北京航空航天大學
BEIHANG UNIVERSITY

飞行力学
Flight Mechanics

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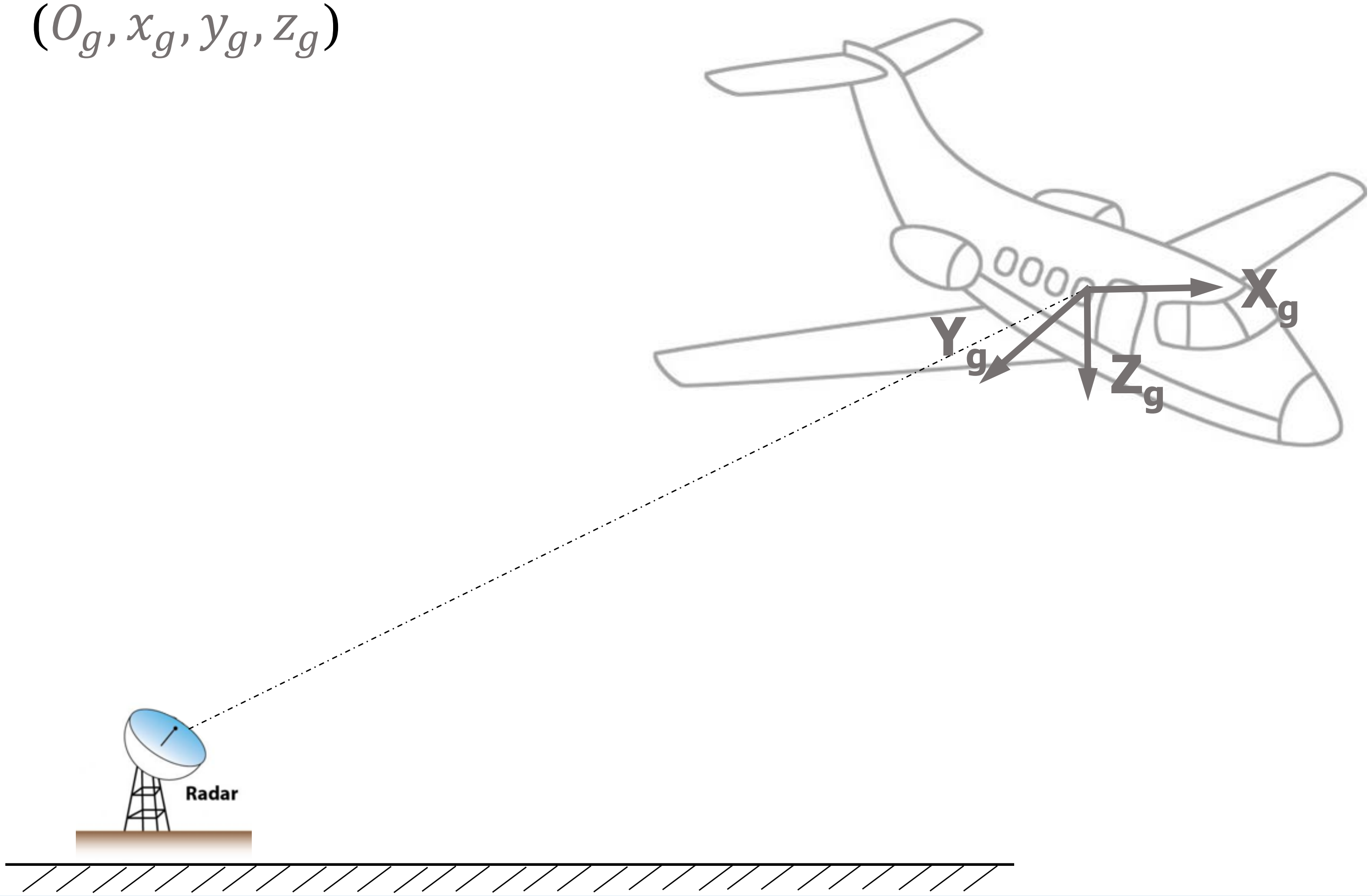
Contents

Question

- How to track a fighter jet
- How to describe its attitude?
- How to describe its motion?

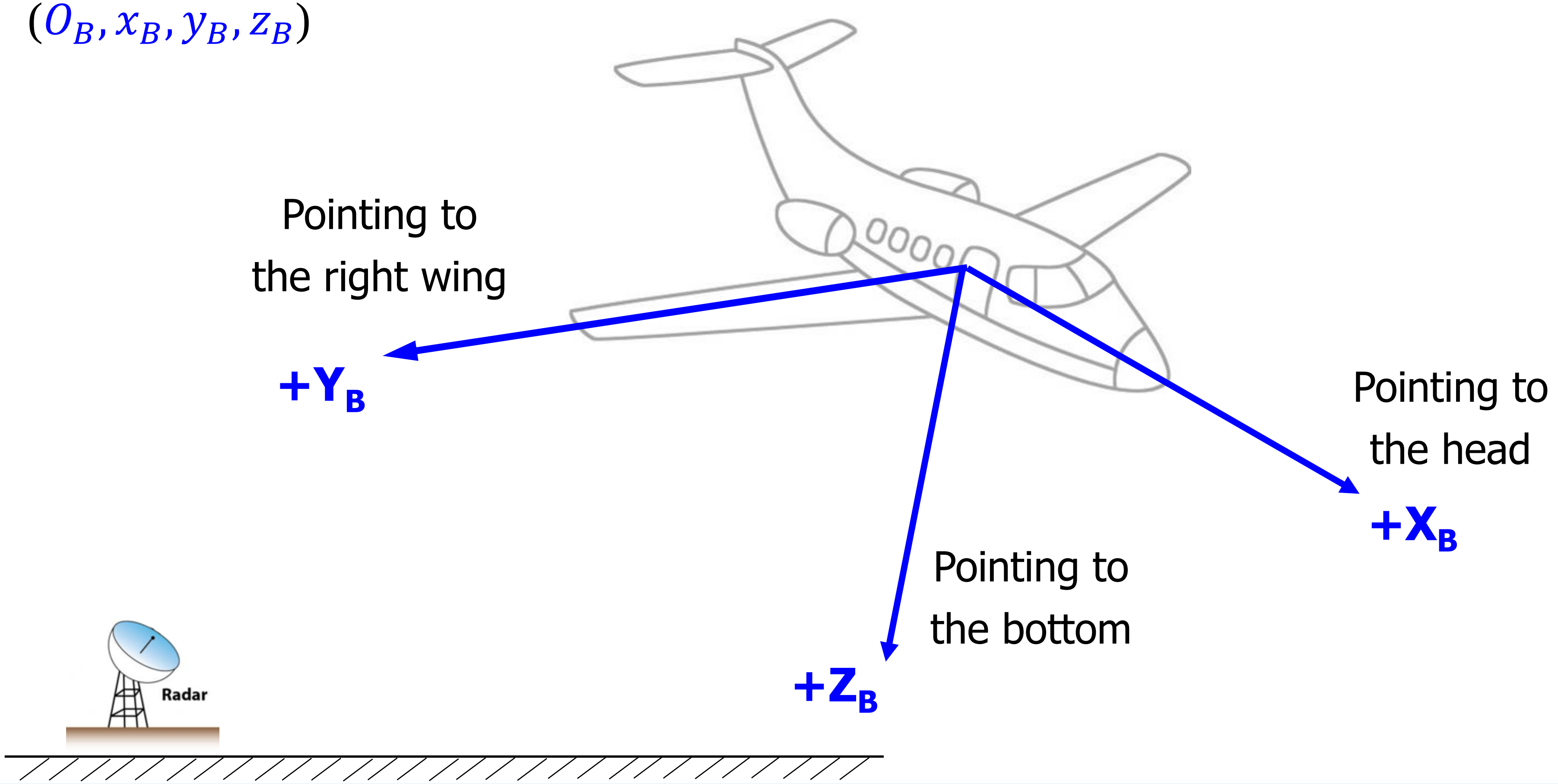
The Earth Axis System

(O_g, x_g, y_g, z_g)



The Body-fixed Frame

(O_B, x_B, y_B, z_B)



The Body-fixed Frame

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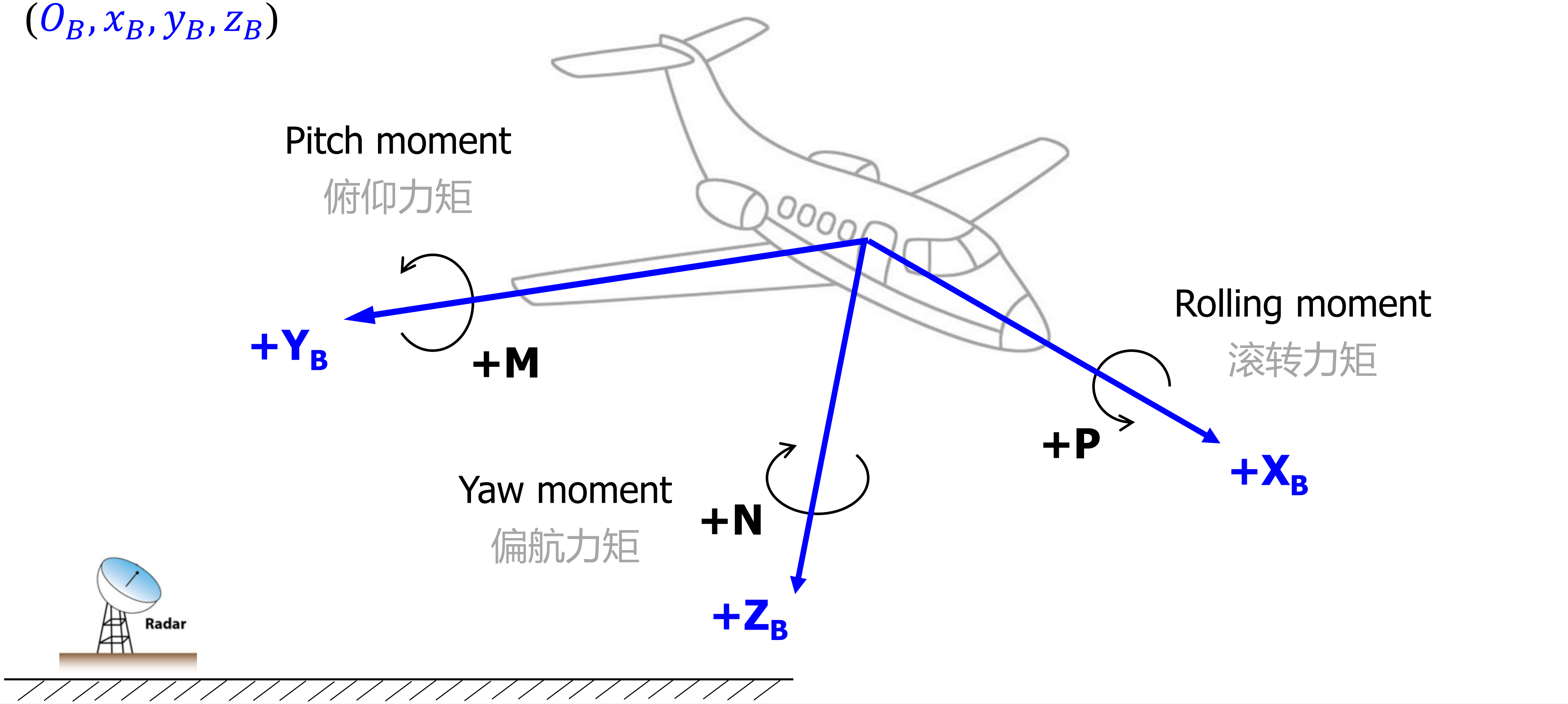
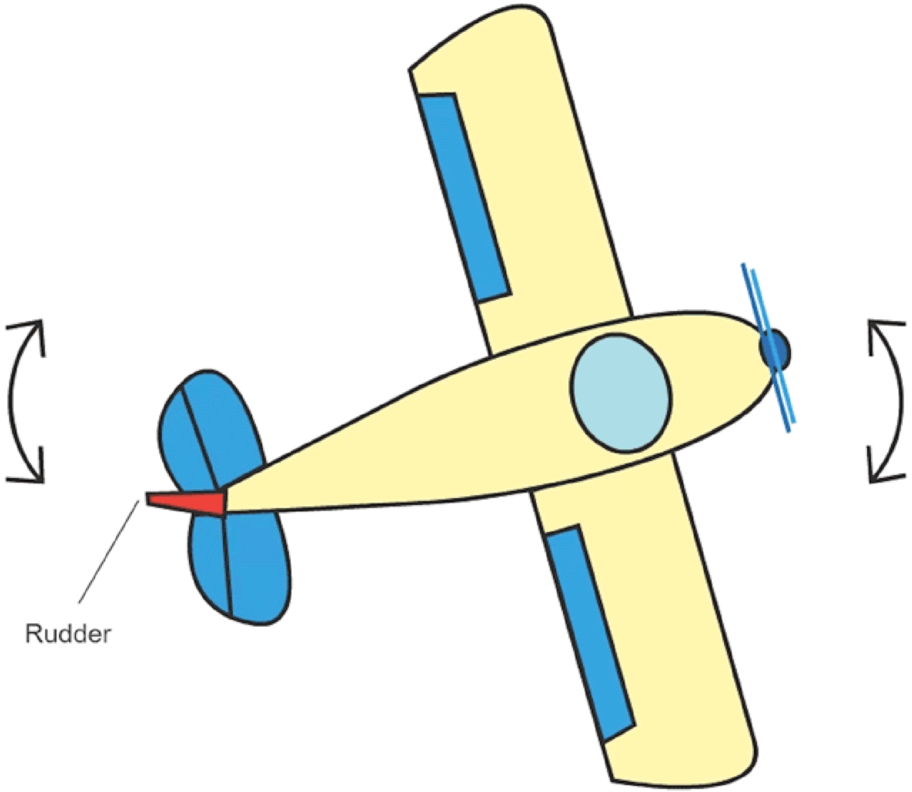
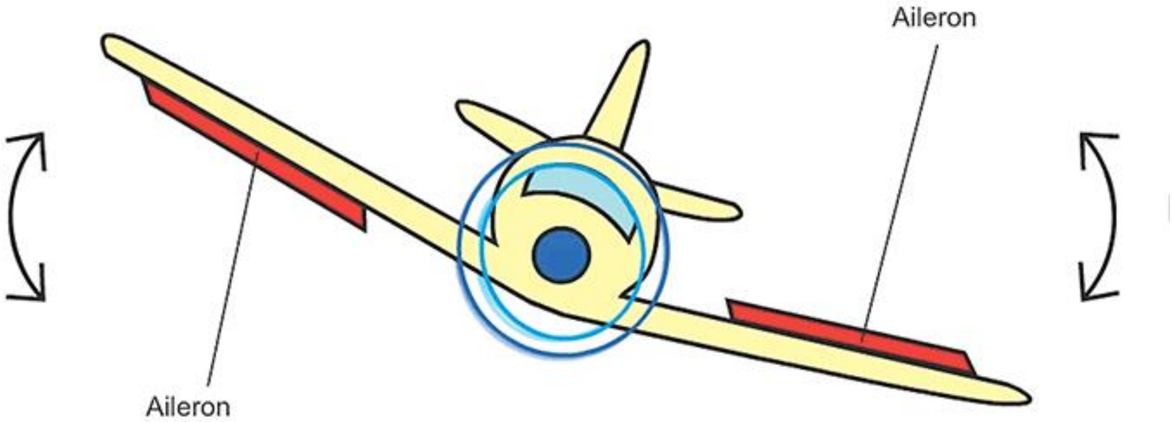


Aircraft Yaw Motion



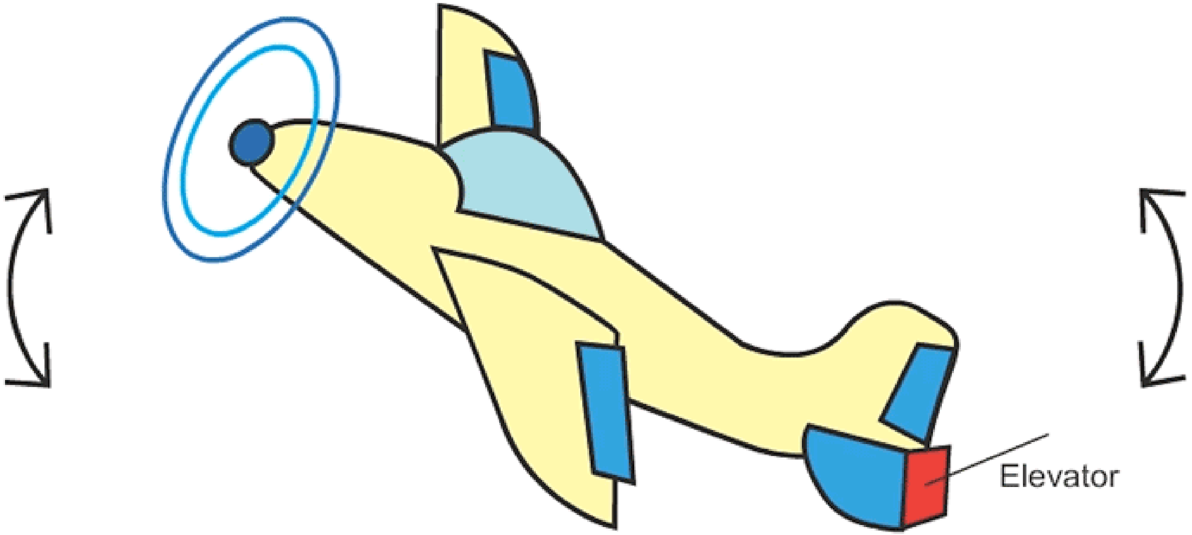
Source: https://howthingsfly.si.edu/flight-dynamics/roll-pitch-and-yaw

Aircraft Roll Motion



Source: https://howthingsfly.si.edu/flight-dynamics/roll-pitch-and-yaw

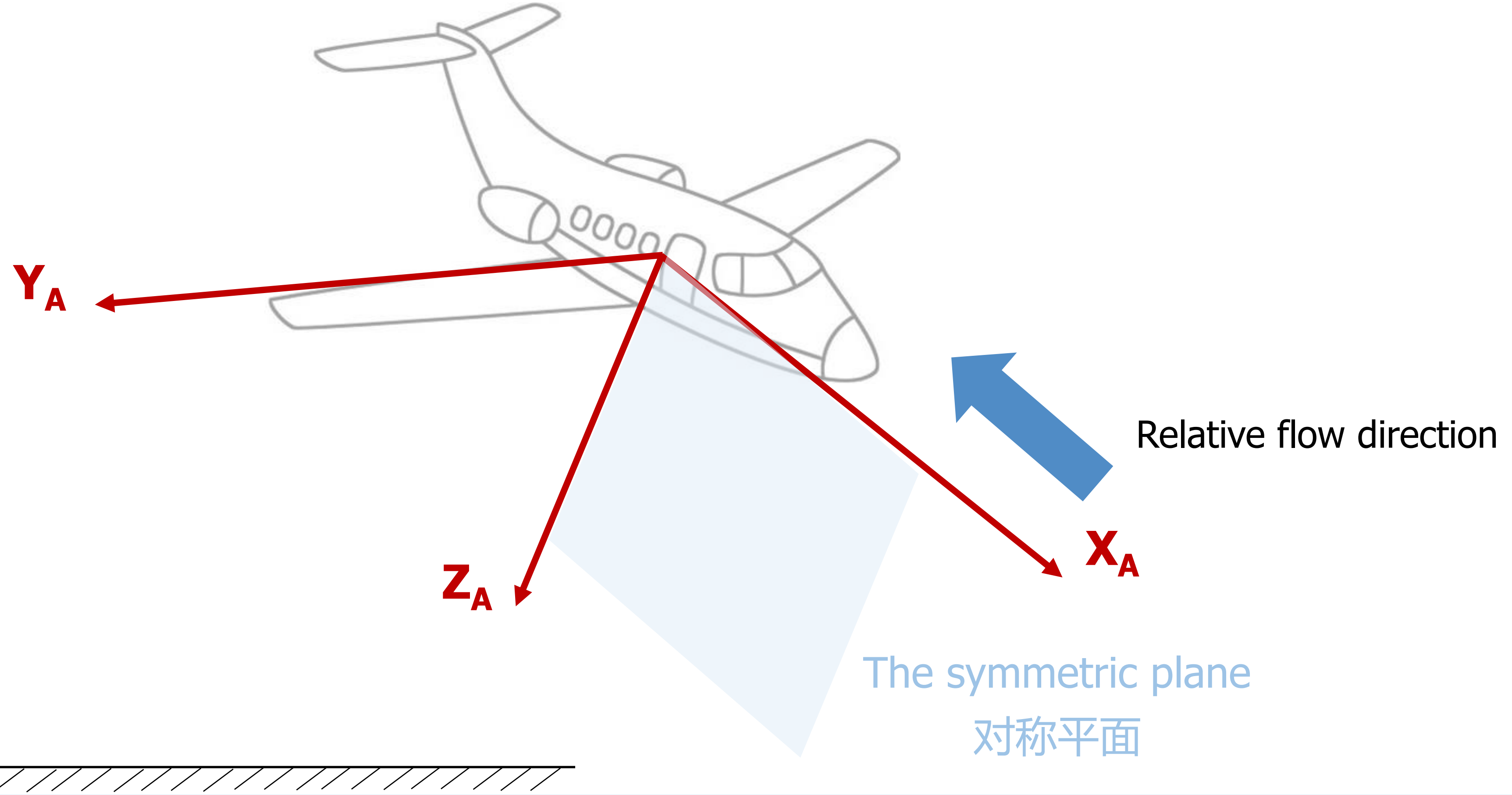
Aircraft Pitch Motion



Source: https://howthingsfly.si.edu/flight-dynamics/roll-pitch-and-yaw

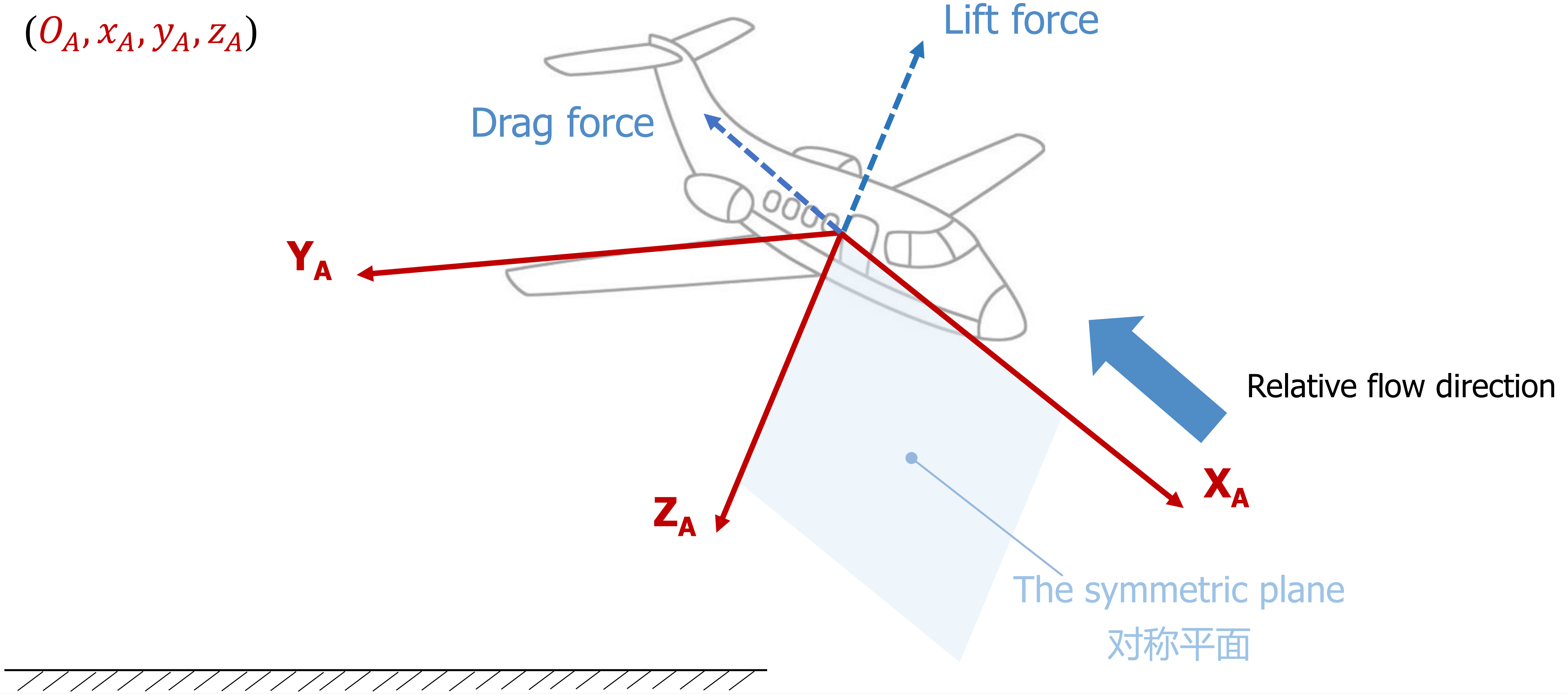
The Aerodynamic Frame

(O_A, x_A, y_A, z_A)



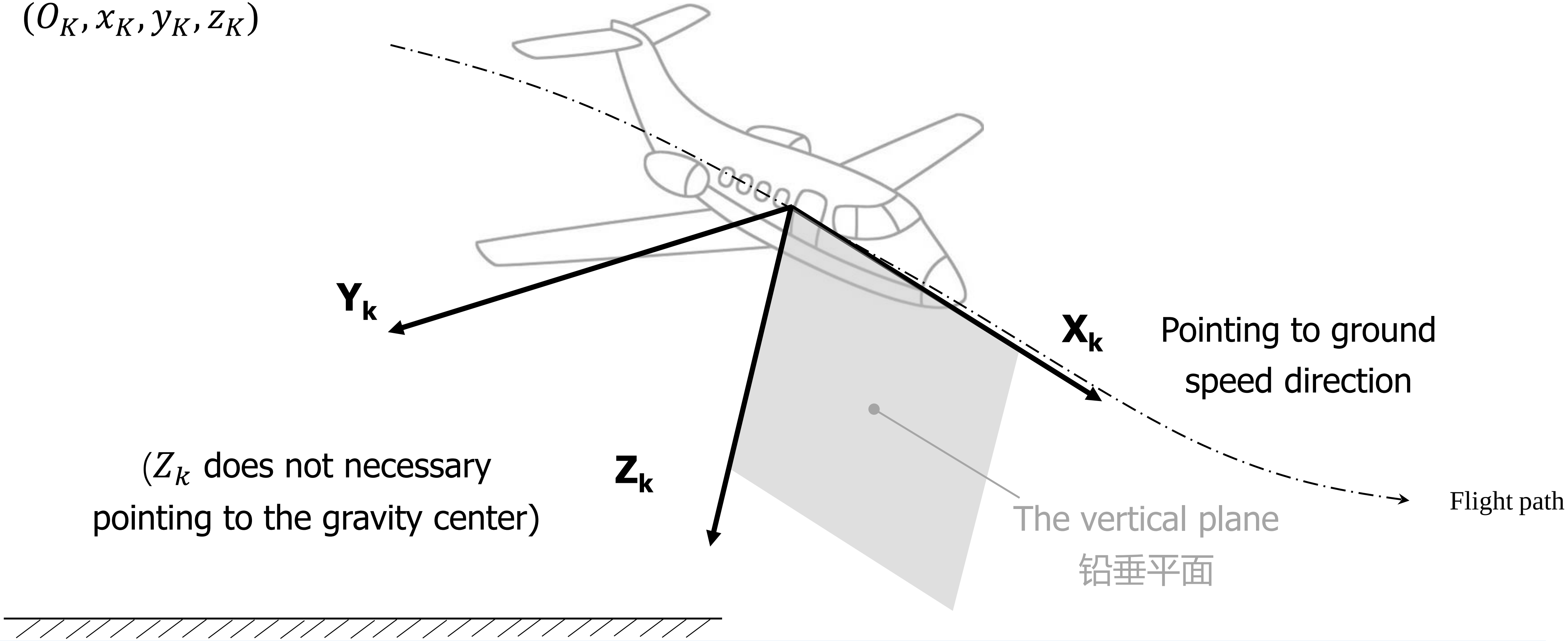
The Aerodynamic Frame

(O_A, x_A, y_A, z_A)

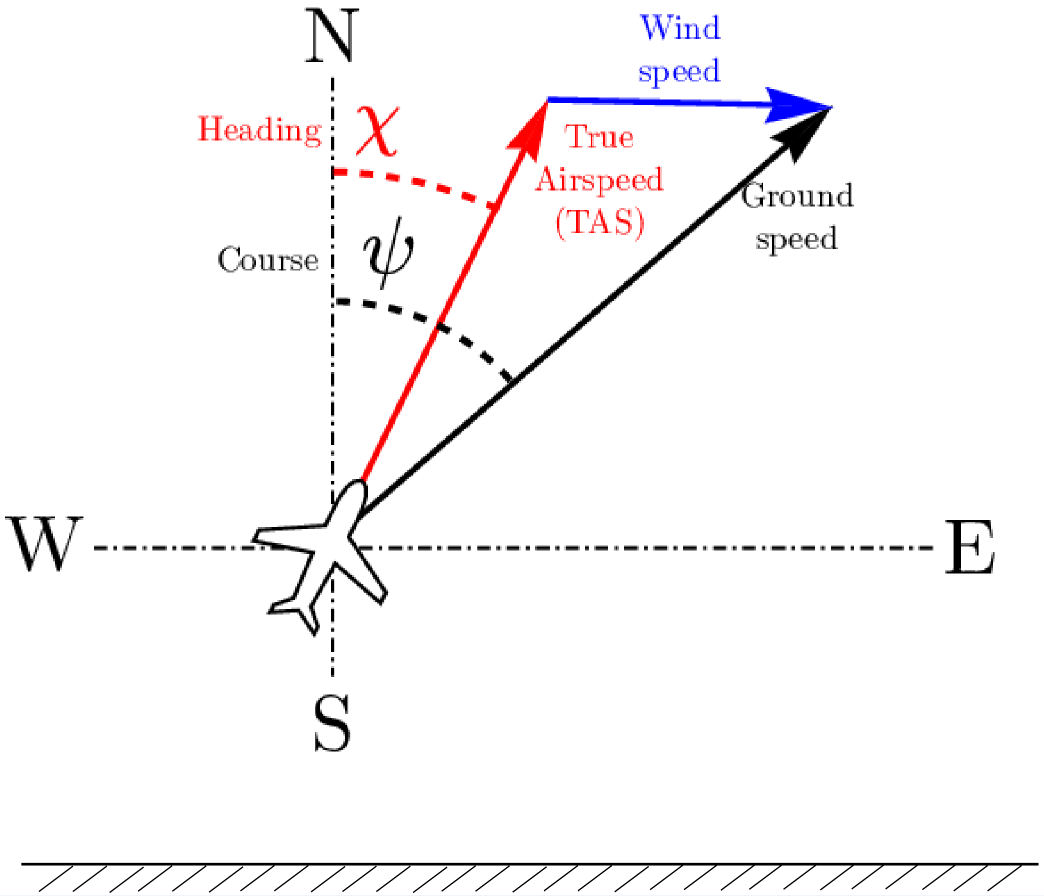


The Kinematic Frame

(O_K, x_K, y_K, z_K)



The Ground Speed



If there's no wind (air is still), the ground speed equals to the true airspeed

Ox_A coincides with Ox_K

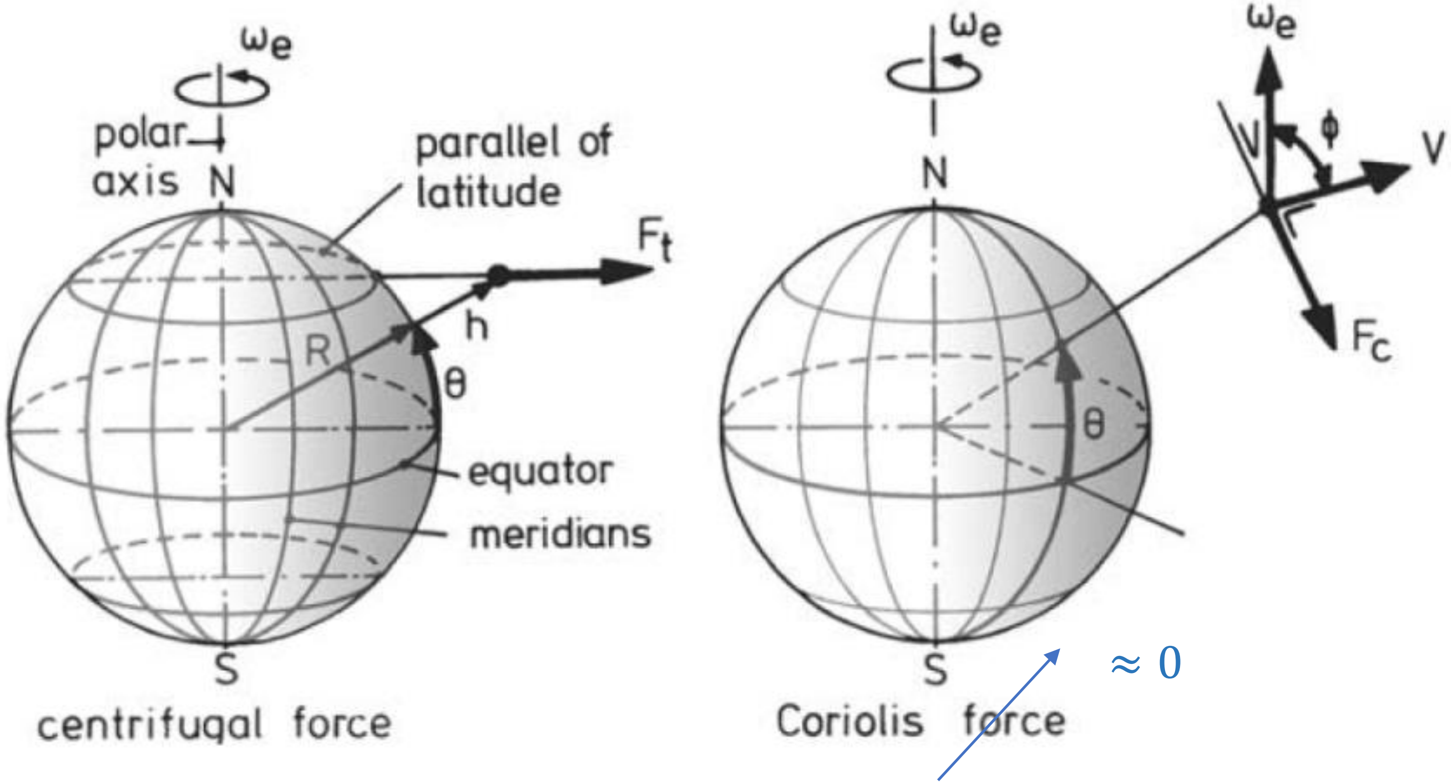
The Newton's Law

- Newton's laws only hold for Inertial frame
- A rotating frame of reference is not an inertial frame
- We assume earth is inertial frame of reference

$$\vec{F} = m\frac{d\vec{V}}{dt}$$

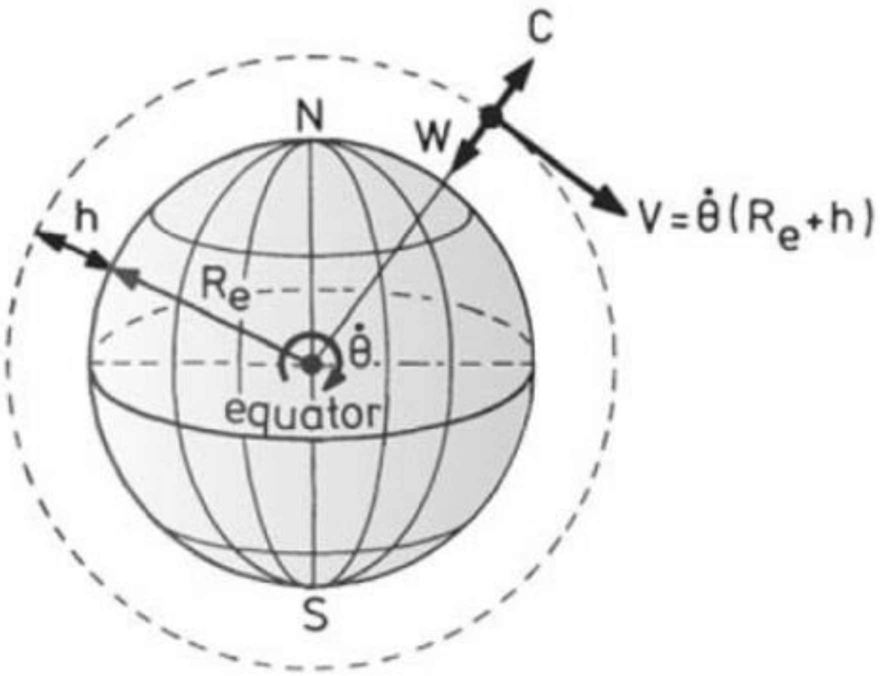
Assumptions

1. the earth is non-rotating



Assumptions

2. the earth is flat



Centrifugal Force $\approx \mathbf{0}$

Assumptions

3. the gravity is constant

Gravitational acceleration at h altitude:

$$g_h = g_0 \frac{R_e^2}{(R_e + h)^2}$$

Since:

$$R_e \gg h$$

We have:

$$g_h \approx g_0$$

Summary

Inertial coordinate frames

- Earth axis system: (O_g, x_g, y_g, z_g)

Other common coordinate frames

- **B**ody-fixed frame: (O_B, x_B, y_B, z_B)
- **A**erodynamic frame: (O_A, x_A, y_A, z_A)
- **K**inematic frame: (O_K, x_K, y_K, z_K)

Summary

Inertial coordinate frames

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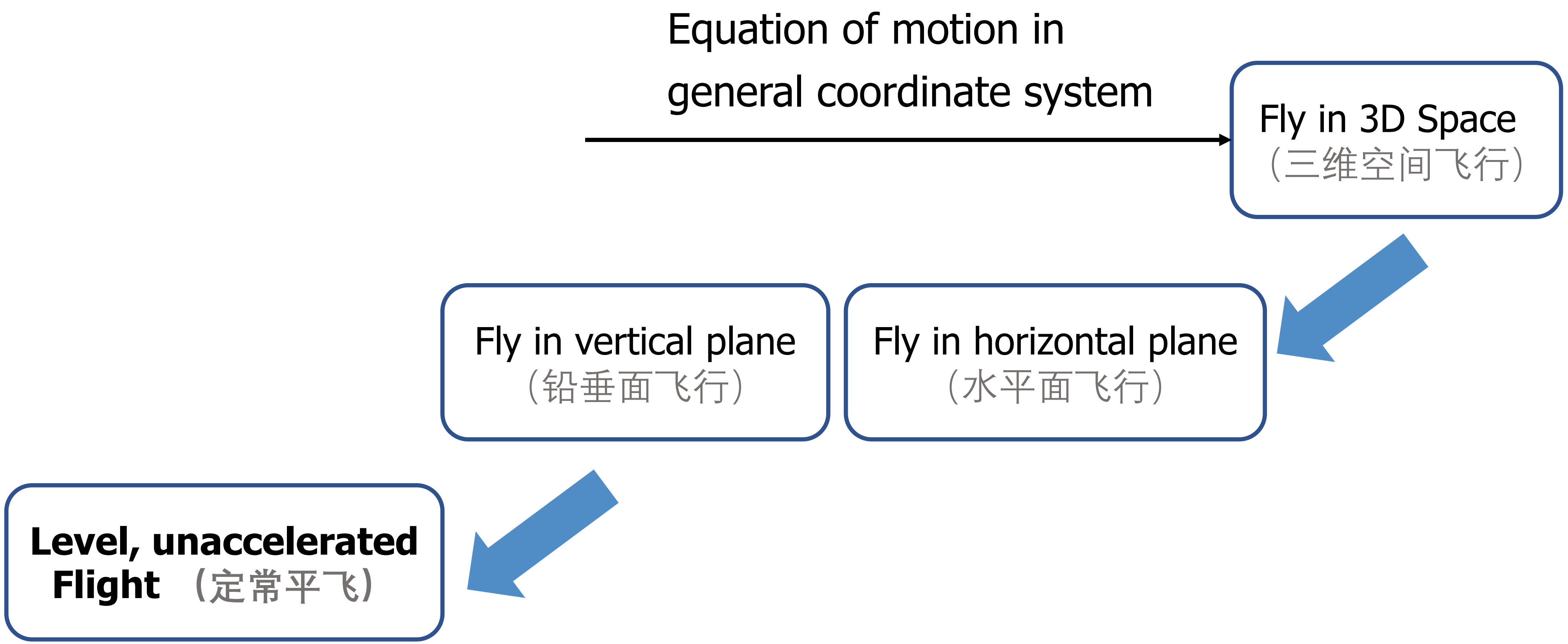
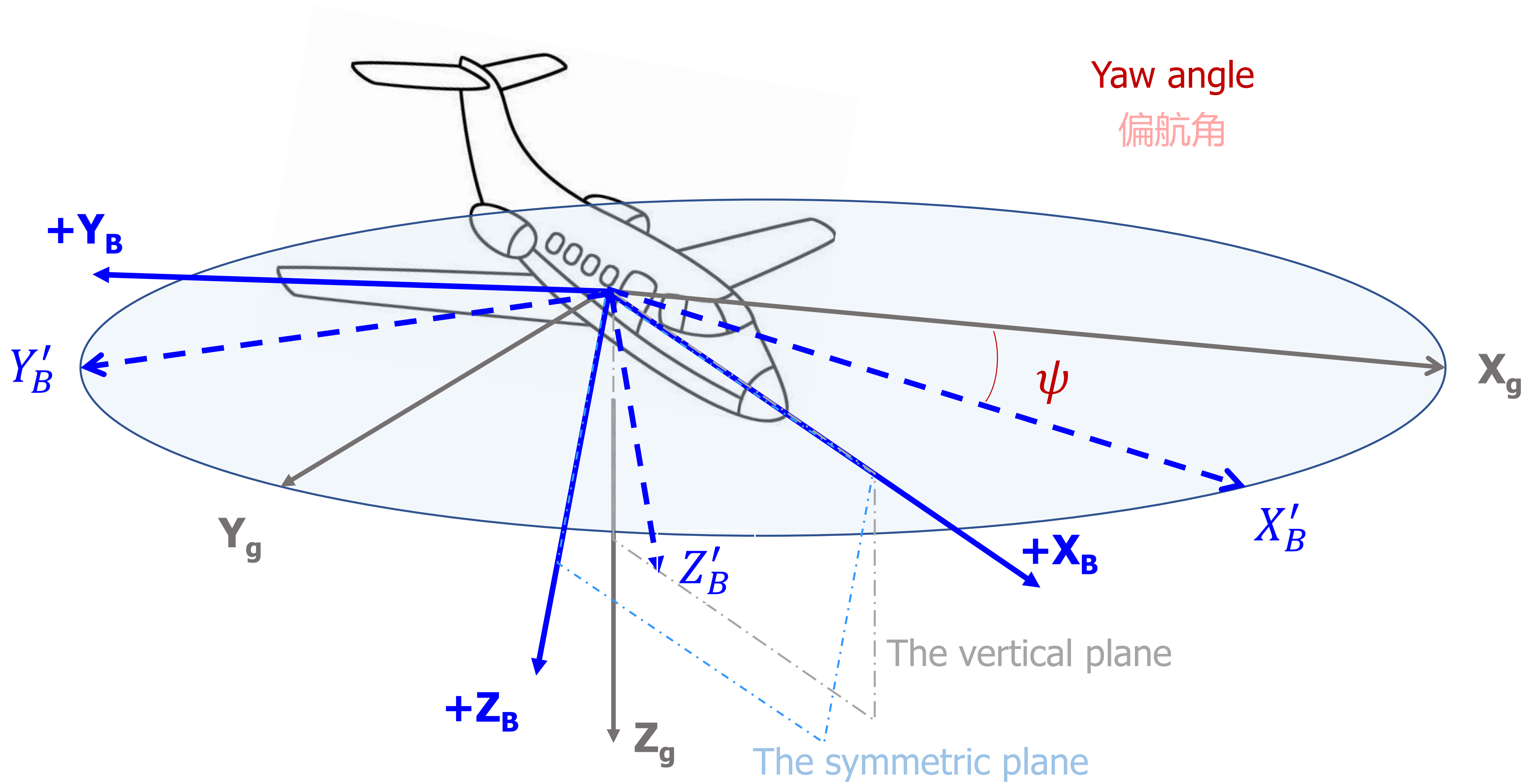
Coordinate transform

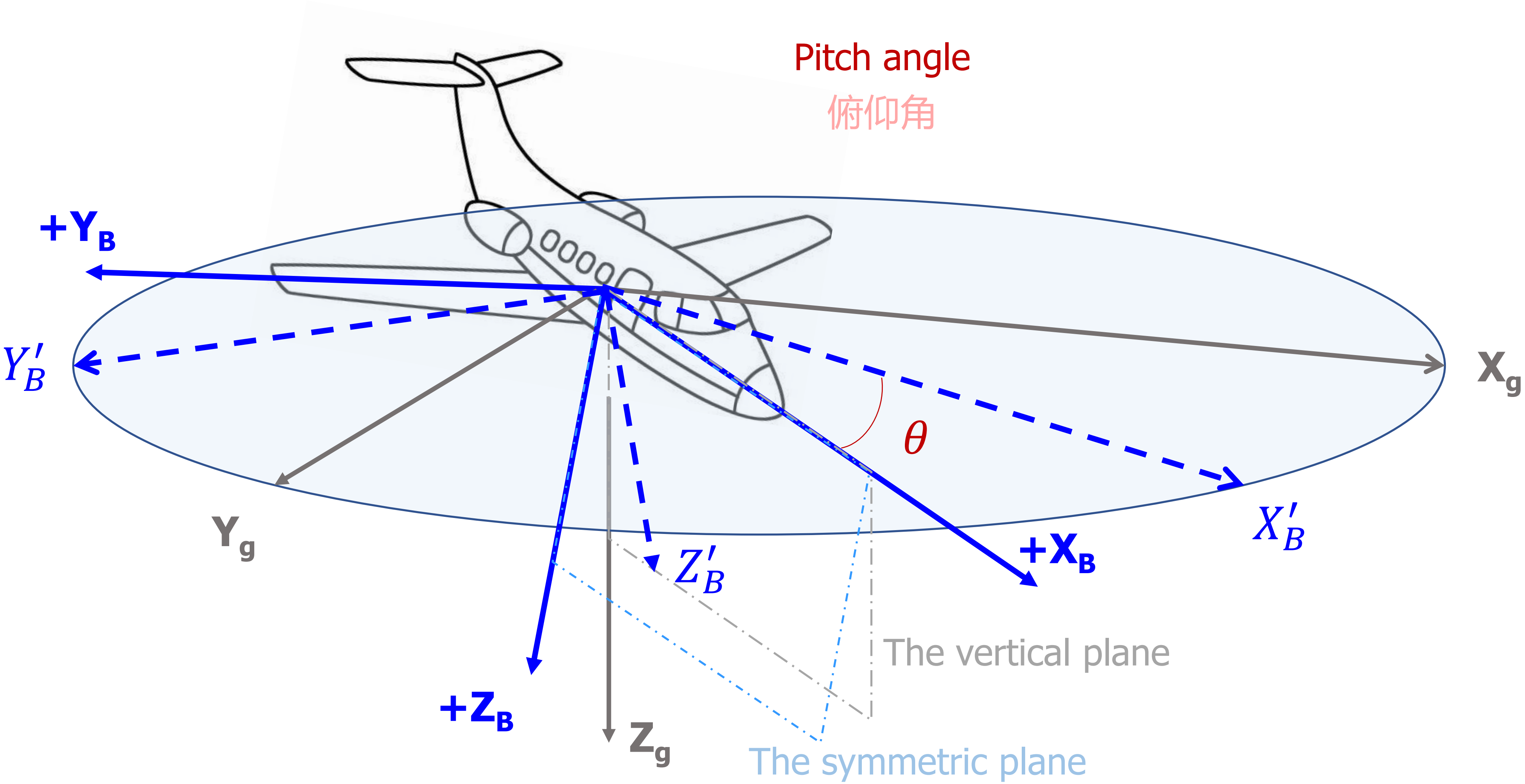
Roadmap


Equation of motion in
general coordinate system
Fly in 3D Space
（三维空间飞行）
Fly in vertical plane
（铅垂面飞行）
Fly in horizontal plane
（水平面飞行）
Level, unaccelerated
Flight （定常平飞）

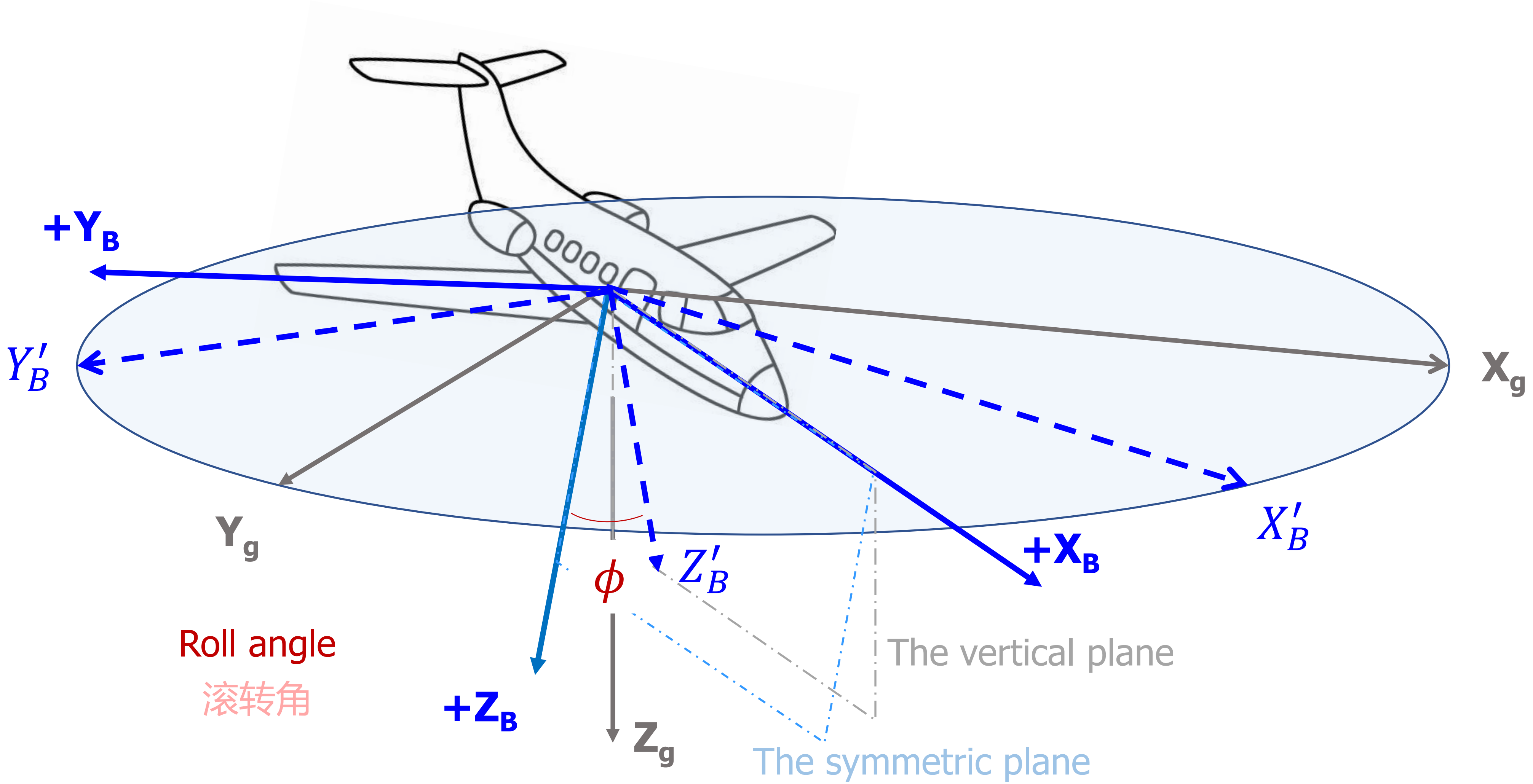
Aircraft Attitude



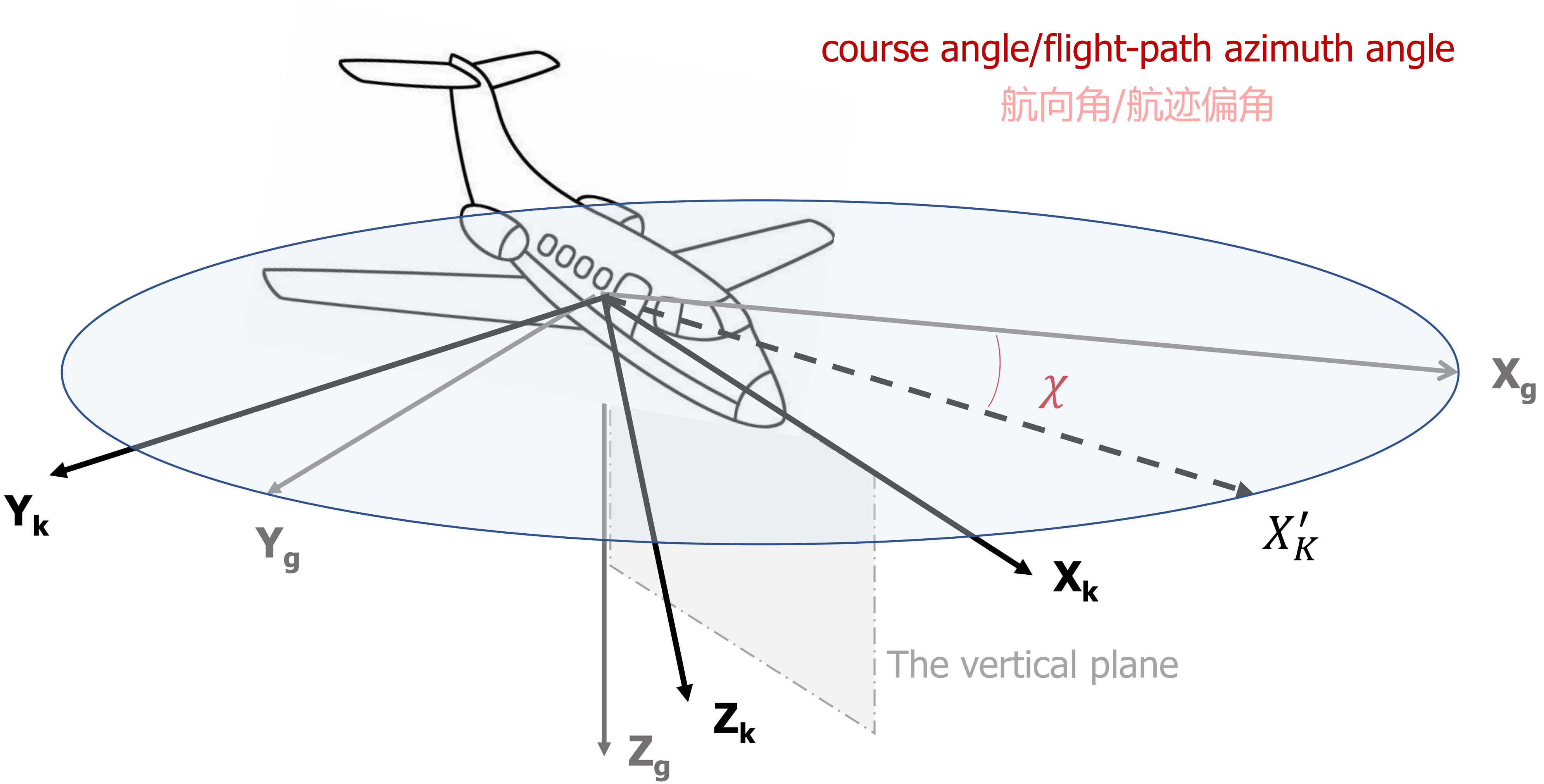
Aircraft Attitude



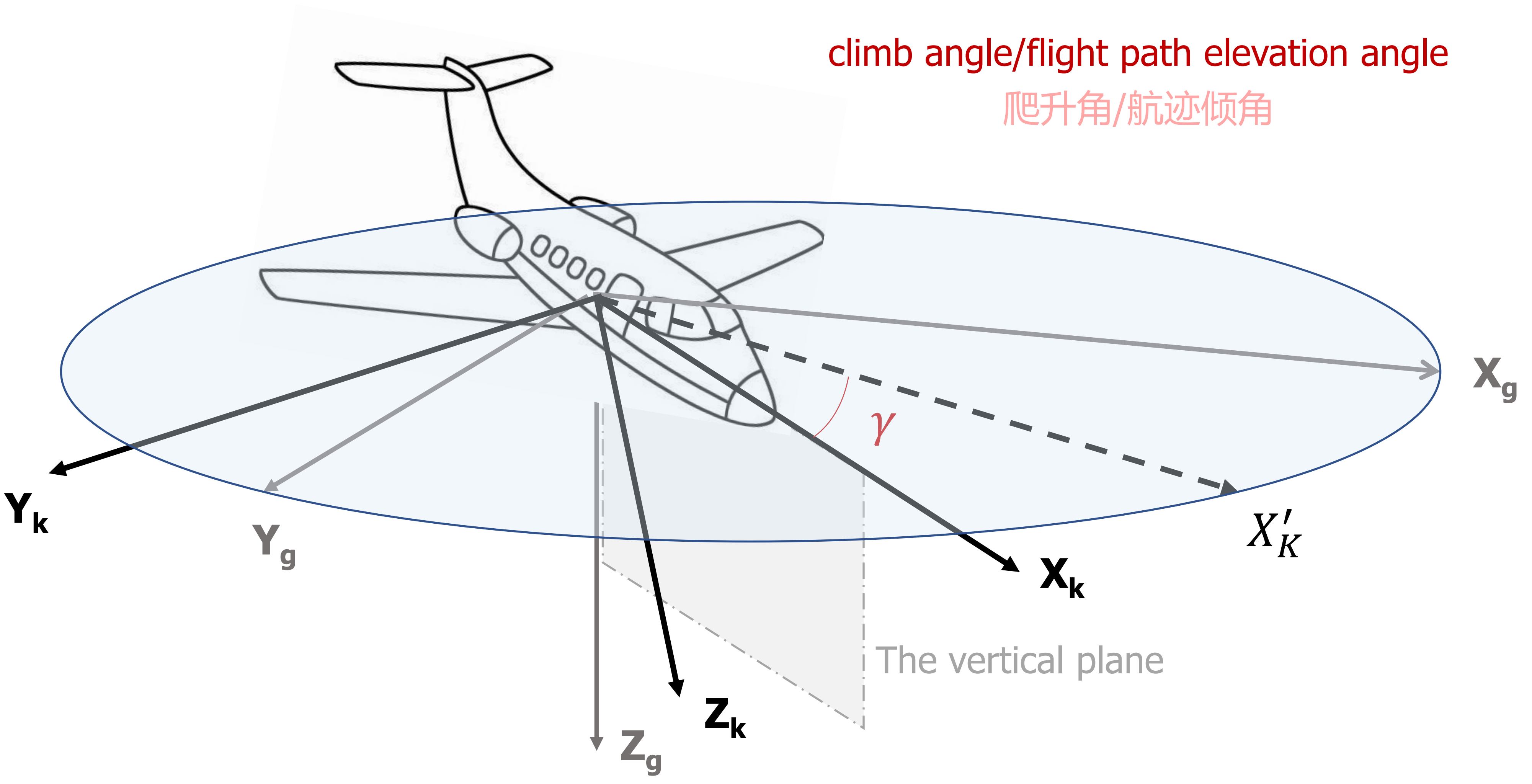
Aircraft Attitude



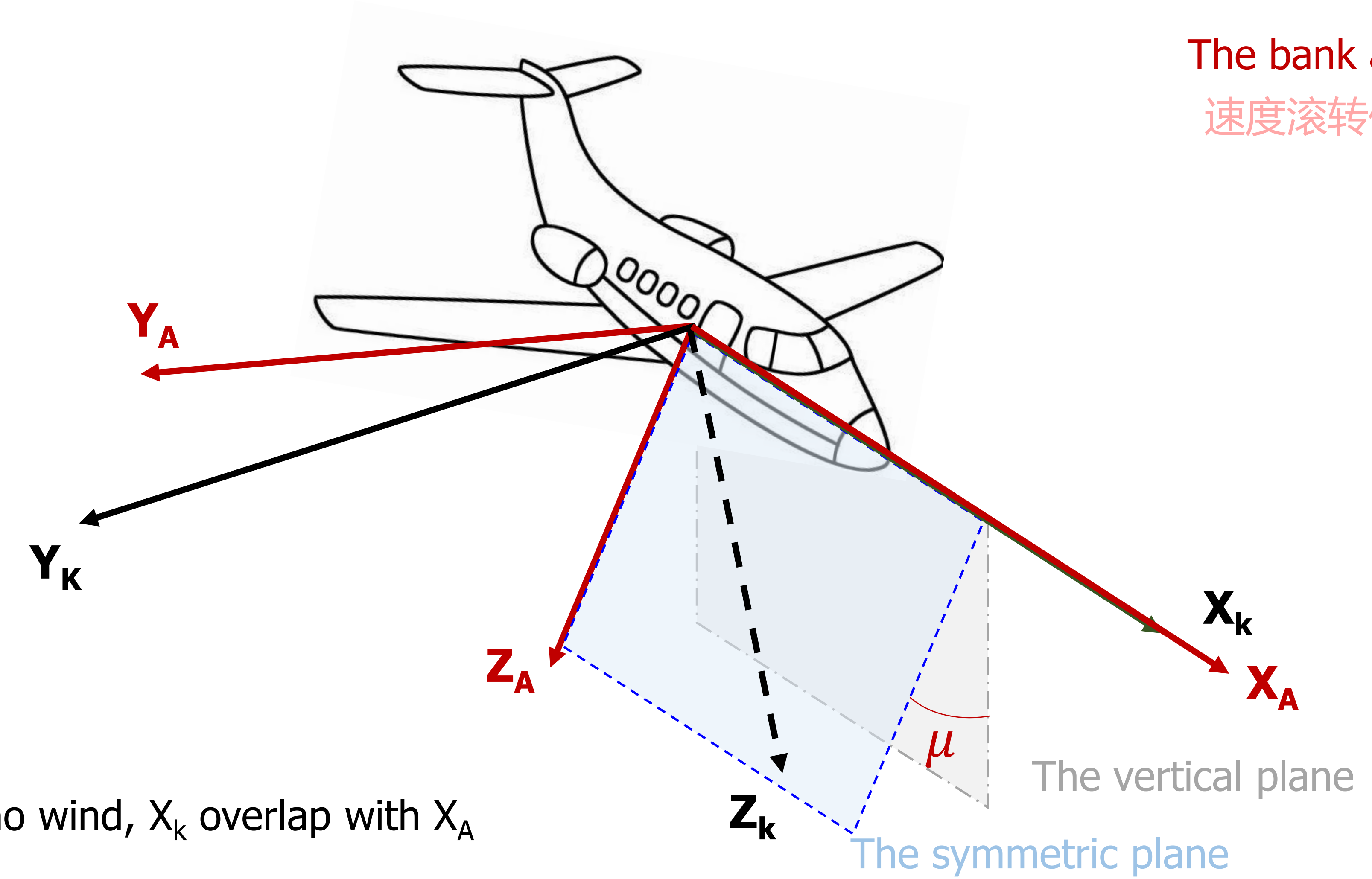
Flight Path Angle



Flight Path Angle



The Bank Angle



Assume no wind, X_k overlap with X_A

Control Surfaces of J-20

Summary

- Relate body-fixed frame to the earth axis system

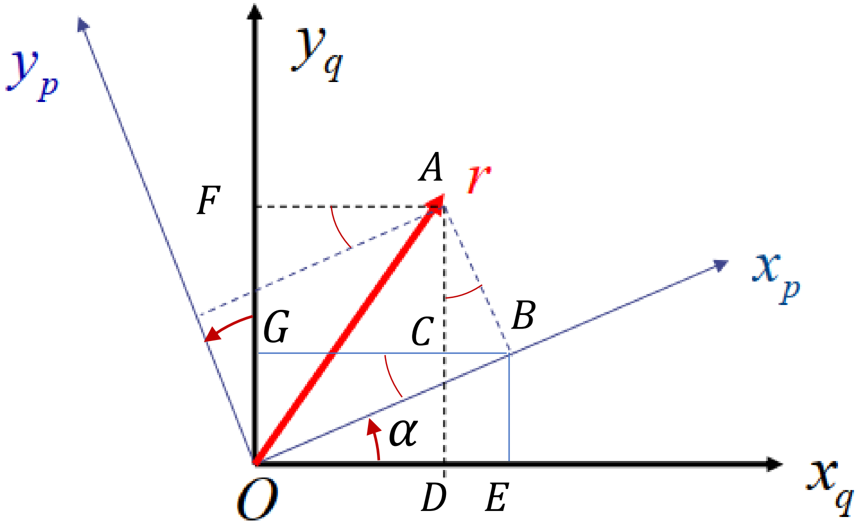
Yaw angle 偏航角	Pitch angle 俯仰角	Roll angle 滚转角

- Relate aerodynamic frame to the earth axis system

Course angle/ Flight path azimuth angle 航向角、航迹偏角	Climb angle/ Flight path elevation angle 爬升角/航迹倾角	Bank angle 速度滚转角

2D Transformation

- Assume the angle between Ox_py_p and Ox_qy_q is α



$$\begin{bmatrix} x_q \\ y_q \end{bmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{bmatrix} x_p \\ y_p \end{bmatrix}$$

Error in the Eq. (1.18)- (1.19) of textbook

Transformation Matrix

$$L_{qp} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \quad p \to q$$

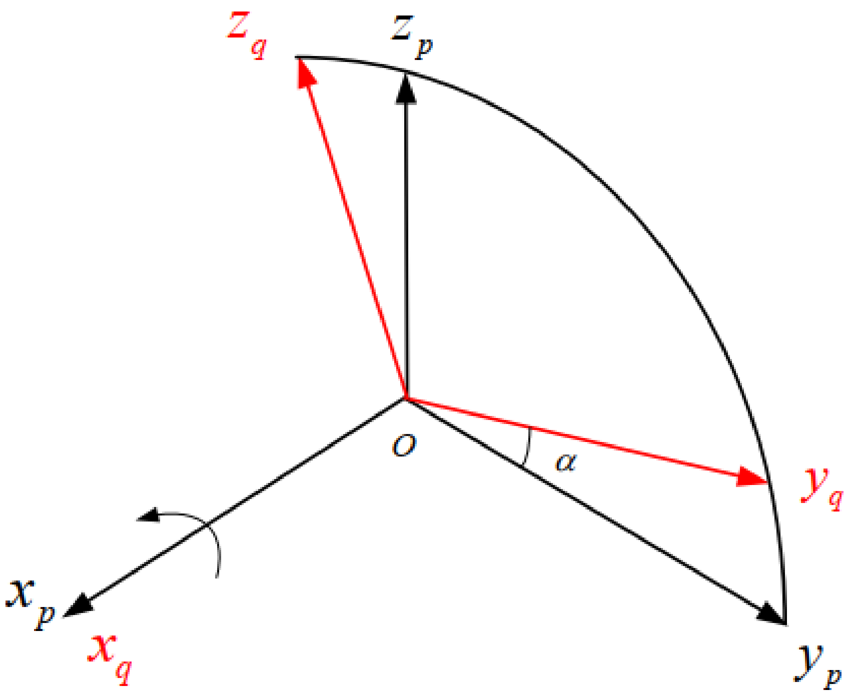
互为转置矩阵 $L_{pq} = (L_{qp})^T$

互为逆矩阵 $L_{pq} = (L_{qp})^{-1}$

正交性 $(L_{pq})^T = (L_{pq})^{-1}$

传导性 $L_{pr} = L_{pq} L_{qr}, L_{rp} = L_{rq} L_{qp}$

3D Transformation



$$p \rightarrow q$$

$$L_x(\alpha)$$

Error in the Eq. (1.24) of textbook

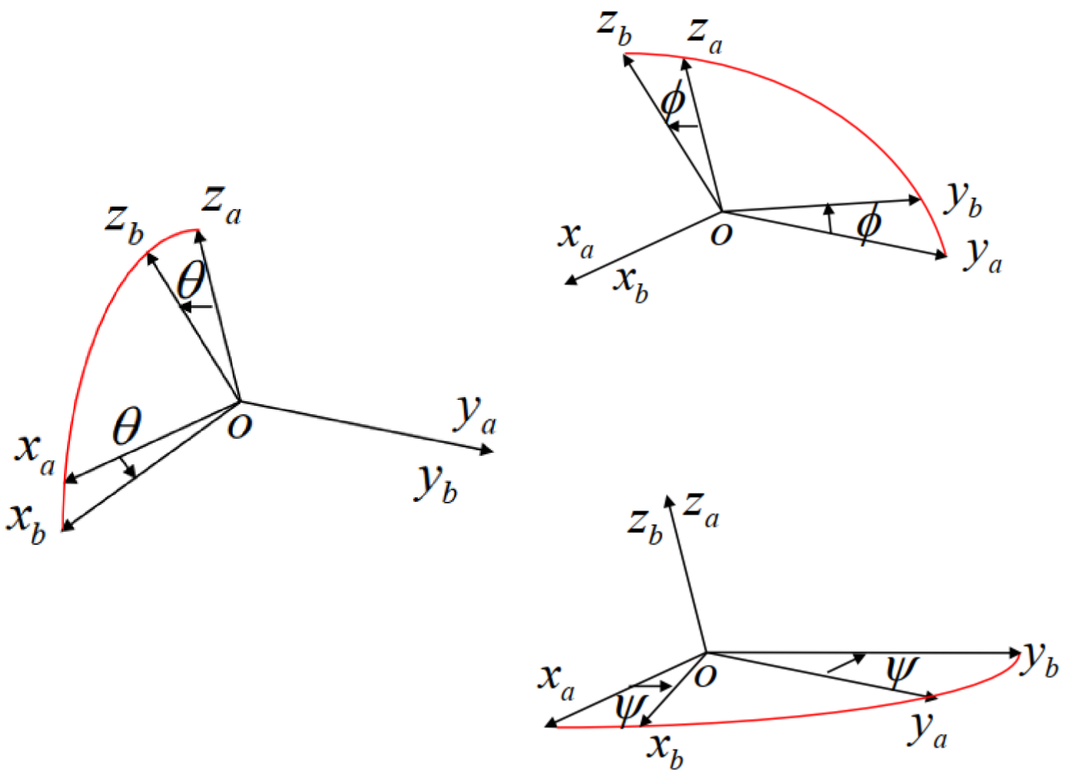
Rotation Rule

The rotation around x axis, y axis and z axis

$$L_x(\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix}$$

$$L_y(\theta) = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$L_z(\psi) = \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



Rotation Rule

Successive rotations around coordinate system axis are <u>not</u> commutative!

Pay attention to the order of rotation when constructing a rotation matrix created through multiplication!

$$\boldsymbol{M_{321} = M_3 \cdot M_2 \cdot M_1 \neq M_1 \cdot M_2 \cdot M_3}$$

Order of rotation: $\boldsymbol{1 - 2 - 3}$

3D Transformation

Assume the angle between $Ox_py_pz_p$and $Ox_qy_qz_q$ is ζ, η, ξ, the transform matrix is

$$L_{qp} = L_x(\xi)L_y(\eta)L_z(\zeta)$$

The 3D L_{qp} have the same properties as 2D case.

3D Transformation

From Earth axis system to body-fixed frame

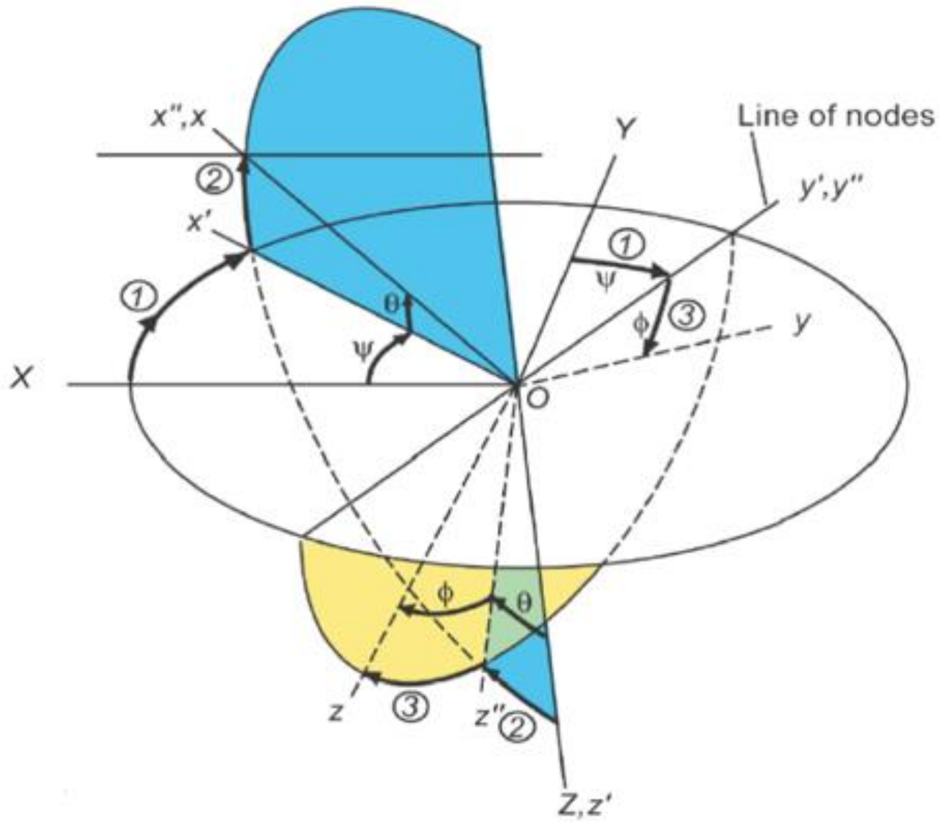
$$(O_g, x_g, y_g, z_g) \longrightarrow (O_B, x_B, y_B, z_B)$$

$$(X, Y, Z) \qquad\qquad (x, y, z)$$

1. Rotate (ψ) around OZ

2. Rotate (θ) around Oy'

3. Rotate (ϕ) around Ox''



3D Transformation

Consider a moving coordinate system with origin at centroid. The absolute speed and rotation speed are $\boldsymbol{V}$ and $\boldsymbol{\omega}$

$$\boldsymbol{V} = V_x\boldsymbol{i} + V_y\boldsymbol{j} + V_z\boldsymbol{k}$$

$$\frac{d\boldsymbol{V}}{dt} = \boxed{\frac{dV_x}{dt}\boldsymbol{i} + \frac{dV_y}{dt}\boldsymbol{j} + \frac{dV_z}{dt}\boldsymbol{k}} + V_x\frac{d\boldsymbol{i}}{dt} + V_y\frac{d\boldsymbol{j}}{dt} + V_z\frac{d\boldsymbol{k}}{dt}$$

$$\Rightarrow \frac{d\boldsymbol{V}}{dt} = \frac{\delta \boldsymbol{V}}{\delta t} + V_x\frac{d\boldsymbol{i}}{dt} + V_y\frac{d\boldsymbol{j}}{dt} + V_z\frac{d\boldsymbol{k}}{dt} \Leftarrow \boldsymbol{\omega} = \omega_x\boldsymbol{i} + \omega_y\boldsymbol{j} + \omega_z\boldsymbol{k}$$

Equation of Motion

$$\frac{dV}{dt} = \frac{\delta V}{\delta t} + V_x \frac{d\boldsymbol{i}}{dt} + V_y \frac{d\boldsymbol{j}}{dt} + V_z \frac{d\boldsymbol{k}}{dt}$$

$$\boldsymbol{\omega} = \omega_x \boldsymbol{i} + \omega_y \boldsymbol{j} + \omega_z \boldsymbol{k}$$

$$\begin{cases} \dfrac{d\boldsymbol{i}}{dt} = \boldsymbol{\omega} \times \boldsymbol{i} = (\omega_x \boldsymbol{i} + \omega_y \boldsymbol{j} + \omega_z \boldsymbol{k}) \times \boldsymbol{i} = -\omega_y \boldsymbol{k} + \omega_z \boldsymbol{j} \\ \dfrac{d\boldsymbol{j}}{dt} = \boldsymbol{\omega} \times \boldsymbol{j} = -\omega_z \boldsymbol{i} + \omega_x \boldsymbol{k} \\ \dfrac{d\boldsymbol{k}}{dt} = \boldsymbol{\omega} \times \boldsymbol{k} = -\omega_x \boldsymbol{j} + \omega_y \boldsymbol{i} \end{cases}$$

$$\frac{d\boldsymbol{V}}{dt} = \frac{\delta \boldsymbol{V}}{\delta t} + \boxed{\boldsymbol{\omega} \times \boldsymbol{V}}$$

$$\begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ \omega_x & \omega_y & \omega_z \\ V_x & V_y & V_z \end{vmatrix}$$

Equation of Motion

$$\frac{d\boldsymbol{V}}{dt} = \frac{\delta \boldsymbol{V}}{\delta t} + \boldsymbol{\omega} \times \boldsymbol{V}$$

Centroid absolute acceleration
Earth axis system

Acceleration in
moving axis system

Acceleration due
to rotation

$$\frac{\delta \boldsymbol{V}}{\delta t} = \frac{dV_x}{dt}\boldsymbol{i} + \frac{dV_y}{dt}\boldsymbol{j} + \frac{dV_z}{dt}\boldsymbol{k}$$

$$\boldsymbol{\omega} \times \boldsymbol{V} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ \omega_x & \omega_y & \omega_z \\ V_x & V_y & V_z \end{vmatrix}$$

Equation of Motion

Project the force in moving axis system ($\boldsymbol{F} = F_x\boldsymbol{i} + F_y\boldsymbol{j} + F_z\boldsymbol{k}$), and substitute into the equation of motion. The equation of motion in general coordinate system is

$$\begin{cases} m(\dfrac{dV_x}{dt} + V_z\omega_y - V_y\omega_z) = F_x \\ m(\dfrac{dV_y}{dt} + V_x\omega_z - V_z\omega_x) = F_y \\ m(\dfrac{dV_z}{dt} + V_y\omega_x - V_x\omega_y) = F_z \end{cases}$$