北京航空航天大學
BEIHANG UNIVERSITY

Avionics Technology

B31353551

— *Inertial Navigation*

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Spring Semester 2023 (30_Mar_T6)

IV. Inertial Navigation

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（1）Some concepts

（2）Accelerometer

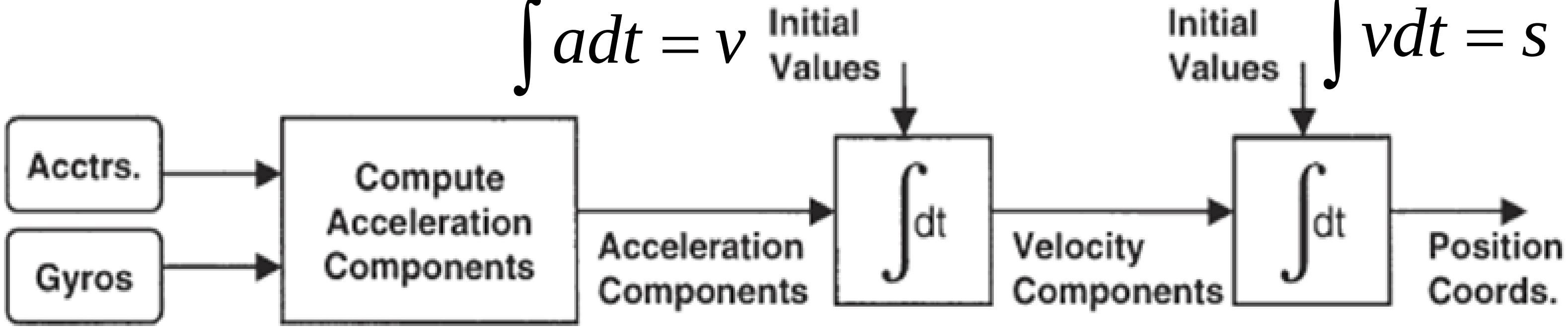
（3）Inertial navigation

(3) Inertial navigation

- We can sense the aircraft's acceleration (and also the gravitational vector) with accelerometers. If the acceleration components are then derived along a known set of axes (relying on gyros), successive integration of the acceleration components with respect to time will yield the velocities and distances travelled along these axes.

$$\int a dt = v \qquad \int v dt = s$$

Inertial navigation schematic



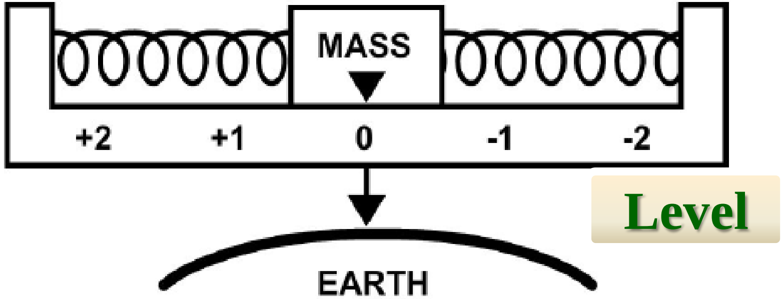
（3）Inertial navigation

- Assume an INS (Inertial navigation system)-equipped train on rail tracks at the equator, i.e. it runs in a straight line east and west only. The INS consists of an accelero-meter, two integrators and a displacement pick-off. The accelerations are detected, and the INS will compute all changes in displacement.

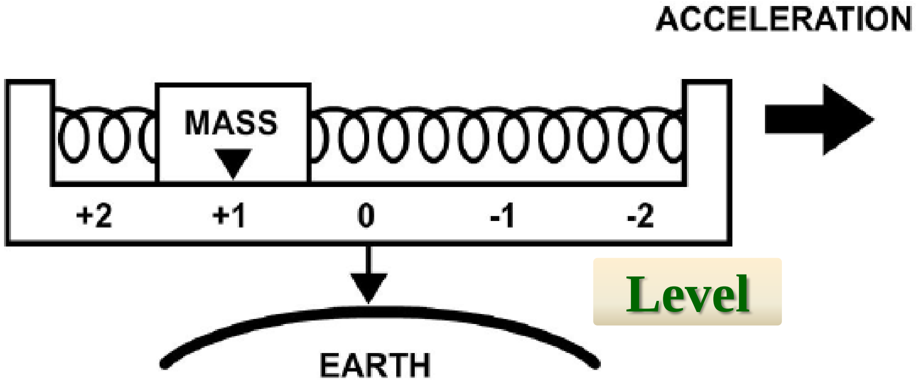
Along equator the direction of movement is normal to gravity

(3) Inertial navigation

- If the accelerometer input axis is exactly orthogonal to the gravity vector (i.e. horizontal) so that there is zero gravitational force component will the accelerometer measure the acceleration component along its input axis.



Accelerometer at rest



Sensing *true acceleration*

(3) Inertial navigation

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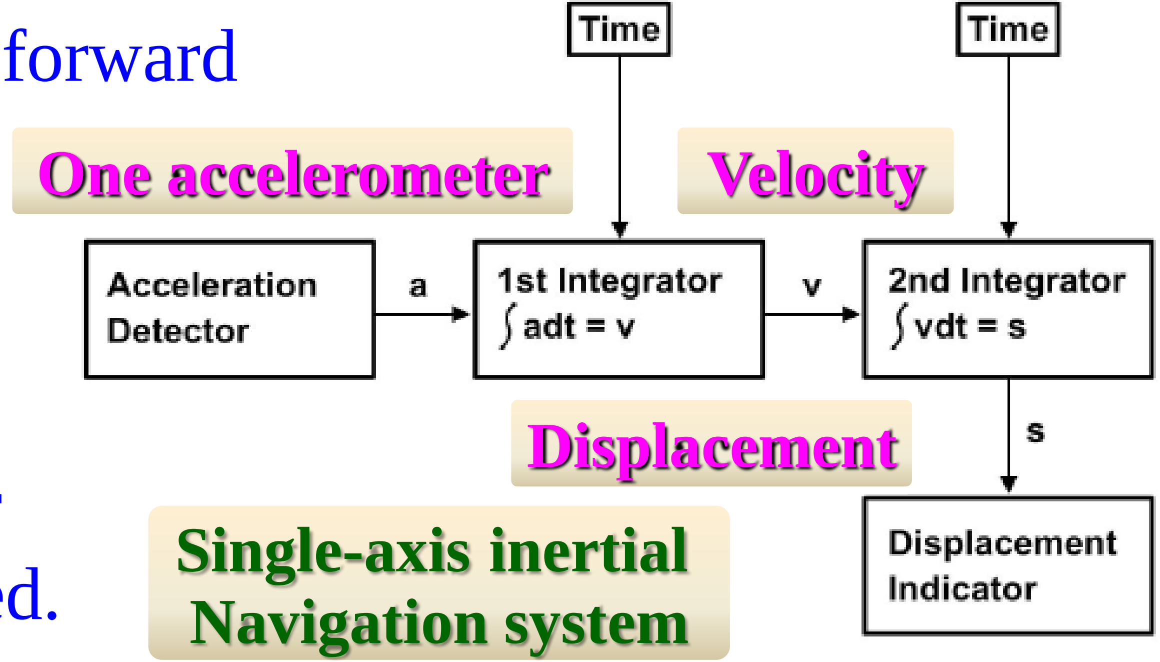
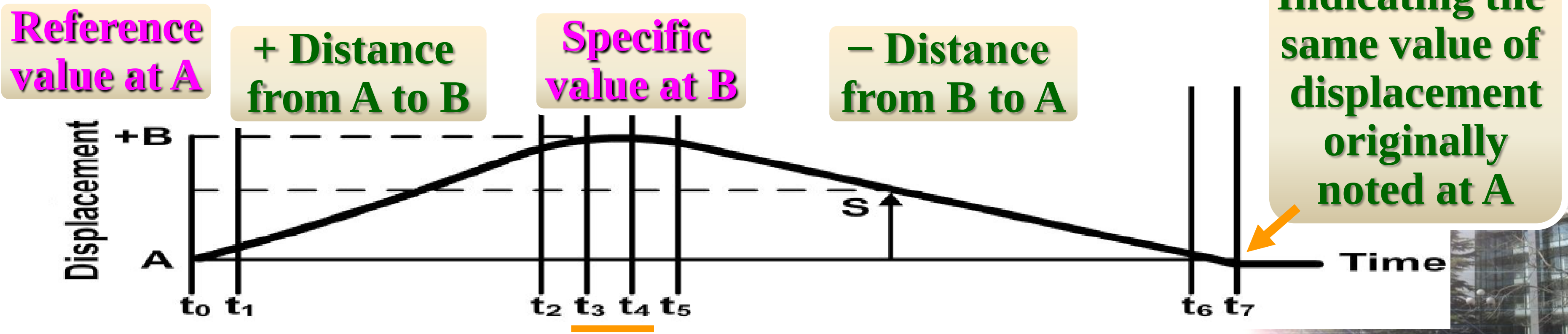
- The accelerometer is oriented in the train with the sensitive axis parallel to the movement direction, so that it detects accelerations when the train is moving forward or backward.
- Since the sensitive axis is known and retained by the rail tracks, no gyros for orientation is needed.



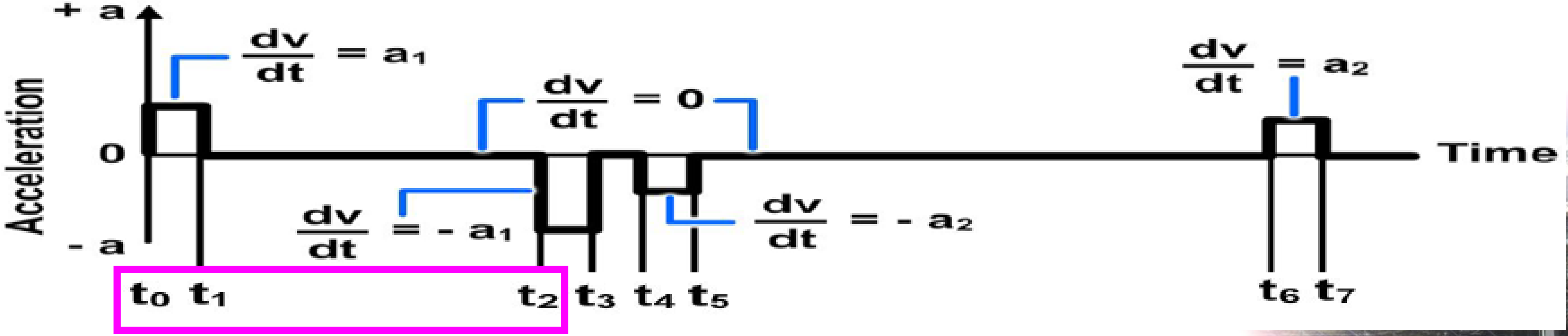
(3) Inertial navigation

- If the train starts moving at point A, a specific reading will show on the displacement indicator. When the train reaches point B and stops, the distance from point A to point B added to the reference value noted at point A will show on the displacement indicator. Then, the train returns to point A by traveling backwards.



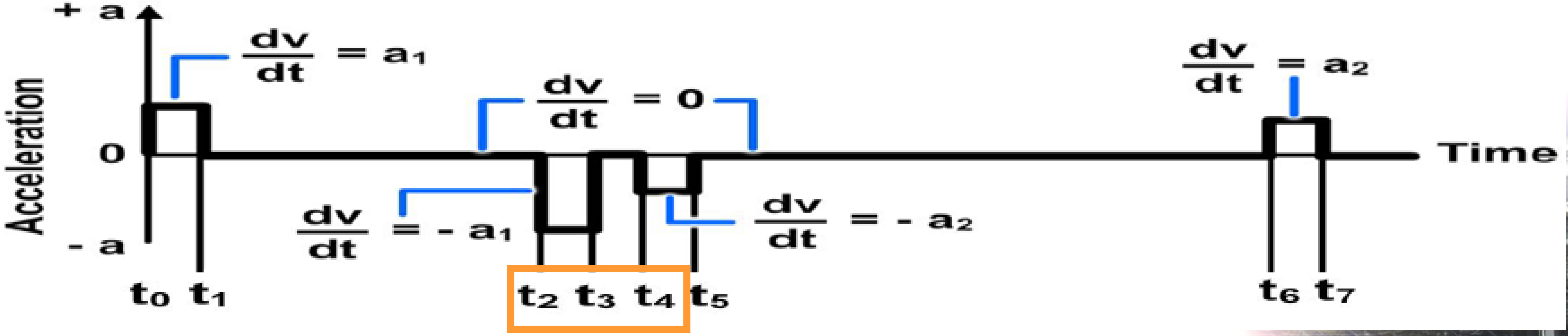
(3) Inertial navigation

- The acceleration curve begins at time t_0 as the train begins to run from point A. The acceleration at t_0 has a value of a_1, and it remains at that value until t_1. At t_1 the train ceases to accelerate, and acceleration goes to 0. At this point, the train reaches a steady velocity. The train continues traveling at a constant velocity until time t_2.



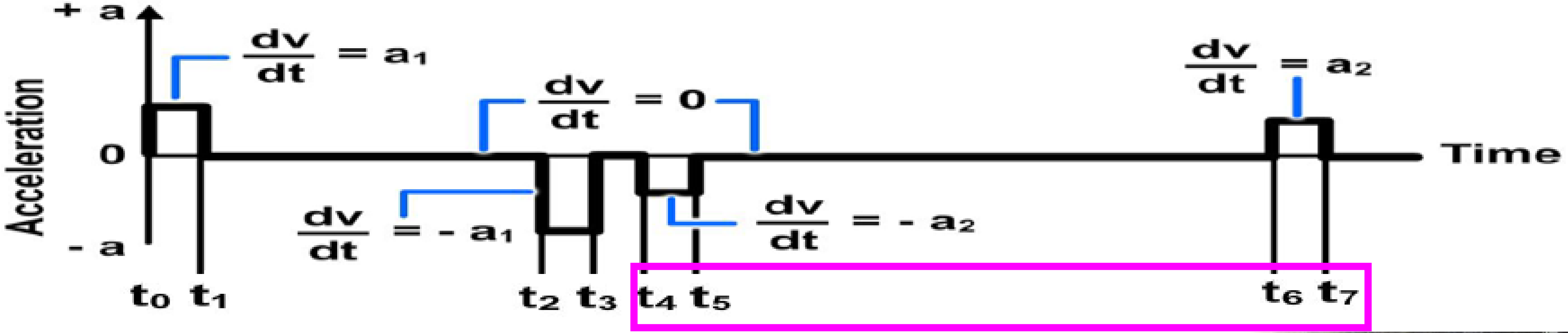
(3) Inertial navigation

- Then, the accelerometer detects an acceleration equal in value to a_1, but its direction is opposite. This acceleration is constant from time t_2 to time t_3. At t_3 the acceleration goes to 0. The train is now stationary and standing at its destination—point B. It is at point B during time interval t_3 to t_4.



(3) Inertial navigation

- In the return trip, the train runs backwards to point A at time t_4. The accelerometer detects an acceleration of $-a_2$ since the movement direction is reversed. At time t_5 it reaches a steady velocity, and acceleration goes to 0. It begins to stop at time t_6 and the accelerometer detects an acceleration of a_2. It comes to a full stop at time t_7.



(3) Inertial navigation

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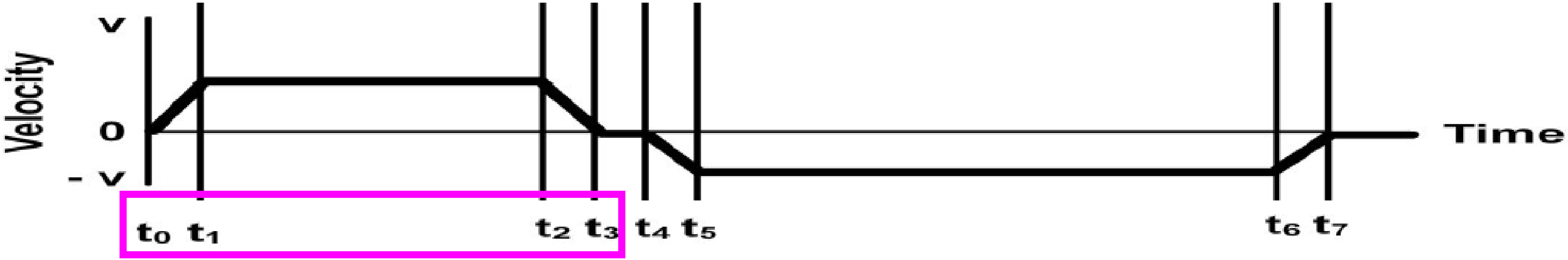
- The velocity curve is the result obtained when the measured acceleration is integrated over the time interval. During the interval t_0 to t_1, velocity is changing in a increasing direction. Velocity is constant during interval t_1 to t_2. During the interval t_2 to t_3, velocity is changing in a decreasing direction. At time t_3, both acceleration and velocity are zero.



(3) Inertial navigation

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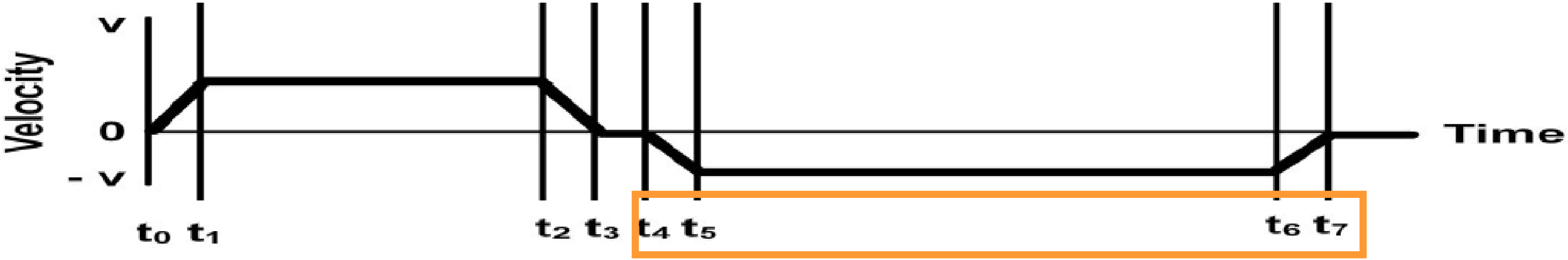
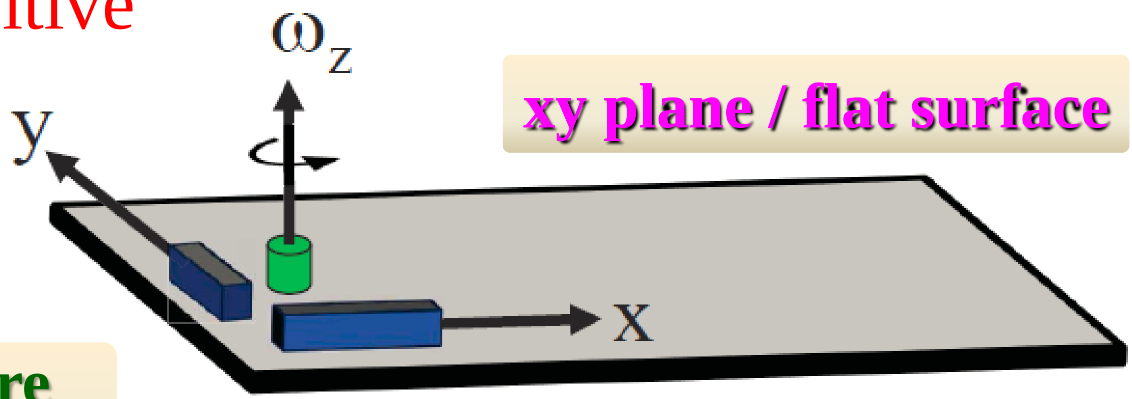
- During the interval t_4 to t_5, velocity is changing in a decreasing direction. Note that velocity is now negative since the movement direction is reversed. Velocity is constant during interval t_5 to t_6, and acceleration goes to 0. During the interval t_6 to t_7, velocity is changing in a increasing direction. At time t_7, both acceleration and velocity are zero.



(3) Inertial navigation

- We can determine position by using a system of two-dimensional coordinate axes on a plane or flat surface (e.g. the earth's surface). In this case we use two single-axis INS, and maintain proper orientation of each accelerometer's sensitive axis relative to the coordinate system.

xy plane / flat surface

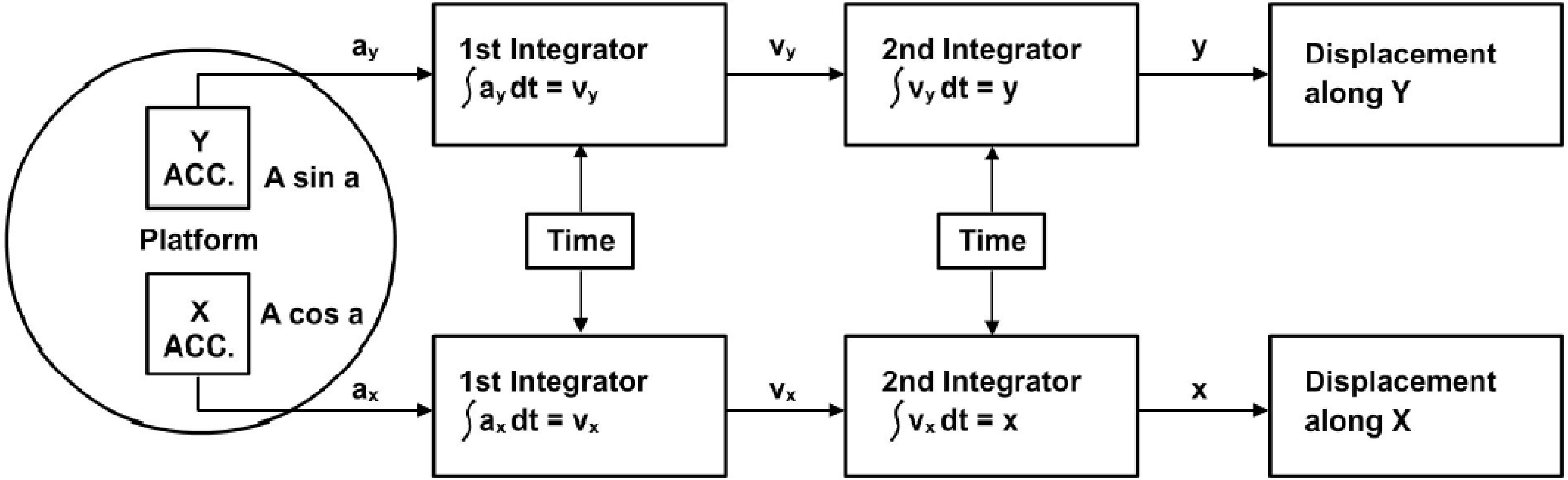


Two sensitive axes are mutually perpendicular

Two-axis inertial navigation system

(3) Inertial navigation

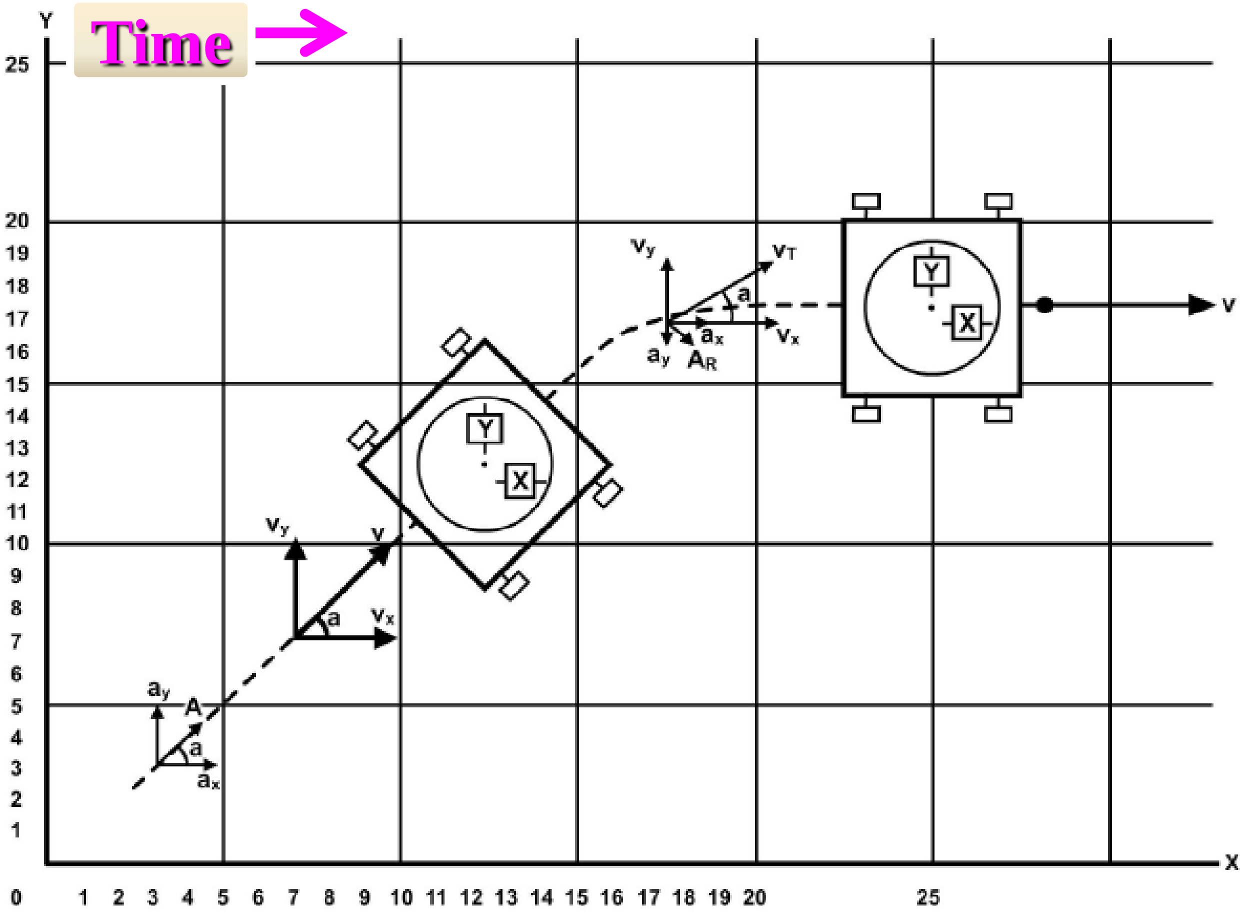
- One accelerometer's sensitive axis lies along the x-axis, and the other accelerometer's sensitive axis lies along the y-axis. The accelerometers will then sense any rate of change of velocity along the coordinate axes.



Inertial platform schematic

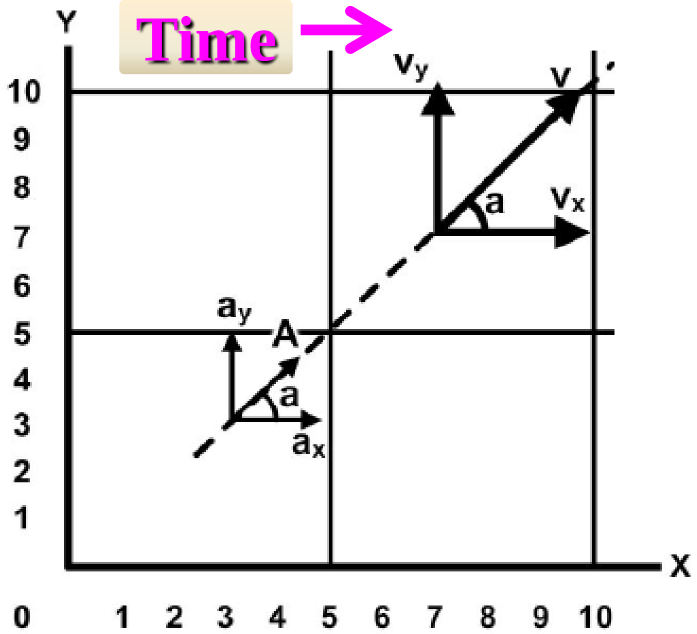
(3) Inertial navigation

- Assume the inertial platform mounted on a vehicle, we can locate the vehicle at any given time by the x and y coordinates.



(3) Inertial navigation

- The vehicle initializes on the coordinate system with a displacement of 3 on the x and y-axes. At time T_3, the vehicle experiences an acceleration, A, in a direction of 45º (i.e. a) from the x-axis. The accelerometers detect only that portion of the acceleration that along its sensitive axis, and sense Acos(a) and Asin(a) respectively. The displacement along x and y are equal due to a = 45º .



(3) Inertial navigation

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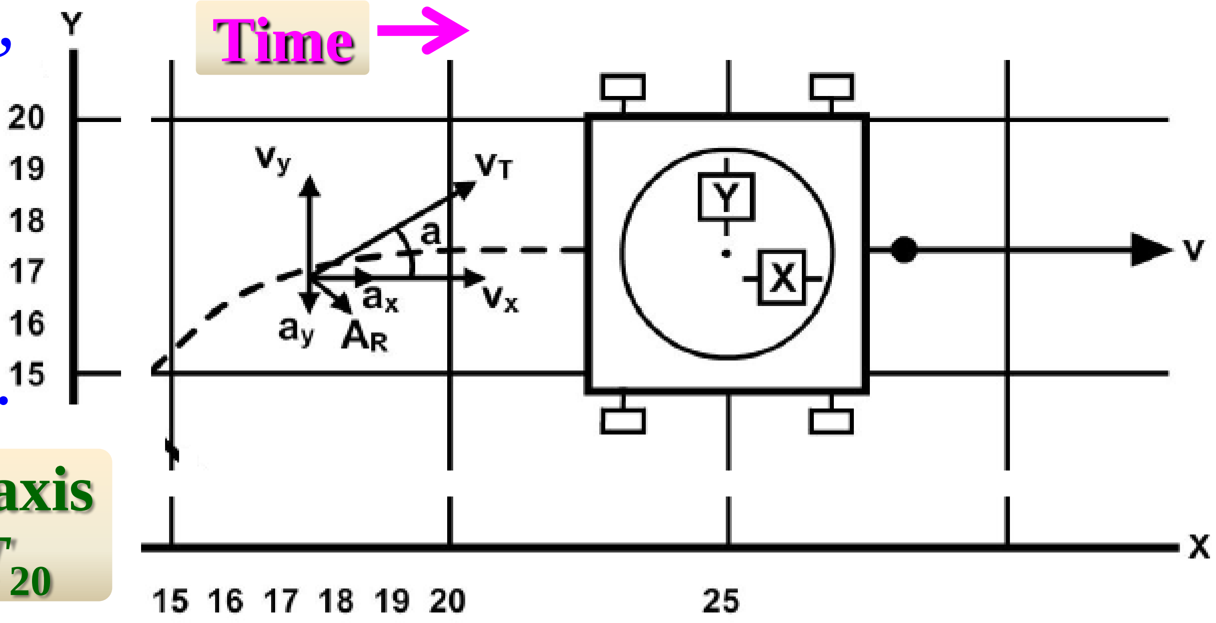
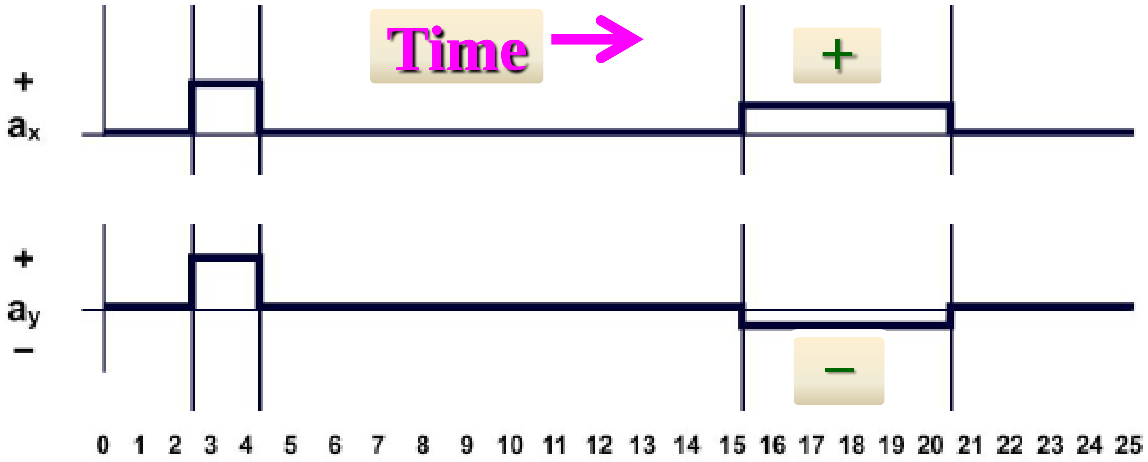
- The vehicle continues in a direction of 45° until time T_{15}. Then, it begins a turn to the right and completes the turn at time T_{20}. The new direction is parallel to the x-axis. Hence at T_{15}, the displacement is (15,15), and (20,15) at T_{20}, (25,15) at T_{25}, etc.



Displacement along y-axis ceases to change at T_{20}

(3) Inertial navigation

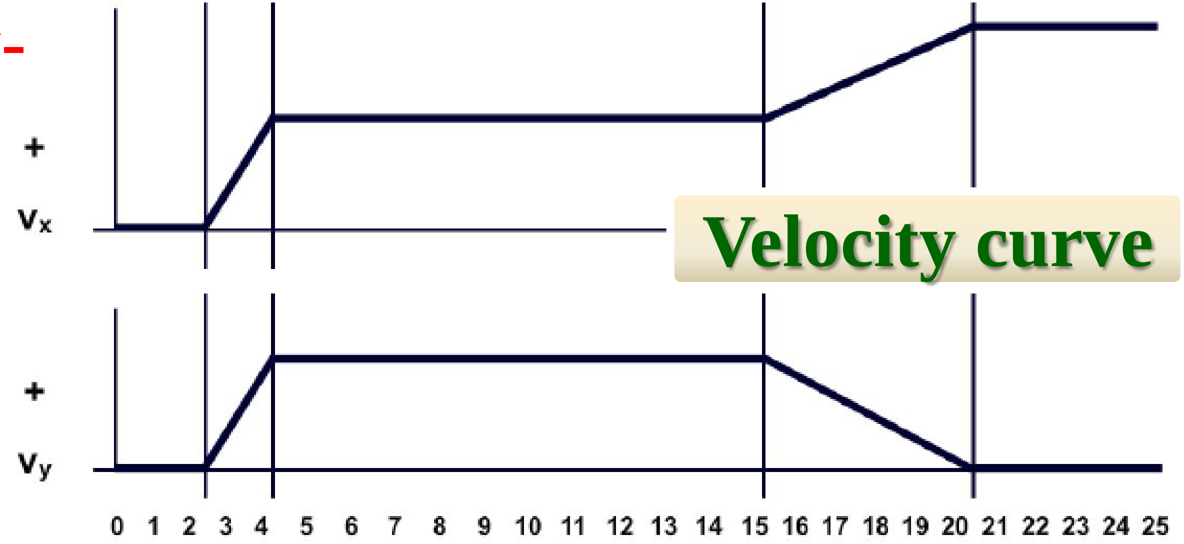
- During the interval T_{15} to T_{20}, the x-accelerometer detects a positive acceleration, while the y-accelerometer detects a negative acceleration. The vehicle maintains a constant speed throughout the turn, then the detected acceleration results from a change in direction. And this acceleration is radial acceleration or centripetal acceleration.



Acceleration curve

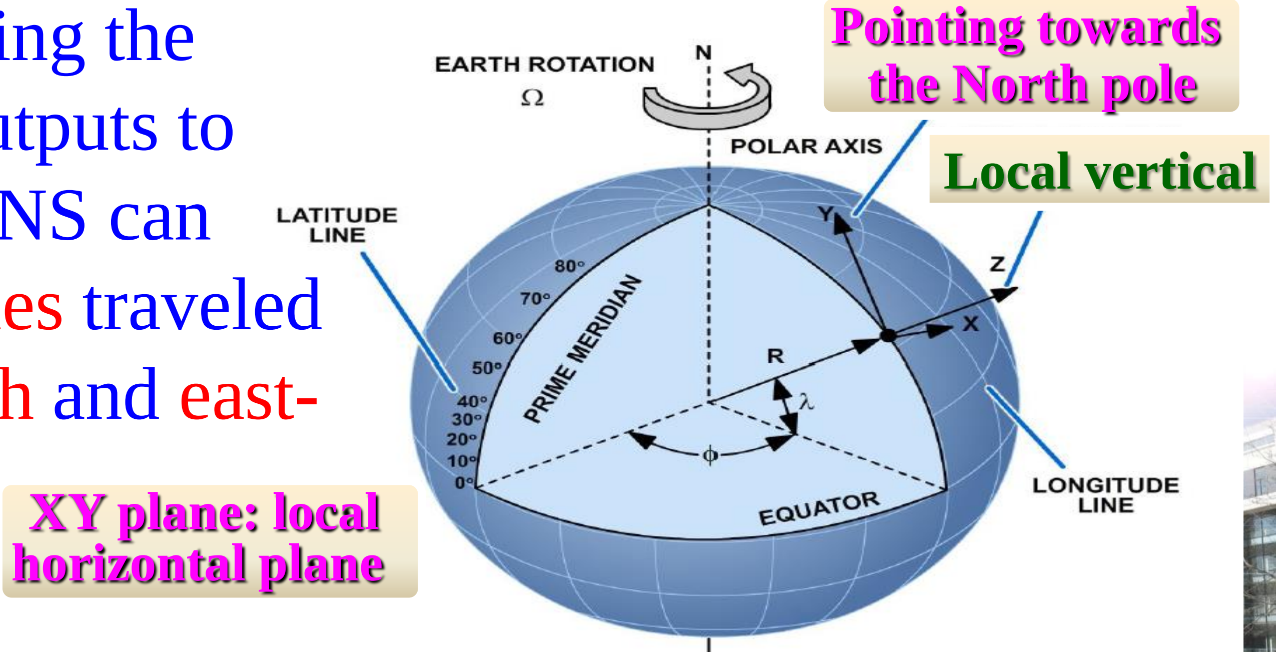
(3) Inertial navigation

- During the interval T_3 to T_{15}, the velocity along the x-axis is equal to the velocity along the y-axis. The integration of the x-component of acceleration for the interval T_{15} to T_{20} shows an increase in the velocity. The integration of the y-component of acceleration over the same interval shows that the velocity goes to 0.



(3) Inertial navigation

- The INS just described can be extended to navigation on the earth, with one horizontal accelerometer pointing north and the other horizontal accelerometer pointing east. By connecting the accelerometer outputs to integrators, the INS can compute velocities traveled in the north-south and east-west directions (i.e. v_N and v_E).

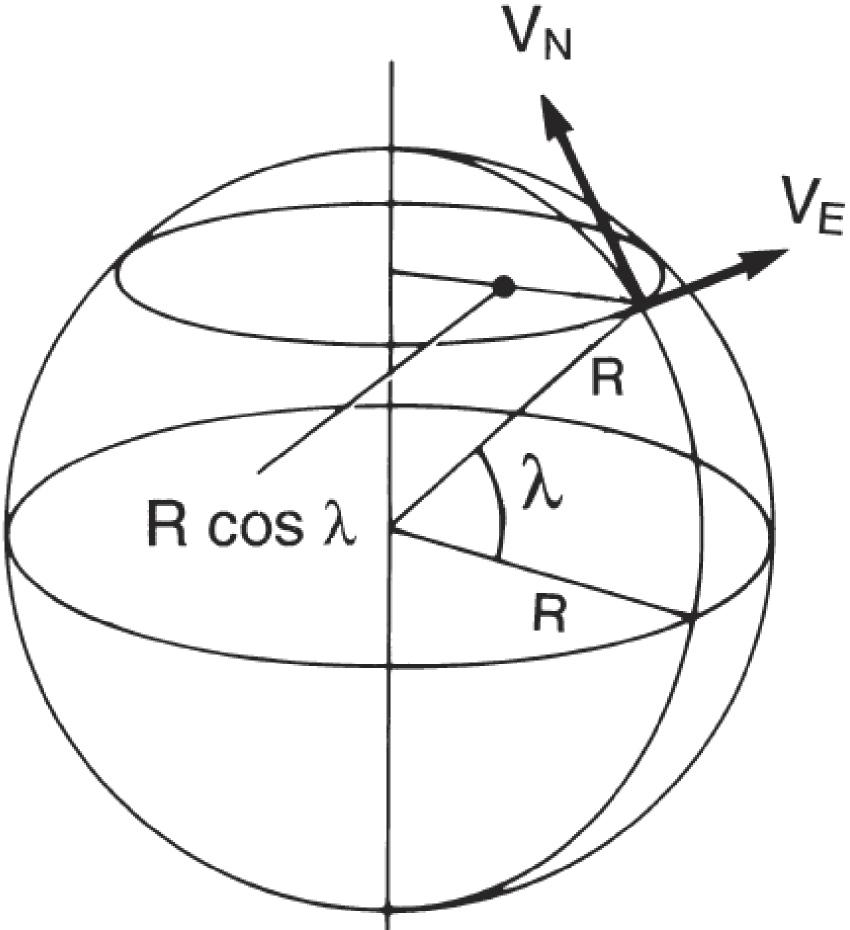


(3) Inertial navigation

- If we knew the initial position in latitude and longitude, the northerly and easterly velocity components of an aircraft, then we can determine the aircraft's present position.

Derivation of rates of change of latitude and longitude:

$$\dot{\lambda} = \frac{v_N}{R} \qquad \dot{\phi} = \frac{v_N}{R\cos\lambda}$$



（3）Inertial navigation

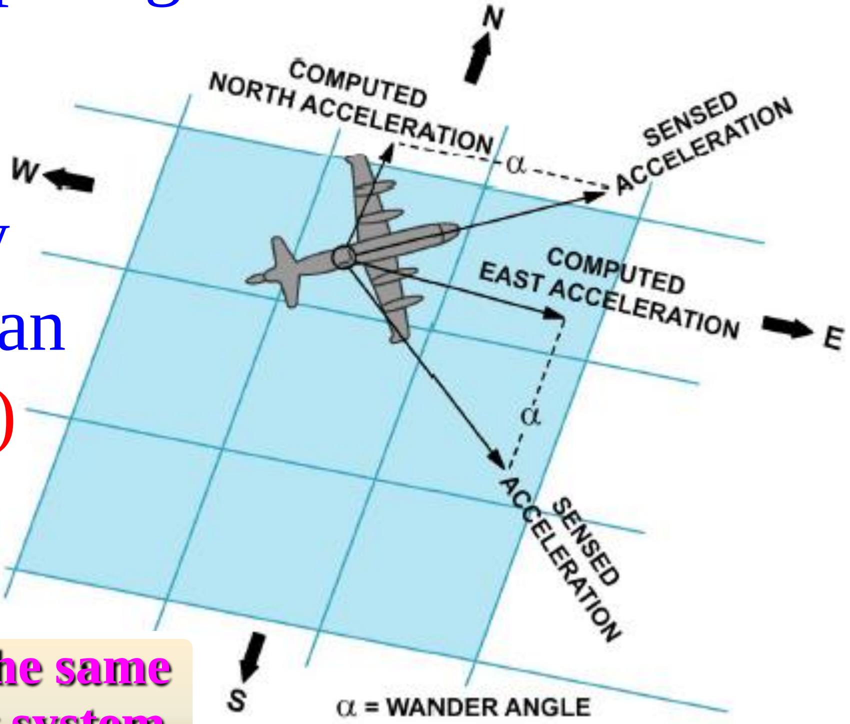
- The change in latitude over time, *t*, is thus equal to $(1/R)\cdot\int_0^t v_N\,dt$ and hence the present latitude at time *t* can be computed given the initial latitude. Similarly, the change in longitude is equal to $(1/R)\cdot\int_0^t v_E \sec\lambda\,dt$ and hence the present longitude can be computed given the initial longitude:

$$\lambda = \lambda_0 + (1/R)\cdot\int_0^t v_N\,dt$$

$$\phi = \phi_0 + (1/R)\cdot\int_0^t v_E \sec\lambda\,dt$$

(3) Inertial navigation

- As λ approaches 90° and sec λ approaches infinity. The method just described of computing the latitude and longitude is hence limited.
- The wander-azimuth inertial system solves this problem by allowing the platform to take an arbitrary angle (wander angle) with respect to true north, which changes as a function of longitude.



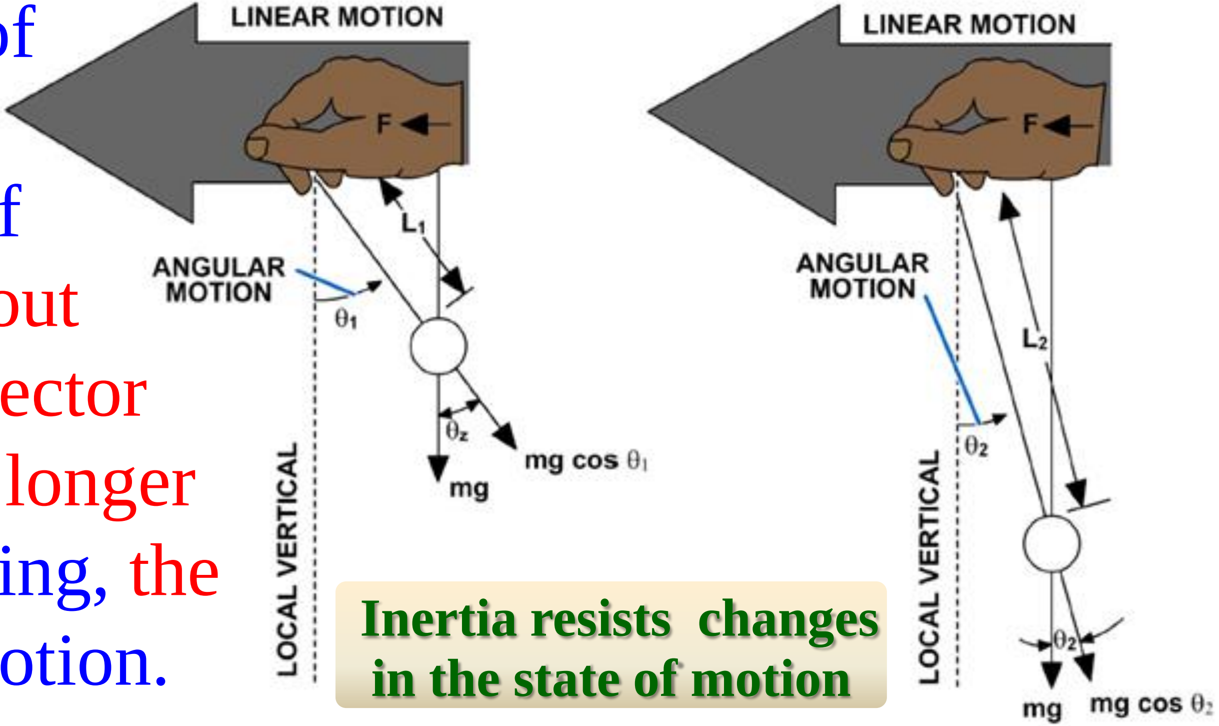
Fundamentals are the same as a north-pointing system

(3) Inertial navigation

- Either for a wander-azimuth system or north-pointing system, it is essential to maintain both accelerometers horizontal to the earth's surface (i.e. the platform normal to the local vertical). If the accelerometers tilt off level, it measures gravitational components, which results in navigation errors.
- A gravity force sensitive pendulum can automatically align with the local vertical, but the accelerated linear motion interferes with this alignment.

(3) Inertial navigation

- Two pendulums are suspended by strings of different lengths (L), and equal forces horizontally accelerate the suspension point of each pendulum, angular motions of the pendulums about the local gravity vector will produce. The longer the suspending string, the less the angular motion.



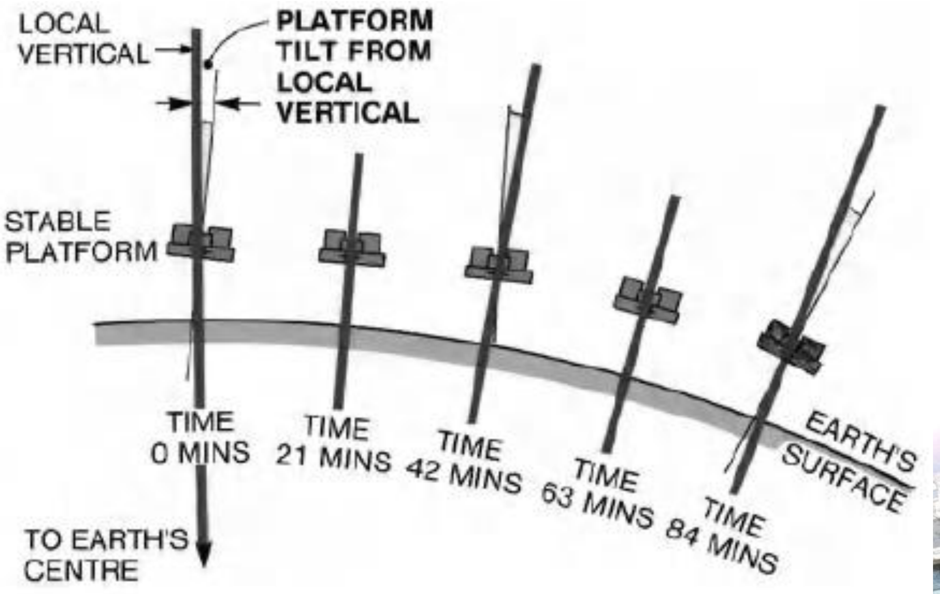
Inertia resists changes in the state of motion

(3) Inertial navigation

- The period of the pendulum is given by $T = 2\pi\sqrt{L/g}$, and the period of Schuler pendulum is

$$T = 2\pi\sqrt{R/g} = 84.4\,\text{min}$$

which is a special case of the pendulum and would indicate the local vertical irrespective of the acceleration of the vehicle carrying it.



Platform with Schuler oscillation

(3) Inertial navigation

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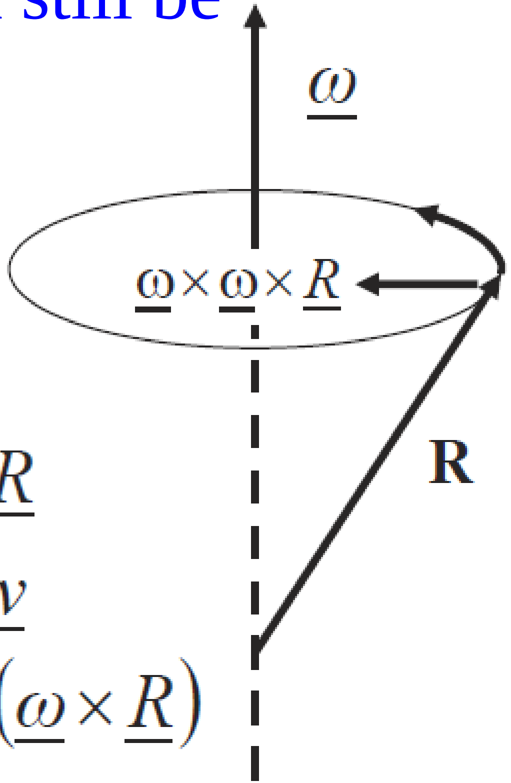
- The earth is a rotating sphere, only points along the equator can be considered to possess uniform linear motion and only at the equator the accelerometer's signals can translate directly into position information. For this reason, it is necessary to provide a corrective device to alter the accelerometer's signals. The device inserts artificial acceleration signals to those already in the accelerometer output circuits for the corrections of centripetal effect and Coriolis.

(3) Inertial navigation

- Centripetal errors has no relationship to Earth dynamics. Even if the earth were stationary, it would still be necessary to insert centripetal correction to the accelerometer's signals. Flying over the earth's surface, the aircraft's linear motion produces a curved flight path in inertial space, and this introduces centrifugal acceleration components.



$$\underline{v} = \underline{\omega}\times\underline{R}$$

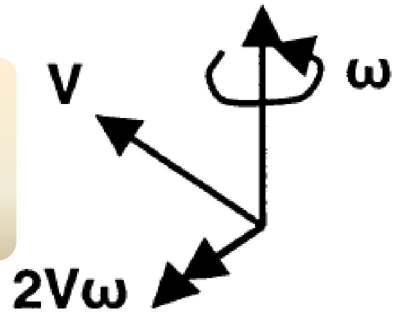
$$\underline{a} = \underline{\omega}\times\underline{v}$$

$$\underline{a} = \underline{\omega}\times\left(\underline{\omega}\times\underline{R}\right)$$

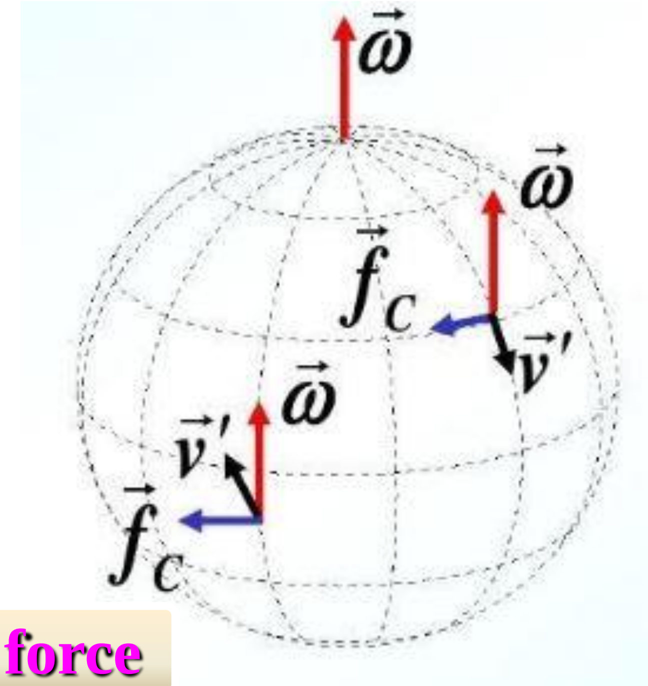
Centripetal motion

- Coriolis errors has relationship to Earth dynamics and generated by the earth's rotation. Coriolis accelerations are introduced because of the linear motion with respect to a rotating axis frame.

Mutually at right angles to the linear velocity and angular velocity



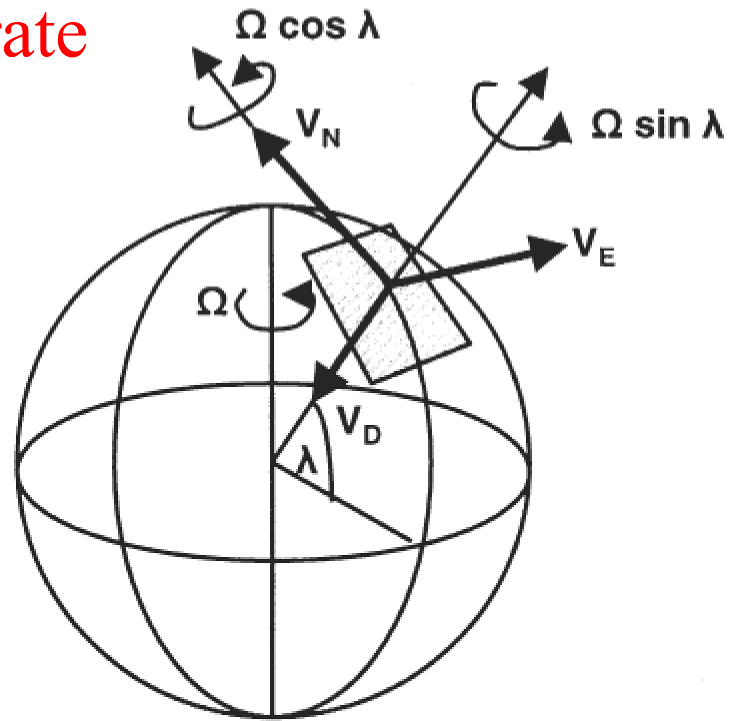
Coriolis acceleration



Coriolis force

- Coriolis acceleration components along the North, East, vertical (Down) axes due to the aircraft's linear velocity components and Earth's rotation rate components are:

North axis	$2V_E\,\Omega \sin\lambda$
East axis	$-2V_N\,\Omega \sin\lambda - 2V_D\,\Omega \cos\lambda$
Vertical axis	$-2V_E\,\Omega \cos\lambda$



Earth referenced axis frame:
local North, East, Down (NED) axes

（3）Inertial navigation

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- The rates of change of the aircraft velocity (acceleration) components along the NED axes are obtained by subtracting the acceleration corrections:

	North Axis	East Axis	Down Axis
Acceleration component			
Coriolis	$2V_E\,\Omega\sin\lambda$	$-2V_N\,\Omega\sin\lambda - 2V_D\,\Omega\cos\lambda$	$-2V_E\,\Omega\cos\lambda$
Centrifugal	$\left(V_E^2\tan\lambda - V_D V_N\right)/R$	$-\left(V_N V_E\tan\lambda + V_D V_E\right)/R$	$\left(V_N^2 + V_E^2\right)/R$
Gravitational			$\frac{R_0^2}{\left(R_0+H\right)^2}g_0$