北京航空航天大學
BEIHANG UNIVERSITY

Avionics Technology

B31353551

— *Inertial Navigation*

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IV. Inertial Navigation

（1）Some concepts

（2）Accelerometer

（3）Inertial navigation

(3) Inertial navigation

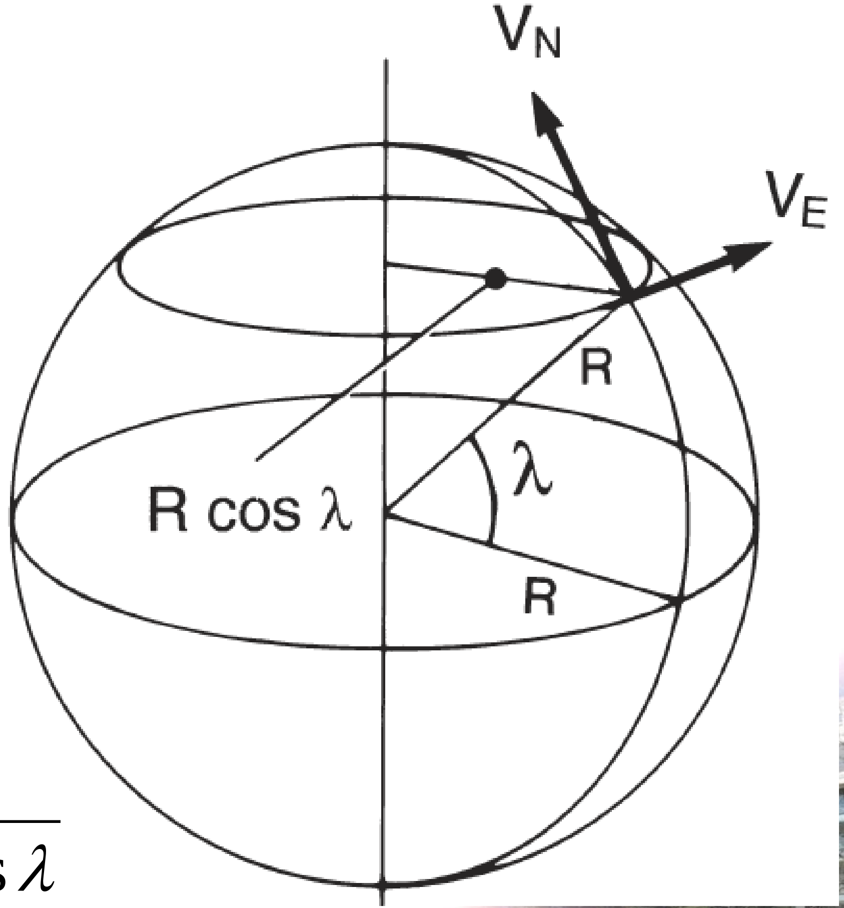
- If we knew the initial position in latitude and longitude, the northerly and easterly velocity components of an aircraft, then we can determine the aircraft's present position.

Rate of change of latitude

$$\dot{\lambda} = \frac{v_N}{R}$$

Rate of change of longitude

$$\dot{\phi} = \frac{v_E}{R\cos\lambda}$$



(3) Inertial navigation

- The change in latitude over time, t, is thus equal to $(1/R)\cdot\int_0^t v_N dt$ and hence the present latitude at time t can be computed given the initial latitude. Similarly, the change in longitude is equal to $(1/R)\cdot\int_0^t v_E \sec\lambda dt$ and hence the present longitude (Φ) can be computed given the initial longitude:

$$\lambda = \lambda_0 + (1/R)\cdot\int_0^t v_N dt$$

$$\phi = \phi_0 + (1/R)\cdot\int_0^t v_E \sec\lambda dt$$

As λ approaches 90°
sec λ approaches infinity

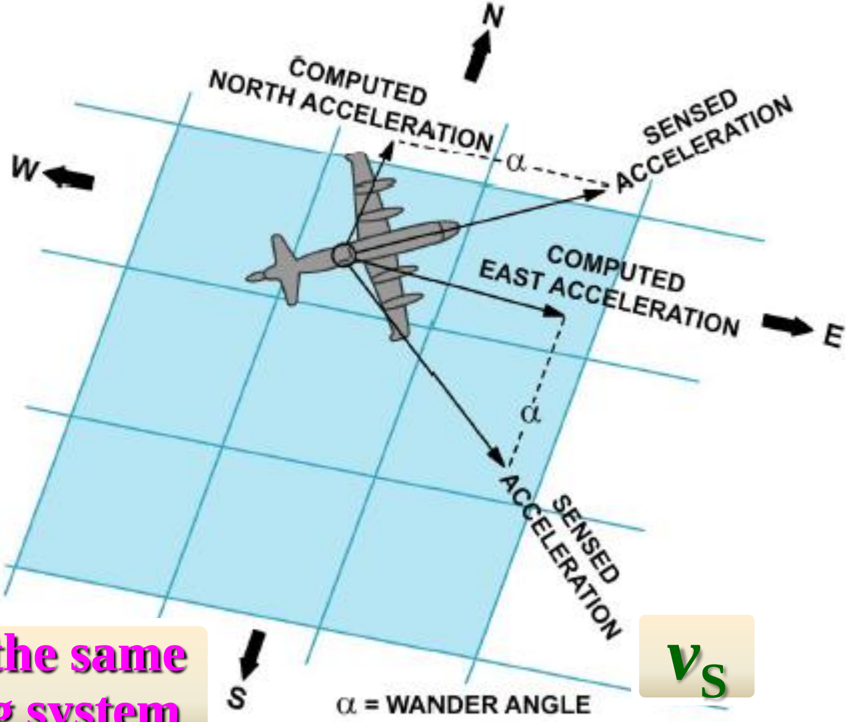
$\sec\lambda = 1/\cos\lambda$

(3) Inertial navigation

- The wander-azimuth inertial system solves this problem by allowing the platform to take an arbitrary angle (wander angle) with respect to true north, which changes as a function of longitude.

$$\dot{\phi}=\frac{v_S\cos\alpha}{R\cos\lambda}=\frac{v_S\cos^2\alpha}{R\cos\lambda\cos\alpha}$$

$$=\frac{2v_S\cos^2\alpha}{R[\cos(\lambda+\alpha)+\cos(\lambda-\alpha)]}$$



Solving the problem of $\lambda = 90^o$ by a proper α

Fundamentals are the same as a north-pointing system

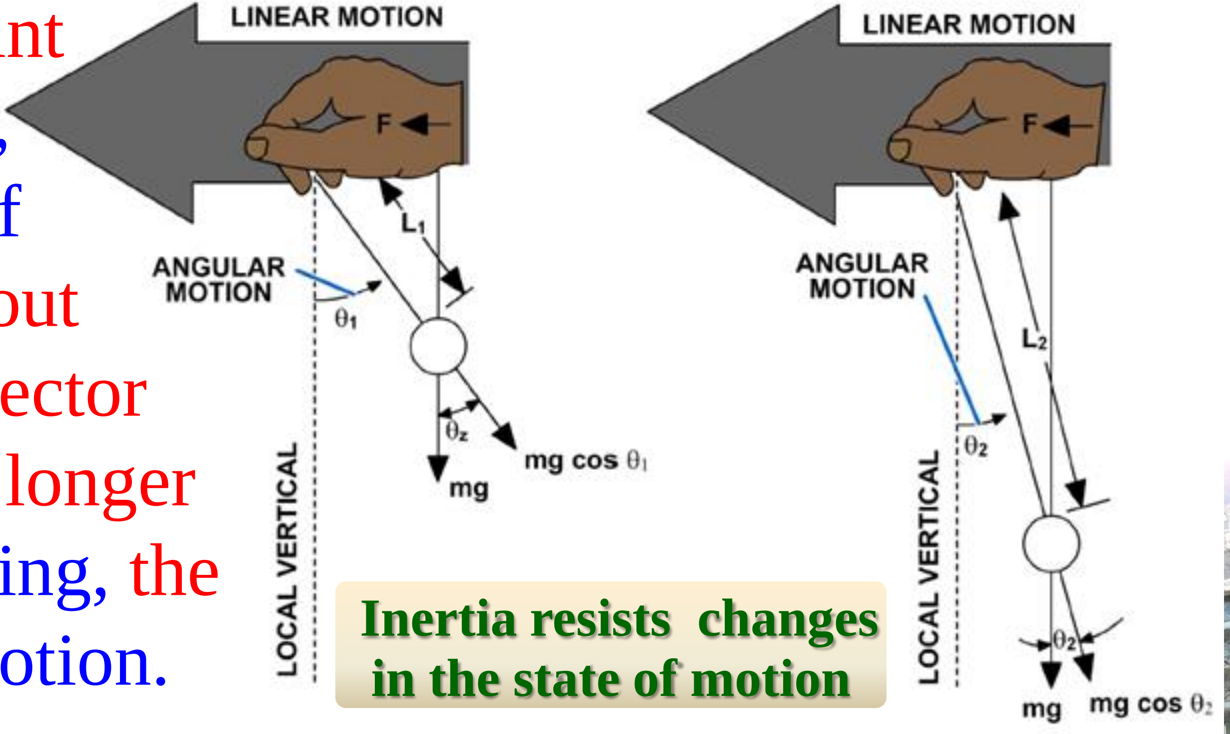
v_S

(3) Inertial navigation

- Either for a wander-azimuth system or north-pointing system, it is essential to maintain both accelerometers horizontal to the earth's surface (i.e. the platform normal to the local vertical). If the accelerometers tilt off level, it measures gravitational components, which results in navigation errors.
- A gravity force sensitive pendulum can automatically align with the local vertical, but the accelerated linear motion interferes with this alignment.

(3) Inertial navigation

- Two pendulums are suspended by strings of different lengths (L_1 or L_2), equal forces horizontally accelerate the suspension point of each pendulum, angular motions of the pendulums about the local gravity vector will produce. The longer the suspending string, the less the angular motion.



Inertia resists changes in the state of motion

(3) Inertial navigation

- The period of pendulum is given by $T = 2\pi\sqrt{L/g}$, and the period of Schuler pendulum:

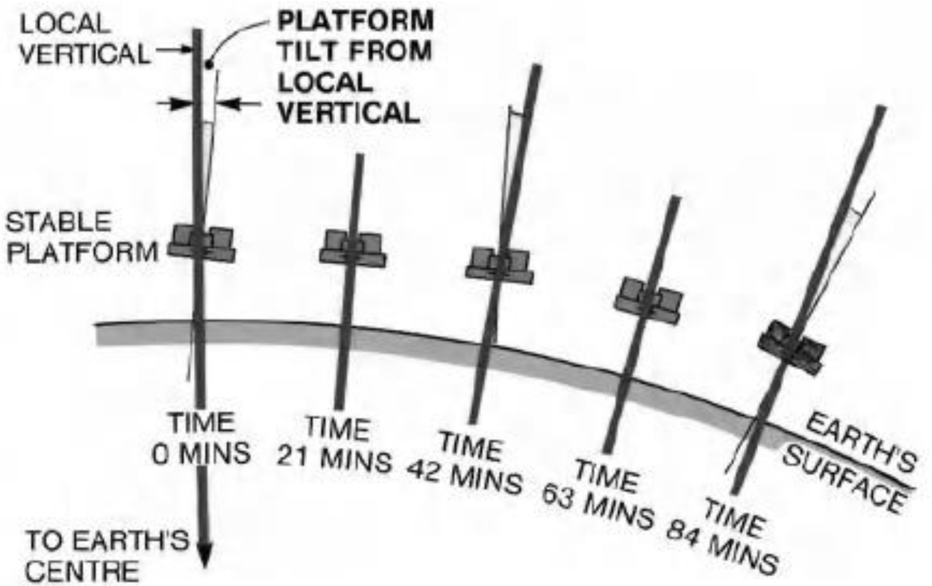
$$T = 2\pi\sqrt{R/g} = 84.4\,\text{min}$$

***R* : the radius of the earth**

which is a special case of the pendulum and would indicate the local vertical irrespective of the acceleration of the vehicle carrying it.

***g* : gravity acceleration**

Platform with Schuler oscillation



Pendulum constructed to oscillate with a period of 84.4 minutes

(3) Inertial navigation

- The earth is a rotating sphere, only points along the equator can be considered to possess uniform linear motion and only at the equator the accelerometer’s signals can translate directly into position information. For this reason, it is necessary to provide a corrective device to alter the accelerometer’s signals. The device inserts artificial acceleration signals to those already in the accelerometer output circuits for the corrections of centripetal effect and Coriolis effect.

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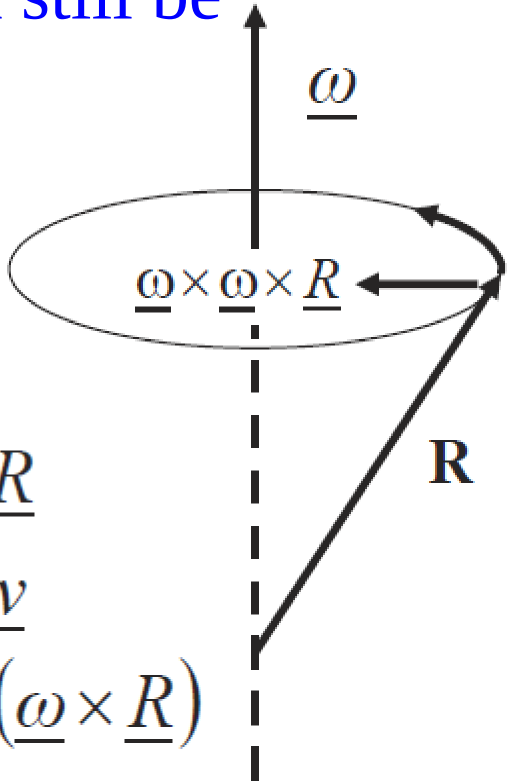
- Centripetal errors has no relationship to Earth dynamics. Even if the earth were stationary, it would still be necessary to insert centripetal correction to the accelerometer's signals. Flying over the earth's surface, the aircraft's linear motion produces a curved flight path in inertial space, and this introduces centripetal acceleration components.



$$\underline{v} = \underline{\omega}\times\underline{R}$$

$$\underline{a} = \underline{\omega}\times\underline{v}$$

$$\underline{a} = \underline{\omega}\times\left(\underline{\omega}\times\underline{R}\right)$$

Centripetal motion

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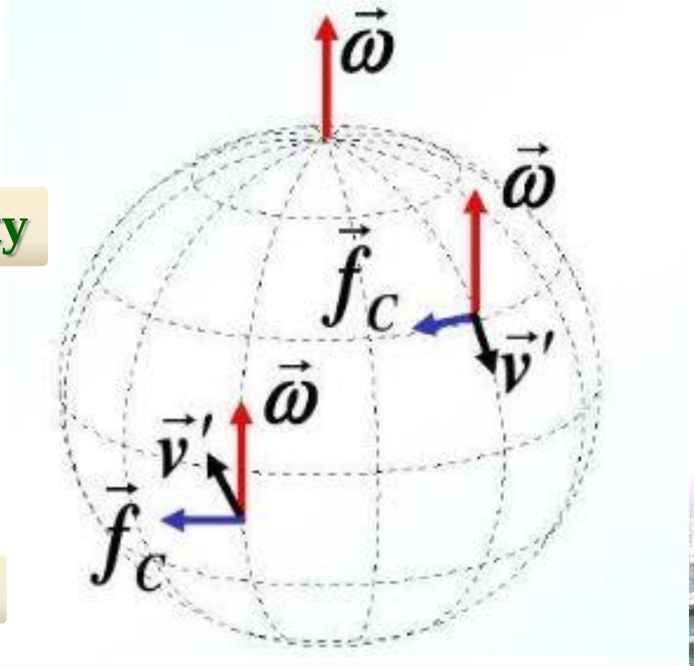
- Coriolis errors has relationship to Earth dynamics and generated by the earth's rotation. Coriolis accelerations are introduced because of the linear motion with respect to a rotating axis frame.



Rotational angular velocity

Coriolis acceleration

Mutually at right angles to the linear velocity and angular velocity

Linear velocity

Coriolis force on Earth's surface

(3) Inertial navigation

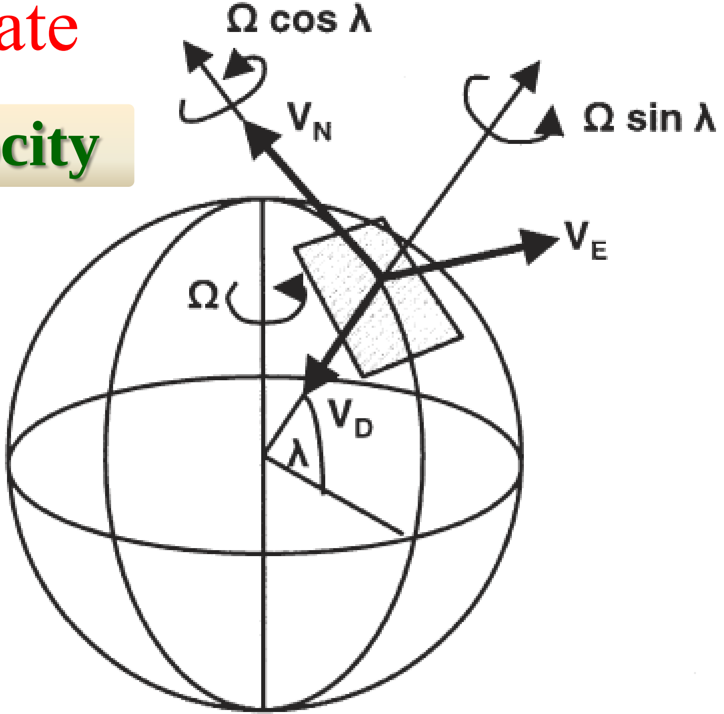
- Coriolis acceleration components along the North, East, vertical (Down) axes due to the aircraft's linear velocity components and Earth's rotation rate components are

V_D : vertical velocity

North axis	$2V_E \Omega \sin \lambda$
East axis	$-2V_N \Omega \sin \lambda -2V_D \Omega \cos \lambda$
Vertical axis	$-2V_E \Omega \cos \lambda$

Ω : Earth's rotation rate
$15^o/h = 7.2717 \times 10^{-5}$ rads/s

Earth referenced axis frame:
local North, East, Down (NED) axes



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- The rates of change of the aircraft velocity (acceleration) components along the NED axes are obtained by subtracting the acceleration corrections:

	North Axis	East Axis	Down Axis
Acceleration component			
Coriolis	$2V_E\,\Omega \sin\lambda$	$-2V_N\,\Omega \sin\lambda - 2V_D\,\Omega \cos\lambda$	$-2V_E\,\Omega \cos\lambda$
Centrifugal **Centripetal**	$\left(V_E^2 \tan\lambda - V_D V_N\right)/R$	$-\left(V_N V_E \tan\lambda + V_D V_E\right)/R$	$\left(V_N^2 + V_E^2\right)/R$
Gravitational			$\dfrac{R_0^2}{\left(R_0 + H\right)^2} g_0$

The end of *Inertial Navigation*