北京航空航天大學
BEIHANG UNIVERSITY

Avionics Technology

B31353551

— Radio Navigation

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Spring Semester 2023 (8_Jun_T16)

VI. Radio Navigation

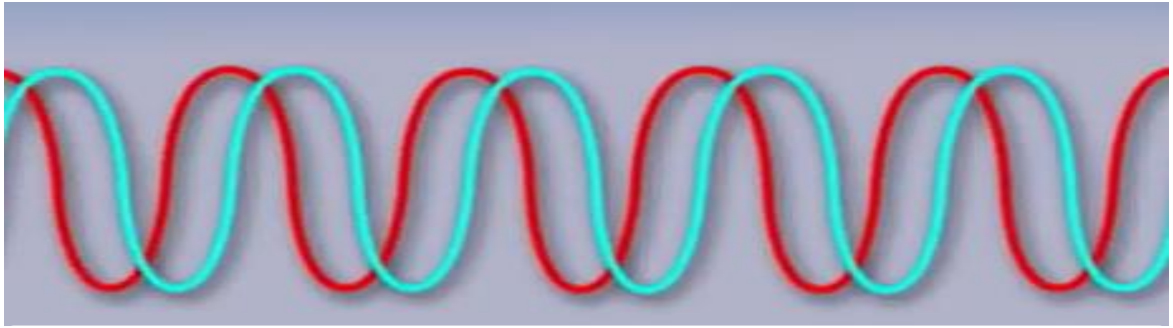
（1）Some concepts

（2）Radio landing systems

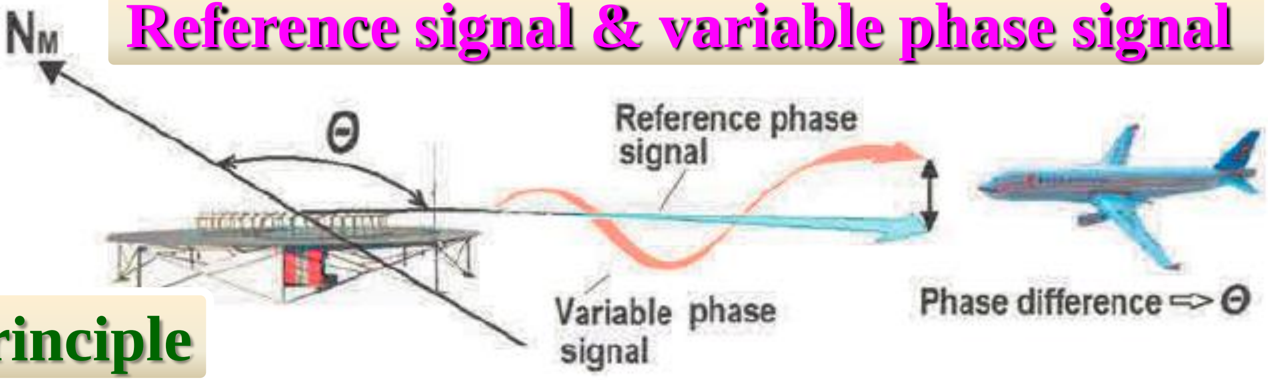
（3）Rho-theta navigation systems

（4）Satellite navigation systems

- The VOR beacon emits the signal of reference phase as well as the signal, which phase of carrier depends on the azimuth, i.e. azimuth signal. Onboard equipment processes signals and measures phase difference proportional to azimuth.



Reference signal & variable phase signal



VOR principle

（3）ρ-θ navigation systems

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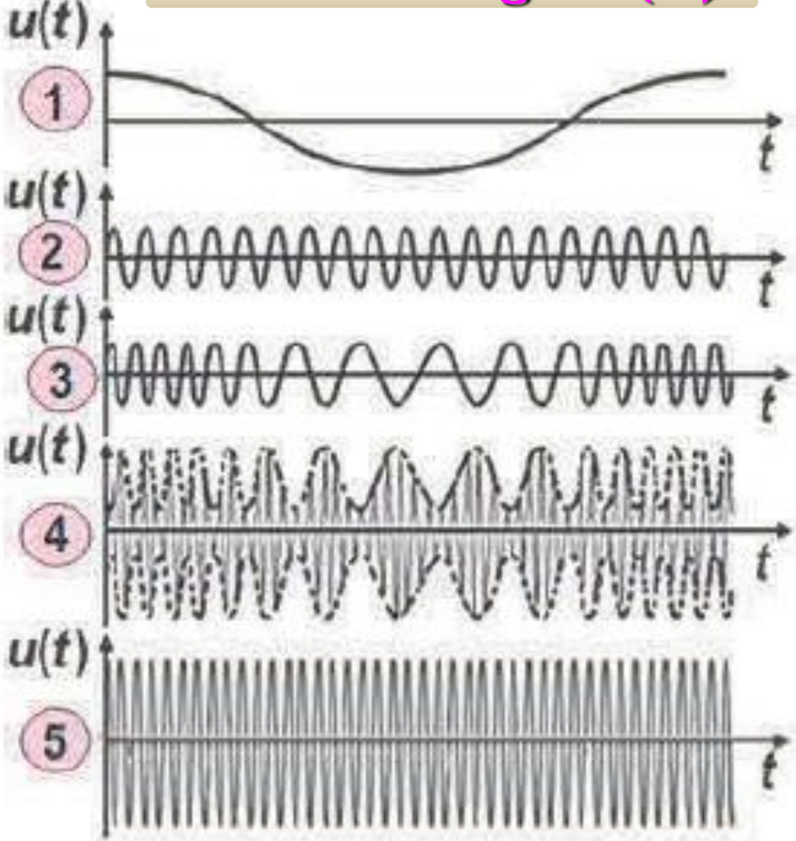
$$r(t) = \cos(\Omega_n t + \frac{\Delta\Omega_n}{\Omega}\sin\Omega t)$$

Reference signal (Ω)



Signal of subcarrier (Ω_n)

FM signal of subcarrier with the modulation coefficient ($\Delta\Omega_n / \Omega$)

AM signal: $E_m[1+m_r r(t)]\cos\omega_c t$

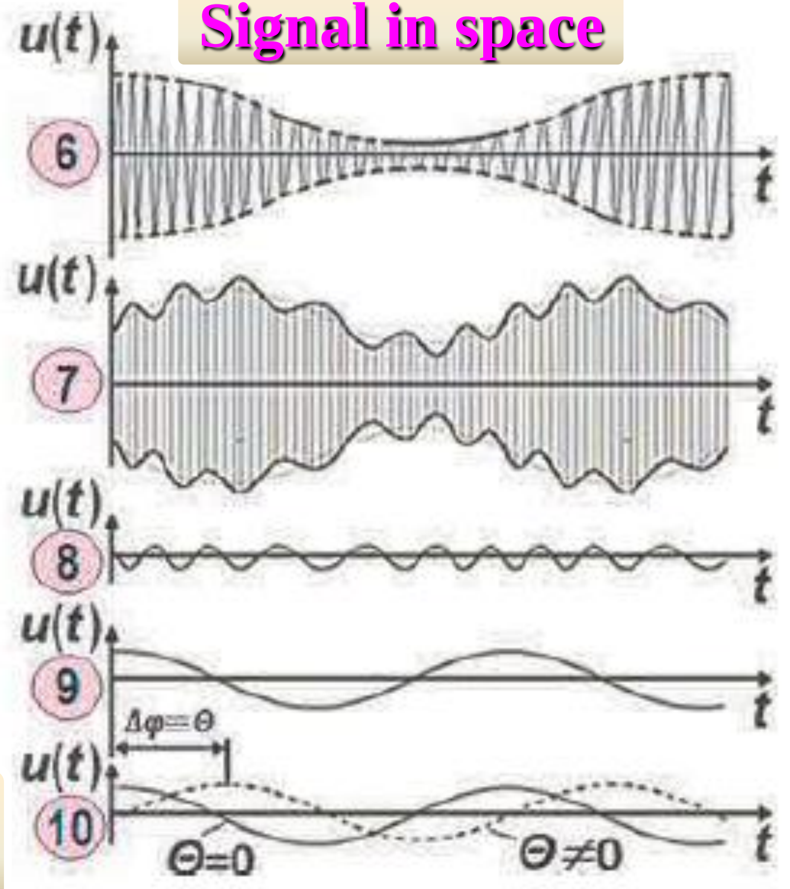
Variable phase signal emitted by rotating antenna

$$v(\theta,t) = A\cos(\Omega t - \theta)\cos\omega_c t$$

(3) ρ-θ navigation systems

$$e(\theta,t)=E_m[1+m_v\cos(\Omega t-\theta)+$$
$$m_r\cos(\Omega_n t+\Delta\Omega_n/\Omega\sin\Omega t)]\cos\omega_c t$$

Signal in space



Received signal onboard

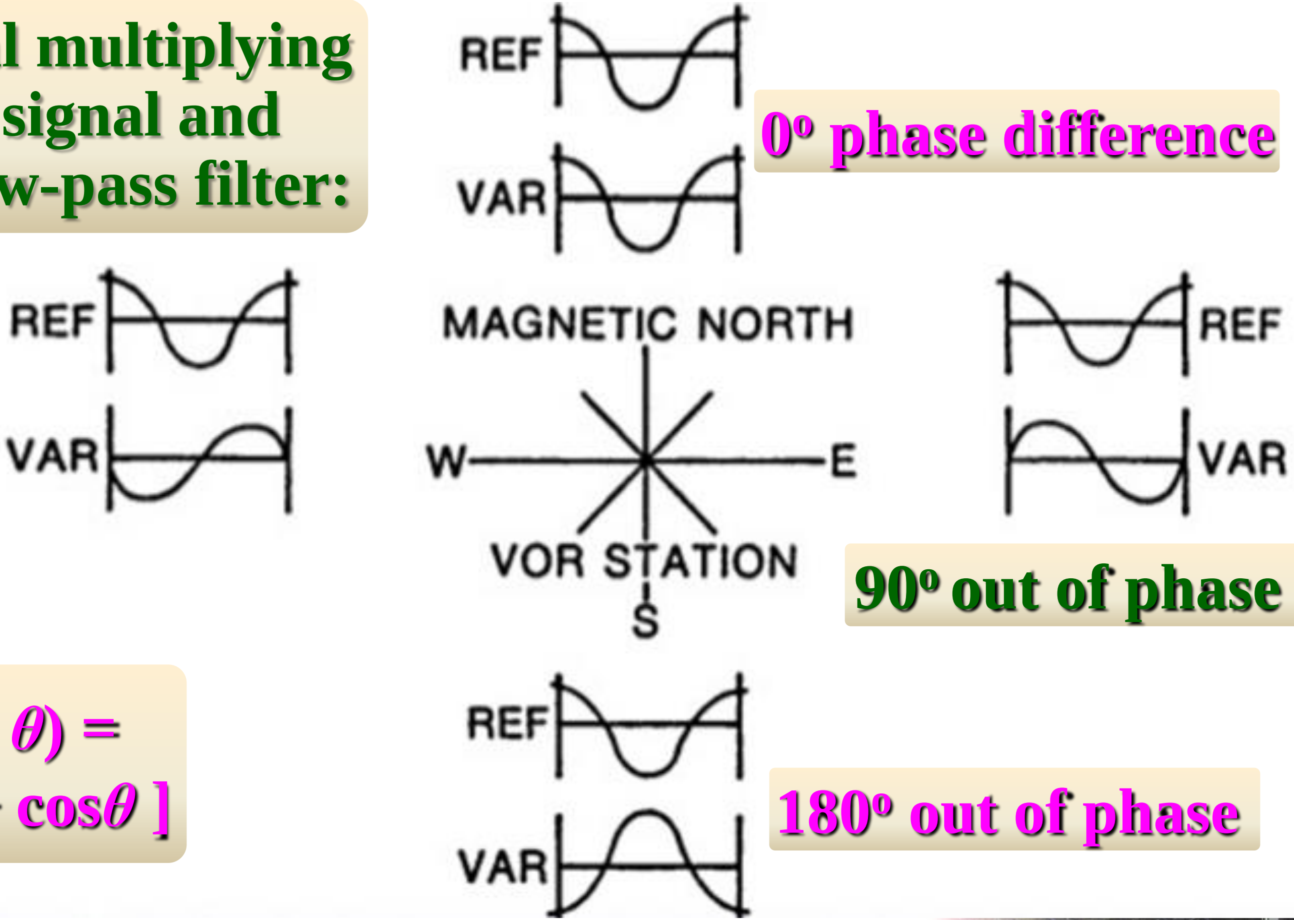
FM signal of subcarrier onboard

Reference (phase) signal after demodulation and filtering: $U\cos\Omega t$

Variable phase signal after demodulation & filtering: $U\cos(\Omega t-\theta)$

(3) ρ-θ navigation systems

Variable phase signal multiplying with the reference signal and passing through a low-pass filter:



0° phase difference

270° out of phase

90° out of phase

180° out of phase

$$U\cos\Omega t \cdot U\cos(\Omega t - \theta) =$$

$$0.5U^2\,[\cancel{\cos(2\Omega t - \theta)} + \cos\theta\,]$$

(4) Satellite Navigation Systems

- *Satellite Navigation Systems* (SNS) are basically radio navigation systems deriving the user's position from radio signals transmitted from a number of satellites moving along orbits around the Earth.

Airborne SNS equipment

SNS satellites

（4）Satellite Navigation Systems

- Modern SNS are GPS (USA), GLONASS (Russia), Galileo (European union) and BeiDou/北斗 (China), having the characteristics of global coverage, precise, three-dimensional positioning and navigation, and superior capabilities to radio navigation systems ground-based.

ACARS messages

LAT, LON, UTC generally come from SNS

┌ QU SHAUOFM
.BJSXCXA 302359
┐ M14
FI FM652/AN B-2153
DT BJS PVG 302359 M20A
- PRESENT POSITION REPORT
CAS 0248,LAT N 45.628,LON E126.266,
ALT 24644, UTC 042105└

（4）Satellite Navigation Systems

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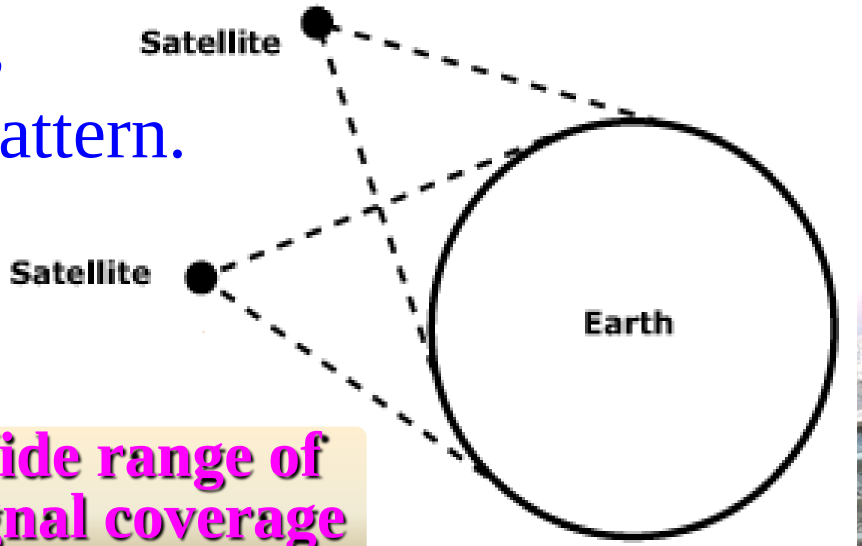
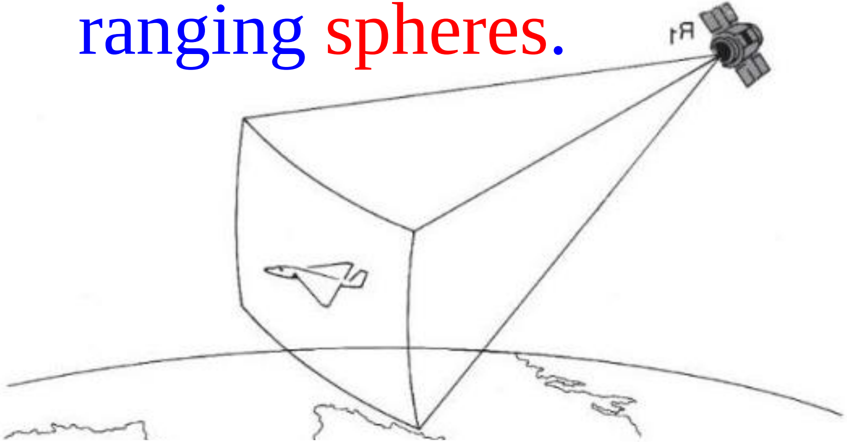
- SNS are space-based radio navigation systems. The navigation satellites are normally not in geostationary orbit like communication satellites, and used as radio beacons provided a user knows their coordinates.
- The orbit tracks over the earth, forming an ‘egg beater’ type pattern. Small rocket boosters are located on each satellite to keep the satellite flying in its correct orbit.



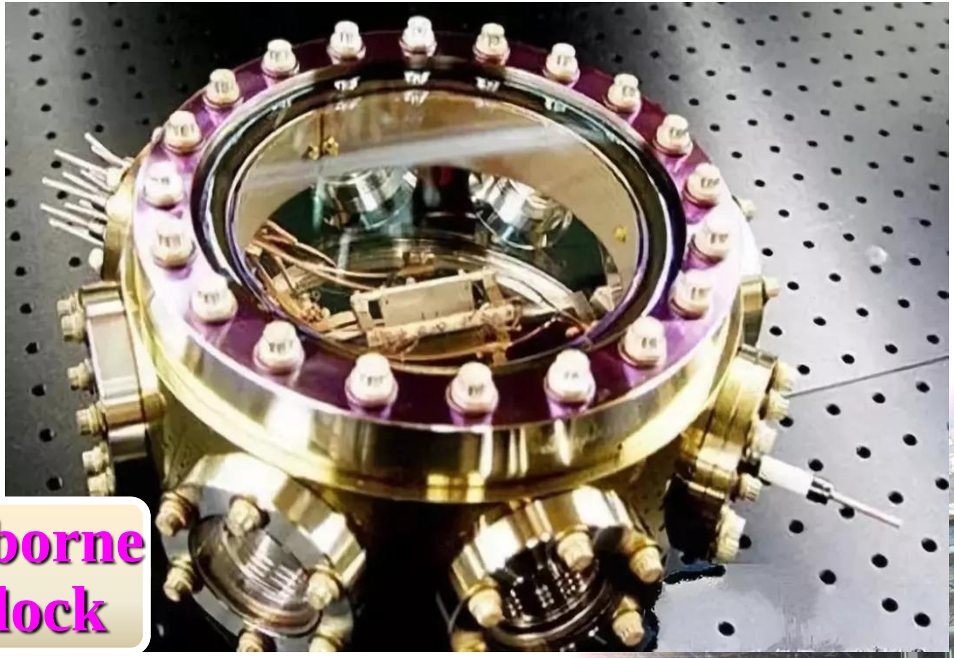
Wide range of signal coverage

(4) Satellite Navigation Systems

- Each satellite contains two or three atomic clocks which are the key components to providing a SNS user (e.g. aircraft en route) with position fix, i.e. the intercept of ranging spheres.



Measuring the time delay for the satellite transmission to the user



Satellite-borne atomic clock

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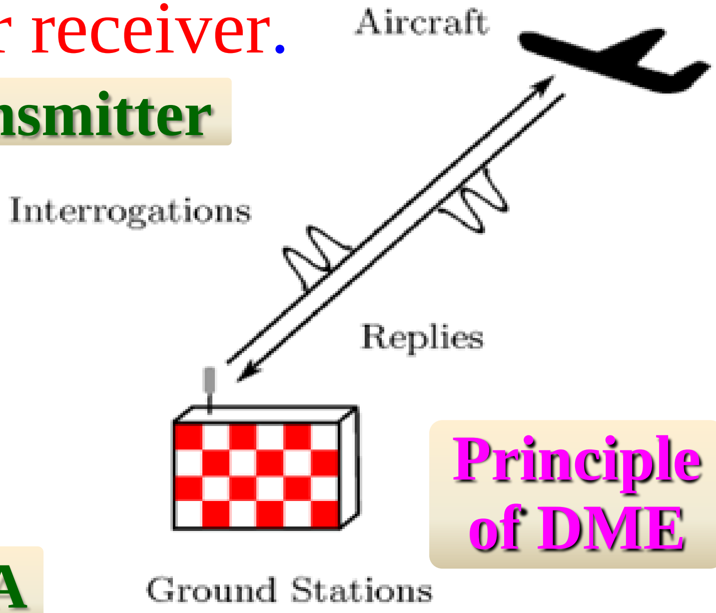
- SNS utilizes the concept of *time-of-arrival* (TOA) ranging to determine user position, i.e. measuring the transit time it takes for a signal transmitted by a satellite at a known location to reach a user receiver.

Range (distance) measuring based on TOA



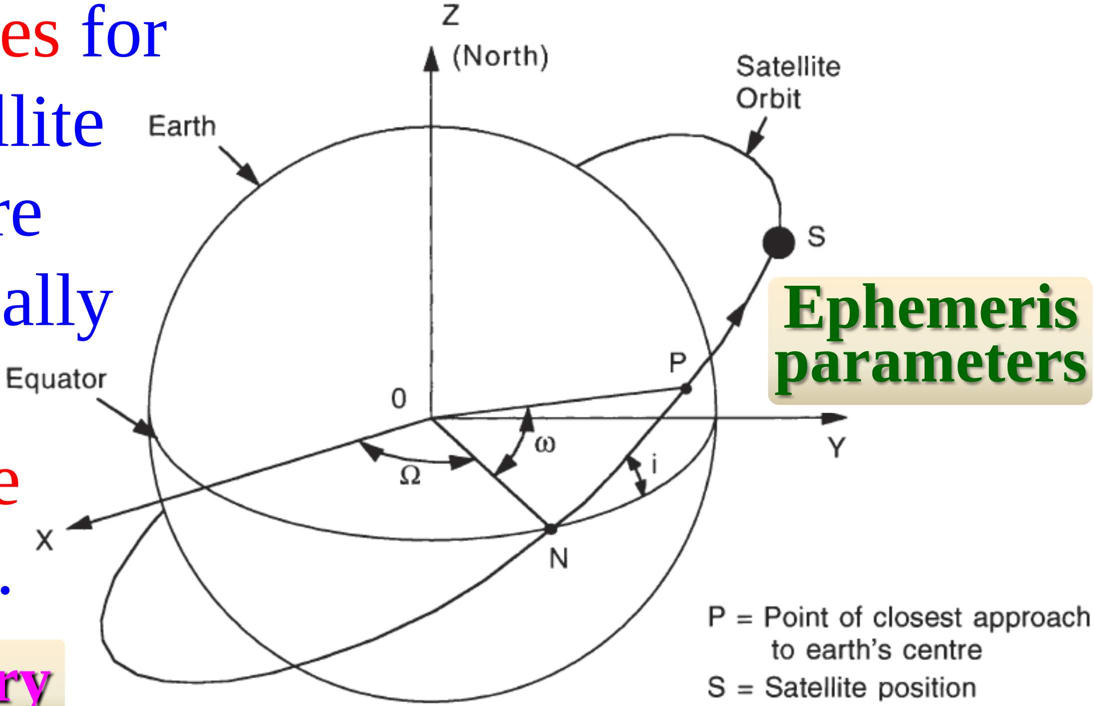
Principle of DME

(4) Satellite Navigation Systems

- The system control element consists of ground-based monitoring stations and master control station. The control element is to ensure the accuracies of the navigation satellite positions and the atomic clocks, the master control station gets data from monitoring stations and able to send data to the satellites to correct for any errors of orbits or clocks.

Monitoring station

- The control segment processes the tracking data acquired from the monitoring stations to generate the predicted orbit estimates for the satellites. The satellite position coordinates are then fitted mathematically over a specified time interval to compute the Ephemeris parameters.



Ephemeris parameters

Satellite-earth geometry

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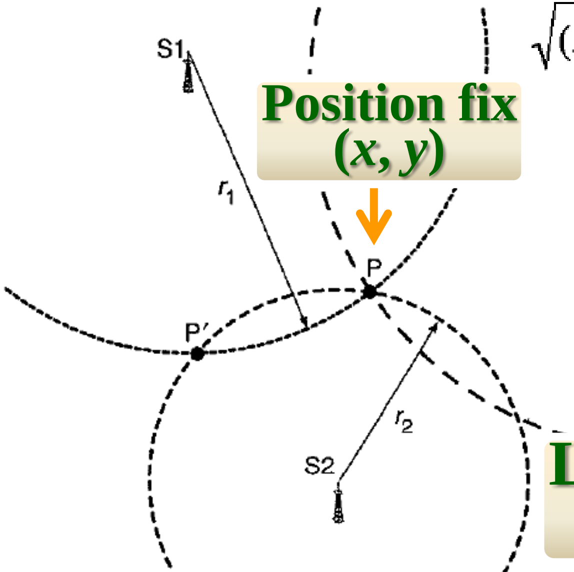
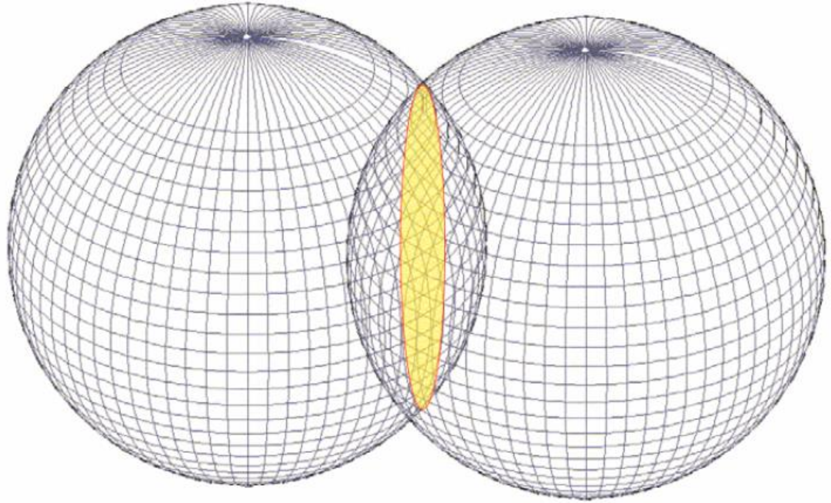
- The SNS user's three-dimensional position fix lies on the surface of ranging spheres. Two ranging spheres intersect a perimeter, then the position located on the circle perimeter.



$$\sqrt{(x_k - x)^2 + (y_k - y)^2} = r_k$$

Position fix (*x*, *y*)

LOP of equivalent distances



Range determination from two navigation satellites

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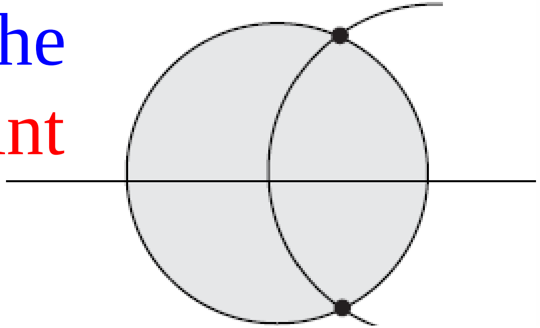
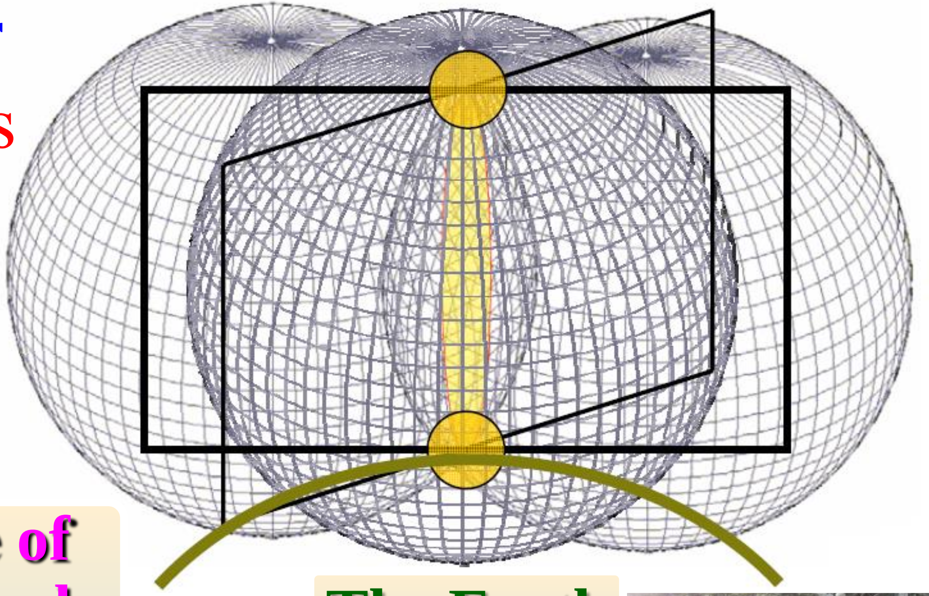
- Using a third satellite, the user is at the intersection of the perimeter of the circle and the surface of the third sphere. This third sphere intersects the shaded circle perimeter at two points. For a user on or near the Earth's surface, the lower point will be the true position.



User located at one of two points on the circle



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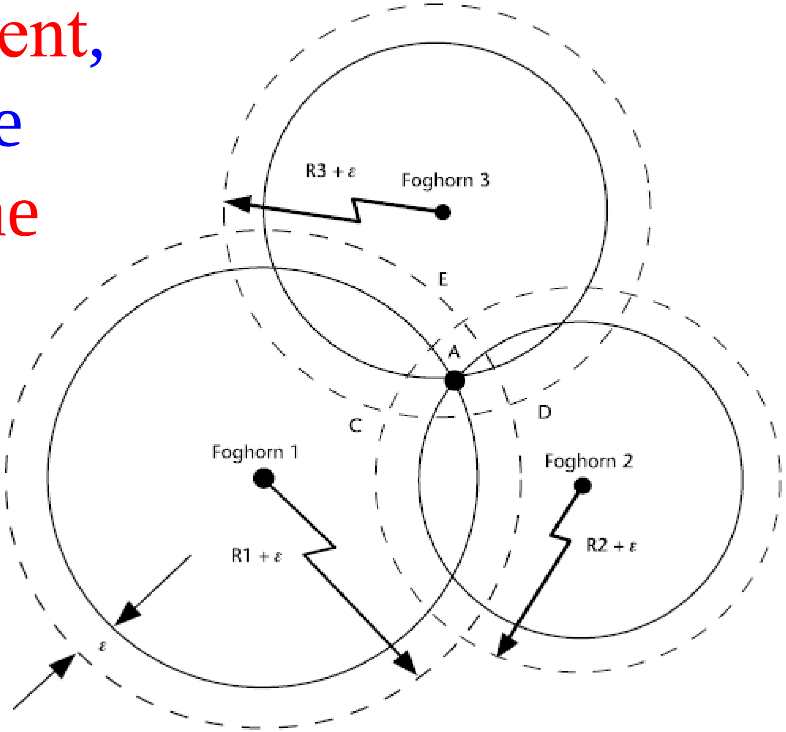
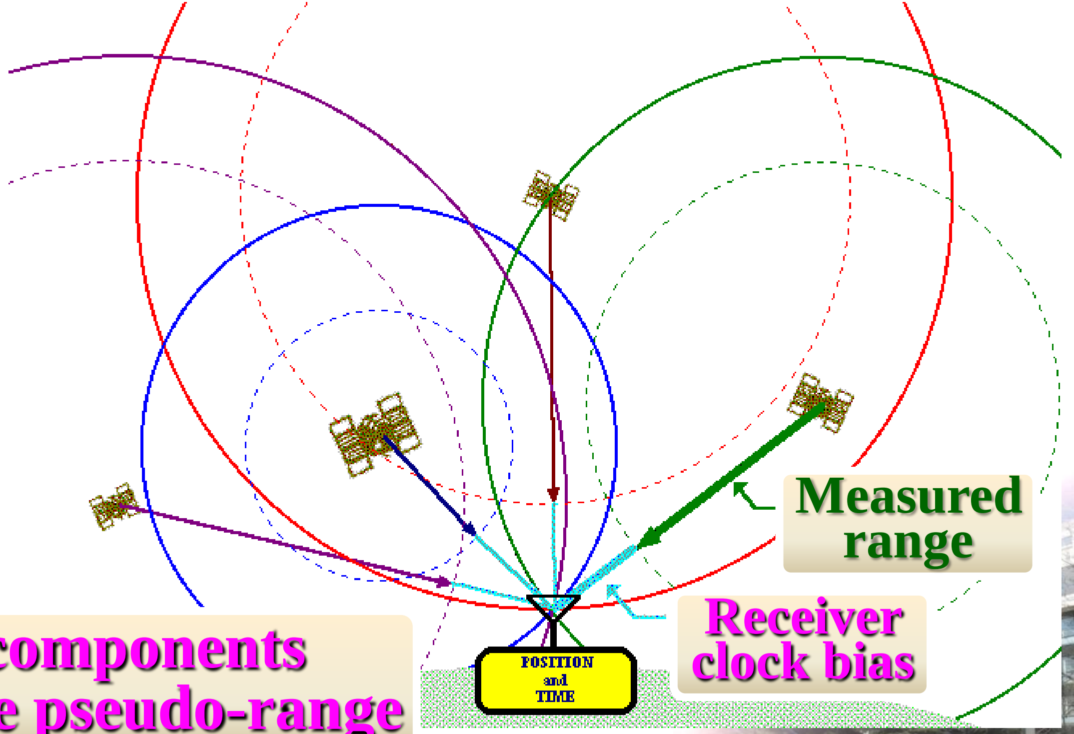
- The orbital positions of the SNS satellites relative to the earth are known to extremely high accuracy, and the transmission time of each satellite sending a signal can be determined. Given a perfect time reference in the user equipment, measurement of the spherical ranges (i.e. time delay relative to the transmission time) of three satellites would be sufficient to determine the user's position.

- Each SNS satellite carries an atomic clock which provides the time reference for the satellite signal transmission. The user's equipment, however, has a crystal clock time reference which introduces a time bias in the measurement of the transit times of the satellite transmissions.



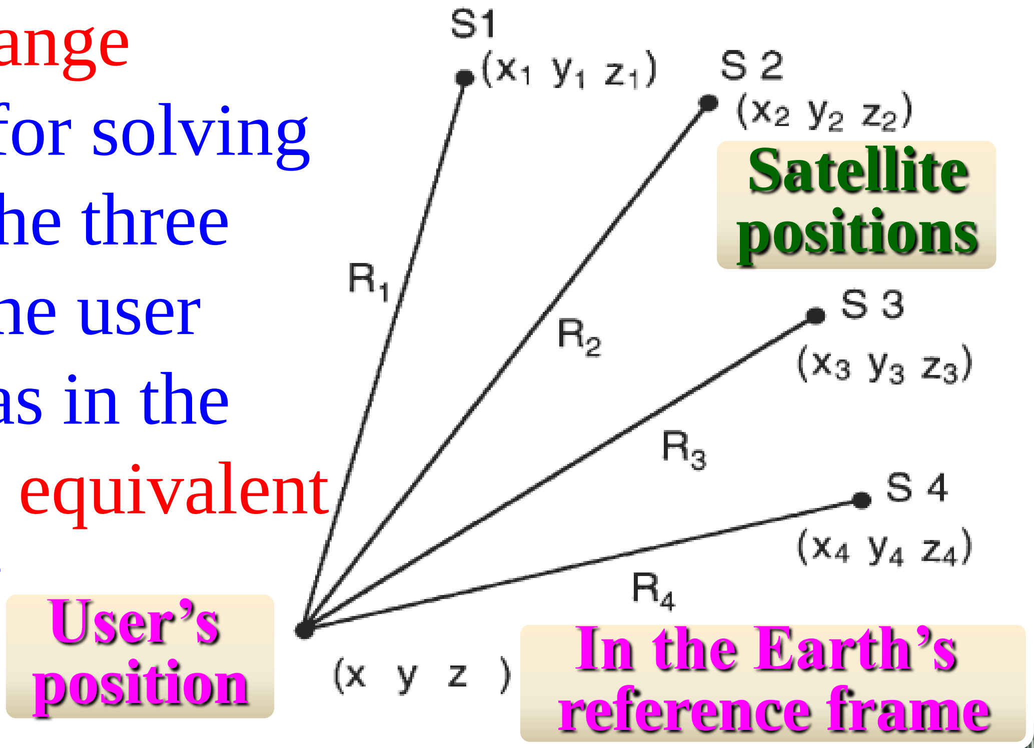
Effect of receiver clock offset on transit time measurements

- The measurement of the time delay is thus made up of two components. The first component is the transit time of the ranging signal and the second component is the time offset between the satellite clock and the receiver clock due to non-synchronization of the clocks.



Two components constitute pseudo-range

- Measuring the spherical ranges from four satellites therefore enables the user's position to be determined and yields four pseudo-range measurement equations for solving the four unknowns, i.e. the three position coordinates of the user (X, Y, Z) and the time bias in the user clock. Let the range equivalent of the user's clock offset equals to T, we have:



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$$R_1 = \left[(X - X_1)^2 + (Y - Y_1)^2 + (Z - Z_1)^2\right]^{1/2} = R_{1p} - T$$

$$R_2 = \left[(X - X_2)^2 + (Y - Y_2)^2 + (Z - Z_2)^2\right]^{1/2} = R_{2p} - T$$

$$R_3 = \left[(X - X_3)^2 + (Y - Y_3)^2 + (Z - Z_3)^2\right]^{1/2} = R_{3p} - T$$

$$R_4 = \left[(X - X_4)^2 + (Y - Y_4)^2 + (Z - Z_4)^2\right]^{1/2} = R_{4p} - T$$

Navigation equations

R_{1p} to R_{4p}: the pseudo-range measurements

R_1 to R_4 : the actual range from the *i* th (i =1, 2, 3, 4) satellite

$T = c\Delta T$: the clock offset range equivalent

- Applying Taylor's series expansion (linearization) to derive solution of navigation equations:

$$R_i \approx \left[(X_u - X_i)^2 + (Y_u - Y_i)^2 + (Z_u - Z_i)^2\right]^{1/2} + h_{iX}\Delta X + h_{iY}\Delta Y + h_{iZ}\Delta Z$$

$$R_{iu} = \left[(X_u - X_i)^2 + (Y_u - Y_i)^2 + (Z_u - Z_i)^2\right]^{1/2}$$

$$R_{ip} - T_u - R_{iu} = \Delta R_i = h_{iX}\Delta X + h_{iY}\Delta Y + h_{iZ}\Delta Z + \delta T$$

where (X_u, Y_u, Z_u) and T_u are estimates about the user's position coordinates and clock offset. Without any *a priori* knowledge, the initial estimates can be (0,0,0) /the Earth's center and 0/ there is no clock offset.

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- Partial differentials of R_i :

$$h_{iX} = \frac{\partial R_i}{\partial X} \;;\; h_{iY} = \frac{\partial R_i}{\partial Y} \;;\; h_{iZ} = \frac{\partial R_i}{\partial Z}$$

$$h_{iX} = (X_u - X_i)/R_{iu}$$

$$h_{iY} = (Y_u - Y_i)/R_{iu}$$

$$h_{iZ} = (Z_u - Z_i)/R_{iu}$$

- $(\Delta X, \Delta Y, \Delta Z)$ and δT are the corrections to the estimated values in iterations of the estimation. The process of iterations is repeated until the correction values become negligible:

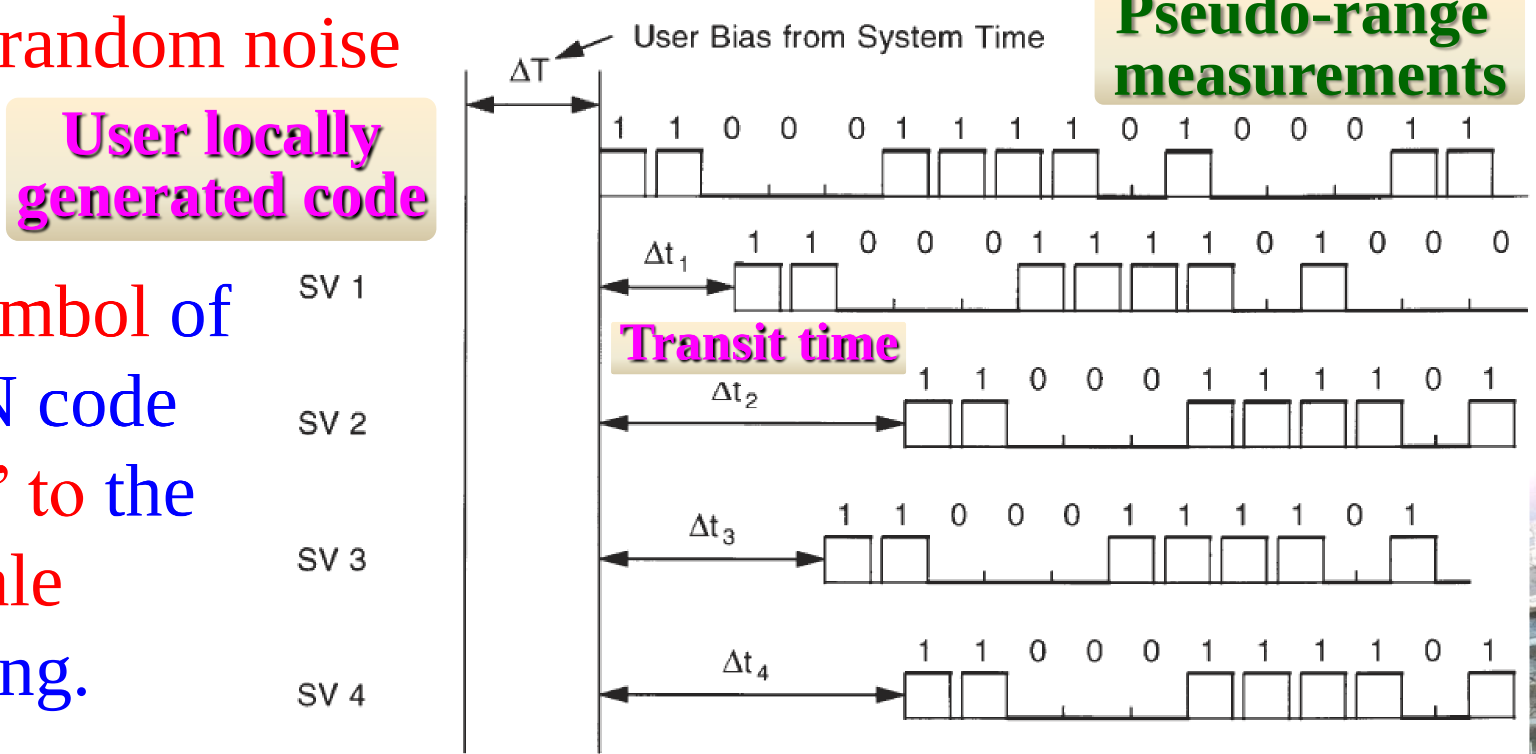
$$X = X_u + \Delta X;\; Y = Y_u + \Delta Y;\; Z = Z_u + \Delta Z;\; T = T_u + \delta t$$

- The linear matrix equation is solved to yield ΔX, ΔY, ΔZ, δT in the iterations:

$$\begin{bmatrix} \Delta R_1 \\ \Delta R_2 \\ \Delta R_3 \\ \Delta R_4 \end{bmatrix} = \begin{bmatrix} h_{1X} & h_{1Y} & h_{1Z} & 1 \\ h_{2X} & h_{2Y} & h_{2Z} & 1 \\ h_{3X} & h_{3Y} & h_{3Z} & 1 \\ h_{4X} & h_{4Y} & h_{4Z} & 1 \end{bmatrix} \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \\ \delta T \end{bmatrix}$$

ΔR_1 to ΔR_4: the range errors between the pseudo-range measurements and the estimated range which dependent on the estimated user coordinates and clock offset range equivalent.

- The SNS signal is continuous carrier wave signal modulated by distance-measuring code, also known as pseudo-random noise (PRN) code. Each symbol of the PRN code is "tied" to the time scale of ranging.



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- For the purpose of measuring transit time (i.e. pseudo-range), the time scale is created in the user's receiver and synchronized with the time scale of the satellites (assumed perfect satellite clocks). In the receiver, a *correlator* device processes a signal from a satellite along with noise: element-by-element comparing implementations of the received signal with a signal copy formed in the receiver itself, i.e. computing their mutual correlation (getting the maximum value if the two PRN code sequences are perfectly aligned).

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- The SNS accuracy of position fixing depends upon accuracy of measuring the range to the satellite. The accuracy of measuring range is greatly affected by an unknown delay and phase shift of satellite signal propagating through the ionosphere.

SNS service provision is about 15m accuracy

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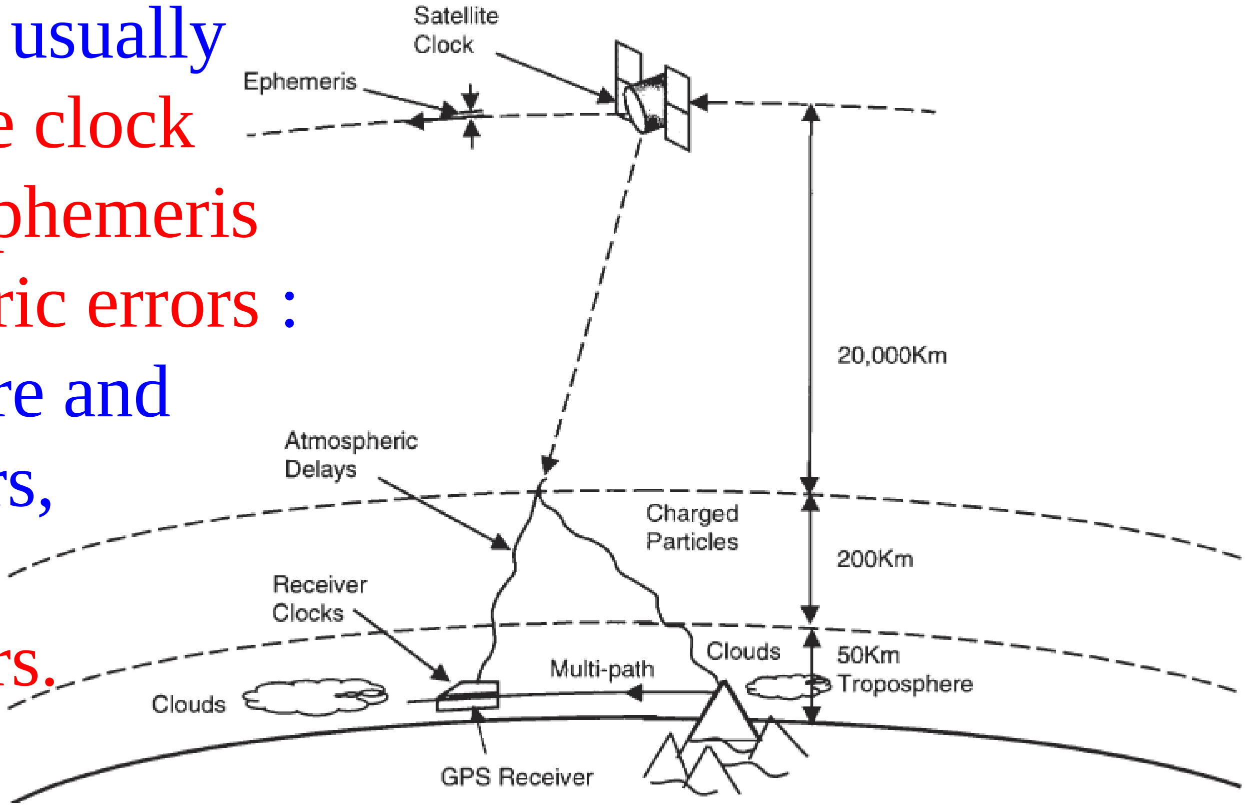
- The errors in the pseudo-range measurements can then be derived. They usually involves: satellite clock errors, satellite ephemeris errors, atmospheric errors : mainly ionosphere and troposphere errors, and multi-path propagation errors.



SNS error sources

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- The required corrections to the measurement errors can be computed and transmitted to the user's receiver over a radio link by ground based reference station or by geostationary orbit satellites. With the increase in accuracy, SNS will be sufficient accurate for the aviation application.

SNS accuracy improvement to about 2m

The end of *Radio Navigation*