北京航空航天大學
BEIHANG UNIVERSITY

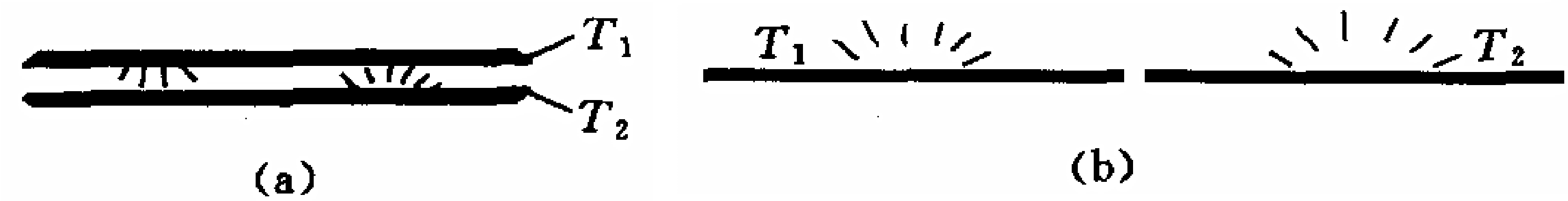
HT: Radiation

L14: heat transfer process

- **Learning Objectives:**
- **Angle factors**
- **irradiation and radiosity**
- **heat transfer within enclosure model**

IV. Angle factor or shape factor

The radiative heat transfer between two surfaces is highly dependent on the relative position of the two surfaces:



When the relative position between the two surfaces is different, the percentage of radiant energy emitted from one surface that falls on the other surface varies, thus affecting the heat transfer.

位置影响?

IV. Angle factor or shape factor

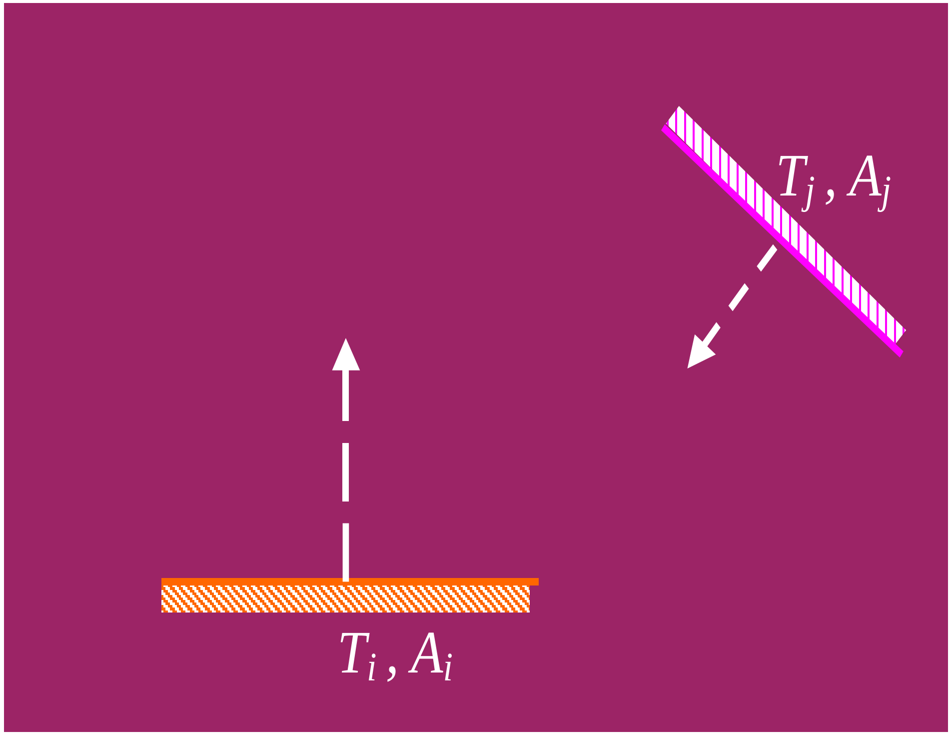
Definition:

The *radiation shape factor* $X_{i\text{-}j}$ of A_i with respect to A_j

A geometrical parameter!

Fraction of energy leaving surface A_i which reaches surface A_j directly.

“发出” — 包含表面1自身的辐射和反射的辐射；
“落到” — 不管表面2是否能够吸收；



$$X_{i-j} = \frac{\text{energy that leaves } A_i \text{ and directly strikes } A_j}{\text{total energy that leaves } A_i}$$

The *radiation shape factor* $X_{j\text{-}i}$ of A_j with respect to A_i

$$X_{j-i} = \frac{\text{energy that leaves } A_j \text{ and directly strikes } A_i}{\text{total energy that leaves } A_j}$$

IV. Angle factor or shape factor

两黑体表面间的辐射传热量 ⇨ $\Phi_{1,2} = A_1 E_{b1} X_{1,2} - A_2 E_{b2} X_{2,1}$

当$\boldsymbol{T_1 = T_2}$时，净辐射传热量为零，即$\boldsymbol{E_{b1} = E_{b2}}$

$$\boxed{A_1 X_{1,2} = A_2 X_{2,1}}$$

故$X_{d1,d2}$和$X_{d2,d1}$不是独立的。

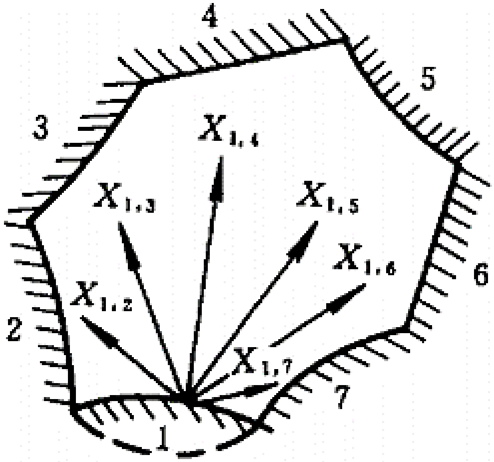
The reciprocity rule of the angle factors。

IV. Angle factor or shape factor

For a closed system composed of n surfaces, energy conservation can be obtained:

The radiant energy emitted from any surface must all fall on the surfaces of the closed system

$$X_{1,1} + X_{1,2} + X_{1,3} + \cdots + X_{1,n} = \sum_{i=1}^{n} X_{1,i} = 1$$

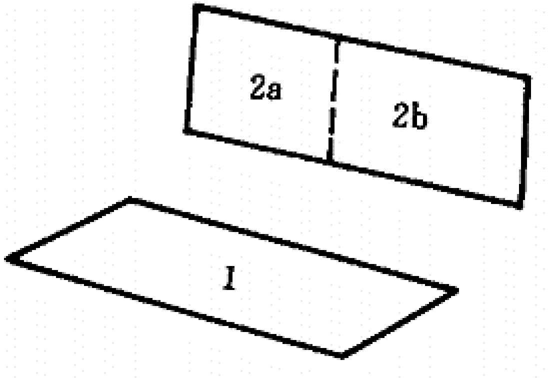


The summation rule of the angle factor。

Concave or convex， $X_{1,1}$ = ?。

IV. Angle factor or shape factor

The total energy from surface 1 falling on surface 2 is equal to the sum of the radiant energy from the parts falling on surface 2, so we have:



$$A_1 E_{b1} X_{1,2} = A_1 E_{b1} X_{1,2a} + A_1 E_{b1} X_{1,2b}$$

$$\Rightarrow X_{1,2} = X_{1,2a} + X_{1,2b}$$

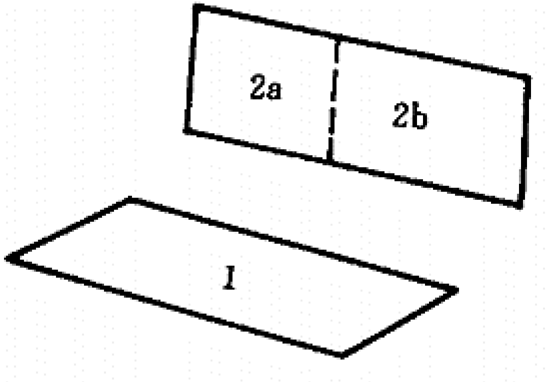
$$X_{1,2} = \sum_{i=1}^{n} X_{1,2i}$$

The superposition rule of the angle factor。

IV. Angle factor or shape factor

The superposition rule is only suitable for a whole surface to other seprate surface and reverse doesn’t available

$$A_2 E_{b2} X_{2,1} = A_{2a} E_{b2} X_{2a,1} + A_{2b} E_{b2} X_{2b,1}$$



$$\Rightarrow \quad A_2 X_{2,1} = A_{2a} X_{2a,1} + A_{2b} X_{2b,1}$$

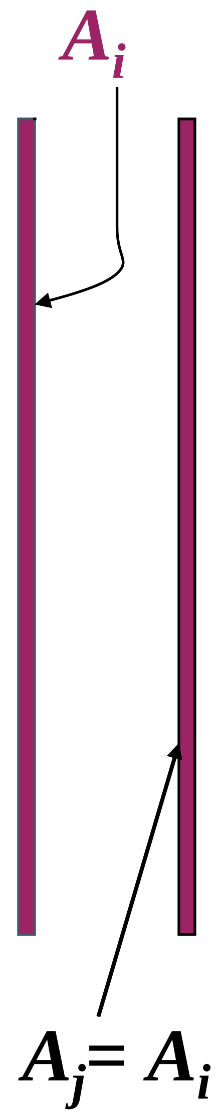
$$\Rightarrow \quad X_{2,1} = X_{2a,1} \frac{A_{2a}}{A_2} + X_{2b,1} \frac{A_{2b}}{A_2}$$

The superposition rule of the angle factor。

IV. Angle factor or shape factor

Two infinite parallel plates

$$X_{i-j} = X_{j-i} = 1$$

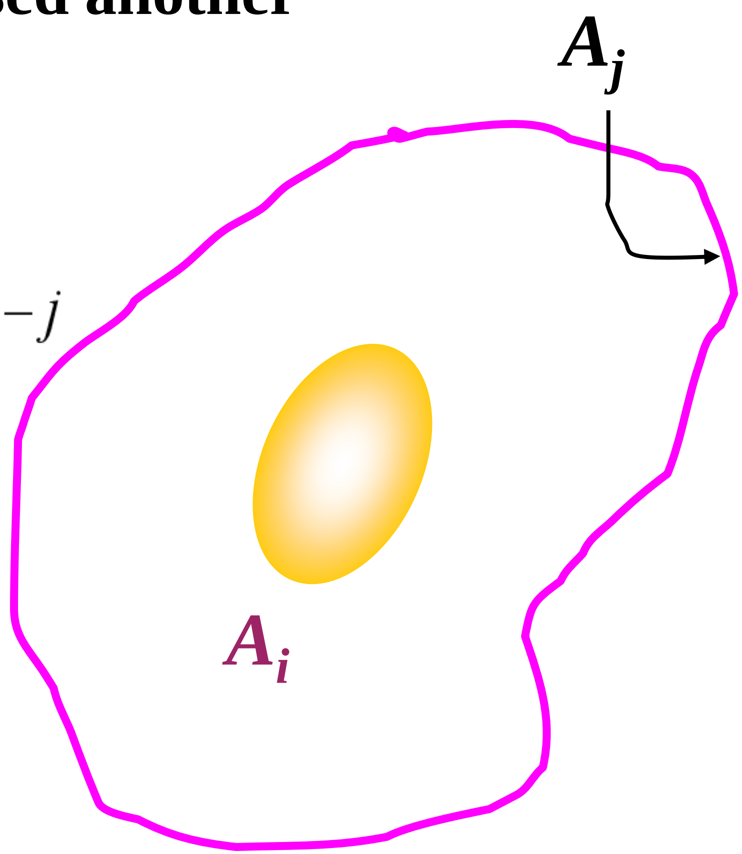


IV. Angle factor or shape factor

Two surfaces one completely enclosed another

$$X_{i-j} = 1 \Rightarrow \quad X_{j-i} = \frac{A_i}{A_j} X_{i-j}$$

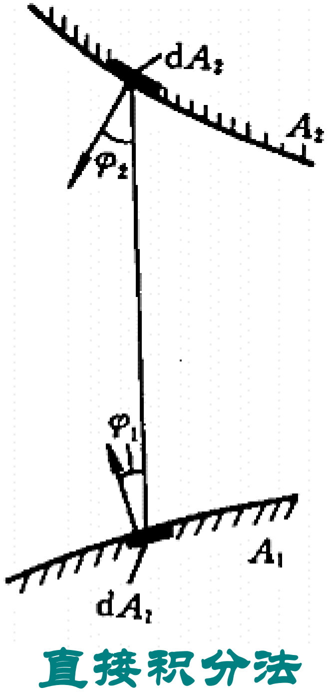
$$X_{j-j} = 1 - X_{j-i}$$



The angle factor of concave is non-zero

IV. Angle factor or shape factor

(1)Directly integrated



$$X_{d1,\ d2} = \frac{\cos\varphi_1 \cos\varphi_2 dA_2}{\pi r^2}$$

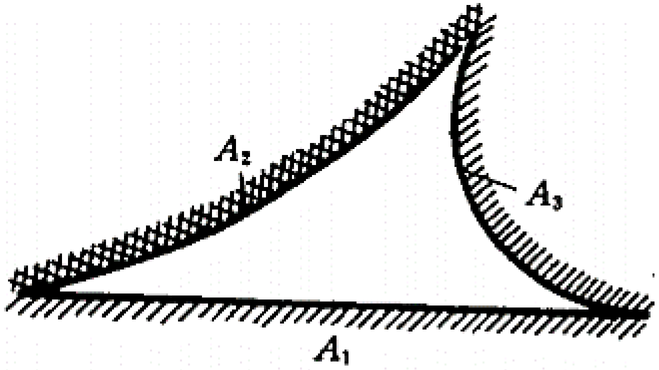
$$X_{d1,2} = \int_{A_2} \frac{\cos\varphi_1 \cos\varphi_2 dA_2}{\pi r^2}$$

$$X_{1,2} = \frac{1}{A_1} \int_{A_1} \int_{A_2} \frac{\cos\varphi_1 \cos\varphi_2 dA_2 dA_1}{\pi r^2}$$

IV. Angle factor or shape factor

(2) Algebraic analysis

$$X_{1,2} + X_{1,3} = 1 \qquad A_1 X_{1,2} = A_2 X_{2,1}$$
$$X_{2,1} + X_{2,3} = 1 \qquad A_1 X_{1,3} = A_3 X_{3,1}$$
$$X_{3,1} + X_{3,2} = 1 \qquad A_2 X_{2,3} = A_3 X_{3,2}$$

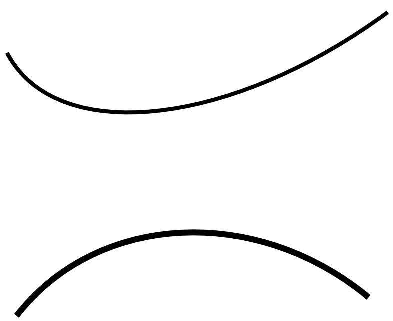


Solve the equations, we could get:

Three convex surface to combine a enclosure system

$$X_{1,2} = \frac{A_1 + A_2 - A_3}{2A_1}$$

$$X_{1,2} = \frac{l_1 + l_2 - l_3}{2l_1}$$



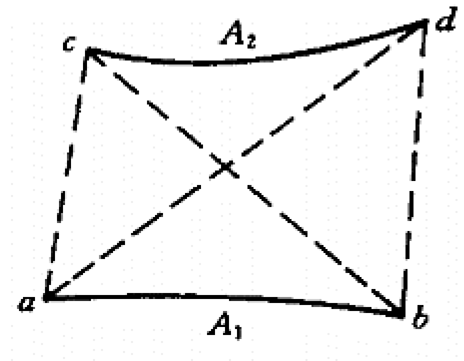
Geometrical factors, independent on T and environments

IV. Angle factor or shape factor

$$X_{ab,cd} = 1 - X_{ab,ac} - X_{ab,bd}$$

$$X_{ab,ac} = \frac{ab + ac - bc}{2ab}$$

$$X_{ab,bd} = \frac{ab + bd - ad}{2ab}$$



两个非凹表面及假想面组成的封闭系统

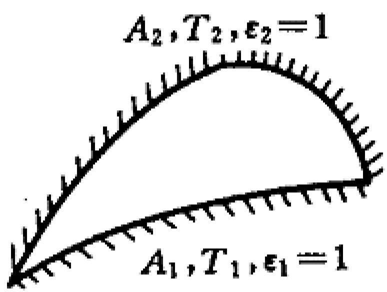
（垂直方向无限长）

$$X_{ab,cd} = \frac{(bc + ad) - (ac + bd)}{2ab}$$

$$= \frac{\text{交叉线之和} - \text{不交叉线之和}}{2 \times \text{表面 } A_1 \text{的断面长度}}$$

V. heat transfer within enclosure model

Heat transfer between two blackbody surfaces:



blackbody

$$\Phi_{1,2} = A_1 E_{b1} X_{1,2} - A_2 E_{b2} X_{2,1}$$

$$= A_1 X_{1,2}(E_{b1} - E_{b2})$$

$$= A_2 X_{2,1}(E_{b1} - E_{b2})$$

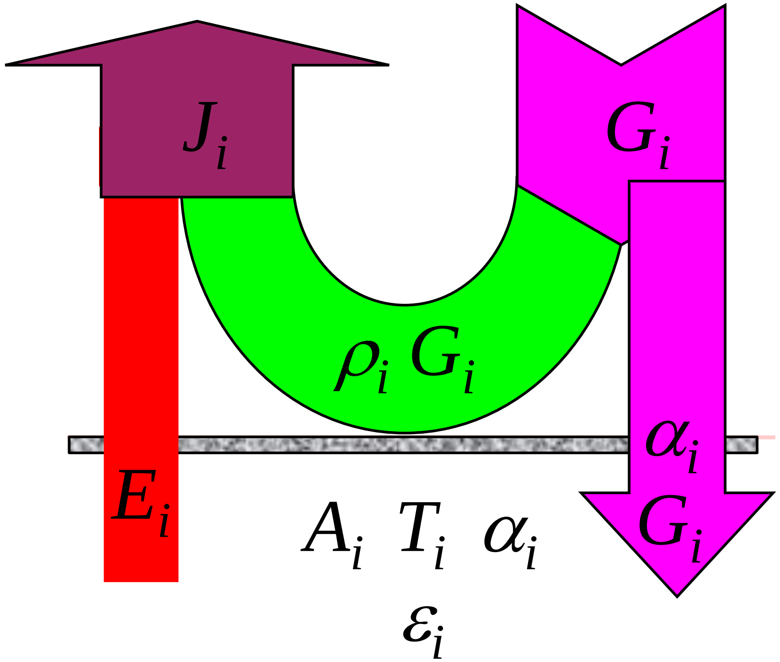
- **The key of blackbody radiation heat transfer calculation lies in the calculation of angle factors**

(1) The absorption ratio of gray body surface is less than 1, and the absorption of radiant energy into gray body surface is not completed at one time, but through many (n) reflections;
(2) The radiant energy emitted outward from a gray body surface includes the reflected radiant force in addition to its own radiant force (self radiation).

V. heat transfer within enclosure model

Heat transfer between two gray surfaces:

- Assumptions:
  - Gray surfaces
  - Diffuse
  - Uniform temperature
  - Uniform reflection, emission



Irradiation and Radiosity

- *Irradiation* G_i: Total *radiation incident* upon A_i per uint time and per unit area
- *Radiosity* J_i: Total *radiation which leaves* A_i per unit time and per unit area

V. heat transfer within enclosure model

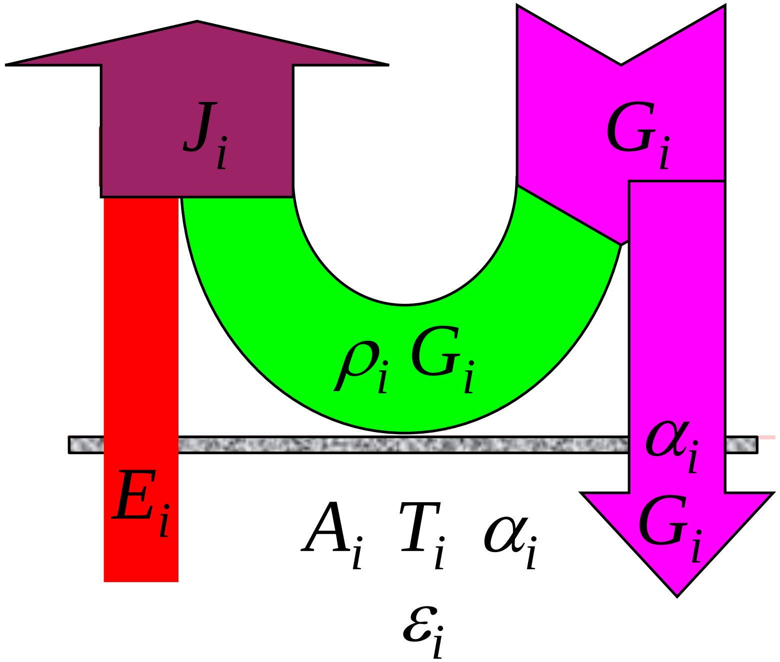
upon surface:

$$J_i = E_i + \rho_i G_i$$

For solid gray surfaces, we have,

$$\rho_i = 1 - \alpha_i = 1 - \varepsilon_i \quad (\text{Why?})$$

$$E_i = \varepsilon_i E_{bi} = \varepsilon_i \sigma T_i^4$$



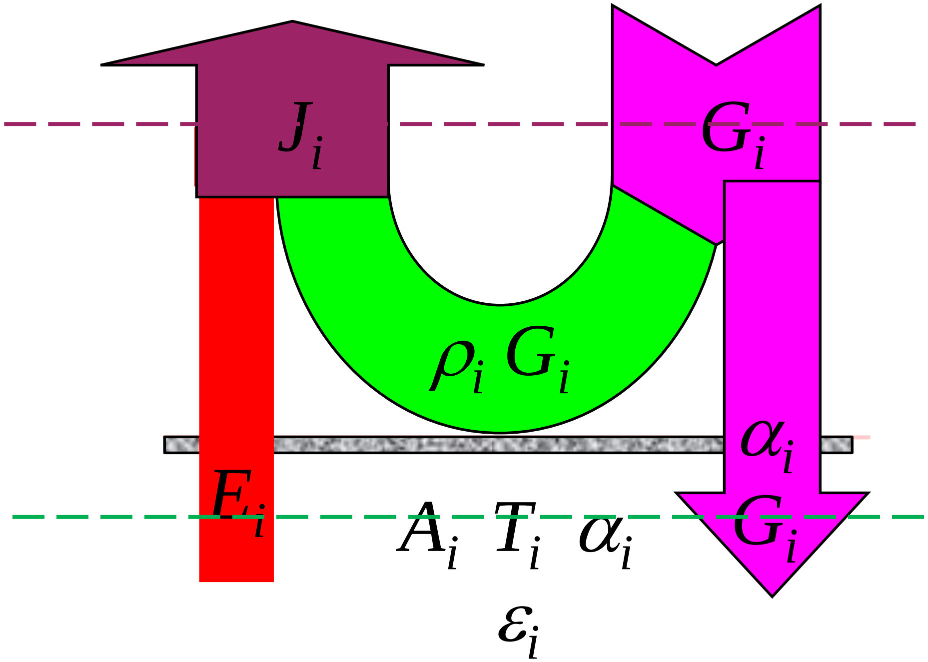
$$J_i = \varepsilon_i E_{bi} + (1 - \varepsilon_i) G_i$$

V. heat transfer within enclosure model

upon surface:

$$q_i = A_i (J_i - G_i)$$

$$q_i = A_i (E_i - \alpha_i G_i)$$



$$q_i = \frac{\varepsilon_i A_i}{1-\varepsilon_i}(E_{bi} - J_i)$$

$$q_i = \frac{E_{bi} - J_i}{\frac{1-\varepsilon_i}{\varepsilon_i A_i}} = \frac{E_{bi} - J_i}{R_a}$$

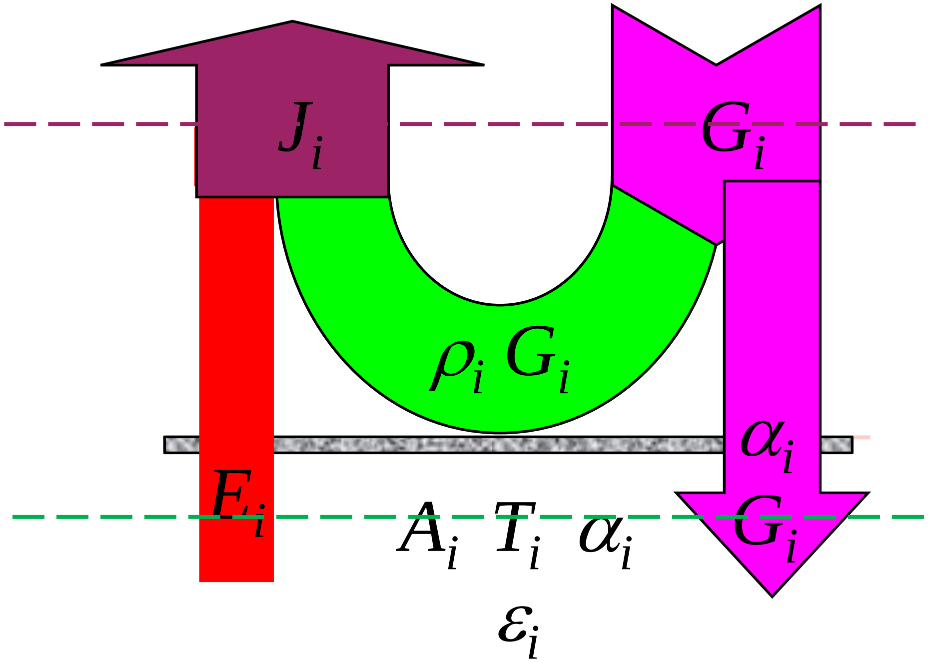
$$R_a = \frac{1-\varepsilon_i}{\varepsilon_i A_i} \text{ --- surface resistance}$$

V. heat transfer within enclosure model

upon surface:

$$q_i = A_i(J_i - G_i)$$

$$q_i = A_i(E_i - \alpha_i G_i)$$



$$q_i = \frac{\varepsilon_i A_i}{1-\varepsilon_i}(E_{bi} - J_i)$$

$$q_i = \frac{E_{bi} - J_i}{\dfrac{1-\varepsilon_i}{\varepsilon_i A_i}} = \frac{E_{bi} - J_i}{R_a}$$

$$R_a = \frac{1-\varepsilon_i}{\varepsilon_i A_i}$$ --- surface resistance

V. heat transfer within enclosure model

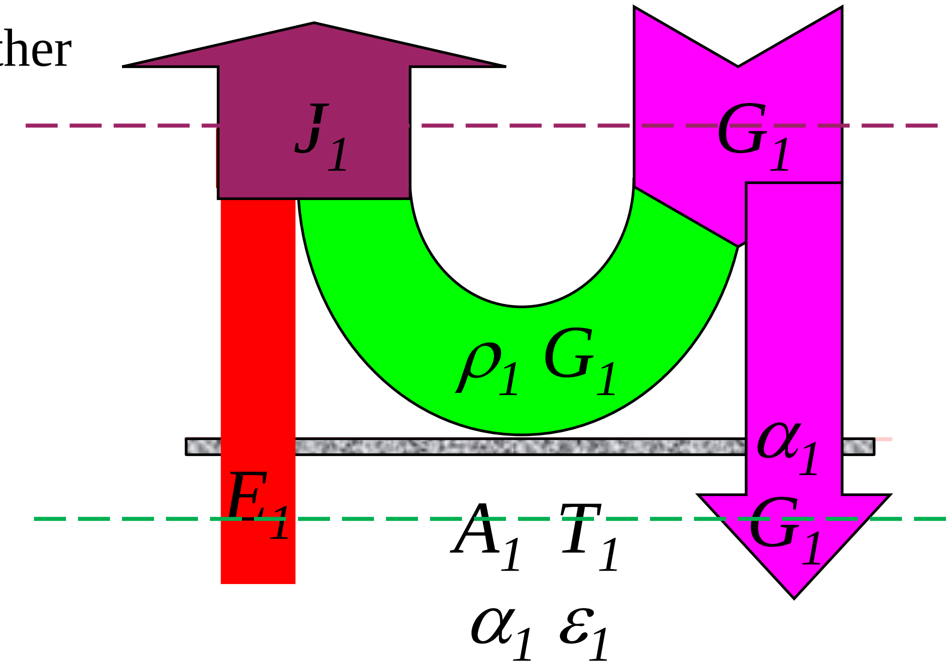
$$J = \frac{E}{\alpha} - \frac{1-\alpha}{\alpha} q = E_b - \left(\frac{1}{\varepsilon} - 1\right) q$$

Heat transfer between two gray surfaces:

If all energy is allocated between each other

$$\Rightarrow \quad \Phi_{1,2} = A_1 J_1 - A_1 G_1$$

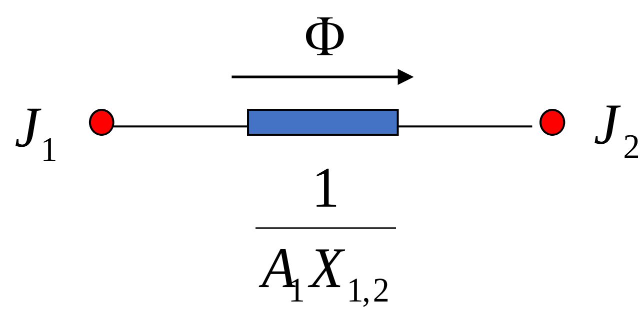
$$\Phi_{1,2} = A_1 J_1 - A_2 J_2$$



If not:

$$\Phi_{1,2} = A_1 J_1 X_{1,2} - A_2 J_2 X_{2,1}$$

$$\Phi_{1,2} = A_1 X_{1,2} (J_1 - J_2) = \frac{J_1 - J_2}{\dfrac{1}{A_1 X_{1,2}}}$$



Space resistance

V. heat transfer within enclosure model

Heat transfer between two gray surfaces:

$$J = \frac{E}{\alpha} - \frac{1-\alpha}{\alpha} q = E_b - \left(\frac{1}{\varepsilon} - 1\right) q$$

$$J_1 A_1 = A_1 E_{b1} - \left(\frac{1}{\varepsilon_1} - 1\right)\Phi_{1,2}$$

$$J_2 A_2 = A_2 E_{b2} - \left(\frac{1}{\varepsilon_2} - 1\right)\Phi_{2,1}$$

Energy conservation:

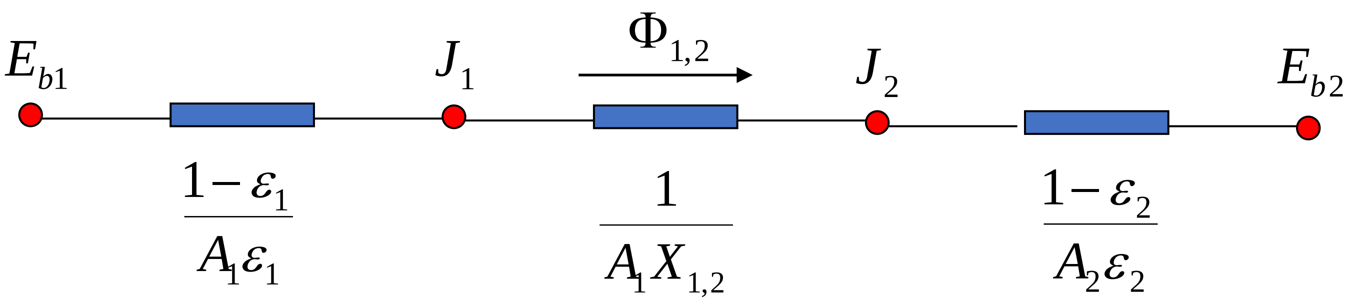
$$\Phi_{1,2} = -\Phi_{2,1}$$

$$\Phi_{1,2} = \frac{E_{b1} - E_{b2}}{\dfrac{1-\varepsilon_1}{\varepsilon_1 A_1} + \dfrac{1}{A_1 X_{1,2}} + \dfrac{1-\varepsilon_2}{\varepsilon_2 A_2}}$$

V. heat transfer within enclosure model

Heat transfer between two gray surfaces:

$$\boxed{\Phi_{1,2} = \frac{E_{b1} - E_{b2}}{\dfrac{1-\varepsilon_1}{\varepsilon_1 A_1} + \dfrac{1}{A_1 X_{1,2}} + \dfrac{1-\varepsilon_2}{\varepsilon_2 A_2}}}$$

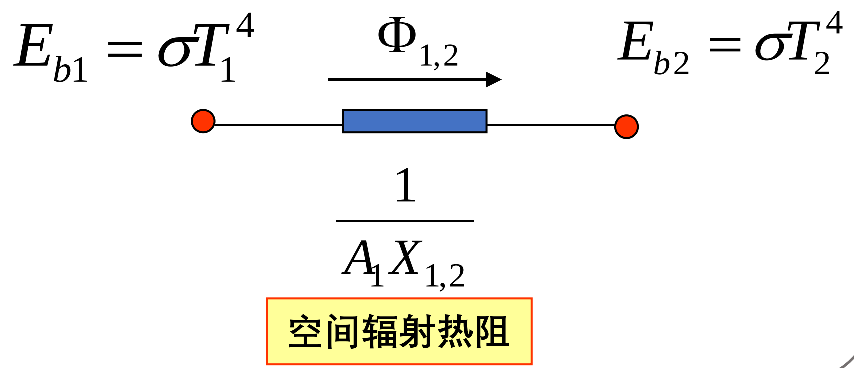


两封闭表面间的辐射传热网络图

V. heat transfer within enclosure model

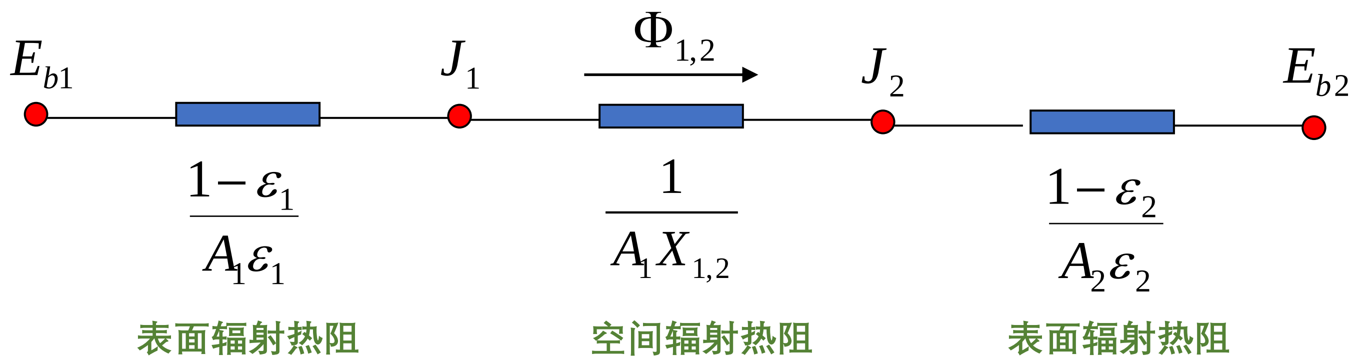
$$\Phi_{1,2} = \frac{(E_{b1} - E_{b2})}{\dfrac{1}{A_1 X_{1,2}}}$$

黑体辐射传热的等效网络



两漫灰面辐射传热的等效网络

$$\Phi_{1,2} = \frac{E_{b1} - E_{b2}}{\dfrac{1-\varepsilon_1}{\varepsilon_1 A_1} + \dfrac{1}{A_1 X_{1,2}} + \dfrac{1-\varepsilon_2}{\varepsilon_2 A_2}}$$



Compare with black body surface, the grey surface includes two surface resistance

V. heat transfer within enclosure model

Heat transfer between two gray surfaces:

$$\boxed{\Phi_{1,2} = \frac{E_{b1} - E_{b2}}{\dfrac{1-\varepsilon_1}{\varepsilon_1 A_1} + \dfrac{1}{A_1 X_{1,2}} + \dfrac{1-\varepsilon_2}{\varepsilon_2 A_2}}}$$

$$\Phi_{1,2} = \varepsilon_s A_1 X_{1,2}(E_{b1} - E_{b2})$$

$$\varepsilon_s = \frac{1}{1 + X_{1,2}\left(\dfrac{1}{\varepsilon_1} - 1\right) + X_{2,1}\left(\dfrac{1}{\varepsilon_2} - 1\right)}$$

系统发射率ε_s： Consider the effects of and concomitant reflection and absorption of grey surface . ε_s<1

V. heat transfer within enclosure model

Heat transfer between two gray surfaces:

$$\boxed{\Phi_{1,2} = \frac{E_{b1} - E_{b2}}{\dfrac{1-\varepsilon_1}{\varepsilon_1 A_1} + \dfrac{1}{A_1 X_{1,2}} + \dfrac{1-\varepsilon_2}{\varepsilon_2 A_2}}}$$

$$\Phi_{1,2} = \varepsilon_s A_1 X_{1,2}(E_{b1} - E_{b2})$$

$$\varepsilon_s = \frac{1}{1 + X_{1,2}\left(\dfrac{1}{\varepsilon_1} - 1\right) + X_{2,1}\left(\dfrac{1}{\varepsilon_2} - 1\right)}$$

系统发射率ε_s： Consider the effects of and concomitant reflection and absorption of grey surface . ε_s<1

Special conditions

(1) Surface 1 is convex or flat， $X_{1,2}=1$， therefore

$$\varepsilon_s = \frac{1}{1 + X_{1,2}\left(\frac{1}{\varepsilon_1} - 1\right) + X_{1,2}\frac{A_1}{A_2}\left(\frac{1}{\varepsilon_2} - 1\right)} \Rightarrow$$

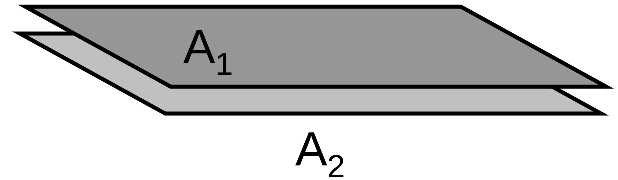
$$\boxed{\varepsilon_s = \frac{1}{\frac{1}{\varepsilon_1} + \frac{A_1}{A_2}\left(\frac{1}{\varepsilon_2} - 1\right)}}$$

$$\Phi_{1,2} = \frac{E_{b1} - E_{b2}}{\frac{1-\varepsilon_1}{\varepsilon_1 A_1} + \frac{1}{A_1 X_{1,2}} + \frac{1-\varepsilon_2}{\varepsilon_2 A_2}} = \frac{A_1(E_{b1} - E_{b2})}{\frac{1}{\varepsilon_1} + \frac{A_1}{A_2}\left(\frac{1}{\varepsilon_2} - 1\right)}$$

V. heat transfer within enclosure model

(2) Surface area A_1 is similar with A_2

特例：两平行平壁间的辐射传热



$$A_1 = A_2 = A \qquad X_{1,2} = X_{2,1} = 1$$

$$\Phi_{1,2} = \frac{E_{b1} - E_{b2}}{\dfrac{1-\varepsilon_1}{\varepsilon_1 A_1} + \dfrac{1}{A_1 X_{1,2}} + \dfrac{1-\varepsilon_2}{\varepsilon_2 A_2}} = \frac{A_1(E_{b1} - E_{b2})}{\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1}$$

$$= \varepsilon_s A \sigma_b (T^4 - T^4)$$

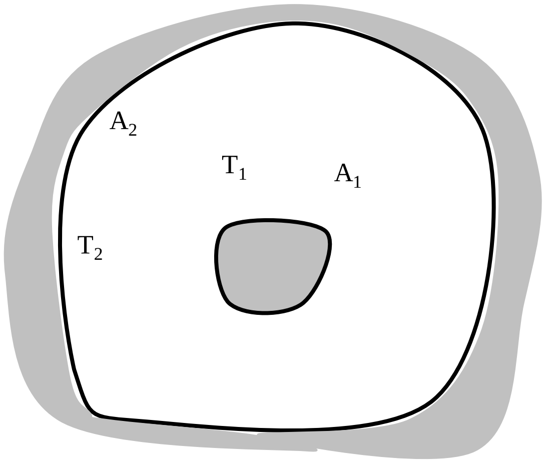
$$\boxed{\varepsilon_s = \frac{1}{\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1}}$$

V. heat transfer within enclosure model

(3) A_2 is much larger than A_1, thereby $A_1/A_2 \rightarrow 0$

特例：空腔与内包壁间的辐射传热

$$\Phi_{1,2} = \frac{A_1(E_{b1} - E_{b2})}{\frac{1}{\varepsilon_1} + \frac{A_1}{A_2}(\frac{1}{\varepsilon_2} - 1)} = \frac{A_1(E_{b1} - E_{b2})}{\frac{1}{\varepsilon_1} + \frac{A_1}{A_2}\left(\frac{1}{\varepsilon_2} - 1\right)}$$



如车间内的采暖板、热力管道，气体容器或管道内的测温传感器等都属于此种情况。

$$\Phi_{1,2} = \varepsilon_1 A_1(E_{b1} - E_{b2})$$

$$\boxed{\varepsilon_s = \varepsilon_1}$$

V. heat transfer within enclosure model

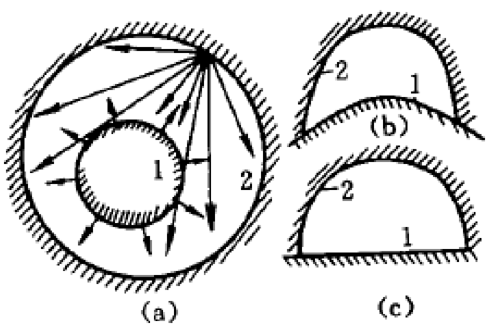
summary

$$\Phi_{1,2} = \varepsilon_s A_1 X_{1,2}(E_{b1} - E_{b2})$$

简化1

表面1为非凹面，$X_{1,1}=0$

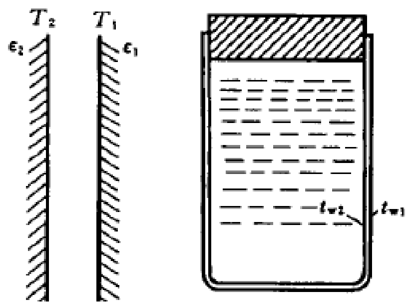
$X_{1,2}=1$



$$\varepsilon_s = \frac{1}{\frac{1}{\varepsilon_1} + \frac{A_1}{A_2}\left(\frac{1}{\varepsilon_2} - 1\right)}$$

简化2

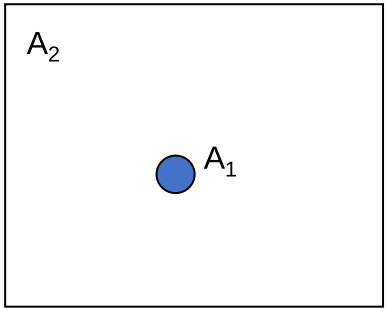
$X_{1,2}=1$，表面积$A_1 \approx A_2$



$$\varepsilon_s = \frac{1}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$$

简化3

$X_{1,2}=1$，表面积$A_1 \ll A_2$



$$\varepsilon_s = \varepsilon_1$$

1. 控制物体表面间辐射传热的方法

$$\Phi_{1,2}=\frac{E_{b1}-E_{b2}}{\frac{1-\varepsilon_1}{\varepsilon_1 A_1}+\frac{1}{A_1 X_{1,2}}+\frac{1-\varepsilon_2}{\varepsilon_2 A_2}}$$

(1) 控制表面热阻

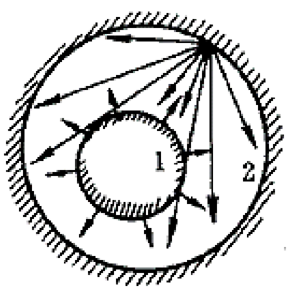
控制表面辐射热阻

强化 $\Phi_{1,2}\uparrow \quad \frac{1-\varepsilon}{\varepsilon A}\downarrow$

1. 增加辐射表面积 A
2. 增加传热表面的发射率 ε

$A_2>A_1$，所以增加表面积为A_1的物体的发射率ε_1更加有效。（减小串联环节的最大热阻项）

例：两同心圆柱表面间的辐射传热



$A_1=1$，$A_2=10$ 表面热阻

$\varepsilon_1=\varepsilon_2=0.5$	1.1
$\varepsilon_1=0.5 \quad \varepsilon_2=0.8$	1.025
$\varepsilon_1=0.8 \quad \varepsilon_2=0.5$	0.35

控制表面辐射热阻

削弱 $\Phi_{1,2}\downarrow \quad \frac{1-\varepsilon}{\varepsilon A}\uparrow$

1. 减小传热表面的发射率 ε

工厂蒸汽管道外敷铝箔

2. 两辐射表面间安插遮热板

$$\Phi_{1,2}=\frac{E_{b1}-E_{b2}}{\frac{1-\varepsilon_1}{\varepsilon_1 A_1}+\frac{1}{A_1 X_{1,2}}+\frac{1-\varepsilon_2}{\varepsilon_2 A_2}}$$

(2)控制空间热阻

实际物体对太阳能的吸收

强化 $\frac{\alpha_s}{\varepsilon}\uparrow$ 太阳能集热器的光谱选择性涂层

削弱 $\frac{\alpha_s}{\varepsilon}\downarrow$ 变压器油漆成浅色；人造卫星采用对太阳能吸收比小的材料作表面涂层

控制空间辐射热阻

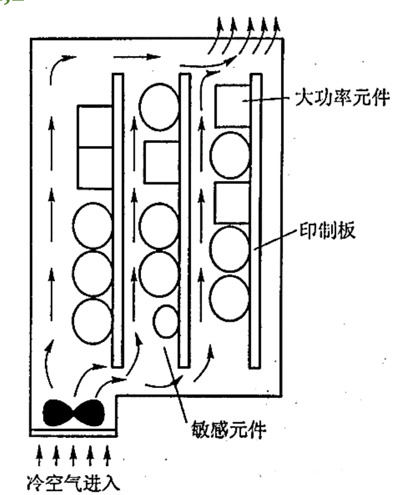
强化 $\Phi_{1,2}\uparrow$ $\frac{1}{A_i X_{i,j}}\downarrow$

1. 增加辐射表面积 A

2. 增加角系数 $X_{1,2}$

图9-39：

将高温部件置于相对冷表面角系数较大的位置。



Chapter 1:

1. Difference between heat transfer and thermodynamics
2. Three types of heat transfer: principle and representative rules
3. What's the thermal resistance
4. Analyze a heat transfer problem with thermal resistance network

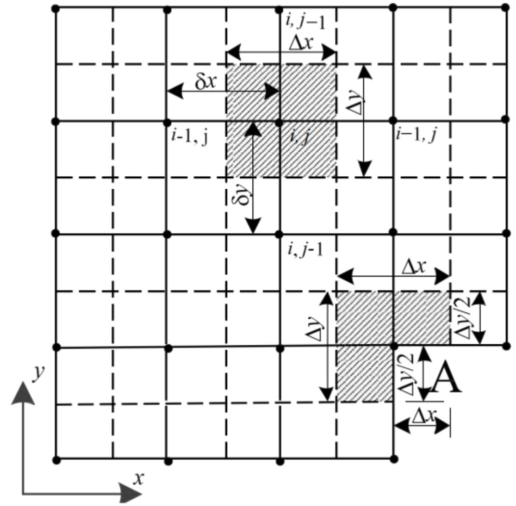
Chapter 2:

1. Definition of thermal conductivity
2. Fourier law
3. Derive the heat diffusion equation (cylindrical and various λ),analyze T distribution
4. Three boundary conditions
5. Multiple wall, insulating layers.

Chapter 3:

1. Bi number
2. Lumped parameter method
3. Semi-infinite plane

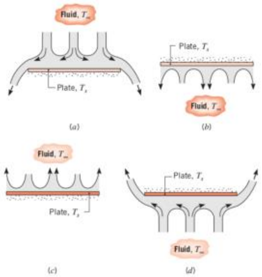
Chapter 4:



Chapter 5:

1. Newton cooling equation
2. Convective heat transfer differential equation
3. Flow state, boundary layer properties(velocity and temperature)
4. Flow through plate, laminar analytical solution. Boundary thickness at x.
5. Nusselt number.
6. Similarity principle to get nu’=nu’’, re’=re’’, Pr’=Pr’’
7. Nu =f (re,Pr)

Chapter 6:
review



Chapter 7:
1.Boiling stages
2.The working principle of heat pipe
3.CHF
4.Condensation, types, properties

Chapter 8-9:
1. Definition
2. G and J
3. Angle factor, calculate
4. Heat transfer between two grey surfaces