第10章 含有耦合电感的电路

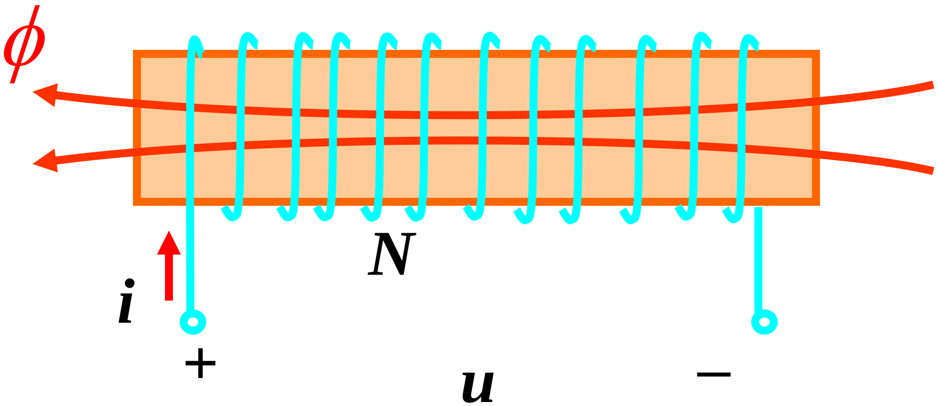
本章重点

1.互感和互感电压

2.有互感电路的计算

3.空心变压器和理想变压器

1. 互感



单独的一个线圈

通入电流→产生磁通 Φ

磁链 $\psi = N\Phi = Li$

感应电压： $u = \dfrac{\mathrm{d}\psi}{\mathrm{d}t} = L\dfrac{\mathrm{d}i}{\mathrm{d}t}$

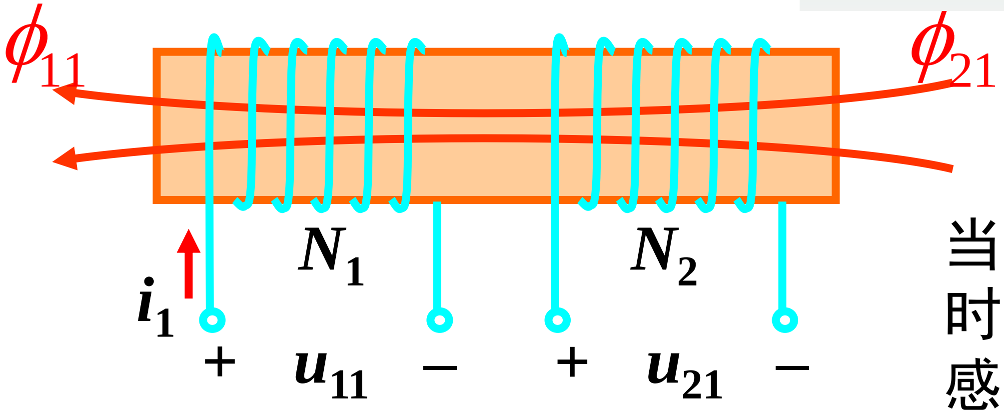
线性电感元件,电压是由自身线圈电流产生的,故叫自感电压，L称为自感系数。

10.1 互 感

载流线圈之间通过彼此的磁场相互联系的物理现象称为磁耦合。
线圈1所产生的磁通通过线圈2的部分称为互感磁通，未穿过线圈2的部分为漏磁通。

1. 互感

- 如果在该 轴芯上再绕一个线圈



$$\psi_{21} = N_2\phi_{21}$$

当周围空间是各向同性线性磁介质时，每一种磁通链都与产生它的施感电流成正比，即有自感磁通链和互感磁通链

$$\psi_{11} = N_1\phi_{11}$$

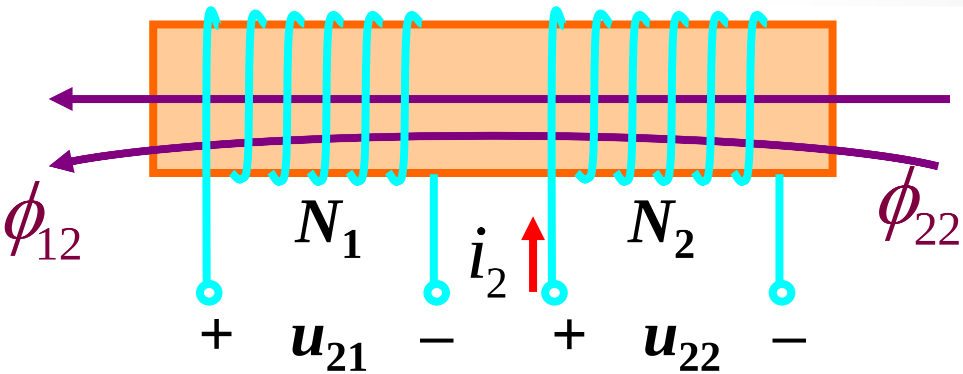
$$\psi_{11} = L_1 i_1$$

$$L_1 = \frac{\psi_{11}}{i_1}$$

互感系数 $M_{21} = \dfrac{\psi_{21}}{i_1}$

双下标的含义

若线圈2通电流



$$\psi_{22} = N_2\phi_{22}$$

$$\psi_{22} = L_2 i_2$$

则线圈1中同样存在互感磁链

$$\psi_{12} = N_1\phi_{12}$$

互感系数 $M_{12} = \dfrac{\psi_{12}}{i_2}$

$M_{12} = M_{21} = M$ 　互感系数，单位亨(H)。

注意： **M值与线圈的形状、匝数、几何位置、空间媒质物理性质有关，满足M_{12}=M_{21}。**

2. 耦合系数　　两个线圈磁耦合的紧密程度。

$$k=\sqrt{\frac{\psi_{21}\psi_{12}}{\psi_{11}\psi_{22}}}=\sqrt{\frac{(N_1\phi_{12})(N_2\phi_{21})}{(N_1\phi_{11})(N_2\phi_{22})}}\leq 1 \quad \begin{pmatrix} \phi_{12}\leq\phi_{22} \\ \phi_{21}\leq\phi_{11} \end{pmatrix}$$

$$\boldsymbol{k \overset{\text{def}}{=} \frac{M}{\sqrt{L_1 L_2}} \leq 1}$$

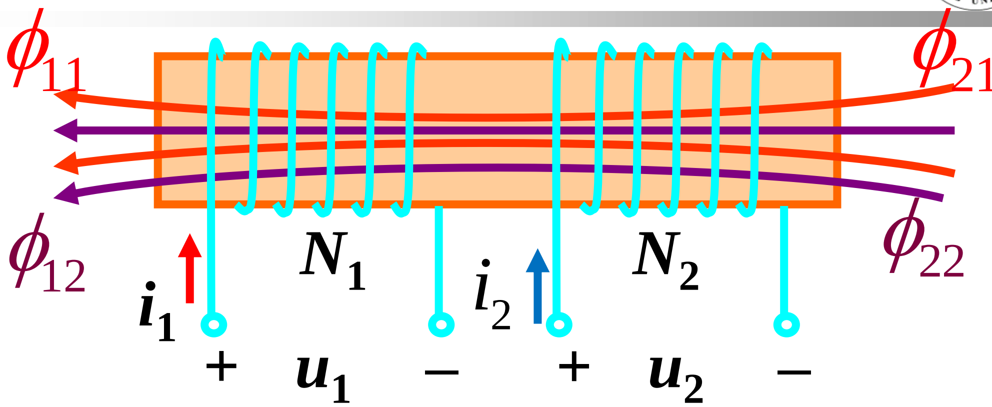
$$\text{相关因素}\begin{cases}\text{线圈的结构}\\ \text{相互几何位置}\\ \text{空间磁介质}\end{cases}$$

k＝1　　全耦合

k≈1　　紧耦合

k小　　松耦合

3. 耦合电感上的电压、电流关系



两个线圈都注入电流

$$\phi_1 = \phi_{11} + \phi_{12}$$

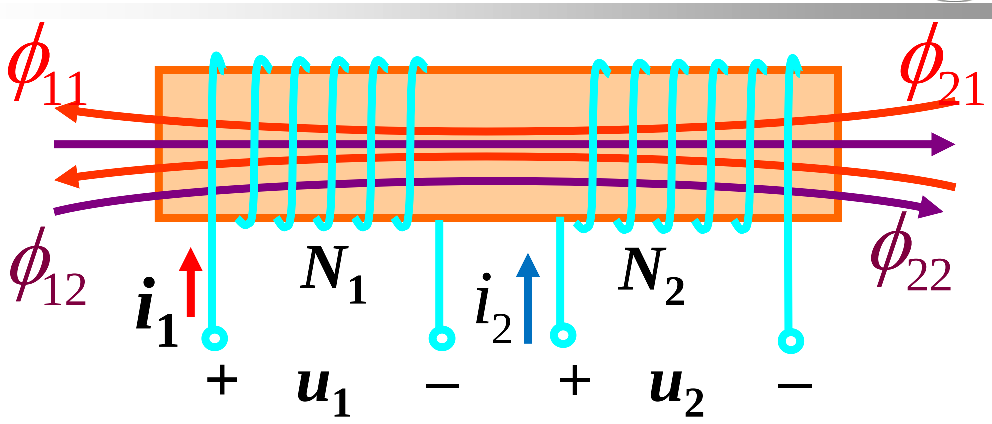
$$\psi_1 = N_1\phi_1 = N_1\phi_{11} + N_1\phi_{12} = \psi_{11} + \psi_{12}$$

$\psi_1 = L_1 i_1 + M i_2$　　**同理**　　$\psi_2 = M i_1 + L_2 i_2$

$$u_1 = \frac{\mathrm{d}\psi_1}{\mathrm{d}t} = L_1\frac{\mathrm{d}i_1}{\mathrm{d}t} + M\frac{\mathrm{d}i_2}{\mathrm{d}t}$$

$$u_2 = \frac{\mathrm{d}\psi_2}{\mathrm{d}t} = M\frac{\mathrm{d}i_1}{\mathrm{d}t} + L_2\frac{\mathrm{d}i_2}{\mathrm{d}t}$$

把线圈2绕线方向变一下



$$\phi_1 = \phi_{11} - \phi_{12}$$

$$\psi_1 = N_1\phi_1 = N_1\phi_{11} - N_1\phi_{12} = \psi_{11} - \psi_{12}$$

$$\psi_1 = L_1 i_1 - M i_2$$

同理 $\psi_2 = -M i_1 + L_2 i_2$

$$u_1 = \frac{\mathrm{d}\psi_1}{\mathrm{d}t} = L_1 \frac{\mathrm{d}i_1}{\mathrm{d}t} - M\frac{\mathrm{d}i_2}{\mathrm{d}t}$$

$$u_2 = \frac{\mathrm{d}\psi_2}{\mathrm{d}t} = -M \frac{\mathrm{d}i_1}{\mathrm{d}t} + L_2\frac{\mathrm{d}i_2}{\mathrm{d}t}$$

互感电压符号原则：

两线圈的自磁链和互磁链方向相同，互感电压取正，否则取负。

电流的参考方向
线圈的相对位置和绕向有关

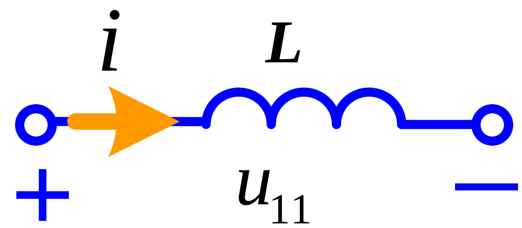
$$\begin{cases} u_1 = u_{11} + u_{12} = L_1 \dfrac{\mathrm{d}i_1}{\mathrm{d}t} \pm M \dfrac{\mathrm{d}i_2}{\mathrm{d}t} \\ u_2 = u_{21} + u_{22} = \pm M \dfrac{\mathrm{d}i_1}{\mathrm{d}t} + L_2 \dfrac{\mathrm{d}i_2}{\mathrm{d}t} \end{cases}$$

$$\begin{cases} \dot{U}_1 = \mathrm{j}\omega L_1 \dot{I}_1 \pm \mathrm{j}\omega M \dot{I}_2 \\ \dot{U}_2 = \pm \mathrm{j}\omega M \dot{I}_1 + \mathrm{j}\omega L_2 \dot{I}_2 \end{cases}$$

4.互感线圈的同名端

自感电压，当u，i 取关联参考方向，u、i与ϕ 符合右螺旋定则，其表达式为

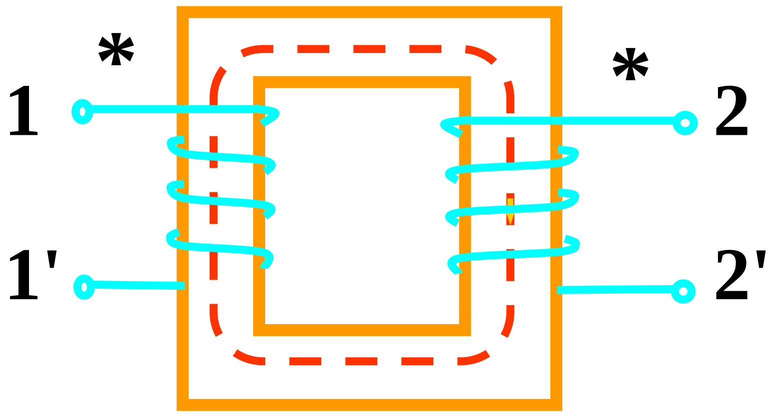
$$u_{11} = \frac{\mathrm{d}\Psi_{11}}{\mathrm{d}t} = N_1 \frac{\mathrm{d}\phi_{11}}{\mathrm{d}t} = L_1 \frac{\mathrm{d}i_1}{\mathrm{d}t}$$

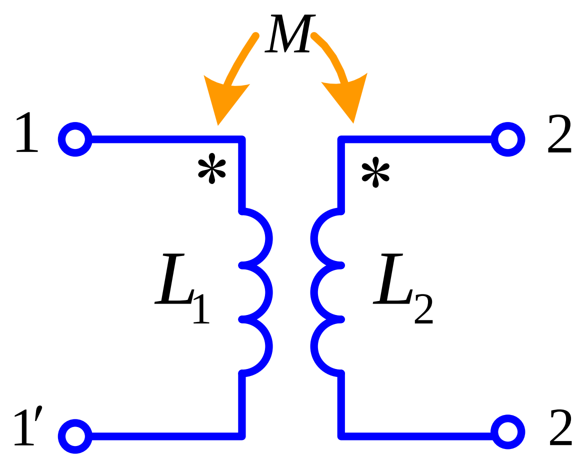


★互感电压极性与绕向、电流参考方向有关：线圈一定则绕向一定，则互感电压极性与施感电流参考方向存在一一对应关系。为体现该关系则引入同名端的概念。

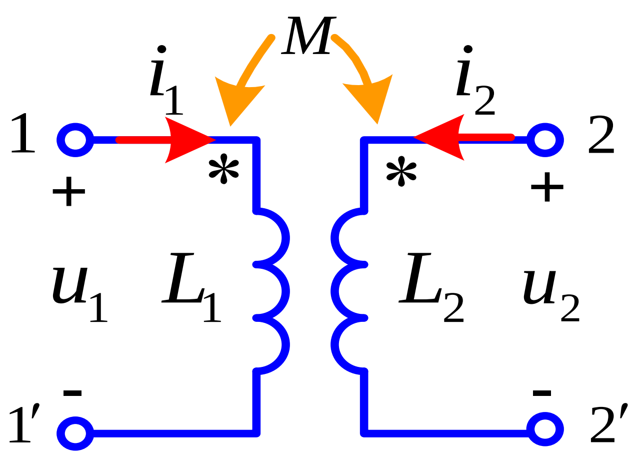
同名端

施感电流的流入端(进端)同互感电压正极性端称为同名端用符号“●”或“△”或“*"表示。



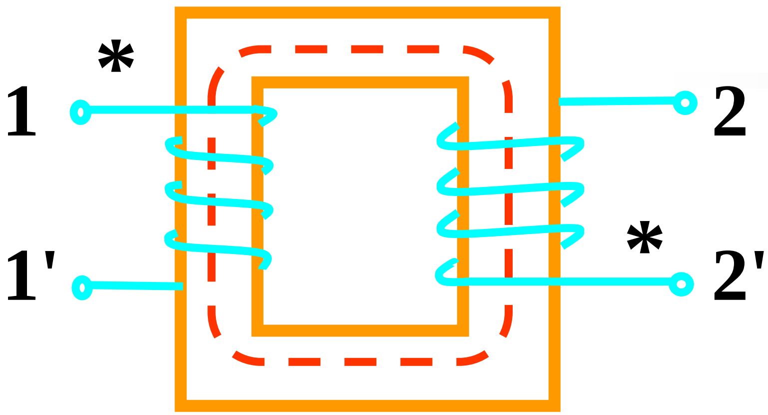


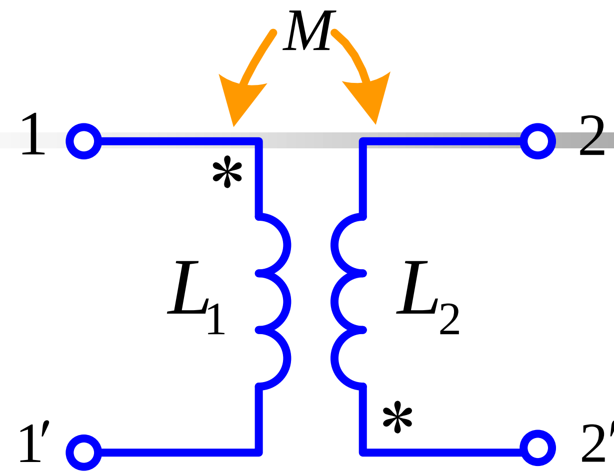
电路符号

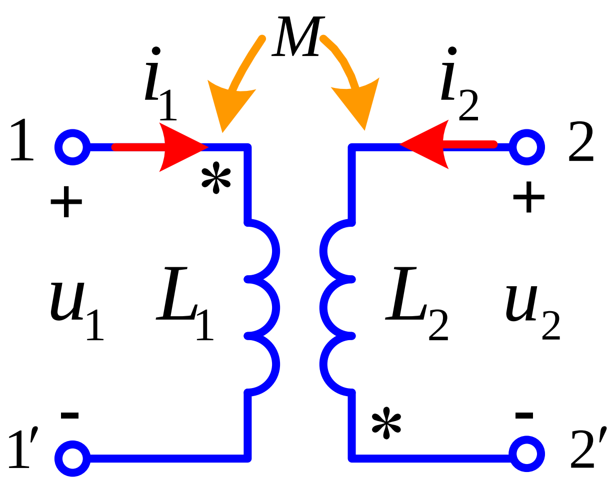


$$u_1 = L_1 \frac{\mathrm{d}\, i_1}{\mathrm{d}\, t} + M \frac{\mathrm{d}\, i_2}{\mathrm{d}\, t}$$

$$u_2 = M \frac{\mathrm{d}\, i_1}{\mathrm{d}\, t} + L_2 \frac{\mathrm{d}\, i_2}{\mathrm{d}\, t}$$







$$u_1 = L_1 \frac{\mathrm{d}\, i_1}{\mathrm{d}\, t} - M \frac{\mathrm{d}\, i_2}{\mathrm{d}\, t}$$

$$u_2 = -M \frac{\mathrm{d}\, i_1}{\mathrm{d}\, t} + L_2 \frac{\mathrm{d}\, i_2}{\mathrm{d}\, t}$$

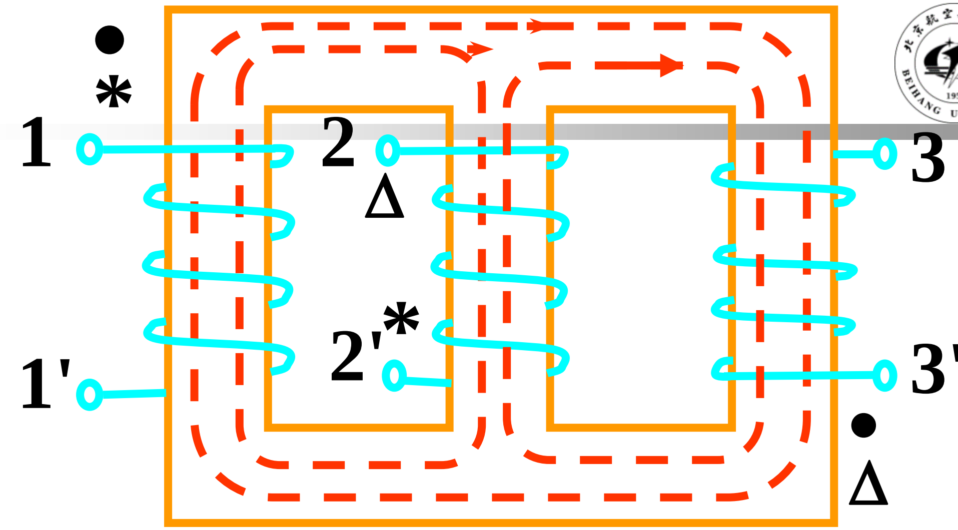
正弦稳态电路

$$\dot{U}_1 = \mathrm{j}\omega L_1 \dot{I}_1 \pm \mathrm{j}\omega M \dot{I}_2$$

$$\dot{U}_2 = \pm \mathrm{j}\omega M \dot{I}_1 + \mathrm{j}\omega L_2 \dot{I}_2$$

确定同名端的方法：

实验的方法测耦合线圈的同名端:

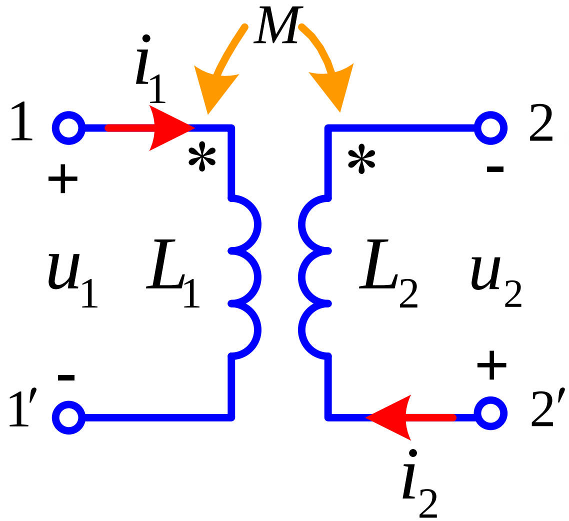


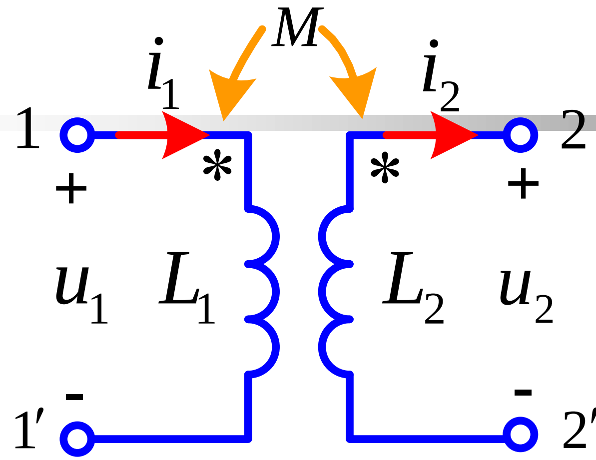
(1) 当随时间增大的时变电流从一线圈的一端流入时，将会引起另一线圈相应同名端的电位升高。

(2) 当两个线圈中电流同时由同名端流入(或流出)时，两个电流产生的磁场相互增强。

(3) 线圈的同名端必须两两确定。

写出图示电路电压电流关系式





$$u_1 = L_1 \frac{\mathrm{d} i_1}{\mathrm{d} t} - M \frac{\mathrm{d} i_2}{\mathrm{d} t}$$

$$u_2 = -M \frac{\mathrm{d} i_1}{\mathrm{d} t} + L_2 \frac{\mathrm{d} i_2}{\mathrm{d} t}$$

$$u_1 = L_1 \frac{\mathrm{d} i_1}{\mathrm{d} t} - M \frac{\mathrm{d} i_2}{\mathrm{d} t}$$

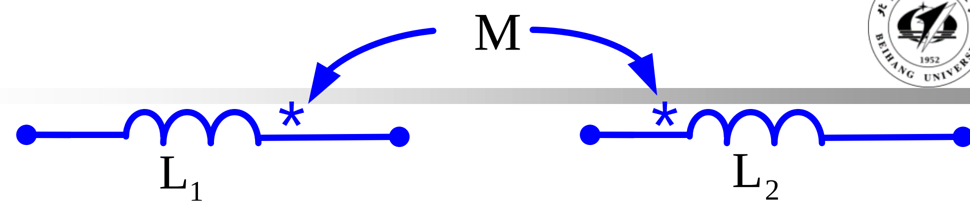
$$u_2 = M \frac{\mathrm{d} i_1}{\mathrm{d} t} - L_2 \frac{\mathrm{d} i_2}{\mathrm{d} t}$$

互感电压的方向 $\left\{ \begin{array}{l} \text{同名端} \\ \text{产生它的电流} \end{array} \right.$

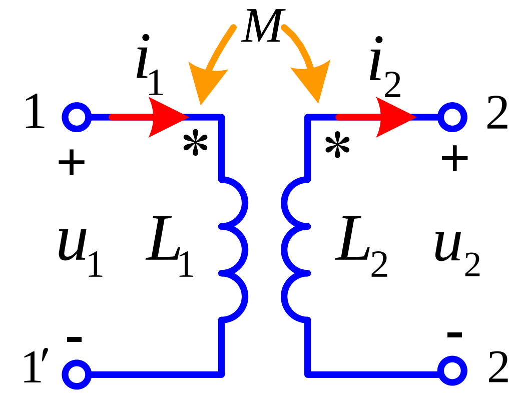
一端电流从*端流入，另端互感电压*端为正

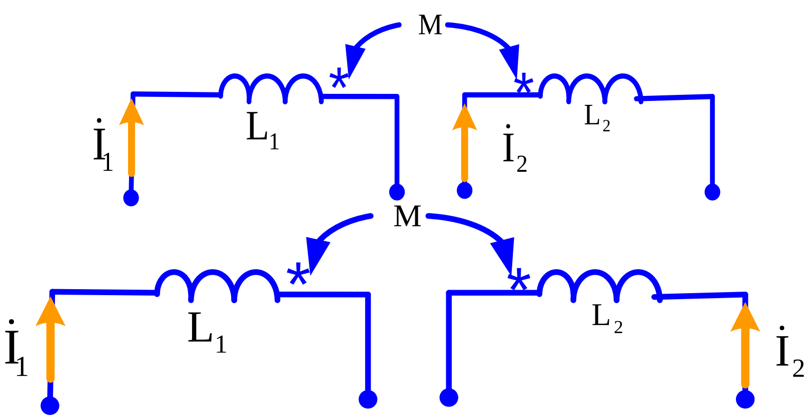
互感前正负号由施感电流方向与互感电压参考方向有关。同名端时取正,否则取负。

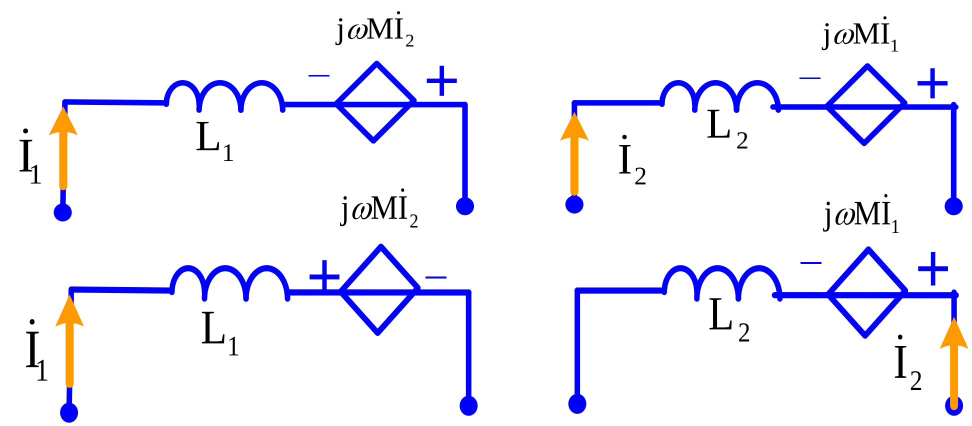
五. 耦合电感的电路符号



- 互感电压的作用可用受控源来表示
  - **CCVS**
  - 受控源极性与施感电流方向有关
  - 受控源极性与同名端有关

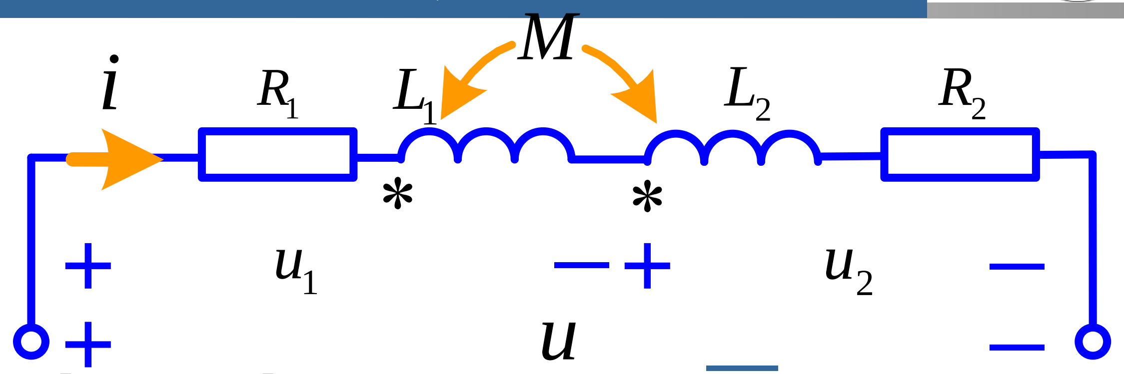






1. 耦合电感的串联

（1）顺接串联



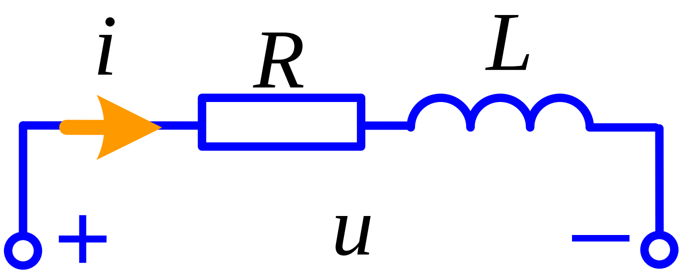
$$u = R_1 i + L_1 \frac{\mathrm{d}i}{\mathrm{d}t} + M\frac{\mathrm{d}i}{\mathrm{d}t} + L_2\frac{\mathrm{d}i}{\mathrm{d}t} + M\frac{\mathrm{d}i}{\mathrm{d}t} + R_2 i$$

$$= (R_1 + R_2)i + (L_1 + L_2 + 2M)\frac{\mathrm{d}i}{\mathrm{d}t}$$

$$= Ri + L\frac{\mathrm{d}i}{\mathrm{d}t}$$

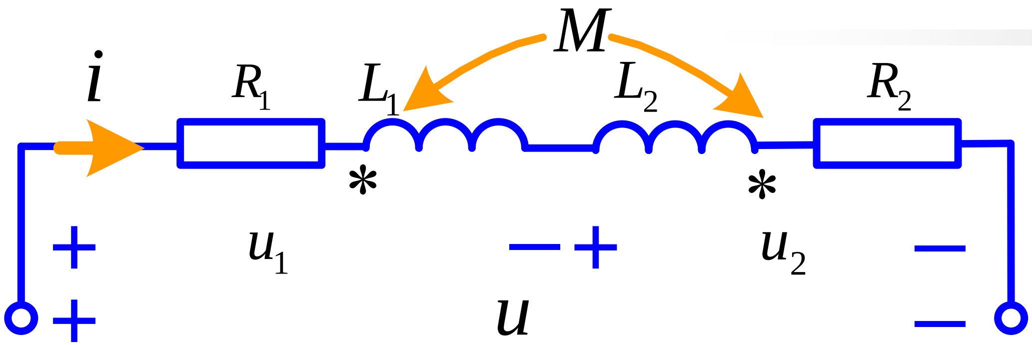
$$R = R_1 + R_2$$

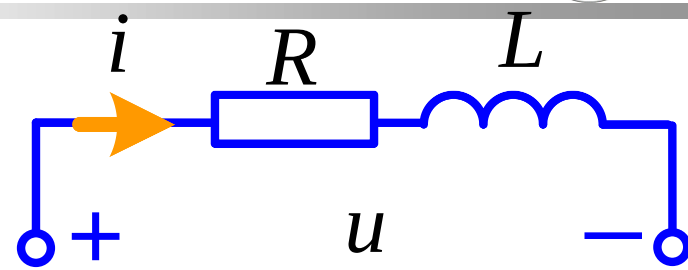
$$L = L_1 + L_2 + 2M$$



去耦等效电路

（2） 反接串联





$$u = R_1 i + L_1\frac{\mathrm{d}i}{\mathrm{d}t} - M\frac{\mathrm{d}i}{\mathrm{d}t} + L_2\frac{\mathrm{d}i}{\mathrm{d}t} - M\frac{\mathrm{d}i}{\mathrm{d}t} + R_2 i$$

$$= (R_1 + R_2)i + (L_1 + L_2 - 2M)\frac{\mathrm{d}i}{\mathrm{d}t} = Ri + L\frac{\mathrm{d}i}{\mathrm{d}t}$$

$$R = R_1 + R_2$$

$$L = L_1 + L_2 - 2M$$

$$M \le \frac{1}{2}(L_1 + L_2)$$

互感不大于两个自感的算术平均值。

$$L = L_1 + L_2 - 2M \ge 0$$

互感系数的测量方法：

顺接一次，反接一次，可测互感系数：

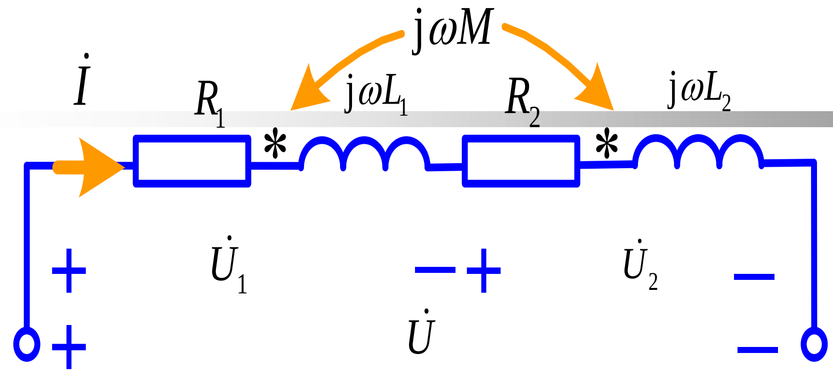
$$M=\frac{L_{顺}-L_{反}}{4}$$

全耦合时 $M=\sqrt{L_1L_2}$

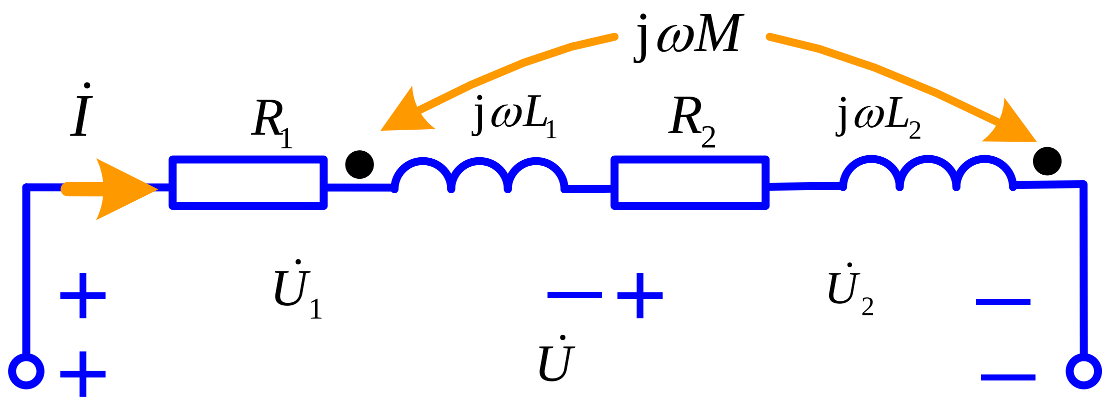
$$L=L_1+L_2\pm 2M=L_1+L_2\pm 2\sqrt{L_1L_2}$$

$$=(\sqrt{L_1}\pm\sqrt{L_2})^2$$

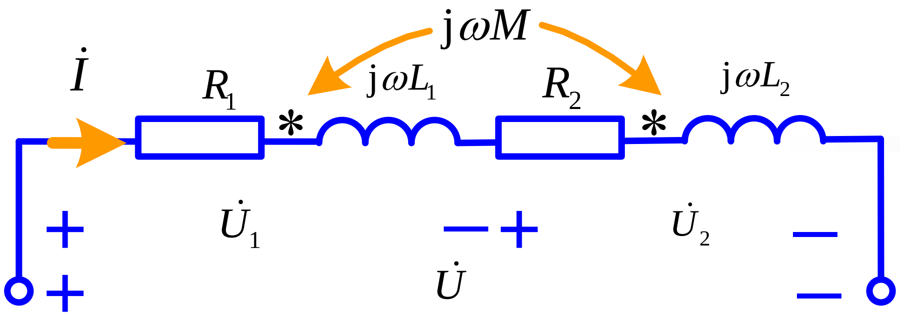
在正弦激励下：



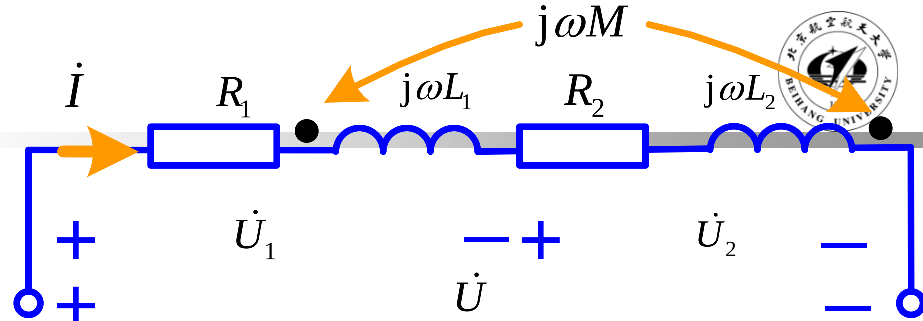
$$\dot{U}=(R_1+R_2)\dot{I}+j\omega(L_1+L_2+2M)\dot{I}$$



$$\dot{U}=(R_1+R_2)\dot{I}+j\omega(L_1+L_2-2M)\dot{I}$$



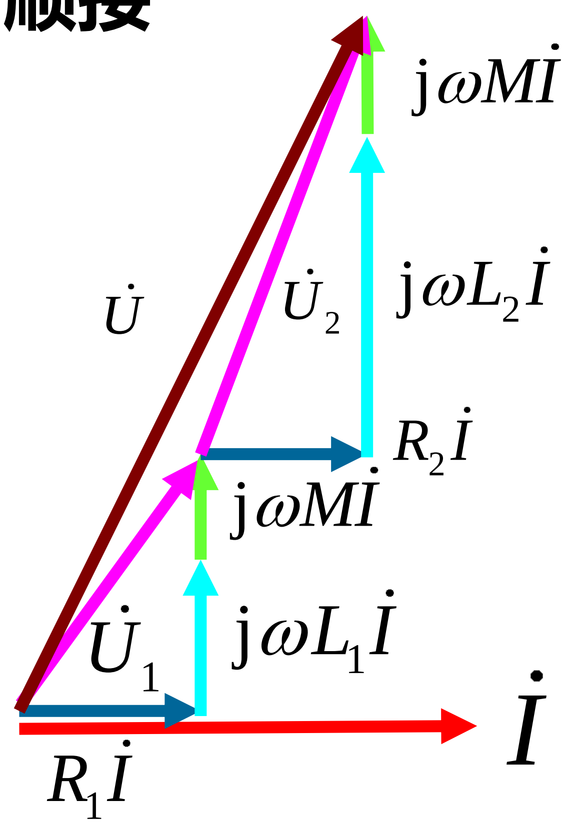
jωM
İ
R1
jωL1
R2
jωL2
U̇1
U̇2
U̇



jωM
İ
R1
jωL1
R2
jωL2
U̇1
U̇2
U̇

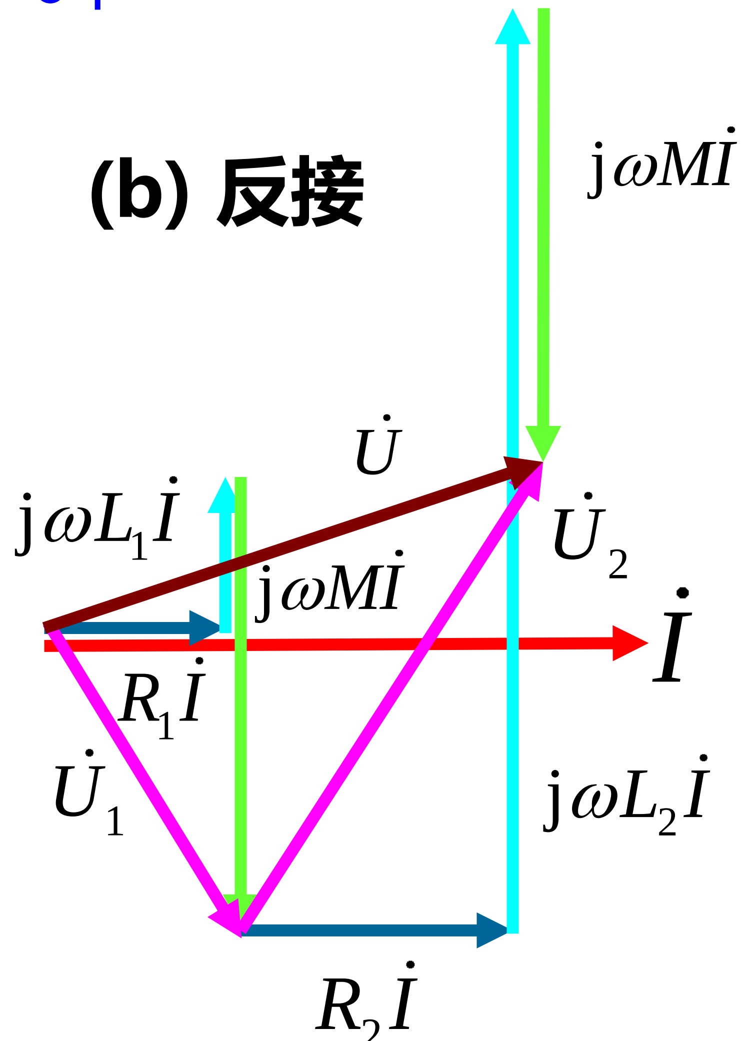
相量图：

(a) 顺接



jωMİ
jωL2İ
U̇2
U̇
R2İ
jωMİ
U̇1
jωL1İ
İ
R1İ

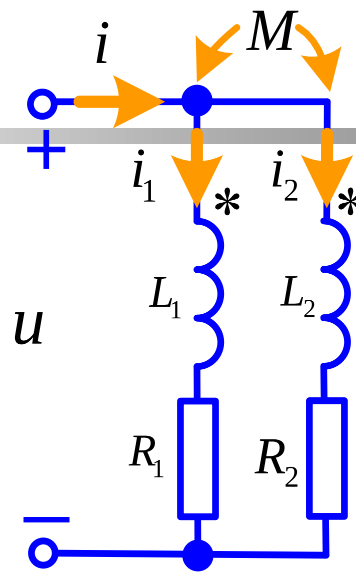
(b) 反接



jωMİ
U̇
jωL1İ
U̇2
jωMİ
İ
R1İ
U̇1
jωL2İ
R2İ

2. 耦合电感的并联

(1) 同侧并联



$$\begin{cases} u = L_1 \dfrac{\mathrm{d}i_1}{\mathrm{d}t} + M \dfrac{\mathrm{d}i_2}{\mathrm{d}t} + R_1 i_1 \\ u = L_2 \dfrac{\mathrm{d}i_2}{\mathrm{d}t} + M \dfrac{\mathrm{d}i_1}{\mathrm{d}t} + R_2 i_2 \end{cases}$$

$$i = i_1 + i_2$$

$$\begin{cases} \dot{U} = \mathrm{j}\omega L_1 \dot{I}_1 + \mathrm{j}\omega M \dot{I}_2 + R_1 \dot{I}_1 \\ \dot{U} = \mathrm{j}\omega L_2 \dot{I}_2 + \mathrm{j}\omega M \dot{I}_1 + R_2 \dot{I}_2 \end{cases}$$

$$Z_1 = j\omega L_1 + R_1$$

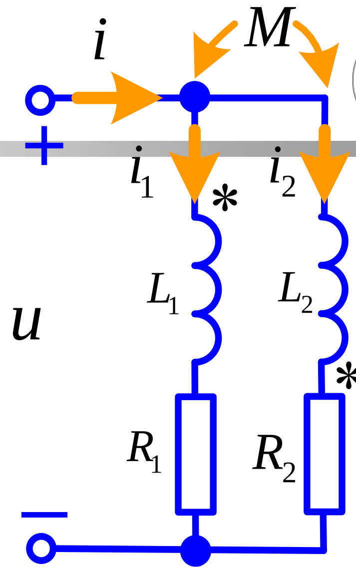
$$Z_2 = j\omega L_2 + R_2$$

$$Z_m = j\omega M$$

$$\dot{I} = \dot{I}_1 + \dot{I}_2 = \dot{U} \frac{Z_1 + Z_2 - 2Z_m}{Z_1 Z_2 - Z_m^{\,2}}$$

等效阻抗 $Z_{eq} = \dfrac{\dot{U}}{\dot{I}} = \dfrac{Z_1 Z_2 - Z_m^{\,2}}{Z_1 + Z_2 - 2Z_m}$

(2) 异侧并联



$$
\begin{cases}
u = L_1 \dfrac{\mathrm{d}i_1}{\mathrm{d}t} - M \dfrac{\mathrm{d}i_2}{\mathrm{d}t} + R_1 i_1 \\
u = L_2 \dfrac{\mathrm{d}i_2}{\mathrm{d}t} - M \dfrac{\mathrm{d}i_1}{\mathrm{d}t} + R_2 i_2
\end{cases}
$$

$$i = i_1 + i_2$$

$$
\begin{cases}
\dot{U} = \mathrm{j}\omega L_1 \dot{I}_1 - \mathrm{j}\omega M \dot{I}_2 + R_1 \dot{I}_1 \\
\dot{U} = \mathrm{j}\omega L_2 \dot{I}_2 - \mathrm{j}\omega M \dot{I}_1 + R_2 \dot{I}_2
\end{cases}
$$

$$Z_1 = j\omega L_1 + R_1$$

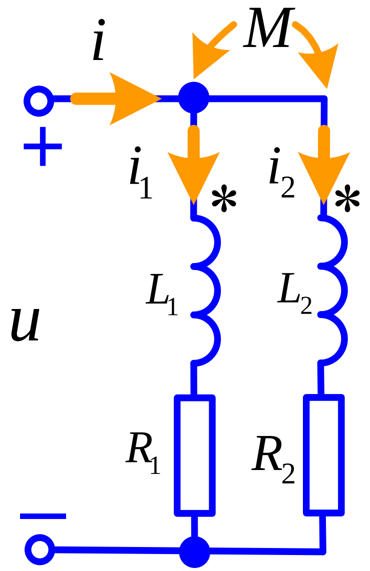
$$Z_2 = j\omega L_2 + R_2$$

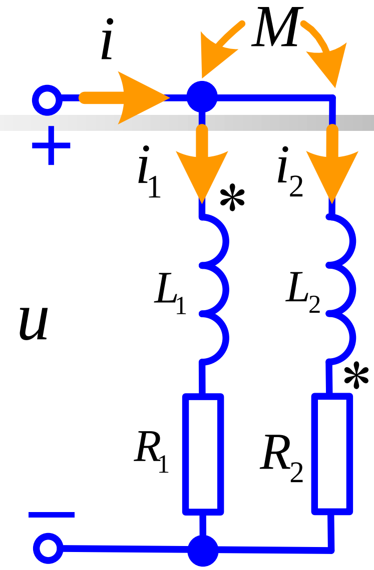
$$Z_m = j\omega M$$

$$\dot{I} = \dot{I}_1 + \dot{I}_2 = \dot{I}_1 = \dot{U} \frac{Z_1 + Z_2 + 2Z_m}{Z_1 Z_2 - Z_m^2}$$

等效阻抗

$$Z_{eq} = \frac{\dot{U}}{\dot{I}} = \frac{Z_1 Z_2 - Z_m^2}{Z_1 + Z_2 + 2Z_m}$$





$$Z_{eq}=\frac{\dot{U}}{\dot{I}}=\frac{Z_1Z_2-Z_m^2}{Z_1+Z_2-2Z_m}$$

$$Z_{eq}=\frac{\dot{U}}{\dot{I}}=\frac{Z_1Z_2-Z_m^2}{Z_1+Z_2+2Z_m}$$

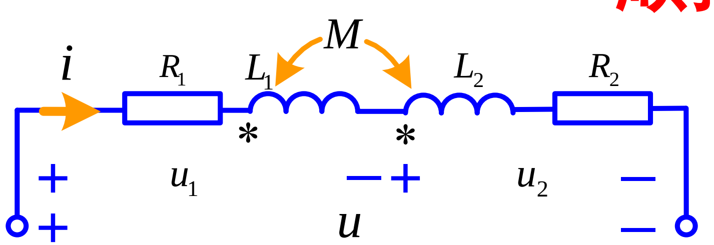
R1=R2=0时 $$L_{eq}=\frac{L_1L_2-M^2}{L_1+L_2\mp 2M}$$

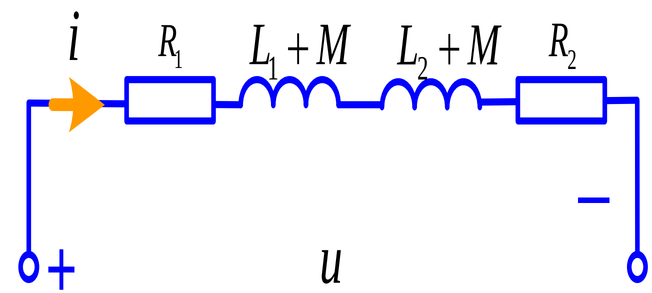
3.互感消去法

两个耦合电感有一端相联接时，可把具有互感的电路快速转化为等效的无互感的电路。

(1) 串联

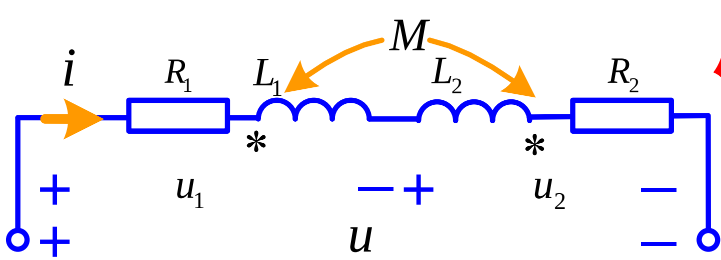
顺接

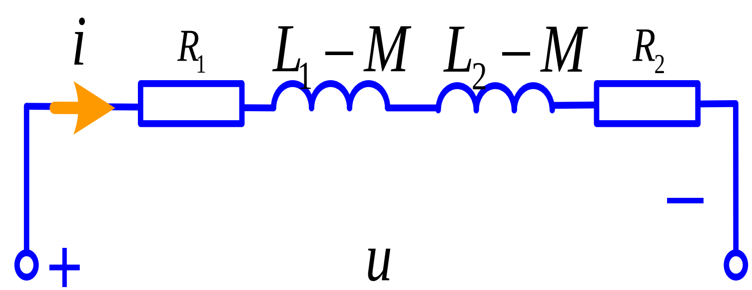




$$u = (R_1 + R_2)i + (L_1 + L_2 + 2M)\frac{\mathrm{d}i}{\mathrm{d}t}$$

反接



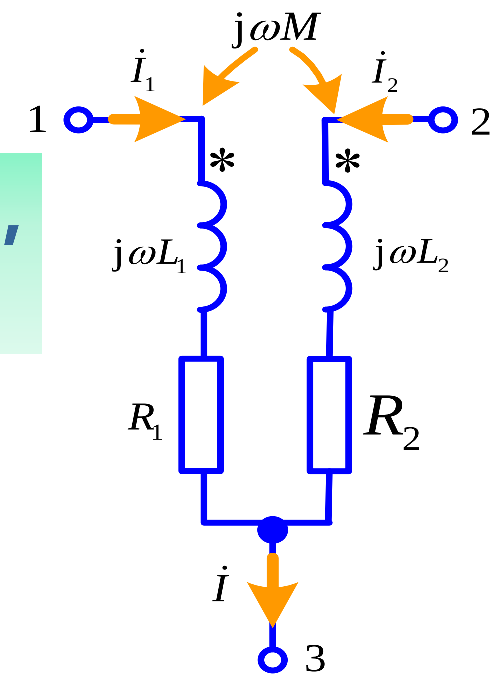


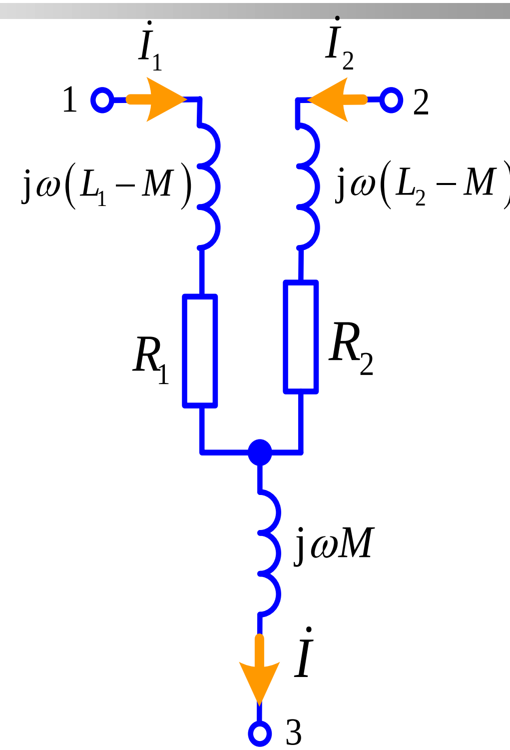
$$u = (R_1 + R_2)i + (L_1 + L_2 - 2M)\frac{\mathrm{d}i}{\mathrm{d}t}$$

3.互感消去法

(2)同名端为共端的T型去耦等效

(1、2端也共点，则为同侧并联)





同侧联接

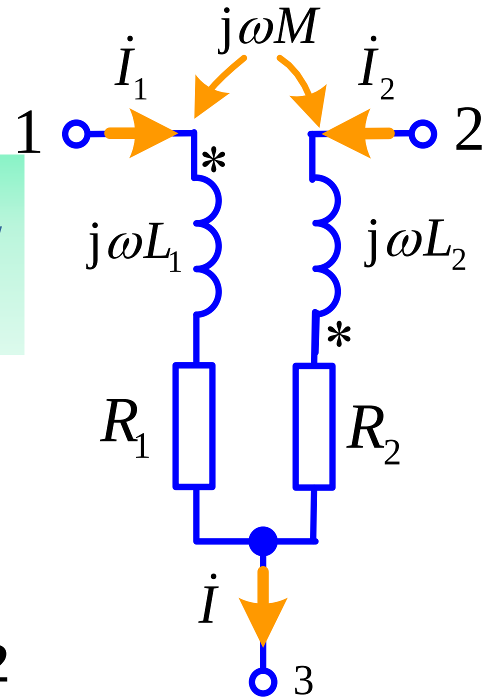
$$\dot{I}=\dot{I}_1+\dot{I}_2$$

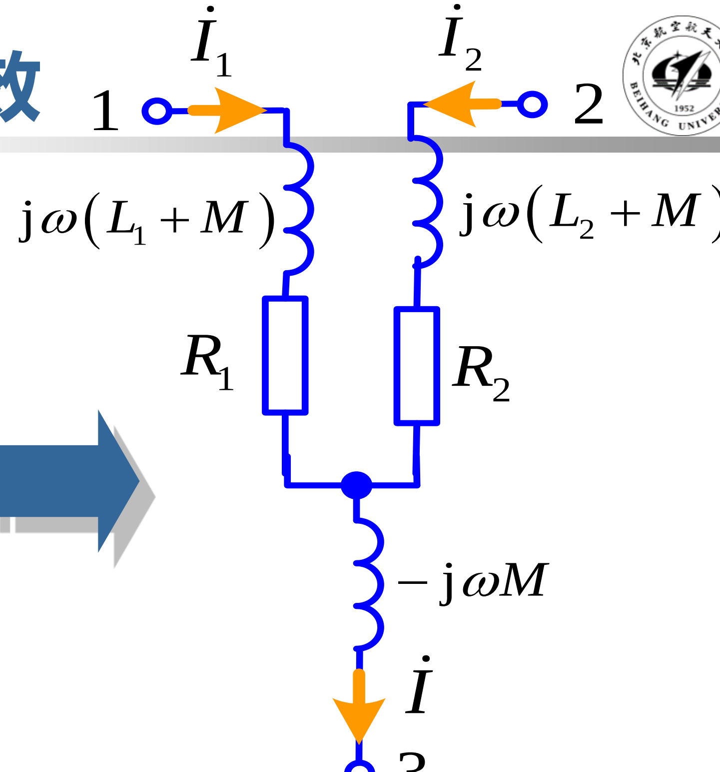
$$\dot{U}_{13}=\mathrm{j}\omega L_1\dot{I}_1+R_1\dot{I}_1+\mathrm{j}\omega M\dot{I}_2=\mathrm{j}\omega(L_1-M)\dot{I}_1+R_1\dot{I}_1+\mathrm{j}\omega M\dot{I}$$

$$\dot{U}_{23}=\mathrm{j}\omega L_2\dot{I}_2+R_2\dot{I}_2+\mathrm{j}\omega M\dot{I}_1=\mathrm{j}\omega(L_2-M)\dot{I}_2+R_2\dot{I}_2+\mathrm{j}\omega M\dot{I}$$

（3） 异名端为共端的T型去耦等效

（1、2端也共点，则为异侧并联）





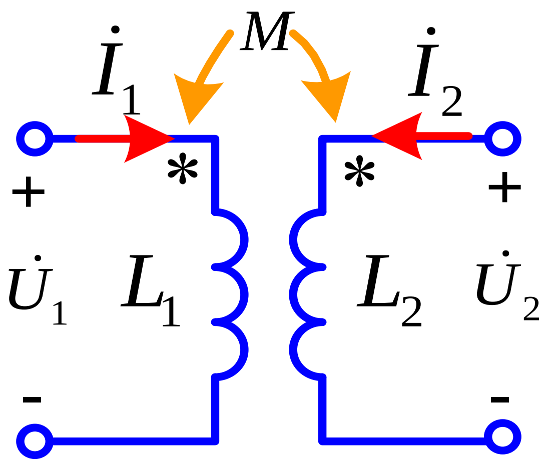
异侧联接

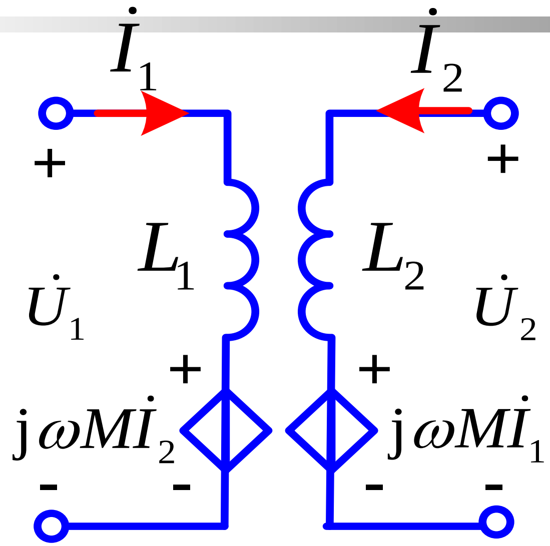
$$\dot{I} = \dot{I}_1 + \dot{I}_2$$

$$\dot{U}_{13} = j\omega L_1\dot{I}_1 + R_1\dot{I}_1 - j\omega M\dot{I}_2 = j\omega(L_1+M)\dot{I}_1 + R_1\dot{I}_1 - j\omega M\dot{I}$$

$$\dot{U}_{23} = j\omega L_2\dot{I}_2 + R_2\dot{I}_2 - j\omega M\dot{I}_1 = j\omega(L_2+M)\dot{I}_2 + R_2\dot{I}_2 - j\omega M\dot{I}$$

4. 受控源等效电路



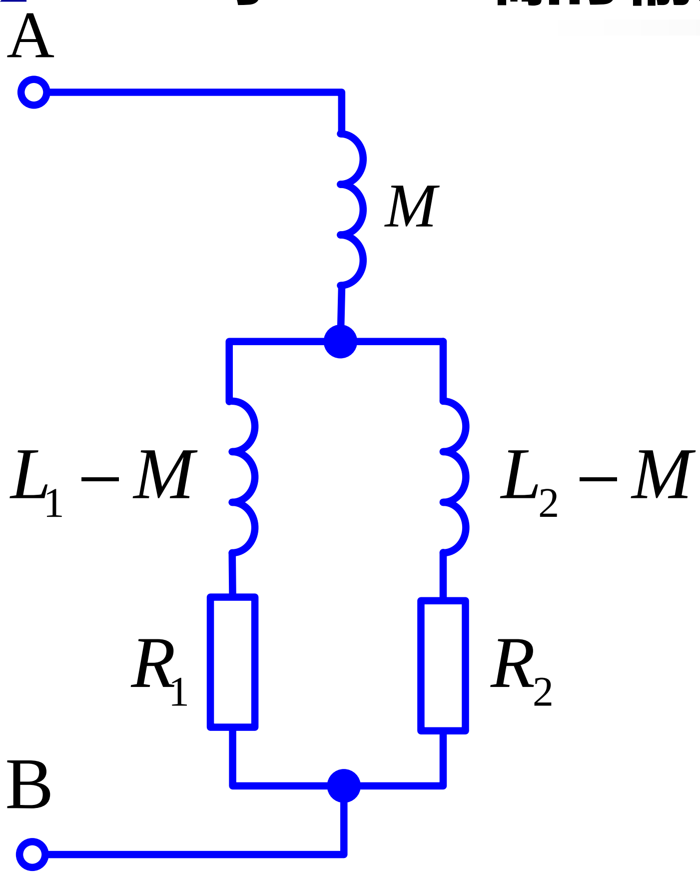


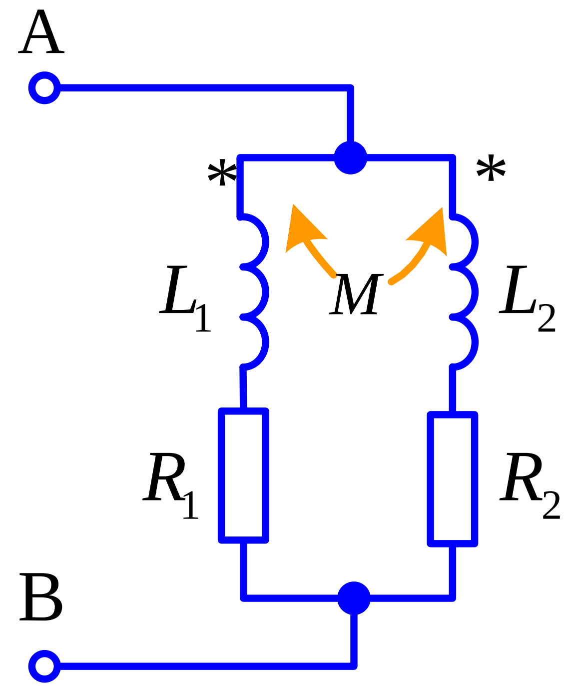
$$\dot{U}_1 = j\omega L_1\dot{I}_1 + j\omega M\dot{I}_2$$

$$\dot{U}_2 = j\omega M\dot{I}_1 + j\omega L_2\dot{I}_2$$

【例】 求：AB端的输入阻抗。

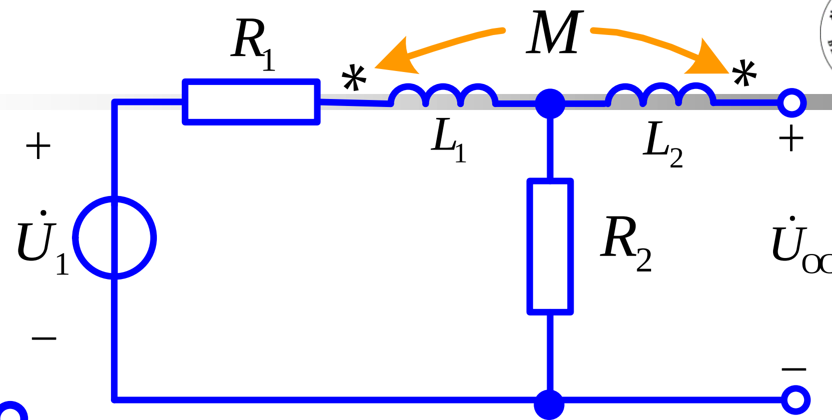
解



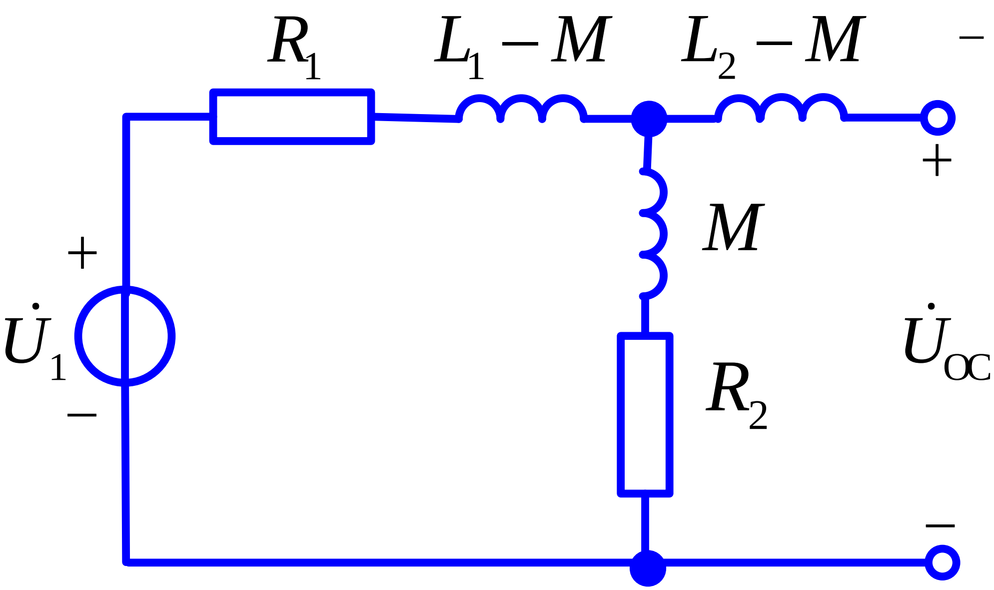


$$Z_{\text{in}} = \mathrm{j}\omega M + \frac{[R_1 + \mathrm{j}\omega(L_1 - M)][R_2 + \mathrm{j}\omega(L_2 - M)]}{R_1 + R_2 + \mathrm{j}\omega(L_1 + L_2 - 2M)}$$

【例】求开路电压 $\dot{U}_{OC}$。



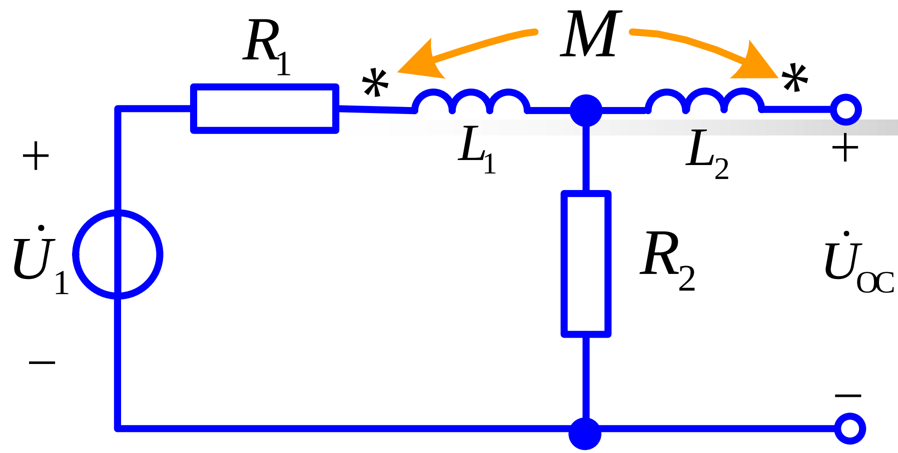
解 解法1:



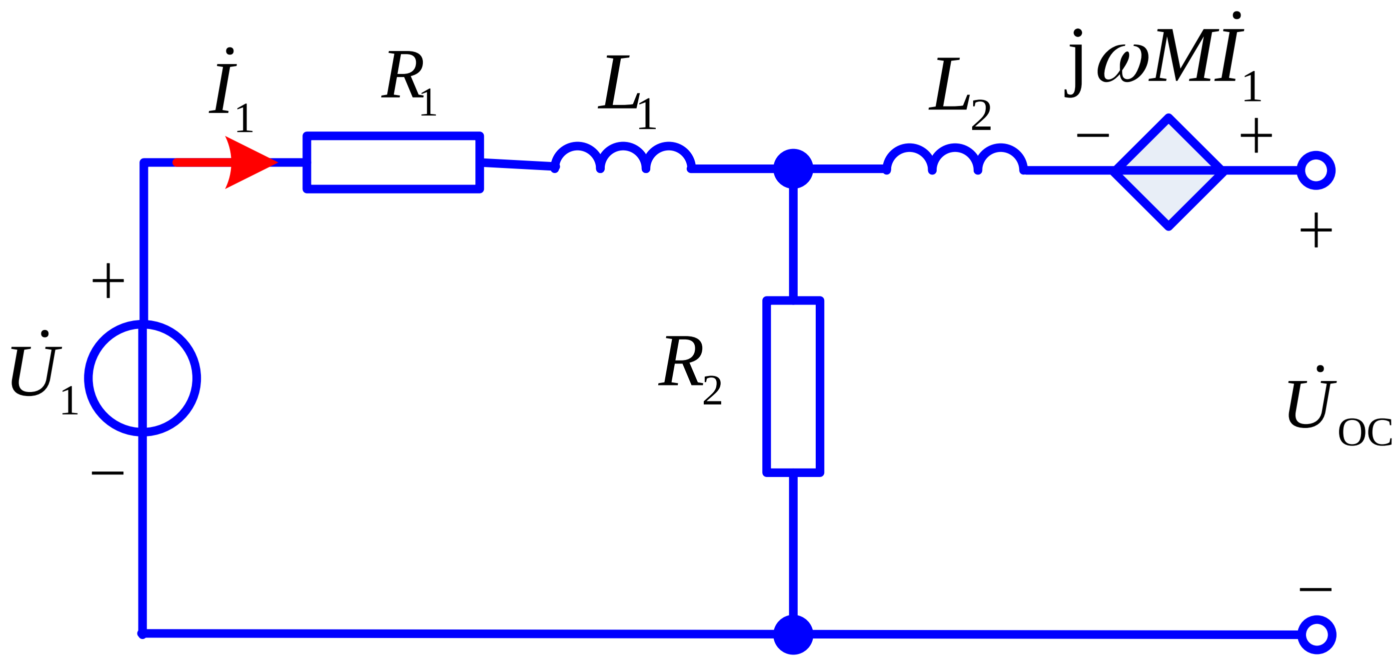
$$\dot{U}_{OC}=\frac{\dot{U}_1(R_2+\mathrm{j}\omega M)}{R_1+R_2+\mathrm{j}\omega(L_1-M+M)}$$

$$=\frac{\dot{U}_1(R_2+\mathrm{j}\omega M)}{R_1+R_2+\mathrm{j}\omega L_1}$$

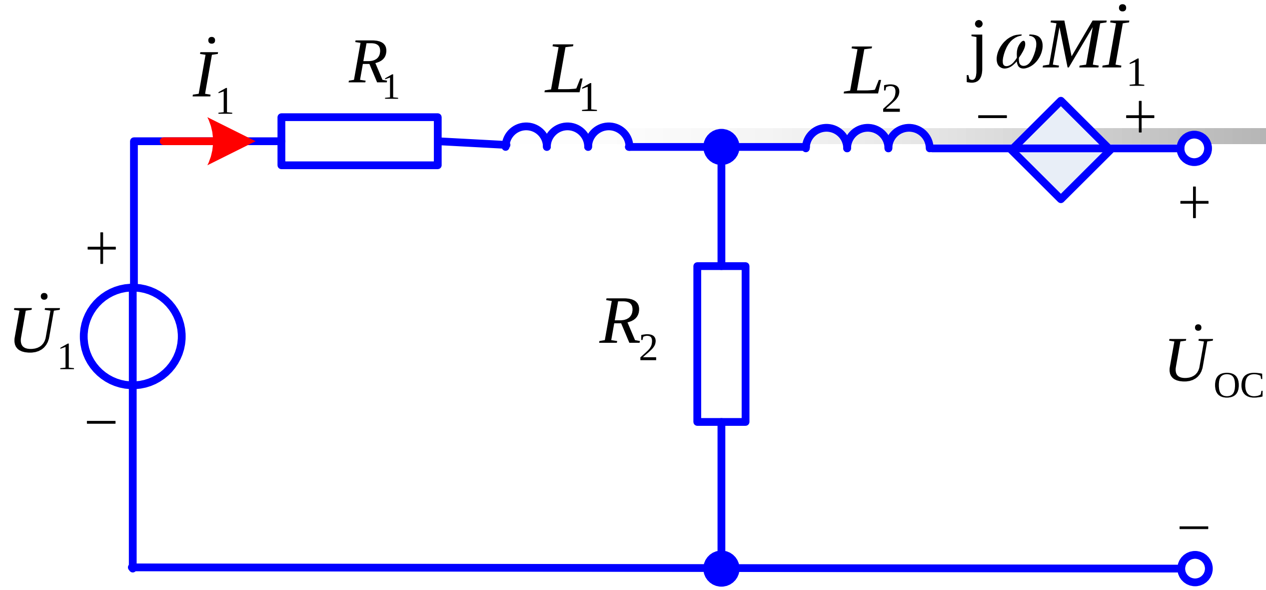
解法2:



M
R_1
*
*
L_1
L_2
+
$\dot{U}_1$
−
R_2
+
$\dot{U}_{OC}$
−



$\dot{I}_1$
R_1
L_1
L_2
$j\omega M\dot{I}_1$
−
+
+
$\dot{U}_1$
−
R_2
$\dot{U}_{OC}$
−

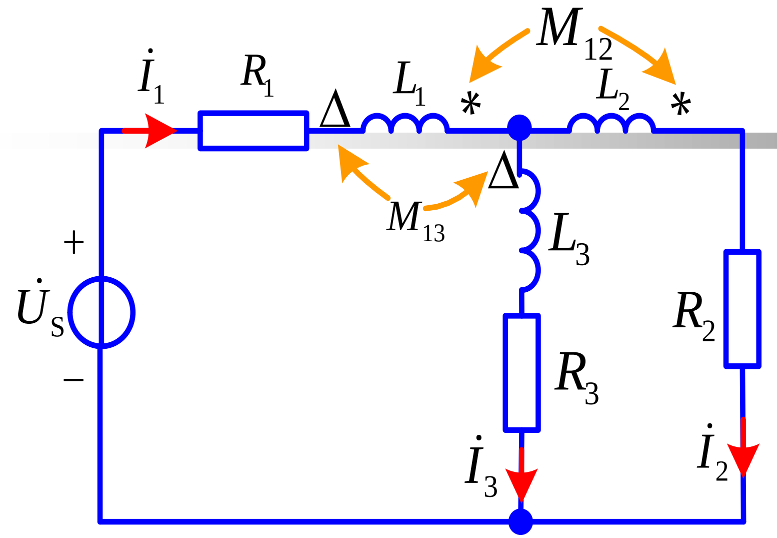


$$\dot{U}_{OC} = \mathrm{j}\omega M\dot{I}_1 + R_2\dot{I}_1$$

$$= (R_2 + \mathrm{j}\omega M)\dot{I}_1$$

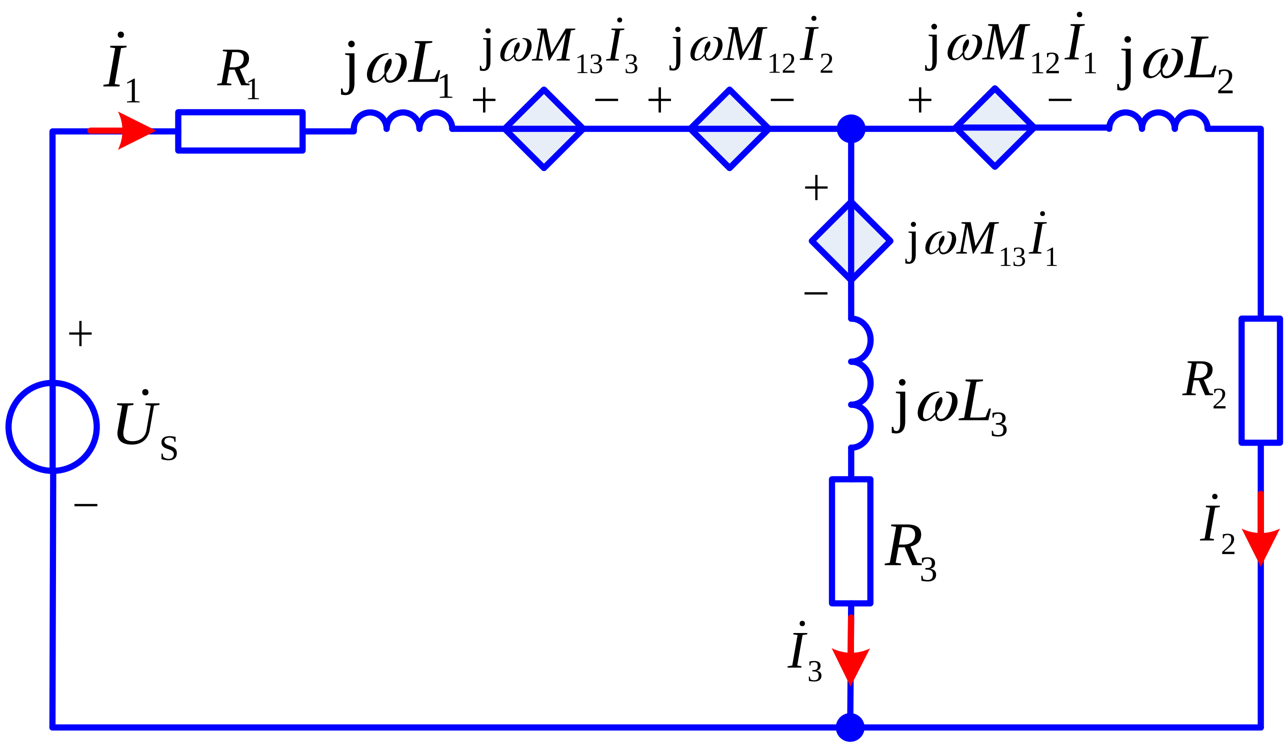
$$= (R_2 + \mathrm{j}\omega M)\frac{\dot{U}_1}{R_1 + R_2 + \mathrm{j}\omega L_1}$$

【例】 列支路法方程

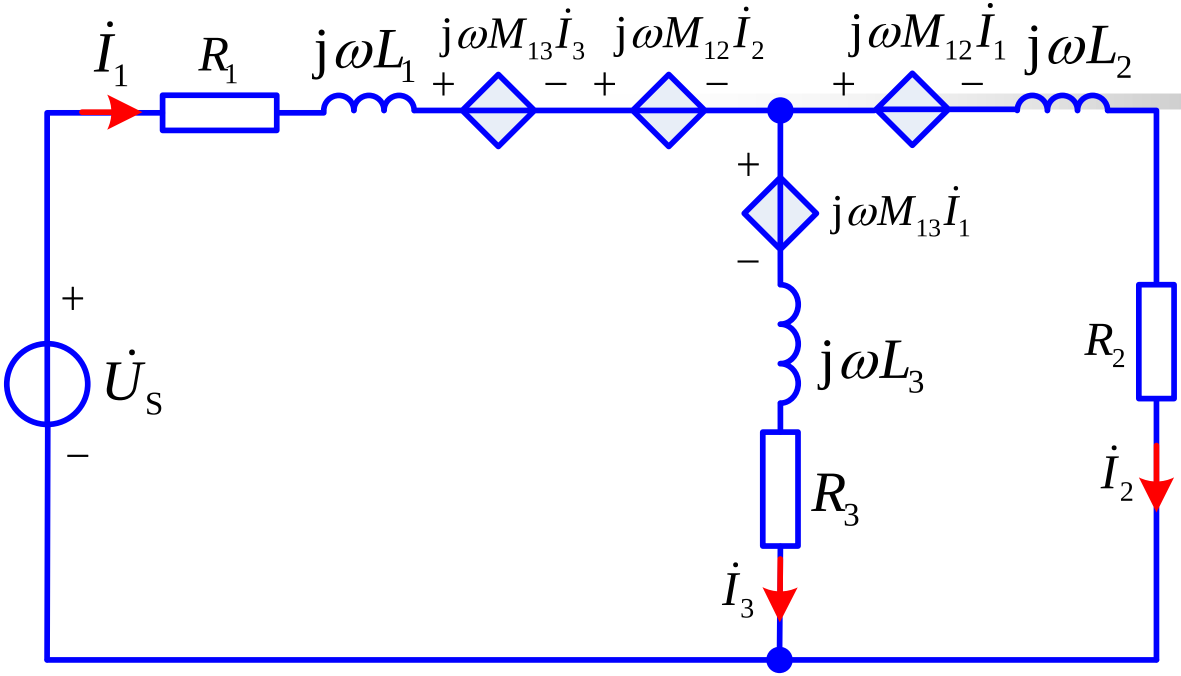


解

方法1:



解

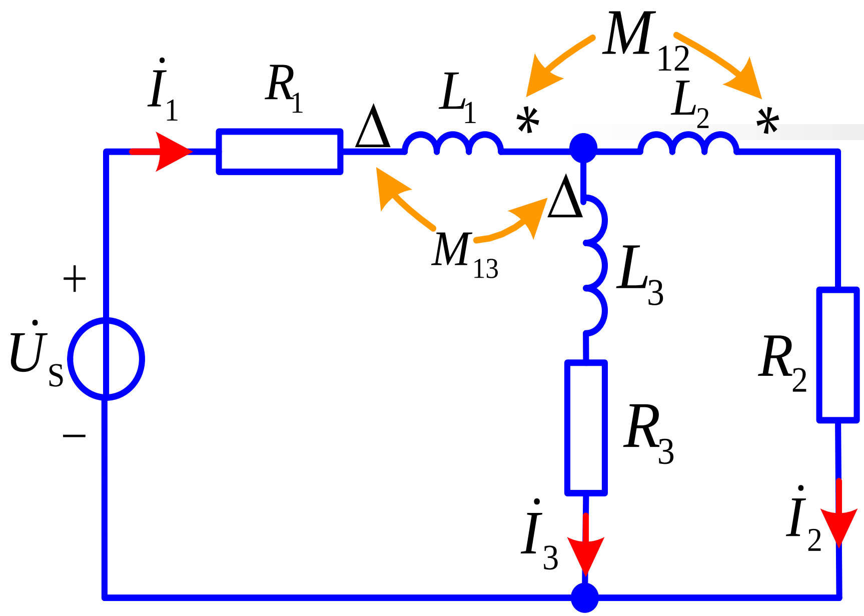


$$\dot{U}_1 = -\dot{U}_S + \dot{I}_1 (R_1 + \mathrm{j}\omega L_1) + \mathrm{j}\omega M_{13}\dot{I}_3 + \mathrm{j}\omega M_{12}\dot{I}_2$$

$$\dot{U}_2 = (R_2 + \mathrm{j}\omega L_2)\dot{I}_2 + \mathrm{j}\omega M_{12}\dot{I}_1$$

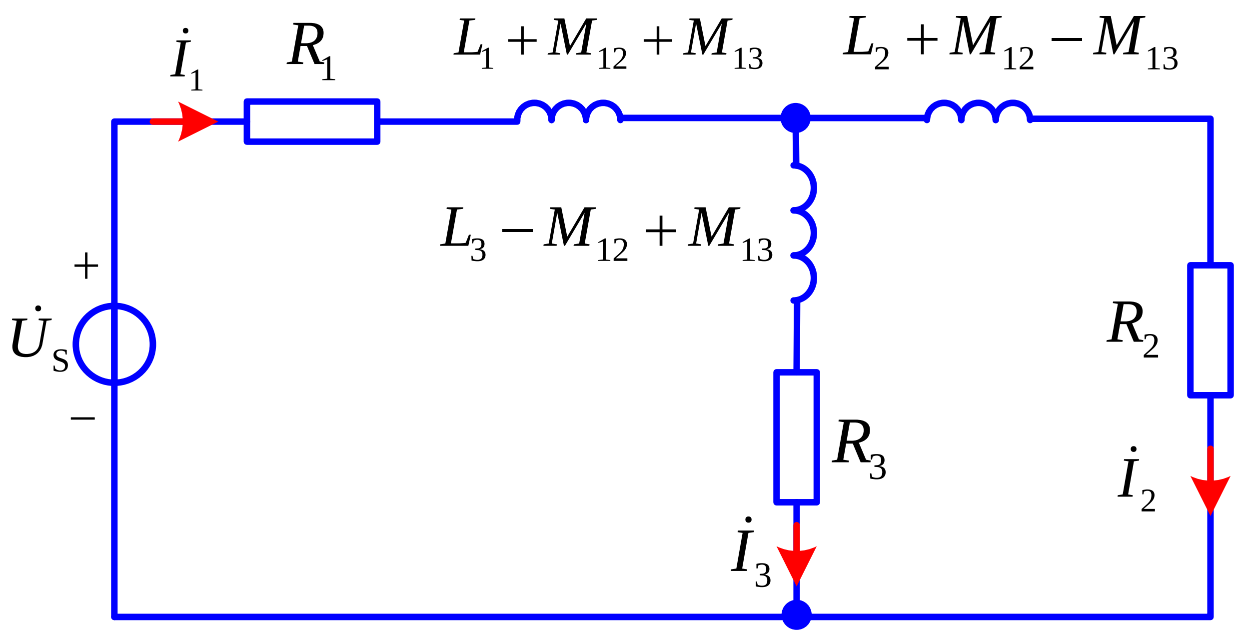
$$\dot{U}_3 = (R_3 + \mathrm{j}\omega L_3)\dot{I}_3 + \mathrm{j}\omega M_{13}\dot{I}_1$$

$$\dot{U}_1 + \dot{U}_2 = 0 \qquad \dot{U}_2 = \dot{U}_3 \qquad \dot{I}_1 = \dot{I}_2 + \dot{I}_3$$



$\dot{I}_1$
R_1
L_1
M_{12}
L_2
M_{13}
L_3
$\dot{U}_S$
R_2
R_3
$\dot{I}_3$
$\dot{I}_2$

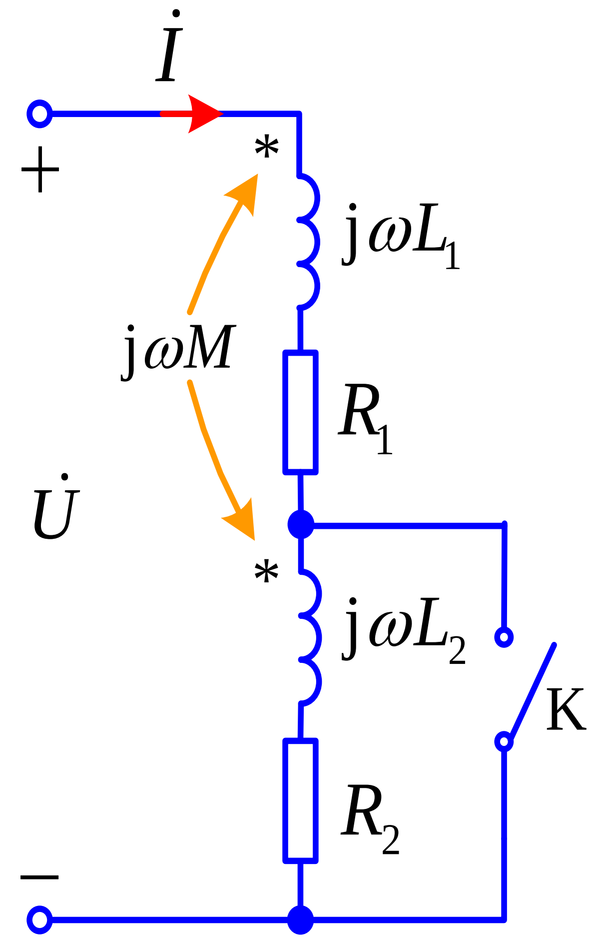
方法2:



$\dot{I}_1$
R_1
$L_1+M_{12}+M_{13}$
$L_2+M_{12}-M_{13}$
$L_3-M_{12}+M_{13}$
$\dot{U}_S$
R_2
R_3
$\dot{I}_3$
$\dot{I}_2$

【例】 已知：R_1、R_2、L_1、L_2、M、ω和 $\dot{U}$

求：开关K打开和闭合时的电流 $\dot{I}$。



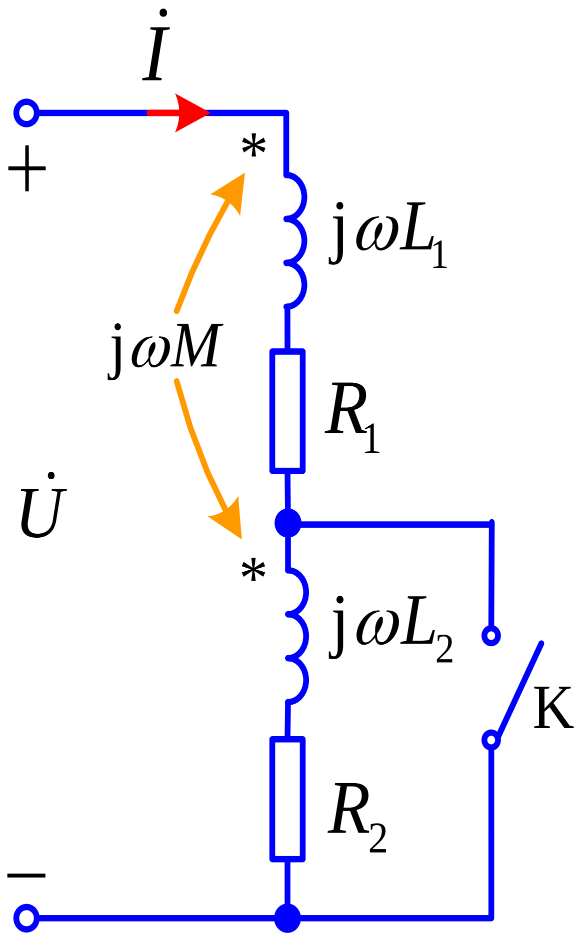
解

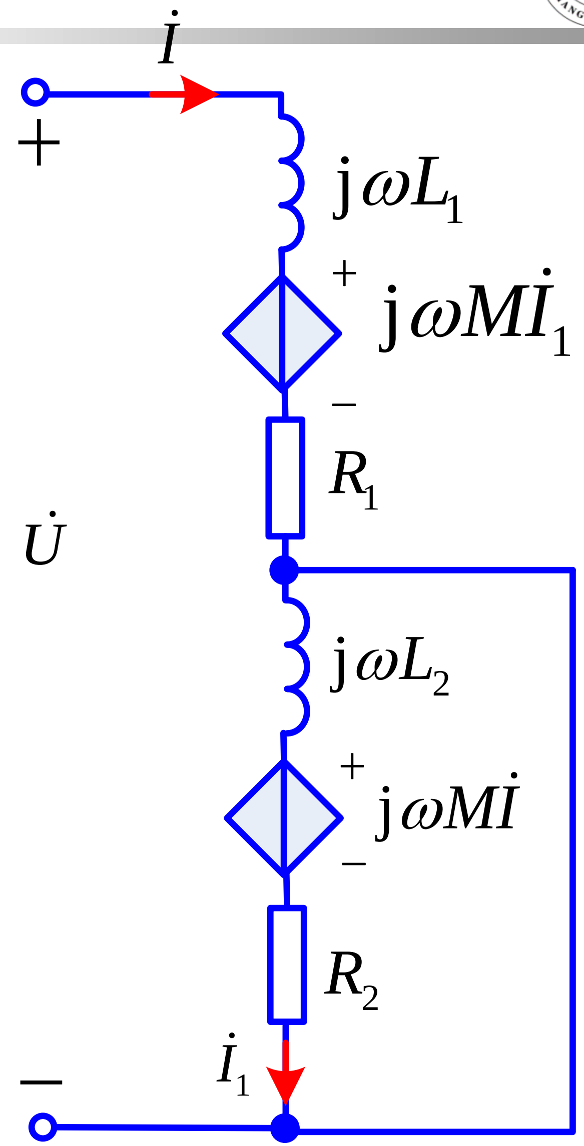
K打开时

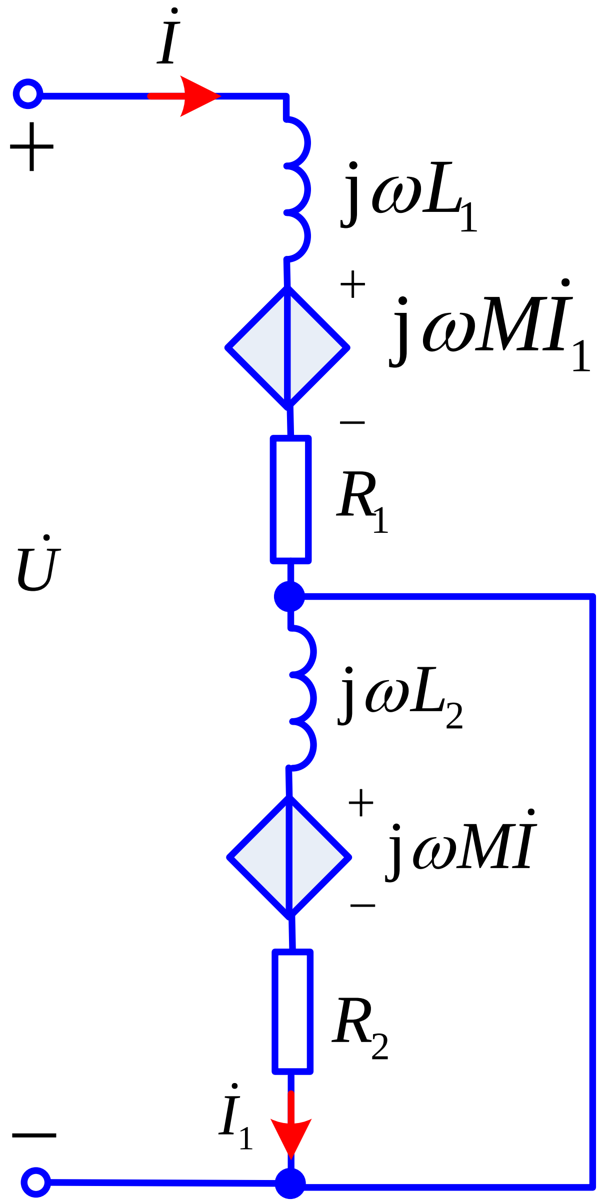
$$\dot{I}=\frac{\dot{U}}{R_1+R_2+\mathrm{j}\omega(L_1+L_2+2M)}$$

K闭合时：

解法1：受控源等效变换







$$\dot U=(R_1+j\omega L_1)\dot I+j\omega M\dot I_1$$

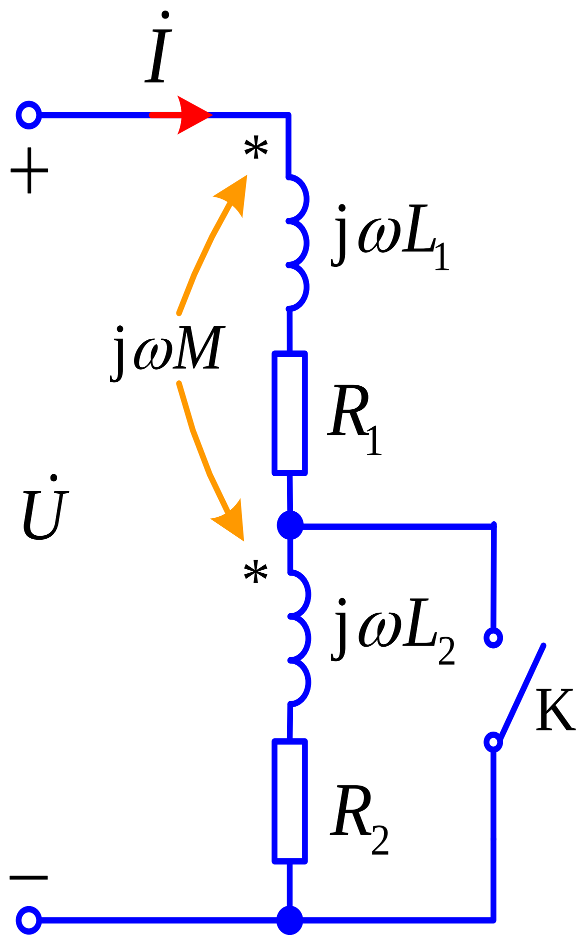
$$(R_2+j\omega L_2)\dot I_1+j\omega M\dot I=0$$

$$\dot I_1=\frac{-j\omega M\dot I}{R_2+j\omega L_2}$$

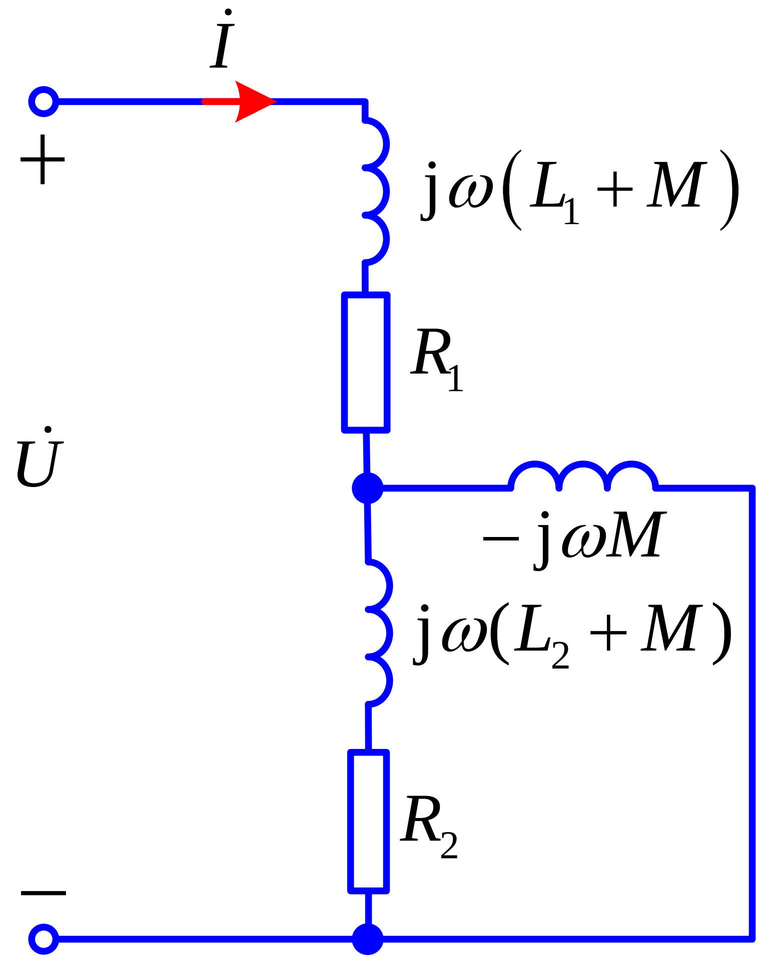
$$\dot U=(R_1+j\omega L_1)\dot I+\frac{\omega^2M^2}{R_2+j\omega L_2}\dot I$$

$$\dot I=\frac{\dot U}{R_1+j\omega L_1+\dfrac{\omega^2M^2}{R_2+j\omega L_2}}$$

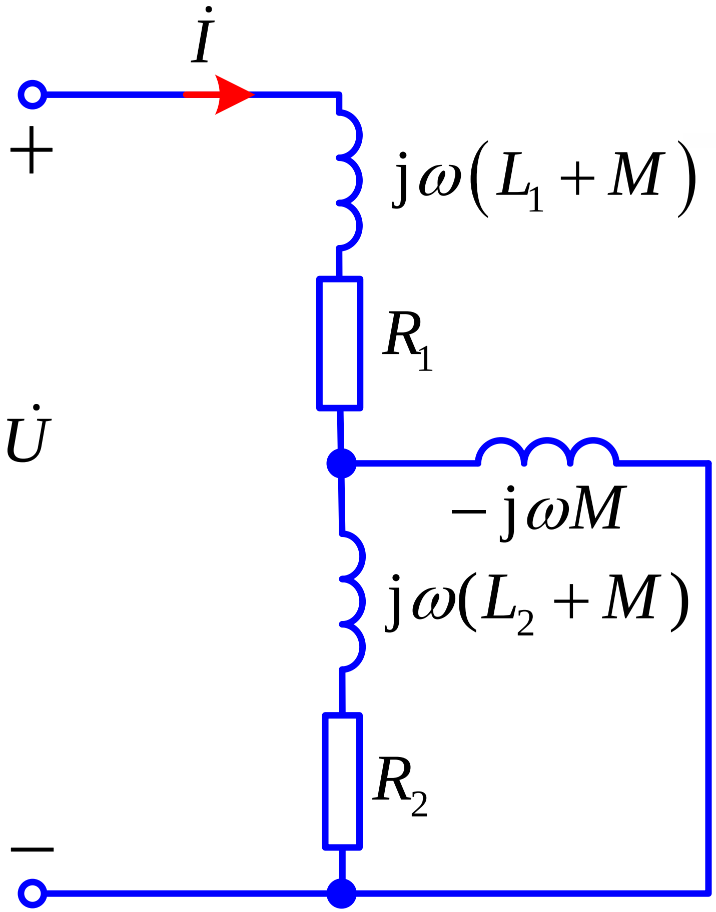
解法2：用互感消去法



$\dot{I}$
+
*
$\mathrm{j}\omega L_1$
$\mathrm{j}\omega M$
R_1
$\dot{U}$
*
$\mathrm{j}\omega L_2$
K
R_2
−



$\dot{I}$
+
$\mathrm{j}\omega(L_1+M)$
R_1
$\dot{U}$
$-\mathrm{j}\omega M$
$\mathrm{j}\omega(L_2+M)$
R_2
−



$$Z = R_1 + \mathrm{j}\omega(L_1+M) + \frac{\left[R_2+\mathrm{j}\omega(L_2+M)\right](-\mathrm{j}\omega M)}{R_2+\mathrm{j}\omega(L_2+M)-\mathrm{j}\omega M}$$

$$\dot{I} = \frac{\dot{U}}{Z}$$

10.2 含有耦合电感电路的计算

含有互感的电路：

★关键在方程中正确计入互感电压（大小、方向）

★互感消去法（去耦法）特殊性：需满足有一端相联的条件，否则只能用受控源方法处理互感

★一般**采用支路法和回路法**，不用结点法计算电路：支路电流不能用结点电压表示出来，（含有互感的原因）只是增加了结点电压未知数而已，若电路用去耦电路等效后可以用结点法。

★使用Y-△变换必须先去耦合

★使用戴维宁定理时，对外解耦合。

作业

【10-1】

【10-2】

【10-3】

【10-4】

【10-5】