飞行力学
Flight Mechanics

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Chapter 2

- Static performance

  Horizontal flight,

  climbing and descending flight,

  Range and endurance.

- Dynamic performance

  Takeoff,

  Landing,

  ……

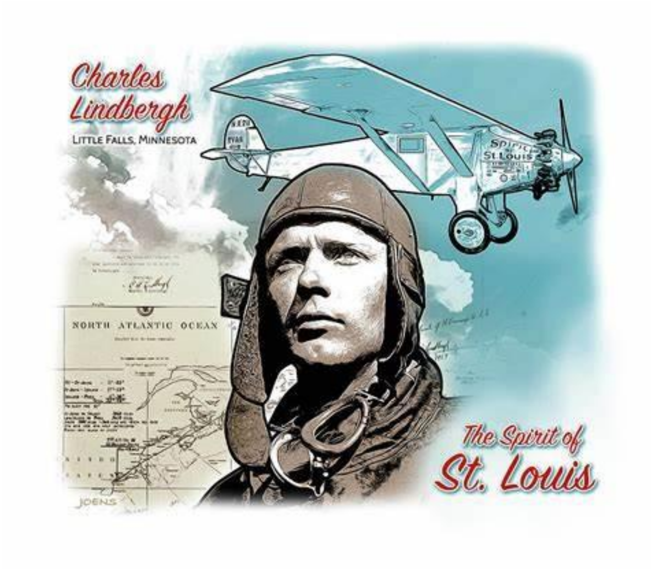
Contents

Range and endurance

The legend of Charles Lindbergh

“ When Charles Lindbergh made his spectacular solo flight across the Atlantic Ocean on May 20–21, 1927, he could not have cared less about maximum velocity, rate of climb, or time to climb. Uppermost in his mind was the maximum distance he could fly on the fuel supply carried by the Spirit of St. Louis. ”

- J.D. Anderson



Charles Lindbergh and his airplane

https://www.thisdayinaviation.com/ryan-nyp-nx211-spirit-st-louis/
https://www.biography.com/historical-figure/charles-lindbergh

Range and endurance

Some definitions

Range (航程): the total distance (measured with respect to the ground) traversed by an airplane on a tank of fuel.

Endurance (航时): the total time that an airplane stays in the air on a tank of fuel.

Specific fuel consumption(SFC, 耗油率): c_f [kg/(W·h)], weight of fuel consumed per unit power per unit time

Thrust-specific fuel consumption(TSFC, 耗油率): c_f [kg/(N·h)], weight of fuel consumed per unit thrust per unit time

Fuel consumption per hour (小时耗油量): $c_{f,t}$ [kg/h]

Fuel consumption per kilometer (千米耗油量): $c_{f,R}$ [kg/km]

Range and endurance

Some definitions

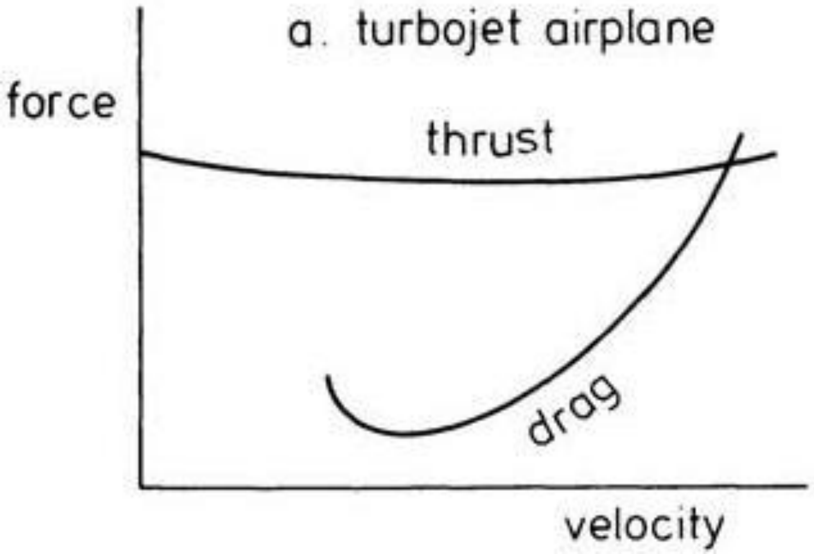
Performance diagram (性能图):

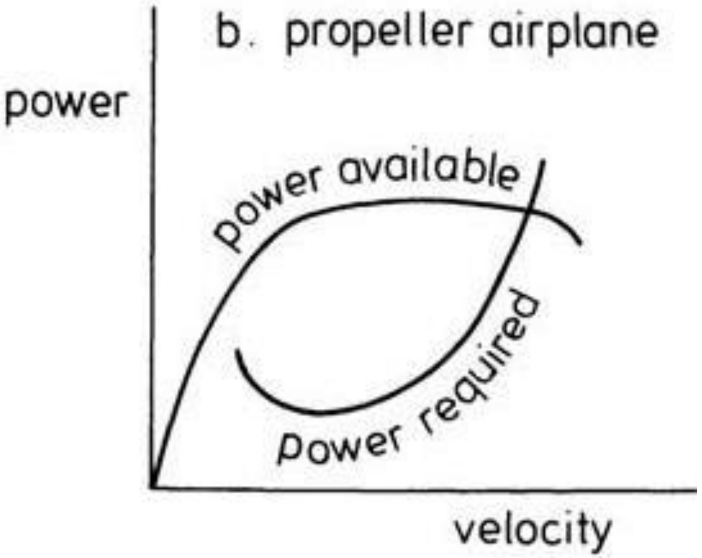
For a given altitude, configuration and engine control setting, the drag or power required and thrust or power available are plotted against airspeed.

(Performance diagram is also called Pernaud diagram, which is very useful to determine aircraft performance parameters)

Range and endurance

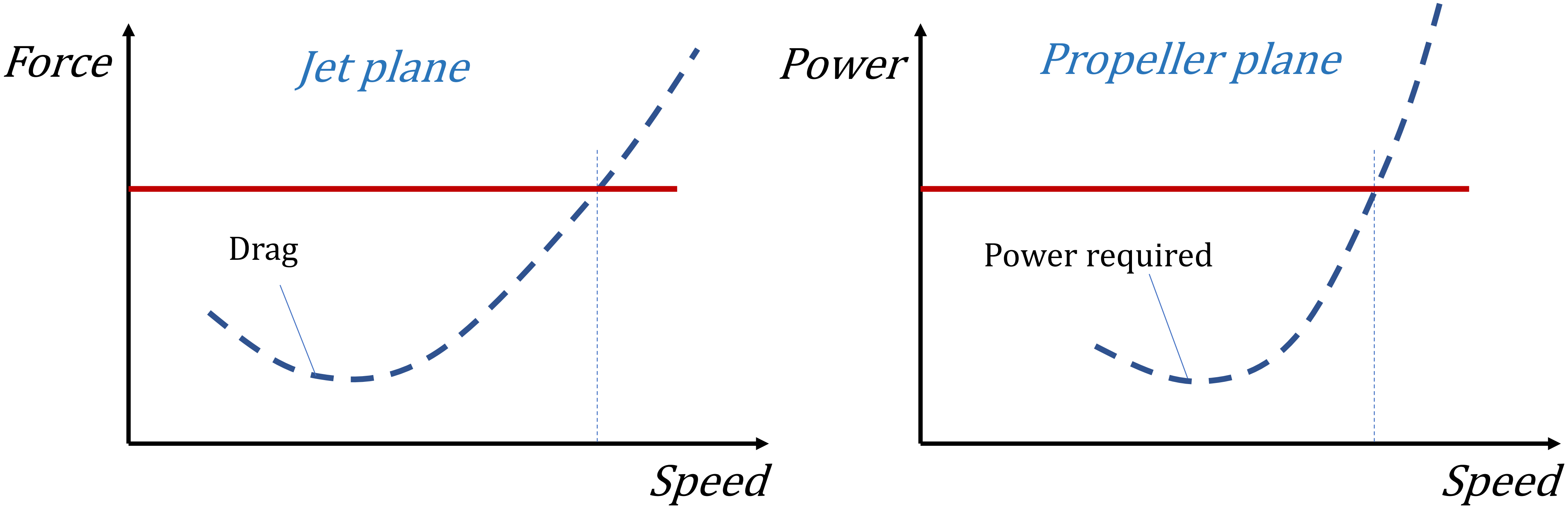
Performance diagram





Range and endurance

Performance diagram - Assumption



Range and endurance

Jet airplane

Fuel consumption per hour [kg/h]

$$C_{f,t} = \frac{C_f T_a}{\eta} = \frac{C_f T_R}{\eta}$$

$$\Rightarrow \left(C_{f,t}\right)_{min} = \left(\frac{C_f T_a}{\eta}\right)_{min} \Rightarrow T_{R,min}$$

Range and endurance

Jet airplane

Fuel consumption per hour [kg/h]

$$C_{f,t} = \frac{C_f T_a}{\eta} = \frac{C_f T_R}{\eta}$$

$$T_{R,min} \Leftrightarrow D_{min} \quad \Rightarrow \left(\frac{C_L}{C_D}\right)_{max}$$

Maximum endurance for a jet airplane occurs when the airplane is flying at a **velocity such that C_L/C_D is at its maximum.**

Range and endurance

Jet airplane

Fuel consumption per kilometer [kg/km]

$$C_{f,R} = \frac{C_f T_a}{\eta V} = \frac{C_f T_R}{\eta V}$$

$$\Rightarrow \left(C_{f,R}\right)_{min} = \left(\frac{C_f T_a}{\eta V}\right)_{min} \Rightarrow \left(\frac{T_R}{V}\right)_{min}$$

Range and endurance

Jet airplane

Fuel consumption per kilometer [kg/km]

$$C_{f,R} = \frac{C_f T_a}{\eta V} = \frac{C_f T_R}{\eta V}$$

$$\left(\frac{T_R}{V}\right)_{min} \Leftrightarrow \left(\frac{D}{V}\right)_{min} \Rightarrow \left(\frac{C_L}{C_D^2}\right)_{max}$$

<u>Maximum range</u> for a jet airplane occurs when the airplane is flying **at a velocity such that C_L/C_D^2 is at its maximum**

Range and endurance

Jet airplane

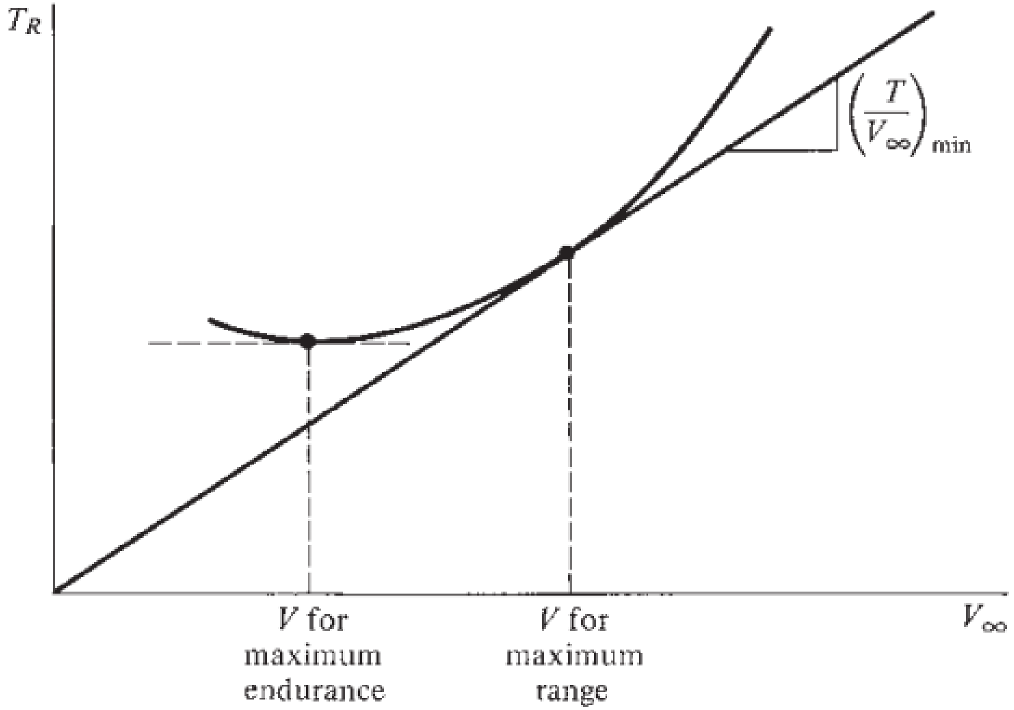


Figure 6.45 Points of maximum range and endurance on the thrust-required curve.

Range and endurance

Propeller airplane

Fuel consumption per hour [kg/h]

$$C_{f,t} = \frac{C_f P_a}{\eta} = \frac{C_f P_R}{\eta} = \frac{C_f DV}{\eta}$$

$$\Rightarrow \left(C_{f,t}\right)_{min} = \left(\frac{C_L^3}{C_D^2}\right)_{max}$$

<u>Maximum endurance</u> for a propeller airplane occurs when the airplane is flying **at a velocity such that C_L^3/C_D^2 is at its maximum**

Range and endurance

Propeller airplane

Fuel consumption per kilometer [kg/km]

$$C_{f,R} = \frac{C_f P_a}{\eta V} = \frac{C_f P_R}{\eta V}$$

$$\Rightarrow \left(C_{f,R}\right)_{min} = \left(\frac{C_f D V}{\eta V}\right)_{min} \Rightarrow D_{min} \quad \Rightarrow \left(\frac{C_L}{C_D}\right)_{max}$$

<u>Maximum range</u> for a propeller airplane occurs when the airplane is flying at a **velocity such that C_L/C_D is at its maximum.**

Range and endurance

Propeller airplane

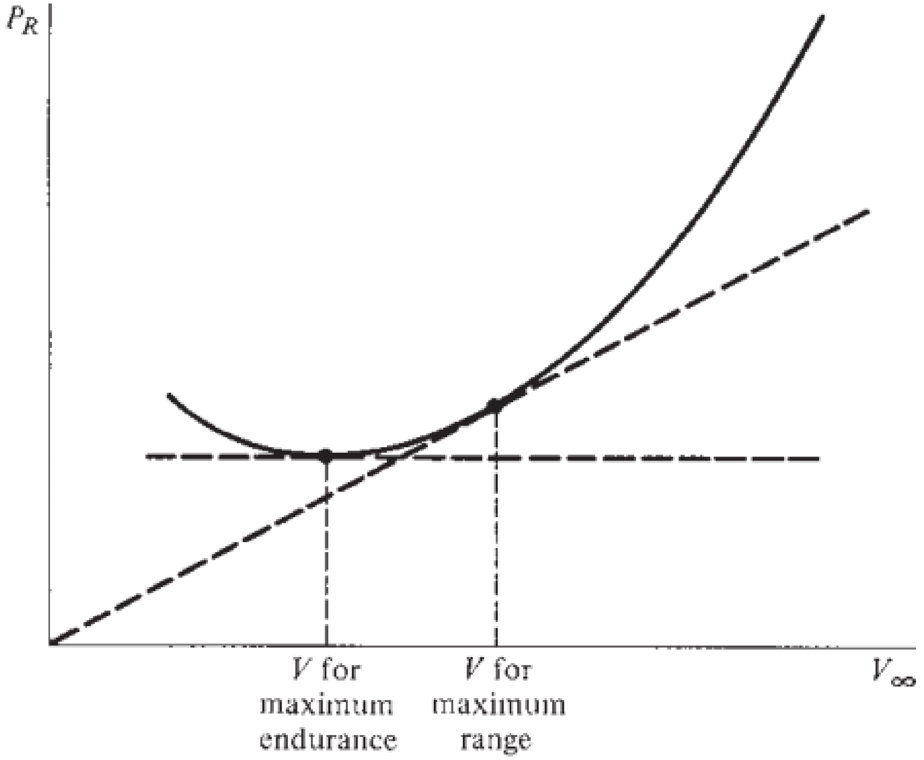
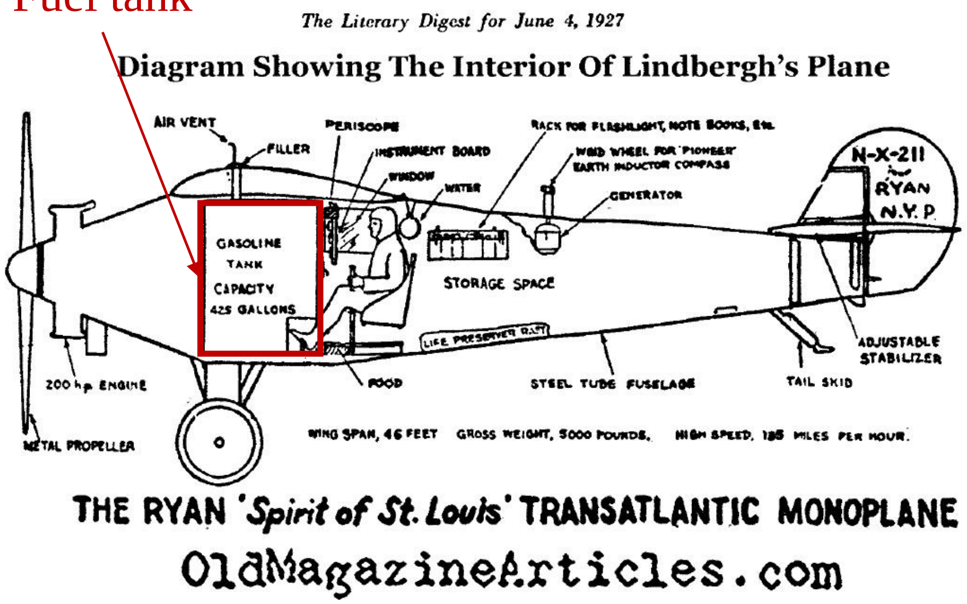


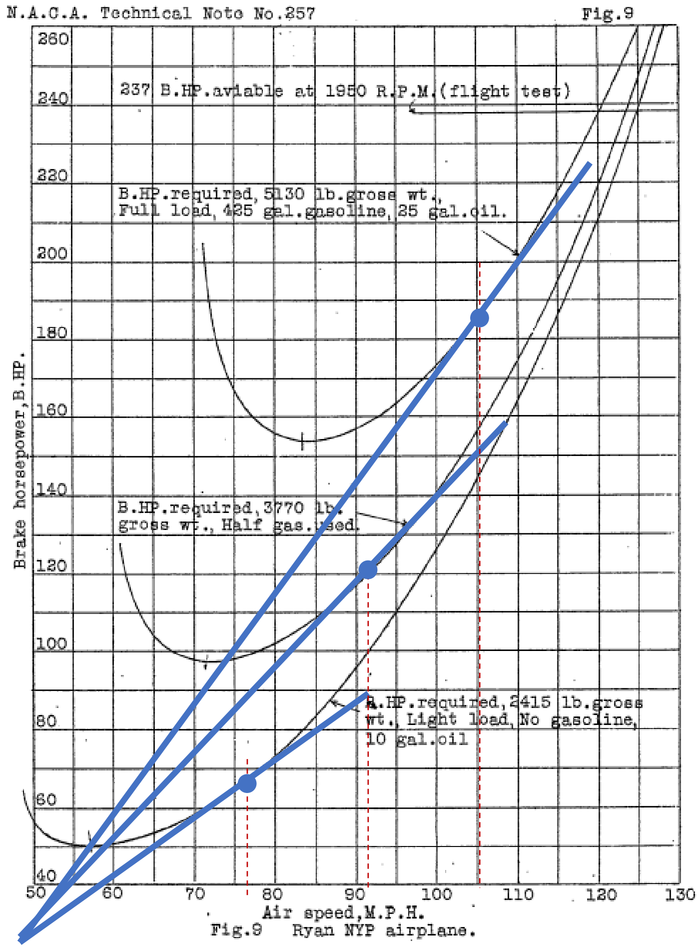
Figure 6.42 Points of maximum range and endurance on the power-required curve for a propeller-driven airplane.

Range and endurance

The legend of Charles Lindbergh

Fuel tank



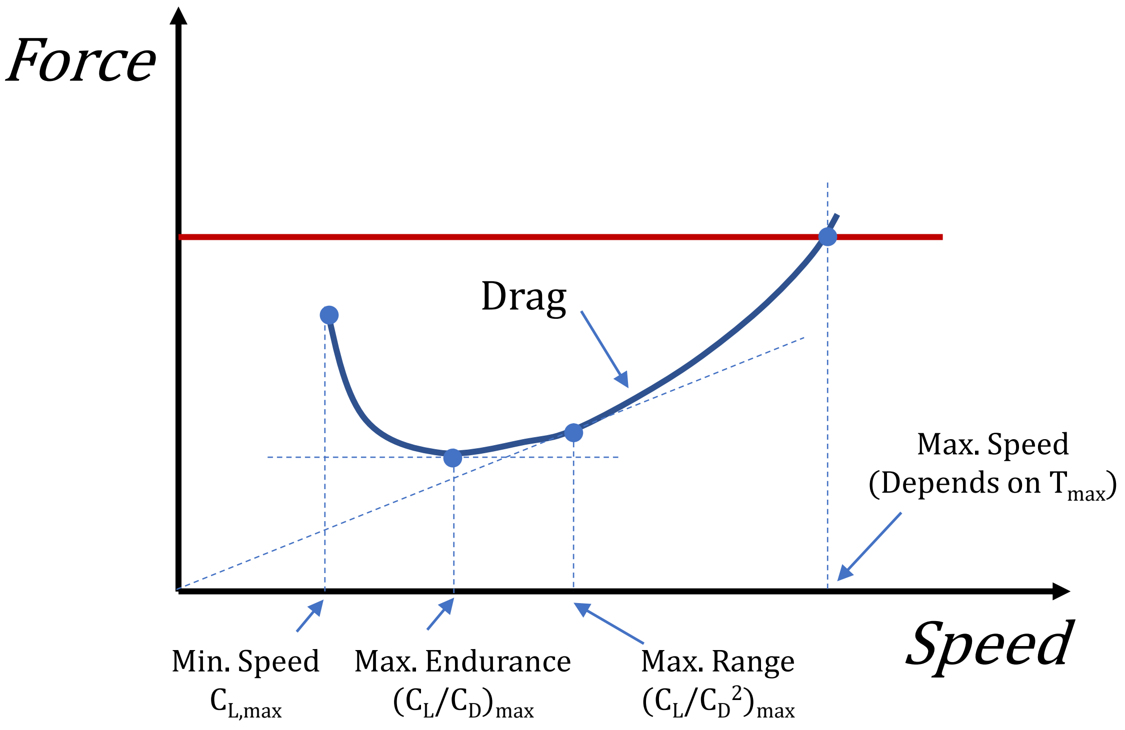


Performance of diagram

http://charleslindbergh.com/plane/naca-tn-257.pdf

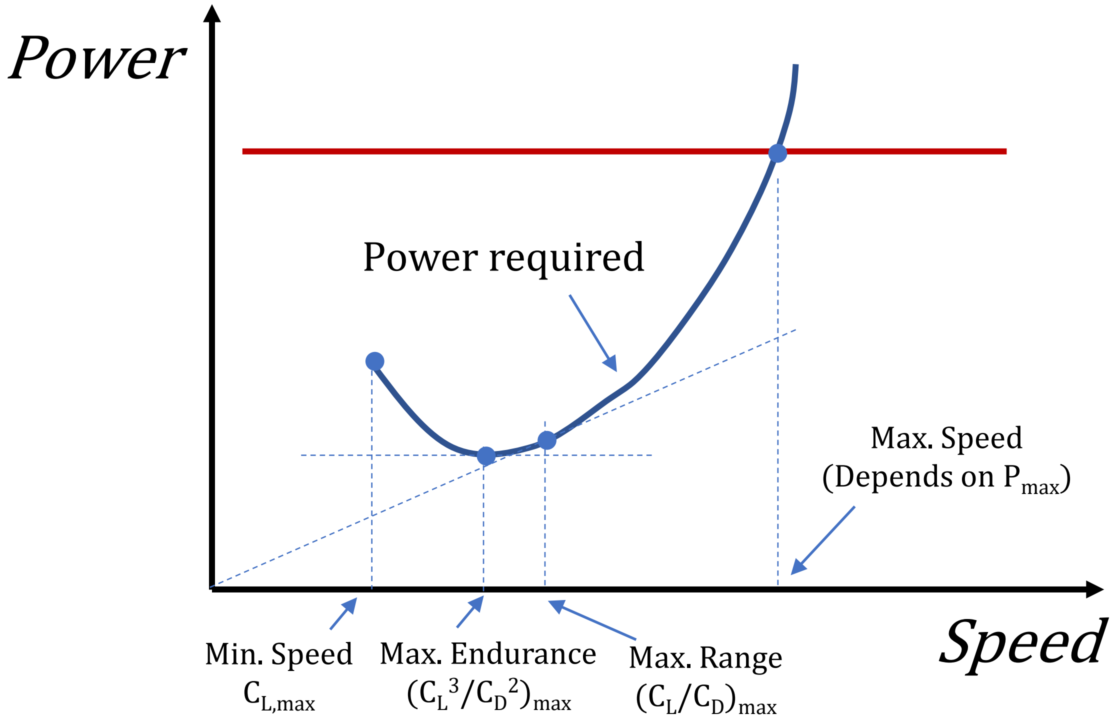
Range and endurance

Summary – Jet



Range and endurance

Summary – Propeller



Range and endurance

Summary – horizontal steady flight

$$T = D$$
$$L = W$$

Minimum airspeed

$$\boxed{C_{L,max}} \Rightarrow V_{min} = \sqrt{\frac{W}{S}\frac{2}{\rho}\frac{1}{C_{L,max}}}$$

Maximum airspeed

$$T_{max} = D\,(Jet)$$
$$P_{a,max} = P_R\,(Prop)$$

$\Rightarrow$ Solve the C_L $\Rightarrow$

$$V_{max} = \sqrt{\frac{W}{S}\frac{2}{\rho}\frac{1}{C_L}}$$

Range and endurance

$$T = D$$
$$L = W$$

Summary – horizontal steady flight

Maximum endurance

$$\begin{tikzcd}
 & \left(\frac{C_L}{C_D}\right)_{max} \arrow[r] & C_L = \sqrt{C_{D_0}\pi\lambda_e} \arrow[dr] & \\
(C_{f,t})_{min} \arrow[ur, "Jet"] \arrow[dr, "Prop."'] & & & V = \sqrt{\frac{W}{S}\frac{2}{\rho}\frac{1}{C_L}} \\
 & \left(\frac{C_L^3}{C_D^2}\right)_{max} \arrow[r] & C_L = \sqrt{3C_{D_0}\pi\lambda_e} \arrow[ur] &
\end{tikzcd}$$

Range and endurance

$$T = D$$
$$L = W$$

Summary – horizontal steady flight

Maximum range

$$\begin{tikzcd}
 & \left(\frac{C_L}{C_D^2}\right)_{max} \arrow[r] & C_L = \sqrt{\frac{1}{3} C_{D_0} \pi \lambda_e} \arrow[dr] & \\
\left(C_{f,R}\right)_{min} \arrow[ur, "Jet"] \arrow[dr, "Prop."'] & & & V = \sqrt{\frac{W}{S}\frac{2}{\rho}\frac{1}{C_L}} \\
 & \left(\frac{C_L}{C_D}\right)_{max} \arrow[r] & C_L = \sqrt{C_{D_0} \pi \lambda_e} \arrow[ur] &
\end{tikzcd}$$

Example

The Gulfstream IV, as shown in the Figure is a twin-turbofan executive transport aircraft.
Data of this aircraft are given below:

$S = 28.2$ [m^2]
$b = 23.7$ [m]
$e = 0.67$
$C_D = C_{D_0} + C_L^2/(\pi\lambda_e)$
$C_{D0} = 0.015$,
$W = 311$ [kN]

The aircraft has two Rolls Royce Tay turbofan engines. Fuel consumption can be represented with the following equation: $C_{f,t} = C_f T$, where $C_f = 0.69$ [N/(h·N)]. The thrust specific fuel consumption is assumed to be independent of airspeed and altitude.

Question:

What is the maximum specific range of this aircraft when flying at 8000m ($\rho = 0.5252$kg/m^3) ?

Homework

1. From the definition of drag polar, prove that

   a) maximum (C_L/C_D) is at $C_L = \sqrt{C_{D0}\pi\lambda_e}$

   b) maximum lift-to-drag ratio $K_{max} = \frac{1}{2}\sqrt{\pi\lambda_e/C_{D0}}$ at maximum (C_L/C_D)

   c) maximum (C_L^3/C_D^2) is at $C_L = \sqrt{3C_{D0}\pi\lambda_e}$

2. A jet plane data is as follows (page 90 of textbook, 2.5).

   - Weight = 30,000 N
   - Wing loading W/S = 1000 N/m^2
   - Assume C_{D0} = 0.015+0.024 C_L^2, and $C_{Lmax} = 1.4$
   - Maximum thrust available T_a = 4,000 N can be assumed independent of airspeed

   Calculate the maximum and minimum speed for steady horizontal flight.

Homework

3. A propeller plane data is given as follows:
   - Drag coefficient at zero lift, C_{D0} = 0.02,
   - Wing area S = 100 m^2, with wing loading W/S = 7000 N/m^2
   - Aspect ratio, λ = 10
   - Span efficiency factor e = 0.85
   - Maximum power available 10435 kW can be assumed independent of airspeed

   a) Calculate the speed at maximum lift-to-drag ratio for steady horizontal flight.

   b) Calculate the maximum rate of climb when the aircraft performs a steady climb flight (ρ=1.225 [kg/m^3]).

Homework

4. China Eastern Airlines flight MU5735 crashed near Wuzhou en route from Kunming (KMG) to Guangzhou (CAN) on Monday, 21 March 2022. The flight was operated by a Boeing 737-800. The data of the aircraft and the flight track are given as follows (pay attention to the units.)

Table 1. Data of Boeing 737-800

Wing area	S	125.0 m^2
Wing span	b	34.34 m
Max takeoff weight	W_t	70,535 Kg
Thrust available	T_a	2*107.6 kN
Cursing speed	V	848 km/h
Max range	R_{max}	3200 miles
Max altitude	H_{max}	12,400 m

The zero-lift coefficient and span efficiency factor can be assumed as C_{D0}=0.02, e = 0.85.

Table 2. First 120 second of Granular ADS-B data

seconds before last contact	lat	lng	altitude (feet)	speed (knots per hour)	Vs (feet per minute)	heading (deg)	nose position
0.000	25.09924	102.91611	7850	174	1792	214	P
120.851	25.01555	102.81168	11925	251	3136	214	P

Homework

Table 3. Final 150 seconds of Granular ADS-B data

seconds before last contact	lat	lng	altitude (feet)	speed (knots per hour)	Vs (feet per minute)	heading (deg)	nose position
3809.248	23.36266	110.76612	29100	457	0	100	N
3811.378	23.36182	110.77128	29100	457	0	100	P
3816.690	23.35987	110.78303	29100	457	0	100	P
3822.580	23.35791	110.79478	29100	457	0	100	P
3827.694	23.35593	110.80695	29100	457	0	100	P
3829.424	23.35529	110.81084	29100	457	0	100	N
3837.354	23.35236	110.82868	29100	457	0	100	P
3842.530	23.35043	110.84042	29100	457	0	100	P
3847.472	23.34874	110.85083	29100	457	0	100	P
3863.590	23.35812	110.88536	27025	433	-21696	98	P
3863.745	23.35810	110.88575	26875	436	-21888	98	N
3867.874	23.35744	110.89615	24925	425	-30784	91	P
3872.908	23.35661	110.90543	22250	386	-30976	100	P
3878.710	23.35263	110.91592	17325	429	-30976	123	P
3883.586	23.34474	110.92432	15325	520	-22528	140	P
3894.436	23.32310	110.94439	12725	551	-16832	127	P
3898.918	23.31711	110.95592	11000	556	-21312	112	P
3904.578	23.31420	110.96949	9150	558	-21888	100	P
3909.336	23.31490	110.98338	7850	590	-15744	81	P
3919.606	23.30773	111.01476	7425	565	3520	75	P
3924.822	23.30940	111.02875	8025	531	7360	84	P
3928.758	23.30974	111.04041	8600	507	8448	88	P
3945.628	23.33031	111.07468	8175	446	-13248	79	P
3955.646	23.33533	111.09754	4375	442	-26752	73	P
3959.730	23.33752	111.10558	3225	376	-30976	87	P

Homework

Assume the atmosphere density and the weight of the aircraft are constant: $\rho = 0.8194\ kg/m^3$, $W = W_t$.

a) Use the data from table 2 to calculate the rate of climb (RC) and climb angle of the aircraft.

b) Use the time and position information of two points from table 2 to calculate heading angle χ_K and speed V during <u>normal cruising period</u>, and check the validity ADS-B data.

c) Assume both engine failed at t=3847.472s, but the pilots can still control the aircraft. Could the aircraft have made it to Guangzhou airport if the pilots had decided to glide there? (You can use commercial digital map to estimate the straight-line distance to the Guanzhou airport)

d) There is a sharp altitude raise around t=3924s with relatively small speed drop in table 2. Use the equation in Page 26 of lecture 5 to estimate whether this is possible with no engine power (P_a=0). If not, what do you think has happened?

e) If you were appointed as the expert to investigate the accident, what do you think are the possible causes of the crash? Please state your reasons.

Useful reference:

1. https://www.flightradar24.com/blog/china-eastern-airlines-flight-5735-crashes-en-route-to-guangzhou/

2. https://www.airliners.net/aircraft-data/boeing-737-800900/96

Due on Friday, 1st April