北京航空航天大学
BEIHANG UNIVERSITY

飞行力学
Flight Mechanics

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2023 - Spring

Introduction

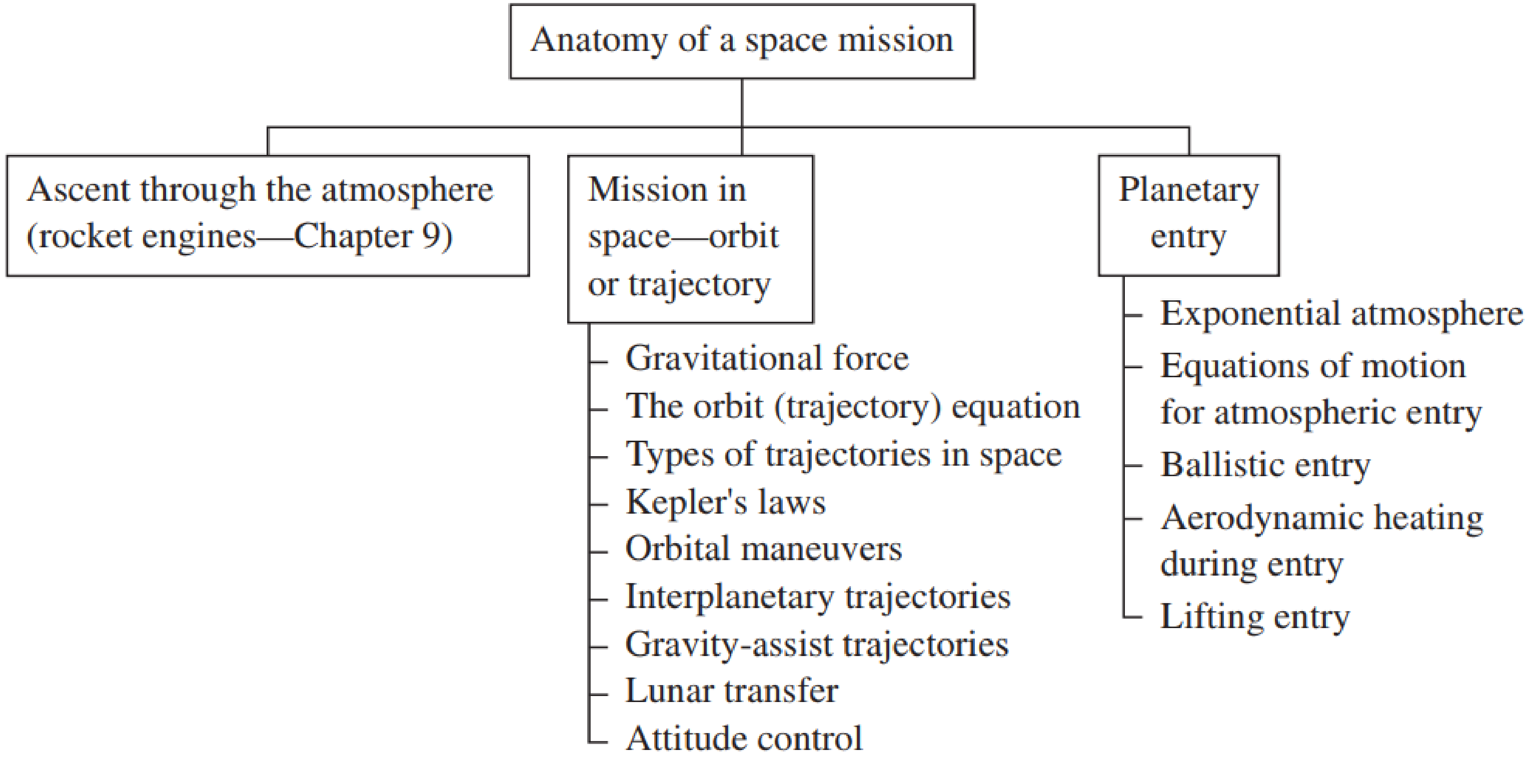


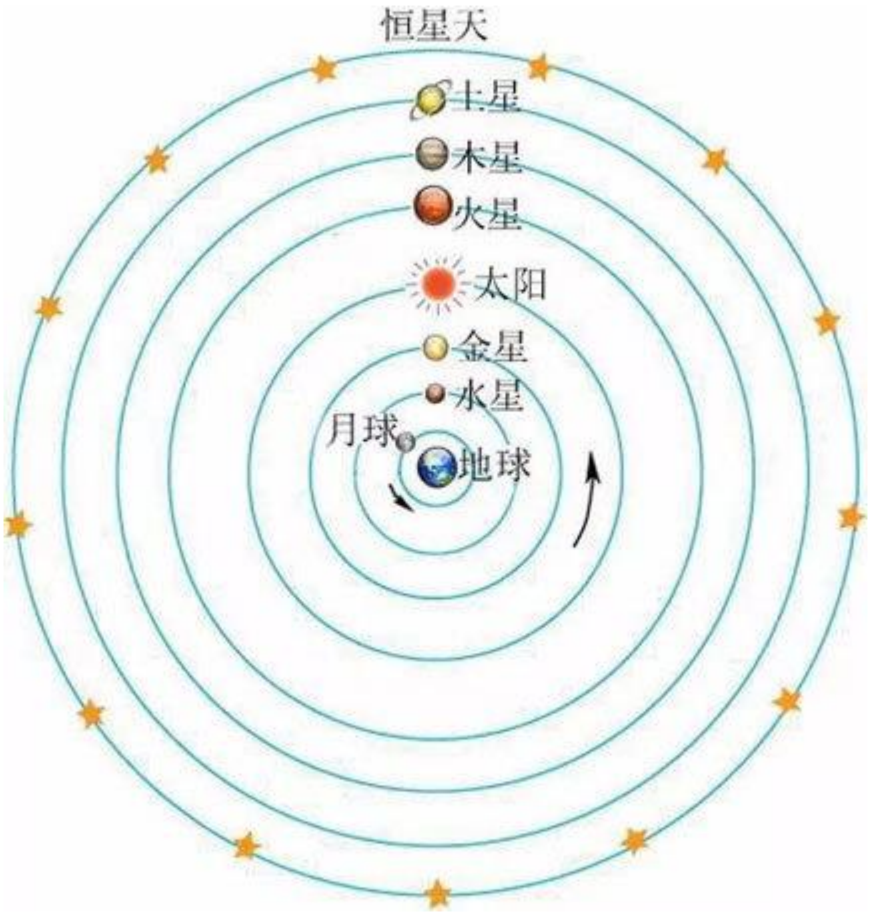
Figure 8.7 Road map for Ch. 8.

Contents

Introduction

Question

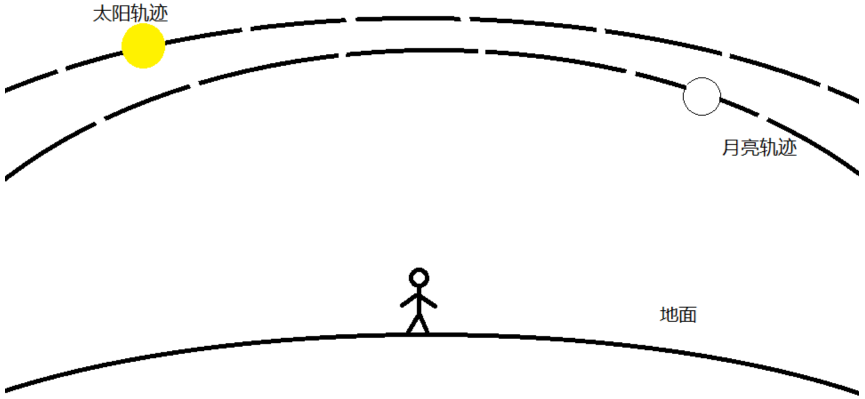
- What's the problem of geocentric theory (地心说) ?
- Why Kepler's laws are so important to the development of modern science?



Introduction

How you were convinced that

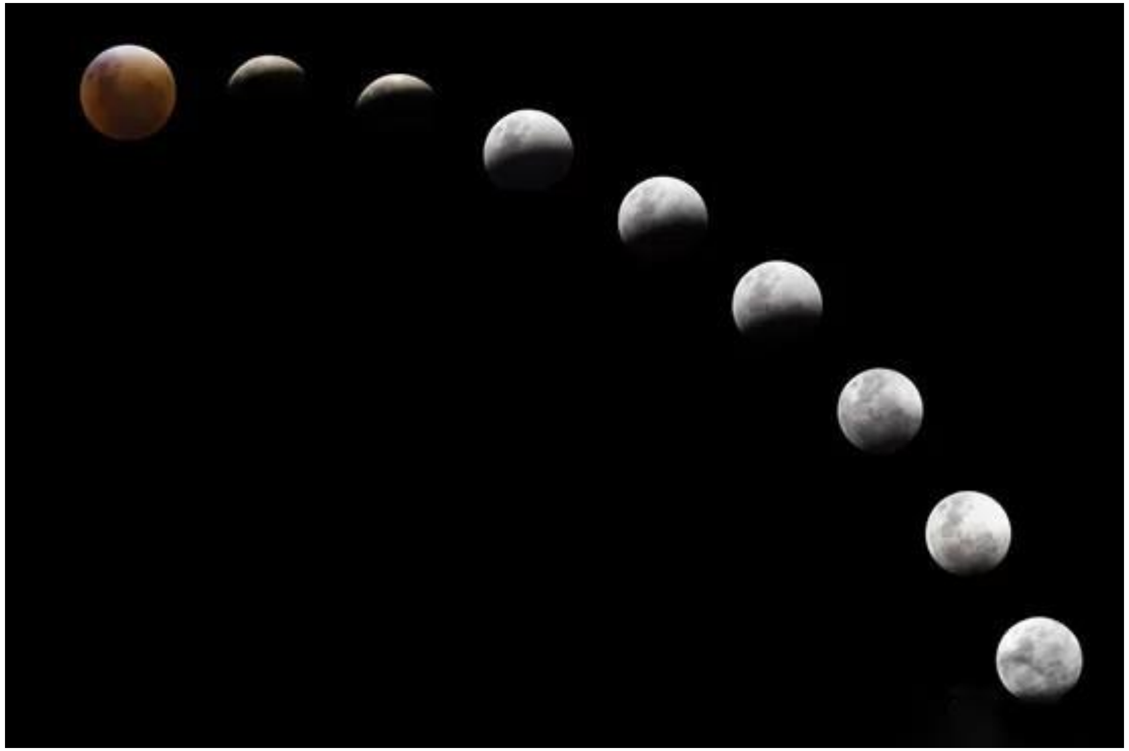
1. The earth is a sphere ?
2. The earth is spinning around its axis ?
3. The earth rotates around the sun ?



Please provide your own observation and reasoning.

Introduction

Lunar eclipse（月食）



Introduction

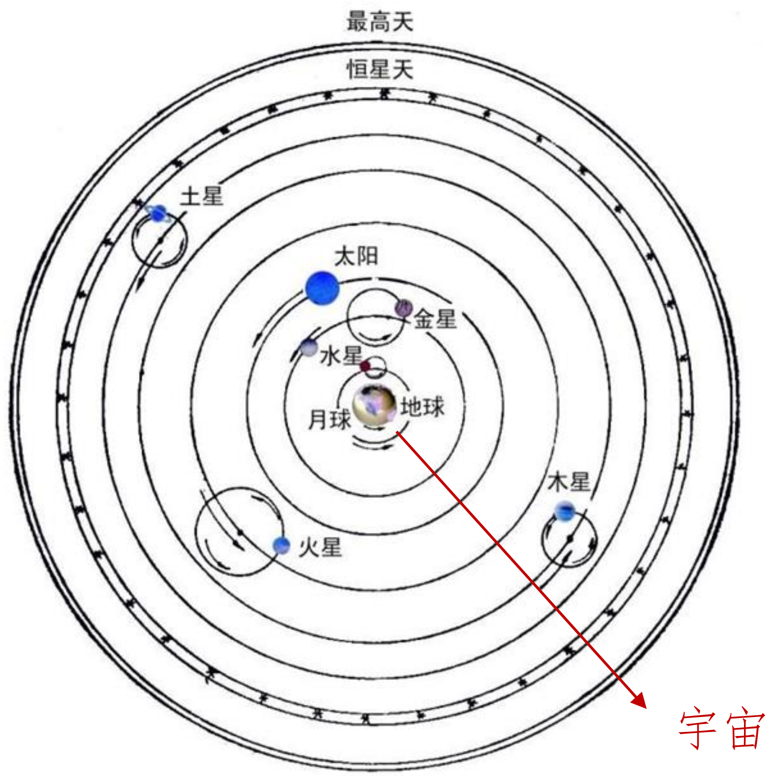
The start rails

All the stars rotate together with fixed relative position.

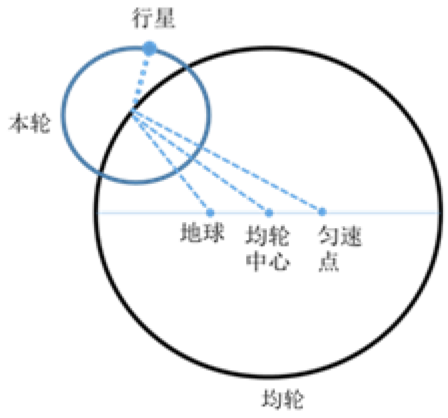
But, there’re five exceptions

Introduction

History: from geocentric to heliocentric model



宇宙中心：地球

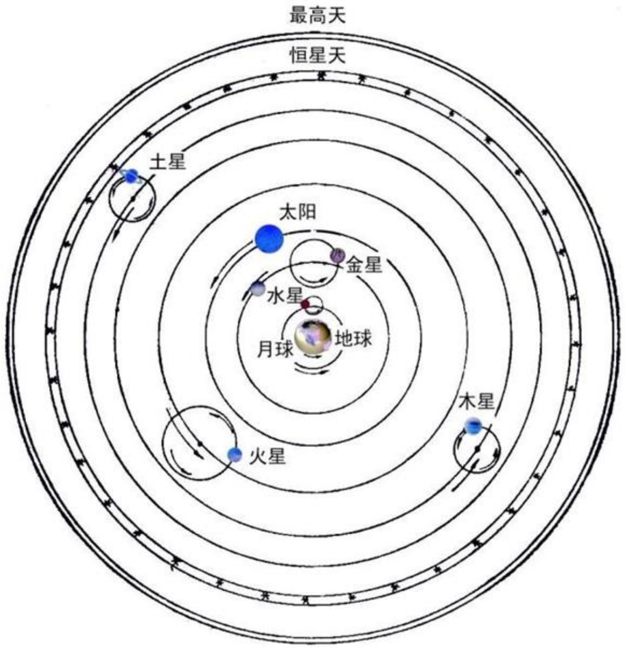


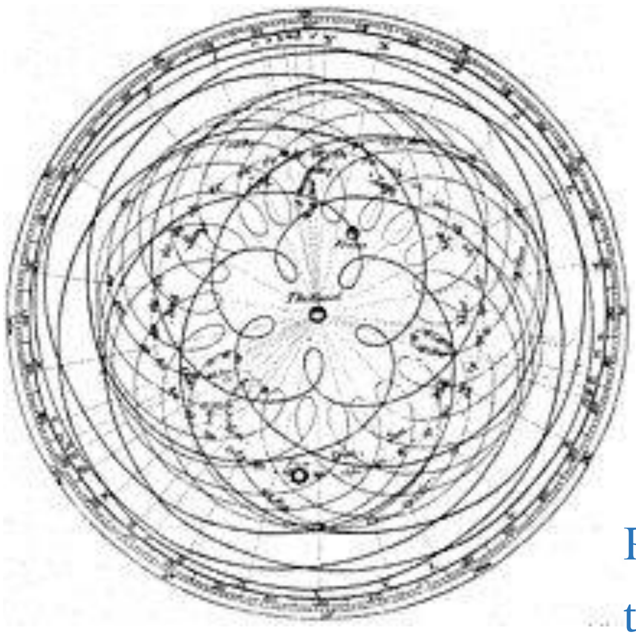
Explanation of planet (行星)

Ptolemy 托勒密，~ 90 B.C.

Introduction

History: development of geocentric model

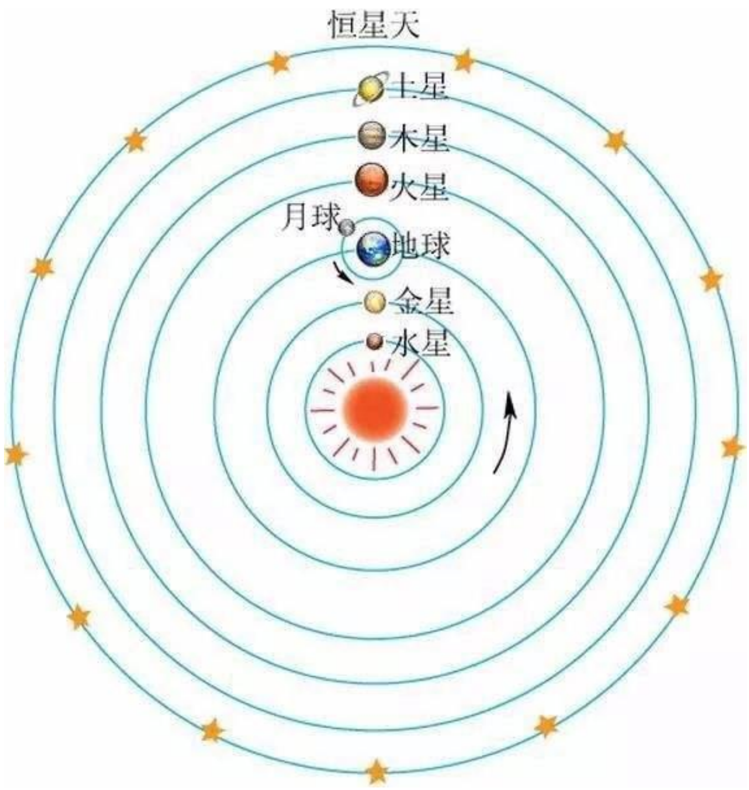




Ptolemy’s geocentric model is in textbooks of European university for more than 1500 years …

Introduction

History: from geocentric to heliocentric model

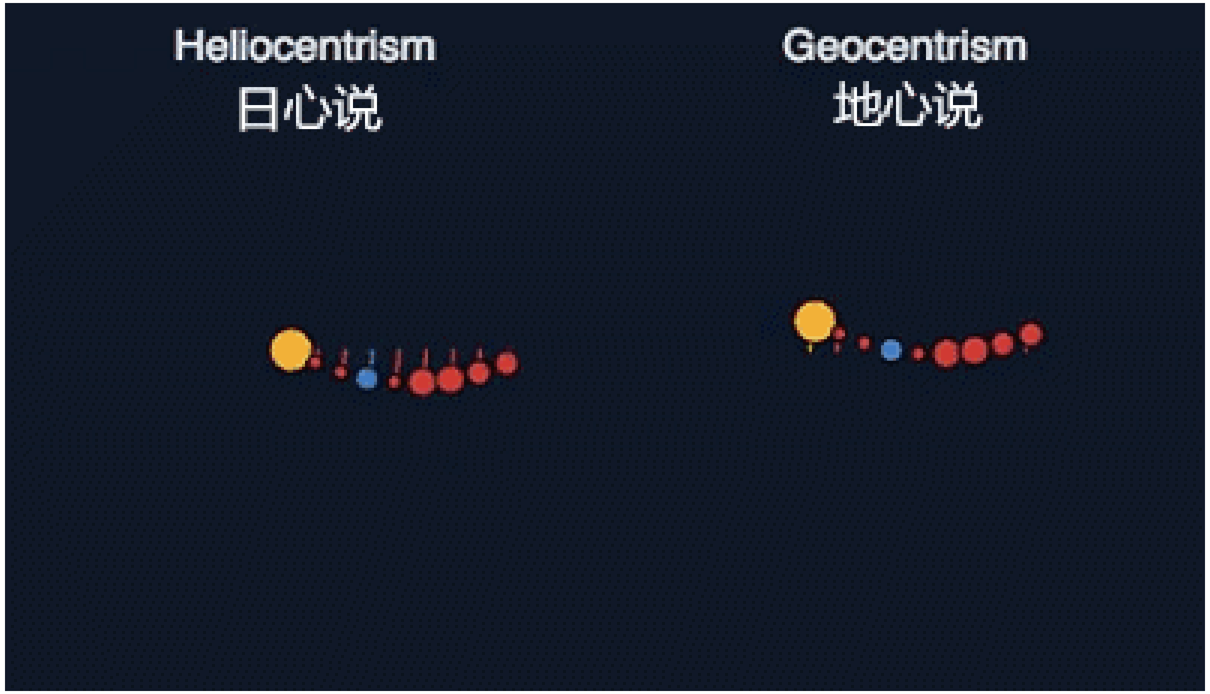


- The sun is at the center, all the planets rotate around the sun.
- Only the moon rotates around the earth.
- The earth spins around its axis, while the star background is still

Copernicus 哥白尼，1473 -1543

Introduction

History: comparison of geocentric heliocentric model



Introduction

History: Skepticism of geocentric model

- 1572: Superstar explosion （超星爆炸）
- 1577：Great comet of 1577（大彗星）

Tycho Brahe (第谷)，1546 －1601

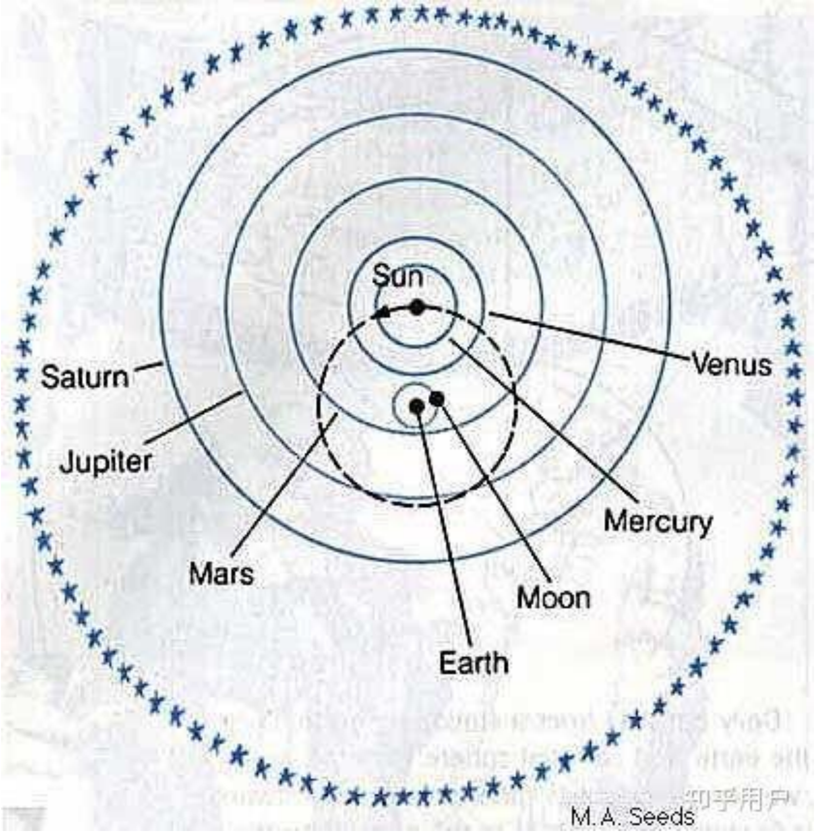
Introduction

History: the model of Tycho (第谷模型)

- The earth is still the center of universe
- Other planets rotates around the sun

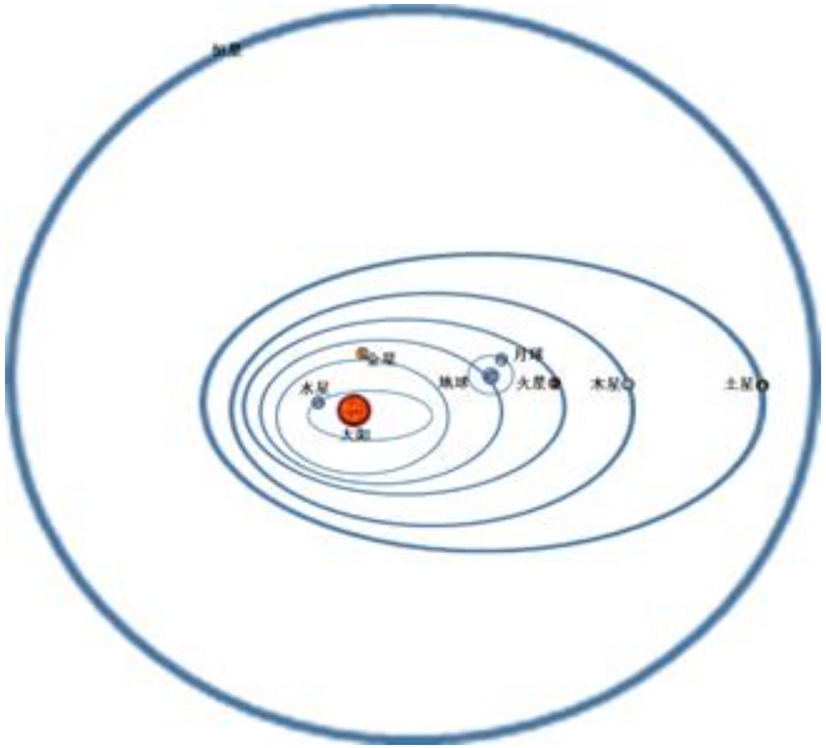
However, Tycho is not good at math.

At the year of 1600, he find Kepler …



Introduction

History: Kepler’s model (开普勒模型)



开普勒，1571 -1630

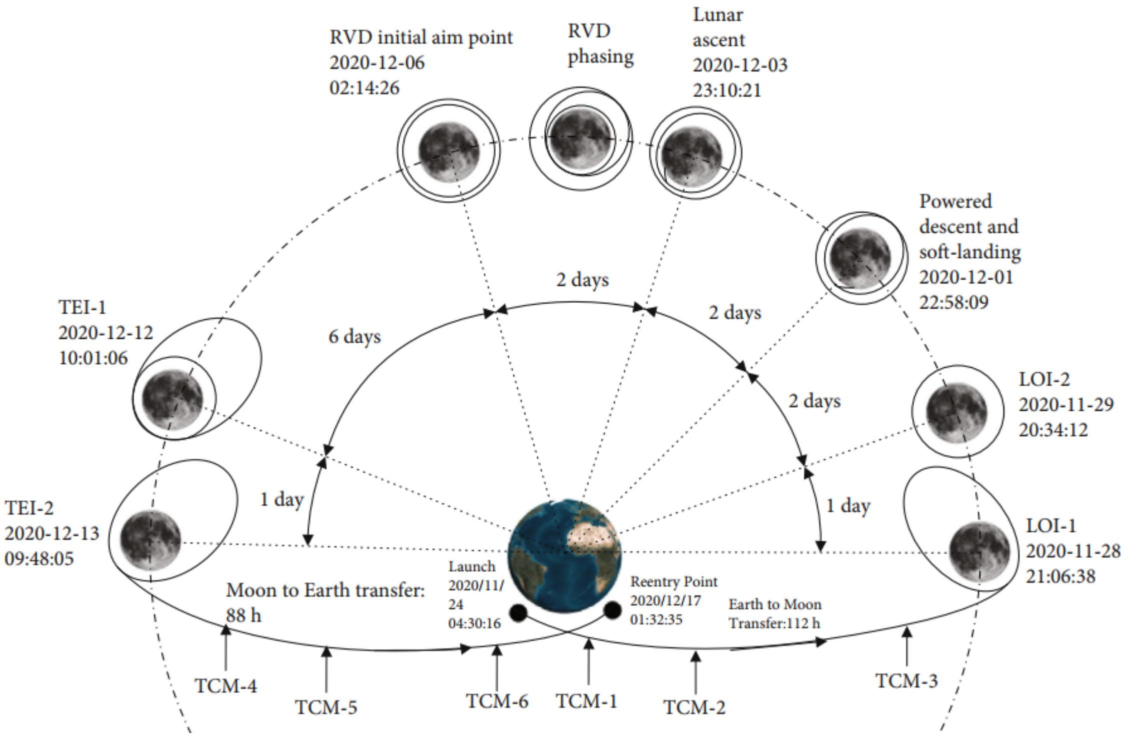
heliocentric theory +

Ellipse orbit

Orbit calculation

Orbit design of Chang’ e 5 mission

Orbit calculation for celestial sphere and spacecraft are similar



Orbit calculation

Orbit design of Chang’ e 5 mission

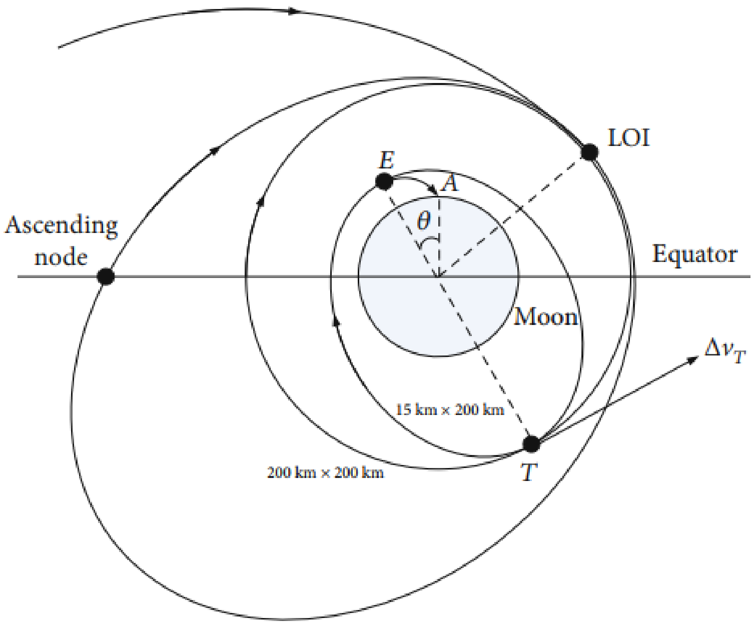


FIGURE 5: Basic orbit geometry.

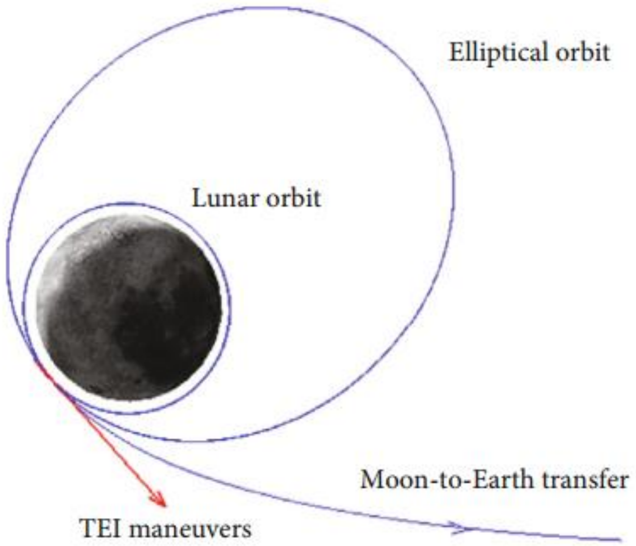


FIGURE 13: Two-impulse TEI strategy illustration.

Zhong-Sheng Wang et al., *Orbit Design Elements of Chang’e 5 Mission*, Space: Science & Technology, 2021

Lagrange equation

Definition

$$\frac{d}{dt}\left(\frac{\partial B}{\partial \dot{x}}\right) - \frac{\partial B}{\partial x} = 0$$

Lagrangian function

$$B \equiv T - \Phi = \frac{1}{2}m(\dot{x})^2 - mgx$$

Kinetic energy

$$T = \tfrac{1}{2}mV^2 = \tfrac{1}{2}m(\dot{x})^2$$

Potential energy

$$\Phi = wx = mgx$$

Lagrange equation

Definition

$$\boxed{\begin{gathered}\frac{d}{dt}\left(\frac{\partial B}{\partial \dot{x}}\right) - \frac{\partial B}{\partial x} = 0 \\ B = \frac{1}{2}m(\dot{x})^2 - mgx\end{gathered}}$$

$$\Rightarrow \frac{\partial B}{\partial \dot{x}} \equiv m\dot{x}, \quad \frac{\partial B}{\partial x} \equiv -mg$$

$$\Rightarrow m\frac{d\dot{x}}{dt} - (-mg) = 0$$

$$\Rightarrow m\ddot{x} + mg = 0$$

$$\Rightarrow \ddot{x} = -g$$

Equivalent to Newton's second law!

Lagrange equation

For general spatial coordinates q_1, q_2, q_3

Velocity: $\dot{q}_1, \dot{q}_2, \dot{q}_2$

Kinetic energy: $\mathrm{T}(q_1, q_2, q_3, \dot{q}_1, \dot{q}_2, \dot{q}_2)$

Potential energy: $\Phi(q_1, q_2, q_3\ )$

$$B \equiv T - \Phi$$

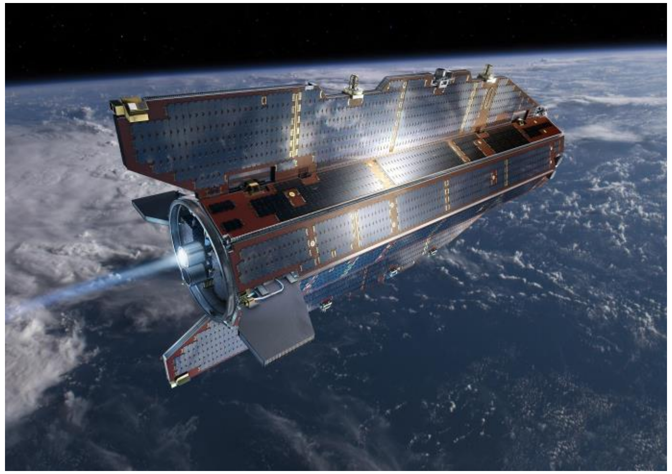
q_1 coordinate: $\dfrac{d}{dt}\left(\dfrac{\partial B}{\partial \dot{q}_1}\right) - \dfrac{\partial B}{\partial q_1} = 0$

q_2 coordinate: $\dfrac{d}{dt}\left(\dfrac{\partial B}{\partial \dot{q}_2}\right) - \dfrac{\partial B}{\partial q_2} = 0$

q_3 coordinate: $\dfrac{d}{dt}\left(\dfrac{\partial B}{\partial \dot{q}_3}\right) - \dfrac{\partial B}{\partial q_3} = 0$

Orbit equation

Force and energy



The average forces acting on GOEC satellite:
(Altitude 250 km)

source	central gravity	flattening Earth	atmospheric drag	Solar radiation	Sun (3rd body)
acceleration $[m/s^2]$	9	1×10^{-2}	3×10^{-6}	5×10^{-9}	5×10^{-7}

Orbit equation

Force and energy

$$F = \frac{GmM}{r^2}$$

$$d\Phi = F\,dr = \frac{GmM}{r^2}\,dr$$

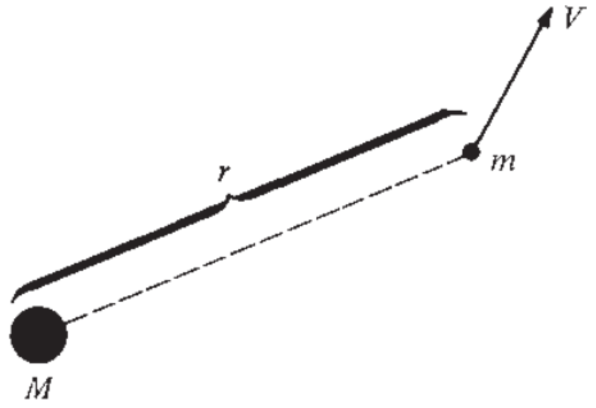


Figure 8.10 Movement of a small mass in the gravitational field of a large mass.

Orbit equation

Force and energy

$$\int_0^{\Phi} d\Phi = \int_{\infty}^{r} \frac{GmM}{r^2} dr$$

$$\Rightarrow \quad \Phi = \frac{-GmM}{r}$$

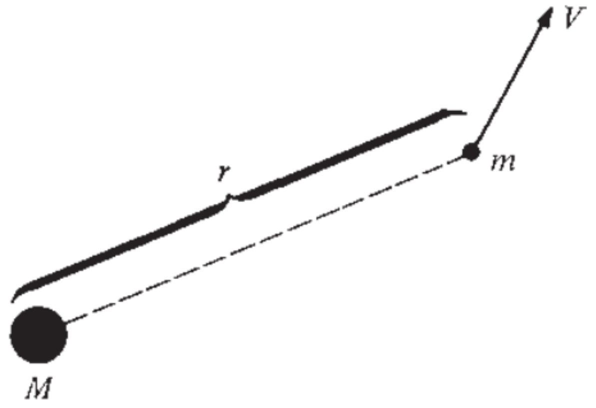


Figure 8.10 Movement of a small mass in the gravitational field of a large mass.

Orbit equation

Force and energy

Kinetic energy of the spacecraft in polar coordinate system:

$$T = \frac{1}{2} m V^2 = \frac{1}{2}[\dot{r}^2 + (r\dot{\theta})^2] m$$

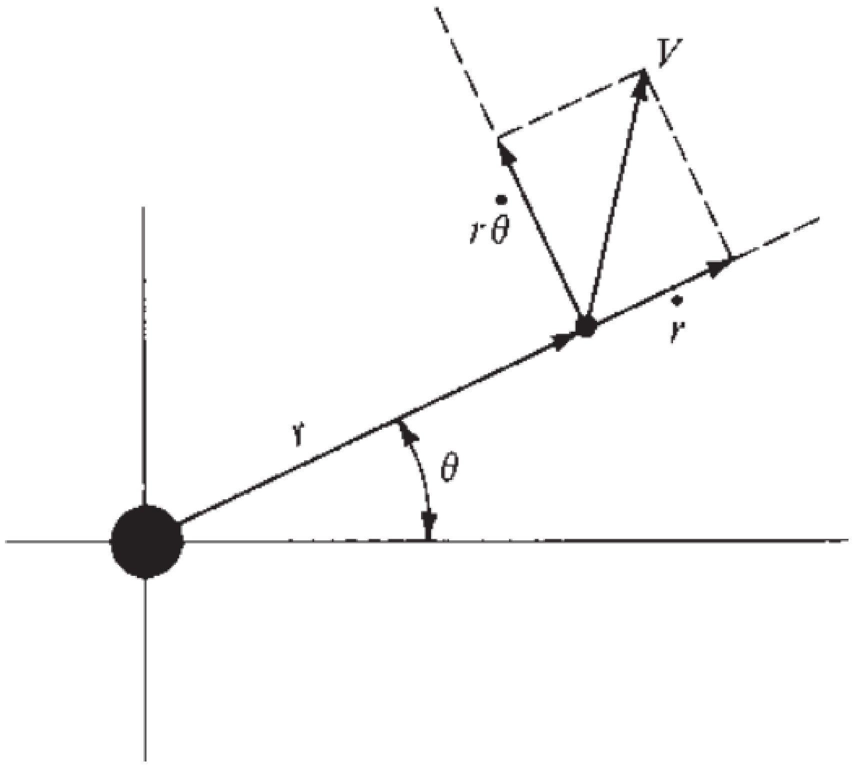


Figure 8.11 Polar coordinate system.

Orbit equation

Equation of motion

$$\boxed{k^2 \equiv GM = 3.986 \times 10^{14}\,\mathrm{m^3/s^2}}$$

The lagrangian function is:

$$B = T - \Phi = \frac{1}{2}m[\dot{r}^2 + (r\dot{\theta})^2] + \frac{GmM}{r}$$

$$\Rightarrow \quad B = \frac{1}{2}m[\dot{r}^2 + (r\dot{\theta})^2] + \frac{mk^2}{r}$$

Orbit equation

Equation of motion

$$\boxed{B = \frac{1}{2}m[\dot{r}^2 + (r\dot{\theta})^2] + \frac{mk^2}{r}}$$

From lagrangian equation :

$$\frac{\partial B}{\partial \dot{\theta}} = mr^2\dot{\theta} \qquad \frac{\partial B}{\partial \theta} = 0$$

$$\frac{d}{dt}\left(\frac{\partial B}{\partial \dot{x}}\right) - \frac{\partial B}{\partial x} = 0 \quad \Rightarrow \quad \frac{d}{dt}(mr^2\dot{\theta}) = 0$$

$$mr^2\dot{\theta} = \text{angular momentum} = \text{const}$$

Orbit equation

Equation of motion

$$\boxed{B = \frac{1}{2}m[\dot{r}^2 + (r\dot{\theta})^2] + \frac{mk^2}{r}}$$

Lagrangian equation in *r* direction:

$$\frac{\partial B}{\partial \dot{r}} = m\dot{r} \qquad \frac{\partial B}{\partial r} = -\frac{mk^2}{r^2} + mr\dot{\theta}^2$$

$$\frac{d}{dt}m\dot{r} + \frac{mk^2}{r^2} - mr\dot{\theta}^2 = 0$$

$$\Rightarrow \quad m\ddot{r} - mr\dot{\theta}^2 + \frac{mk^2}{r^2} = 0$$

Orbit equation

Equation of motion

$$\boxed{m\ddot{r} - mr\dot{\theta}^2 + \frac{mk^2}{r^2} = 0}$$

Define $\qquad r^2\dot{\theta} \equiv h = \text{angular momentum per unit mass}$

$$\Rightarrow \qquad \ddot{r} - \frac{h^2}{r^3} + \frac{k^2}{r^2} = 0$$

Solution r = ?

Orbit equation

Equation of motion

Solution for orbit equation

$$r = \frac{h^2/k^2}{1 + A(h^2/k^2)\cos(\theta - C)}$$

where A and C are constants

Orbit equation

Discussion

Standard form of conic section（圆锥截面）in Polar coordinate

$$r = \frac{p}{1 + e\cos(\theta - C)}$$

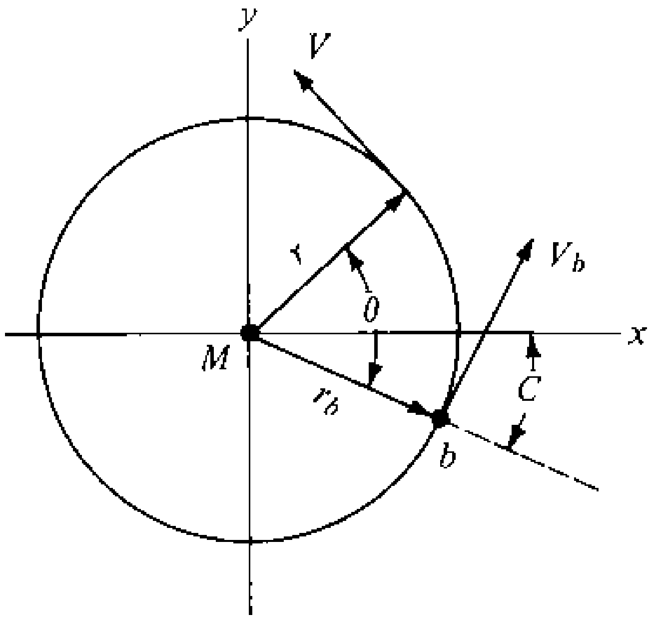
where $p = h^2/k^2$. $e = A(h^2/k^2)$ is eccentricity

If $e = 0$, the path is a *circle*.
If $e < 1$, the path is an *ellipse*.
If $e = 1$, the path is a *parabola*.
If $e > 1$, the path is a *hyperbola*.

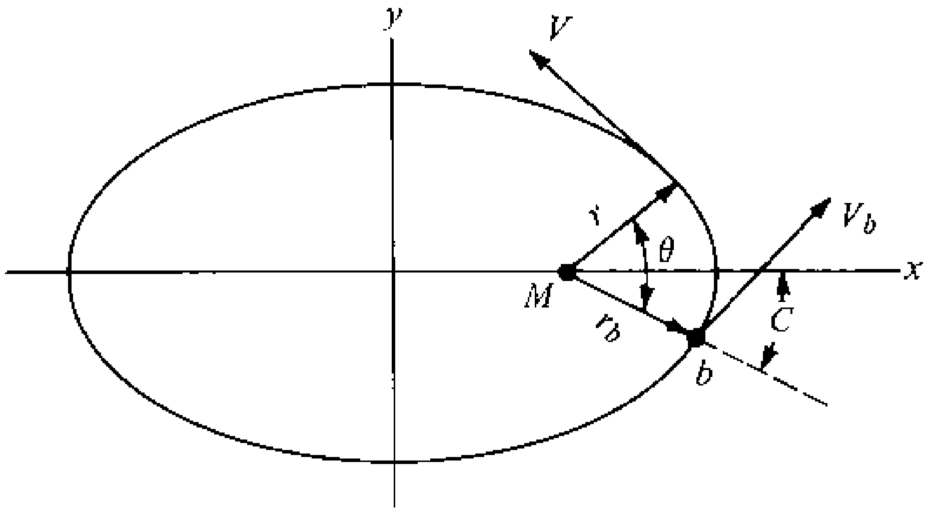
Orbit equation

Discussion

$$r = \frac{p}{1 + e\cos(\theta - C)}$$



Circle $e = 0$

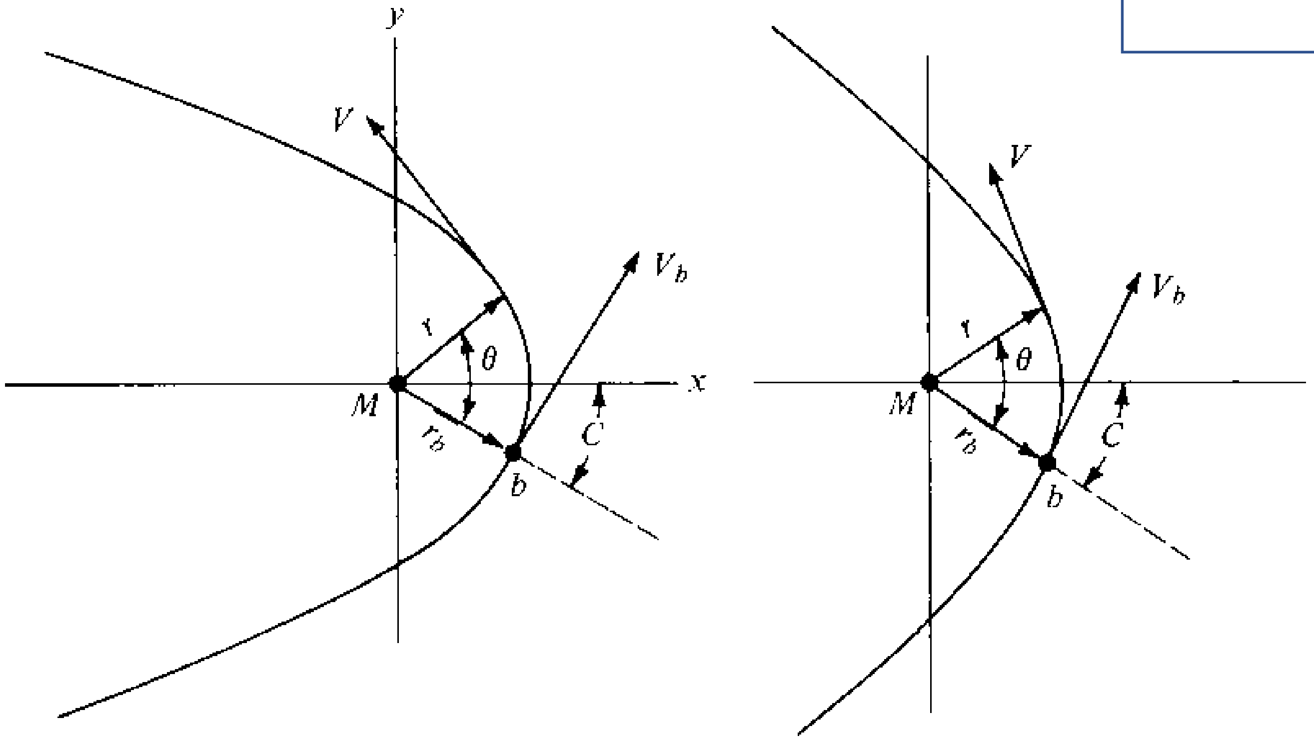


Ellipse $e < 1$

Orbit equation

$$r = \frac{p}{1 + e\cos(\theta - C)}$$

Discussion



Parabola $e = 1$ Hyperbola $e > 1$

Orbit equation

With some algebraic manipulations, we have

Eccentricity（偏心率） $e = \sqrt{1 + \dfrac{2h^2 H}{mk^4}}$

$T = |\Phi| \Rightarrow H = 0, e = 1$

where $H \equiv \boxed{T - |\Phi|} = -\dfrac{1}{2} m \dfrac{k^4}{h^2}(1 - e^2)$

Orbit equation

Type of trajectory

Type of Trajectory	e	Energy Relation
Ellipse	< 1	$\frac{1}{2}mV^2 < \frac{GMm}{r}$
Parabola	$= 1$	$\frac{1}{2}mV^2 = \frac{GMm}{r}$
Hyperbola	> 1	$\frac{1}{2}mV^2 > \frac{GMm}{r}$

Transfer orbit

Orbit equation

$$H \equiv T - |\Phi| = -\frac{1}{2} m \frac{k^4}{h^2}(1-e^2)$$

$$r = \frac{p}{1 + e\cos(\theta - C)}$$

where $p = h^2/k^2$.

Discussion

For circular orbit $\quad e = \sqrt{1 + \frac{2h^2 H}{mk^4}} = 0$

$$\Rightarrow \quad H = -\frac{mk^4}{2h^2}, \text{ and } \quad r = \frac{h^2}{k^2}$$

$$\Rightarrow \quad \frac{1}{2} m V^2 = -\frac{m}{2}\frac{k^2}{r} + \frac{k^2 m}{r} = \frac{k^2 m}{2r}$$

Circular velocity:

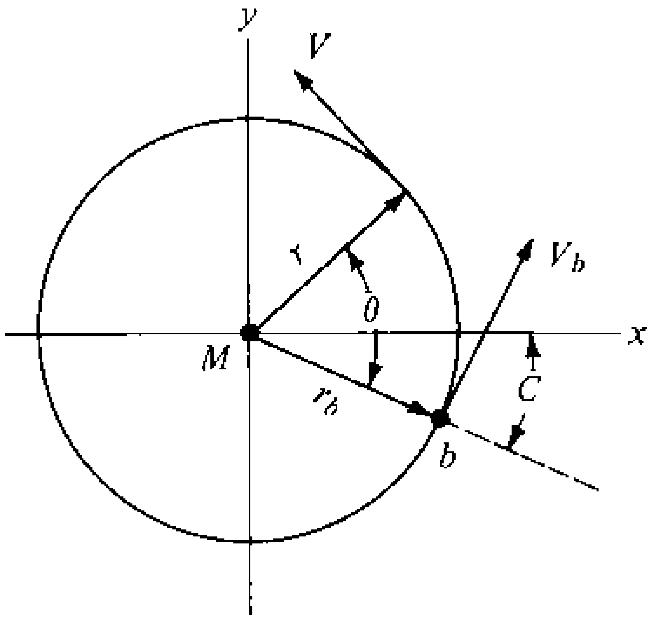
$$\Rightarrow \quad V = \sqrt{\frac{k^2}{r}} = \sqrt{\frac{GM}{r}}$$

Orbit equation

We can also obtain directly by

$$\frac{GmM}{r^2} = m\frac{V^2}{r}$$

$$\Longrightarrow V = \sqrt{\frac{GM}{r}} = 7.9\ km/s$$



Circle $e = 0$

For earth, $GM = 3.986 \times 10^{14}\ m^3/s^2, r = 6.4 \times 10^6 m$

Orbit equation

Parabolic trajectory (e = 1)

$$e = \sqrt{1 + \frac{2h^2 H}{mk^4}} = 1 \qquad \Rightarrow H \equiv T - |\Phi| = 0$$

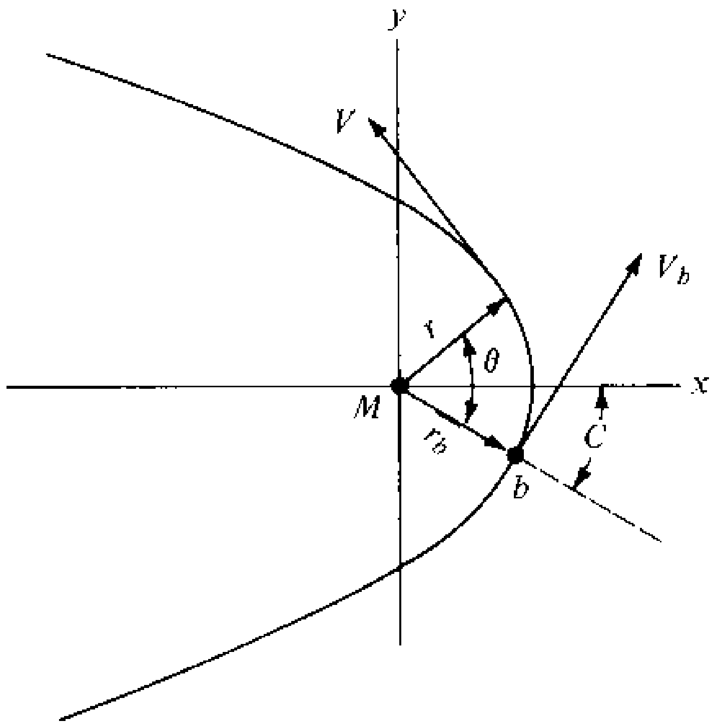
$$\Rightarrow \quad \frac{1}{2} mV^2 = \frac{GMm}{r} = \frac{k^2 m}{r}$$

Orbit equation

Parabolic velocity

$$V = \sqrt{\frac{2k^2}{r}} = \sqrt{\frac{2GM}{r}}$$

$$= 11.2\ km/s$$ Escape Velocity



Parabola $e = 1$

Practice

Example 8.1

At the end of a rocket launch of a space vehicle, the burnout velocity is 9 km/s in a direction due north and 3° above the local horizontal. The altitude above sea level is 500 mi. The burnout point is located at the 27th parallel (27°) above the equator. Calculate and plot the trajectory of the space vehicle.

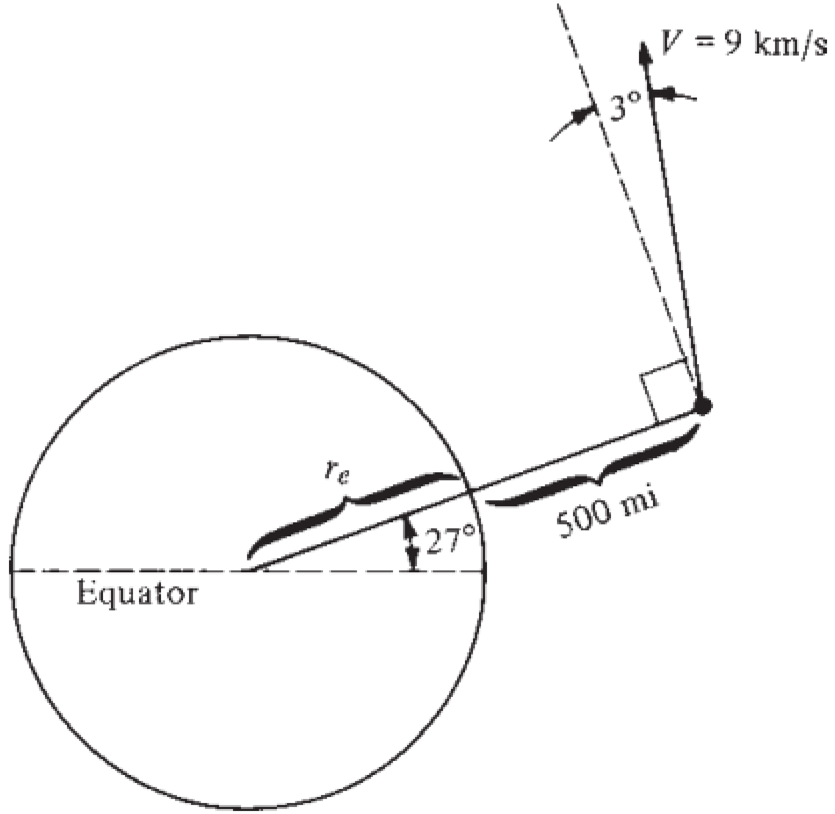


Figure 8.14 Burnout conditions for Example 8.1.

The Kepler's laws

A list of Kepler's First

- First to correctly explain planetary motion, thereby, becoming founder of celestial mechanics and the first "natural laws" in the modern sense;
- First to explain the process of vision by refraction within the eye;
- First to formulate eyeglass designing for nearsightedness and farsightedness;
- First to explain the use of both eyes for depth perception.
- First to explain the principles of **how** a telescope works;
- First to explain that the tides are caused by the Moon.
- First to derive the birth year of Christ, that is now universally accepted.
- ….

Source: https://www.nasa.gov/kepler/education/johannes

The Kepler's laws

Kepler's Laws of Planetary Motion

1. Planets move in ellipses with the Sun at one focus (**The First Law**).
2. The radius vector describes equal areas in equal times (**The Second Law**).
3. The period of a planet's orbit squared is equal to the size semi-major axis of the orbit cubed when it is expressed in astronomical units (**The Third Law**).

Source: https://www.nasa.gov/kepler/education/johannes

The Kepler's laws

The second law

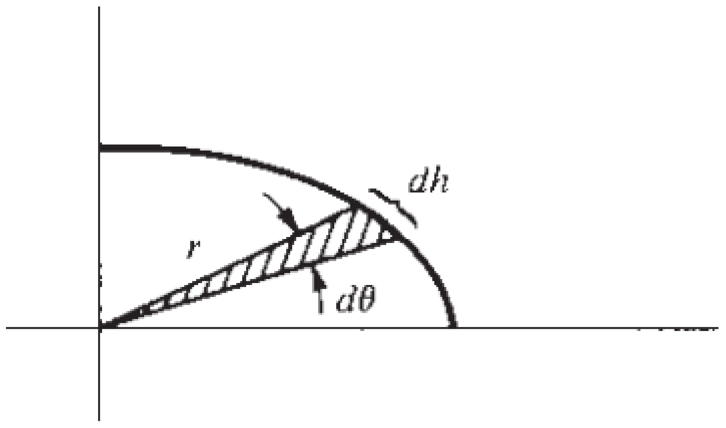


Figure 8.16 Area swept out by the radius vector in moving through angle $d\theta$.

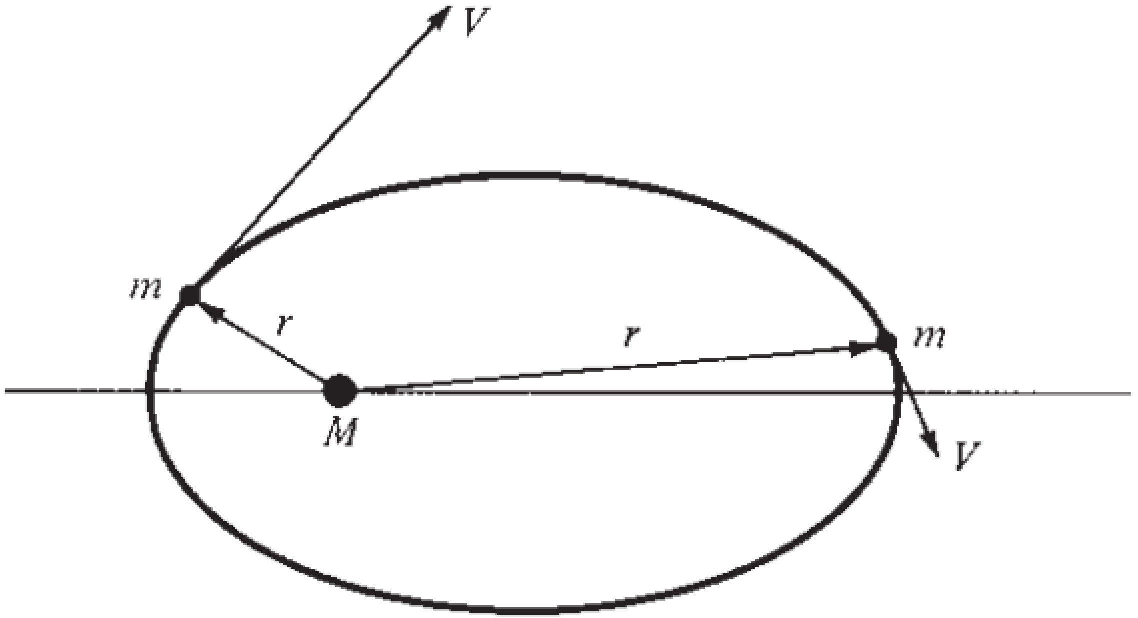


Figure 8.17 Illustration of the variation in velocity at different points along the orbit.

The Kepler's laws

The second law

$$\boxed{mr^2\dot{\theta} = \text{angular momentum} = \text{const}}$$

$$\frac{dA}{dt} = \frac{\frac{1}{2}r^2 d\theta}{dt} = \frac{1}{2}r^2\dot{\theta}$$

$$\Rightarrow \quad \frac{dA}{dt} = \text{const}$$

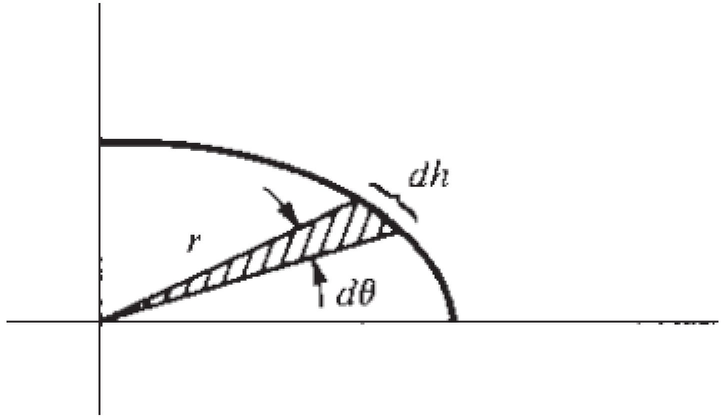


Figure 8.16 Area swept out by the radius vector in moving through angle $d\theta$.

The Kepler's laws

The third law

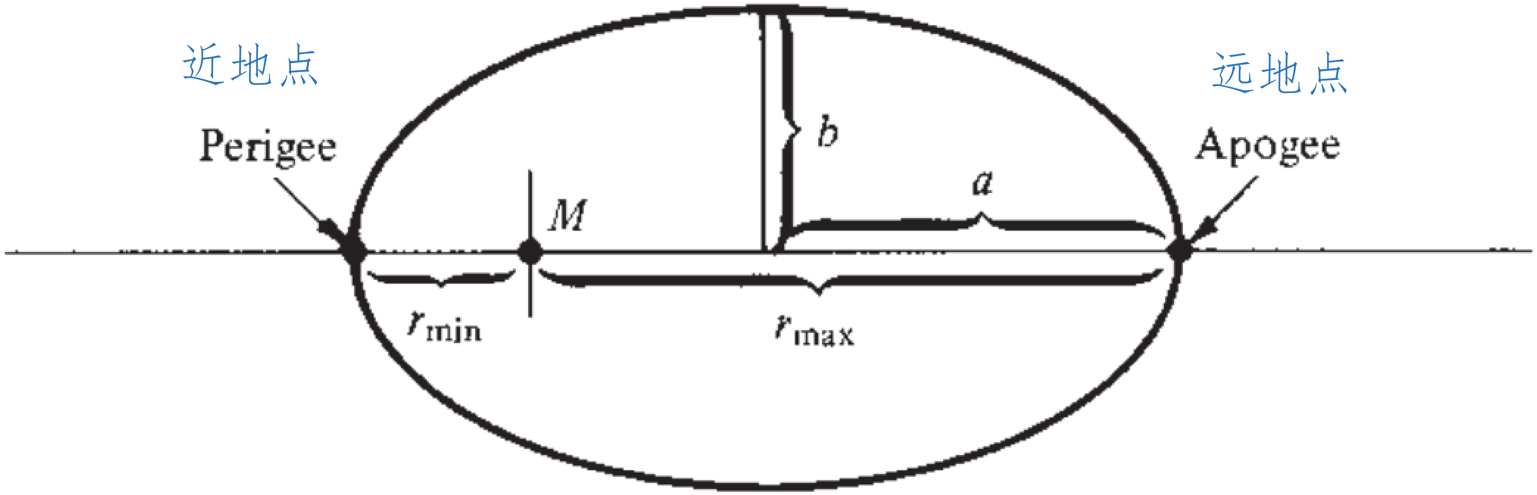


Figure 8.18 Illustration of apogee, perigee, and semimajor and seminor axes. 远地点、近地点和半长轴和半短轴

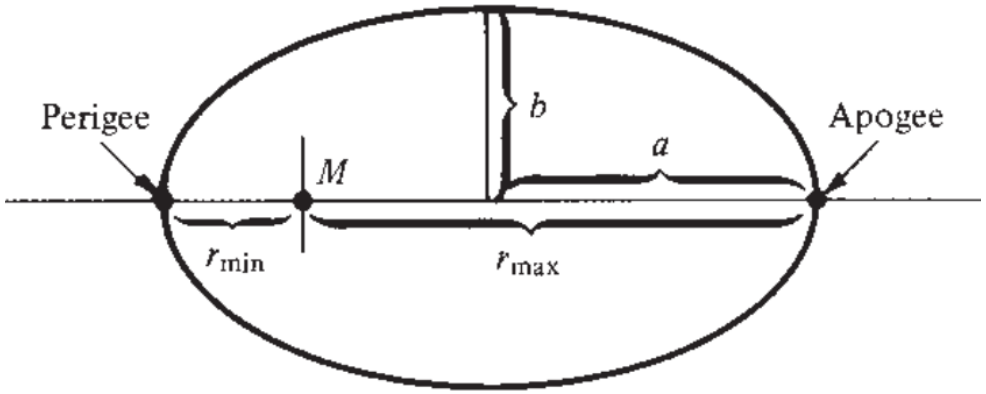
The Kepler's laws

The third law

$$r = \frac{p}{1 + e\cos(\theta - C)}$$

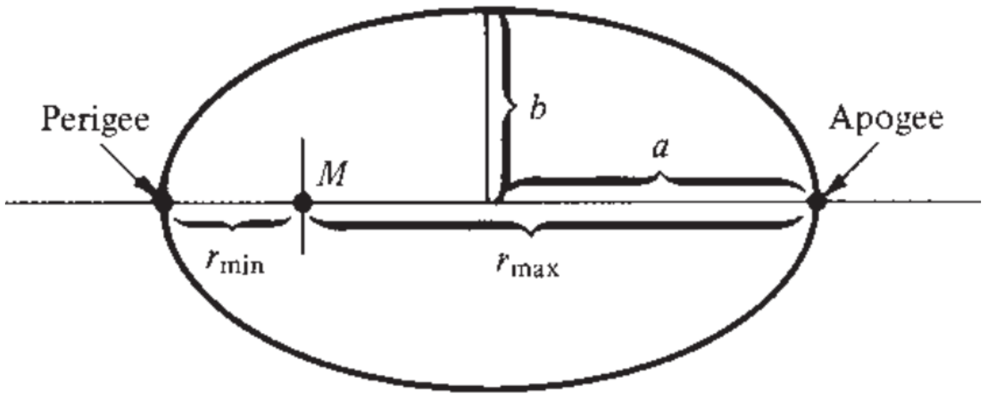
$$r_{\max} = \frac{h^2/k^2}{1-e} \quad \cos\theta = -1, C = 0$$

$$r_{\min} = \frac{h^2/k^2}{1+e} \quad \cos\theta = 1, C = 0$$



The Kepler's laws

The third law



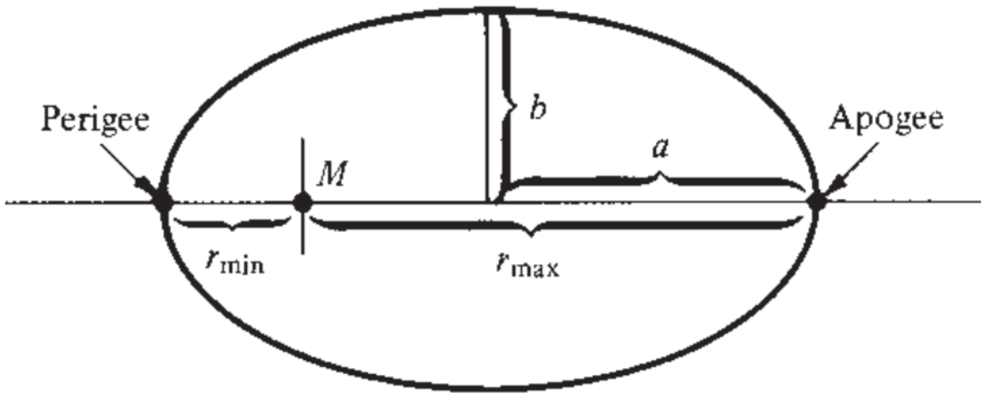
Period（周期）: $$\tau^2 = \frac{4\pi^2}{k^2} a^3$$

$$a = \frac{1}{2}(r_{max} + r_{min}) = \frac{1}{2}\frac{h^2}{k^2}\left(\frac{1}{1-e} + \frac{1}{1+e}\right) = \frac{h^2/k^2}{1-e^2}$$

The Kepler's laws

The third law

$$\frac{\tau_1^2}{\tau_2^2} = \frac{a_1^3}{a_2^3} = \mathrm{Const}$$



Practice

Example 8.2

The period of revolution of the earth about the sun is 365.256 days. The semimajor axis of the earth's orbit is 1.49527×10^{11} m. The semimajor axis of the orbit of Mars is 2.2783×10^{11} m. Calculate the period of Mars.

From Kepler's third law:

$$\tau_2 = \tau_1 \left(\frac{a_2}{a_1} \right)^{3/2}$$

The Mars

TED interview of Elon Mask