北京航空航天大學
BEIHANG UNIVERSITY

飞行力学
Flight Mechanics

Chen, Song（陈松）

School of General Engineering (SGE)

chensong@buaa.edu.cn； Office: D-1109

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Chapter 2

- Static performance

  Horizontal flight,

  climbing and descending flight,

  Range and endurance.

- Dynamic performance

  Takeoff,

  Landing,

  ……

Contents

Questions

Aircraft performance

- **How high can an aircraft fly?**
- How long can an aircraft stay in air?
- How far can an aircraft reach?

"Dragon lady"

U-2 reconnaissance aircraft built by Lockheed

Equation of Motion

Some definitions

Straight flight: flight in which the center of gravity of the aircraft travels along a straight line (dγ/dt = 0)

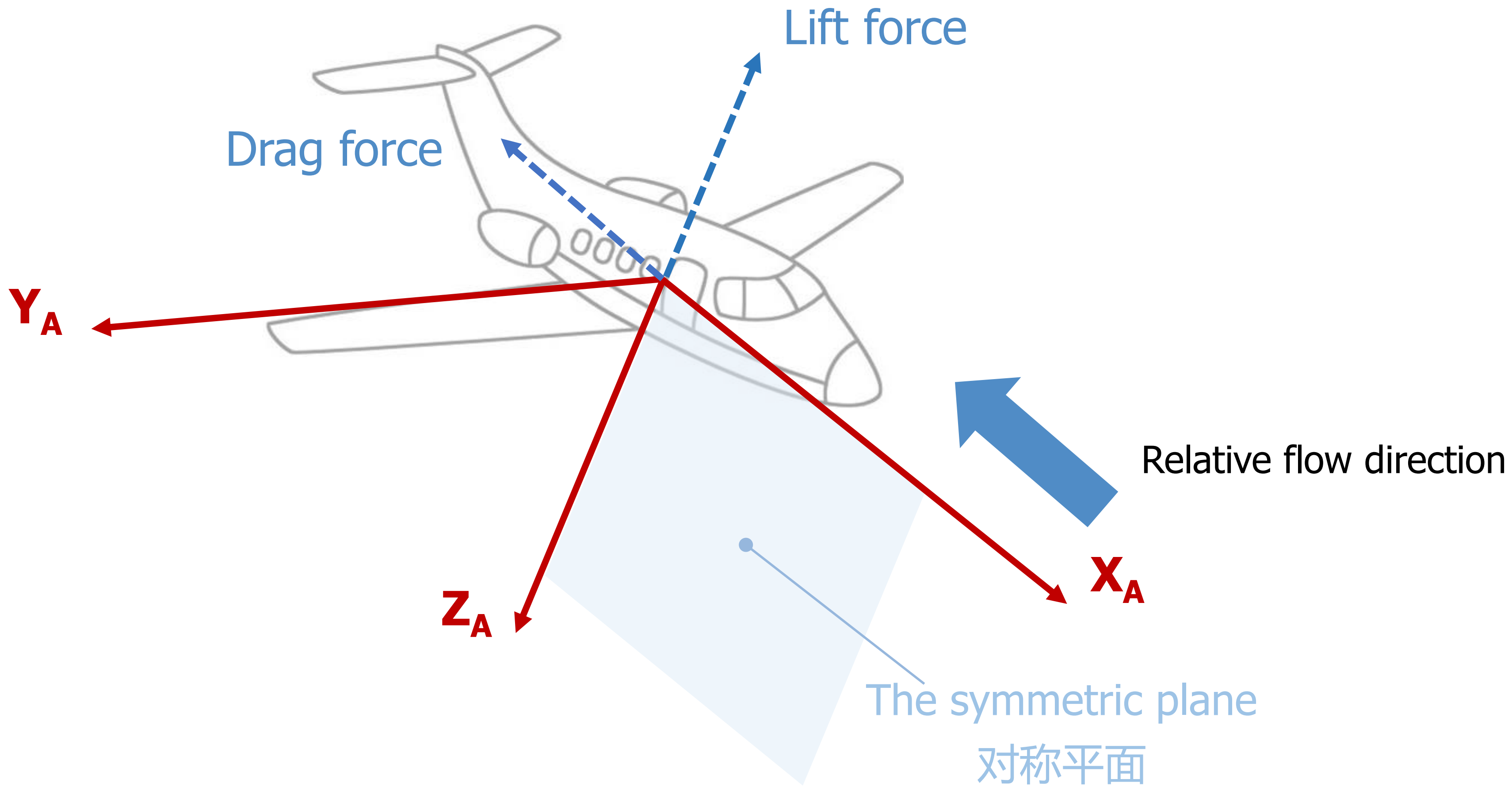
Steady flight: Flight in which the forces and moments acting on the aircraft do not vary in time, neither in magnitude, nor in direction (dV/dt = 0)

Horizontal flight: The aircraft remains at a constant altitude (γ = 0)

Symmetric flight: flight in which both the angle of sideslip is zero and the plane of symmetry of the aircraft is perpendicular to the earth (β = 0 and the aircraft is not turning)

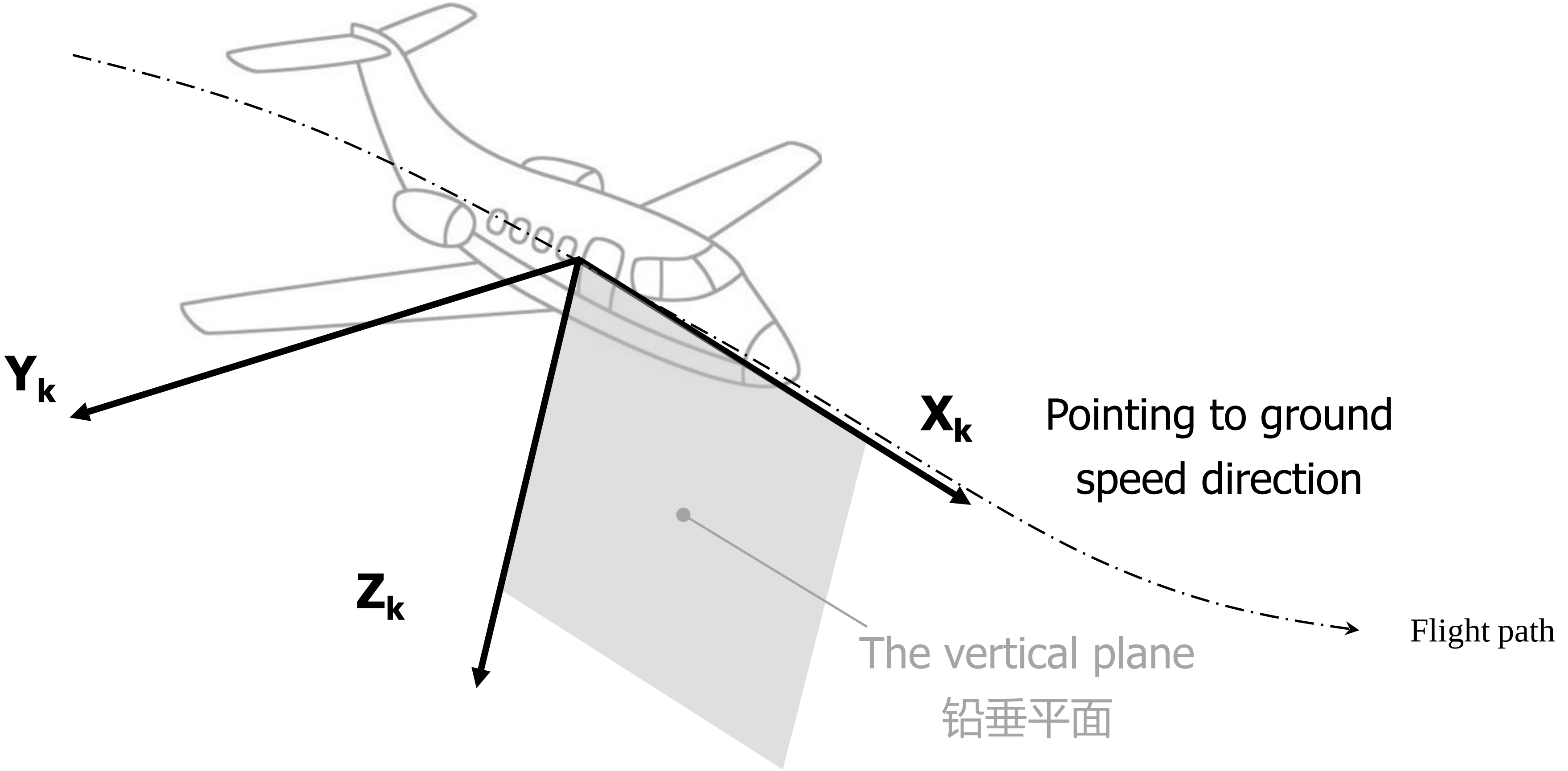
Review of frames

(O_A, x_A, y_A, z_A)



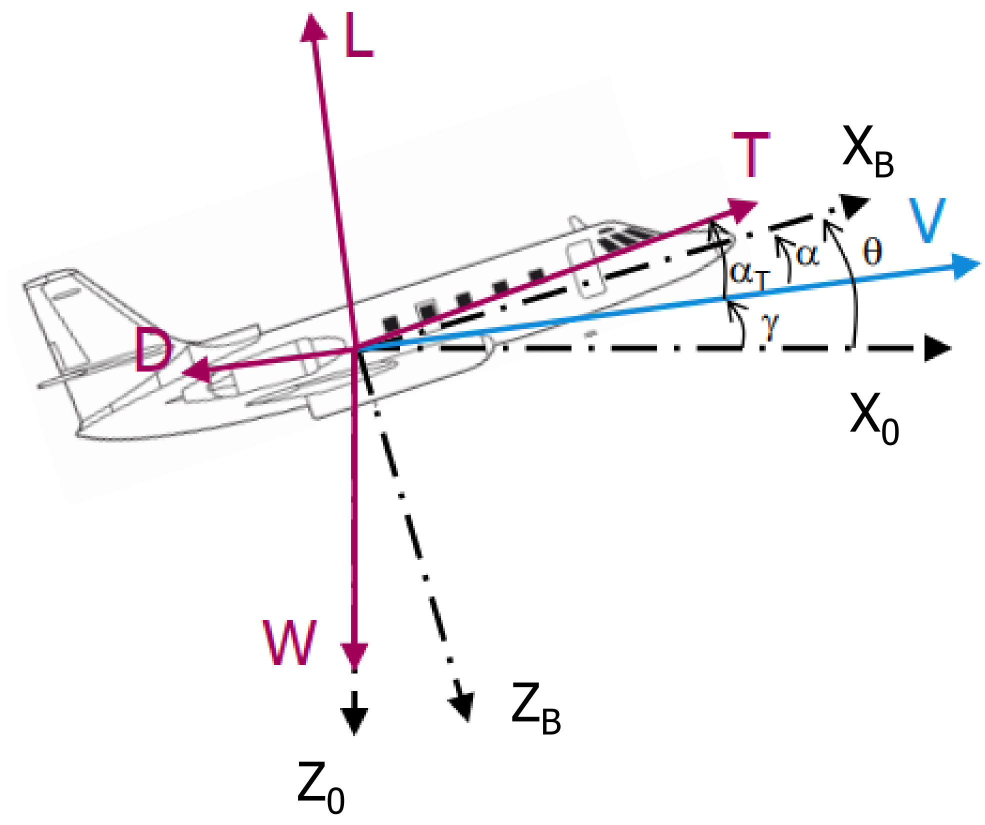
Review of frames

(O_K, x_K, y_K, z_K)

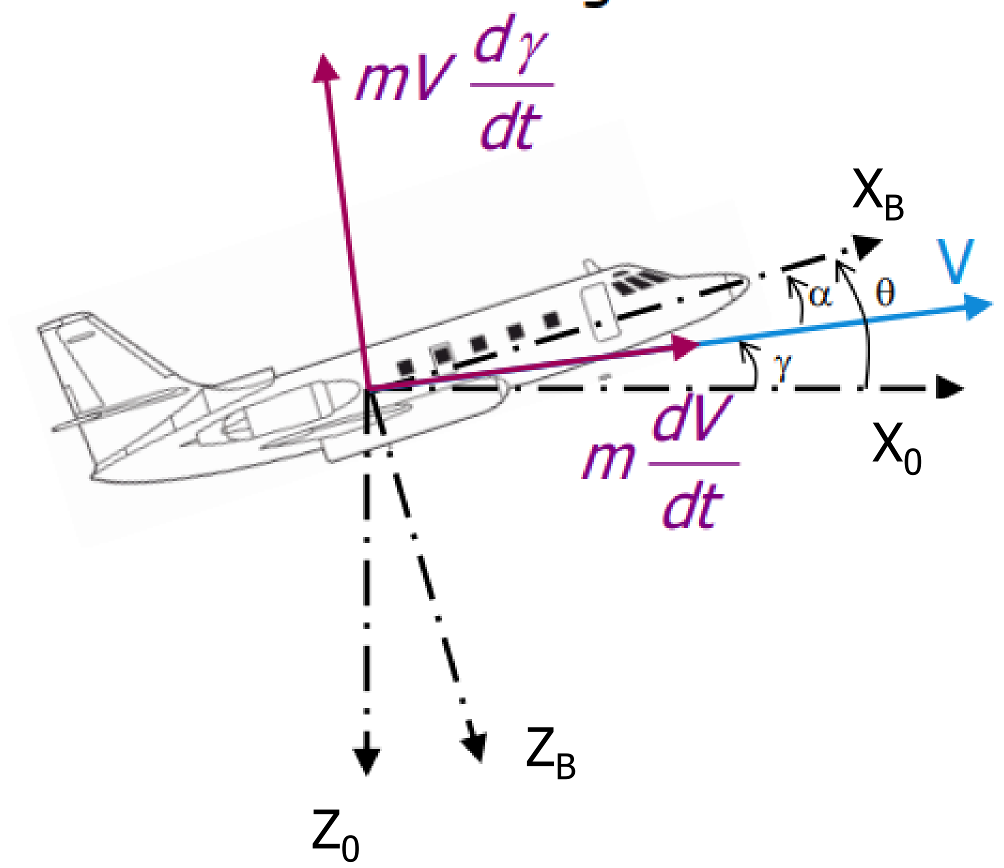


Equation of Motion

Free Body Diagram



Kinetic Diagram



Equation of Motion

$$\parallel V:\ T\cos\alpha_T - D - W\sin\gamma = m\frac{dV}{dt}$$

$$\perp V:\ L - W\cos\gamma + T\sin\alpha_T = mV\frac{d\gamma}{dt}$$

Equation of Motion

General equation for symmetric flight

- Equation of motion in two directions
- The aircraft aerodynamics can be represented by the drag polar

$$\parallel V:\ T\cos\alpha_T - D - W\sin\gamma = \frac{W}{g}\frac{dV}{dt}$$

$$\perp V:\ L - W\cos\gamma + T\sin\alpha_T = \frac{W}{g}V\frac{d\gamma}{dt}$$

$$C_D = C_{D0} + \frac{C_L^2}{\pi\lambda_e}$$

effective span ratio (textbook page 4):

$$\lambda_e = \lambda\frac{1}{1+S_b/S} = \lambda e,\quad e = \frac{1}{1+S_b/S} \in [0,1]$$

Equation of Motion

Steady, horizontal, symmetric flight

$$\parallel V:\ T\underbrace{\cos\alpha_T}_{=1} - D - W\underbrace{\sin\gamma}_{=0} = \frac{W}{g}\underbrace{\frac{dV}{dt}}_{=0}$$

$$\perp V:\ L - W\underbrace{\cos\gamma}_{=1} + T\underbrace{\sin\alpha_T}_{=0} = \frac{W}{g}V\underbrace{\frac{d\gamma}{dt}}_{=0}$$

$$\boxed{\begin{aligned}\parallel V&:\ T = D\\ \perp V&:\ L = W\end{aligned}}$$

Equation of Motion

Calculation of Thrust Required T_R

$$T_R = \mathrm{D} = C_D \frac{1}{2}\rho V^2 S$$

$$W = \mathrm{L} = C_L \frac{1}{2}\rho V^2 S$$

$$\Rightarrow \begin{cases} \dfrac{D}{W} = \dfrac{C_D}{C_L} = \dfrac{1}{K} \\ T_R = D = \dfrac{W}{K} \end{cases}$$

Equation of Motion

Calculation of Thrust Required T_R

$$T_R = D = \left(C_{D0} + \frac{C_L^2}{\pi\lambda_e}\right)\frac{1}{2}\rho V^2 S$$

$$= C_{D0}\,\frac{1}{2}\rho V^2 S + \frac{2W^2}{\pi\lambda_e \rho V^2 S}$$

$$= D_0 + D_i$$

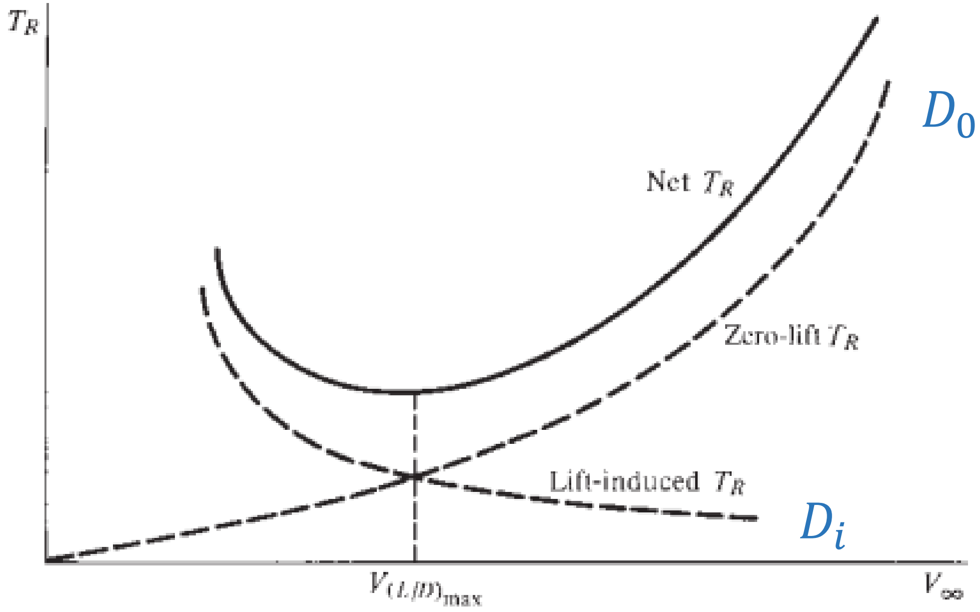


Figure 6.9 Comparison of lift-induced and zero-lift thrust required.

Equation of Motion

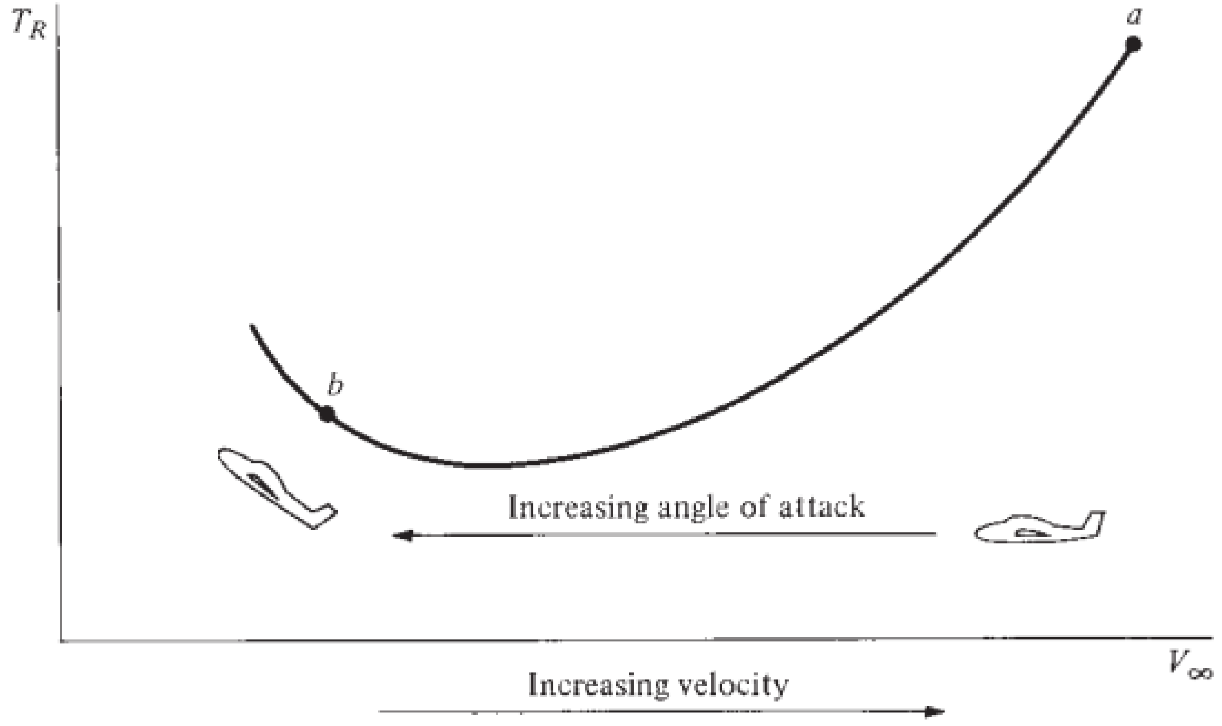


Figure 6.8 Thrust-required curve with associated angle-of-attack variation.

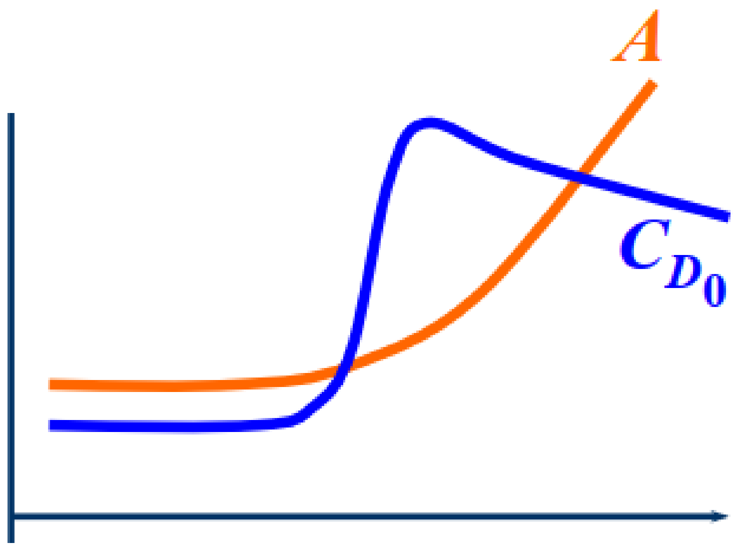
$$T_R = C_{D0}\frac{1}{2}\rho V^2 S + \frac{2W^2}{\pi\lambda_e\rho V^2 S}$$

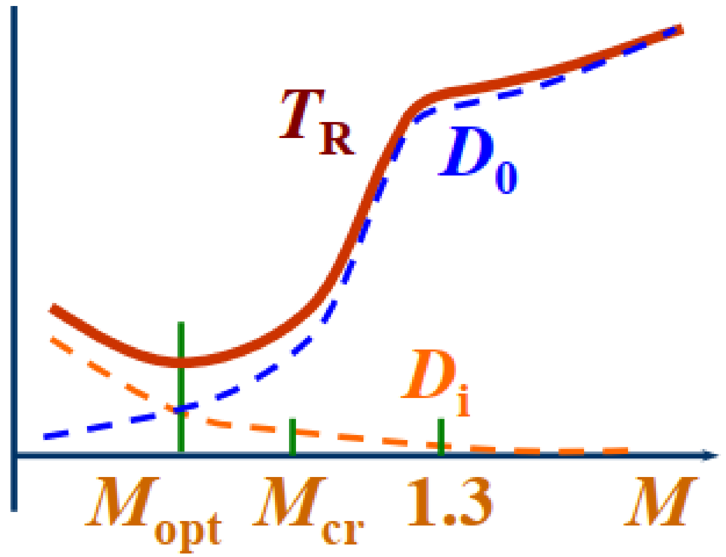
Question:

What are the impact factors for T_R?

Equation of Motion

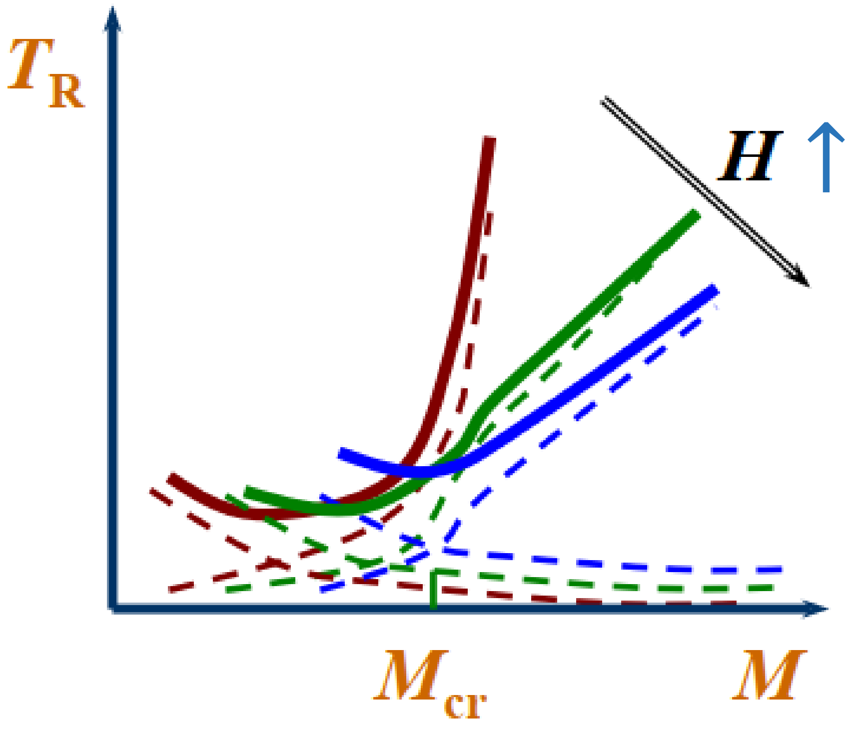
The impact of A and C_{D0}





Equation of Motion

The impact of H



$$T_R = C_{D0}\frac{1}{2}\rho V^2 S + \frac{2W^2}{\pi\lambda_e \rho V^2 S}$$

The Maximum Speed

The basic relationship

$$L = W$$

change of V $\Leftrightarrow$ change of C_L $\Leftrightarrow$ change of α

$$T = D$$

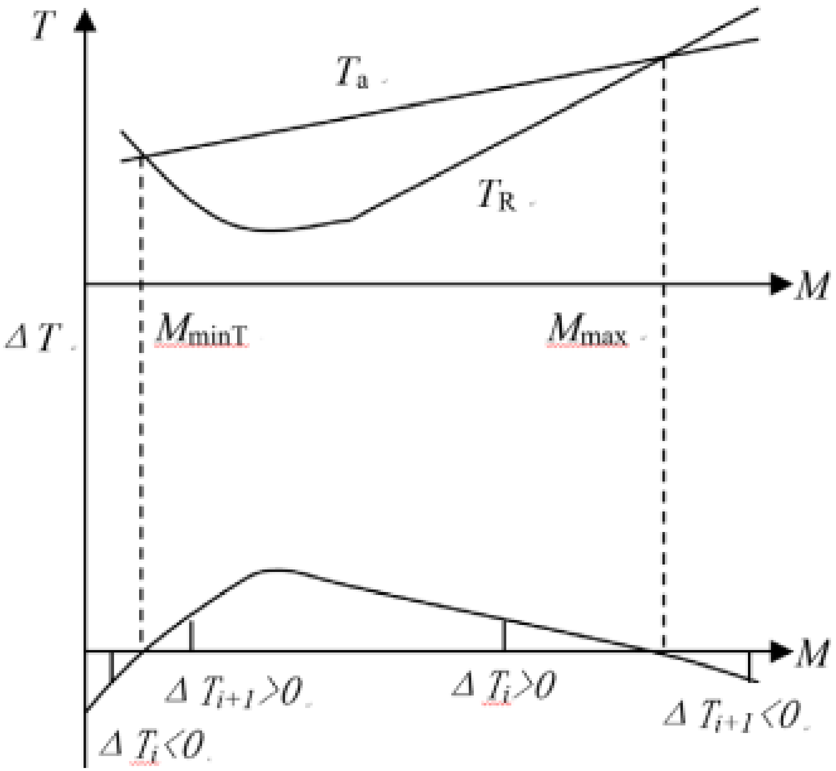
change of V $\Leftrightarrow$ change of D $\Leftrightarrow$ change of T

Flight envelop: V_{max} (M_{max}) , V_{min}, H_{max}

The Maximum Speed

Simple thrust method

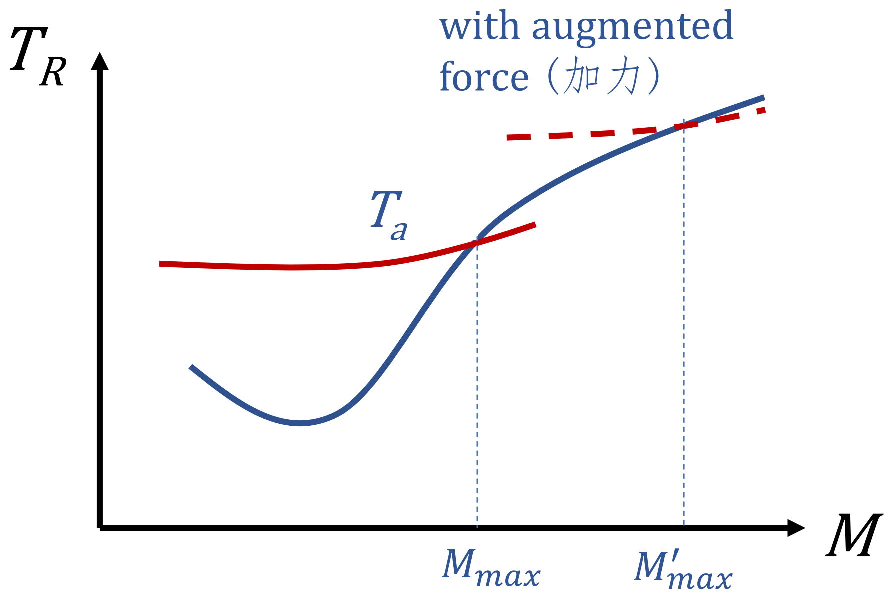
- $T_R = T_a$
- The V_{min} and V_{max} equal to the positions where two lines intersect.



The Maximum Speed

Simple thrust method

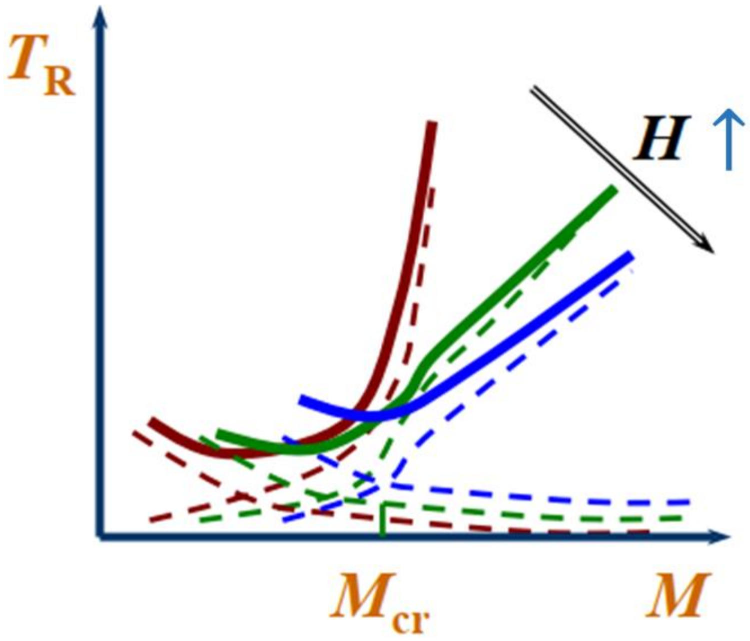
- For $M < M_{max}$, set $T=T_R$ by changing engine throttle.
- If $M > M_{max}$, airplane cannot maintain steady flight

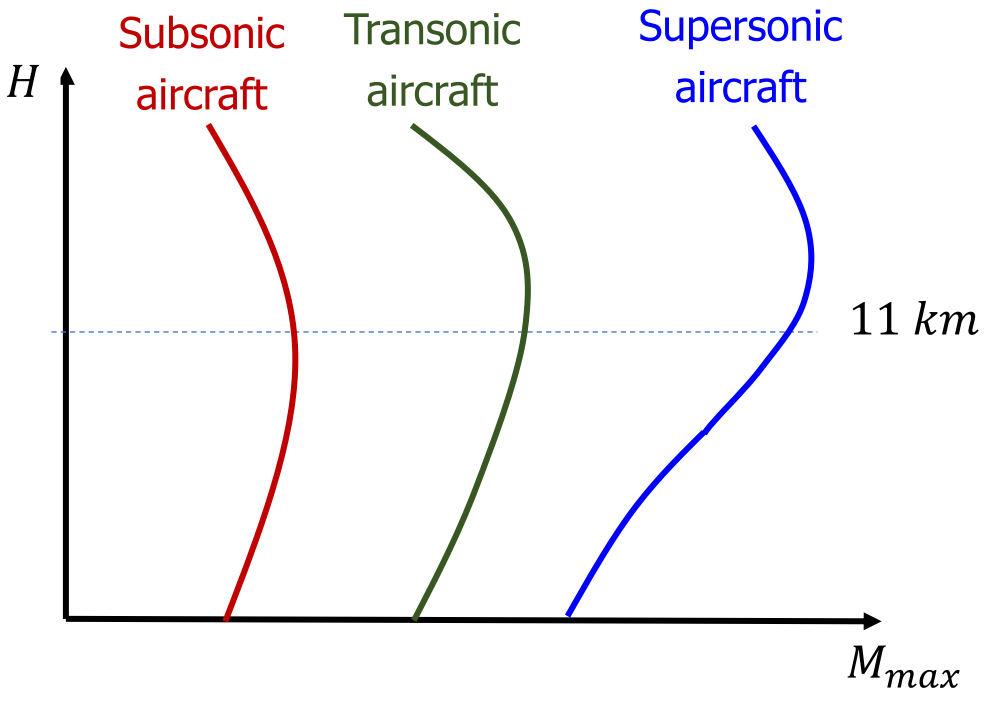


T_R-M diagram at a given altitude

The Maximum Speed

The impact of H





The Maximum Speed

How to calculate V_{max}?

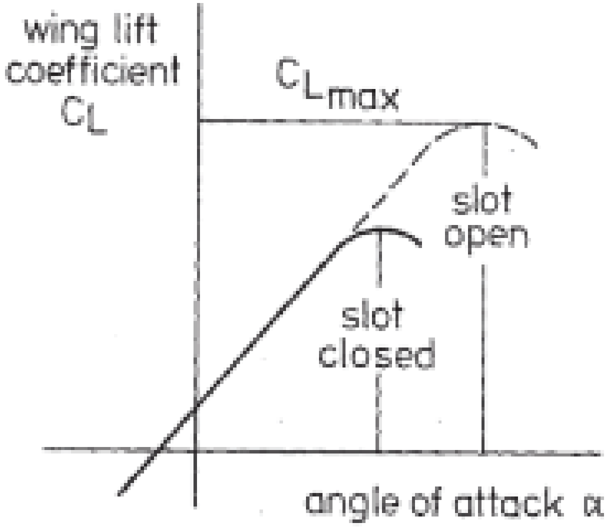
The following data is known of the Cessna Citation II (subsonic jet)

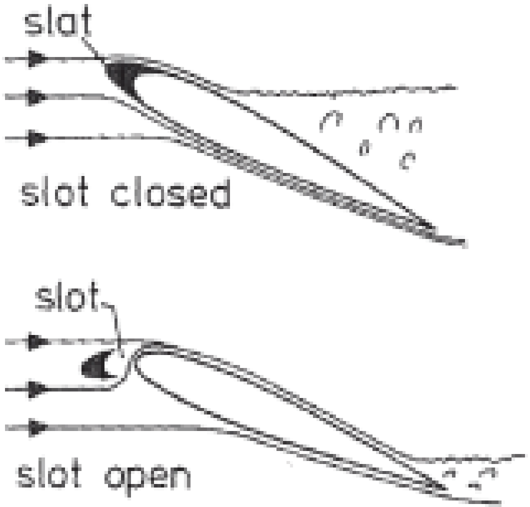
Aircraft Weight : W = 60 kN,
Wing area : S = 30 m^2,
(Parabolic) Lift-Drag polar : $C_D = C_{Do} + kCL^2$; $C_{Do} = 0.022$, $k = 0.047$, $C_{Lmax} = 1.35$,
Maximum Thrust at 0 m ISA :T0 = 12 kN.
The aircraft is flying at an altitude of H = 0 m in the International Standard
Atmosphere (ρ_0 = 1.225 kg/m^3)
Thrust is assumed to be independent of the airspeed

Calculate (1) the V_{max} of this aircraft when flying at H = 0 m and (2) the corresponding Ma

The Minimum Speed

How to calculate V_{min}?





$$L = W$$

$$C_L \tfrac{1}{2} \rho V^2 S = W$$

$$V = \sqrt{\frac{W}{S} \frac{2}{\rho} \frac{1}{C_L}}$$

$$V_{min} = \sqrt{\frac{W}{S} \frac{2}{\rho} \frac{1}{C_{L_{max}}}}$$

The Minimum Speed

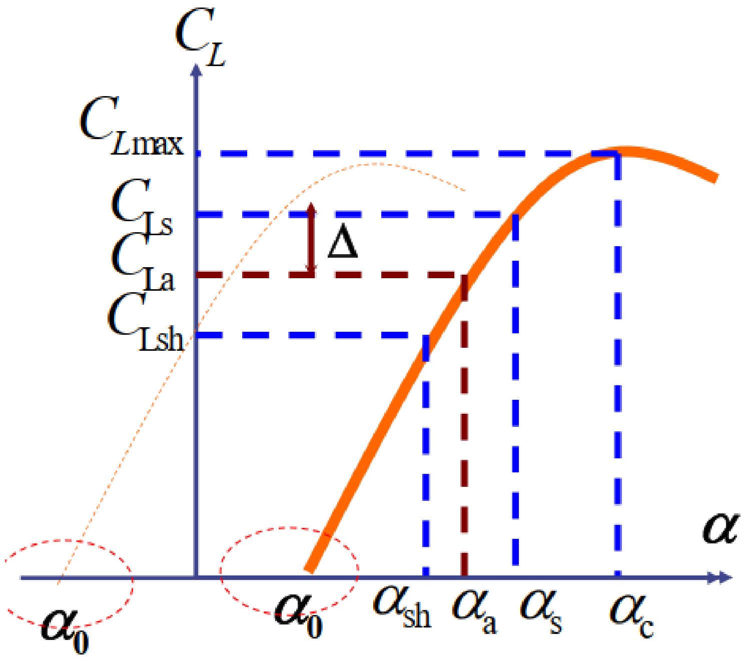
From the 1st lecture

$$f(\boldsymbol{\alpha_c}) \to C_{Lmax}$$

$$f(\boldsymbol{\alpha_s}) \to C_{Ls}$$

$$f(\boldsymbol{\alpha_a}) \to C_{La}$$

$$f(\boldsymbol{\alpha_{sh}}) \to C_{Lsh}$$



The Minimum Speed

Example

An aircraft has a wing loading (W/S) 2400 N/m2 and C_{Lmax} = 1.4. Find the airspeed at which stall occurs (minimum airspeed) at

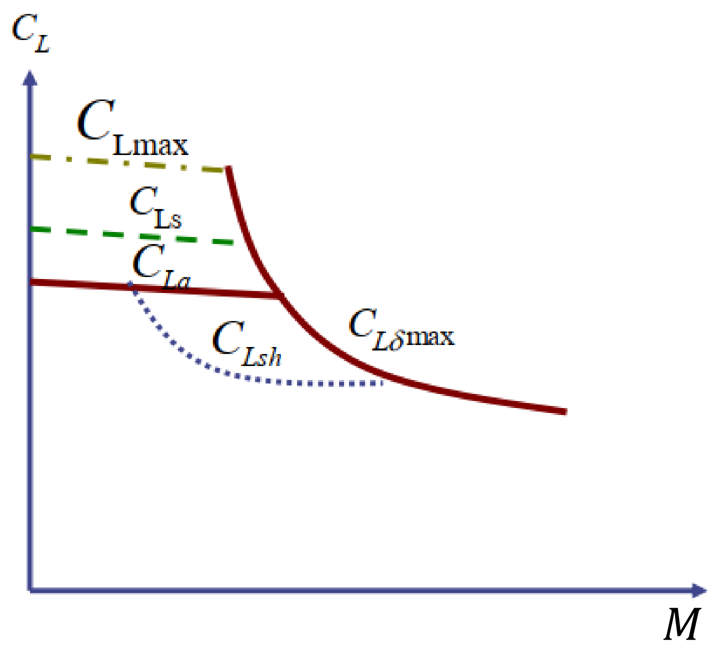
(1) sea level (ρ = 1.225 kg/m3) and **(2)** at 5000m (ρ = 0.737 kg/m3)

The Minimum Speed

From the 1st lecture

$$\delta_{e,max} \longrightarrow \alpha_{max} \longrightarrow C_{L\delta,max}$$

$$\boxed{C_{La} = \min\{C_{Ls} - \Delta, C_{L\delta,max}\}}$$



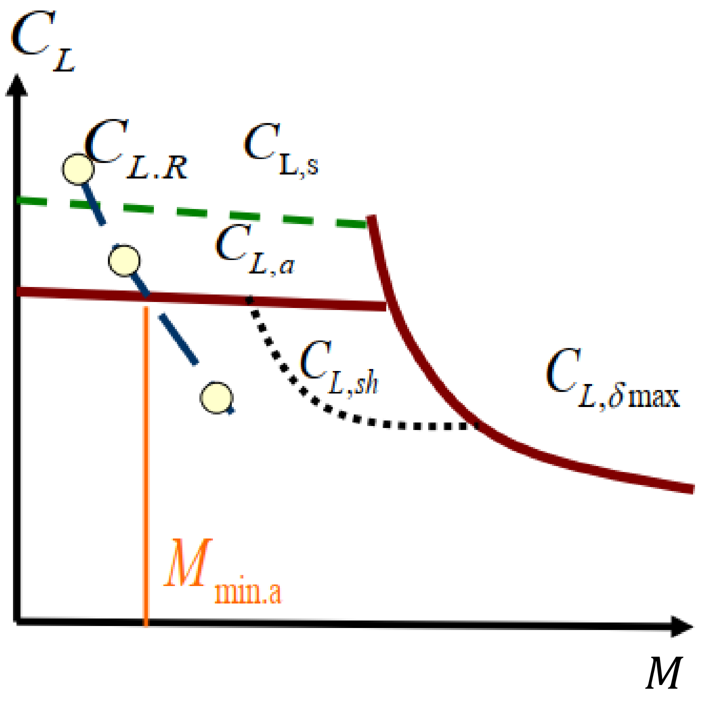
The Minimum Speed

Graphical method

$$\boxed{\begin{aligned} &L = W \\ &C_L \tfrac{1}{2}\rho V^2 S = W \\ &V = \sqrt{\frac{W}{S}\frac{2}{\rho}\frac{1}{C_L}} \end{aligned}}$$

$$\boxed{V_{min} = \sqrt{\frac{W}{S}\frac{2}{\rho}\frac{1}{C_{L_{max}}}}}$$

$$\Rightarrow C_L = \frac{2W}{\rho c^2 S}\frac{1}{M^2}$$



1) Calculate $C_{L,R}$ curve based on a series of M

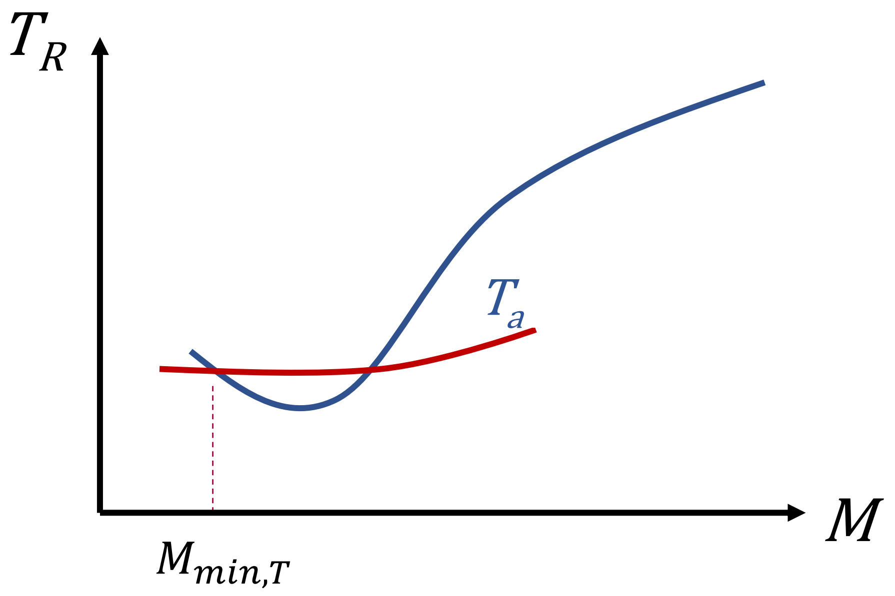
2) Plot the $C_{L,R}$ curve on the $C_{L,a}$ ~ M map, marked the intersection point as $M_{min,a}$

The Minimum Speed

Graphical method

1) Calculate $C_{L,R}$ curve based on a series of M
2) Plot the $C_{L,R}$ curve on the $C_{L,a}$ ~ M map, marked the intersection point as $M_{min,a}$
3) Find the left intersection point of T_R ~ M map, marked as $M_{min,T}$

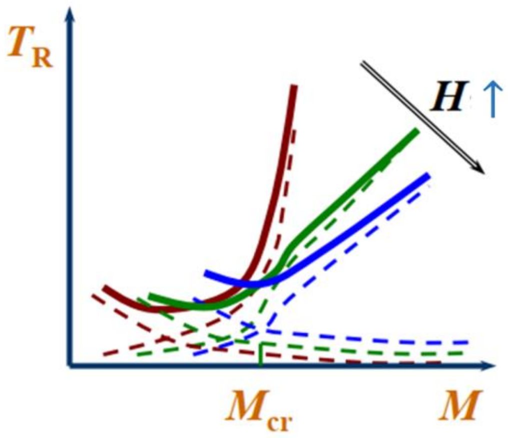
$$M_{min} = \max\{M_{min,a}, M_{min,T}\}$$



The Minimum Speed

Discussion

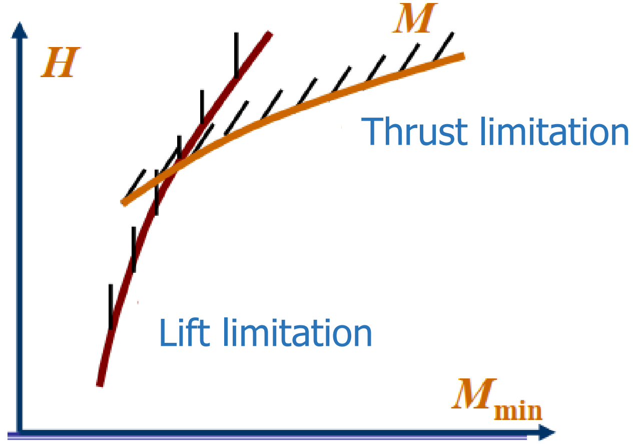
$$\boxed{V_{\min} = \sqrt{\frac{W}{S}\frac{2}{\rho}\frac{1}{C_{L_{\max}}}}}$$



As H increases， density ρ decreases.

1) $V_{min,a}$ increase

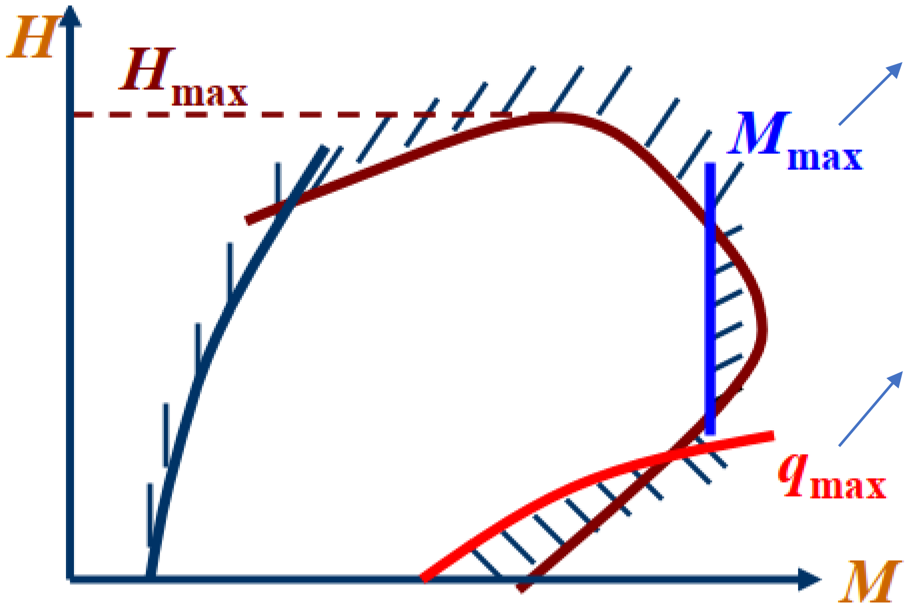
2) As H increases, $M_{min,T}$ increases



At low altitude, the minimum speed is limited by $V_{min,a}$

At high altitude, the minimum speed is limited by $V_{min,T}$

Flight envelop



Aerodynamic heating limitation

Dynamic pressure limitation

Flight envelop

Flight envelop of Bird?
A good research topic.