北京航空航天大學
BEIHANG UNIVERSITY

飞行力学
Flight Mechanics

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2022 - Spring

About the exam

- Exam (60%) + Course project (25%) + Homework (15%)
- Online open-book exam (在线开卷考试), 2h
- The exam questions will be based on lecture slides
- Exam date: between week 17~18

Follow-up arrangements

13		23	24	25	26	27	28	29
14		~~30~~	31					
				1	2	~~3 端午节~~	4	5
15		6	7	8	9	10	11	12
16	六	13	14	15	16	17	18	19
17		20	21	22	23	24	25	26
18		27	28	29	30			
	七					1	2	3

Contents

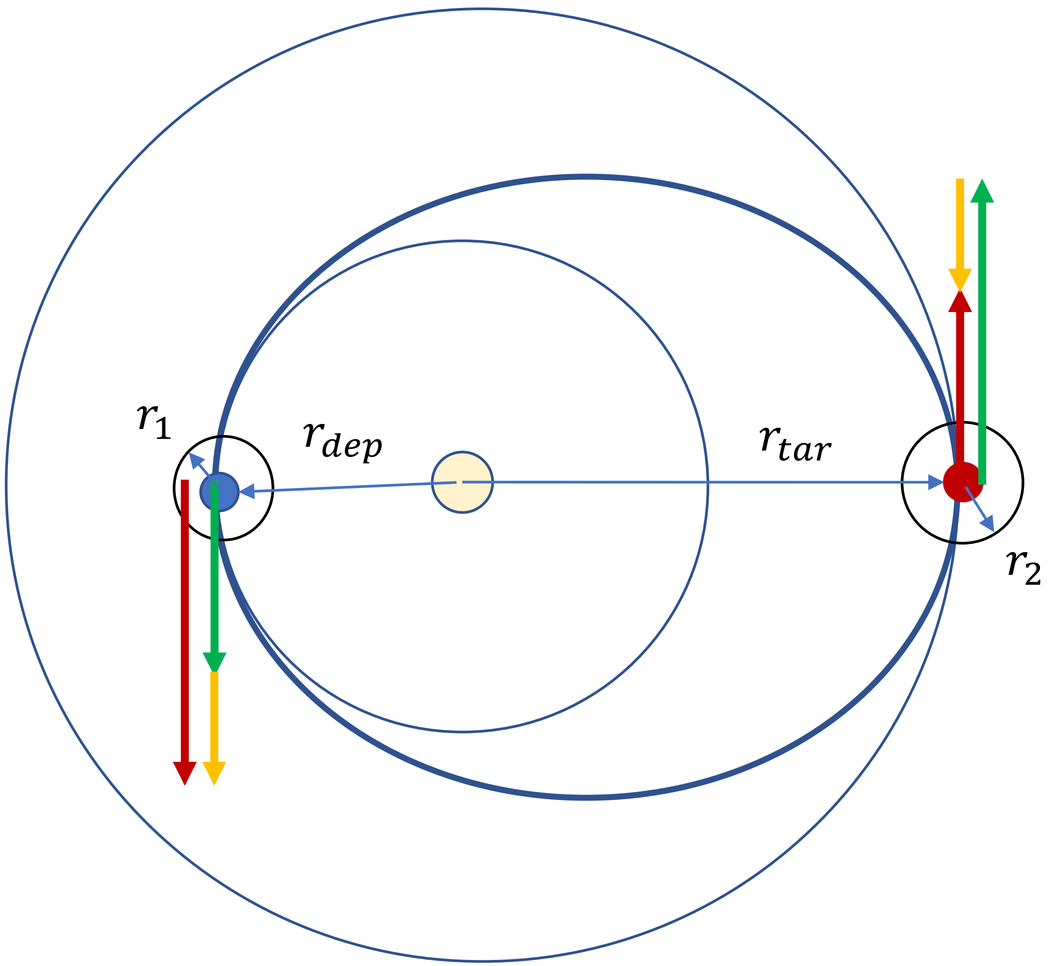
Interplanetary flight

Geometric relationship

$$r_1 = R_{dep} + H_1 \qquad r_2 = R_{tar} + H_2$$

$$a_{tr} = \frac{1}{2}\left(r_{dep} + r_{tar}\right)$$

$$e = \left|r_{dep} - r_{tar}\right| / \left(r_{dep} + r_{tar}\right)$$



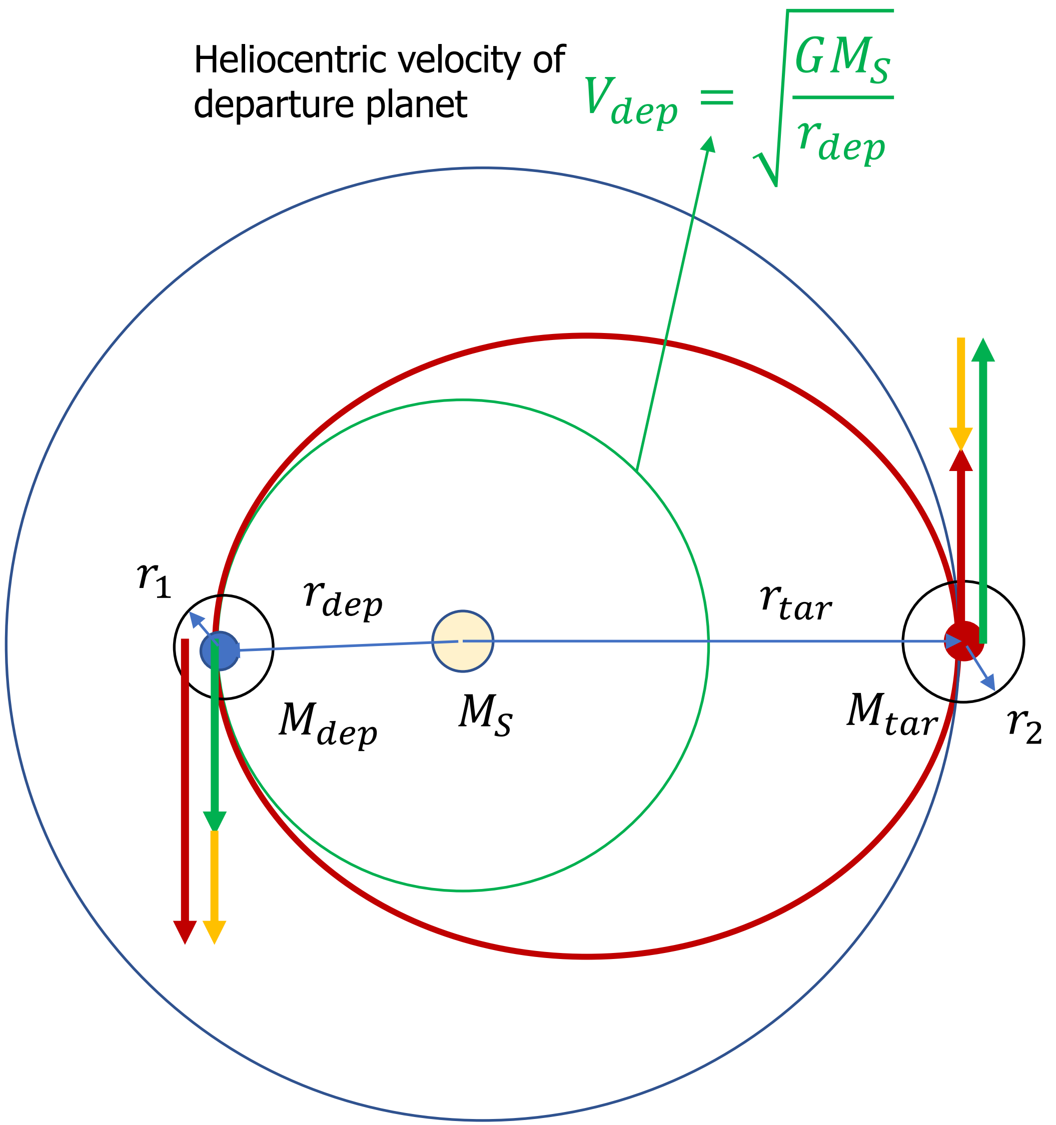
Interplanetary flight

Velocity at periapsis on transfer orbit

$$V_{pt} = \sqrt{GM_S\left(\frac{2}{r_{dep}} - \frac{1}{a_{tr}}\right)}$$

Excess velocity at departure planet

$$V_{\infty,1} = \left|\boldsymbol{V}_{pt} - \boldsymbol{V}_{dep}\right|$$

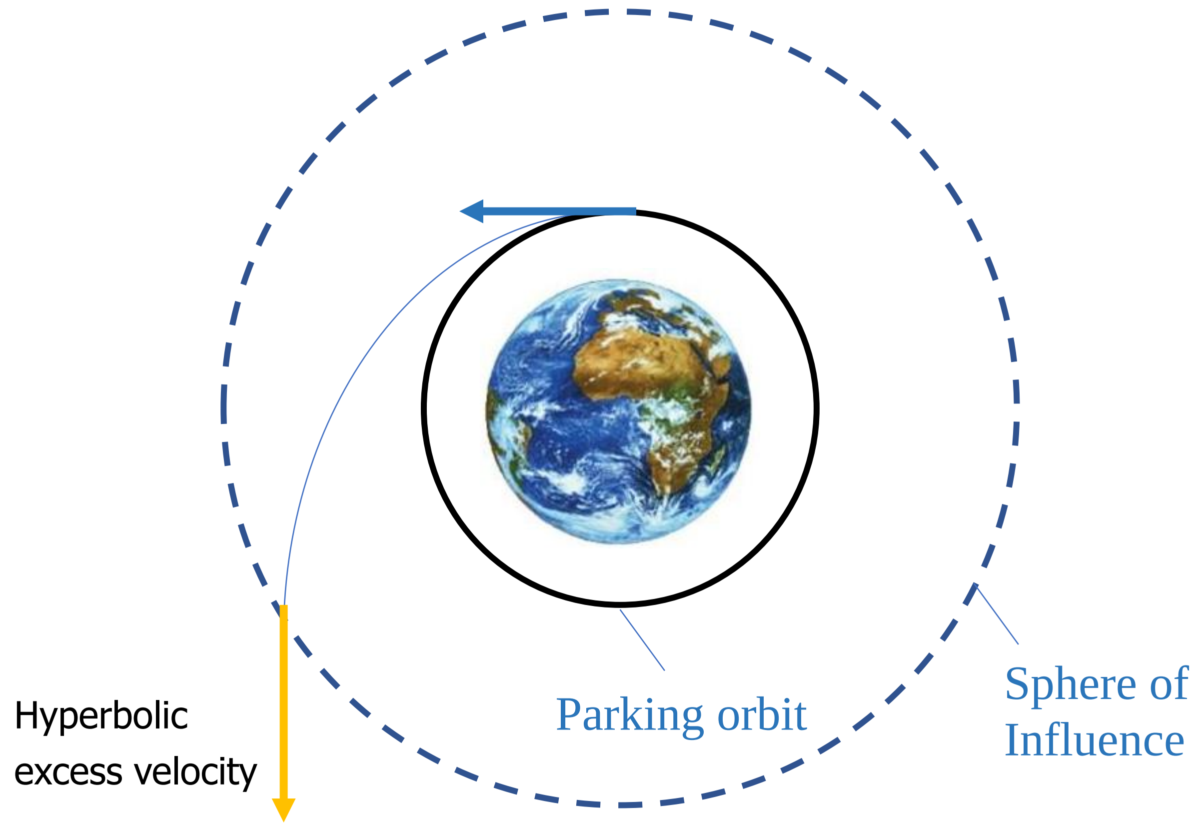


Interplanetary flight

Circular velocity around the earth:

$$V_{c1} = \sqrt{\frac{GM_{dep}}{r_1}}$$

Hohmann transfer (Earth scale)



$$V_{\infty,1} = |V_{pt} - V_{dep}|$$

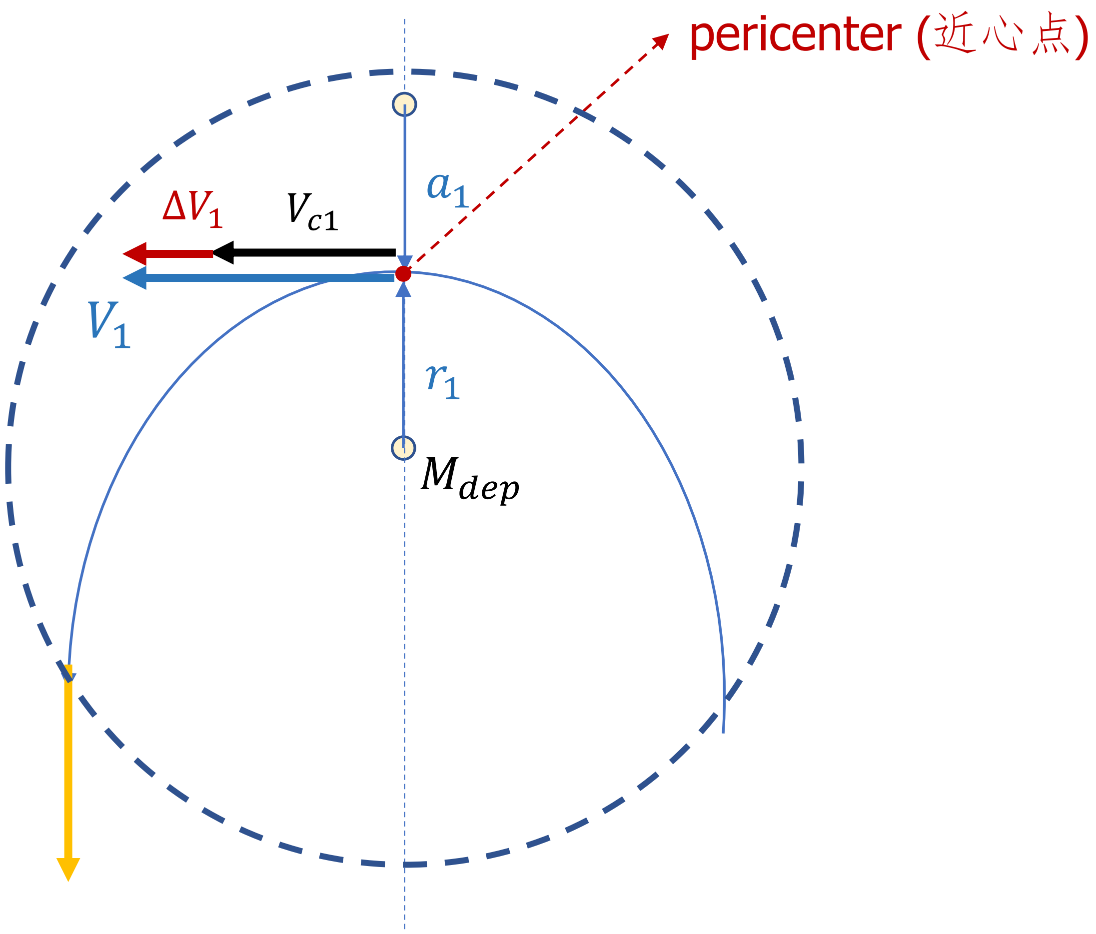
Interplanetary flight

Velocity in pericenter of <u>hyperbola</u> around departure planet

$$V_1 = \sqrt{\frac{2k^2}{r_1} - \frac{k^2}{a_1}} = \sqrt{\frac{2GM_{dep}}{r_1} + V_{\infty,1}^2}$$

Maneuver in pericenter around departure planet

$$\Delta V_1 = |V_1 - V_{c1}|$$



Interplanetary flight

Practice (10 mins)

Consider a Hohmann transfer from Earth (E) to Mercury (M). Begin and end of the transfer is in a parking orbit at 500 km altitude, for both cases.

a) What are the semi-major axis and the eccentricity of the transfer orbit?

b) What is the trip time?

c) What are the excess velocities at Earth and at Mercury (i.e., heliocentric)?

d) What are the circular velocities in the parking orbit around Earth and Mercury (i.e., planetocentric)?

e) What are the ΔV's of the maneuvers to be executed at Earth and Mercury? What is the total ΔV?

Data: $GM_S = 1.3271 \times 10^{11}$ km^3/s^2; GM_E= 398,600 km^3/s^2; GM_M = 22,034 km^3/s^2;
R_E = 6378 km; R_M=2440 km; distance Earth-Sun = 1 AU; distance Mercury-Sun = 0.387 AU; 1 AU = 149.6 × 10^6 km.

Interplanetary flight

An example

Consider a Hohmann transfer from Earth (E) to Mercury (M). Begin and end of the transfer is in a parking orbit at 500 km altitude, for both cases.

Answers:

a) $a = 0.6935$ AU $= 103.7 \times 10^6$ km; $e = 0.442$

b) $T_H = 9.11 \times 10^6$ sec $= 0.289$ year

c) V_∞(Earth) = 7.535 km/s; V_∞(Mercury) = 9.615 km/s

d) V_c(Earth) = 7.613 km/s; V_c(Mercury) = 2.738 km/s

e) ΔV(Earth) = 5.528 km/s; ΔV(Mercury) = 7.627 km/s; ΔV_{tot} = 13.155 km/s

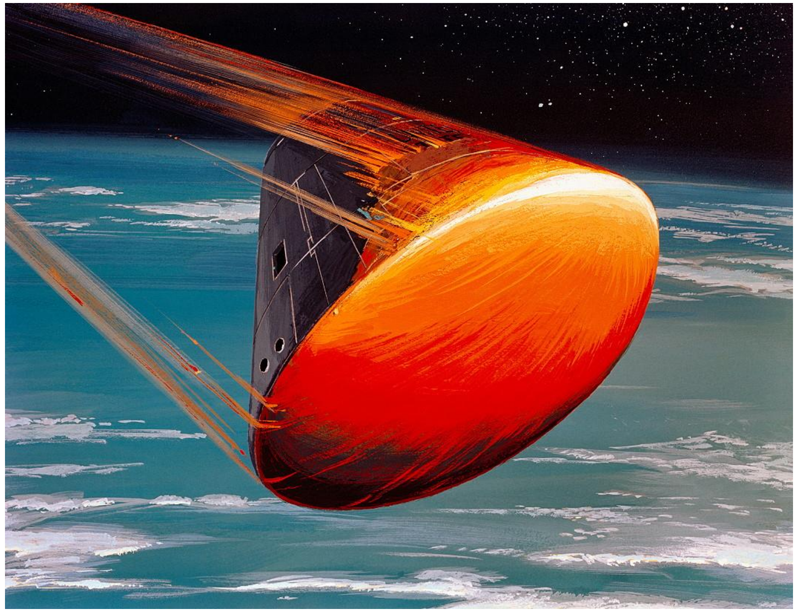
The reentry process

The life of a typical spacecraft:

1. Launch from the surface of the earth
2. Travel in space
3. Return to earth, or landing on some other planet

Introduction

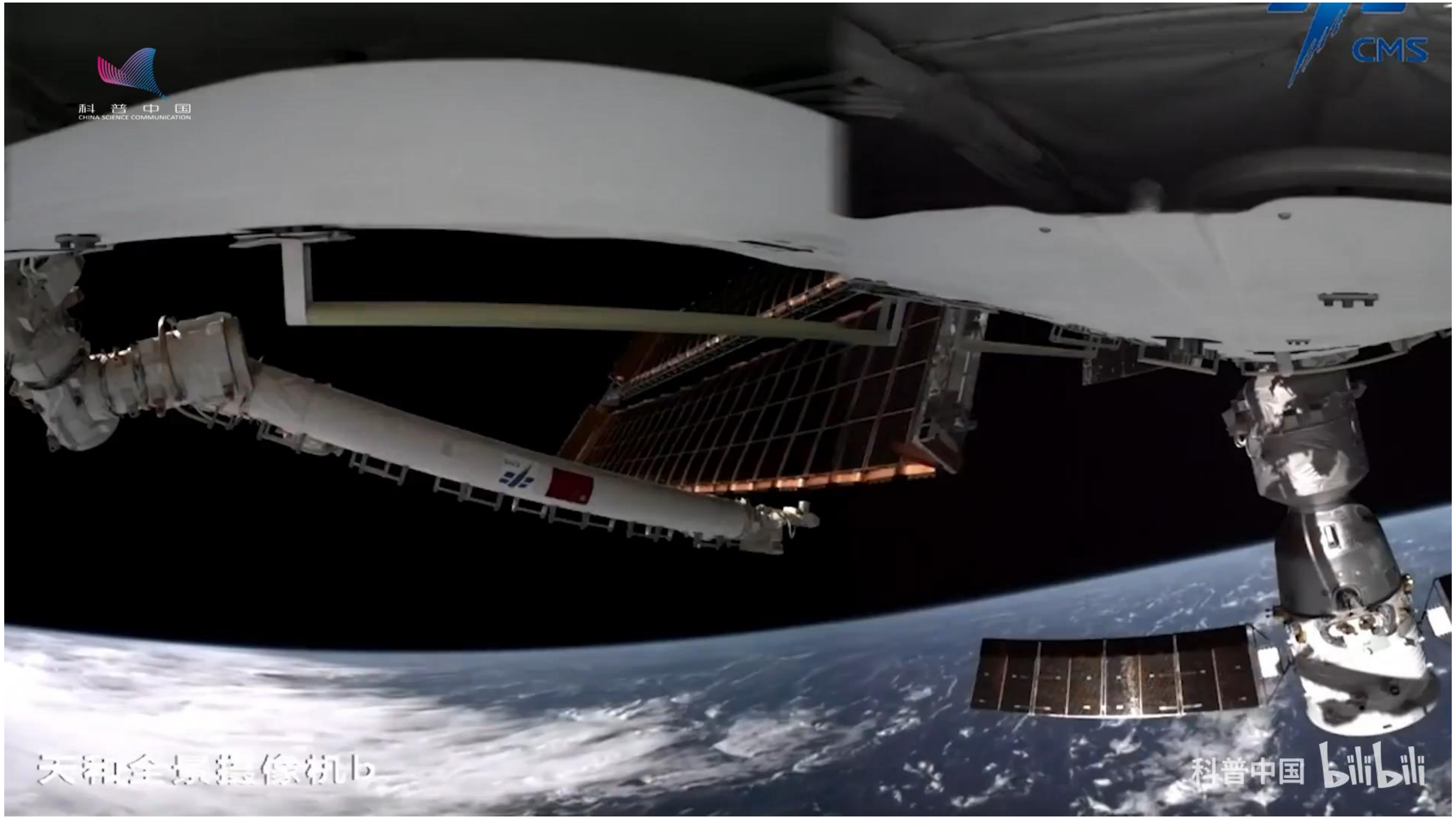
Spacecraft reentry module



Apollo reentry module

Shenzhou reentry module

The reentry process



The entry paths

1. **Ballistic entry**: vehicle has little or no aerodynamic lift
2. **Glide entry**: Space shuttle (L/D > 4)
3. **Skip entry**: not used yet

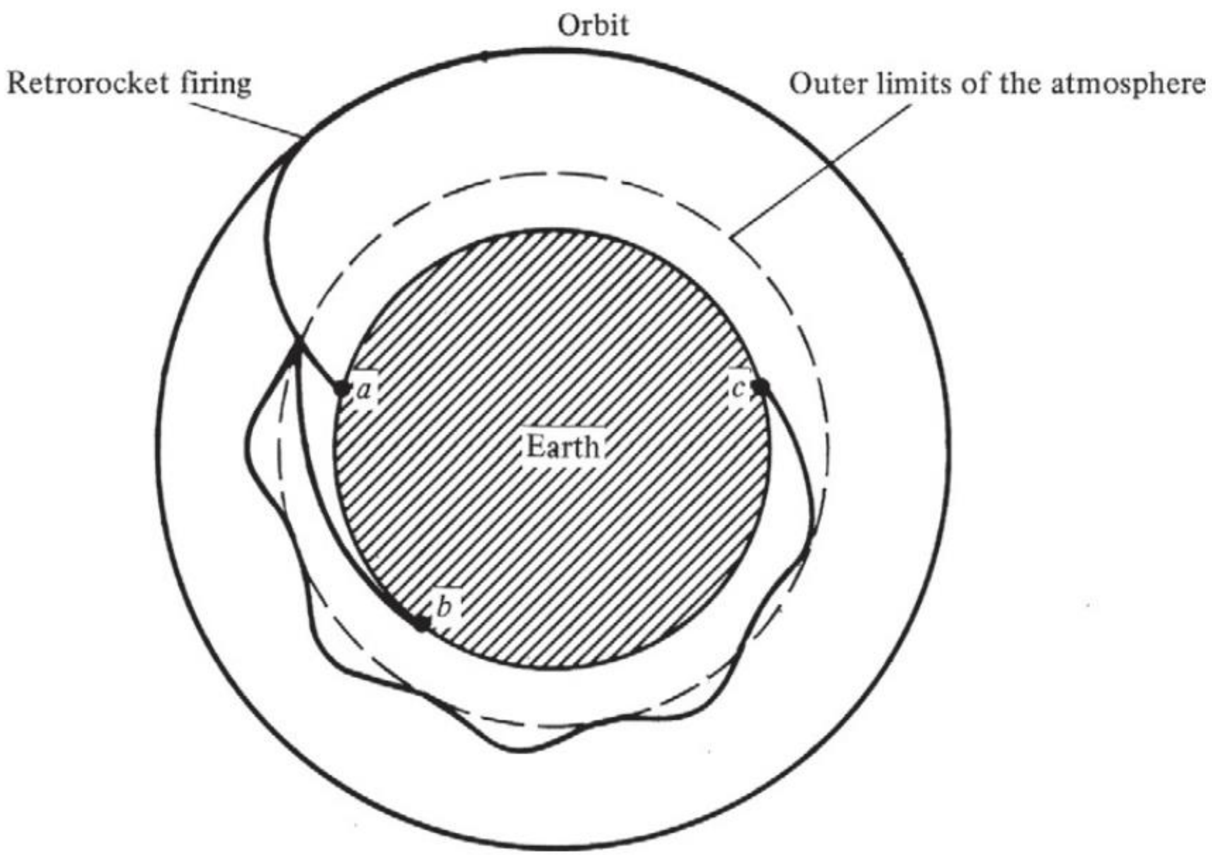
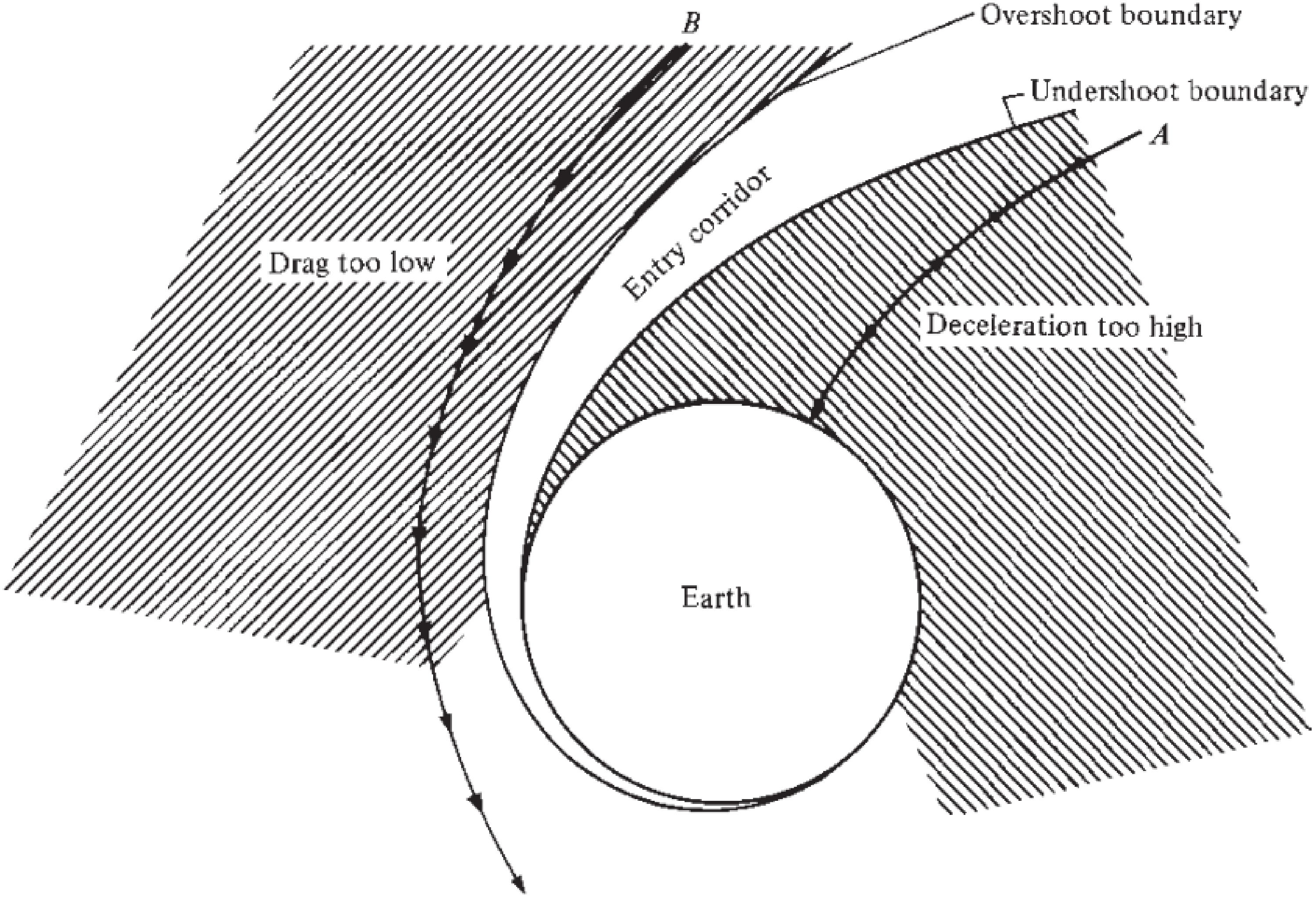


Figure 8.37 Three types of entry paths: (*a*) ballistic; (*b*) glide; (*c*) skip.

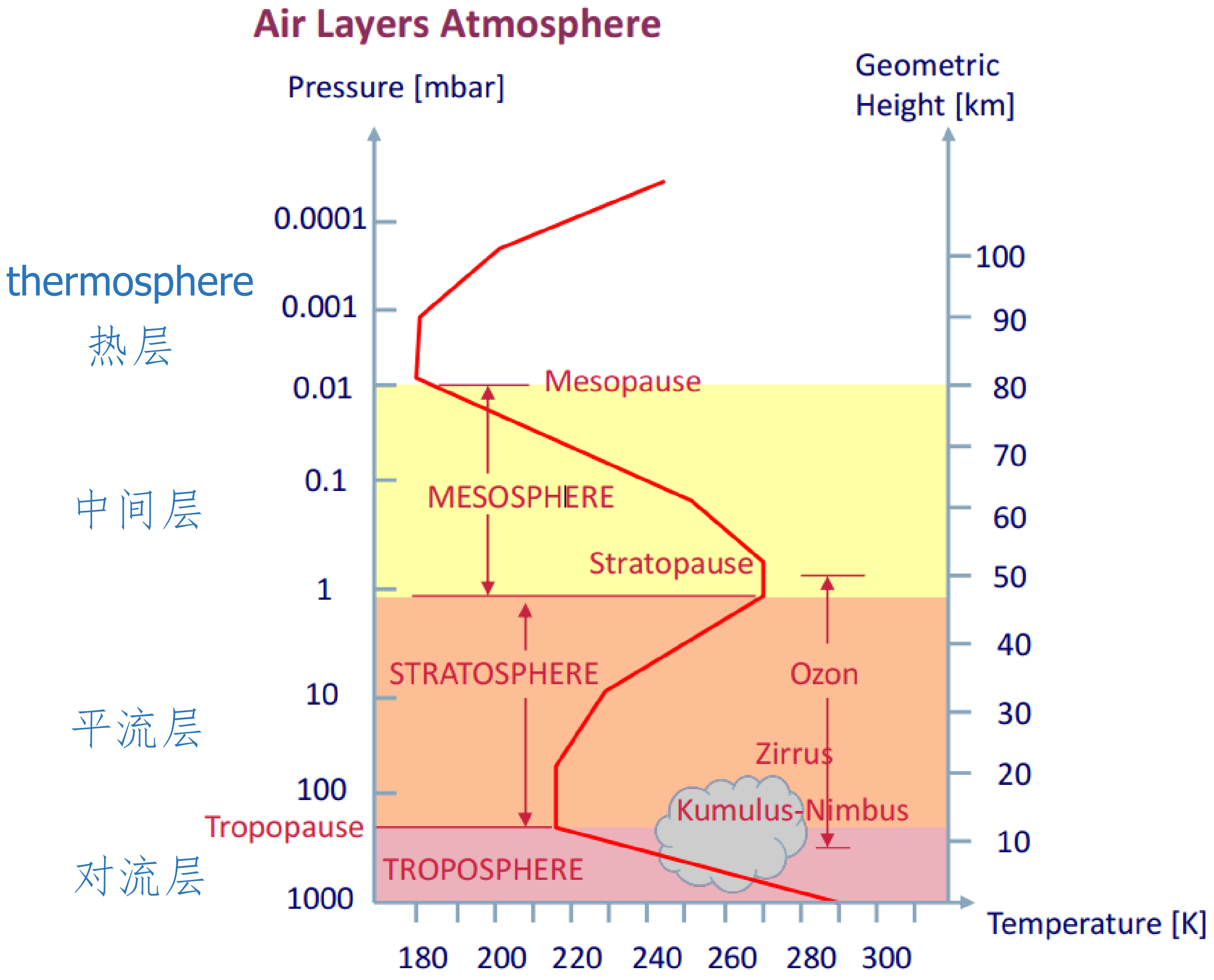
The entry paths



The atmosphere

Components of dry air at sea level (relative to volume)

- Nitrogen 78.1 %
- Oxygen 20.9 %
- Argon 0.9 %
- Carbon dioxide 0.04 %
- Other elements only as traces (ppm)



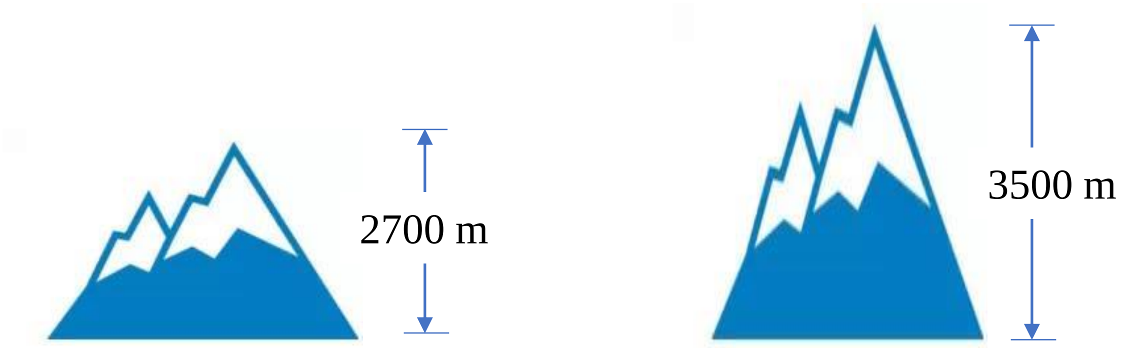
The atmosphere

U.S. Standard Atmosphere Air Properties - SI Units

Geo potential Altitude above Sea Level - *h* - (m)	Temperature - *t* - (^oC)	Acceleration of Gravity - *g* - (m/s^2)	Absolute Pressure - *p* - (10^4 N/m^2)	Density - *ρ* - (kg/m^3)	Dynamic Viscosity - *μ* - (10^{-5} $N\ s/m^2$)
-1000	21.50	9.810	11.39	1.347	1.821
0	15.00	9.807	10.13	1.225	1.789
1000	8.50	9.804	8.988	1.112	1.758
2000	2.00	9.801	7.950	1.007	1.726
3000	-4.49	9.797	7.012	0.9093	1.694
4000	-10.98	9.794	6.166	0.8194	1.661
5000	-17.47	9.791	5.405	0.7364	1.628
6000	-23.96	9.788	4.722	0.6601	1.595
7000	-30.45	9.785	4.111	0.5900	1.561
8000	-36.94	9.782	3.565	0.5258	1.527
9000	-43.42	9.779	3.080	0.4671	1.493
10000	-49.90	9.776	2.650	0.4135	1.458
15000	-56.50	9.761	1.211	0.1948	1.422
20000	-56.50	9.745	0.5529	0.08891	1.422
25000	-51.60	9.730	0.2549	0.04008	1.448
30000	-46.64	9.715	0.1197	0.01841	1.475
40000	-22.80	9.684	0.0287	0.003996	1.601
50000	-2.5	9.654	0.007978	0.001027	1.704
60000	-26.13	9.624	0.002196	0.0003097	1.584
70000	-53.57	9.594	0.00052	0.00008283	1.438
80000	-74.51	9.564	0.00011	0.00001846	1.321

The atmosphere

High altitude effects



Potala Palace: 3700 m above sea level

The atmosphere

Exponential model atmosphere $\rho = f(h)$

$$\frac{\rho}{\rho_0} = e^{-g_0 h/(RT)}$$

Where temperature T can be assumed as constant of 288 K. The approximation is valid for the condition up to about 140 km

Equation of motion

Enter atmosphere at inclined angle θ

$$-D + W\sin\theta = m\frac{dV}{dt}$$

$$L - W\cos\theta = m\frac{V^2}{r_c}$$

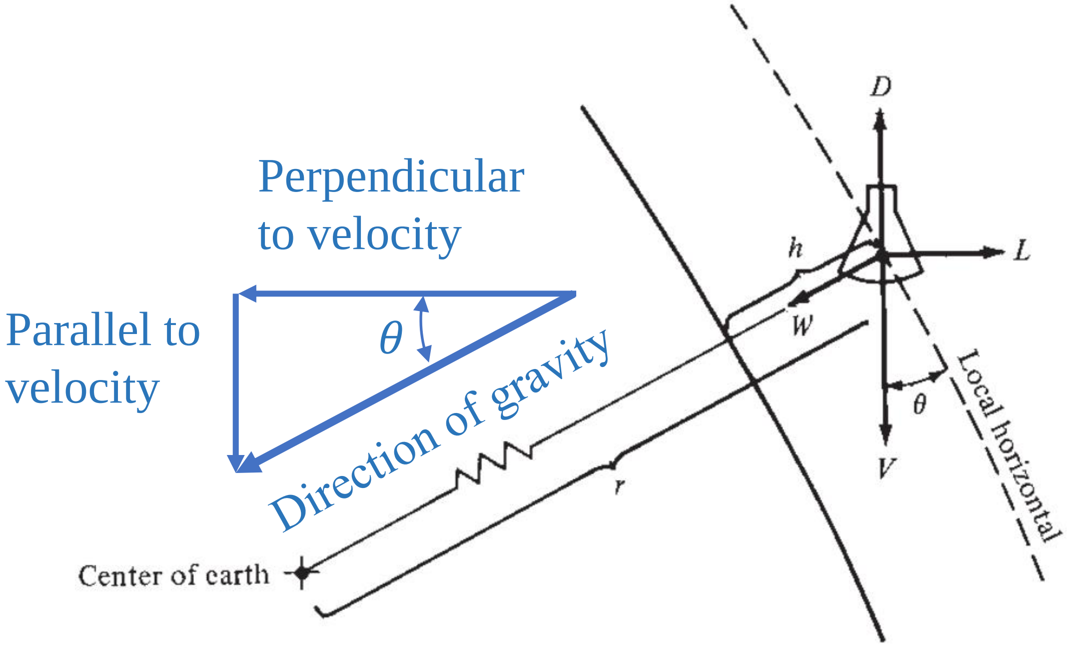


Figure 8.39 Geometry of entry vehicle forces and motion.

Equation of motion

Velocity as a function of altitude

$$\boxed{ds = -\frac{dh}{\sin\theta}}$$

$$-D + W\sin\theta = m\frac{dV}{dt} = m\frac{dV}{ds}\frac{ds}{dt} = mV\frac{dV}{ds}$$

$$-D + W\sin\theta = \frac{1}{2}m\frac{dV^2}{ds}$$

$$\boxed{D = \tfrac{1}{2}\rho V^2 S C_D}$$

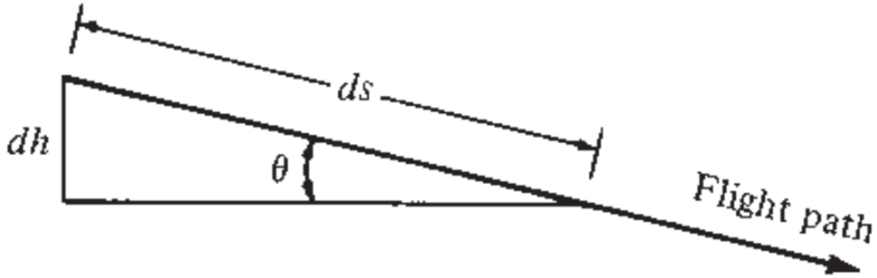


Figure 8.40 Flight path geometry.

Equation of motion

Velocity as a function of altitude

$$-\frac{1}{2}\boxed{\rho}V^2SC_D + W\sin\theta = -\frac{1}{2}m\sin\theta\frac{dV^2}{\boxed{dh}}$$

?

Equation of motion

Density relation

$$\boxed{\frac{\rho}{\rho_0} = e^{-g_0 h/(RT)}}$$

$$\frac{d\rho}{\rho_0} = e^{-Zh}(-Z\,dh) = \frac{\rho}{\rho_0}(-Z\,dh)$$

$$dh = -\frac{d\rho}{Z\rho}$$

Where $Z \equiv g_0/RT$

Equation of motion

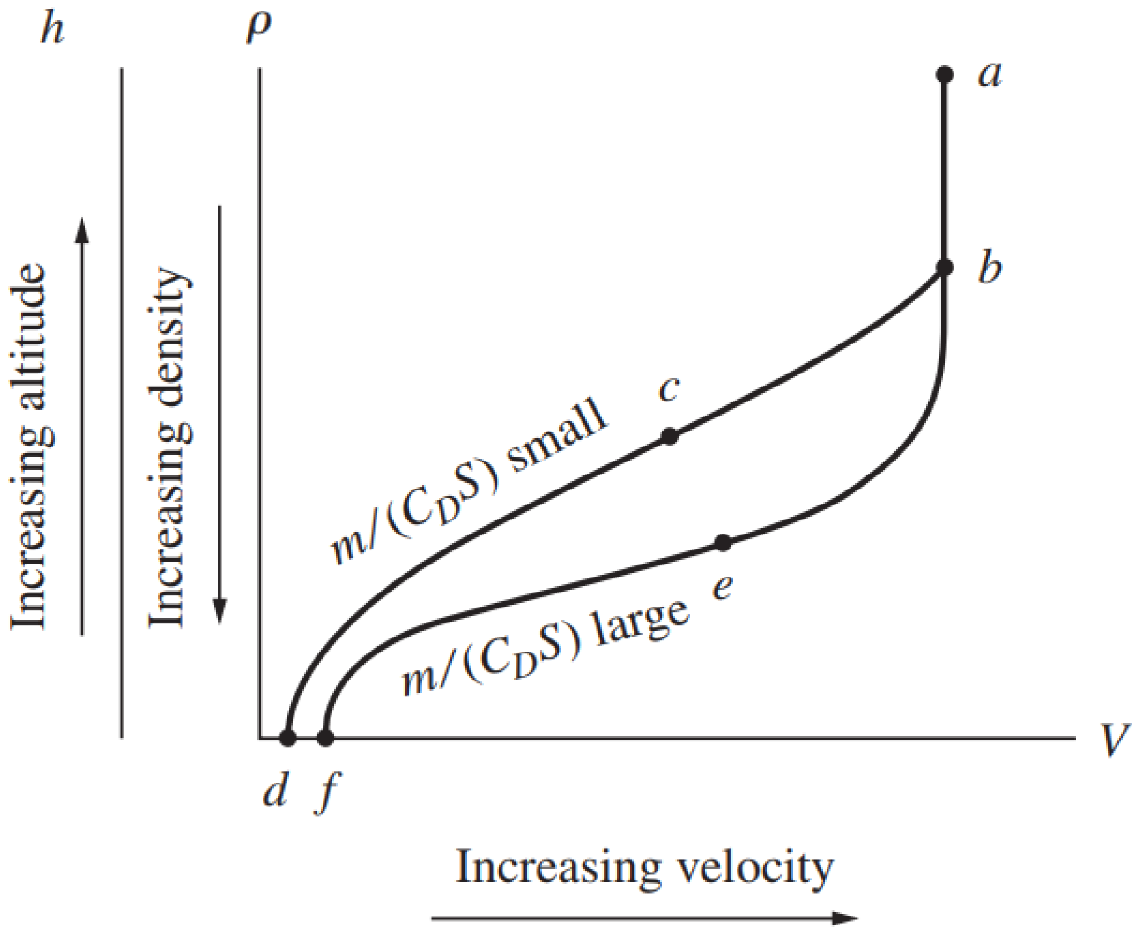
Velocity as a function of altitude

$$\frac{dV^2}{d\rho} + \frac{1}{\boxed{m/(C_D S)}} \frac{V^2}{Z\sin\theta} = \frac{2g}{Z\rho}$$

Ballistic parameter

Exact equation for spacecraft entering atmosphere. Where $Z \equiv g_0/RT$, $\rho = f(h)$

The entry path



The entry path

Gupta et al., NASA report, 1990

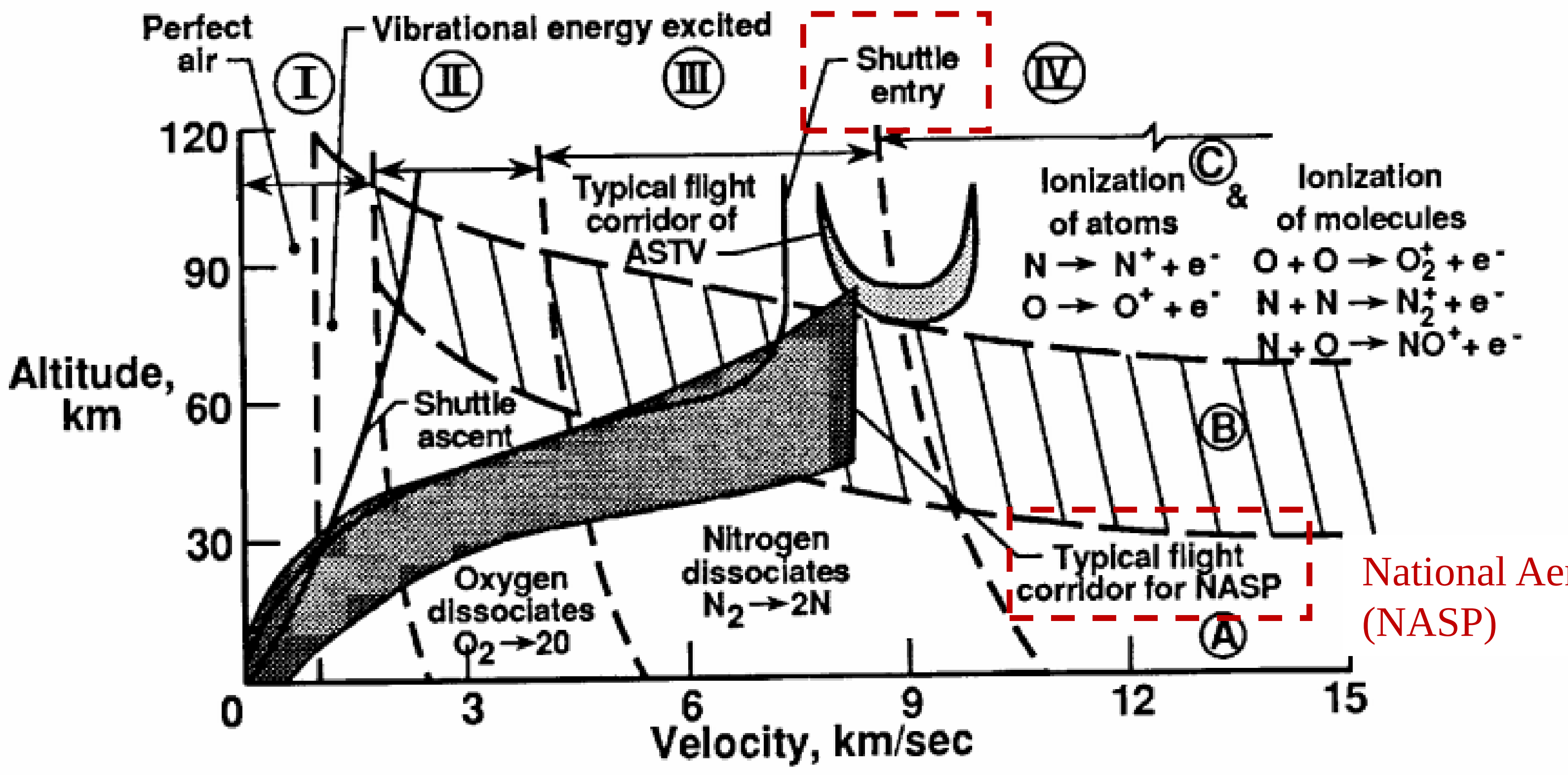
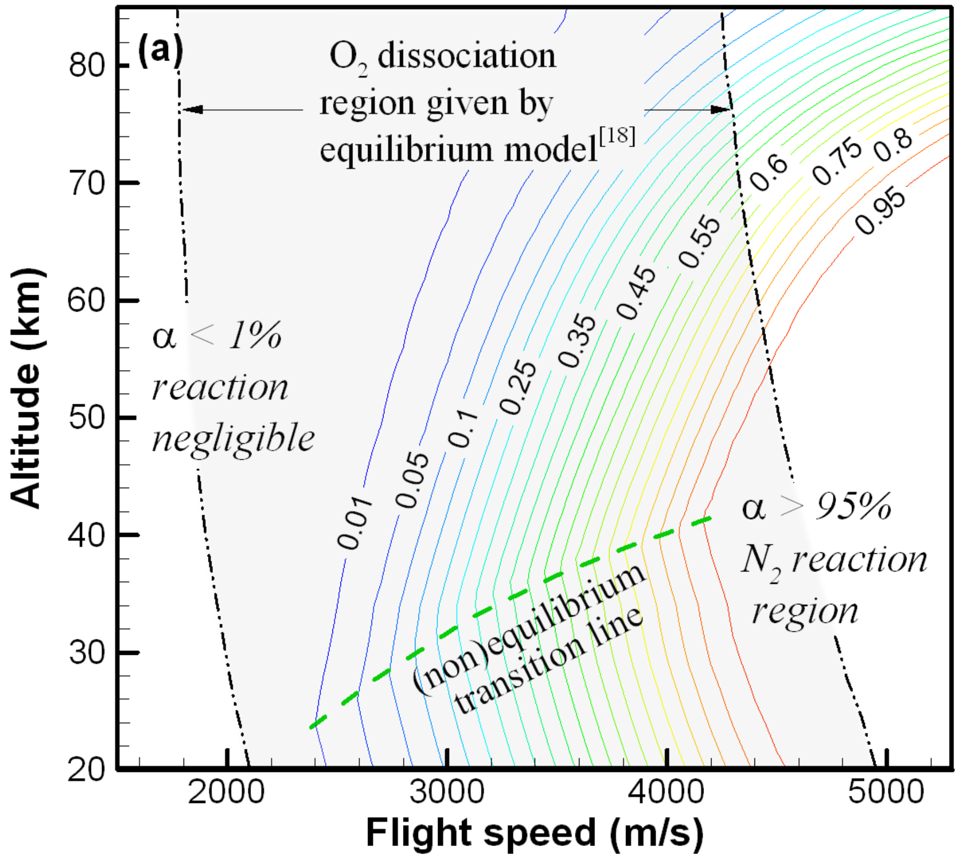


Figure 1. Flight stagnation region air chemistry for a 30.5-cm radius sphere (adapted from ref. 5).

National Aero-Space Plane (NASP)

Relevant research work



[1] 陈松，孙泉华，高超声速飞行流场中最大氧气离解度分析,力学学报,46(1):20-27，2014.

Ballistic entry (弹道再入)

Approximation ($L \approx 0; D \gg W$)

$$\boxed{-D + W\sin\theta = m\frac{dV}{dt}}$$

$$-D = m\frac{dV}{dt}$$

$$\Rightarrow \quad \frac{dV^2}{d\rho} + \frac{1}{m/(C_D S)}\frac{V^2}{Z\sin\theta} = 0$$

Ballistic entry

Solution

$$\frac{dV^2}{d\rho} + \frac{1}{m/(C_D S)} \frac{V^2}{Z \sin\theta} = 0$$

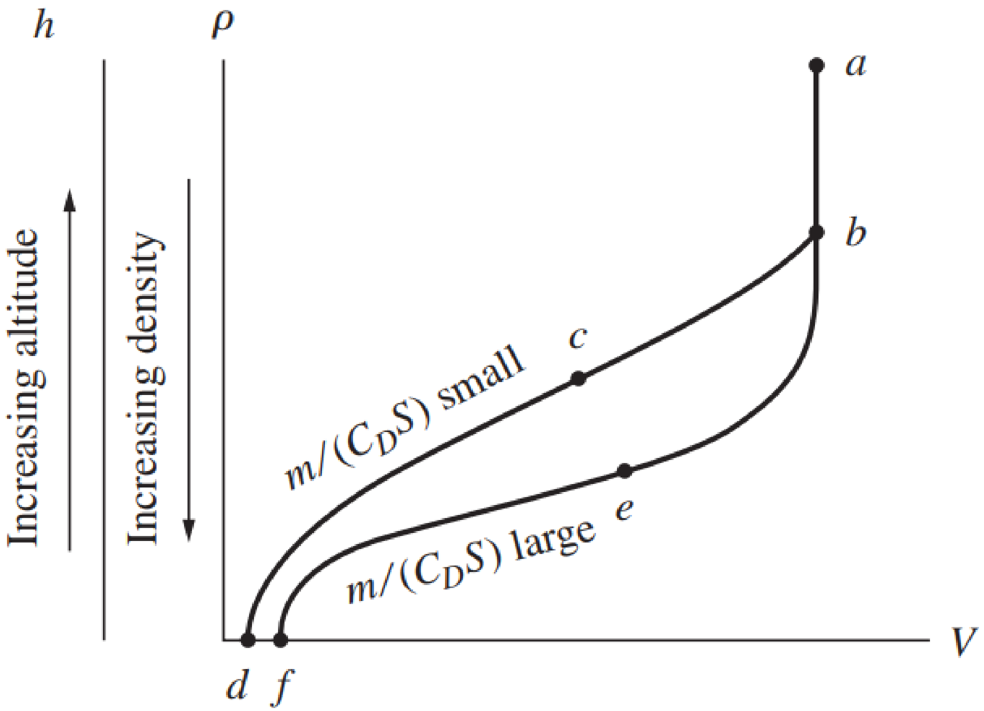
$$\frac{V}{V_E} = e^{-\rho/2[m/(C_D S)]Z \sin\theta}$$

Where V_E is the initial entry velocity

Ballistic entry

Discussion

$$\frac{V}{V_E} = e^{-\rho/2[m/(C_D S)]Z\sin\theta}$$



1. As ρ increases (namely, as the altitude decreases), V decreases

2. If $m/(C_D S)$ is larger, the exponential term does not have strong effect until ρ becomes larger

3. For manned space mission, $m/(C_D S)$ should be sufficiently small.

Ballistic entry

Maximum Deceleration

$$\boxed{-D = m\frac{dV}{dt}}$$

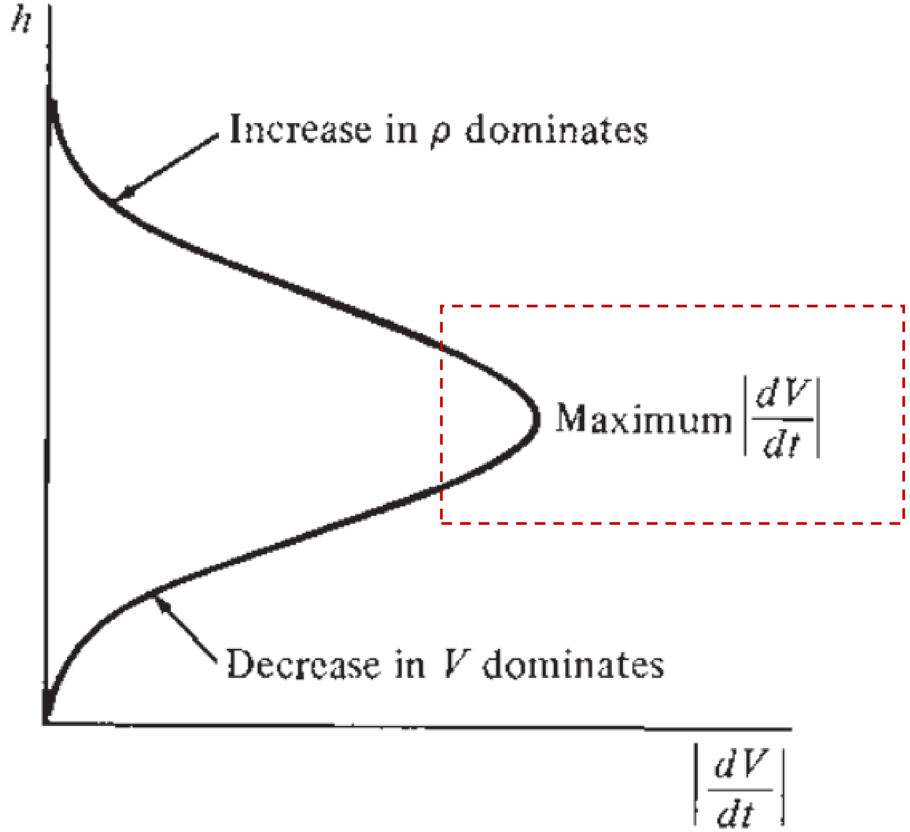
$$\text{Deceleration} = \left|\frac{dV}{dt}\right| = \frac{D}{m}$$

$$\Rightarrow \quad \left|\frac{dV}{dt}\right| = \frac{\rho V^2 S C_D}{2m}$$

Ballistic entry

Maximum Deceleration

$$\left|\frac{dV}{dt}\right| = \frac{\rho V^2 S C_D}{2m}$$



?

Ballistic entry

Maximum Deceleration

$$\boxed{\left|\frac{dV}{dt}\right| = \frac{\rho V^2 S C_D}{2m}}$$

$$\frac{\partial}{\partial t}$$

$$\left|\frac{d^2V}{dt^2}\right| = \frac{SC_D}{2m}\left(2\rho V\frac{dV}{dt} + V^2\frac{d\rho}{dt}\right)$$

Solution ?

Ballistic entry

Maximum Deceleration

$$\boxed{\frac{V}{V_E} = e^{-\rho/2[m/(C_D S)]Z\sin\theta}}$$

$$\rho = \frac{m}{C_D S} Z \sin\theta$$

$$\left|\frac{dV}{dt}\right|_{\max} = \frac{1}{2} V^2 Z \sin\theta$$

$$\Rightarrow \quad V = V_E e^{-1/2}$$

$$\left|\frac{dV}{dt}\right|_{\max} = \frac{V_E^2 Z \sin\theta}{2e}$$

Ballistic entry

Discussion

$$\boxed{\left|\frac{dV}{dt}\right|_{\max} = \frac{V_E^2 Z \sin\theta}{2e}}$$

$$\left|\frac{dV}{dt}\right|_{\max} \propto V_E^2 \quad \text{and} \quad \left|\frac{dV}{dt}\right|_{\max} \propto \sin\theta$$

1. Entry with a higher initial velocity, V_E, will experience much more severe deceleration;
2. A smaller entry angle, θ, reduces the maximum deceleration.

Ballistic entry

Practice – example 8.10

Consider a solid iron sphere entering the earth's atmosphere at 13 km/s (slightly above escape velocity) and at an angle of 15° below the local horizontal. The sphere diameter is 1 m. The drag coefficient for a sphere at hypersonic speeds is approximately 1. The density of iron is 6963 kg/m^3. Calculate (*a*) the altitude at which maximum deceleration occurs, (*b*) the value of the maximum deceleration, and (*c*) the velocity at which the sphere would impact the earth's surface.

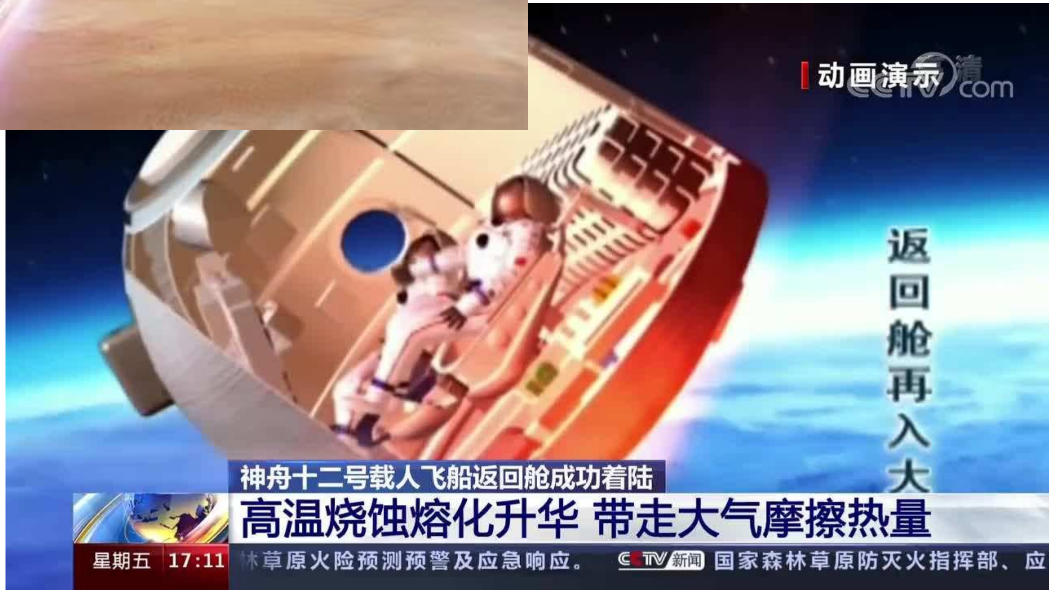
Aerodynamic heating

Question

- What’s highest temperature at the blunt body surface of the entry capsule?

Aerodynamic heating

Heat shield



Aerodynamic heating

A dimensionless heat transfer coefficient C_H

Stanton number (C_H) is defined as the ratio of heat transfer in the fluid to the heat capacity of the fluid.

$$C_H = \frac{dQ/dt}{\rho_\infty V_\infty (h_0 - h_w) S}$$

Aerodynamic heating

Aerodynamic heating rate

$$C_H = \frac{dQ/dt}{\rho_\infty V_\infty (h_0 - h_w) S}$$

$$h_0 = h_\infty + \frac{V_\infty^2}{2} \approx \frac{V_\infty^2}{2} \qquad h_0 \gg h_w \approx 0$$

$$\Rightarrow \quad \frac{dQ}{dt} = \frac{1}{2}\rho_\infty \boxed{V_\infty^3} S C_H$$

Conclusion: the aerodynamic heating rate varies as the cube of the velocity

Aerodynamic heating

The Fay-Riddell formula

$$q_w = \left.\frac{dQ}{dt}\right|_s \propto \frac{1}{\sqrt{R_N}}$$

Where q_w is the heat rate at the stagnation point of the blunt body, R_N is the radius.

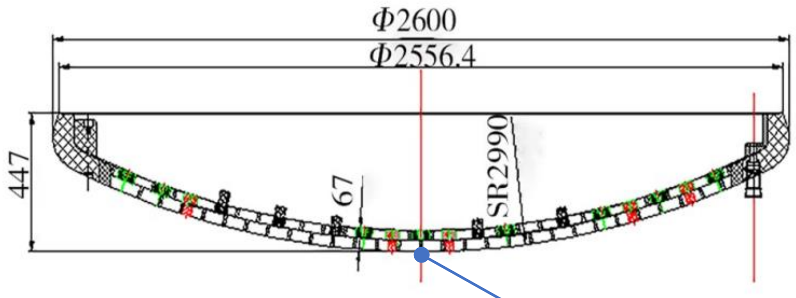


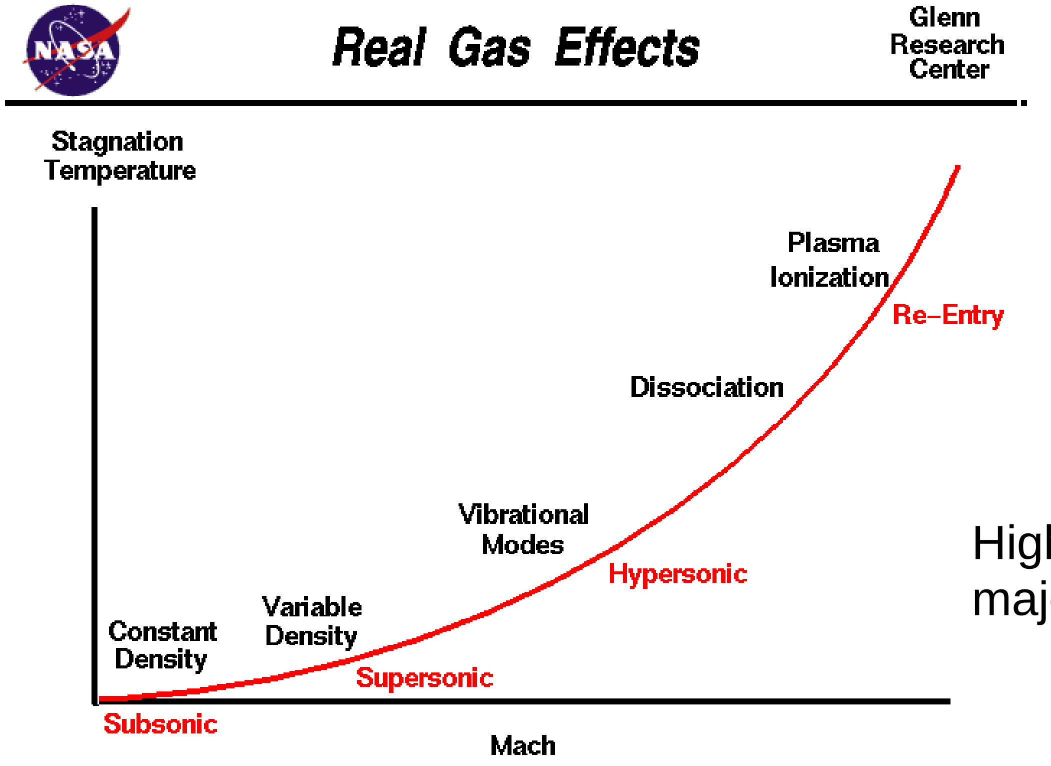
图 1 防热大底
Fig. 1 Heatshield

stagnation point

Source of the figure: 神舟飞船防热大底钻孔工艺技术研究，宇航材料工艺，2020

Aerodynamic heating

The high temperature real gas effects



High temperature [real gas effects]() play a major role in [hypersonic aerodynamics]()

Aerodynamic heating

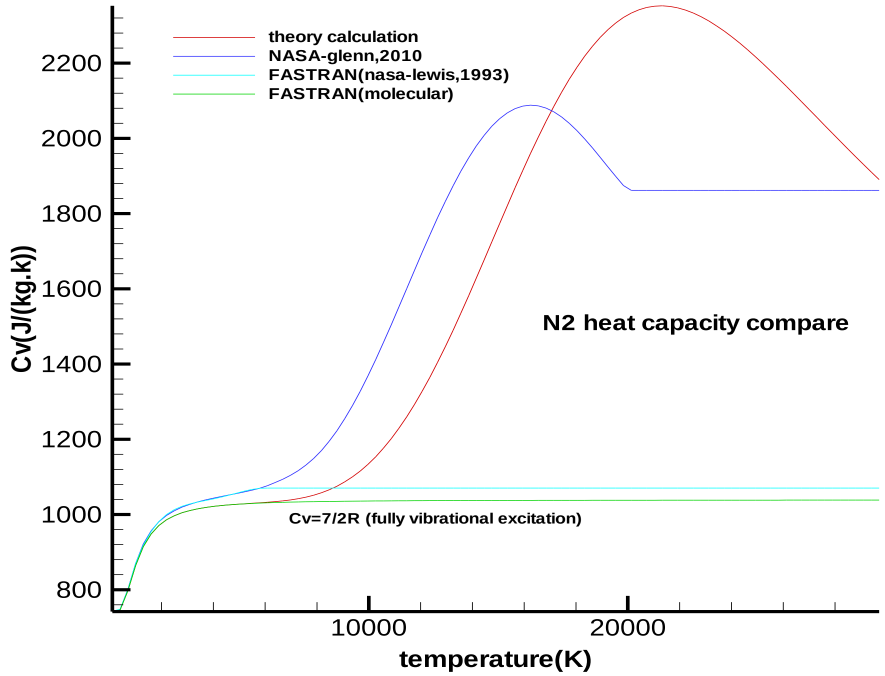
Heat capacity

$$Cv_{tr,s} = \begin{cases} \frac{5}{2}\frac{R_u}{M_s}\mathrm{T} & \textit{for diatomic} \\ \frac{3}{2}\frac{R_u}{M_s}\mathrm{T} & \textit{for monatomic} \end{cases}$$

Perfect gas

$$Cv_{v,s} = \frac{R_u}{M_s}\frac{\left(\theta_{v,s}/T_v\right)^2 \exp\left(\theta_{v,s}/T_v\right)}{\left[\exp\left(\theta_{v,s}/T_v\right)-1\right]^2}$$

Vibrational excitation

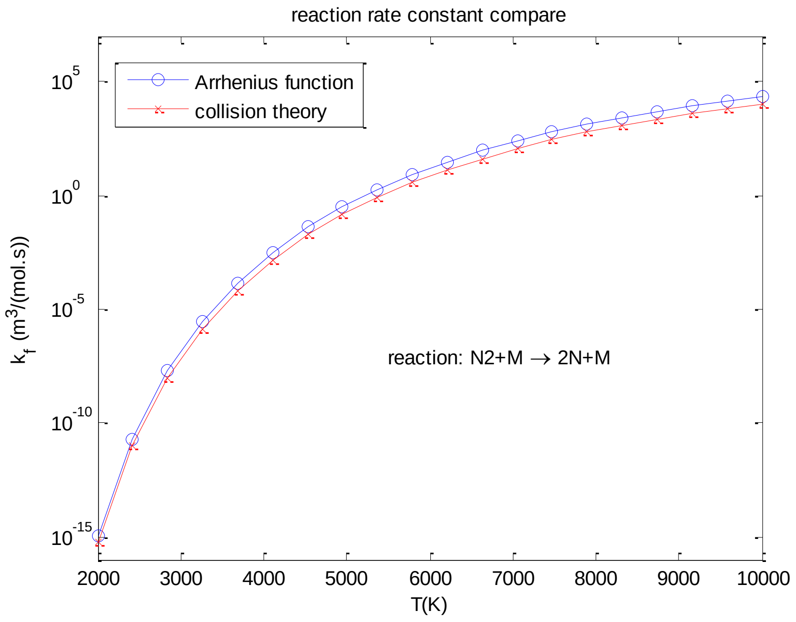


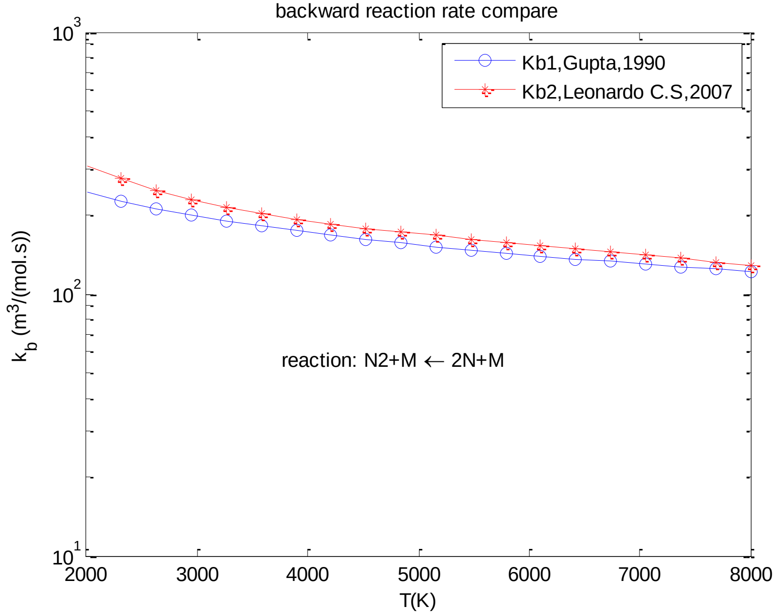
$$Cv_{e,s} = \frac{R_u}{M_s}\left\{\frac{\sum_{i=1}^{\infty} g_{i,s}\left(\theta_{el,i,s}/T_e\right)^2 \exp\left(-\theta_{el,i,s}/T_e\right)}{\sum_{i=0}^{\infty} g_{i,s}\exp\left(-\theta_{el,i,s}/T_e\right)} - \frac{\left[\sum_{i=0}^{\infty} g_{i,s}\exp\left(-\theta_{el,i,s}/T_e\right)\right]\left[\sum_{i=0}^{\infty} g_{i,s}\exp\left(\theta_{el,i,s}/T_e^2\right)\exp\left(-\theta_{el,i,s}/T_e\right)\right]}{\left[\sum_{i=0}^{\infty} g_{i,s}\exp\left(-\theta_{el,i,s}/T_e\right)\right]^2}\right\}$$

electronical excitation

Aerodynamic heating

Dissociation





Arrhenius empirical formula

$$k = CT^{\alpha} e^{-E_a/R_u T}$$

Gas Kinetic theory

$$k_f = \frac{2\sigma_{ref}}{\pi^{1/2}} \left(\frac{T}{T_{ref}} \right)^{1-\omega} \left(\frac{2kT}{m_r} \right) \left(1 + \frac{E_d}{\left(\overline{\zeta} + 3/2 - \omega \right)} \right) \exp\left(-\frac{E_d}{kT} \right)$$

Aerodynamic heating

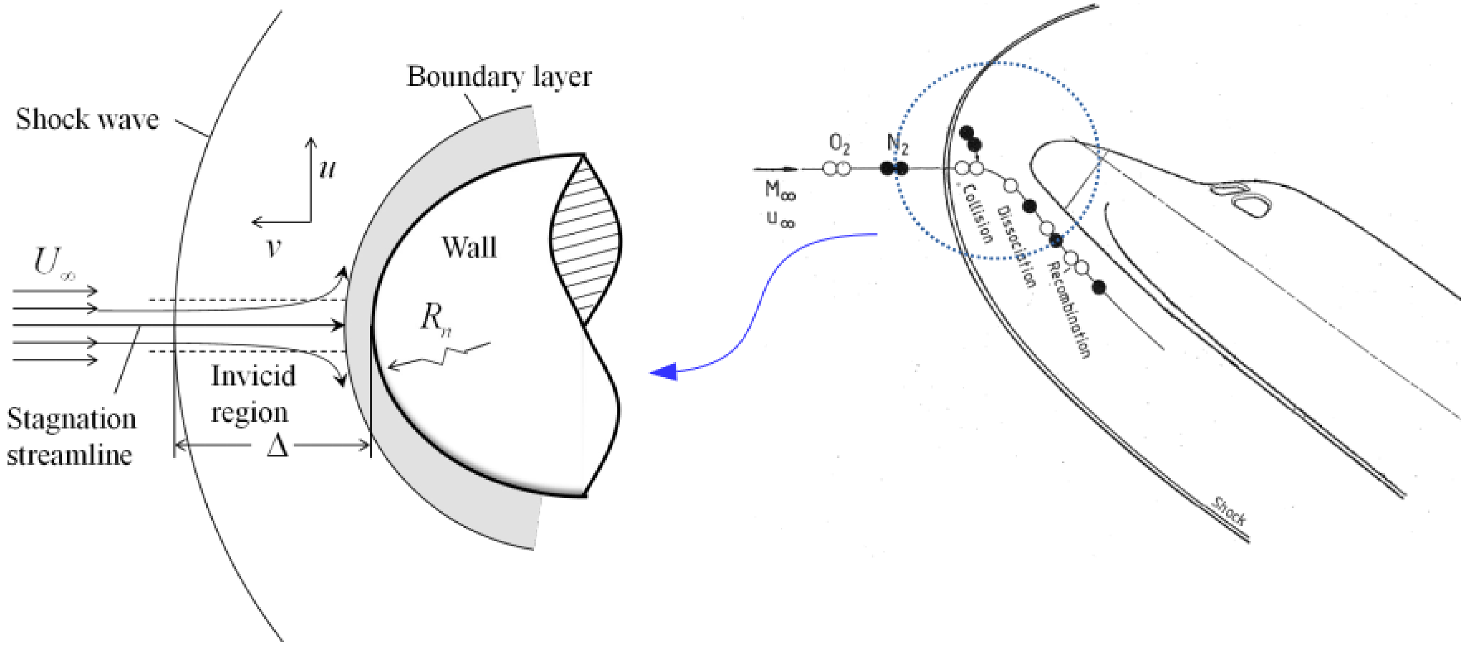
Reactions of air

$$\sum_{s=1}^{N} v'_{s,r} X_s \leftrightharpoons \sum_{s=1}^{N} v''_{s,r} X_s, \qquad r = 1, 2, \ldots, nr$$

Reaction rate coefficients: $k = C T_x^{\eta} \exp(-\theta_d / T_x)$					
Reaction	M_i	k	$C(\mathrm{m^3-kmol-s})$	η	θ_d (K)
$N_2 + M_i \rightarrow N + N + M_i$	N_2	k_{f11}	4.80×10^{14}	-0.5	113000
	O_2	k_{f12}	1.92×10^{14}	-0.5	113000
	NO	k_{f13}	1.92×10^{14}	-0.5	113000
	N	k_{f14}	4.16×10^{19}	-1.5	113000
	O	k_{f15}	1.92×10^{14}	-0.5	113000
$N + N + M_i \rightarrow N_2 + M_i$	N_2	k_{b11}	2.72×10^{10}	-0.5	0
	O_2	k_{b12}	1.10×10^{10}	-0.5	0
	NO	k_{b13}	1.10×10^{10}	-0.5	0
	N	k_{b14}	2.27×10^{15}	-1.5	0
	O	k_{b15}	1.10×10^{10}	-0.5	0
$O_2 + M_i \rightarrow O + O + M_i$	N_2	k_{f21}	7.21×10^{15}	-1.0	59500
	O_2	k_{f22}	3.25×10^{16}	-1.0	59500
	NO	k_{f23}	3.61×10^{15}	-1.0	59500
	N	k_{f24}	3.61×10^{15}	-1.0	59500
	O	[illegible]	9.02×10^{16}	-1.0	59500

Aerodynamic heating

The quasi-one-dimensional model (Chen & Sun, 2016)



[1] **Chen, S.,** Sun Q., A quasi-one-dimensional model for hypersonic reactive flow along the stagnation streamline. Chinese Journal of Aeronautics, 29(6): 1517-1526, 2016.

Entry of Mars