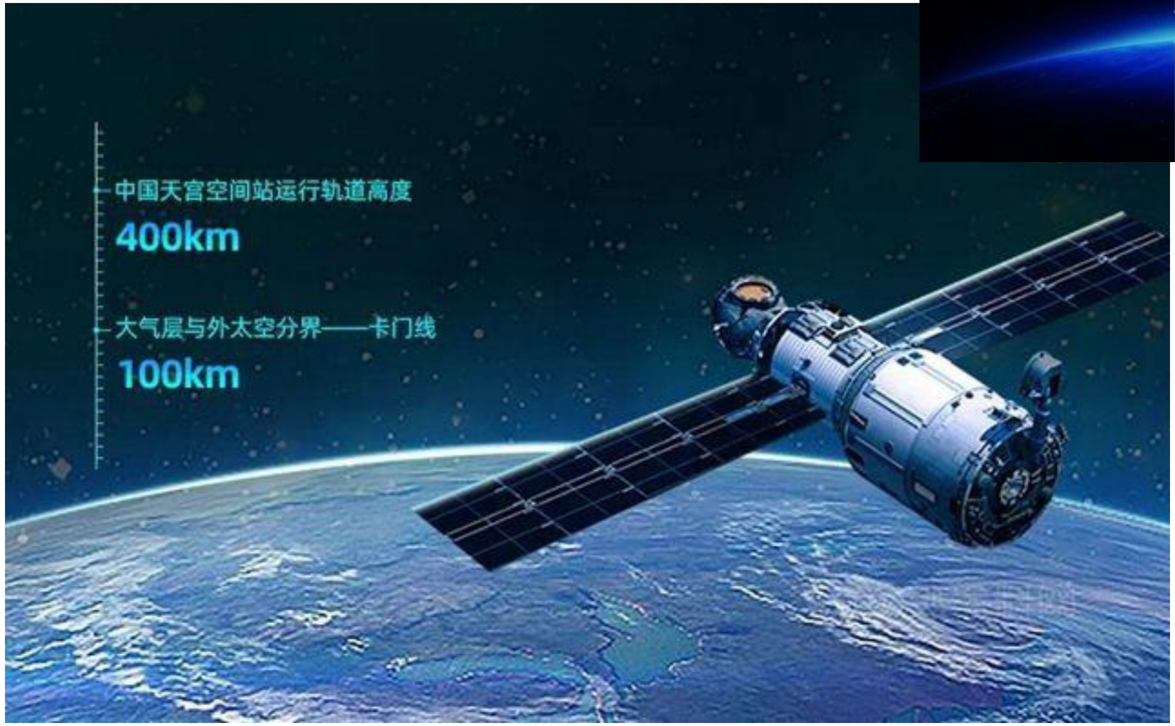
$$h_n = h_{\mathrm{ac}_{\mathrm{wb}}} + V_H \frac{a_t}{a}\left(1 - \frac{\partial \varepsilon}{\partial \alpha}\right)$$

$$\delta_{\mathrm{trim}} = \frac{C_{M,0} + (\partial C_{M,\mathrm{cg}}/\partial \alpha_a)\alpha_n}{V_H(\partial C_{L,t}/\partial \delta_e)}$$

Space flight & orbital mechanics

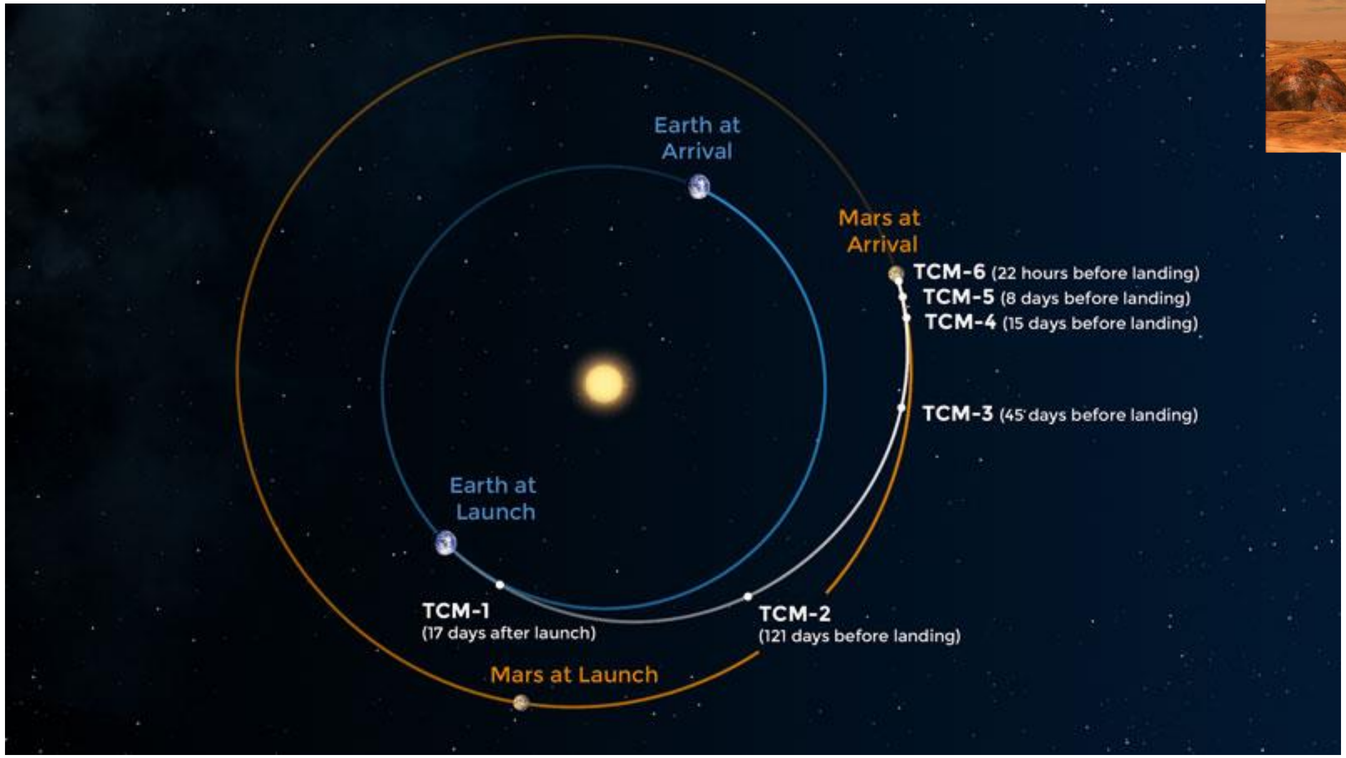
Tiangong Space Station





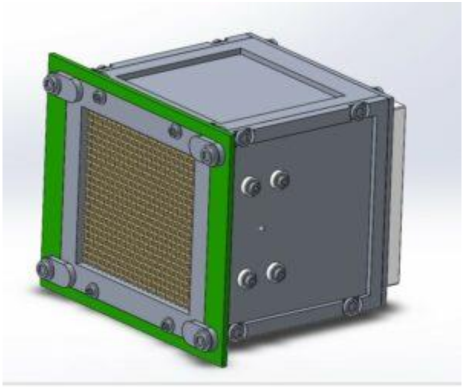
Introduction

Tianwen-1 and Zhurong

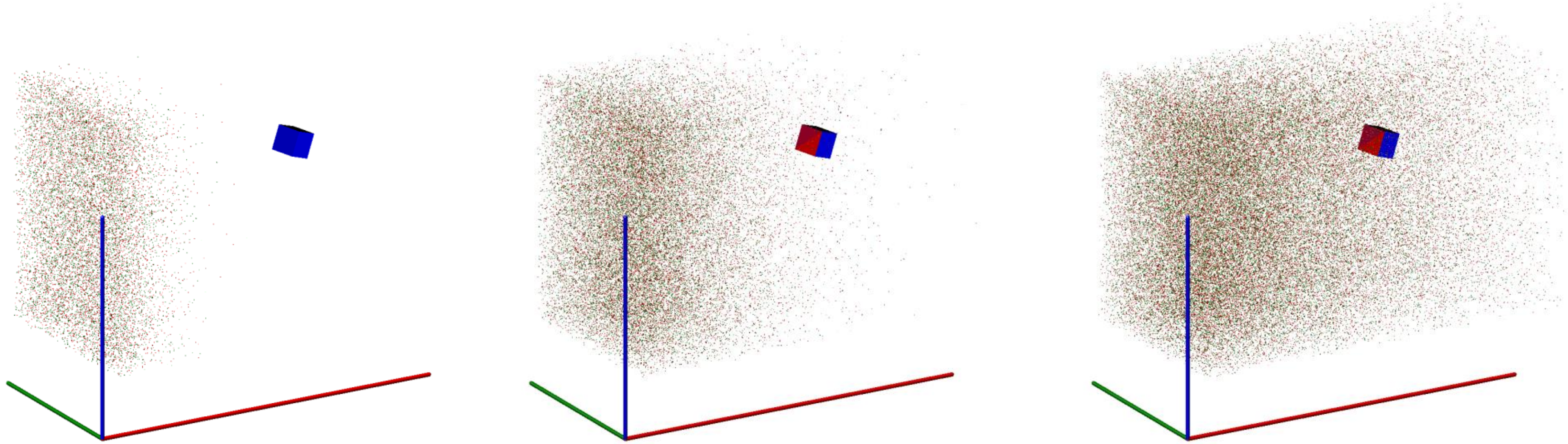


Introduction

Research on CubeSat



TUM CubeSat



TUM MOVE-II project: https://www.move2space.de/MOVE-II/

Introduction

Question

- What’s the purpose and meaning for space explore?

Introduction

[Starship Explosion: How Elon Musk's SpaceX Got Here | Timeline | WSJ]