Convective Heat Transfer

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Beihang University, 2021 Spring

What is convection?

- Background
  - Thermodynamics: flow of heat; deal with equilibrium, potential, does not consider rate.
  - Fluid dynamics: flow of fluid, normally at a fixed T.
  - Heat transfer: whenever there is T difference (q-T relationship): rate-dependent, non-equilibrium
- Conduction
  - Random molecular motion, vibration of atoms: gas/solid/liquid
  - Governing equations
    - Fourier’s law: 1d /2d /steady /transient

$$q''_x = -k dT / dx$$

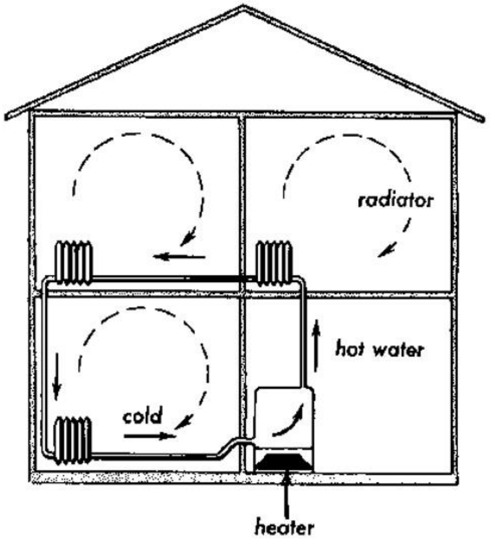
- Convection
  - Macroscopic motions, molecular velocity / macroscopic velocity
  - Conduction + advection.
  - Governing equations
    - Nervier –stokes (NS) equation
    - Newton’s law of cooling

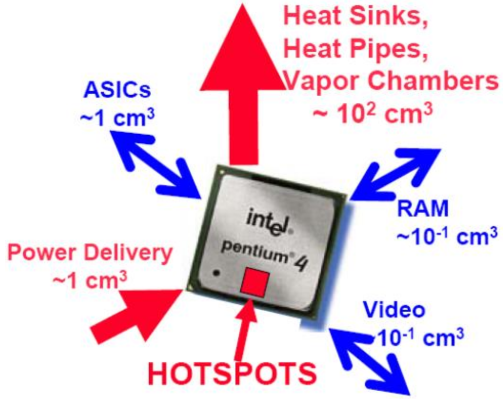
$$q'' = h(T_s - T_{ref})$$

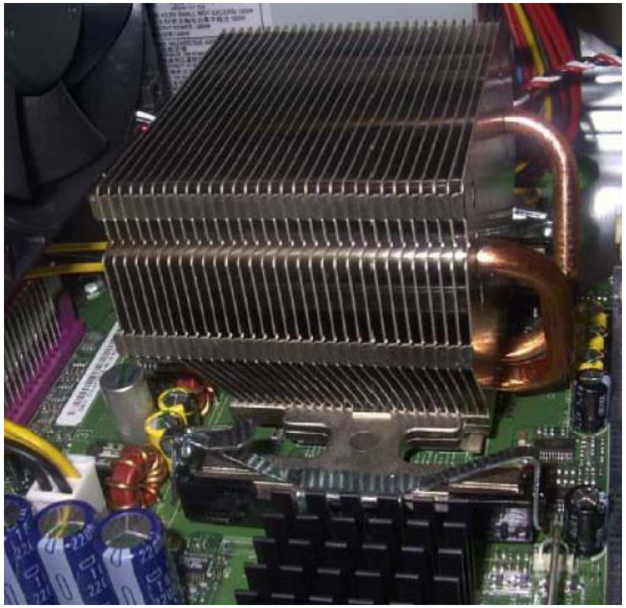
- Radiation
  - Electromagnetic wave:
  - Governing equations
    - Stephan Boltzmann equation: black body, grey body

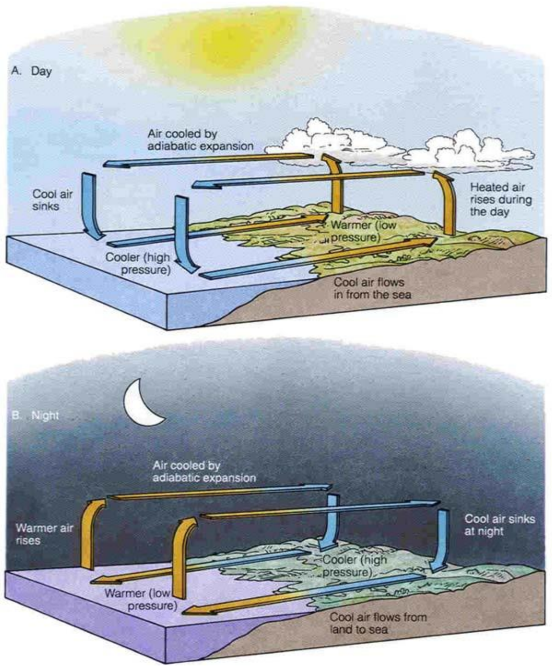
$$q'' = \varepsilon\sigma T^4$$

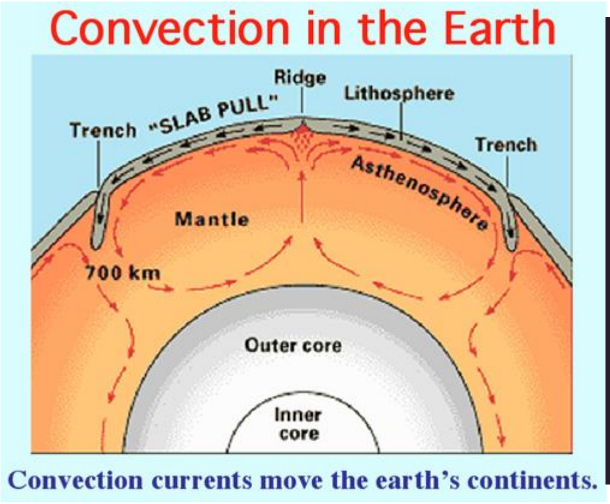
Examples of convections

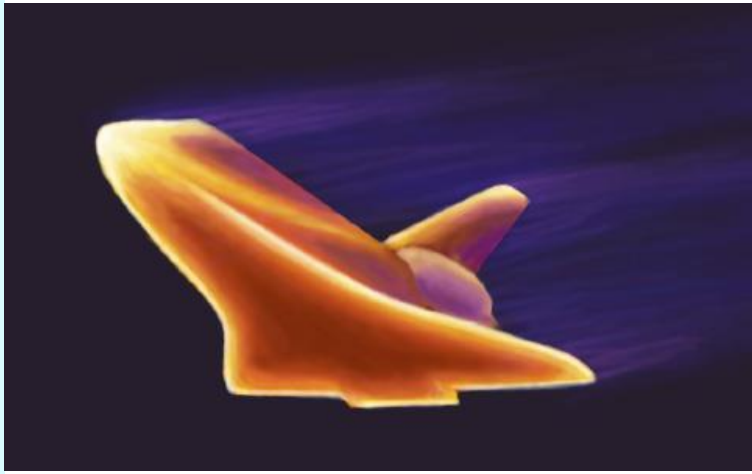








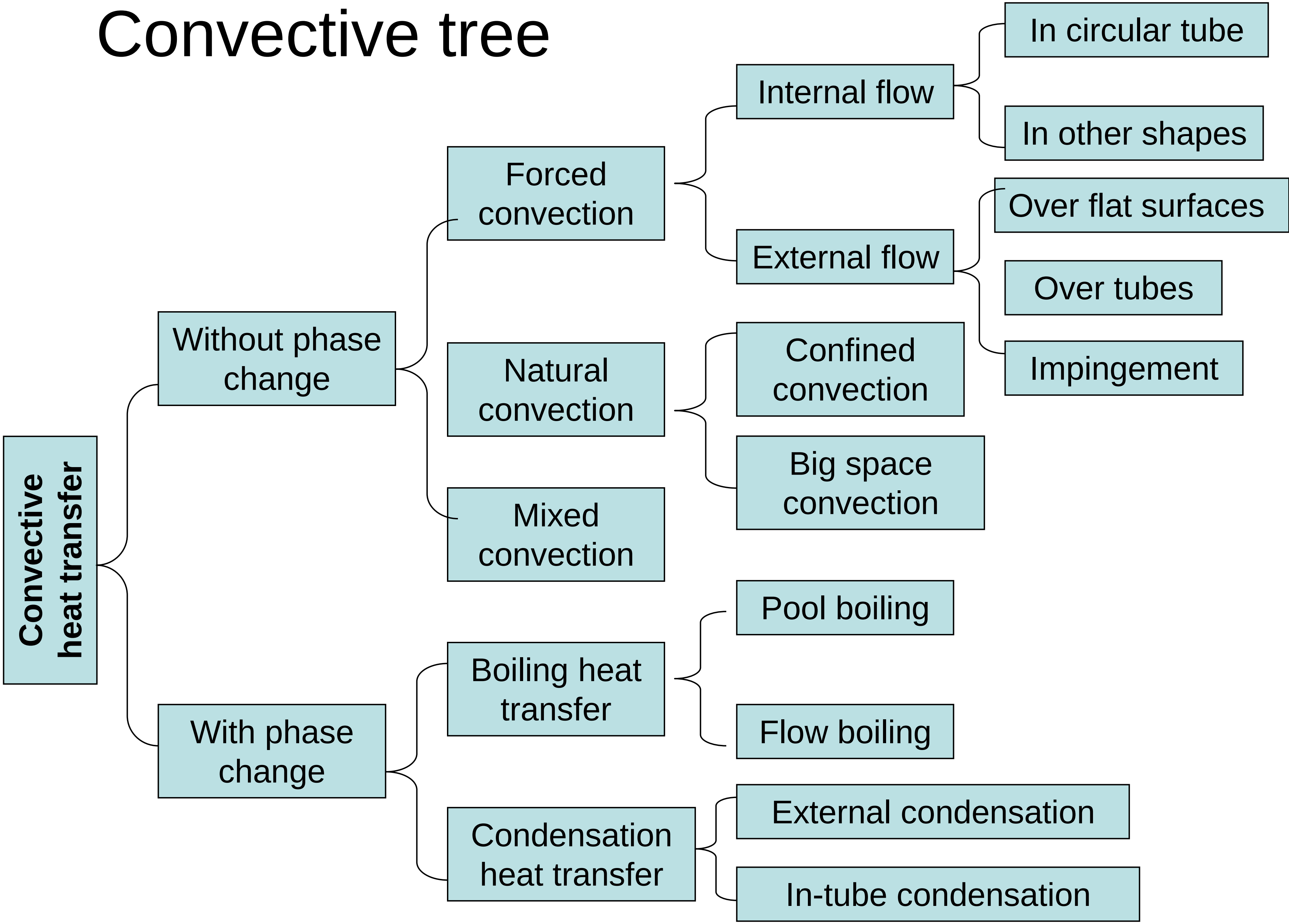




Affecting factors of the convective flow and heat transfer

- Fluids property
  - Thermal conductivity, viscosity, heat capacity, density …
- Flow conditions
  - Forced convection, natural convection
  - Laminar flow, turbulent flow
  - Single phase flow, multiphase flow / phase change
- Geometry constraints
  - External flow
  - Internal flow
  - Other complex geometries

Convective tree



Order of magnitudes of *h*

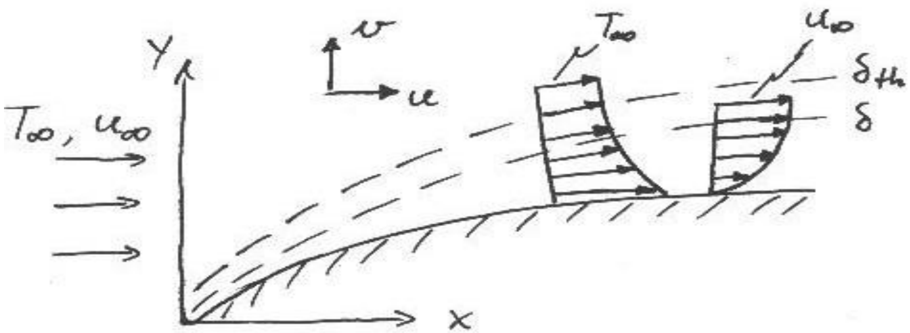
Situation	$\overline{h}$, W/m^{2}K
Natural convection in gases	
• 0.3 m vertical wall in air, $\Delta T = 30°C$	4.33
Natural convection in liquids	
• 40 mm O.D. horizontal pipe in water, $\Delta T = 30°C$	570
• 0.25 mm diameter wire in methanol, $\Delta T = 50°C$	4,000
Forced convection of gases	
• Air at 30 m/s over a 1 m flat plate, $\Delta T = 70°C$	80
Forced convection of liquids	
• Water at 2 m/s over a 60 mm plate, $\Delta T = 15°C$	590
• Aniline-alcohol mixture at 3 m/s in a 25 mm I.D. tube, $\Delta T = 80°C$	2,600
• Liquid sodium at 5 m/s in a 13 mm I.D. tube at 370°C	75,000
Boiling water	
• During film boiling at 1 atm	300
• In a tea kettle	4,000
• At a peak pool-boiling heat flux, 1 atm	40,000
• At a peak flow-boiling heat flux, 1 atm	100,000
• At approximate maximum convective-boiling heat flux, under optimal conditions	10^6
Condensation	
• In a typical horizontal cold-water-tube steam condenser	15,000
• Same, but condensing benzene	1,700
• Dropwise condensation of water at 1 atm	160,000

How to get *h*

- Mathematical method
  - Derivation of governing equations: normally partial differential equations (PDEs)
    - Solve the PDEs
      - Analytical solutions
      - Integration method
      - Numerical solutions
  - Similarity between flow and heat transfer
    - Reynolds analogy
- Experimental method
  - Dimensional analysis
  - Experiments and correlations of flow and heat transfer

Convection transfer equations

- **Key points: At each point in the fluid, conservation of mass, energy and momentum must be satisfied.**



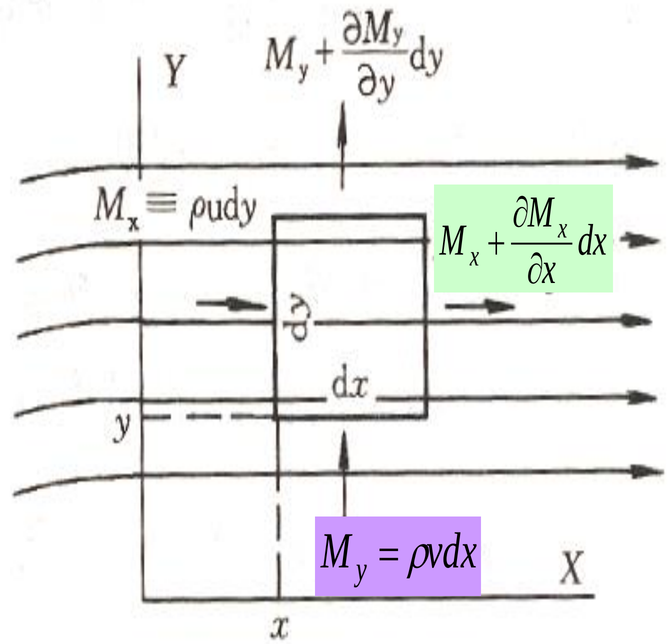
- Consider steady, 2-D flow of a viscous, incompressible Newtonian fluid $\tau = \eta \frac{\partial u}{\partial y}$ with constant properties (ρ、$\mathbf{c_p}$、$\boldsymbol{k}$、η).

- Four unknowns: **u、v, T, p**
- Four equations are needed: mass, momentum (x, y) and energy

Mass conservation

M: mass flow rate **[kg/s]** $$M_x = \rho u\, dy$$

At position x+dx $$M_{x+dx} = M_x + \frac{\partial M_x}{\partial x} dx$$



Mass gain in x direction at unit time:

$$M_x - M_{x+dx} = -\frac{\partial M_x}{\partial x} dx = -\frac{\partial (\rho u)}{\partial x} dxdy$$

Mass gain in y direction at unit time:

$$M_y - M_{y+dy} = -\frac{\partial M_y}{\partial y} dy = -\frac{\partial (\rho v)}{\partial y} dxdy$$

Mass change rate $$\frac{\partial (\rho dxdy)}{\partial t} = \frac{\partial \rho}{\partial t} dxdy$$

Mass conservation $$-\frac{\partial (\rho u)}{\partial x} dxdy - \frac{\partial (\rho v)}{\partial y} dxdy = \frac{\partial \rho}{\partial t} dxdy$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho \mathrm{v})}{\partial y} = 0$$

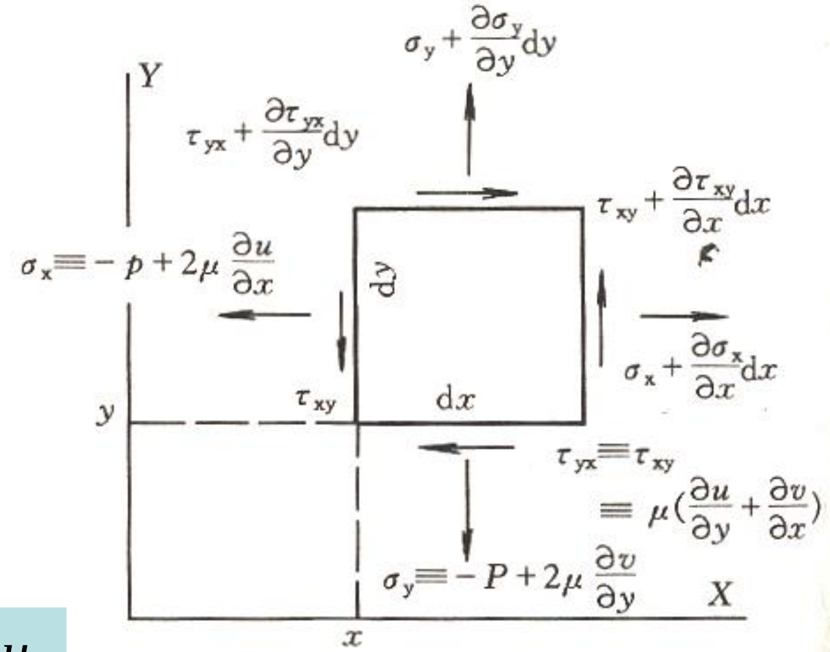
For incompressible flow $$\frac{\partial u}{\partial x} + \frac{\partial \mathrm{v}}{\partial y} = 0$$

Momentum conservation

Newton's second law: ***F=ma***

Volumetric force: gravity, centrifugal, electr-magnetic force

Viscosity force: Newtonian shear force



For incompressible flow with constant viscosity

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = F_x - \frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = F_y - \frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)$$

(1) (2) (3) (4)

(1)— inertial force（*ma*）
(2) —volumetric force
(3) — pressure gradient
(4) — viscosity force

$$F_x = \rho g_x; \quad F_y = \rho g_y$$

When the volumetric force is gravity only

Energy conservation (1)

- First law of thermodynamics

$$Q = \Delta E + W$$

$$Q \;\text{—}\; Q_{cnd} + Q_{cov} + Q_{int}$$

$$\Delta E \;\text{—}\; \Delta U_{th} + \Delta U_{K}$$

W — Work through gravity, surface tension, viscous force etc

Assumptions:

1) There is no work output by the fluids (no viscous dissipation) $\Longrightarrow$ $\mathbf{W = 0}$
2) Non-compressible flow
3) No chemical reaction, internal heat source is zero $\Longrightarrow$ $\boldsymbol{Q_{int} = 0}$
4) Velocity is relatively slow, the kinetic energy is negligible $\Longrightarrow$ $\boldsymbol{\Delta U_K = 0}$

Energy conservation (2)

- Heat by conduction

$$Q_{\mathrm{cnd}} = k\frac{\partial^2 T}{\partial x^2}dxdy + k\frac{\partial^2 T}{\partial \mathrm{y}^2}dxdy$$

- Heat by convection, x direction

$$Q_x'' - Q_{x+dx}'' = Q_x'' - \left(Q_x'' + \frac{\partial Q_x''}{\partial x}dx\right) = -\frac{\partial Q_x''}{\partial x}dx = -\rho c_p \frac{\partial (uT)}{\partial x}dxdy$$

- Heat by convection, y direction

$$Q_y'' - Q_{y+dy}'' = Q_y'' - \left(Q_y'' + \frac{\partial Q_y''}{\partial y}dy\right) = -\frac{\partial Q_y''}{\partial y}dy = -\rho c_p \frac{\partial (vT)}{\partial y}dydx$$

- Internal energy change

$$\Delta U = \rho c_p \frac{\partial T}{\partial t}dxdy$$

- Energy conservation

$$\frac{k}{\rho c_p}\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial \mathrm{y}^2}\right] = u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + \frac{\partial T}{\partial t}$$

Four equations of convective heat transfer

$$
\left\{
\begin{aligned}
&\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \\
&\rho \left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = F_x - \frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \\
&\rho \left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = F_y - \frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \\
&\rho c_p \left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right)
\end{aligned}
\right.
$$

Note: 1) assumptions: 2D, constant property, incompressible, no internal heat source, no viscous heating, Newtonian fluids

2) Applicable to both laminar and turbulent flow

3) Four equations with four unknowns, the heat transfer coefficient can be calculated once the temperature field is got.

4) The flow and temperature fields are coupled.