System Dynamics and Vibrations

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Chapter 7: Applications of Two degree-of-freedom systems

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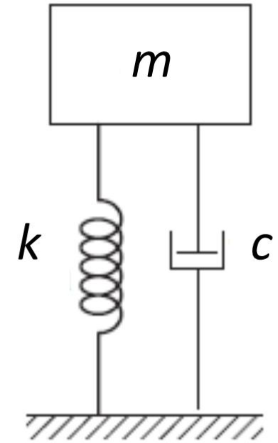
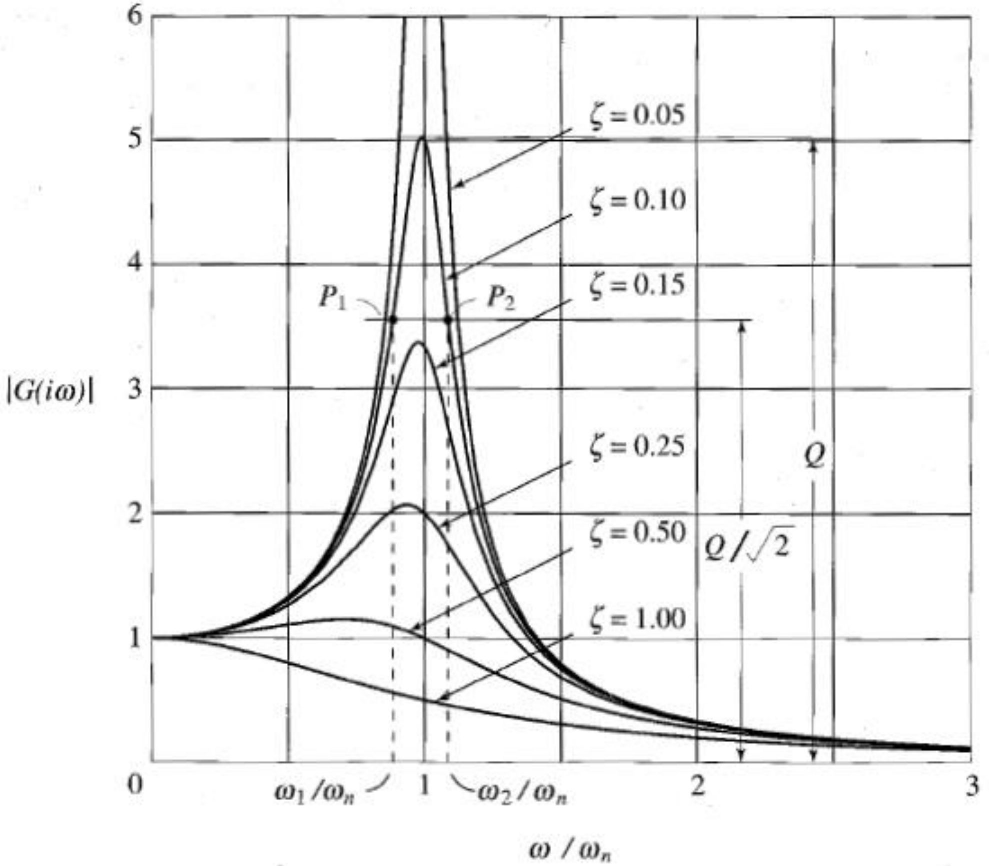
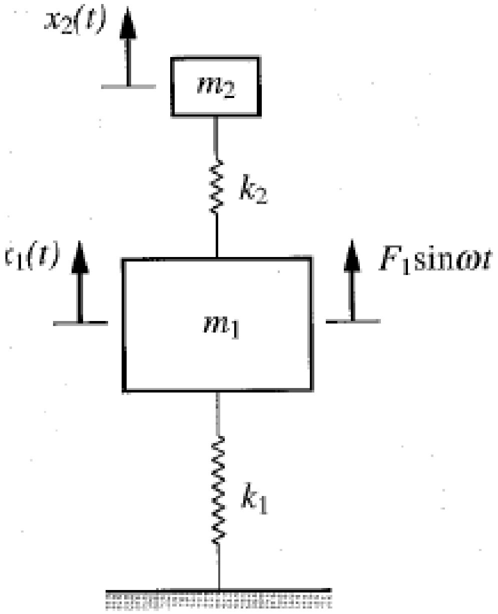
Introduction



- When rotating machinery operates at a frequency close to resonance, violent vibration is induced.
- Assuming that the system can be represented by a single-degree-of-freedom system subjected to harmonic excitation, the situation can be alleviated by changing either the mass or the spring constant.
- At times, however, this may not be possible.

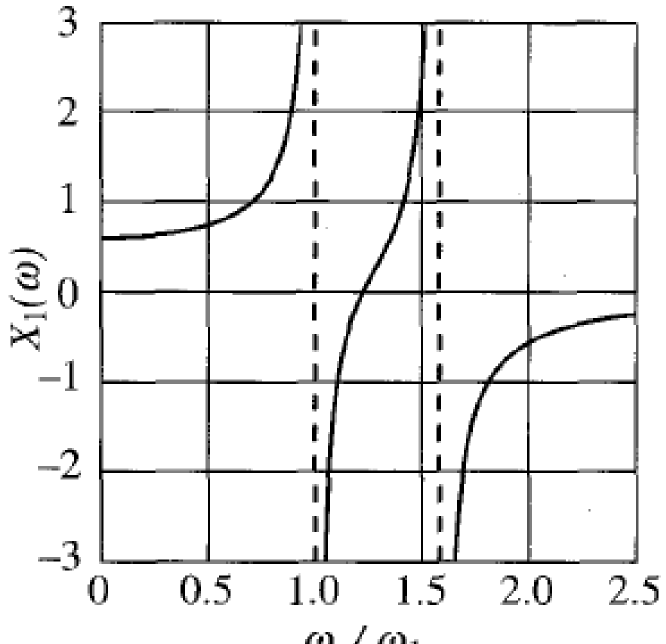


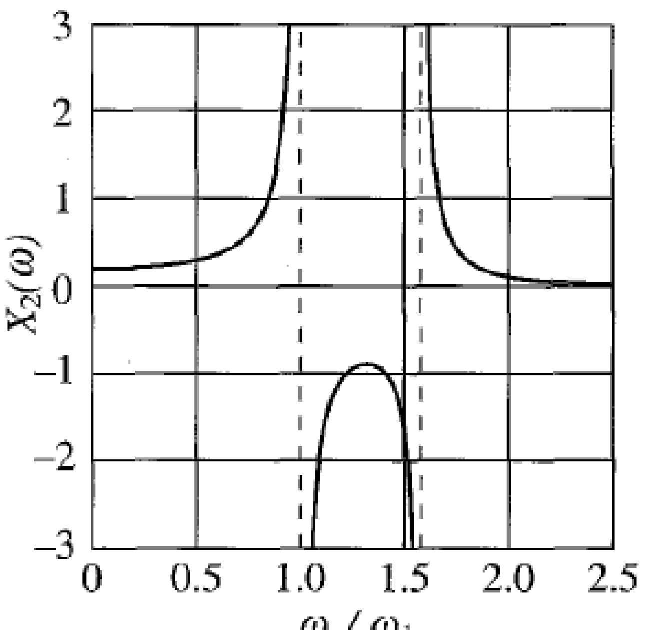
Introduction



- A second mass and spring can be added to the system, where the added mass and spring are so designed as to produce a two-degree-of-freedom system whose frequency response is zero at the excitation frequency.
- A point at which the frequency response is zero does indeed exist.

- The new two-degree-of-freedom system has two resonant frequencies, but these frequencies generally present no problem because they are reasonably far removed from the operating frequency.

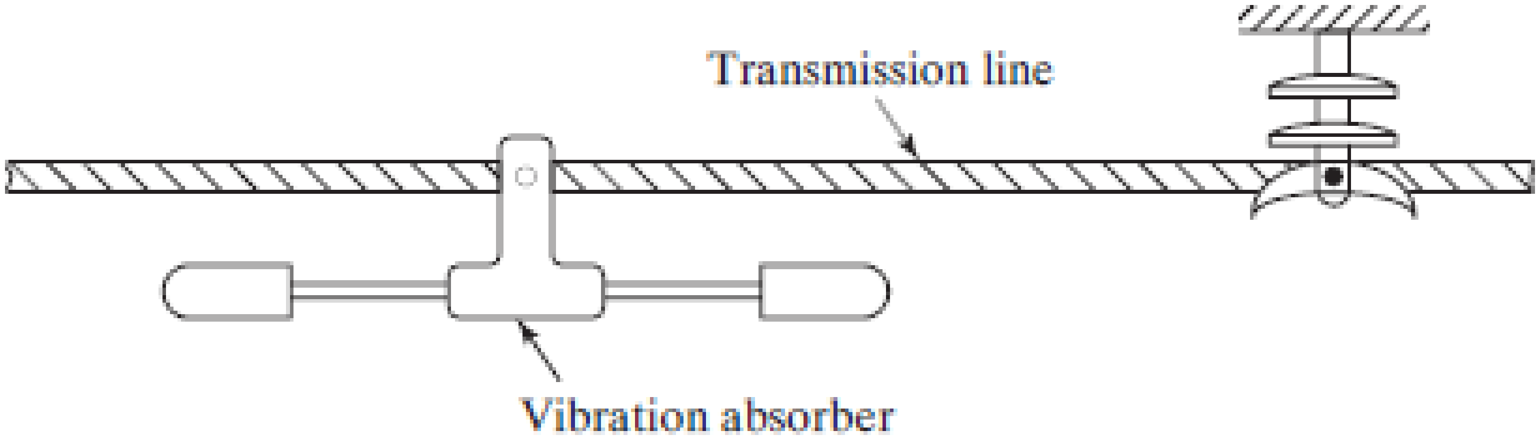




Introduction

Applications:

- Machinery that operates at a constant speed, for instance reciprocating internal combustion engines
- High-voltage transmission lines
- High buildings



Vibration absorbers

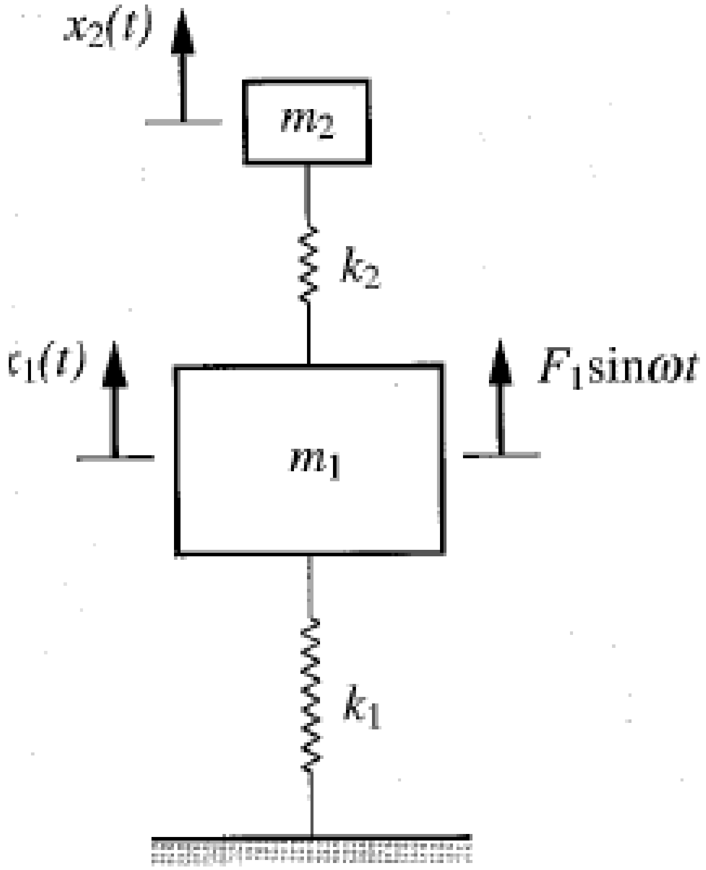
- Original SDOF system: main system ➔ m_1, k_1
- Added system: absorber ➔ m_2, k_2
- The equations of motion of the combined system are:

$$m_1\ddot{x}_1 + \left(k_1 + k_2\right)x_1 - k_2 x_2 = F_1 \sin\omega t$$

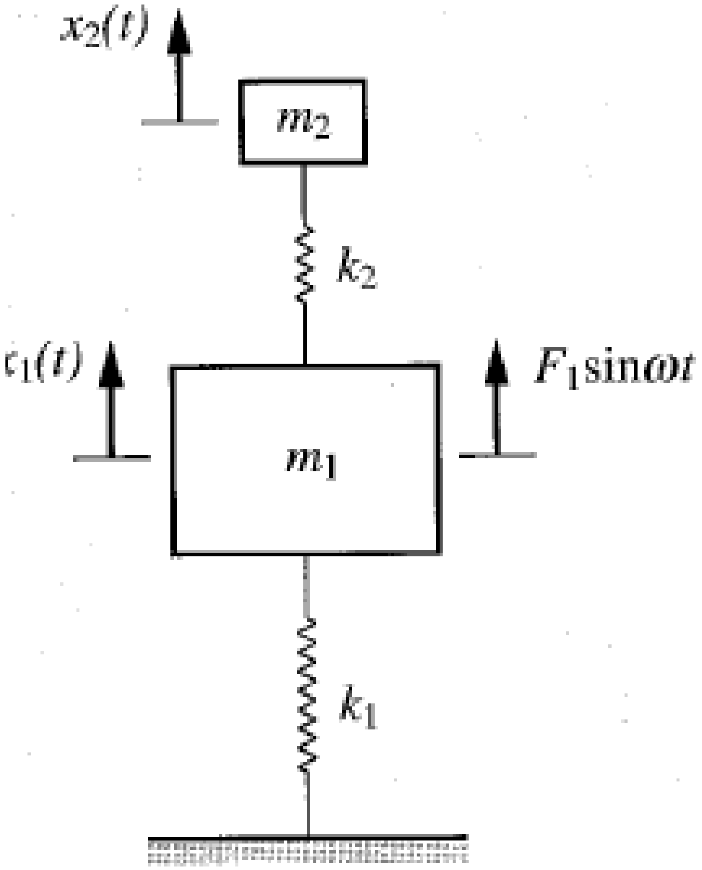
$$m_2\ddot{x}_2 - k_2 x_1 + k_2 x_2 = 0$$

- And the solution:

$$x_1(t) = X_1 \sin\omega t, \quad x_2(t) = X_2 \sin\omega t$$



Vibration absorbers



- Two algebraic equations in X_1 and X_2:

$$\begin{bmatrix} k_1 + k_2 - \omega^2 m_1 & -k_2 \\ -k_2 & k_2 - \omega^2 m_2 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} F_1 \\ 0 \end{bmatrix}$$

- Solution for a two-degree-of-freedom system subjected to harmonic excitations:

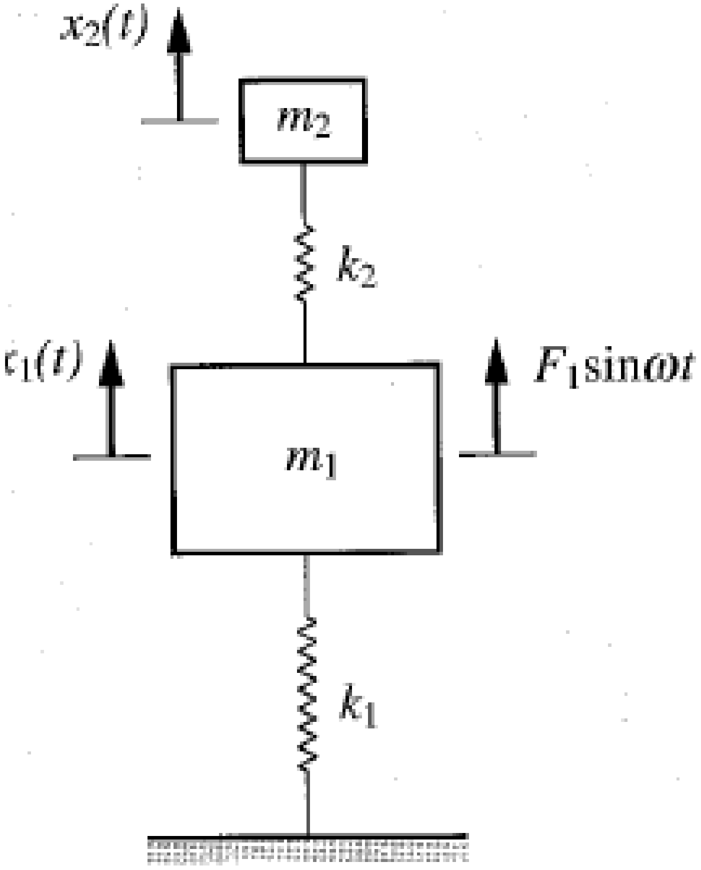
$$X_1(\omega) = \frac{\left(k_{22} - \omega^2 m_2\right) F_1 - k_{12} F_2}{\left(k_{11} - \omega^2 m_1\right)\left(k_{22} - \omega^2 m_2\right) - k_{12}^2}, \quad X_2(\omega) = \frac{-k_{12} F_1 + \left(k_{11} - \omega^2 m_1\right) F_2}{\left(k_{11} - \omega^2 m_1\right)\left(k_{22} - \omega^2 m_2\right) - k_{12}^2}$$

Vibration absorbers

- Solution for a two-degree-of-freedom system subjected to harmonic excitations:

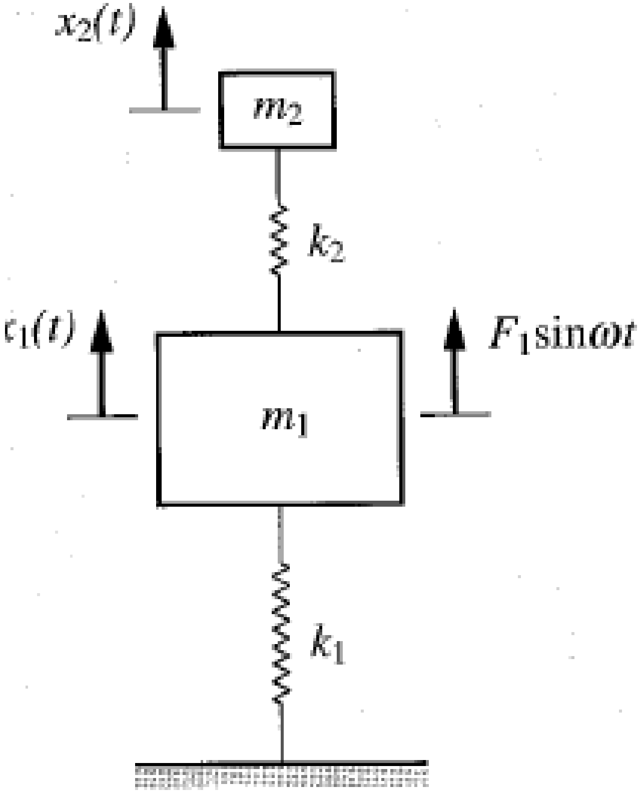
$$X_1(\omega) = \frac{\left(k_2 - \omega^2 m_2\right) F_1}{\left(k_1 + k_2 - \omega^2 m_1\right)\left(k_2 - \omega^2 m_2\right) - k_2^2}$$

$$X_2(\omega) = \frac{k_2 F_{12}}{\left(k_1 + k_2 - \omega^2 m_1\right)\left(k_2 - \omega^2 m_2\right) - k_2^2}$$



Vibration absorbers

- Introducing the notation:



$$\omega_n = \sqrt{k_1/m_1}$$ natural frequency of the main system alone

$$\omega_a = \sqrt{k_2/m_2}$$ natural frequency of the absorber alone

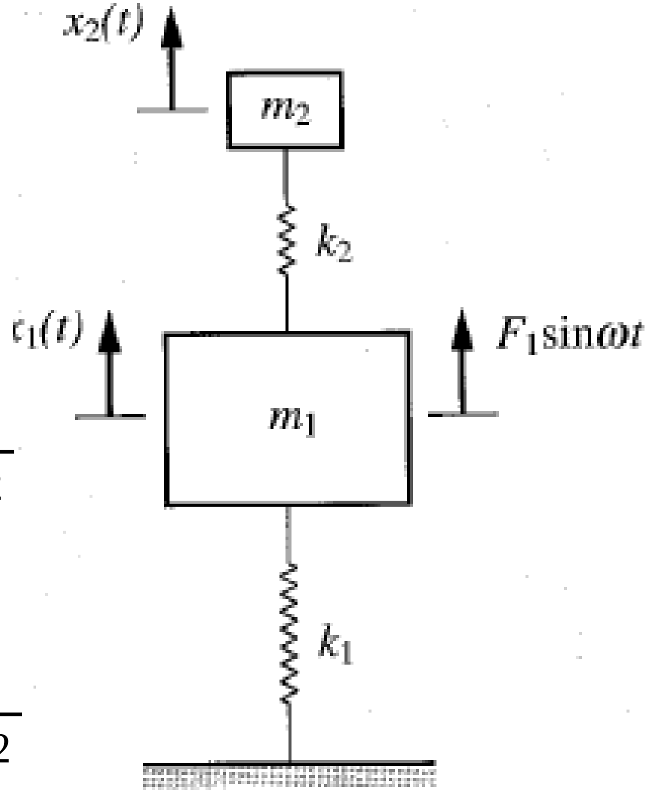
$$x_{st} = F_1/k_1$$ static deflection of the main system

$$\mu = m_2/m_1$$ ratio of the absorber mass to the main mass

Vibration absorbers

$$X_1 = \frac{\left[1-(\omega/\omega_a)^2\right]x_{st}}{\left[1+\mu(\omega_a/\omega_n)^2-(\omega/\omega_n)^2\right]\left[1-(\omega/\omega_a)^2\right]-\mu(\omega_a/\omega_n)^2}$$

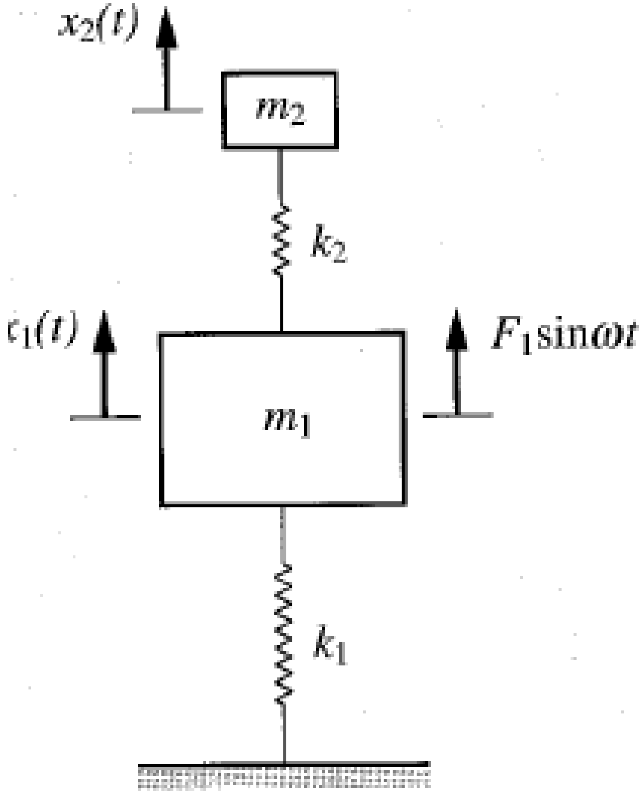
$$X_2 = \frac{x_{st}}{\left[1+\mu(\omega_a/\omega_n)^2-(\omega/\omega_n)^2\right]\left[1-(\omega/\omega_a)^2\right]-\mu(\omega_a/\omega_n)^2}$$



For $\omega_a = \omega$ the amplitude X_1 of the main mass reduces to zero:

➔ the absorber indeed eliminate the vibration of the main mass

Vibration absorbers



For $\omega_a = \omega$

$$X_2 = -\left(\frac{\omega_n}{\omega_a}\right)^2 \frac{x_{st}}{\mu} = -\frac{F_1}{k_2}$$

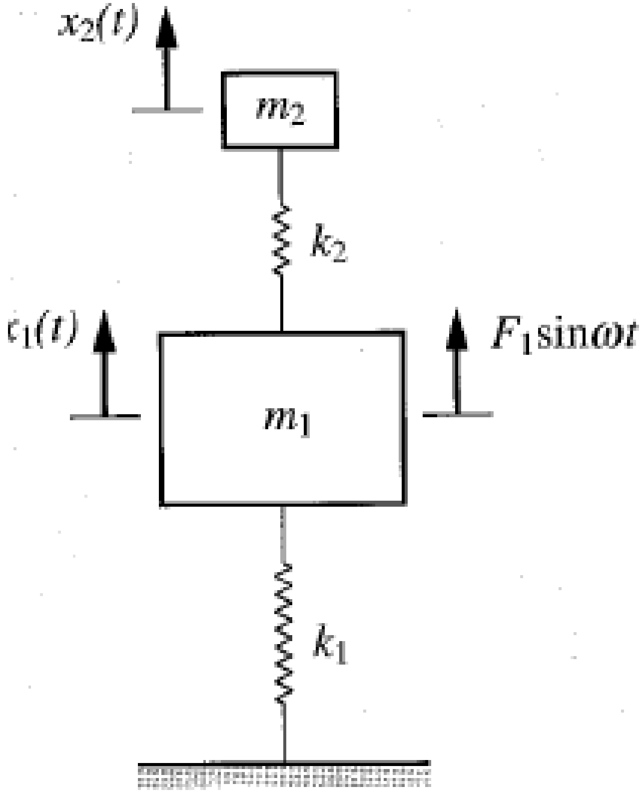
$$x_2(t) = -\frac{F_1}{k_2} \sin \omega t$$

The force in the absorber spring at any time is:

$$k_2 x_2(t) = -F_1 \sin \omega t$$

➔ the absorber exerts on the main mass a force which balances exactly the applied force

Vibration absorbers



The absorber is designed for a given operating frequency

$$\omega_a = \omega$$

which can be achieved by a wide choice of absorber parameters

The vibration absorber can perform satisfactorily for operating frequencies close in value to ω

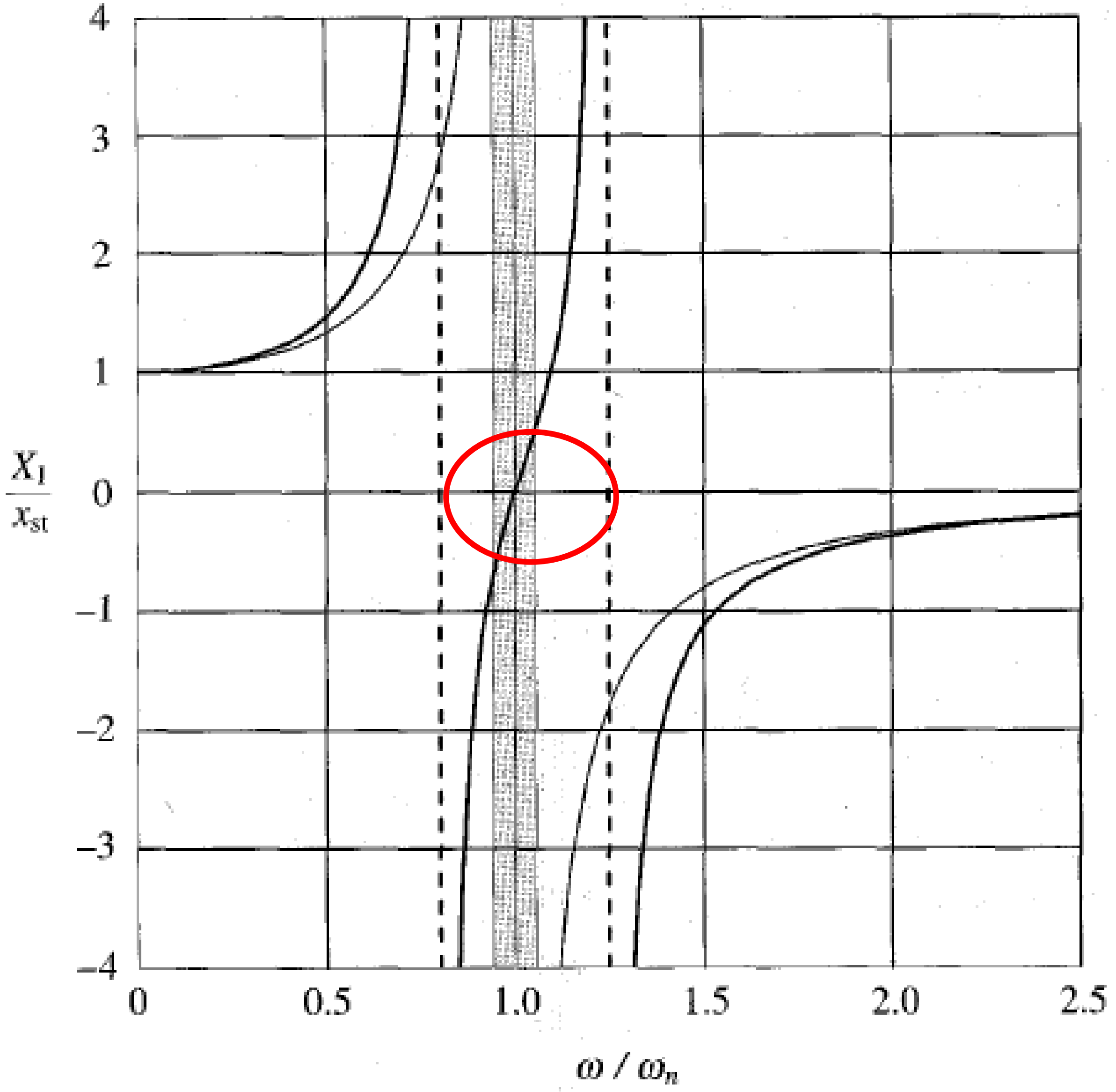
➔ in this case the motion of m_1 is not zero, but its amplitude is very small

Vibration absorbers

$$\omega_n = \omega_a$$

$$\mu = 0.2$$

- One disadvantage of the vibration absorber is that two new resonant frequencies are created.
- To reduce the amplitude at the resonant frequencies, damping can be added, but this results in an increase in amplitude in the neighborhood of the operating frequency



Example

A diesel engine, weighing 3000 N, is supported on a pedestal mount. It has been observed that the engine induces vibration into the surrounding area through its pedestal mount at an operating speed of 6000 rpm.

Determine the parameters of the vibration absorber that will reduce the vibration when mounted on the pedestal. The magnitude of the exciting force is 250 N, and the amplitude of motion of the auxiliary mass is to be limited to 2 mm.

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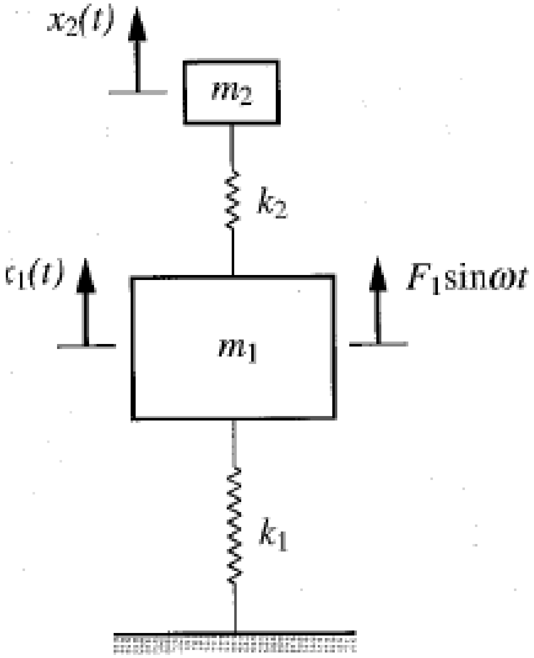


$$f = \frac{6000}{60} = 100 Hz \longrightarrow \omega = 628.32 \text{ rad/s}$$

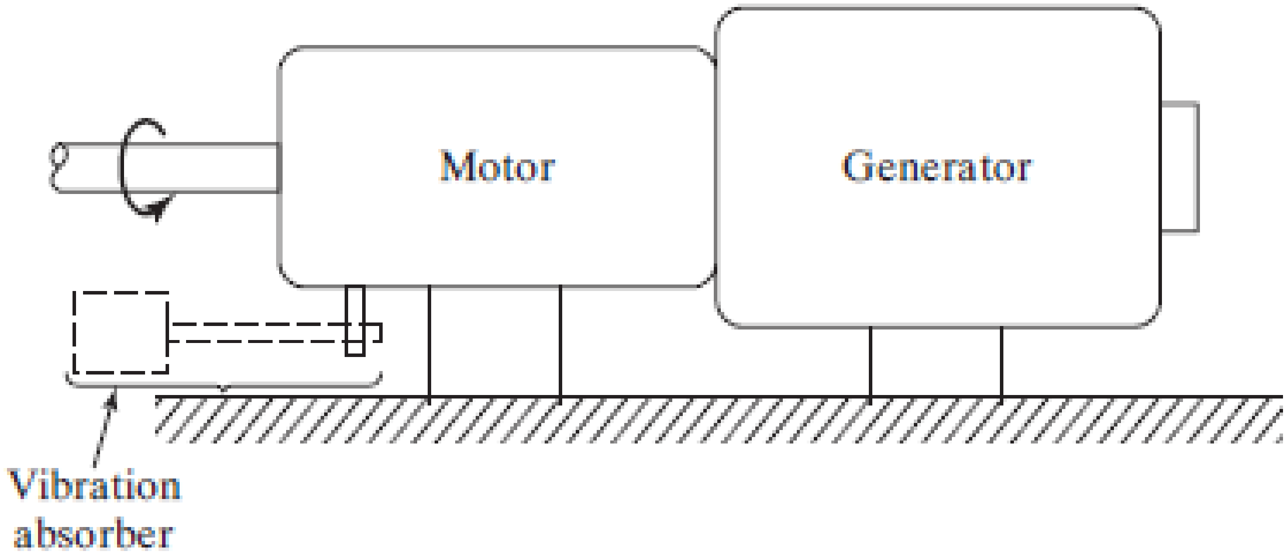
$$X_2 = -\frac{F_1}{k_2} \longrightarrow F_1 = m_2 \omega^2 X_2 \longrightarrow m_2 = \frac{250}{0.002 \times (628.32)^2} = 0.316 \text{ kg}$$

$$\longrightarrow k_2 = \omega^2 m_2 = (628.32)^2 \times 0.316 = 125009 \text{N/m}$$

Example

A motor-generator set is designed to operate in the speed range of 2000 to 4000 rpm. However, the set is found to vibrate violently at a speed of 3000 rpm due to a slight unbalance in the rotor. It is proposed to attach a cantilever mounted lumped-mass absorber system to eliminate the problem. When a cantilever carrying a trial mass of 2 kg tuned to 3000 rpm is attached to the set, the resulting natural frequencies of the system are found to be 2500 rpm and 3500 rpm.

Design the absorber to be attached (by specifying its mass and stiffness) so that the natural frequencies of the total system fall outside the operating-speed range of the motor-generator set.



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Design the absorber to be attached (by specifying its mass and stiffness) so that the natural frequencies of the total system fall outside the operating-speed range of the motor-generator set.

$$\omega_1 = \sqrt{k_1/m_1}$$

$$\omega_2 = \sqrt{k_2/m_2}$$

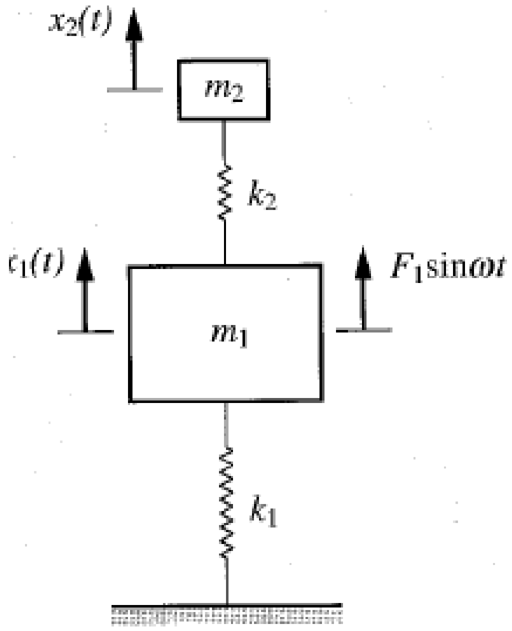
$$x_{st} = F_1/k_1$$

$$\mu = m_2/m_1$$

$$r_1 = \frac{\Omega_1}{\omega_2}$$

$$r_2 = \frac{\Omega_2}{\omega_2}$$

resonant frequencies of the combined system



Example

$$\left.\begin{matrix}\left(\dfrac{\Omega_1}{\omega_2}\right)^2 \\ \left(\dfrac{\Omega_2}{\omega_2}\right)^2\end{matrix}\right\} = \frac{\left\{\left[1+\left(1+\dfrac{m_2}{m_1}\right)\left(\dfrac{\omega_2}{\omega_1}\right)^2\right] \mp \left\{\left[1+\left(1+\dfrac{m_2}{m_1}\right)\left(\dfrac{\omega_2}{\omega_1}\right)^2\right]^2 - 4\left(\dfrac{\omega_2}{\omega_1}\right)^2\right\}^{1/2}\right\}}{2\left(\dfrac{\omega_2}{\omega_1}\right)^2}$$

$$r_1^2, r_2^2 = \left(1+\frac{\mu}{2}\right) \pm \sqrt{\left(1+\frac{\mu}{2}\right)^2 - 1}$$

$$r_1 = \frac{\Omega_1}{\omega_2} = \frac{261.80}{314.16} = 0.8333$$

$$r_2 = \frac{\Omega_2}{\omega_2} = \frac{366.52}{314.16} = 1.1667$$

Example

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$$\mu = \left(\frac{r_1^4+1}{r_1^2}\right) - 2 = 0.1345$$

$$\Downarrow$$

$$\mu = \frac{m_2}{m_1}$$

$$\Rightarrow \quad m_1 = \frac{m_2}{\mu} = 14.8699 \text{ kg}$$

Example

The specified lower limit of Ω_1 is 2000 rpm or 209.44 rad/s

Then

$$r_1 = \frac{\Omega_1}{\omega_2} = \frac{209.44}{314.16} = 0.6667 \qquad \mu = \frac{m_2}{m_1} = 0.6942 \qquad m_2 = 10.3227 \text{ kg}$$

$$r_2^2 = \left(1 + \frac{\mu}{2}\right) + \sqrt{\left(1 + \frac{\mu}{2}\right)^2 - 1} = 2.2497 \qquad \Omega_2 \simeq 4499.4 \text{ rpm}$$

➔ larger than the specified upper limit of 4000 rpm

The spring stiffness of the absorber is given by $k_2 = \omega_2^2 m_2 = \left(314.16\right)^2 \times 10.3227 = 1.0188 \times 10^6 \text{ N/m}$