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Heat Transfer

Conduction Heat Transfer

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HT Week 1 Recall

- Introduction -

- **Three modes of heat transfer**

  - **Conduction:** **Energy exchange through a fixed body or across bodies at the point of contact.**
  - **Convection:** **Energy conveyance by the bulk motion of a fluid accompanied by conduction between the fluid and the bodies it comes in contact with.**
  - **Radiation:** **Energy exchange through electromagnetic radiation and absorption.**

  - **In general, heat transfer mixes three modes mentioned above.**

- **We will study all of these in this course**
- **First part – Conduction HT**

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HT Week 1 Recall

- Introduction -

TABLE 1.5 Summary of heat transfer processes

Mode	Mechanism(s)	Rate Equation	Equation Number	Transport Property or Coefficient
Conduction	Diffusion of energy due to random molecular motion	$q''_x(\mathrm{W/m^2}) = -k\dfrac{dT}{dx}$	(1.1)	k (W/m · K)
Convection	Diffusion of energy due to random molecular motion plus energy transfer due to bulk motion (advection)	$q''(\mathrm{W/m^2}) = h(T_s - T_\infty)$	(1.3a)	h (W/m^2 · K)
Radiation	Energy transfer by electromagnetic waves	$q''(\mathrm{W/m^2}) = \varepsilon\sigma(T_s^4 - T_{sur}^4)$ or $q\,(\mathrm{W}) = h_r A(T_s - T_{sur})$	(1.7) (1.8)	ε h_r (W/m^2 · K)

- *Conservation of Energy*

$$\Delta E_{st} = E_{in} - E_{out} + E_g \quad (1.12b)$$

- *Using a dot over a term to indicate a rate*

$$\dot{E}_{st} \equiv \frac{dE_{st}}{dt} = \dot{E}_{in} - \dot{E}_{out} + \dot{E}_g \quad (1.12c)$$

- The *simplified steady-flow thermal energy equation*

$$q = \dot{m}c_p(T_{out} - T_{in}) \quad (1.12e)$$

- *The Surface Energy Balance*

$$\dot{E}_{in} - \dot{E}_{out} = 0 \quad (1.13)$$

HT Week 1 Recall

- Introduction -

WK1: Introduction to heat transfer

WK2: Conduction 1: Introduction of Conduction Heat transfer

WK3: Conduction 2: One-dimensional steady state conduction

WK4: Conduction 3: Two-dimensional steady state conduction

WK5: Conduction 4: Transient conduction

WK6: Conduction 5: Numerical heat transfer introduction

WK7: Numerical heat transfer – case study: conduction

WK8: Convection 1: Introduction and equations

WK9: Convection 2: Boundary layer and BL integration

WK10: Convection 3: Other solutions of BL equations

WK11: Convection 4: Internal flow

WK12: Convection 5: Other convective modes

WK13: Numerical heat transfer – case study: convection

WK14: Radiation: Introduction and fundamentals

WK15: Radiation: Radiation between surfaces.

WK16: Revision week

Syllabus

L2. Introduction to Condution

- 2.1 Fourier’s Law
- 2.2 Thermal Conductivity
- 2.3 Heat Diffusion Equation
- 2.4 Boundary and Initial Conditions

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HT: Conduction

L2: Introduction to Conduction

Learning Objectives:

- **Explain the Fourier’s law**
- **Describe the thermal conductivity**
- **Develop the general heat conduction equation**
- **B.C. & I.C.**

Conduction HT – Basic Concepts

- Conduction
  - Simplest among the three modes of heat transfer
  - It occurs all three phases of matter (gas, liquid, and solid)
  - The driving force of heat conduction is temperature gradient.
    - Whenever there is temperature gradient, there exists heat conduction.
    - The basic law of heat conduction is Fourier’s law.

Conduction HT – Basic Concepts

- **Temperature field:**

$$T = T(x, y, z, \tau)$$

Steady-state conduction:

$$\frac{\partial T}{\partial \tau} = 0 \qquad T = T(x, y, z)$$

Transient-state conduction:

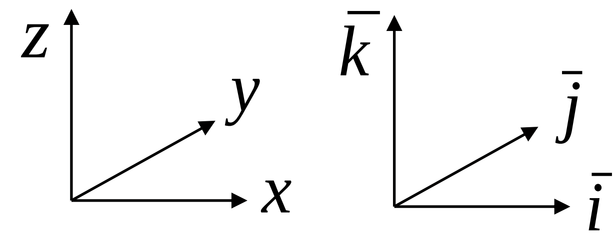
$$T = T(x, y, z, \tau)$$

Conduction HT – Basic Concepts

- **Temperature gradient:**
  - Since we will be dealing with other coordinates also, realize that the gradient takes on other forms:

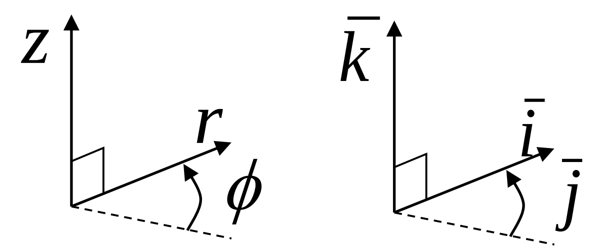
<u>Cartesian</u>

$$\bar{\nabla} T = \bar{i}\,\frac{\partial T}{\partial x} + \bar{j}\,\frac{\partial T}{\partial y} + \bar{k}\,\frac{\partial T}{\partial z}$$



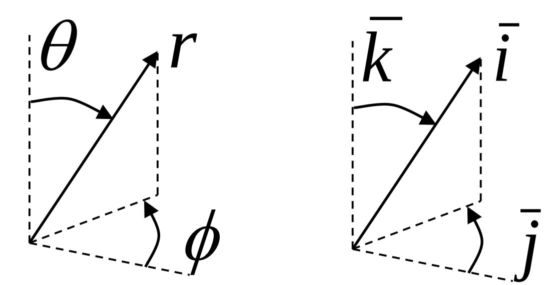
<u>Cylindrical</u>

$$\bar{\nabla} T = \bar{i}\,\frac{\partial T}{\partial r} + \bar{j}\,\frac{1}{r}\frac{\partial T}{\partial \phi} + \bar{k}\,\frac{\partial T}{\partial z}$$



<u>Spherical</u>

$$\bar{\nabla} T = \bar{i}\,\frac{\partial T}{\partial r} + \bar{j}\,\frac{1}{r}\frac{\partial T}{\partial \theta} + \bar{k}\,\frac{1}{r\sin\theta}\frac{\partial T}{\partial \phi}$$

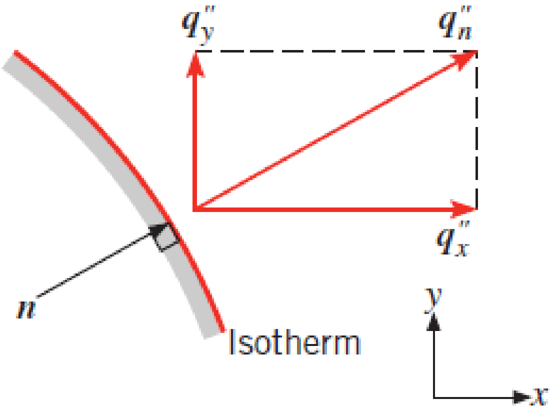


Conduction HT – Basic Concepts

- **Isothermal surfaces**

$$T = T(x, y, z)$$

$$\overline{\nabla} T = \bar{i}\frac{\partial T}{\partial x} + \bar{j}\frac{\partial T}{\partial y} + \bar{k}\frac{\partial T}{\partial z}$$



- Temperature field is a scalar
- Temperature gradient is a **vector** (perpendicular to the **isothermal surfaces** or **isotherms**, and from <u>low T to high T</u>)

- Heat is transferred from <u>high T to low T</u>.

Conduction HT – Basic Concepts

- **Heat Flux, *q*** q''

Heat flow is measured as the amount of energy transferred through any given plane per unit area and per unit time, which is called heat flux

$$heat\ \ flux = q = \frac{energy}{area \cdot time} = \frac{\phi}{A \cdot t} = \frac{J}{m^2 \cdot s}$$

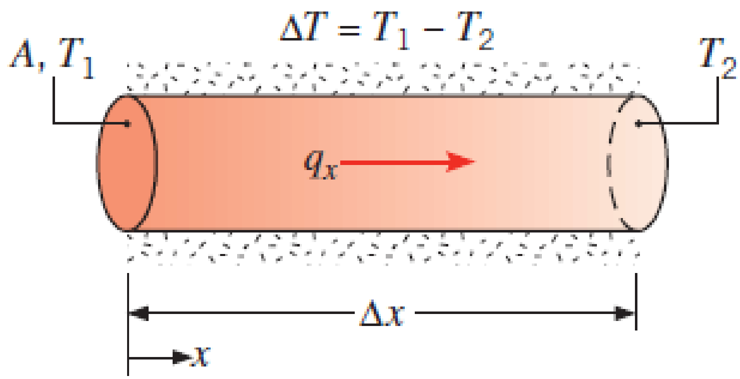
- **Heat Rate, *Q*** q

$$Q = q \cdot A = \frac{energy}{time} = \frac{\phi}{t}$$

2.1 Fourier's Law

- **Fourier's law is phenomenological**

The temperature difference causes conduction heat transfer



(Steady-state heat conduction experiment, 1822)

B Fourier, France (1768-1830)

If it is **one-dimensional, steady-state:**

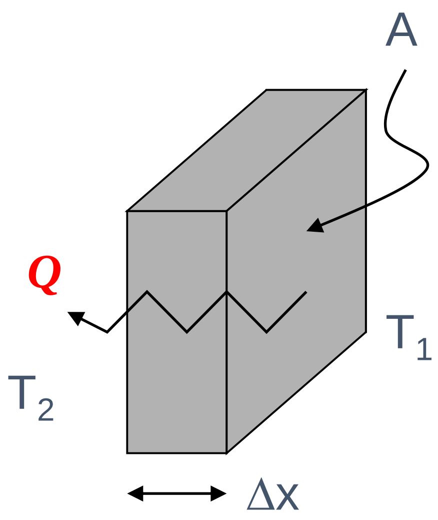
$$\boxed{Q_x \propto A\frac{\Delta T}{\Delta x}}$$

q_x : the heat transfer rate
A : the cross-sectional area
ΔT : the temperature difference
Δx : the rod length

Conduction HT – Basic Concepts

- **Fourier's Law**

  - **When two differing temperatures occur on opposing sides of a material, the rate of heat transfer through the material is directly proportional to the surface area and temperature difference but inversely proportional to the thickness.**



- Mathematically:

$$Q \propto A\Delta T / \Delta x$$

$Q =$ *heat transfer rate or heat flux (J/s or W)*

$A =$ *area* (m^2)

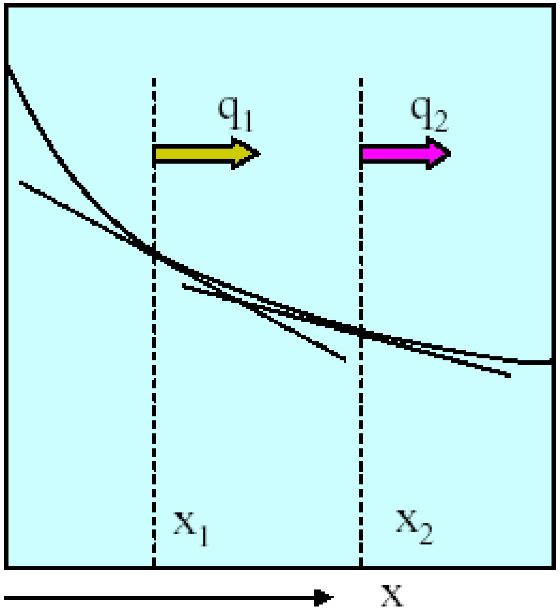
$\Delta T =$ *temperature difference* $= T_2 - T_1 \ ({}^{o}C \text{ or } {}^{o}K)$

$\Delta x =$ *thickness (m)*

2.1 Fourier's Law (cont.)

- **The heat transfer rate is** material-dependent. (e.g., plastic, metal, etc.)

- The constant of proportionality is called **thermal conductivity *k***, W/m·K



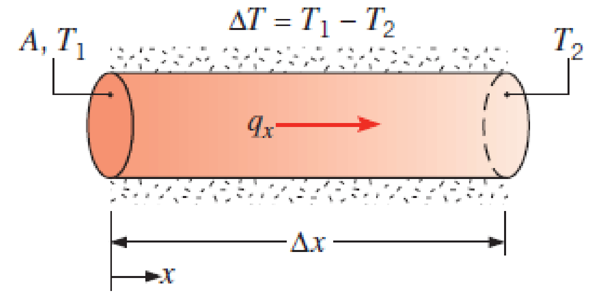
heat rate: $$Q_x = -kA\frac{dT}{dx}$$

heat flux: $$\boxed{q_x = \frac{Q_x}{A} = -k\frac{dT}{dx}}$$ ← **Fourier's law**

Note:
the minus sign – the heat is transferred in the direction of decreasing temperature

2.1 Fourier's Law (cont.)

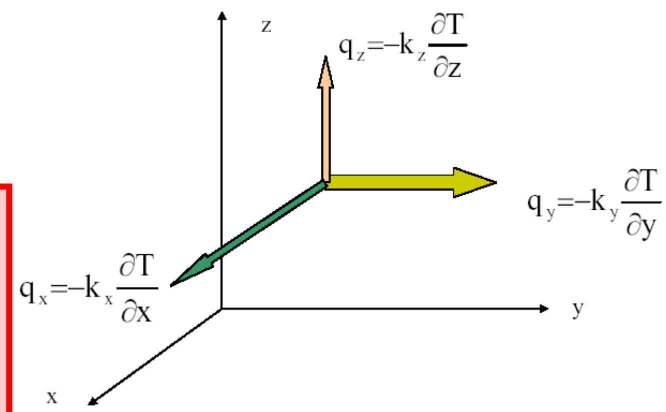
- One dimensional: $$q_x = \frac{Q_x}{A} = -k\frac{dT}{dx}$$



- **A general statement of Fourier's law**

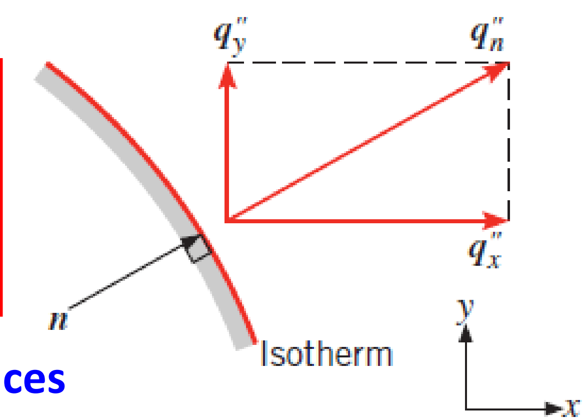
three-dimensional, isotropic material

$$\boxed{q = -k\nabla T = -k\left(\frac{\partial T}{\partial x}\vec{i} + \frac{\partial T}{\partial y}\vec{j} + \frac{\partial T}{\partial z}\vec{k}\right)}$$



(An alternative form)

$$\boxed{\boldsymbol{q} = q_n\boldsymbol{n} = -k\frac{\partial T}{\partial n}\boldsymbol{n}}$$



Note:
the heat flux – vector, perpendicular to the isothermal surfaces

L2. Introduction to Condution

- 2.1 Fourier’s Law
- **2.2 Thermal Conductivity**
- 2.3 Heat Diffusion Equation
- 2.4 Boundary and Initial Conditions

2.2 Thermal conductivity

$$q = -k\nabla T = -k\left(\frac{\partial T}{\partial x}\vec{i} + \frac{\partial T}{\partial y}\vec{j} + \frac{\partial T}{\partial z}\vec{k}\right)$$

$$\boxed{k_x \equiv k_y \equiv k_z \equiv k}$$

- **Isotropic material:** k is independent of the coordinate direction

$$q = q_x\vec{i} + q_y\vec{j} + q_z\vec{k} = -\left(k_x\frac{\partial T}{\partial x}\vec{i} + \frac{\partial T}{\partial y}\vec{j} + \frac{\partial T}{\partial z}\vec{k}\right), \quad q_x = -k_x\frac{\partial T}{\partial x}\vec{i}$$

- Thermal conductivity depends strongly upon the material and usually also varies temperature.

In general: $k_{\text{metal}} > k_{\text{non-metal}}$; $k_{\text{solid}} > k_{\text{liquid}} > k_{\text{gas}}$

For fluids (gasses and liquids) conduction occurs through the random motion of the fluid particles.

2.2 Thermal conductivity

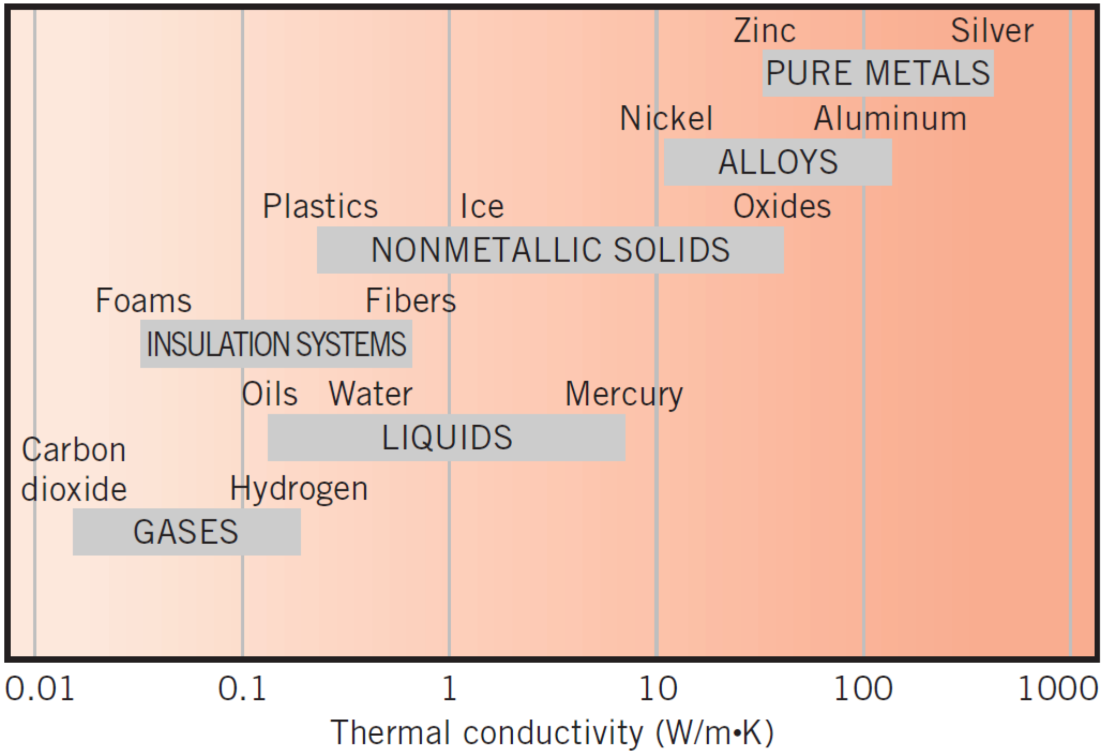


FIGURE 2.4 Range of thermal conductivity for various states of matter at normal temperatures and pressure.

2.2 Thermal conductivity

- **The Solid State**

For solids, there are two mechanisms of heat transfer: **the migration of free electrons** and **crystal lattice vibration**.

➢ The migration of free electrons is similar to the conduction by random particle motion in gasses.

Since the number of free electrons is proportional to the electrical conductance of the material, better electrical conductors are better heat conductors.

➢ Lattice vibration is associate with vibrations of the atoms and molecules bound in the structure of solids. Basic, shake one side of a crystal and the other side moves in response.

2.2 Thermal conductivity

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- How thermal conductivity depends on temperature?

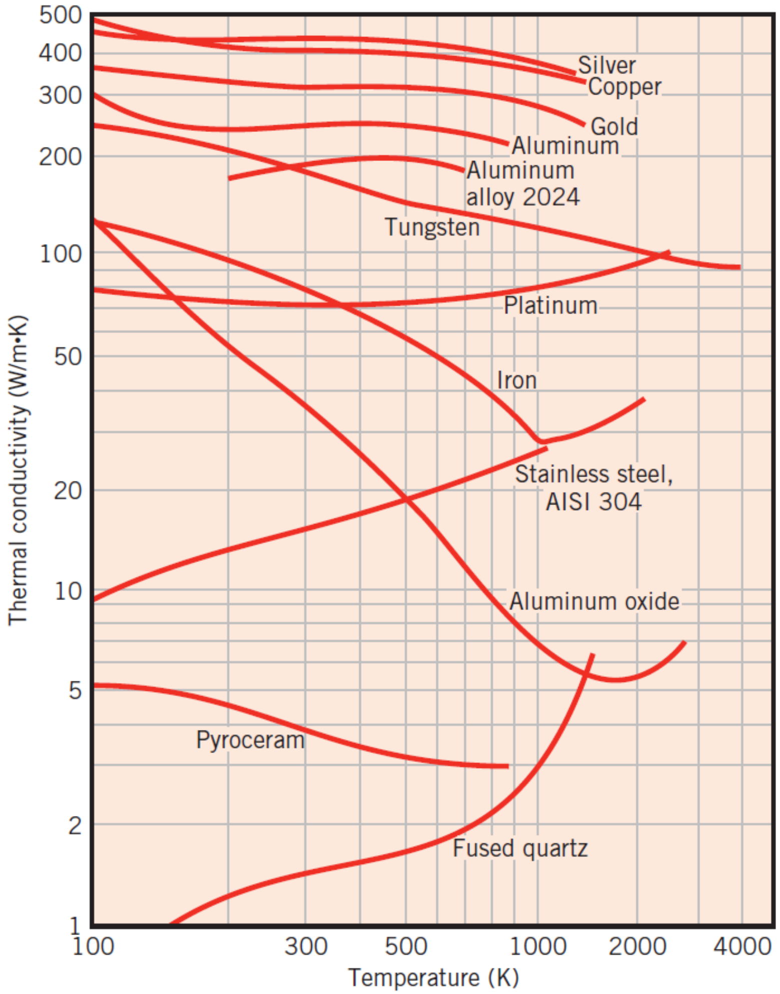
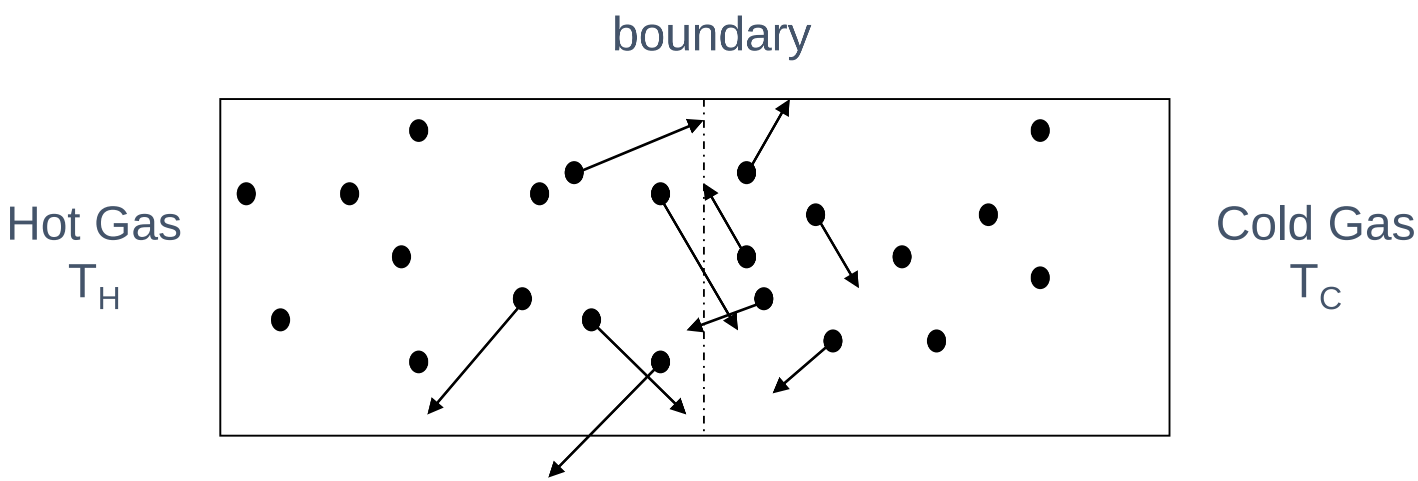


FIGURE 2.5 The temperature dependence of the thermal conductivity of selected solids.

2.2 Thermal conductivity

- **Fluids**

Consider the flux across an imaginary boundary between two fluids (gasses and liquids) at different temperatures.



- For fluids (gasses and liquids) conduction occurs through the random motion of the fluid particles.
- As a result of this random motion, energy is transfer from side of the partition to the other - this is conduction.
- It also follows that as temperature increases, there is more random motion, and thus the conduction rate increases.

2.2 Thermal conductivity

- **The Fluid State**

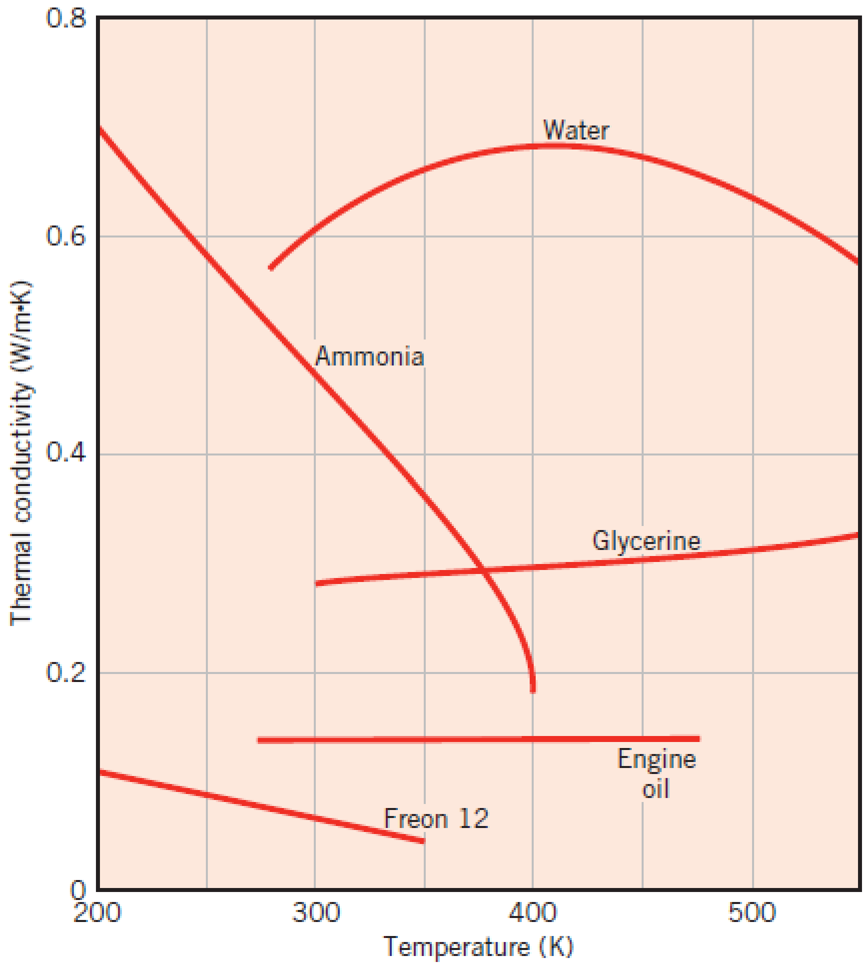


FIGURE 2.9 The temperature dependence of the thermal conductivity of selected nonmetallic liquids under saturated conditions.

2.2 Thermal conductivity

- **The Gas State**

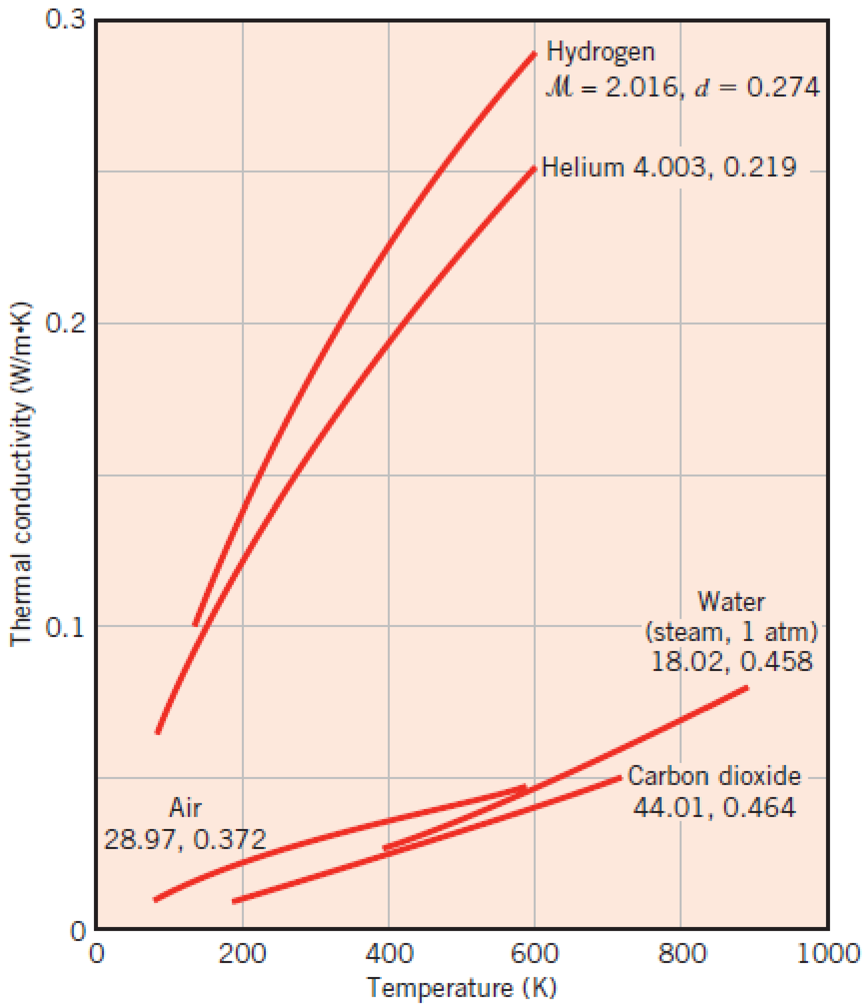


FIGURE 2.8 The temperature dependence of the thermal conductivity of selected gases at normal pressures. Molecular diameters (d) are in nm [10]. Molecular weights ($\mathcal{M}$) of the gases are also shown.

- Other thermal properties?

e.g. thermal diffusivity α

$$\alpha = \frac{k}{\rho c_p}$$

TABLE A.1 Thermophysical Properties of Selected Metallic Solids[a]

		Properties at 300 K				Properties at Various Temperatures (K)									
						k (W/m · K)/c_p (J/kg · K)									
Composition	Melting Point (K)	ρ (kg/m^3)	c_p (J/kg · K)	k (W/m · K)	$\alpha \cdot 10^6$ (m^2/s)	100	200	400	600	800	1000	1200	1500	2000	2500
Aluminum															
Pure	933	2702	903	237	97.1	302	237	240	231	218					
						482	798	949	1033	1146					
Alloy 2024-T6 (4.5% Cu, 1.5% Mg, 0.6% Mn)	775	2770	875	177	73.0	65	163	186	186						
						473	787	925	1042						
Alloy 195, Cast (4.5% Cu)		2790	883	168	68.2			174	185						
								—	—						
Beryllium	1550	1850	1825	200	59.2	990	301	161	126	106	90.8	78.7			
						203	1114	2191	2604	2823	3018	3227	3519		
Bismuth	545	9780	122	7.86	6.59	16.5	9.69	7.04							
						112	120	127							
Boron	2573	2500	1107	27.0	9.76	190	55.5	16.8	10.6	9.60	9.85				
						128	600	1463	1892	2160	2338				
Cadmium	594	8650	231	96.8	48.4	203	99.3	94.7							
						198	222	242							
Chromium	2118	7160	449	93.7	29.1	159	111	90.9	80.7	71.3	65.4	61.9	57.2	49.4	
						192	384	484	542	581	616	682	779	937	
Cobalt	1769	8862	421	99.2	26.6	167	122	85.4	67.4	58.2	52.1	49.3	42.5		
						236	379	450	503	550	628	733	674		
Copper															
Pure	1358	8933	385	401	117	482	413	393	379	366	352	339			
						252	356	397	417	433	451	480			
Commercial bronze (90% Cu, 10% Al)	1293	8800	420	52	14		42	52	59						
							785	460	545						
Phosphor gear bronze (89% Cu, 11% Sn)	1104	8780	355	54	17		41	65	74						
							—	—	—						
Cartridge brass (70% Cu, 30% Zn)	1188	8530	380	110	33.9	75	95	137	149						
							360	395	425						
Constantan (55% Cu, 45% Ni)	1493	8920	384	23	6.71	17	19								
						237	362								
Germanium	1211	5360	322	59.9	34.7	232	96.8	43.2	27.3	19.8	17.4	17.4			
						190	290	337	348	357	375	395			

2.3 Heat Diffusion Equation

- **Objective:**

  To determine the temperature distribution

- **Approach:**

  Fourier's law + Energy conservation equation

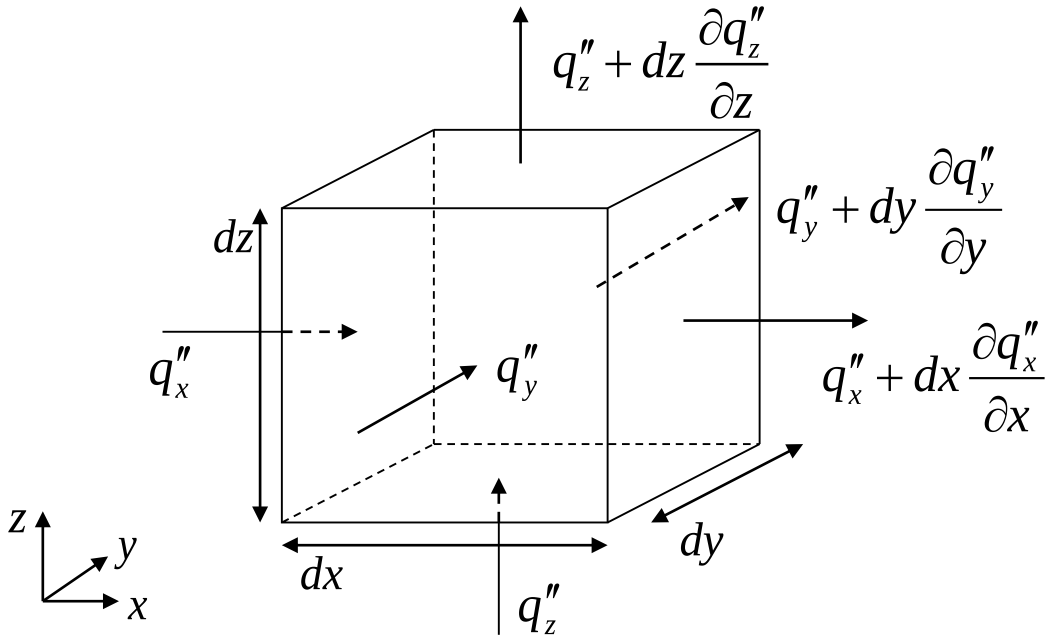
2.3 Heat Diffusion Equation

- In Cartesian coordinates:

$$\bar{q} = -kA\bar{\nabla}T = \bar{i}q_x + \bar{j}q_y + \bar{k}q_z$$

$$q_x = -kA\frac{\partial T}{\partial x} \qquad q_y = -kA\frac{\partial T}{\partial y} \qquad q_z = -kA\frac{\partial T}{\partial z}$$

- To develop a generalized governing law for heat conduction, we begin by consider the fluxes in an element as shown:



2.3 Heat Diffusion Equation (cont.)

- A statement of energy conservation for this control mass (similar to the 1st Law) would be:

$$\begin{bmatrix} \text{Rate of flux} \\ \text{of energy into} \\ \text{the element} \end{bmatrix} + \begin{bmatrix} \text{Rate of energy} \\ \text{generation in} \\ \text{the element} \end{bmatrix} = \begin{bmatrix} \text{Rate of change} \\ \text{of energy storage in the} \\ \text{element} \end{bmatrix}$$

- Let’s consider this statement term by term.

2.3 Heat Diffusion Equation (cont.)

- (1) consider the heat flux through the element:

$$\begin{bmatrix}\text{Rate of flux} \\ \text{of energy into} \\ \text{the element}\end{bmatrix} = \left[\text{Heat influx}\right] - \left[\text{Heat outflux}\right]$$

$$= \left[q''_x dydz + q''_y dxdz + q''_z dydx\right] -$$

$$\left[\left(q''_x + \frac{\partial q''_x}{\partial x}dx\right)dydz + \left(q''_y + \frac{\partial q''_y}{\partial y}dy\right)dxdz + \left(q''_z + \frac{\partial q''_z}{\partial z}dz\right)dxdy\right]$$

$$= -\left(\frac{\partial q''_x}{\partial x} + \frac{\partial q''_y}{\partial y} + \frac{\partial q''_z}{\partial z}\right)dxdydz$$

$$\frac{\partial q''_x}{\partial x} + \frac{\partial q''_y}{\partial y} + \frac{\partial q''_z}{\partial z} = \boxed{\frac{\partial}{\partial x}\left(-k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(-k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(-k\frac{\partial T}{\partial z}\right) = \bar{\nabla}\cdot\left(-k\bar{\nabla}T\right)}$$

2.3 Heat Diffusion Equation (cont.)

- (2) consider the heat generation term:

$$\begin{bmatrix} \text{Rate of energy} \\ \text{production in} \\ \text{the element} \end{bmatrix} = \boxed{\dot{q}dxdydz}$$

$$\dot{q} \equiv \text{heat generation per unit mass}$$
$$(\mathrm{W/m^3})$$

- Sources of heat generation could be electrical resistance, chemical reactions, nuclear reactions, or even radiation absorption like in a microwave.

2.3 Heat Diffusion Equation (cont.)

(3) the rate of change of energy:

$$\begin{bmatrix} \text{Rate of change} \\ \text{of energy storage in the} \\ \text{element} \end{bmatrix} = \boxed{\rho c_p \frac{\partial T}{\partial t} dxdydz}$$

2.3 Heat Diffusion Equation (cont.)

- Finally, putting all these together, we get the heat diffusion equation:

$$\boxed{\rho c_p \frac{\partial T}{\partial t} = \overline{\nabla} \cdot \left(k \overline{\nabla} T \right) + \dot{q}}$$

- Note that this is a 2nd order differential equation!

- One common simplification of this equation is to assume that the material conductivity, k, is independent of position.This is valid if:
  - objects are made of a single material
  - k is not a strong function of T or T is nearly constant
- In this case, the diffusion equation can be written as:

$$\boxed{\frac{1}{\alpha} \frac{\partial T}{\partial t} = \nabla^2 T + \frac{\dot{q}}{k}}$$

- where

$$\alpha = \frac{k}{\rho c_p} \equiv \text{thermal diffusivity } (\text{m}^2/\text{sec}) \qquad \nabla^2 = \overline{\nabla} \cdot \overline{\nabla} \equiv \text{Laplace differential operation}$$

2.3 Heat Diffusion Equation (cont.)

$$\boxed{\frac{1}{\alpha}\frac{\partial T}{\partial t} = \nabla^2 T + \frac{\dot{q}}{k}}$$

Cartesian

$$\nabla^2 T = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}$$

Cylindrical

$$\nabla^2 T = \frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2}$$

Spherical

$$\nabla^2 T = \frac{1}{r}\frac{\partial^2}{\partial r^2}(rT) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial T}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 T}{\partial \phi^2}$$

L2. Introduction to Condution

- 2.1 Fourier’s Law
- 2.2 Thermal Conductivity
- 2.3 Heat Diffusion Equation
- **2.4 Boundary and Initial Conditions**

2.4 Boundary and Initial Conditions

$$\boxed{\frac{1}{\alpha}\frac{\partial T}{\partial t} = \nabla^2 T + \frac{\dot{q}}{k}}$$

- Requirements:

  - Two **Boundary Conditions (2)**

    the heat equation is **second order in the spatial coordinates**

  - One **Initial Condition (1)**

    the heat equation is **first order in time**

2.4 Boundary and Initial Conditions

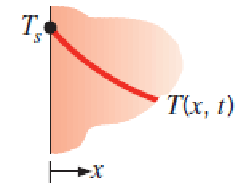
- **IC:**
  - $T(x_w,0) = T_i$

- **Three** types of **BCs**

TABLE 2.2 Boundary conditions for the heat diffusion equation at the surface ($x = 0$)

1. Constant surface temperature

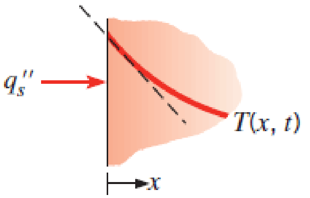
$$T(0, t) = T_s \tag{2.31}$$



2. Constant surface heat flux

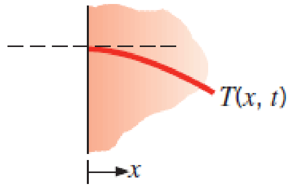
(a) Finite heat flux

$$-k\frac{\partial T}{\partial x}\bigg|_{x=0} = q_s'' \tag{2.32}$$



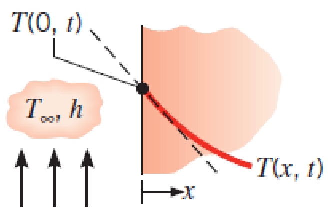
(b) Adiabatic or insulated surface

$$\frac{\partial T}{\partial x}\bigg|_{x=0} = 0 \tag{2.33}$$



3. Convection surface condition

$$-k\frac{\partial T}{\partial x}\bigg|_{x=0} = h[T_\infty - T(0, t)] \tag{2.34}$$

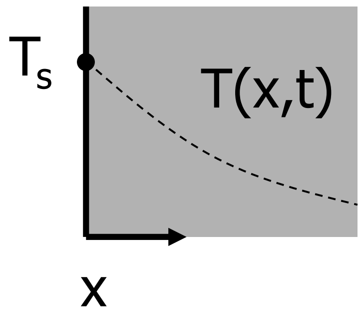


2.4 Boundary and Initial Conditions

- **BCs**

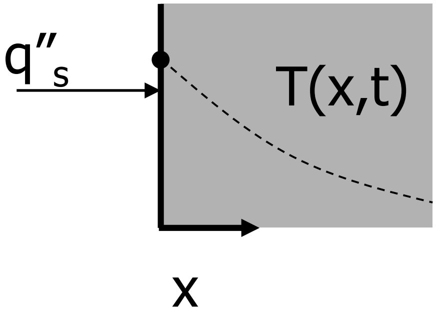
  – Fixed temperature (First B.C.):

    - $T(0, t) = T_s$



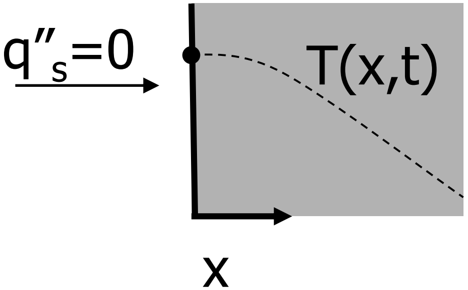
2.4 Boundary and Initial Conditions

- **BCs**
  - Fixed heat flux (Second B. C.)
    - $-k(dT/dx)_{x=0}=q''_s$



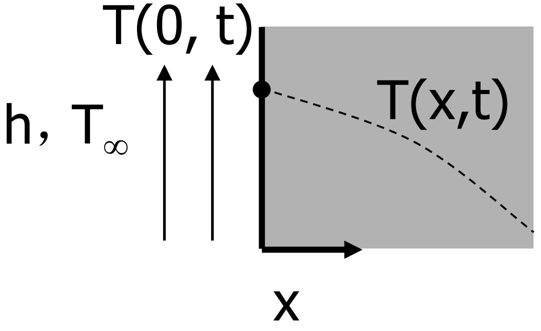
2.4 Boundary and Initial Conditions

- **BCs**
  - Adiabatic Wall (Second B. C.)
    - $-k(dT/dx)_{x=0}=0$



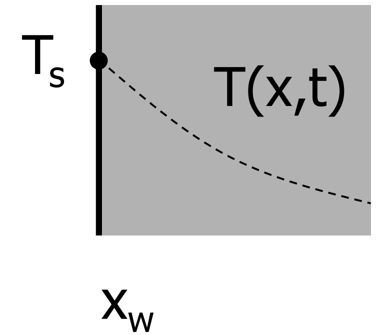
2.4 Boundary and Initial Conditions

- **BCs**
  - Convectively cooled (Third B. C.)
    - $-k(dT/dx)_{x=0}=h[T_\infty - T(0, t)]$

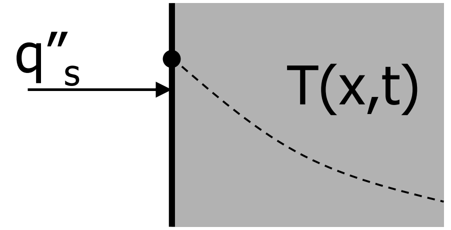


2.4 Boundary and Initial Conditions

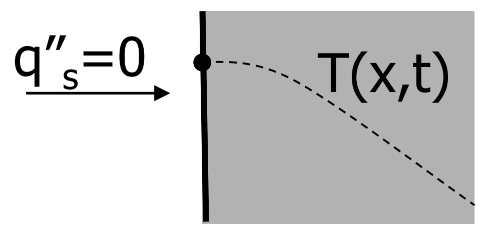
- **BCs**
  - Fixed temperature (First B.C.):
    - $T(x_w,t) = T_s$



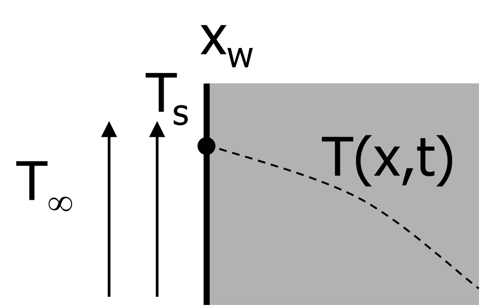
  - Fixed heat flux (Second B. C.)
    - $-k(dT/dx)_{xw}=q''_s$



  - Adiabatic Wall (Second B. C.)
    - $-k(dT/dx)_{xw}=0$



  - Convectively cooled (Third B. C.)
    - $-k(dT/dx)_{xw}=h(T_\infty -T_s)$



HT Week 2 Wrap Up

- Fourier's Law

$$q = -k\nabla T = -k\left(\frac{\partial T}{\partial x}\vec{i} + \frac{\partial T}{\partial y}\vec{j} + \frac{\partial T}{\partial z}\vec{k}\right)$$

- Thermal Conductivity *k*, thermal diffusivity $\boldsymbol{\alpha}$

- Heat Diffusion Equation

$$\rho c_p \frac{\partial T}{\partial t} = \overline{\nabla} \cdot \left(k\overline{\nabla} T\right) + \dot{q}$$

- Boundary and Initial Conditions

Homework

- Develop the heat diffusion equation in cylindrical and spherical coordinates.

- P97: 2.12

- P101: 2.34 *(optional)*