北京航空航天大学
BEIHANG UNIVERSITY

飞行力学
Flight Mechanics

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Flight orbit

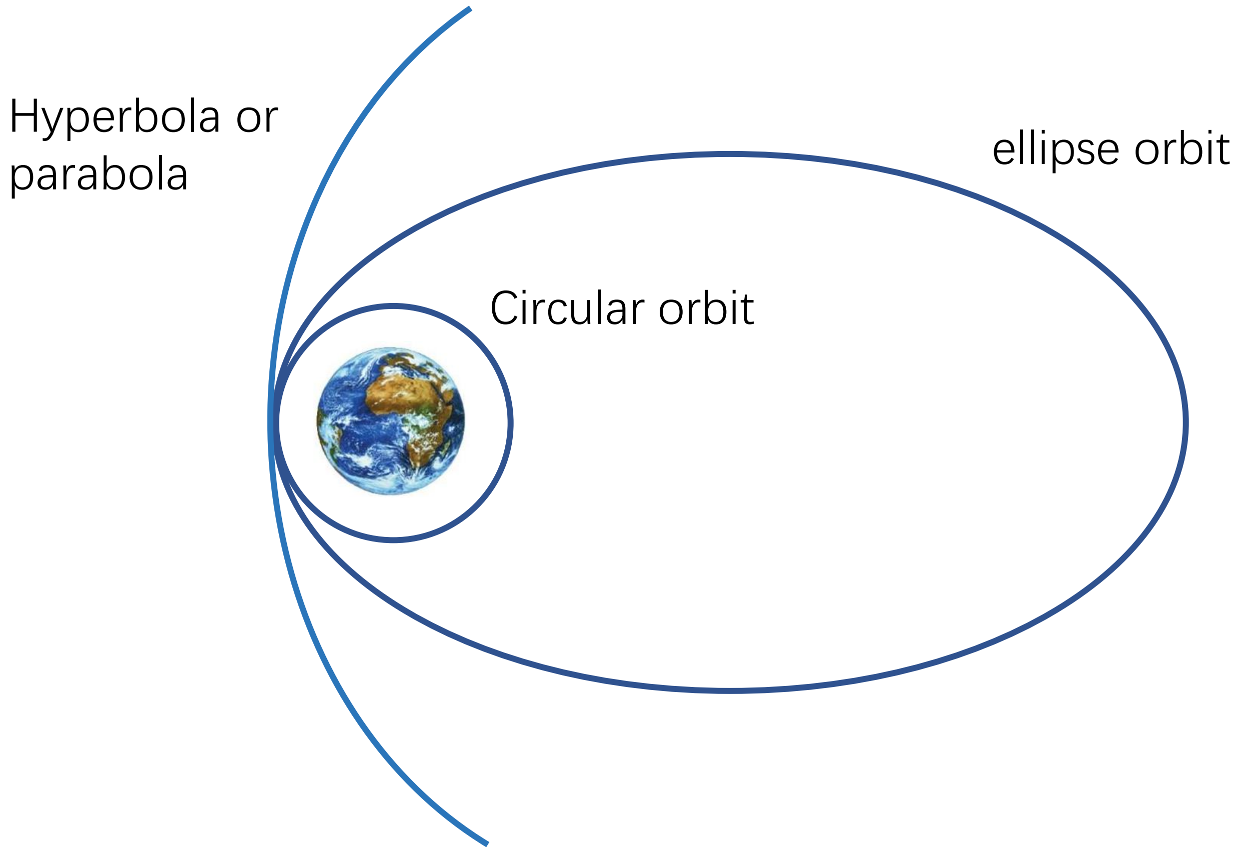
Review

Type of Trajectory	e	Energy Relation
Ellipse	< 1	$\frac{1}{2}mV^2 < \frac{GMm}{r}$
Parabola	$= 1$	$\frac{1}{2}mV^2 = \frac{GMm}{r}$
Hyperbola	> 1	$\frac{1}{2}mV^2 > \frac{GMm}{r}$

escape orbit

Flight orbit

Review



Flight orbit

Hyperbolic orbit

Semi-major axis

$a < 0$

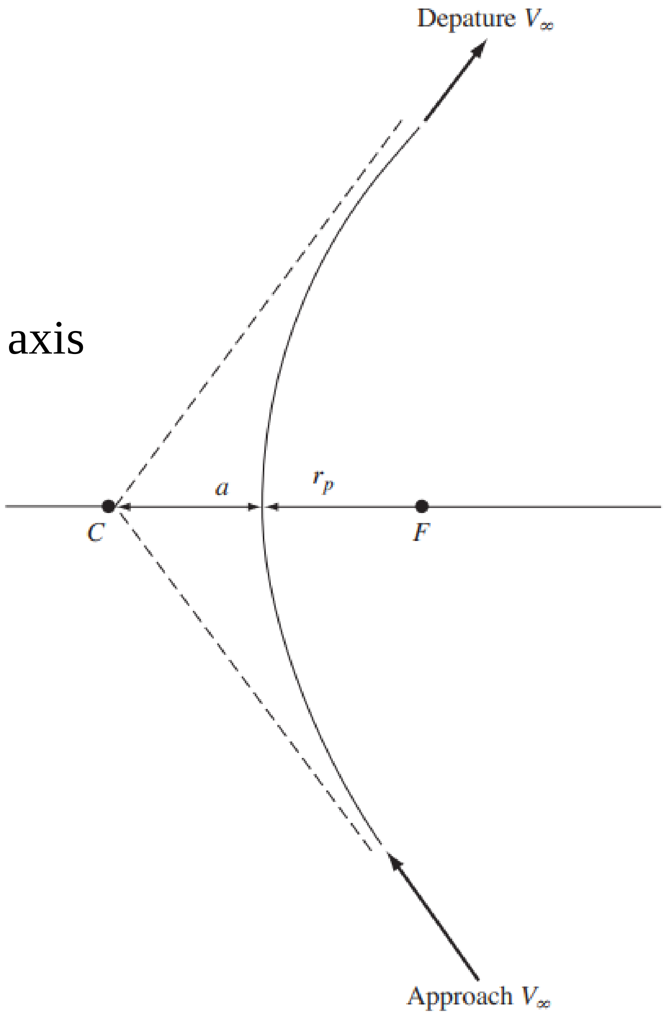


Figure 8.30 Hyperbolic trajectory.

Flight orbit

$$H = -m\frac{k^2}{2a}$$

Specific energy

$$E = \frac{H}{m} = -\frac{k^2}{2a}$$

Ellipse orbit ($a > 0$) , E < 0.

Hyperbolic orbit ($a < 0$) , E > 0.

Flight orbit

Parameters

$$k^2 = GM = 3.986 \times 10^{14}\ m^3/s^2$$

symbol	meaning	ellipse	hyperbola
a	semi-major axis	> 0	< 0
e	eccentricity	< 1	> 1
E	(specific) energy	< 0	> 0
$r(\theta)$	radial distance	$a(1-e^2)/(1+e\cos\theta)$	
r_{min}	minimum distance (pericenter)	$a(1-e)$	
r_{max}	maximum distance (apocenter)	$a(1+e)$	∞
V	velocity	$\sqrt{2k^2/r-k^2/a}$	$\sqrt{V_{es}^2-V_\infty^2}$

Two Line Elements (TLE)

Definition

The Two Line Elements or TLE format is the standard way to describe satellites in orbit above Earth. Understanding the TLE format allows users to make predictions about future or past satellite passes.

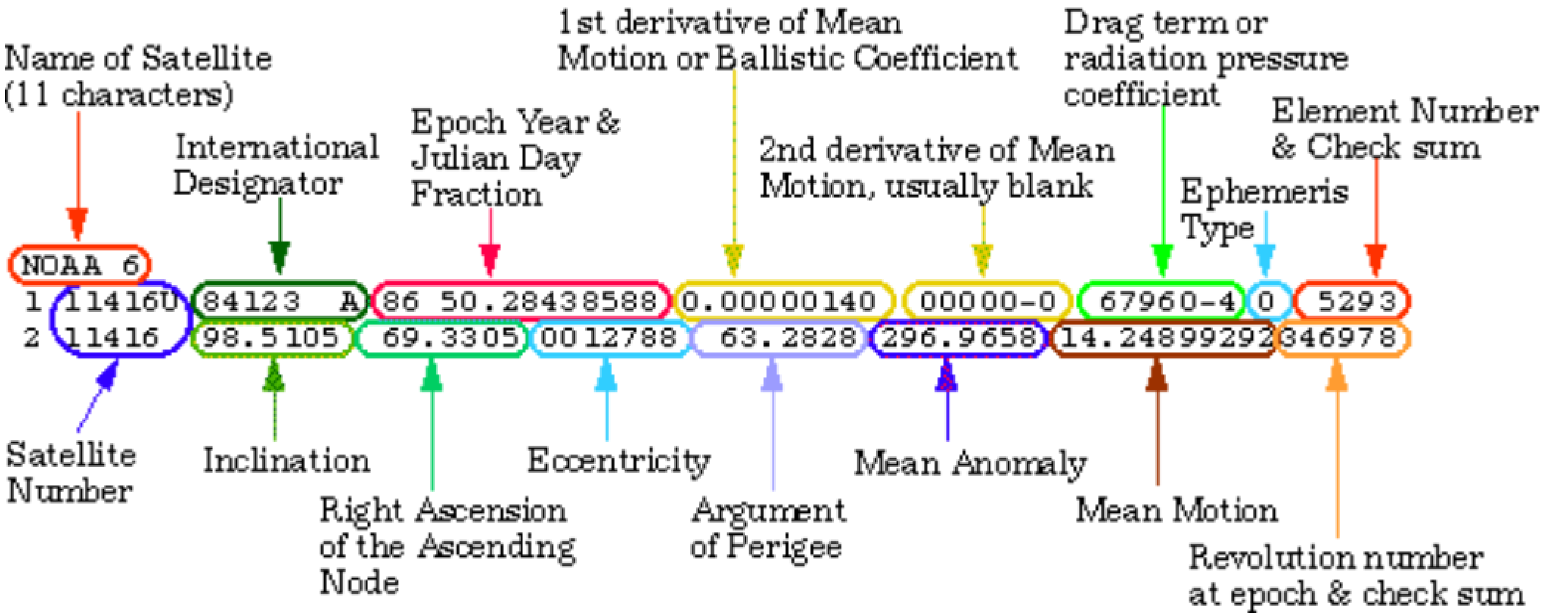
```
ISS (ZARYA)
1 25544U 98067A 20331.01187177 .00003392 00000-0 69526-4 0 9990
2 25544 51.6456 267.7478 0001965 82.1336 12.7330 15.49066632257107

SKCUBE
1 42789U 17036AA 19084.74586761 .00001246 00000-0 56588-4 0 9992
2 42789 97.3621 144.5852 0012165 160.6847 199.4853 15.22573301 97347
```

TLE data for most spacecraft can be downloaded from https://www.space-track.org/

Two Line Elements (TLE)

Data format



[Source: Two Line Elements (TLE) – Kaitlyn's Tech Logs]

Two Line Elements (TLE)

TLE of **Tiangong**

中国空间站轨道参数

发布日期：2022-05-19　　信息来源：中国载人航天工程办公室

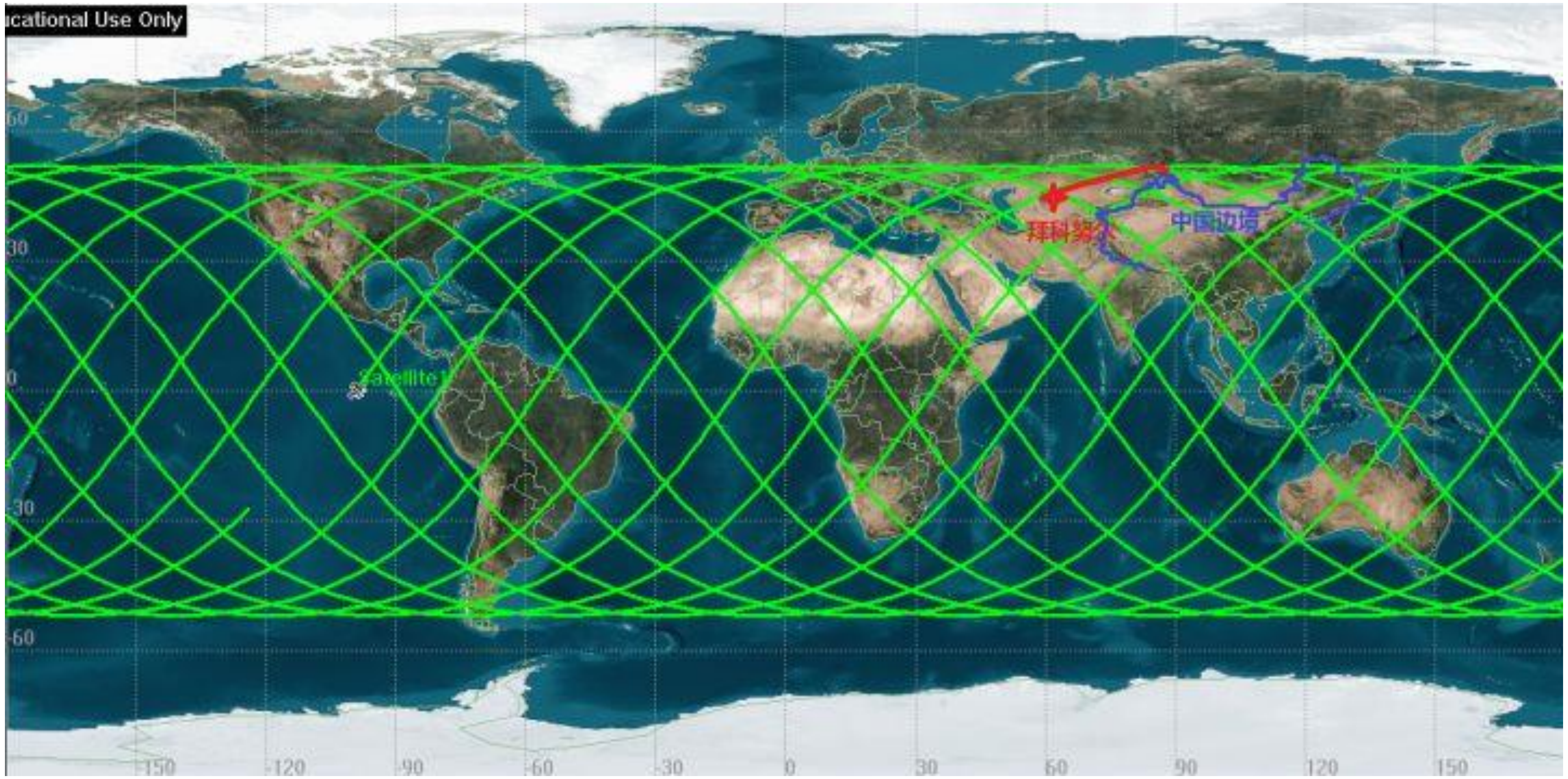
```
1 48274U 21035A   22139.00000000  .00028715  00000-0  34150-3 0  9992
2 48274  41.4712  36.3623 0002231   1.8855 335.0230 15.59814191 60213
```

Mean motion
(revolutions/day)

[Source: 中国空间站轨道参数_中国载人航天官方网站 (cmse.gov.cn)](#)

Two Line Elements (TLE)

Trajectory of ISS



Drawn by Satellite Tool Kit (STK)

Two Line Elements (TLE)

Orbit parameters of **Tiangong**

$$\tau = \frac{1}{n}[day] = \frac{86164}{15.598}[s] = \mathbf{5524}\ s$$

$$a = \sqrt[3]{\frac{\tau^2}{4\pi^2}GM} = 6.754 \times 10^6[m] = \mathbf{6754}\ km$$

$$h = a - r_{earth} = 6754 - 6378\ [km] = \mathbf{376}\ km$$

$$E = -\frac{GM}{2a} = -\mathbf{29.5}\ km^2/s^2$$

Flight orbit

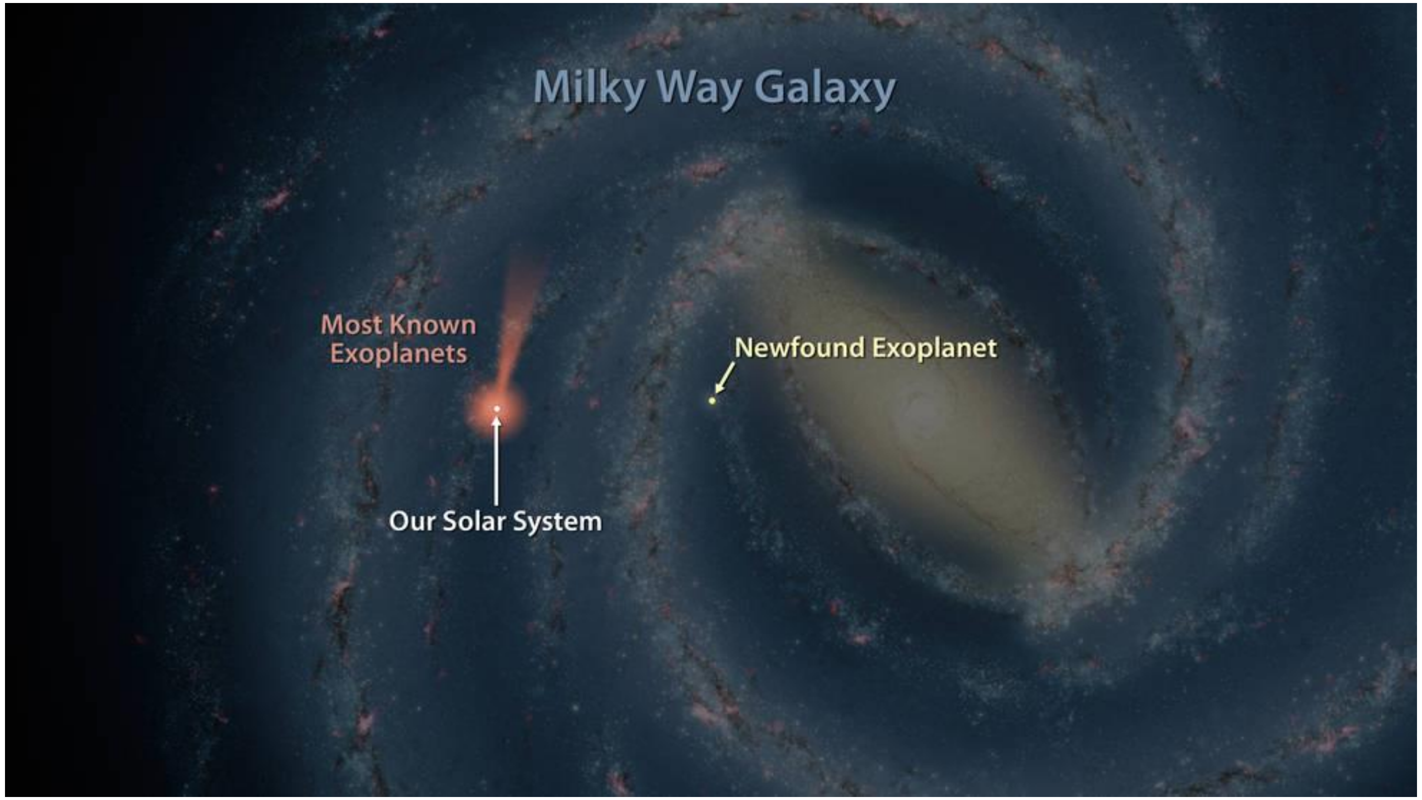
Specific energy comparison

satellite	altitude [km]	specific energy [km^2/s^2]
NVISAT	800	-27.8
LAGEOS	5,900	-16.2
GEO	35,900	-4.7
Moon	384,000	-0.5
Tiangong	376	-29.5
in parking orbit	185	-30.4
Hohmann orbit to Mars	185 (after 1st ΔV)	+4.3

Interplanetary flight

The Galaxy

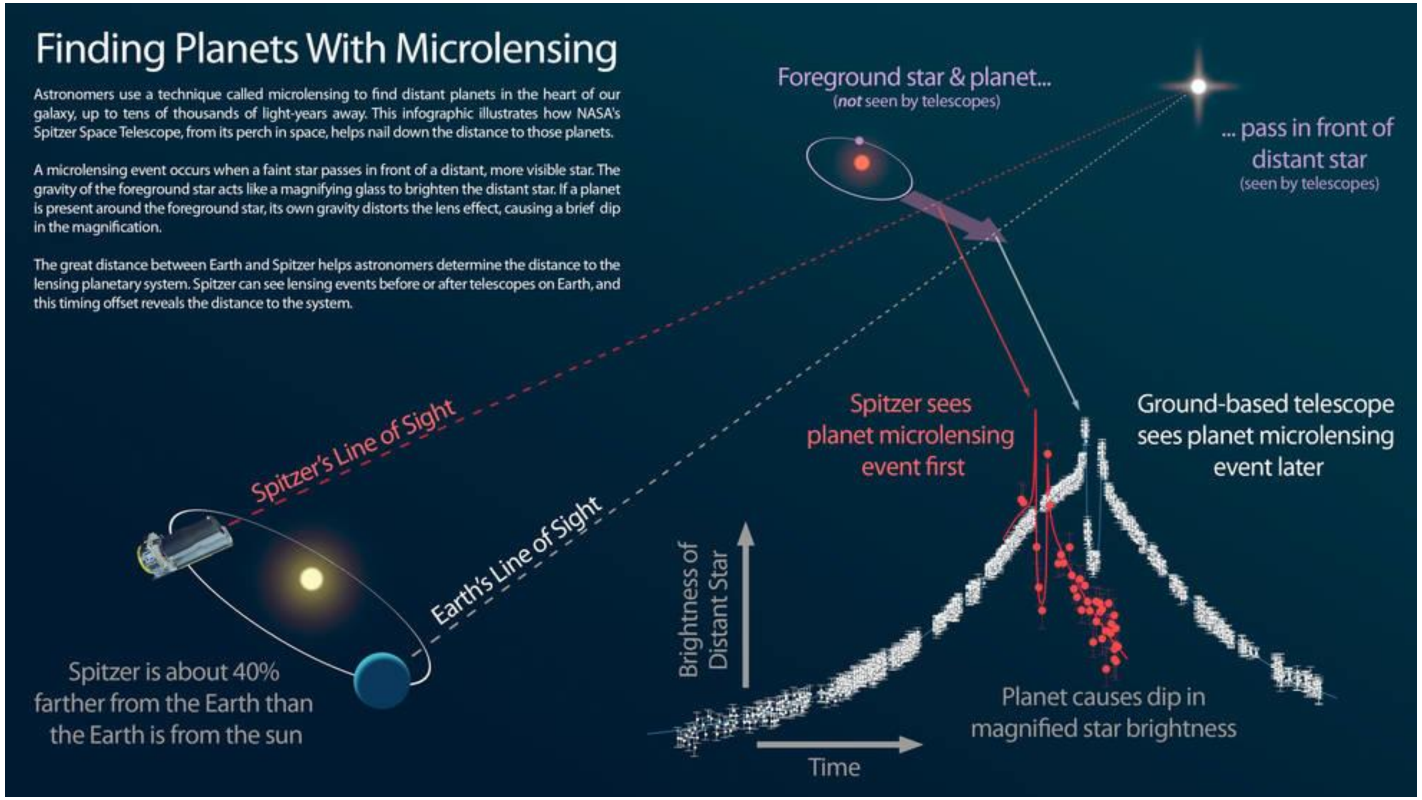
Source: NASA



Interplanetary flight

The Galaxy

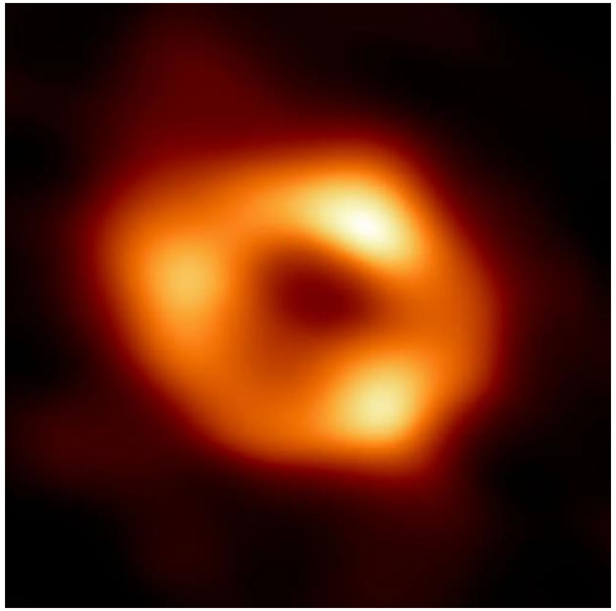
Source: NASA



The NASA's Spitzer Space Telescope

Interplanetary flight

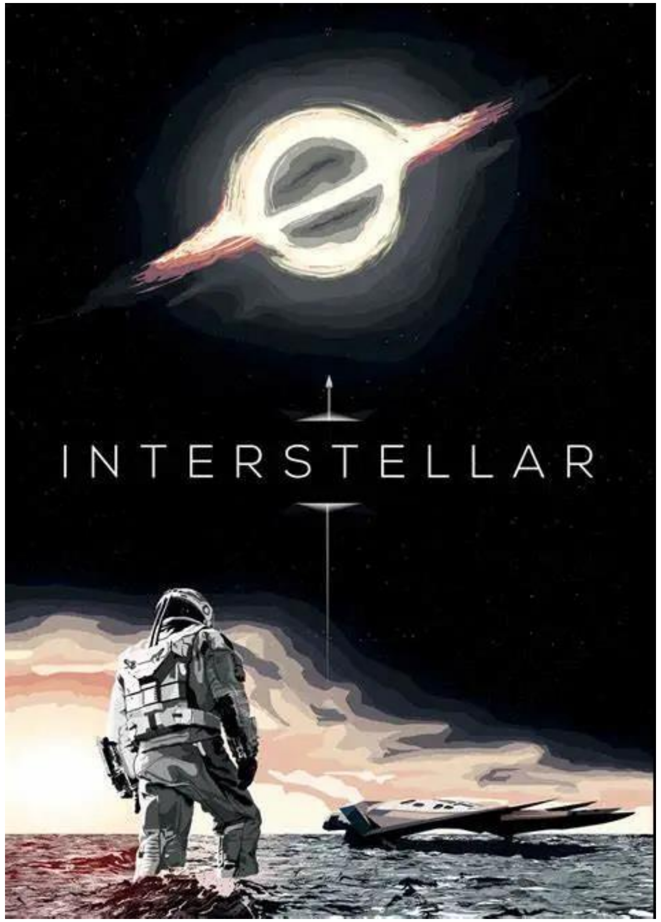
The blackhole of Galaxy



Source: 新华网

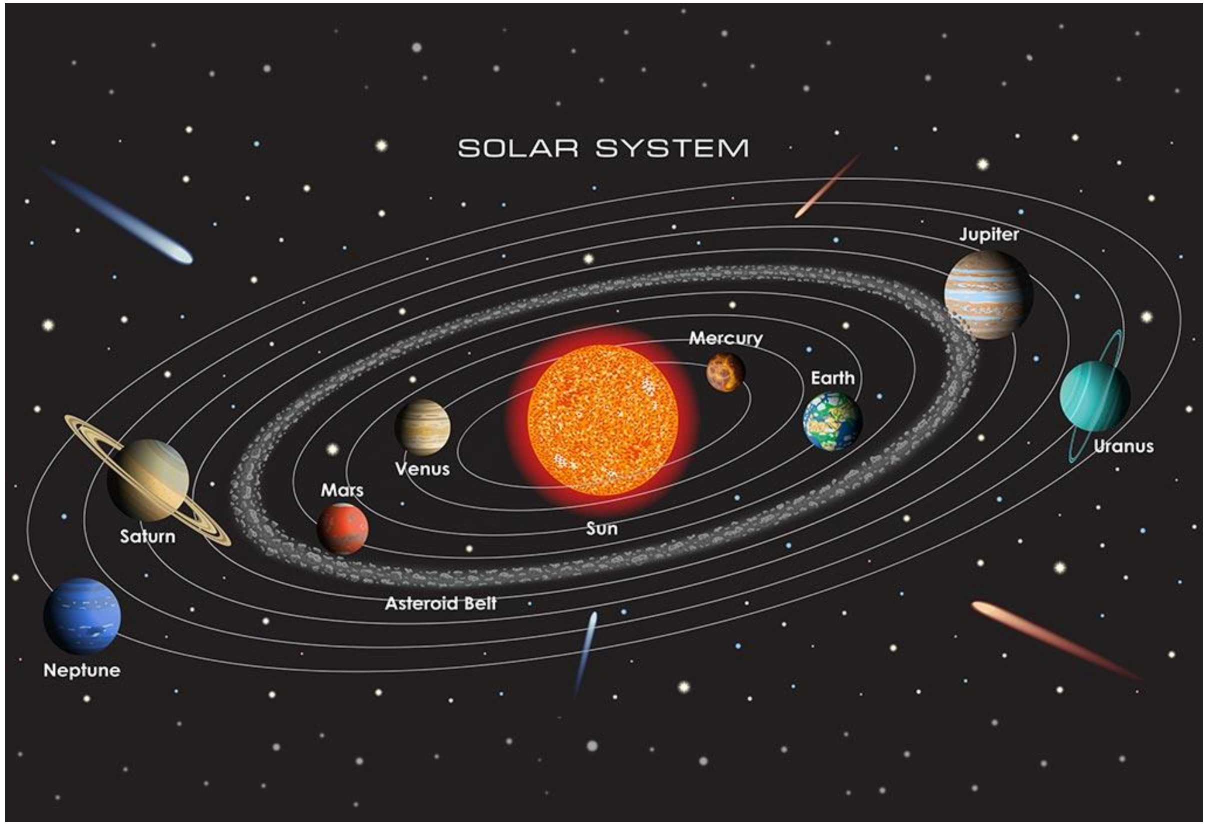
Blackhole at the Galaxy center (2022.05.13)

Source: internet



Interplanetary flight

Solar system



[Source: WorldAtlas]

Interplanetary flight

Solar system

planet	mean distance [AU]	eccentricity [-]	inclination [°]
Mercury (水星)	0.387	0.206	7.0
Venus (金星)	0.723	0.007	3.4
Earth (地球)	1.000	0.017	0.0
Mars (火星)	1.524	0.093	1.9
Jupiter (木星)	5.203	0.048	1.3
Saturn (土星)	9.537	0.054	2.5
Uranus (天王星)	19.191	0.047	0.8
Neptune (海王星)	30.069	0.009	1.8
Pluto *	39.482	0.249	17.1

Interplanetary flight

Solar system - conclusions

- Orbits of planets more or less circular (Except Mercury)
- Orbits of planets more or less coplanar
- 2-dimensional situation with circular orbit → good 1st order model
- Scale of interplanetary travel >> scale of earth-bound missions

Interplanetary flight

Question

- How can we escape from earth gravity?
- How can we travel to other planets/asteroids in the most efficient way?
- How can we reach beyond 10 AU?

Hohmann transfer

Two impulses maneuver

$$\boxed{(\Delta V)^2 = V_1^2 + V_2^2 - 2\,V_1 V_2 \cos\alpha}$$

$$\alpha \Rightarrow 0$$

$\Delta V_1 =$ impulse at point 1
$\Delta V_2 =$ impulse at point 2
$V_{pt} =$ velocity at periapsis on the transfer orbit
$V_{at} =$ velocity at apoapsis on the transfer orbit
$V_1 =$ velocity at point 1 on orbit 1
$V_2 =$ velocity at point 2 on orbit 2

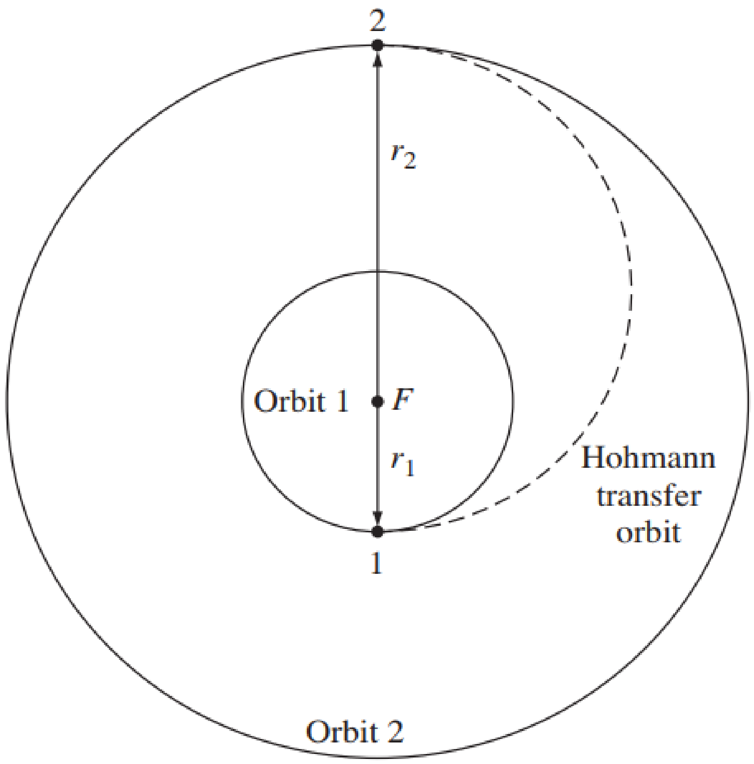


Figure 8.29 Illustration of the Hohmann transfer orbit.

Hohmann transfer

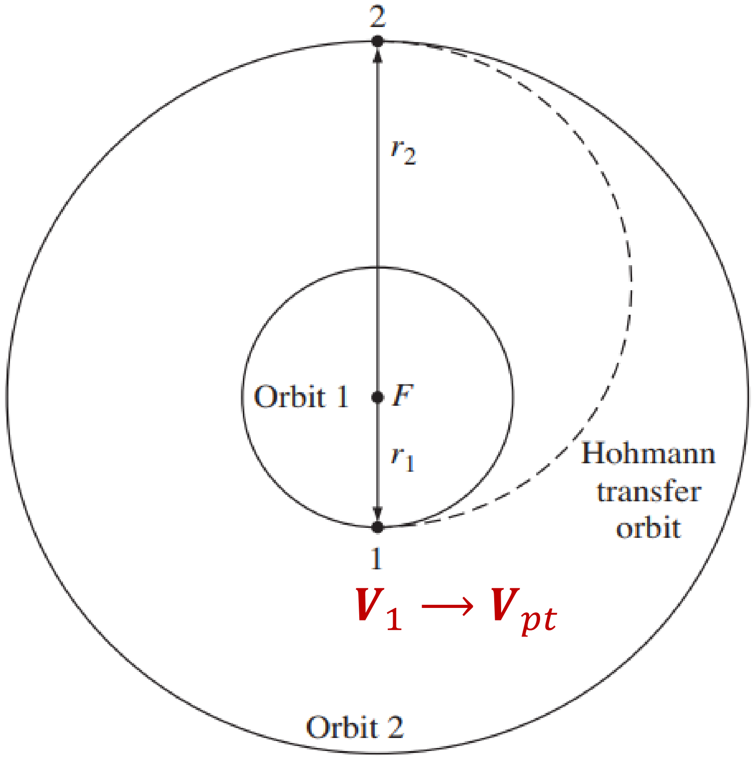
Advantages

- Minimum energy transfer

$$\Delta \boldsymbol{V}_1 = \boldsymbol{V}_{pt} - \boldsymbol{V}_1$$

$$\Delta \boldsymbol{V}_2 = \boldsymbol{V}_2 - \boldsymbol{V}_{at}$$

$\boldsymbol{V}_2 \longleftarrow \boldsymbol{V}_{at}$



$\boldsymbol{V}_1 \longrightarrow \boldsymbol{V}_{pt}$

Hohmann transfer

Example 8.8

Consider the Space Shuttle in a low-earth circular orbit at an altitude of 200 km above sea level. The payload of the shuttle is a satellite to be boosted by means of a Hohmann transfer into geosynchronous circular orbit at an altitude of 35,700 km above sea level. Calculate the total impulse $\Delta \boldsymbol{V}$ required for this transfer.

$$\Delta \boldsymbol{V}_1 = \boldsymbol{V}_{pt} - \boldsymbol{V}_1$$

$$\Delta \boldsymbol{V}_2 = \boldsymbol{V}_2 - \boldsymbol{V}_{at}$$

Hohmann transfer

Example 8.8

$$\boxed{V_1 = \sqrt{\frac{2k^2}{r_1} - \frac{k^2}{a_1}}}$$

$$r_1 = 6.4 \times 10^6 + 2 \times 10^5 = 6.6 \times 10^6 \text{ m}$$

$$r_2 = 3.57 \times 10^7 + 6.4 \times 10^6 = 4.21 \times 10^7 \text{ m}$$

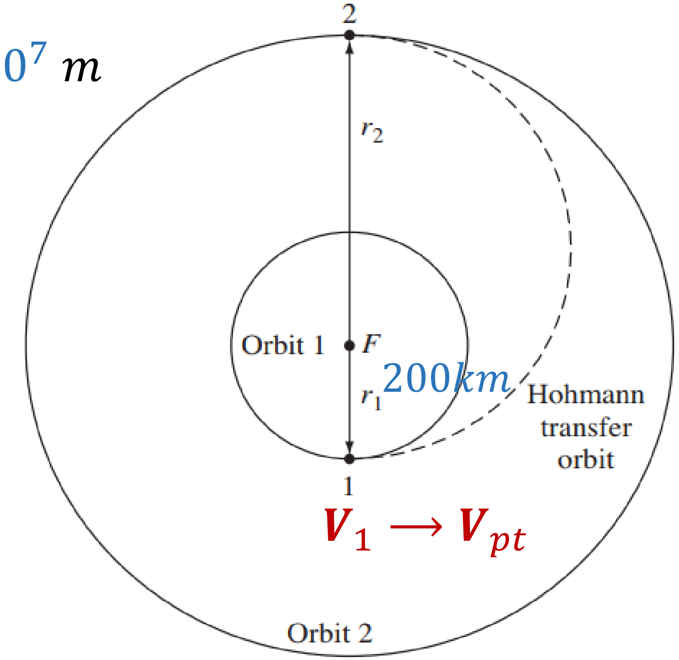
$$a = \frac{r_1 + r_2}{2} = 2.435 \times 10^7 \ m$$

$$V_1 = \sqrt{\frac{3.986 \times 10^{14}}{6.6 \times 10^6}} = 7771 \text{m/s}$$

$$V_{pt} = \sqrt{\frac{2k^2}{r_1} - \frac{k^2}{a_1}} = \sqrt{\frac{2(3.986 \times 10^{14})}{6.6 \times 10^6} - \frac{3.986 \times 10^{14}}{2.435 \times 10^7}}$$

$$= \sqrt{1.2079 \times 10^8 - 0.1637 \times 10^8} = 10,219 \text{m/s}$$

$$\Delta V_1 = V_{pt} - V_1$$



Hohmann transfer

Example 8.8

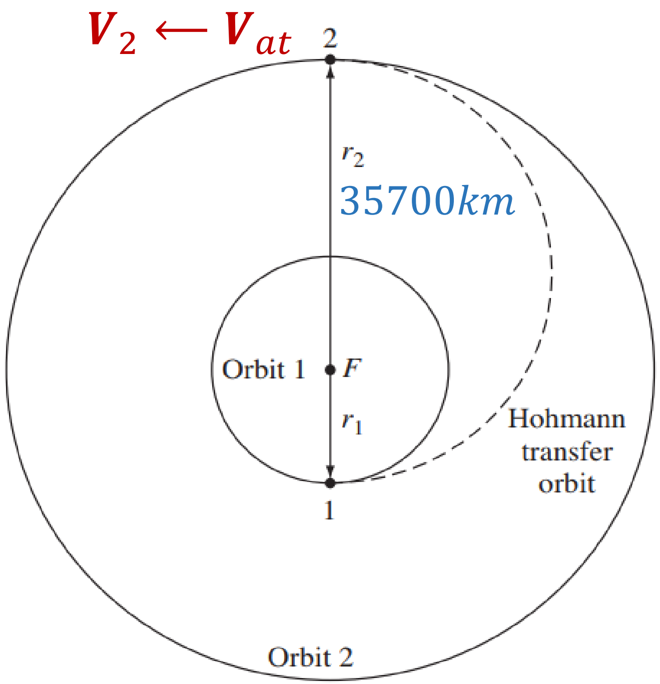
$$V_1 = \sqrt{\frac{2k^2}{r_1} - \frac{k^2}{a_1}}$$

$$V_2 = \sqrt{\frac{k^2}{r_2}} = \sqrt{\frac{3.986\times10^{14}}{4.21\times10^7}} = 3077\,\mathrm{m/s}$$

$$V_{at} = \sqrt{\frac{2k^2}{r_2} - \frac{k^2}{a}} = \sqrt{\frac{2(3.986\times10^{14})}{4.21\times10^7} - \frac{3.986\times10^{14}}{2.435\times10^7}}$$

$$= \sqrt{1.8936\times10^7 - 1.637\times10^7} = 1602\,\mathrm{m/s}$$

$$\Delta \boldsymbol{V}_2 = \boldsymbol{V}_2 - \boldsymbol{V}_{at}$$



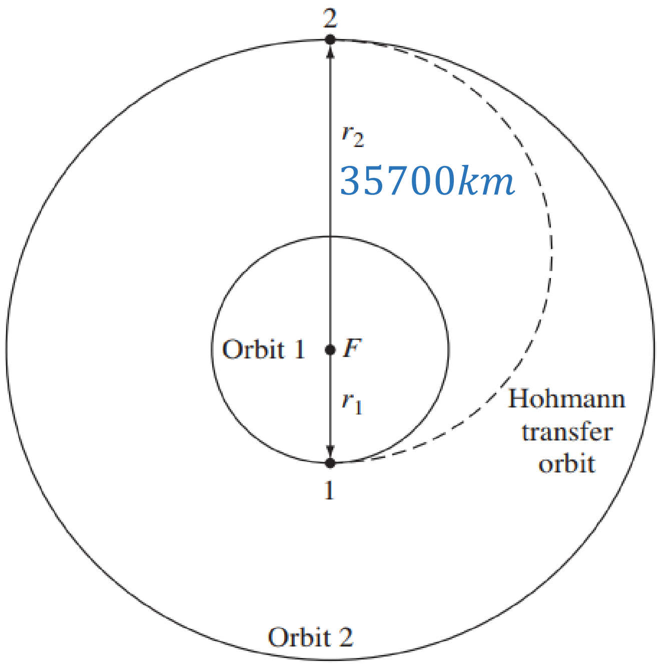
$$\Delta \boldsymbol{V}_1 = \Delta \boldsymbol{V}_1 + \Delta \boldsymbol{V}_2$$

The problem is solved !

Hohmann transfer

Example 8.8 – extra practice

Calculate the time from orbit 1 to orbit 2.

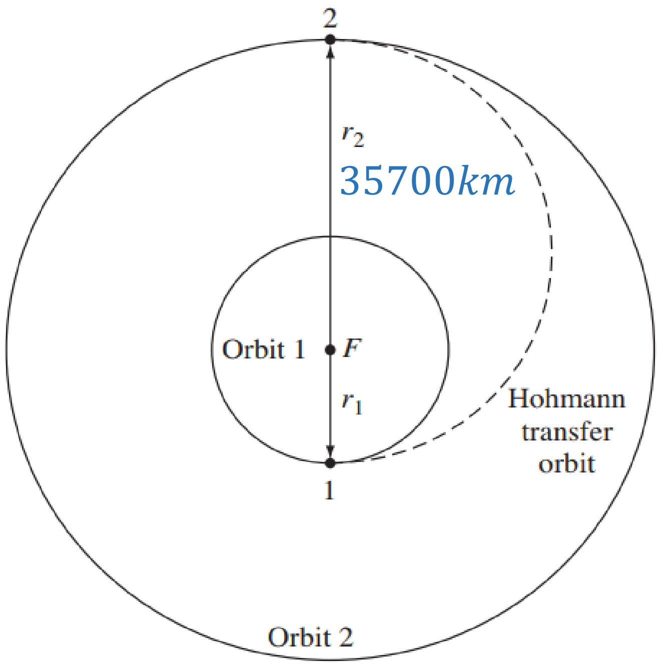


Hohmann transfer

Example 8.8 – extra practice

The time of transit in the Hohmann orbit is one half the period of the full elliptic orbit. Kepler's third law gives:

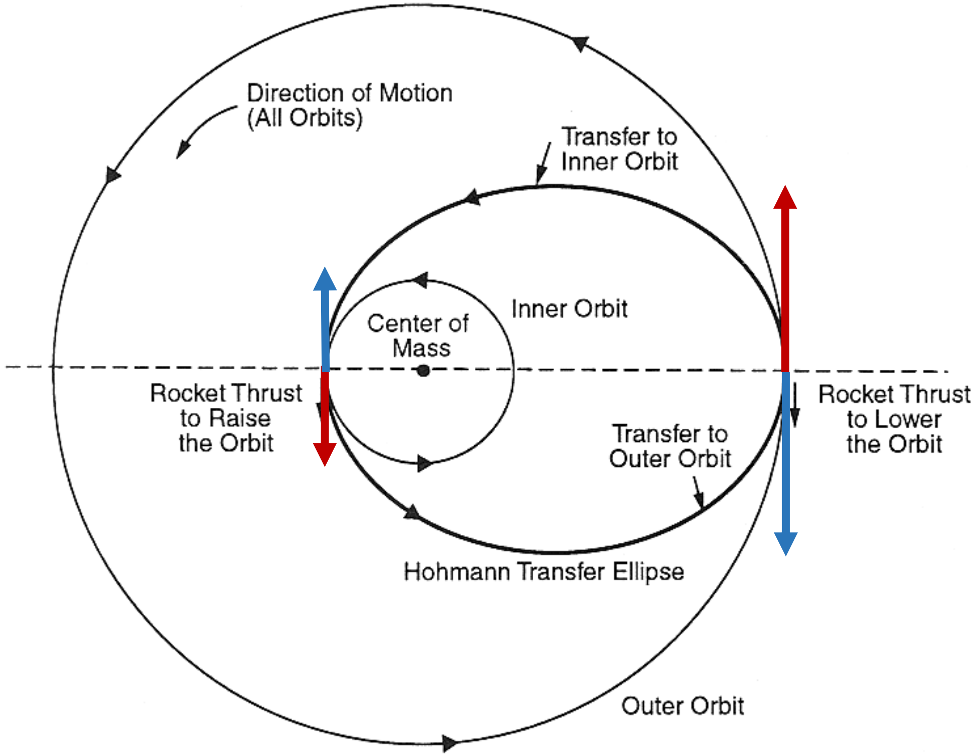
$$\tau_H = \frac{\tau}{2} = \pi\sqrt{\frac{a^3}{k^2}} = \pi\sqrt{\frac{(r_1 + r_2)^3}{8GM}}$$



Hohmann transfer

Hohmann transfer between orbits around Earth:

- Coplanar orbits
- Impulsive shots
- Transfer orbit touches tangentially
- Minimum energy



Hohmann transfer

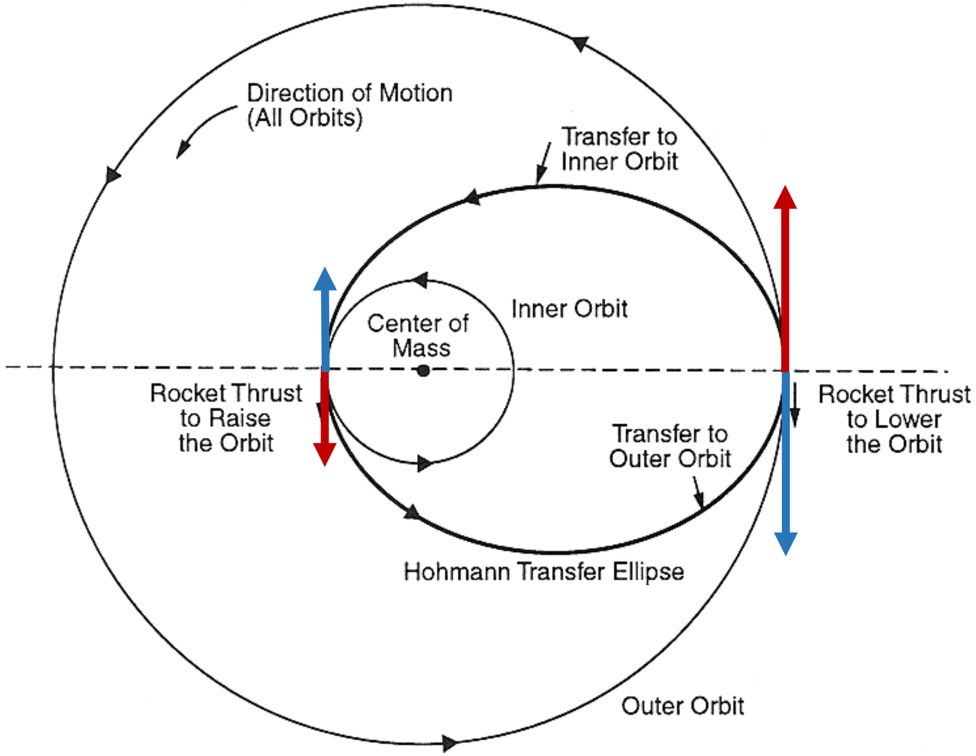
Hohmann transfer between orbits around Earth:

$$a = \frac{r_1 + r_2}{2}$$

$$\Delta \boldsymbol{V}_1 = \boldsymbol{V}_{pt} - \boldsymbol{V}_1 = \sqrt{GM_E\left(\frac{2}{r_1} - \frac{1}{a}\right)} - \sqrt{\frac{GM_E}{r_1}}$$

$$\Delta \boldsymbol{V}_2 = \boldsymbol{V}_2 - \boldsymbol{V}_{at} = \sqrt{\frac{GM_E}{r_2}} - \sqrt{GM_E\left(\frac{2}{r_2} - \frac{1}{a}\right)}$$

$$\tau_t = \frac{1}{2}\tau = \pi^2\sqrt{\frac{a^3}{GM_E}}$$



Hohmann transfer

Hohmann transfer between orbits around Sun:

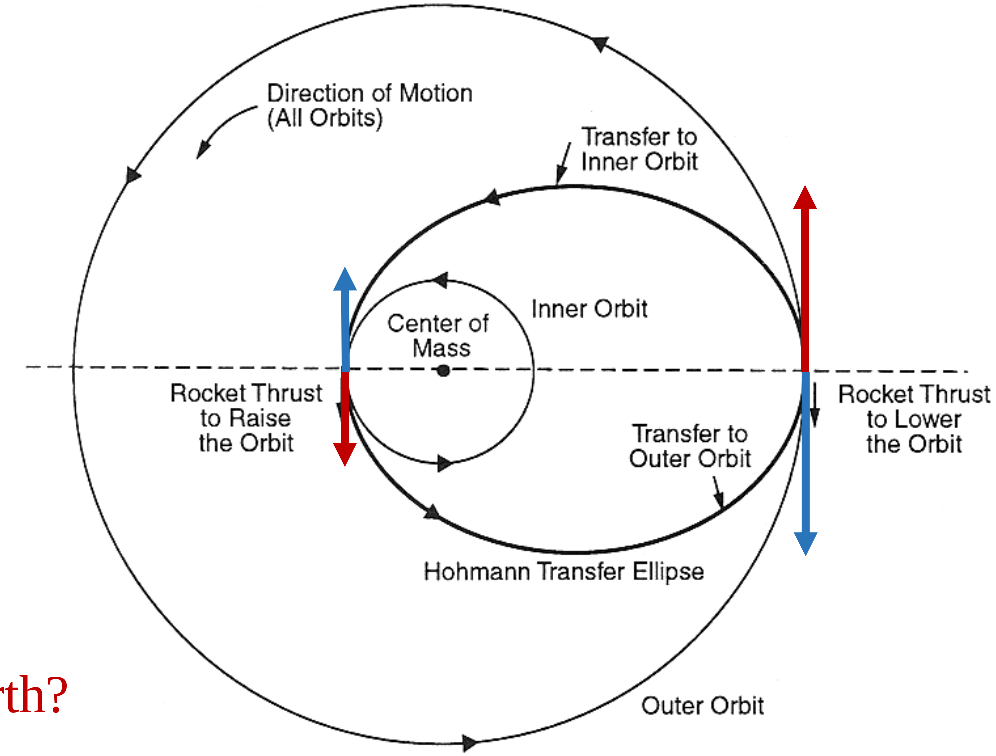
$$a = \frac{1}{2}\left(r_{dep} + r_{arr}\right)$$

$$\boldsymbol{V}_{\infty,1} = \boldsymbol{V}_{pt} - \boldsymbol{V}_{dep} = \sqrt{GM_S\left(\frac{2}{r_{dep}} - \frac{1}{a}\right)} - \sqrt{\frac{GM_S}{r_{dep}}}$$

$$\boldsymbol{V}_{\infty,2} = \boldsymbol{V}_{tar} - \boldsymbol{V}_{at} = \sqrt{\frac{GM_S}{r_{tar}}} - \sqrt{GM_S\left(\frac{2}{r_{tar}} - \frac{1}{a}\right)}$$

$$\tau_t = \frac{1}{2}\tau = \pi\sqrt{\frac{a^3}{GM_S}}$$

Identical to the earth?



Hohmann transfer

Interplanetary trajectory

three-body problem

Due to the three-body problem, the interplanetary trajectory is much more complicated. Hohmann transfer orbit does not have to be restricted followed.

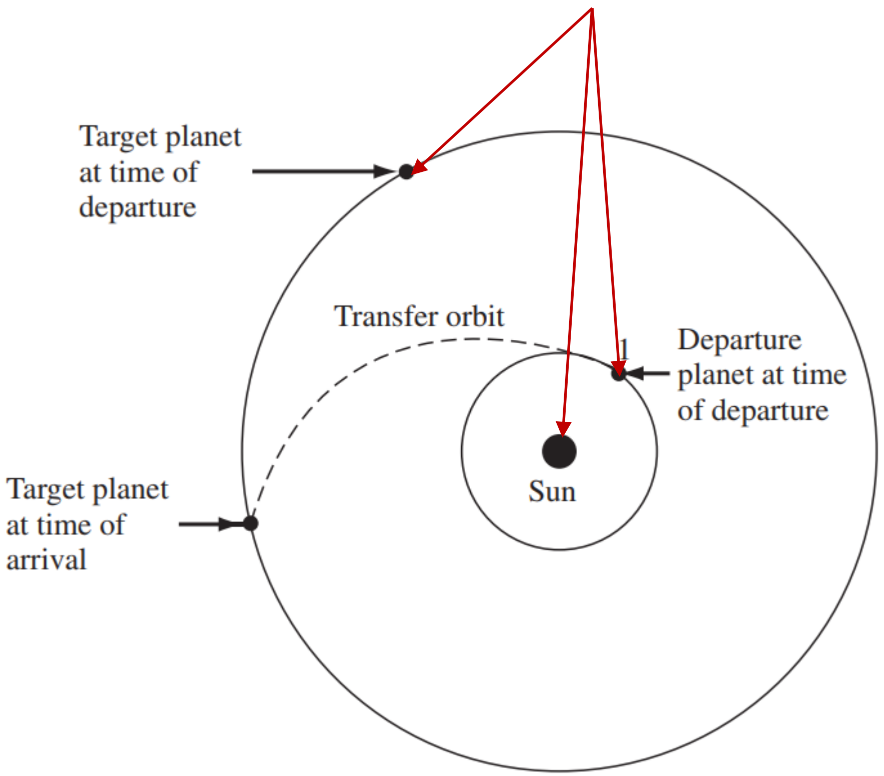
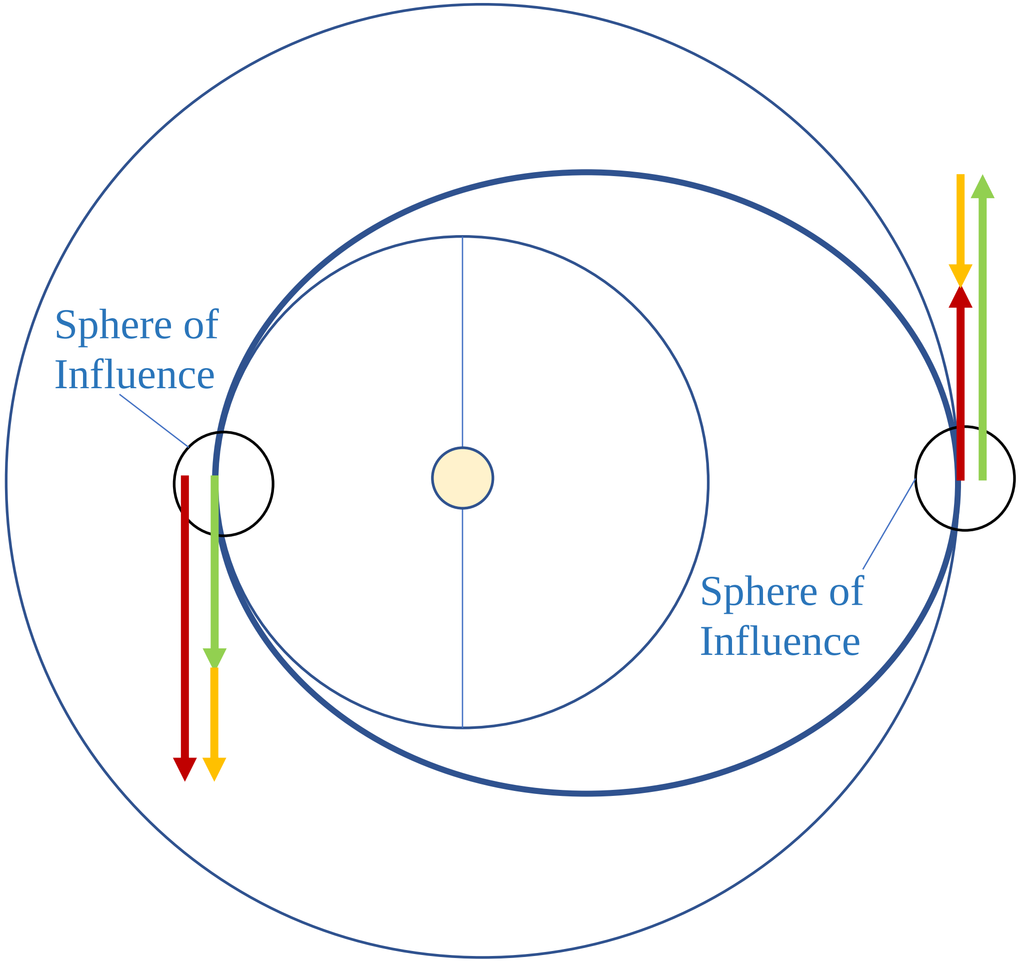


Figure 8.31 Heliocentric transfer orbit.

Hohmann transfer

Heliocentric velocities:

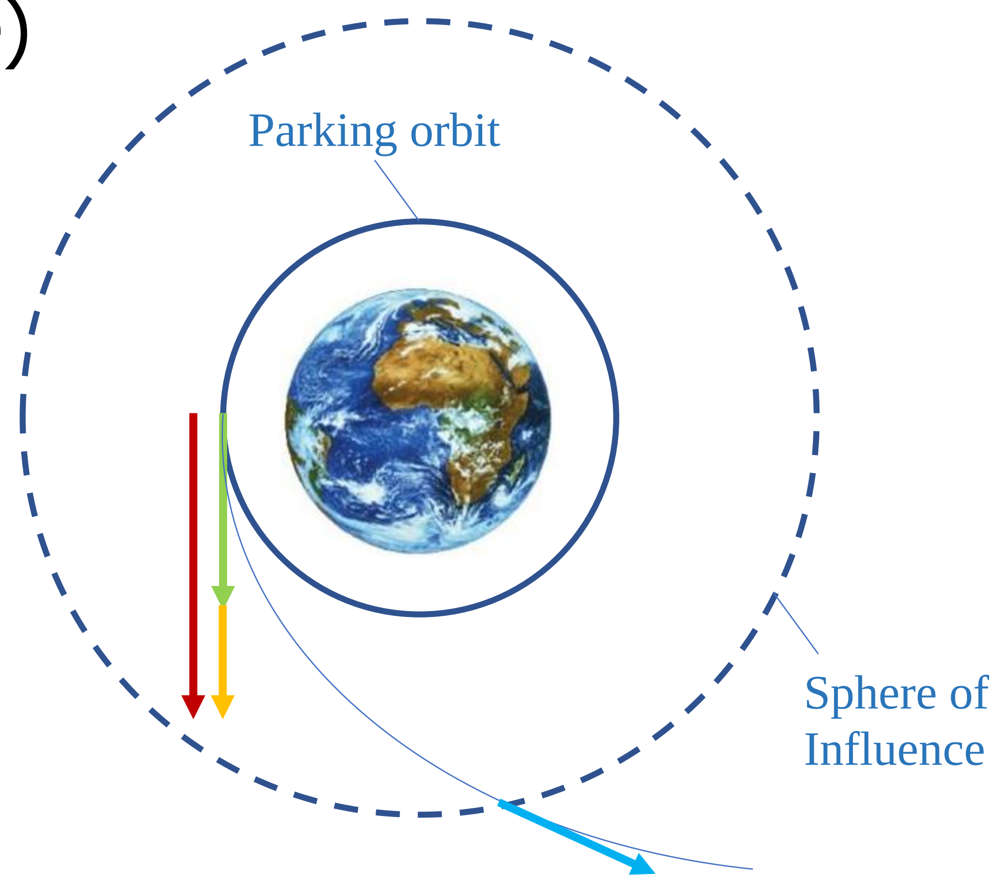
- V_{dep}, V_{tar} (planets)
- V_1, V_2 (Spacecraft)
- V_∞ (relative)



Hohmann transfer

Planetocentric velocities (Earth scale)

- V_c (Parking orbit)
- V_0 (Hyperbola)
- ΔV (Maneuver)
- V_∞ (excess velocity)



Hohmann orbit

Hohmann transfer

$$V = \sqrt{\frac{2k^2}{r} - \frac{k^2}{a}}$$

Escape velocity and excess velocity

$$V = \sqrt{\frac{2k^2}{r}} = V_{es}$$ Parabola orbit ($a \to \infty$)

$$V = \sqrt{\frac{2k^2}{r} \boxed{- \frac{k^2}{a}}}$$

Excess velocity ($r \to \infty$) : $V_\infty = \sqrt{-\frac{k^2}{a}}$

Hohmann transfer

Example 8.9

The initial hyperbolic trajectory of the *Viking I* Mars Lander upon departure from the earth had a semimajor axis of -1.885×10^4 km. Calculate the hyperbolic excess velocity provided by the space vehicle's *Titan IIIE* launch vehicle.

Hohmann transfer

Hohmann transfer between planets around sun

- Transfer starts in parking orbit around departure planet
- Planetocentric until leaving Sphere of influence (SoI)
- Relative velocity when crossing SoI: V_∞
- V_∞ achieved by maneuver ΔV in parking orbit
- Similarly around target planet
- Succession of 3 two-body problems

Hohmann transfer

Essential difference between around Earth and around Sun

- Earth missions: ΔV directly changes velocity from V_{circle} to V_{per} (or V_{apo}) of Hohmann transfer orbit
- Interplanetary missions: ΔV changes velocity from V_{circle} to value (larger than) V_{escape}, which results in V_{∞}

Hohmann transfer

Earth (185 km) → Mars (500 km)

Recipe (1-2):

step	parameter	expression	example
1	V_{dep} (heliocentric velocity of departure planet)	$V_{dep} = \sqrt{(GM_S/r_{dep})}$	29.785 km/s
2	V_{tar} (heliocentric velocity of target planet)	$V_{tar} = \sqrt{(GM_S/r_{tar})}$	24.130 km/s
3	V_{c0} (circular velocity around departure planet)	$V_{c0} = \sqrt{(GM_{dep}/r_0)}$	7.793 km/s
4	Vc3 (circular velocity around target planet)	$V_{c3} = \sqrt{(GM_{tar}/r_3)}$	3.315 km/s
5	atr (semi-major axis of transfer orbit)	$a_{tr} = (r_{dep} + r_{tar})/2$	188.77×10^6 km
6	e_{tr} (eccentricity of transfer orbit)	$e_{tr} = \lvert r_{tar} - r_{dep} \rvert / (r_{tar} + r_{dep})$	0.208
7	V_1 (heliocentric velocity at departure position)	$V_1 = \sqrt{[GM_S(2/rdep-1/atr)]}$	32.729 km/s
8	V_2 (heliocentric velocity at target position)	$V_2 = \sqrt{[GM_S(2/r_{tar}-1/a_{tr})]}$	21.481 km/s

Hohmann transfer

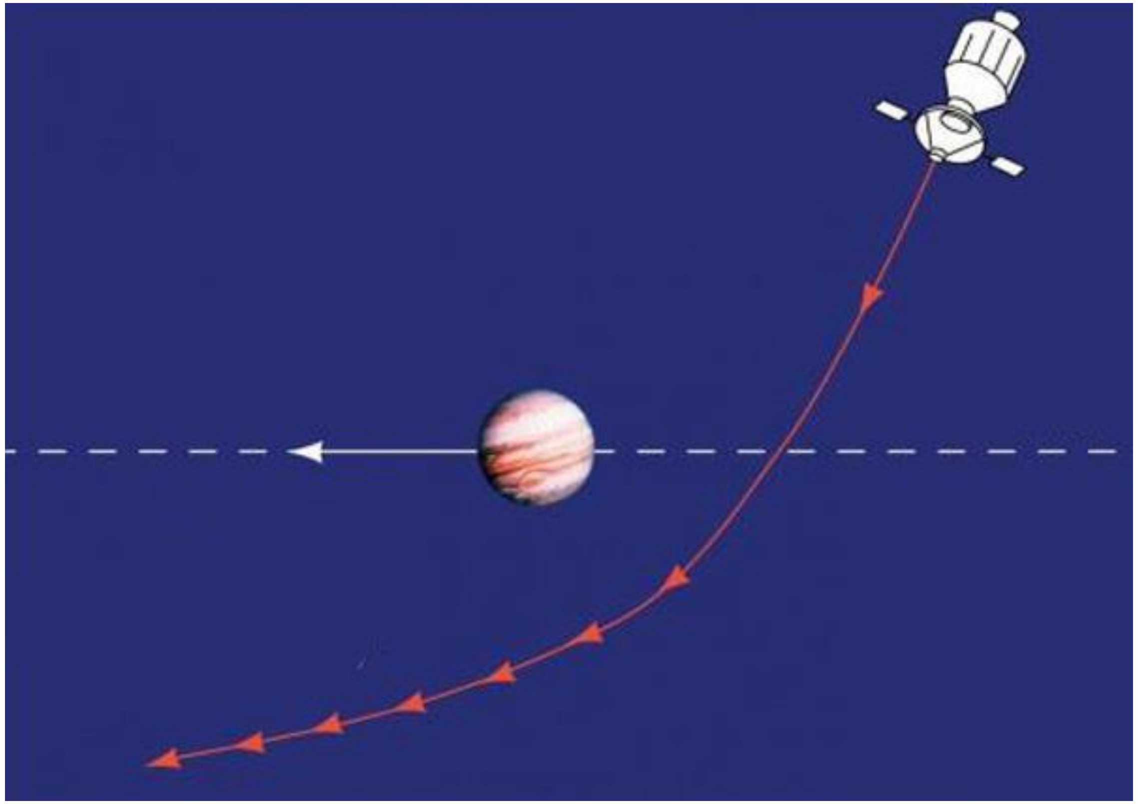
Earth (185 km) → Mars (500 km)

Recipe (2-2):

step	parameter	expression	example
9	$V_{\infty,1}$ (excess velocity at departure planet)	$V_{\infty,1} = \lvert V_1 - V_{dep} \rvert$	2.945 km/s
10	$V_{\infty,2}$ (excess velocity at target planet)	$V_{\infty,2} = \lvert V_2 - V_{tar} \rvert$	2.649 km/s
11	V_0 (velocity in pericenter of hyperbola around departure planet)	$V_0 = \sqrt{(2GM_{dep}/r_0 + V_{\infty,1}^2)}$	11.408 km/s
12	V_3 (velocity in pericenter of hyperbola around target planet)	$V_3 = \sqrt{(2GM_{tar}/r_3 + V_{\infty,2}^2)}$	5.385 km/s
13	ΔV_0 (maneuver in pericenter around departure planet)	$\Delta V_0 = \lvert V_0 - V_{c0} \rvert$	3.615 km/s
14	ΔV_3 (maneuver in pericenter around target planet)	$\Delta V_3 = \lvert V_3 - V_{c3} \rvert$	2.070 km/s
15	ΔV_{tot} (total velocity increase)	$\Delta V_{tot} = \Delta V_0 + \Delta V_3$	5.684 km/s
16	T_{tr} (transfer time)	$T_{tr} = \pi \sqrt{(a_{tr}^3/GM_S)}$	0.709 year

Interplanetary flight

Gravity-Assist (引力助推/引力弹弓)



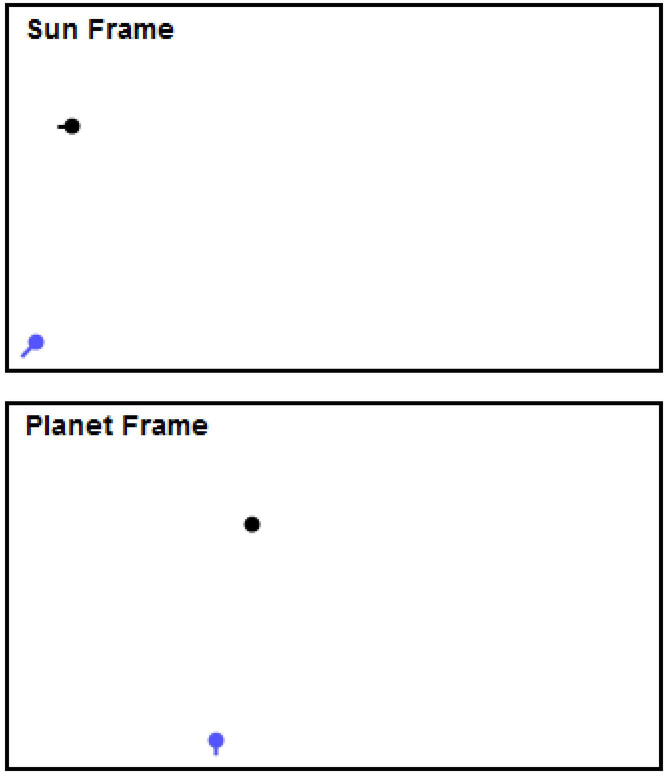
Interplanetary flight

Gravity-Assist

Interplanetary flight

Gravity-Assist (引力助推/引力弹弓)

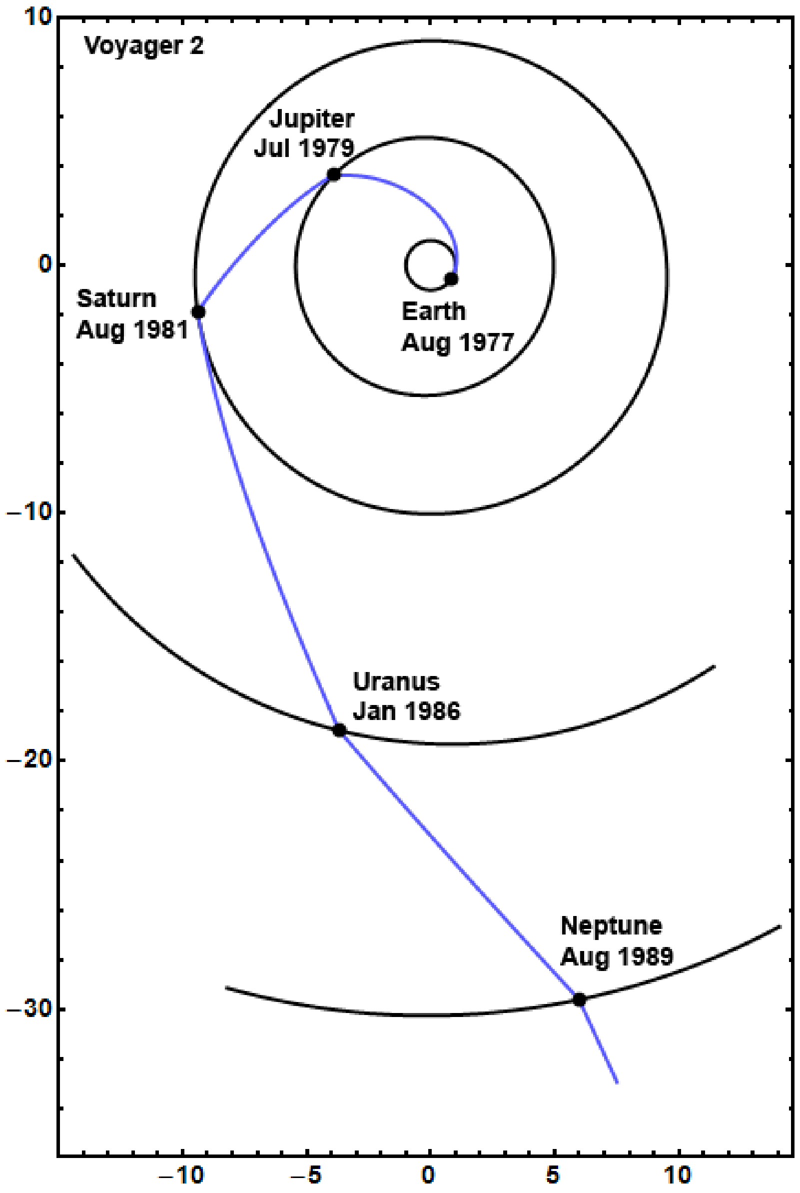
EXAMPLE ENCOUNTER



Source: NASA

Interplanetary flight

THE PATH OF VOYAGER 2 FROM ITS LAUNCH FROM EARTH IN 1977 THROUGH ITS ENCOUNTER WITH NEPTUNE 12 YEARS LATER

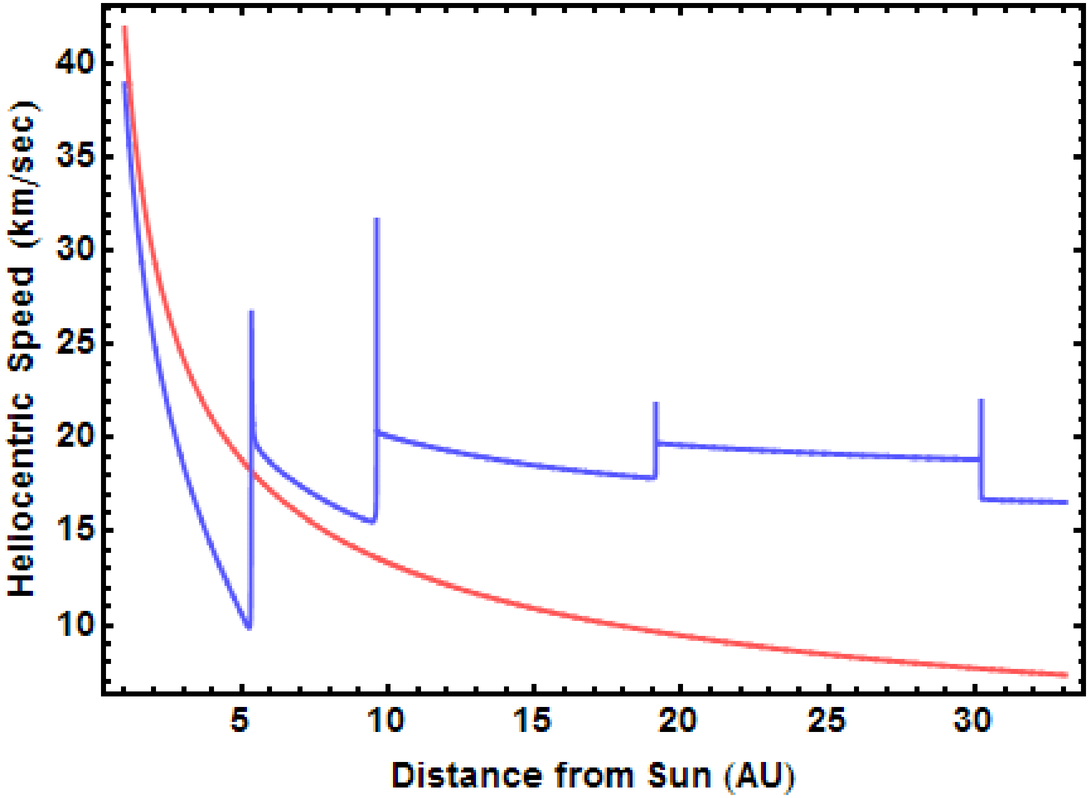


Source: NASA

Interplanetary flight

Gravity-Assist

VOYAGER 2 SPACECRAFT SPEED AS A FUNCTION DISTANCE FROM THE SUN



Source: NASA

Interplanetary flight

Gravity-Assist

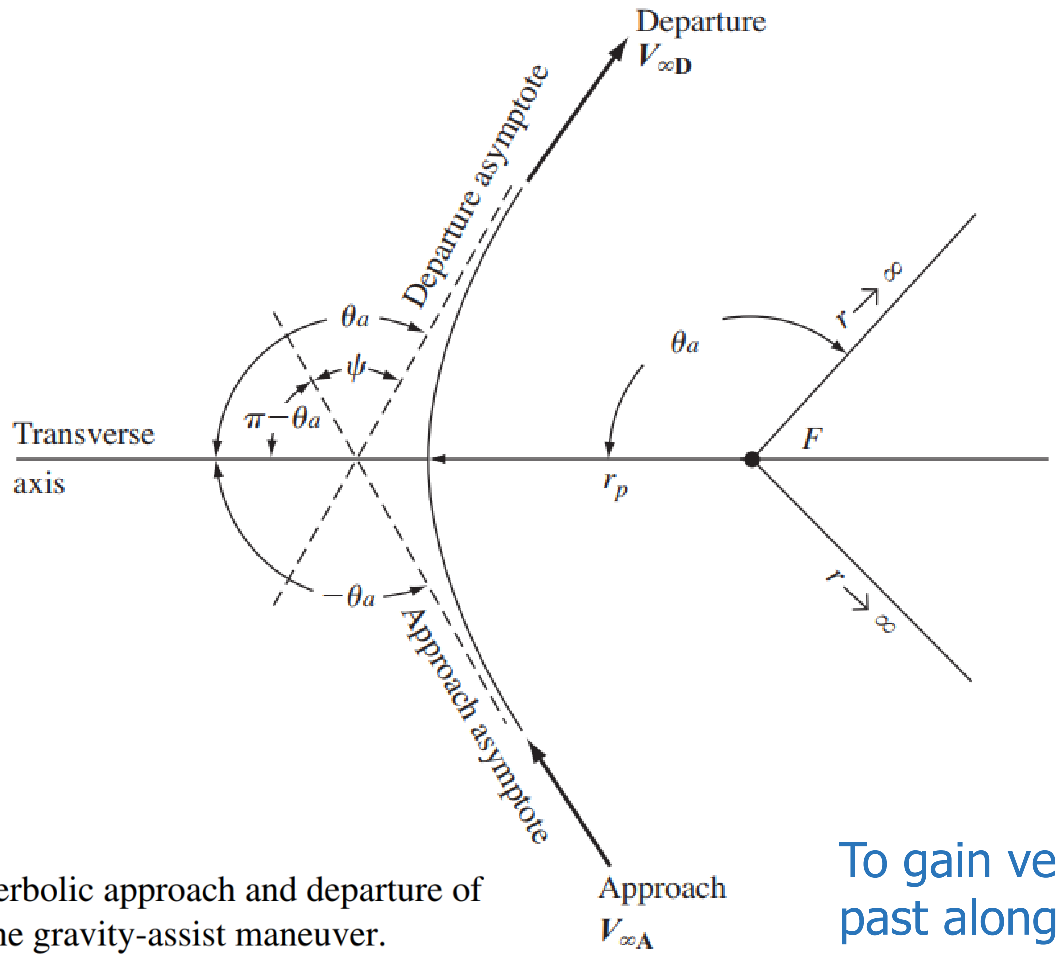


Figure 8.32 Geometry for the hyperbolic approach and departure of a spacecraft to and from a planet: the gravity-assist maneuver.

To gain velocity, the spacecraft should past along the direction of the planet

Interplanetary flight

旅行者号探测器 -纪录片-哔哩哔哩