北京航空航天大學
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# 飞行力学
# Flight Mechanics

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# Summary

- **Chapter 1**

✓ Lift, Drag and Thrust

✓ Drag polar

$$C_D = C_{D0} + \frac{C_L^2}{\pi \lambda_e}$$

✓ Coordinate frames

✓ Transformation of coordinate frames

✓ Equation of motion

# Summary

- **Chapter 2**

- Static performance
  - ✓ Horizontal flight
  - ✓ Climbing and descending flight
  - ✓ Range and endurance
- Dynamic performance
  - ✓ Takeoff
  - ✓ Landing

# Summary

- **Chapter 3**

✓ Maneuverability at the vertical plane

✓ Maneuverability at the horizontal plane

✓ Turning performance

# Chapter 6

- Equation of Motion for Rigid body Aircraft

✓ Dynamic equation for rigid body aircraft（刚体飞机动力学方程）

✓ Kinematic equation for rigid body aircraft（刚体飞机运动学方程）

✓ Linearization of the equations of motion（运动方程线性化）

✓ Longitudinal/Lateral small perturbation equation system（纵向/横向小扰动方程组）

# Contents

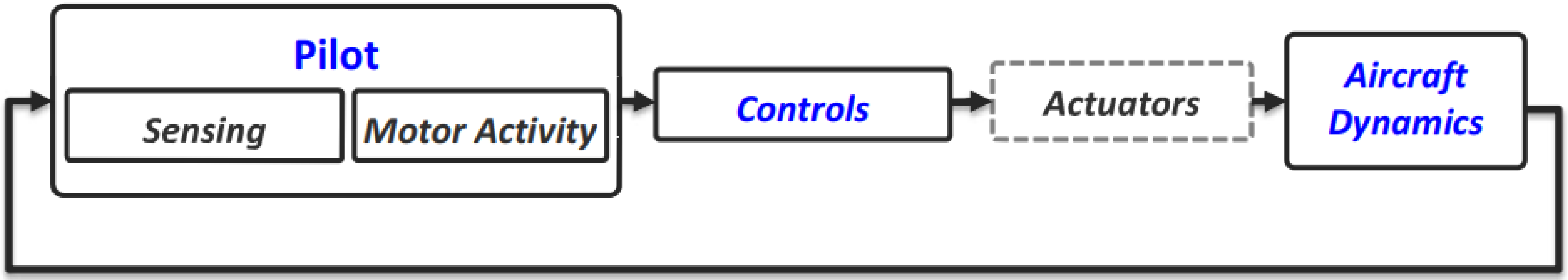
# Introduction

## Question

- What assumptions are made in Chapters 1-3 ?
- How aircraft responds to the control signals ?
- Why we need to consider perturbations ?

# Introduction

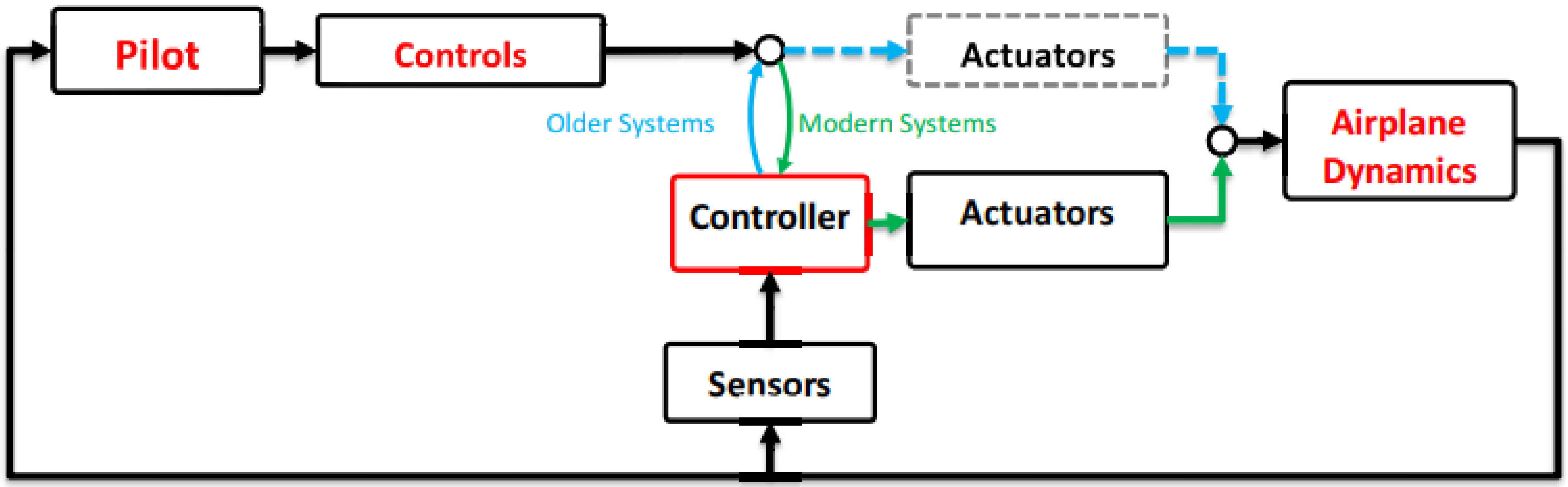
## Block diagram of manual control




- Pilot responsible for control and guidance of airplane
- Pilot senses airplane motions
- Controls are utilized via muscle force
- Actuators to help pilot with bigger airplanes (less muscle force necessary)

# Introduction

Overview of flight control



# Introduction

## Example for unstable configurations


**High Performance Airplanes:**

Usually unstable, i.e. unstable aerodynamic characteristics

Perturbations and control are important!

# Introduction

## Some definitions

**Equilibrium**（平衡）: Flight data do not change as time.

**Stability**（稳定性）: The ability of aircraft to return to equilibrium state after being disturbed by external perturbation.

**Control performance**（操纵性）: The ability to switch to a new desired flight state under the control of pilot.

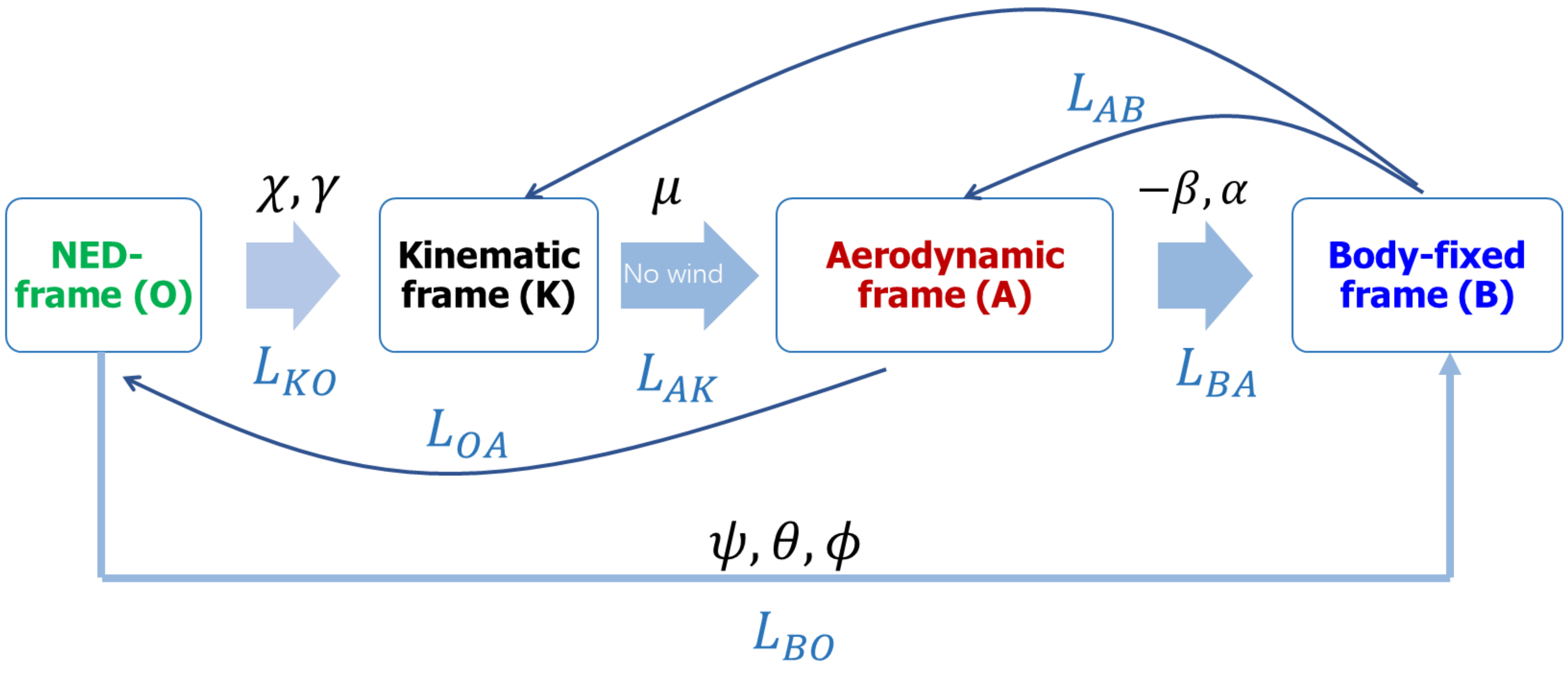
# Introduction

## Coordinate Systems/Frames

- ECEF frame $(O_E, x_E, y_E, z_E)$
- WSG-84 frame $(O_E, x_E, y_E, z_E)$
- NED frame $(O_O, x_O, y_O, z_O)$
- **B**ody fixed frame: $(O_B, x_B, y_B, z_B)$
- **A**erodynamic frame: $(O_A, x_A, y_A, z_A)$
- **K**inematic frame: $(O_K, x_K, y_K, z_K)$

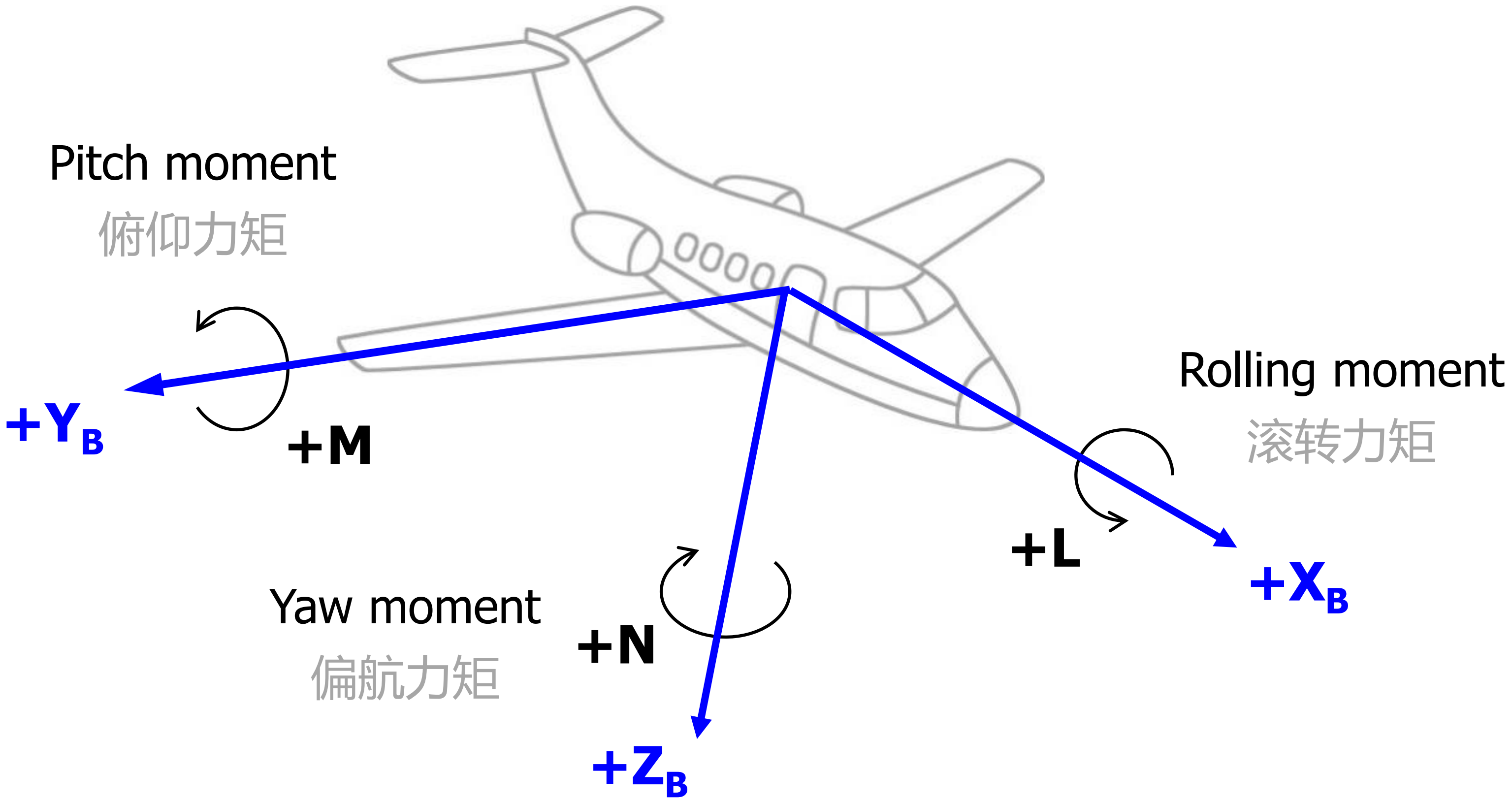
# Introduction

## Coordinate Transformation



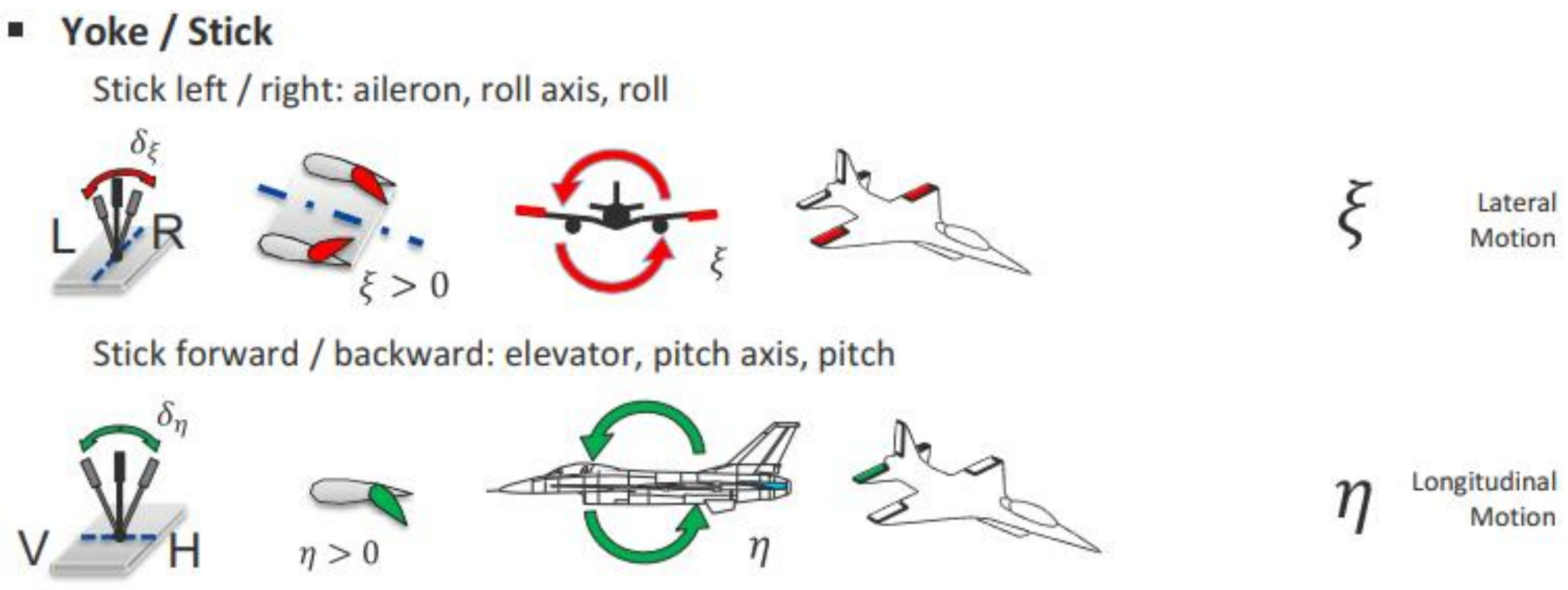
# Introduction

## Rotation motion



# Introduction

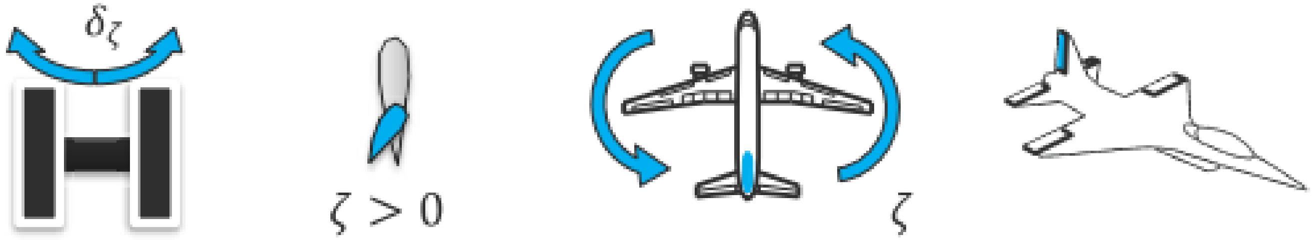
## Control Variables of a classic Fixed Wing Aircraft



# Introduction

## Control Variables of a classic Fixed Wing Aircraft

- **Pedals** left / right: rudder, yaw axis, yaw




$\zeta$ Lateral Motion

- **Throttle** $\delta_T$: Engine thrust

$\delta_T$ Longitudinal Motion

# Equation of motion

The equation of motion for center of mass

$$\begin{cases} m(\dfrac{\mathrm{d}V_x}{\mathrm{d}t}+V_z\omega_y - V_y\omega_z) = F_x \\ m(\dfrac{\mathrm{d}V_y}{\mathrm{d}t}+V_x\omega_z - V_z\omega_x) = F_y \\ m(\dfrac{\mathrm{d}V_z}{\mathrm{d}t}+V_y\omega_x - V_x\omega_y) = F_z \end{cases}$$

Eq. (1.35)

# Equation of motion in kinematic frame

Equations system for center of mass of aircraft

$$\begin{cases} m\dfrac{\mathrm{d}V}{\mathrm{d}t} = T\cos(\alpha+\varphi)\cos\beta - D - mg\sin\gamma \\ mV\cos\gamma\dfrac{\mathrm{d}\chi}{\mathrm{d}t} = T\left[\sin(\alpha+\varphi)\sin\mu - \cos(\alpha+\varphi)\sin\beta\cos\mu\right] + C\cos\mu + L\sin\mu \\ -mV\dfrac{\mathrm{d}\gamma}{\mathrm{d}t} = T\left[-\sin(\alpha+\varphi)\cos\mu - \cos(\alpha+\varphi)\sin\beta\sin\mu\right] + C\sin\mu - L\cos\mu + mg\cos\gamma \end{cases}$$

Eq. (1.36)

# Equation of motion in body-fixed frame

Equations system for center of mass of aircraft

$$\begin{cases} m(\dfrac{\mathrm{d}u}{\mathrm{d}t}+qw-rv)=T\cos\varphi-D\cos\alpha\cos\beta-C\cos\alpha\sin\beta+L\sin\alpha-mg\sin\theta \\ m(\dfrac{\mathrm{d}v}{\mathrm{d}t}+ru-pw)=-D\sin\beta+C\cos\beta+mg\sin\phi\cos\theta \\ m(\dfrac{\mathrm{d}w}{\mathrm{d}t}+pv-qu)=-T\sin\varphi-D\sin\alpha\cos\beta-C\sin\alpha\sin\beta-L\cos\alpha+mg\cos\phi\cos\theta \end{cases} \quad \text{Eq. (6.1)}$$

# Equation of motion in body-fixed frame

Another form for center of mass of aircraft

$$\begin{cases} u = V\cos\alpha\cos\beta \\ v = V\sin\beta \\ w = V\sin\alpha\cos\beta \end{cases}$$

$$\begin{cases} \dfrac{\mathrm{d}u}{\mathrm{d}t} = \dfrac{\mathrm{d}V}{\mathrm{d}t}\cos\alpha\cos\beta - \dfrac{\mathrm{d}\alpha}{\mathrm{d}t}V\sin\alpha\cos\beta - \dfrac{\mathrm{d}\beta}{\mathrm{d}t}V\cos\alpha\sin\beta \\ \dfrac{\mathrm{d}v}{\mathrm{d}t} = \dfrac{\mathrm{d}V}{\mathrm{d}t}\sin\beta + \dfrac{\mathrm{d}\beta}{\mathrm{d}t}V\cos\beta \\ \dfrac{\mathrm{d}w}{\mathrm{d}t} = \dfrac{\mathrm{d}V}{\mathrm{d}t}\sin\alpha\cos\beta + \dfrac{\mathrm{d}\alpha}{\mathrm{d}t}V\cos\alpha\cos\beta - \dfrac{\mathrm{d}\beta}{\mathrm{d}t}V\sin\alpha\sin\beta \end{cases}$$

# Aircraft rotation motion

## Momentum Theorem

$$\frac{d\vec{h}}{dt} = \vec{M}$$

Textbook p. 177~178

Time derivative of moment of momentum
动量矩对时间导数

Torque with respect to the origin point
外力对原点的合力矩

# Aircraft rotation motion

Components of moment of momentum

$$\begin{cases} h_x = \omega_x I_x - \omega_y I_{xy} - \omega_z I_{zx} \\ h_y = \omega_y I_y - \omega_x I_{xy} - \omega_z I_{yz} \\ h_z = \omega_z I_z - \omega_x I_{zx} - \omega_y I_{yz} \end{cases}$$

# Aircraft rotation motion

Components of moment of momentum

$$\begin{cases} I_x = \int \left(y^2 + z^2\right) \mathrm{d}m \\ I_y = \int \left(z^2 + x^2\right) \mathrm{d}m \\ I_z = \int \left(x^2 + y^2\right) \mathrm{d}m \end{cases}$$

惯性矩

$$\begin{cases} I_{xy} = \int xy \mathrm{d}m \\ I_{yz} = \int yz \mathrm{d}m \\ I_{zx} = \int zx \mathrm{d}m \end{cases}$$

惯性积

# Aircraft rotation motion

## Question

$$\boxed{\frac{d\vec{h}}{dt} = \vec{M}}$$

What is the scalar form of the moment equation of momentum in moving coordinate system?

$$\frac{\mathrm{d}\vec{h}}{\mathrm{d}t} = \frac{\delta\vec{h}}{\delta t} + \vec{\omega} \times \vec{h}$$

# Aircraft rotation motion

The scalar form of rotational equation

$$\begin{cases} h_x = \omega_x I_x - \omega_y I_{xy} - \omega_z I_{zx} \\ h_y = \omega_y I_y - \omega_x I_{xy} - \omega_z I_{yz} \\ h_z = \omega_z I_z - \omega_x I_{zx} - \omega_y I_{yz} \end{cases}$$

$$\frac{\delta \vec{h}}{\delta t} + \vec{\omega} \times \vec{h} = \vec{M} \Rightarrow \begin{cases} \dfrac{\mathrm{d}h_x}{\mathrm{d}t} + \left(\omega_y h_z - \omega_z h_y\right) = M_x \\ \dfrac{\mathrm{d}h_y}{\mathrm{d}t} + \left(\omega_z h_x - \omega_x h_z\right) = M_y \\ \dfrac{\mathrm{d}h_z}{\mathrm{d}t} + \left(\omega_x h_y - \omega_y h_x\right) = M_z \end{cases}$$

# Aircraft rotation motion

The final rotational motion in moving coordinate frame

$$\begin{cases} I_x \dfrac{d\omega_x}{dt} + \left(I_z - I_y\right)\omega_y\omega_z + I_{yz}\left(\omega_z^2 - \omega_y^2\right) + I_{xy}\left(\omega_x\omega_z - \dfrac{d\omega_y}{dt}\right) - I_{zx}\left(\omega_x\omega_y + \dfrac{d\omega_z}{dt}\right) = M_x \\ I_y \dfrac{d\omega_y}{dt} + \left(I_x - I_z\right)\omega_z\omega_x + I_{zx}\left(\omega_x^2 - \omega_z^2\right) + I_{yz}\left(\omega_y\omega_x - \dfrac{d\omega_z}{dt}\right) - I_{xy}\left(\omega_y\omega_z + \dfrac{d\omega_x}{dt}\right) = M_y \\ I_z \dfrac{d\omega_z}{dt} + \left(I_y - I_x\right)\omega_x\omega_y + I_{xy}\left(\omega_y^2 - \omega_x^2\right) + I_{zx}\left(\omega_z\omega_y - \dfrac{d\omega_x}{dt}\right) - I_{yz}\left(\omega_z\omega_x + \dfrac{d\omega_y}{dt}\right) = M_z \end{cases}$$

Eq. (6.12)

# Aircraft rotation motion

The rotational motion in Body-fixed frame

$$\begin{cases} I_x \dfrac{\mathrm{d}p}{\mathrm{d}t} + \left(I_z - I_y\right) qr - I_{zx} \left( pq + \dfrac{\mathrm{d}r}{\mathrm{d}t} \right) = L \\ I_y \dfrac{\mathrm{d}q}{\mathrm{d}t} + \left(I_x - I_z\right) rp + I_{zx} \left( p^2 - r^2 \right) = M \\ I_z \dfrac{\mathrm{d}r}{\mathrm{d}t} + \left(I_y - I_x\right) pq + I_{zx} \left( qr - \dfrac{\mathrm{d}p}{\mathrm{d}t} \right) = N \end{cases} \qquad \text{Eq. (6.14)}$$

# Kinematic equation of aircraft

The kinematic equation of center of mass in ground frames

$$\begin{cases} \dfrac{dx_o}{dt} = V\cos\gamma\cos\chi \\ \dfrac{dy_o}{dt} = V\cos\gamma\sin\chi \\ \dfrac{dz_o}{dt} = -V\sin\gamma \end{cases}$$

Error in the p.180 of textbook： $L_{qk} \rightarrow L_{gk}$

# Kinematic equation of aircraft

The kinematic equation of center of mass in ground frames

$$\begin{cases} \dfrac{dx_o}{dt} = V_{x_o} = \cos\theta\cos\psi + v(\sin\theta\sin\phi\cos\psi - \cos\phi\sin\psi) \\ \qquad\qquad + w(\sin\theta\cos\phi\cos\psi + \sin\phi\sin\psi) \\ \dfrac{dy_o}{dt} = V_{y_o} = \cos\theta\sin\psi + v(\sin\theta\sin\phi\sin\psi + \cos\phi\cos\psi) \\ \qquad\qquad + w(\sin\theta\cos\phi\sin\psi - \sin\phi\cos\psi) \\ \dfrac{dy_o}{dt} = V_{y_o} = u\sin\theta + v\sin\phi\cos\theta + w\cos\phi\sin\theta \end{cases}$$

Eq. (6.16)

# kinematic equation of rotation

Relationship between rotational angular velocity and $[\phi, \theta, \psi]$

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = L_x(\phi) L_y(\theta) \begin{bmatrix} 0 \\ 0 \\ \dot{\psi} \end{bmatrix} + L_x(\phi) \begin{bmatrix} 0 \\ \dot{\theta} \\ 0 \end{bmatrix} + \begin{bmatrix} \dot{\phi} \\ 0 \\ 0 \end{bmatrix}$$

Error in the p.181:
图1. 15 →图1. 25

# kinematic equation of rotation

Relationship between rotational angular velocity and $[\phi, \theta, \psi]$

$$\Rightarrow \begin{cases} p = \dot{\phi} - \dot{\psi}\sin\theta \\ q = \dot{\theta}\cos\phi + \dot{\psi}\sin\phi\cos\theta \\ r = -\dot{\theta}\sin\phi + \dot{\psi}\cos\phi\cos\theta \end{cases}$$

$$\Rightarrow \begin{cases} \dot{\phi} = p + \tan\theta\left(q\sin\phi + r\cos\phi\right) \\ \dot{\theta} = q\cos\phi - r\sin\phi \\ \dot{\psi} = 1/\cos\theta\left(q\sin\phi + r\cos\phi\right) \end{cases}$$

# Practice

1) Prove:

The relationship between rotational angular velocity (p, q, r) and attitude angle ($\theta$, $\psi$, $\phi$)

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} \dot{\phi} - \dot{\psi}\sin\theta \\ \dot{\theta}\cos\phi + \dot{\psi}\sin\phi cos\theta \\ -\dot{\theta}\sin\phi + \dot{\psi}\cos\phi cos\theta \end{bmatrix}$$

# Linearization of the equation of motion

## Some definitions

**Reference motion**（基准运动）：Under the control of a pilot, the aircraft fly according to the predetermined rules without any external disturbances;

**Disturbed motion**（扰动运动）：Due to various external disturbances, the motion parameters of the aircraft deviate from the reference motion, and does not follow a predetermined rule.

Aircraft motion = reference motion + disturbed motion

# Linearization of the equation of motion

## Small perturbation assumption (小扰动假设)

If the external disturbance acting on the aircraft and the variation of motion parameters are small, the second order and higher order of the parameters can be ignored.

What is the basis for the small perturbation assumption?

# Linearization of the equation of motion

## Further assumption

1) The reference motion is steady straight-line flight.

Decoupling of motion (运动解耦)： Vertical + horizontal

2) Aircraft has symmetry plane, the gyroscopic effect of the rotating parts are ignored;

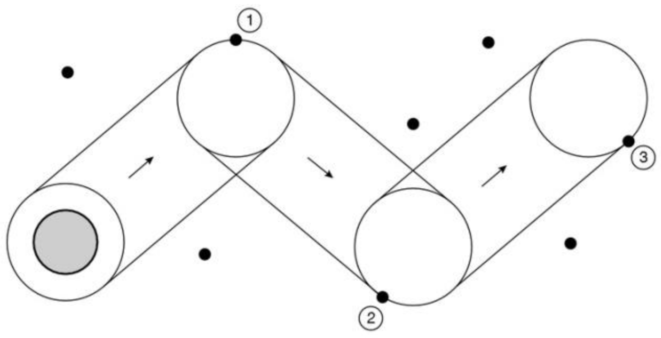
3) In the reference motion, the symmetry plane is in vertical position ($\phi = 0$), and the motion plane coincides with the symmetry plane ($\beta = 0$)

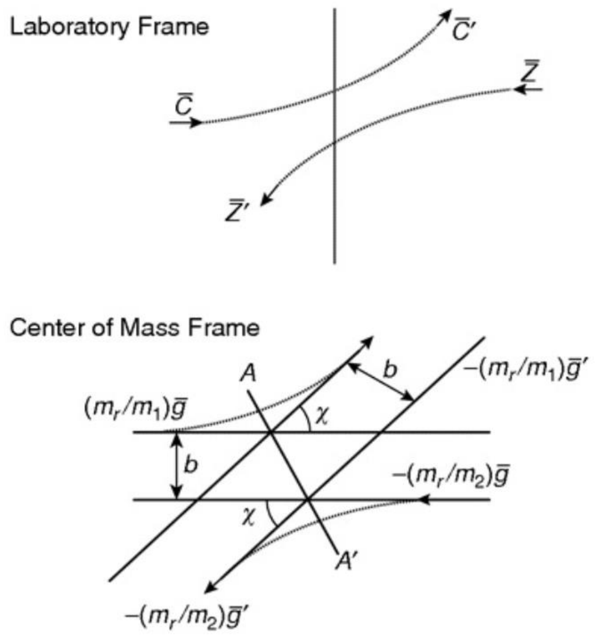
# Example – decoupling of motion

An key assumption for complex physical problem

Direct Simulation Monte Carlo Method (DSMC)

Assumption: decoupling of move and collision




Boyd and Schwartzentruber, *Nonequilibrium Gas Dynamics and Molecular Simulation*, 2017

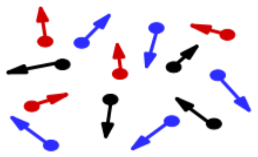
Molecular collision (分子碰撞示意图)

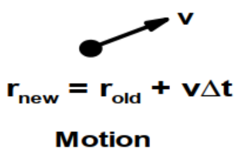
# Example – decoupling of motion

## Main features DSMC method

- Particles posses microscopic properties
- Particles move and collision are decoupled
- Applications include hypersonic flow & rarefied gas dynamics

**Flow** = **representative particles** + **movement and collisions** + **averaging microscopic properties**



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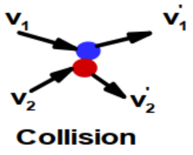


$$\vec{U} = \sum_{i=1}^{N} m_i \vec{v}_i \Big/ N$$

# Linearization of the equation of motion

## linearization method

Assume $x_i = x_{i*} + \Delta x_i$

$$\Rightarrow f(x_{1*}, x_{2*}, \dots, x_{n*}) = 0$$

$$\Rightarrow f(x_{1*} + \Delta x_1, x_{2*} + \Delta x_2, \dots, x_{n*} + \Delta x_n) = 0$$

# Linearization of the equation of motion

## linearization method

Tylor expansion

$$f(x_{1*}, x_{2*}, \dots x_{n*}) + \left(\frac{\partial f}{\partial x_1}\right)_* \Delta x_1 + \left(\frac{\partial f}{\partial x_2}\right)_* \Delta x_2 + \dots + \left(\frac{\partial f}{\partial x_n}\right)_* \Delta x_n = 0$$

What assumption have made?

# Linearization of the equation of motion

The linearized small perturbation equation

Determined by reference motion

Deviation to the reference motion

$$\Rightarrow \left(\frac{\partial f}{\partial x_1}\right)_* \Delta x_1 + \left(\frac{\partial f}{\partial x_2}\right)_* \Delta x_2 + \cdots + \left(\frac{\partial f}{\partial x_n}\right)_* \Delta x_n = 0$$

# Linearization of the equation of motion

Linearization of force and moment

Assume $$A = \mathrm{A}(x_1, x_2, \dots, x_n)$$

Deviation $$\Delta A = \left(\frac{\partial A}{\partial x_1}\right)_* \Delta x_1 + \left(\frac{\partial A}{\partial x_2}\right)_* \Delta x_2 + \cdots + \left(\frac{\partial A}{\partial x_n}\right)_* \Delta x_n$$

# Linearization of the equation of motion

Linearization of force and moment

$$A = \mathrm{A}(x_1, x_2, \ldots, x_n)$$

$$\Delta A = \boxed{\left(\frac{\partial A}{\partial x_1}\right)_*} \Delta x_1 + \left(\frac{\partial A}{\partial x_2}\right)_* \Delta x_2 + \cdots + \left(\frac{\partial A}{\partial x_n}\right)_* \Delta x_n$$

Determined by
reference motion

# Linearization of the equation of motion

Linearized expression of force and moment

$$\begin{cases} \Delta T = T_V \Delta V + T_H \Delta H + T_{\delta_p} \Delta \delta_p \\ \Delta D = D_V \Delta V + D_H \Delta H + D_\alpha \Delta \alpha + D_{\delta_e} \Delta \delta_e \\ \Delta L = L_V \Delta V + L_H \Delta H + L_\alpha \Delta \alpha + L_{\delta_e} \Delta \delta_e + L_{\dot\alpha} \Delta \dot\alpha + L_q \Delta q \\ \Delta C = D_\beta \Delta \beta + D_p \Delta p + D_r \Delta r + D_{\delta_r} \Delta \delta_r \end{cases}$$

$$\begin{cases} \Delta L = L_\beta \Delta \beta + L_p \Delta p + L_r \Delta r + L_{\delta_a} \Delta \delta_a + L_{\delta_r} \Delta \delta_r \\ \Delta M = M_V \Delta V + M_H \Delta H + M_\alpha \Delta \alpha + M_{\dot\alpha} \Delta \dot\alpha + M_q \Delta q + M_{\delta_e} \Delta \delta_e \\ \Delta N = N_\beta \Delta \beta + N_p \Delta p + N_r \Delta r + N_{\delta_a} \Delta \delta_a + N_{\delta_r} \Delta \delta_r \end{cases}$$

# Linearization of the equation of motion

Derivative of velocity

$$T_V = \left(\frac{\partial T}{\partial V}\right)_* = \left[\frac{\partial\left(\frac{1}{2}\rho V^2 S C_T\right)}{\partial V}\right]_*$$

$$= \frac{\partial C_T}{\partial Ma}\left(\frac{\mathrm{d}Ma}{\mathrm{d}V}\right)_* \frac{1}{2}\rho_* V_*^2 S + C_{T*}\rho_* V_* S = \rho_* V_* S\left(\frac{1}{2}C_{TMa}Ma_* + C_{T*}\right)$$

# Linearization of the equation of motion

Derivative of altitude

$$T_H = \left(\frac{\partial T}{\partial H}\right)_* = \left[\frac{\partial\left(\frac{1}{2}\rho V^2 S C_T\right)}{\partial H}\right]_*$$

$$= \frac{\partial C_T}{\partial H}\frac{1}{2}\rho_* V_*^2 S + C_{T*}\frac{1}{2}V_*^2 S\left(\frac{\partial \rho}{\partial H}\right)_* = \frac{1}{2}\rho_* V_*^2 S\left(C_{TH} + C_{T*}\frac{\rho^H}{\rho^*}\right)$$

# Linearization of the equation of motion

Derivative of angle

$$D_\alpha = \left(\frac{\partial D}{\partial \alpha}\right)_* = C_{D\alpha}\frac{1}{2}\rho_* V_*^2 S$$

# Linearization of the equation of motion

Derivative of angular velocity

$$M_q = \left(\frac{\partial M}{\partial q}\right)_* = C_{m_q} \frac{1}{4} \rho_* V_*^2 S c^2$$

mean aerodynamic chord

# Linearization of kinematic equation

The choice of equation system

Equation of forces – From kinematic frame or body-fixed frame

Equation of moments – Body-fixed frame

# Linearization of kinematic equation

## Linearized equation in kinematic frame

Drag force equation

$$m\frac{\mathrm{d}\Delta V}{\mathrm{d}t}=\Delta T\cos(\alpha_*+\varphi)\cos\beta_* - T_*\sin(\alpha_*+\varphi)\cos\beta_*\Delta\alpha$$
$$- T_*\cos(\alpha_*+\varphi)\sin\beta_*\Delta\beta-\Delta D - mg\cos\gamma_*\Delta\gamma$$

Lift force equation

$$mV_*\frac{\mathrm{d}\Delta\gamma}{\mathrm{d}t}=\left(T_V\Delta V+T_H\Delta H+T_{\delta_p}\Delta\delta_e\right)\sin(\alpha_*+\varphi)+T_*\cos(\alpha_*+\varphi)\Delta\alpha$$
$$+\left(L_V\Delta V+L_H\Delta H+L_\alpha\Delta\alpha+L_{\dot\alpha}\Delta\dot\alpha+L_q\Delta q+T_{\delta_e}\Delta\delta_e\right)+mg\sin\gamma_*\gamma$$

Side force equation

$$mV_*\frac{\mathrm{d}\Delta\beta}{\mathrm{d}t}+mV_*\Delta r-mV_*\alpha_*\Delta p = -D\Delta\beta+\left(C_\beta\Delta\beta+C_p\Delta p+C_r\Delta r+C_{\delta_r}\Delta\delta_r\right)$$

Textbook p.193-196

# Linearization of kinematic equation

Linearized dynamic equation for rotational motion around center of mass

$$\begin{cases} I_x \dfrac{\mathrm{d}\Delta p}{\mathrm{d}t} - I_{zx} \dfrac{\mathrm{d}\Delta r}{\mathrm{d}t} = L_\beta \Delta\beta + L_p \Delta p + L_r \Delta r + L_{\delta_a} \Delta\delta_a + L_{\delta_r} \Delta\delta_r \\ I_y \dfrac{\mathrm{d}\Delta q}{\mathrm{d}t} = M_V \Delta V + M_H \Delta H + M_\alpha \Delta\alpha + M_{\dot\alpha} \Delta\dot\alpha + L_q \Delta q + L_{\delta_e} \Delta\delta_e \\ I_z \dfrac{\mathrm{d}\Delta r}{\mathrm{d}t} - I_{zx} \dfrac{\mathrm{d}\Delta p}{\mathrm{d}t} = N_\beta \Delta\beta + N_p \Delta p + N_r \Delta r + N_{\delta_a} \Delta\delta_a + N_{\delta_r} \Delta\delta_r \end{cases}$$

# Linearization of kinematic equation

Linearized kinematic equation for center of mass

$$\begin{cases} \dfrac{d\Delta x_o}{dt} = \cos\gamma_* V - V_* \sin\gamma_* \Delta\gamma \\ \dfrac{d\Delta y_o}{dt} = V_* \cos\gamma_* \Delta\chi \\ \dfrac{d\Delta z_o}{dt} = -\sin\gamma_* V - V_* \cos\gamma_* \Delta\gamma \end{cases}$$

# Linearization of kinematic equation

Linearized kinematic equation for rotation around center of mass

$$\begin{cases} \dfrac{\mathrm{d}\Delta\phi}{\mathrm{d}t} = \Delta p + \tan\theta_* \Delta r \\ \dfrac{\mathrm{d}\Delta\theta}{\mathrm{d}t} = \Delta q \\ \dfrac{\mathrm{d}\Delta\psi}{\mathrm{d}t} = \dfrac{1}{\cos\theta_*}\Delta r \end{cases}$$

# Linearization of kinematic equation

Linearized geometric relationship

$$\begin{cases} \Delta\alpha = \Delta\theta - \Delta\gamma \\ \Delta\beta = \Delta\psi - \Delta\chi \\ \Delta\mu = \Delta\phi \end{cases}$$

# Linearization of kinematic equation

## Summary

7 equations involving vertical motion parameters. Thus the equation system is called vertical small perturbation equation

$$\Delta V, \Delta\gamma, \Delta x_{g}, \Delta z_{g}, \Delta\alpha, \Delta q, \Delta\theta, \Delta\delta_{e}, \Delta\delta_{p}$$

7 equations involving horizontal motion parameters. So the equation system is called horizontal small perturbation equation

$$\Delta\beta, \Delta\phi, \Delta p, \Delta r, \Delta\psi, \Delta\chi, \Delta\mu, \Delta y_{g}, \Delta\delta_{a}, \Delta\delta_{r}$$

# Practice

2) Prove:

$$\chi = \psi + \frac{\beta - \sin\alpha_* \phi}{\cos\gamma_*} \approx \psi + \beta$$

# Practice

Textbook p.204, 6.4

Show that the linearized equation of side force in kinematic frame:

$$mV\cos\gamma\frac{d\chi}{dt} = T[\sin(\alpha+\varphi)\sin\mu - \cos(\alpha+\varphi)\sin\beta\cos\mu] + C\cos\mu + L\sin\mu$$

# Practice

## Textbook p.205, 6.7

An aircraft model is tested in wind tunnel and has the following data:

$A_x = 79N, A_y = 12N, A_z = -333N,$ under

$\alpha = 30°, \beta = 10°, \phi = 10°,$

Calculate:

1) transformation matrix $L_{AB}$;

2) Lift force L, drag force D and side force C on the model.