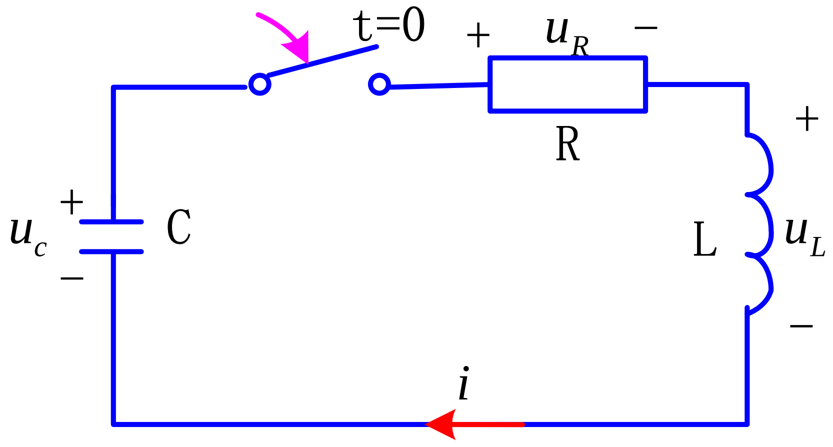
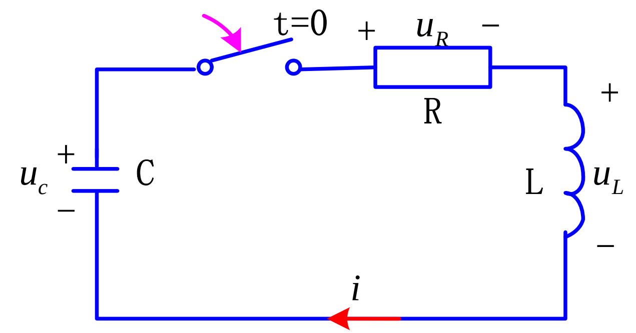
零输入响应

二阶电路在无外加激励的情况下，换路后仅由电容、电感储能元件所储存的初始能量作用于电路而引起的响应。

分析方法:经典法



$$Ri + u_L = u_C, \quad i = -C\frac{du_C}{dt}, \quad u_L = L\frac{di}{dt}$$



$$LC\frac{d^2u_C}{dt^2} + RC\frac{du_C}{dt} + u_C = 0$$

$$u_C(0-) = U_0$$

求解二阶微分方程

（1）求初值

由换路定理：

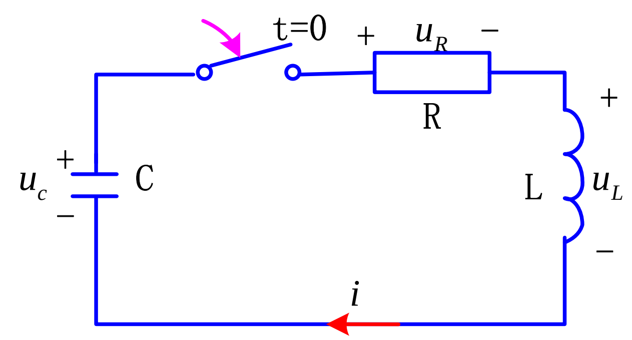
$$u_C(0+) = u_C(0-) = U_0$$

$$i(0+) = i(0-) = 0$$

$$\left.\frac{du_C}{dt}\right|_{0+} = -\frac{1}{C}i(0+) = 0$$

7.5 二阶电路的零输入响应

$$LC\frac{d^2u_C}{dt^2}+RC\frac{du_C}{dt}+u_C=0$$



$$u_C(0-)=U_0$$

求解二阶微分方程

（2）求微分方程解

特征方程： $LCp^2+RCp+1=0$

特征根： $p_{1,2}=\frac{-R\pm\sqrt{R^2-4L/C}}{2L}=-\frac{R}{2L}\pm\sqrt{(\frac{R}{2L})^2-\frac{1}{LC}}$

$$\boxed{u_C(t)=A_1e^{p_1t}+A_2e^{p_2t}}$$

代入初始值 $u_C(0+)$ 和 $\frac{du_C}{dt}(0+)$ 确定系数A_1和A_2

$$\begin{cases} u_C(0+)=A_1+A_2 \\ \frac{du_C}{dt}(0+)=A_1p_1+A_2p_2 \end{cases}$$

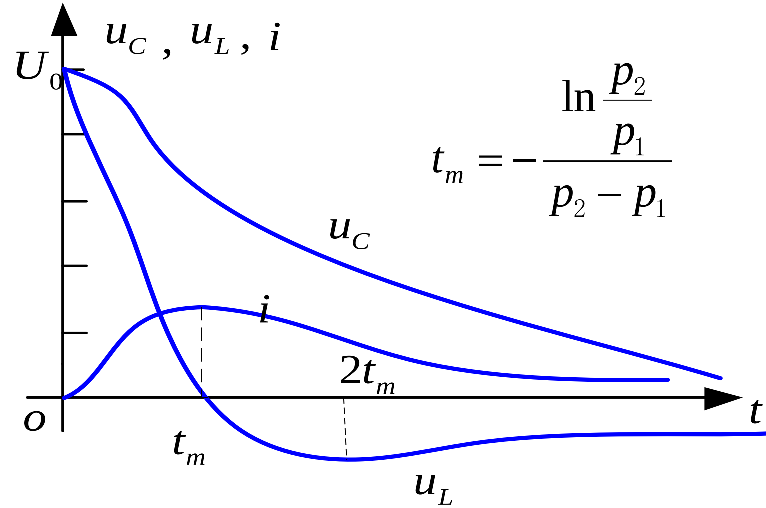
系数A_1和A_2方程不一定是实数

7.5 二阶电路的零输入响应

情况一： $R > 2\sqrt{\dfrac{L}{C}}$

p_1和p_2为两个不相等实数

$$\boxed{u_C(t) = \frac{U_0}{p_2 - p_1}\left(p_2\, e^{p_1 t} - p_1\, e^{p_2 t}\right)}$$



$$t_m = -\frac{\ln \dfrac{p_2}{p_1}}{p_2 - p_1}$$

$$i(t) = -C\frac{du_C}{dt} = -\frac{U_0 C p_1 p_2}{p_2 - p_1}\left(e^{p_1 t} - e^{p_2 t}\right) = -\frac{U_0}{L(p_2 - p_1)}\left(e^{p_1 t} - e^{p_2 t}\right)$$

$$u_L(t) = L\frac{di}{dt} = -\frac{U_0}{(p_2 - p_1)}\left(p_1\, e^{p_1 t} - p_2\, e^{p_2 t}\right)$$

$t = t_m$时，$u_L = 0$，i取得极值（最大值），u_C取得拐点

过阻尼情况，非振荡放电过程

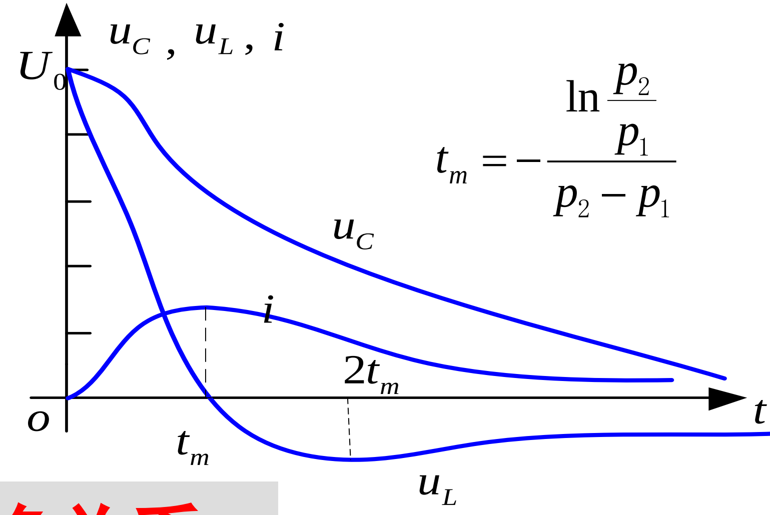
7.5 二阶电路的零输入响应

情况一： $R > 2\sqrt{\dfrac{L}{C}}$

$$u_C(t) = \frac{U_0}{p_2 - p_1}(p_2\,\mathrm{e}^{p_1 t} - p_1\,\mathrm{e}^{p_2 t})$$

$$i(t) = -\frac{U_0}{L(p_2 - p_1)}(\mathrm{e}^{p_1 t} - \mathrm{e}^{p_2 t})$$

$$u_L(t) = -\frac{U_0}{(p_2 - p_1)}(p_1\,\mathrm{e}^{p_1 t} - p_2\,\mathrm{e}^{p_2 t})$$

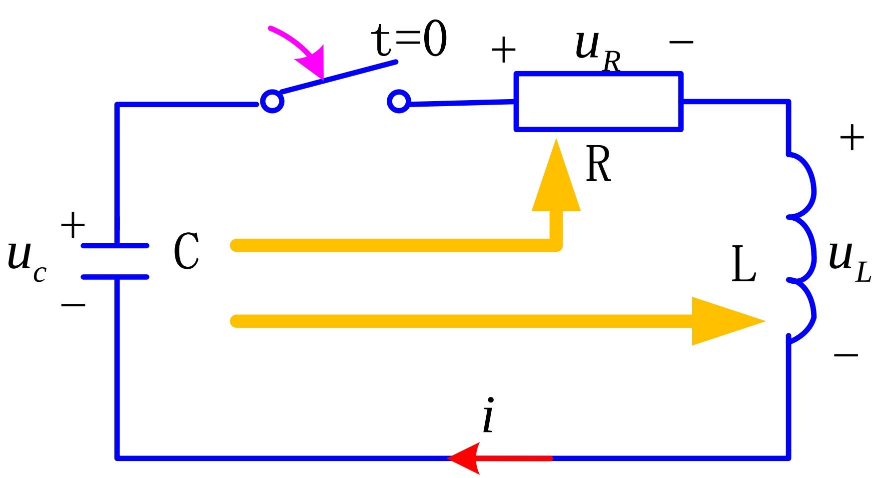


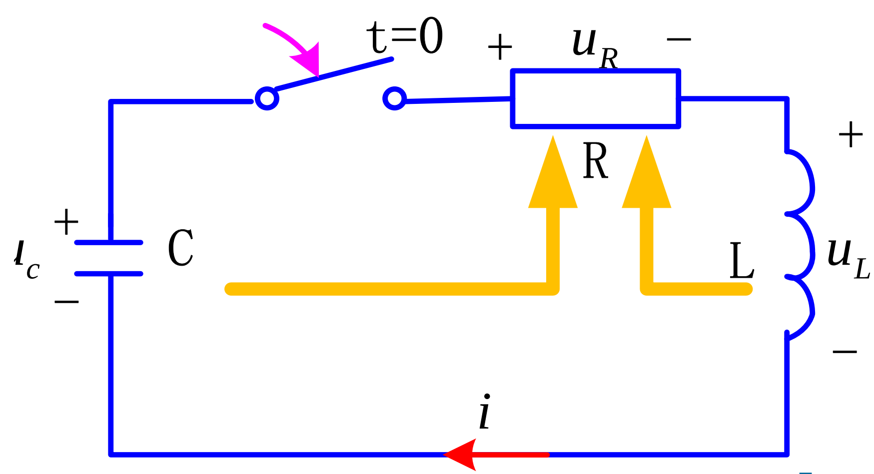
$$t_m = -\frac{\ln\dfrac{p_2}{p_1}}{p_2 - p_1}$$

能量转换关系

$0 < t < t_m, i > 0,\ \ u_L > 0$

$t > t_m, i > 0,\ \ u_L < 0$



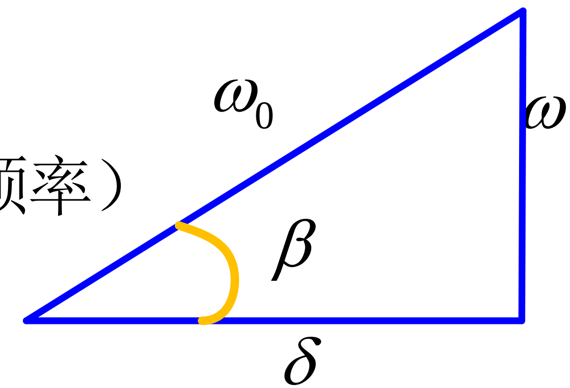


7.5 二阶电路的零输入响应

情况二： $R < 2\sqrt{\dfrac{L}{C}}$

p_1和p_2为共轭复数 $p_{1,2} = -\delta \pm j\omega$

$$\delta = \frac{R}{2L}\text{(衰减系数)},\ \omega = \sqrt{\omega_0{}^2 - \delta^2}\text{（振荡角频率）}$$



$$\omega_0 = \sqrt{\frac{1}{LC}}\text{（谐振角频率）}$$

$$u_C(t) = A_1' \mathrm{e}^{p_1 t} + A_2' \mathrm{e}^{p_2 t}$$

A_1'和A_2'也为共轭复数

$$\boxed{u_C(t) = e^{-\delta t}(A_1 \cos\omega t + A_2 \sin\omega t)}$$

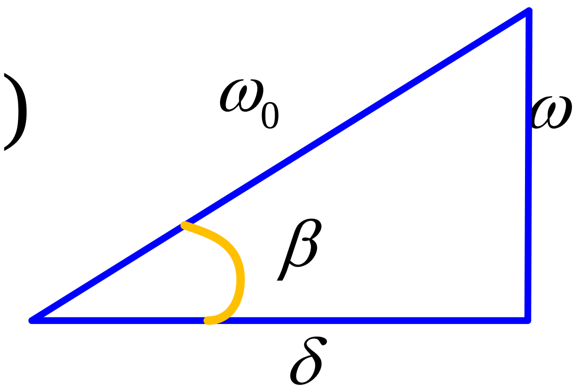
A_1和A_2为实数

$$\begin{cases} u_C(0+) = A_1 = U_0 \\ \dfrac{du_C}{dt}(0+) = -A_1\delta + A_2\omega = 0 \end{cases} \qquad \begin{cases} A_1 = U_0 \\ A_2 = \dfrac{A_1\delta}{\omega} \end{cases}$$

7.5 二阶电路的零输入响应

情况二： $R < 2\sqrt{\dfrac{L}{C}}$

$$u_C(t) = e^{-\delta t}(U_0 \cos \omega t + \frac{U_0 \delta}{\omega} \sin \omega t)$$



$$\boxed{u_C(t) = \frac{\omega_0}{\omega} U_0 e^{-\delta t} \sin(\omega t + \beta)}$$

$$i(t) = -C\frac{du_C}{dt} = -C\frac{\omega}{\omega_0} U_0[-\delta\, \mathrm{e}^{-\delta t} \sin(\omega t + \beta) + \omega\, \mathrm{e}^{-\delta t} \cos(\omega t + \beta)]$$

$$i(t) = \frac{U_0}{\omega L} \mathrm{e}^{-\delta t} \sin \omega t$$

$$u_L(t) = L\frac{di}{dt} = \frac{U_0}{\omega}(-\delta\, \mathrm{e}^{-\delta t} \sin \omega t + \omega\, \mathrm{e}^{-\delta t} \cos \omega t)$$

$$u_L(t) = -\frac{\omega_0}{\omega} U_0\, \mathrm{e}^{-\delta t} \sin(\omega t - \beta)$$

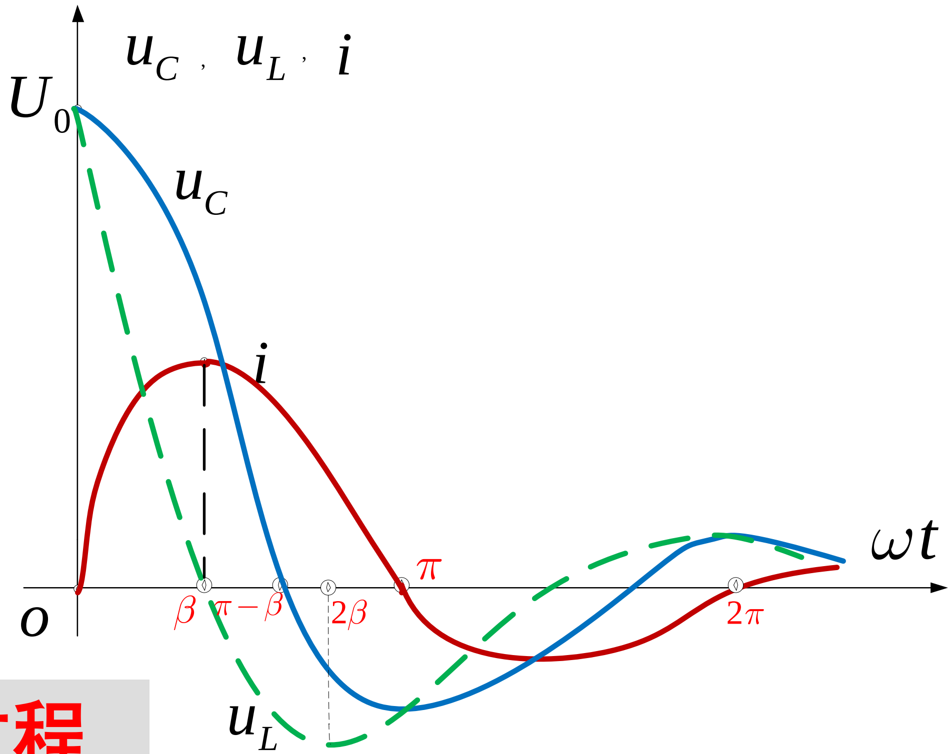
情况二：

$$R < 2\sqrt{\frac{L}{C}}$$

$$u_C(t) = \frac{\omega_0}{\omega} U_0 e^{-\delta t} \sin(\omega t + \beta)$$

$$i(t) = \frac{U_0}{\omega L} e^{-\delta t} \sin \omega t$$

$$u_L(t) = -\frac{\omega_0}{\omega} U_0 e^{-\delta t} \sin(\omega t - \beta)$$



欠阻尼情况，振荡放电过程

$\omega t = k\pi - \beta, k = 0,1,2,\ldots, u_C = 0$

$\omega t = k\pi, k = 0,1,2,\ldots, i = 0$， u_C取得极值点

$\omega t = k\pi + \beta, k = 0,1,2,\ldots, u_L = 0$， i取得极值点，u_C取得拐点

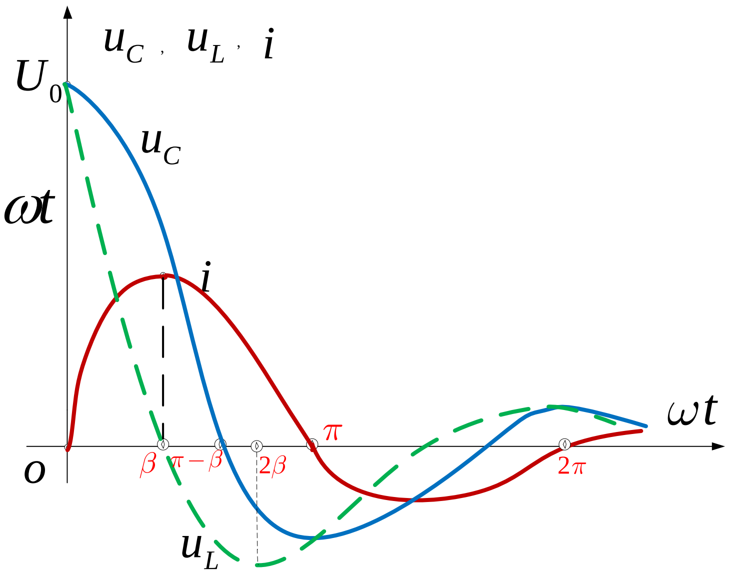
$\omega t = k\pi + 2\beta, k = 0,1,2,\ldots, u_L$取得极值点

7.5 二阶电路的零输入响应

情况二： $R<2\sqrt{\dfrac{L}{C}}$

$$u_C(t)=\frac{\omega_0}{\omega}U_0e^{-\delta t}\sin(\omega t+\beta)\quad i(t)=\frac{U_0}{\omega L}\mathrm{e}^{-\delta t}\sin\omega t$$

$$u_L(t)=-\frac{\omega_0}{\omega}U_0\,\mathrm{e}^{-\delta t}\sin(\omega t-\beta)$$



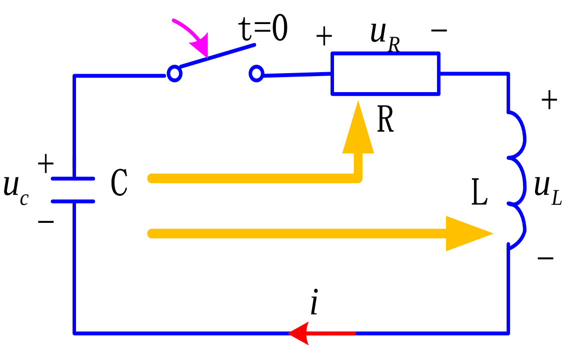
能量转换关系

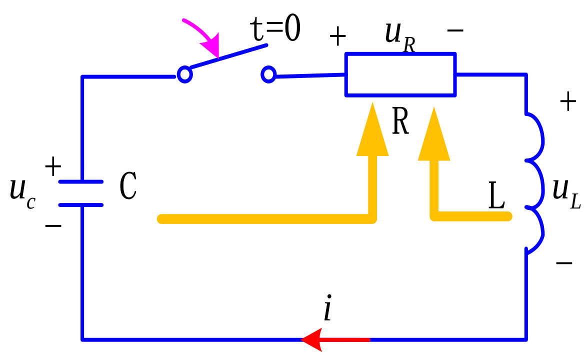
$0<\omega t<\beta$ $\quad$ $u_C>0, i>0,\ u_L>0$

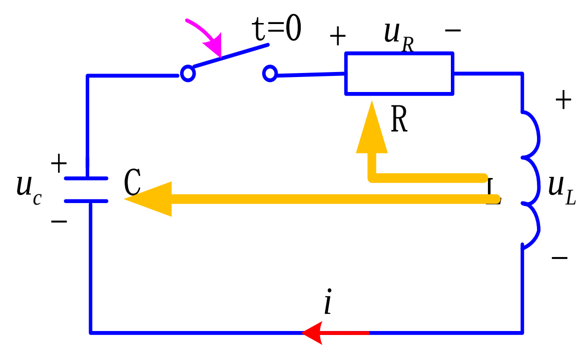
$\beta<\omega t<\pi-\beta$ $\quad$ $u_C>0, i>0,\ u_L<0$

$\pi-\beta<\omega t<\pi$ $\quad$ $u_C<0, i>0,\ u_L<0$

••••••







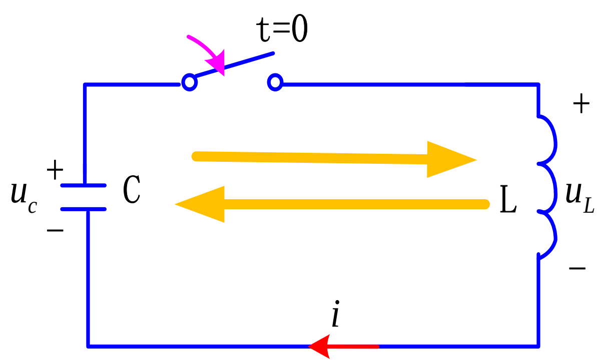
特殊情况：

$$R=0, \delta=-\frac{R}{2L}=0, \qquad \beta=arctg(\frac{\omega}{\delta})=\frac{\pi}{2}$$

$$u_C(t)=U_0\sin(\omega t+\frac{\pi}{2})$$

$$i(t)=\frac{U_0}{\omega L}\sin\omega t$$

$$u_L(t)=-U_0\sin(\omega t-\frac{\pi}{2})=U_0\sin(\omega t+\frac{\pi}{2})$$



零阻尼情况，等幅振荡过程

情况三： $R=2\sqrt{\dfrac{L}{C}}$

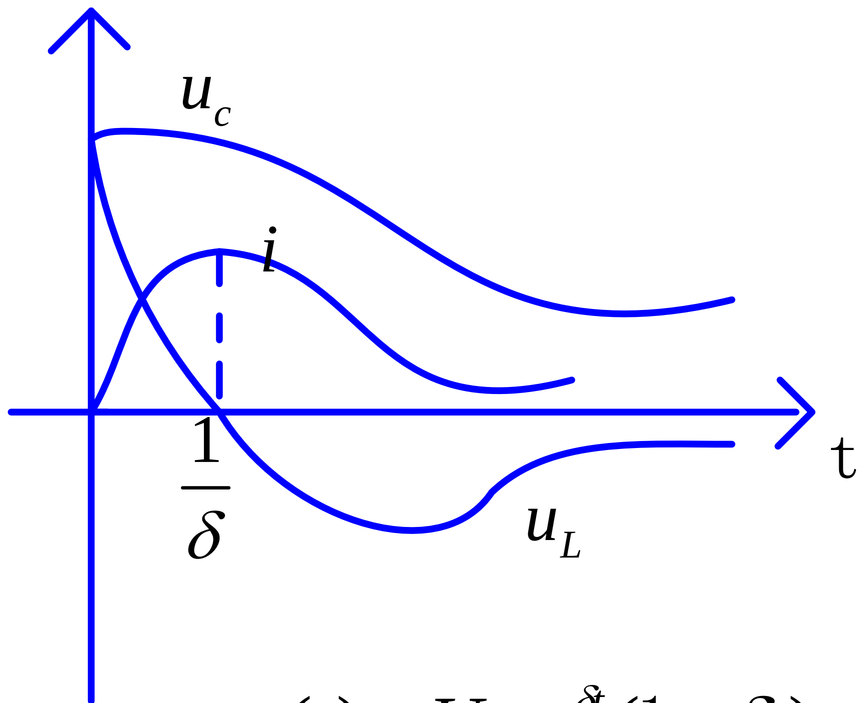
p_1和p_2为相等实根

$$p_1=p_2=-\frac{R}{2L}=-\delta$$

$$\boxed{u_C(t)=e^{-\delta t}(A_1+A_2t)}$$

A_1和A_2为实数

$$\begin{cases} u_C(0+)=A_1=U_0 \\ \dfrac{du_C}{dt}(0+)=-A_1\delta+A_2=0 \end{cases} \qquad \begin{cases} A_1=U_0 \\ A_2=\delta U_0 \end{cases}$$



$$u_C(t)=U_0e^{-\delta t}(1+\delta t)$$

$$i(t)=\frac{U_0}{L}te^{-\delta t}$$

$$u_L(t)=U_0e^{-\delta t}(1-\delta t)$$

临界阻尼情况，非振荡放电过程

结论： p_1和p_2为特征方程特征根

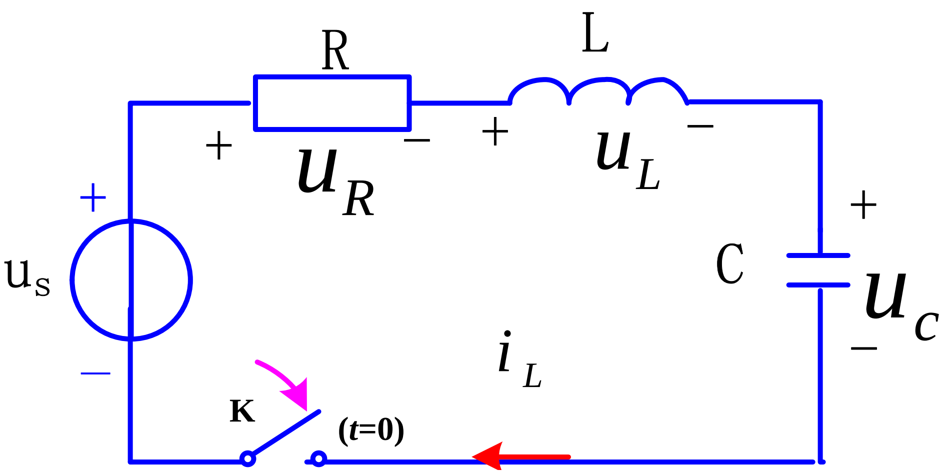
$R > 2\sqrt{\frac{L}{C}}$	$p_1 \neq p_2$两个不等实根 $u_{0i}(t) = A_1 e^{p_1 t} + A_2 e^{p_2 t}$ 过阻尼情况,非振荡放电（衰减）
$R < 2\sqrt{\frac{L}{C}}$	$p_1 = p_2^* = -\delta + j\omega$共轭复根 $u_{0i}(t) = e^{-\delta t}(A_1\cos\omega t + A_2\sin\omega t)$ 欠阻尼情况，振荡放电（衰减）
$R = 2\sqrt{\frac{L}{C}}$	$p_1 = p_2 = p$相等实根 $u_{0i}(t) = e^{pt}(A_1 + A_2 t)$ 临界阻尼情况，非振荡放电（衰减）
$R = 0$	$p_1 = p_2^* = j\omega$ $u_{0i}(t) = A_1\cos\omega t + A_2\sin\omega t$ 零阻尼情况，等幅振荡（非衰减）

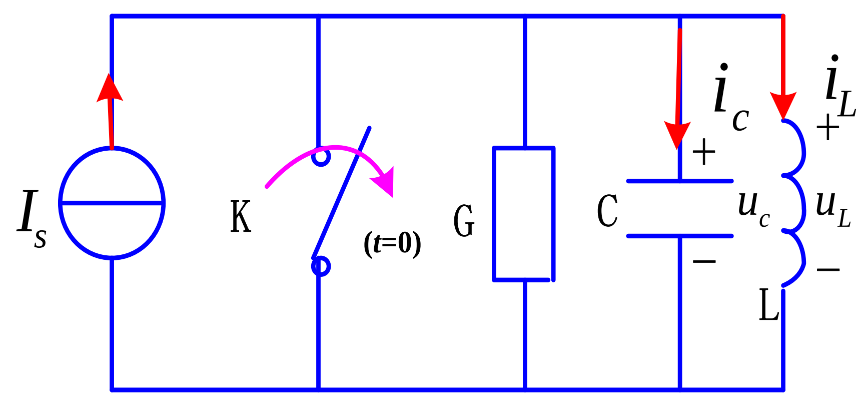
A_1、A_2是实数

7.6 二阶电路的零状态响应和全响应

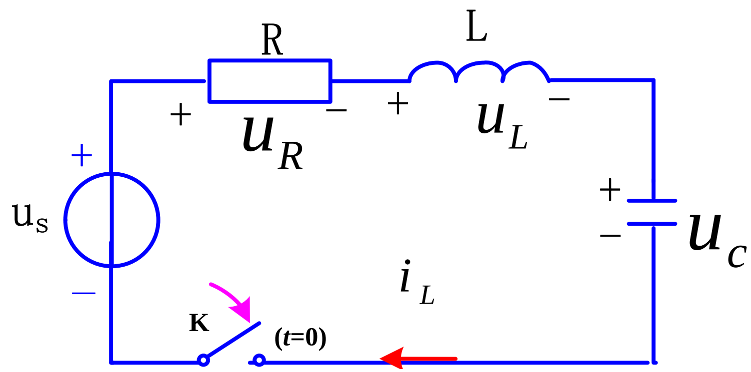
零状态响应

二阶电路在**零初始储能**的条件下, 在$t>0$时仅由施加于电路的激励所引起的响应。





零状态响应



$$u_C(0_-)=0, i_L(0_-)=0$$

$$u_R+u_C+u_L=U_S$$

$$LC\frac{d^2u_C}{dt^2}+RC\frac{du_C}{dt}+u_C=U_S$$

非齐次线性常微分方程

$$\frac{du_C}{dt}(0+)=\frac{1}{C}i_L(0+)=0, u_C(0+)=0$$

解的形式为： $u_C=u_C'+u_C''$

非齐次方程特解

齐次方程通解

u_C' → 特解（强制分量，稳态分量）

$$LC\frac{d^2u_C}{dt^2}+RC\frac{du_C}{dt}+u_C=U_S$$ 的特解 → $u_C'=U_S$

与输入激励的变化规律有关，为电路的稳态解。

u_C'' → 通解（自由分量，暂态分量）

$$LC\frac{d^2u_C}{dt^2}+RC\frac{du_C}{dt}+u_C=0$$ 的解 → $u_C''(t)=A_1\,\mathrm{e}^{p_1t}+A_2\,\mathrm{e}^{p_2t}$

变化规律由电路参数和结构决定。

全解 $$u_C(t)=u_C'+u_C''=U_S+A_1\,\mathrm{e}^{p_1t}+A_2\,\mathrm{e}^{p_2t}$$

由初始条件 $u_C(0_+)=0$ 、 $\frac{\mathrm{d}u_C}{\mathrm{d}t}(0+)=0$ 定常数 A_1、A_2

$$U_S+A_1+A_2=0$$

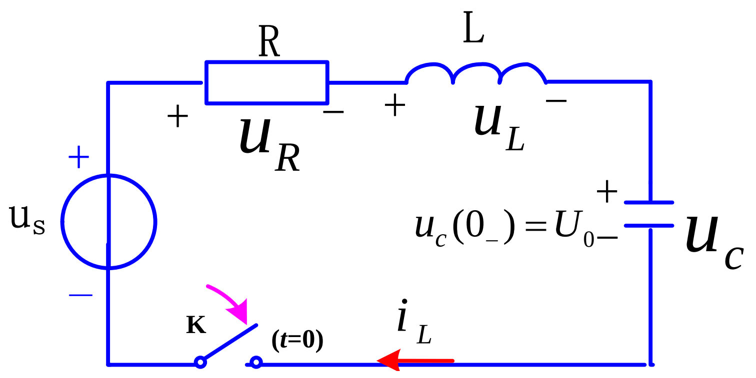
$$A_1p_1+A_2p_2=0$$

$$u_C=U_S+A_1\,\mathrm{e}^{p_1t}+A_2\,\mathrm{e}^{p_2t}\quad(t\geq 0)$$

全响应

由外施激励和初始储能共同作用引起的响应。

电路的初始状态不为零，同时又有外加激励源作用时电路中产生的响应。



7.6 二阶电路的零状态响应和全响应

$$u_C = u'_C + u''_C = u'_C + \underbrace{A_1 e^{p_1 t} + A_2 e^{p_2 t}} \quad (t \geq 0)$$

强制分量

（稳态响应）

非齐次方程的特解

自由分量

暂态响应

齐次方程的通解

全响应= 强制分量 + 自由分量

全响应= 零输入响应+ 零状态响应

*列写二阶电路方程

*根据特征方程的特征根判断相应的四种情况

$R > 2\sqrt{\frac{L}{C}}$

$p_1 \neq p_2$两个不等实根,过阻尼情况

$u(t) = u'(t) + A_1 e^{p_1 t} + A_2 e^{p_2 t}$

A_1、A_2是实数

$R < 2\sqrt{\frac{L}{C}}$

$p_1 = p_2^* = -\delta + j\omega$共轭复根,欠阻尼情况

$u(t) = u'(t) + e^{-\delta t}(A_1 \cos\omega t + A_2 \sin\omega t)$

$R = 2\sqrt{\frac{L}{C}}$

$p_1 = p_2 = p$相等实根,临界阻尼情况

$u(t) = u'(t) + e^{pt}(A_1 + A_2 t)$

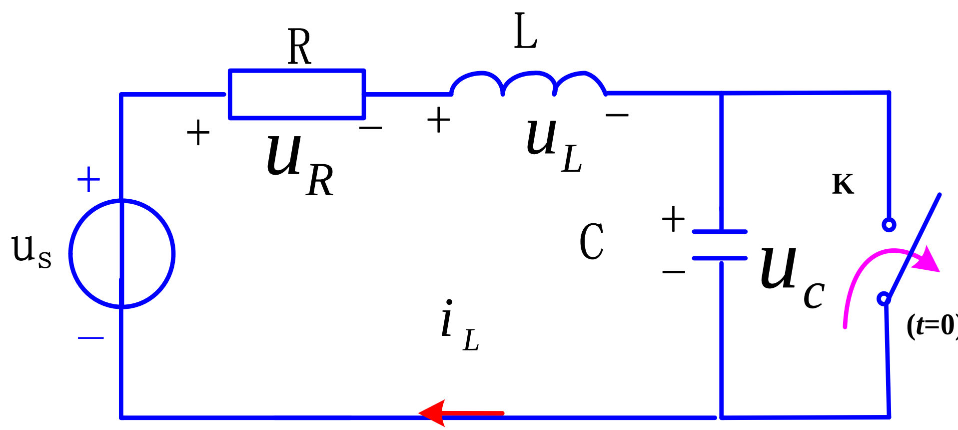
$R = 0$

$p_1 = p_2^* = j\omega$,零阻尼情况

$u(t) = u'(t) + A_1 \cos\omega t + A_2 \sin\omega t$

【例】已知u_S=100V, R=10Ω, L=0.5mH, C=2μF，开关K打开前电路处于稳态。求t>0时$u_C(t)$。

$$u_C(0_-)=0, i_L(0_-)=\frac{100}{10}=10\text{A}$$



$$u_R+u_C+u_L=u_S$$

$$LC\frac{d^2u_C}{dt^2}+RC\frac{du_C}{dt}+u_C=u_S$$

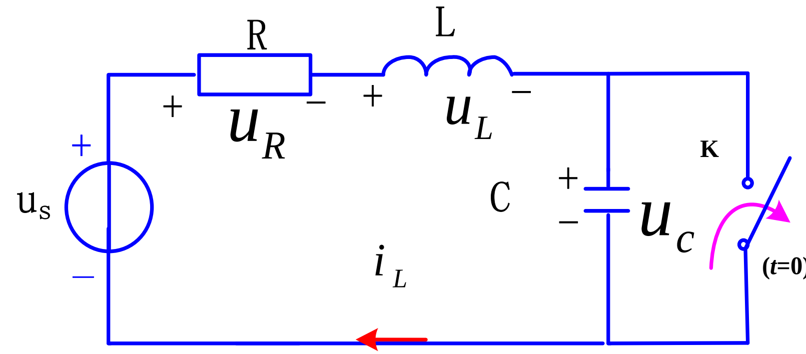
$$10^{-9}\frac{d^2u_C}{dt^2}+2\times10^{-5}\frac{du_C}{dt}+u_C=u_S$$

$$\frac{du_C}{dt}(0+)=\frac{1}{C}i_L(0+)=5\times10^6, u_C(0+)=0$$

7.6 二阶电路的零状态响应和全响应

$$10^{-9}\frac{d^2u_C}{dt^2}+2\times10^{-5}\frac{du_C}{dt}+u_C=u_S$$

$$\frac{du_C}{dt}(0+)=\frac{1}{C}i_L(0+)=5\times10^6,u_C(0+)=0$$



特征方程: $10^{-9}p^2+2\times10^{-5}p+1=0$

$$p_1=-10^4+j3\times10^4,\quad p_2=-10^4-j3\times10^4$$

$$u_C(t)=u_C'(t)+u_C''(t)=100+e^{-10^4t}(A_1\cos3\times10^4t+A_2\sin3\times10^4t)$$

$$100+A_1=0,\qquad -10^4A_1+3\times10^4A_2=5\times10^6$$

$$u_C(t)=100+e^{-10^4t}(-100\cos3\times10^4t+200\sin3\times10^4t)$$

$$=100+167e^{-10^4t}\sin(3\times10^4t-36.9^\circ)(\mathrm{V})$$

作业

【7-9】如图所示电路在开关S打开之前已达稳态；t=0时开关S打开，求t>0时电压$u_C(t)$。