A, B卷答案

一　填空题(每小题3分，共27 分)

1. 答案:$\frac{1}{8}$

2. 答案:$\frac{2}{5}$

3. 答案:$\frac{99}{100}$

4. 答案:$2(n-1)\sigma^4$

5. 答案:$\hat{\lambda}=\frac{n}{\sum\limits_{i=1}^{n}X_i^{\alpha}}$

6. 答案:$\frac{1}{p}, \frac{1-p}{p^2}$

7. 答案:$f(u)=0.3f(u-1)+0.7f(u-2)$

8. 答案:$\frac{1}{2(n-1)}$

9. 答案:$\frac{1}{4}$

二　选择题(每小题3分，共27 分)

1. 答案:B

2. 答案:D

3. 答案:B

4. 答案:B

5. 答案:A

6. 答案:C

7. 答案:D

8. 答案:B

9. 答案:A

三 (本题16分)

解答:(1)$f_X(x)=\begin{cases}\int_0^2(x^2+\frac{1}{3}xy)dy=2x^2+\frac{2}{3}x, & 0<x<1\\ 0, & \text{其他}\end{cases}$ ....(2分)

$f_Y(y)=\begin{cases}\int_0^1(x^2+\frac{1}{3}xy)dx=\frac{1}{3}+\frac{1}{6}y, & 0<y<2\\ 0, & \text{其他}\end{cases}$ ....(4分)

(2)$y\in(0,2)$时,条件密度为:

$f_{X|Y}(x|y)=\dfrac{f(x,y)}{f_Y(y)}=\begin{cases}\dfrac{x^2+\frac{1}{3}xy}{\frac{1}{3}+\frac{1}{6}y}=\dfrac{6x^2+2xy}{2+y}, & 0<x<1\\ 0, & \text{其他}\end{cases}$ ....(6分)

$y\notin(0,2)$时,条件密度不存在............(8分)

(3)$f_{X|Y}(x|1)=\begin{cases}2x^2+\dfrac{2x}{3}, & 0<x<1\\ 0, & \text{其他}\end{cases}$ ....(10分)

$P(X<\frac{1}{3}|Y=1)=\int_0^{\frac{1}{3}}(2x^2+\frac{2x}{3})dx=\frac{5}{81}$............(12分)

(4)$P(X<\frac{1}{3}|Y<1)=\dfrac{P(X<\frac{1}{3},Y<1)}{P(Y<1)}$............(14分)

$=\dfrac{\int_0^{\frac{1}{3}}\int_0^1(x^2+\frac{1}{3}xy)dydx}{\int_0^1(\frac{1}{3}+\frac{1}{6}y)dy}=\dfrac{\frac{7}{324}}{\frac{5}{12}}=\dfrac{7}{135}$............(16分)

四 解答题(本题14分)

解答:(1)$E(X)=\int_0^\theta x\cdot\frac{3}{\theta^3}x^2dx=\frac{3}{4}\theta,\frac{3}{4}\hat{\theta}=\overline{X},\hat{\theta}=\frac{4}{3}\overline{X}$............(2分)

(2)由(1)知,$E\frac{4}{3}\overline{X}=\theta$............(4分)

X的分布函数$F(x)=\begin{cases}0, & x\le 0\\ \dfrac{x^3}{\theta^3}, & 0\le x\le\theta\\ 1, & \text{其他}\end{cases}$

Y_n的密度$f_n(x)=\begin{cases}n(\dfrac{x^3}{\theta^3})^{n-1}\cdot\dfrac{3}{\theta^3}x^2=\dfrac{3nx^{3n-1}}{\theta^{3n}}, & 0\le x\le\theta\\ 0, & \text{其他}\end{cases}$ ....(6分)

$E(Y_n)=\int_0^\theta x\cdot\frac{3nx^{3n-1}}{\theta^{3n}}dx=\frac{3n}{3n+1}\theta$

$E\frac{3n+1}{3n}Y_n=\theta$............(8分)

(3)$E(X^2)=\int_0^\theta x^2\cdot\frac{3}{\theta^3}x^2dx=\frac{3}{5}\theta^2$

$D(X)=E(X^2)-(EX)^2=\frac{3}{5}\theta^2-(\frac{3}{4}\theta)^2=\frac{3}{80}\theta^2$ .... (10分)

$D(\frac{4}{3}\overline{X})=\frac{16}{9}D(\overline{X})=\frac{16}{9}\cdot\frac{1}{n}D(X)=\frac{\theta^2}{15n}$

$E(Y_n^2)=\int_0^\theta x^2\cdot\frac{3nx^{3n-1}}{\theta^{3n}}dx=\frac{3n}{3n+2}\theta^2$

$D(Y_n)=E(Y_n^2)-(EY_n)^2=\frac{3n}{(3n+2)(3n+1)^2}\theta^2$

$D(\frac{(3n+1)}{3n}Y_n)=\frac{(3n+1)^2}{9n^2}D(Y_n)=\frac{\theta^2}{3n(3n+2)}$ .... (12分)

$D(\frac{(3n+1)}{3n}Y_n)<D(\frac{4}{3}\overline{X})$

$\frac{(3n+1)}{3n}Y_n$比$\frac{4}{3}\overline{X}$更有效. .... (14分)

[五] (本题学《概率统计A》的学生做，学《概率统计B》的学生不做,本题8分)

解答:$E(X(t))=0$为常数, ... (2分)

$R_X(t,t+\tau)=E(X(t)X(t+\tau))$

$=E(X+tY+t^2Z)(X+(t+\tau)Y+(t+\tau)^2Z)$ .... (4分)

$=E(X^2+t(t+\tau)Y^2+t^2(t+\tau)^2Z^2)=1+t(t+\tau)+t^2(t+\tau)^2$ .... (6分)

$\neq R_X(\tau)$,故不是平稳分布. .... (8分)

五 (本题学《概率统计B》的学生做，学《概率统计A》的学生不做,本题8 分)

解:设$X\sim N(\mu,\sigma^2)$

$H_0:\mu=70;\qquad H_1:\mu\neq 70$ .... (2分)

拒绝域$|t|=\dfrac{|\bar{x}-70|}{s}\sqrt{n}\geq t_{1-\frac{\alpha}{2}(n-1)}$ .... (4分)

由$n=36,\hat{x}=66.5,s=15,t_{0.975}(35)=2.0301$,得

$|t|=\dfrac{|66.5-70|}{15}\sqrt{36}=1.4<2.0301$ .... (6分)

接受原假设,可以认为平均成绩70分 .... (8分)

[六] (本题学《概率统计A》的学生做，学《概率统计B》的学生不做,本题8分)

解答:(1)状态空间$\{1,2,3\}$ .... (2分)

$$P=\begin{pmatrix}\frac{2}{4} & \frac{1}{4} & \frac{1}{4}\\ \frac{2}{5} & \frac{2}{5} & \frac{1}{5}\\ \frac{1}{2} & \frac{1}{3} & \frac{1}{6}\end{pmatrix}$$ .... (4分)

$P\{X_1=3,X_2=2,X_4=1\}=P\{X_1=3\}\cdot P_{32}\cdot P_{21}^{(2)}$ .... (6分)

$=\dfrac{1}{4}\cdot\dfrac{1}{3}\cdot\sum_{k=1}^{3}P_{2k}\cdot P_{k1}=\dfrac{1}{4}\cdot\dfrac{1}{3}\cdot(\dfrac{2}{5}\cdot\dfrac{1}{2}+\dfrac{2}{5}\cdot\dfrac{2}{5}+\dfrac{1}{5}\cdot\dfrac{1}{2})=\dfrac{23}{600}$ .... (8分)

六 (本题学《概率统计B》的学生做，学《概率统计A》的学生不做,本题8分)

解答:(1)$f(x,y)=\begin{cases}\dfrac{1}{\pi}, & x^2+y^2\leq 1\\ 0, & x^2+y^2>1\end{cases}$ .... (2分)

$EX=EY=EXY=0,cov(X,Y)=EXY-EXEY=0$ .... (4分)

(2)$f_X(x)=\begin{cases}\dfrac{2\sqrt{1-x^2}}{\pi}, & |x|\leq 1\\ 0, & |x|>1\end{cases}$ .... (6分)

$f_Y(y)=\begin{cases}\dfrac{2\sqrt{1-y^2}}{\pi}, & |y|\leq 1\\ 0, & |y|>1\end{cases}$

$f(x,y)\neq f_X(x)f_Y(y)$ .... (8分)