§ 6-3 Forced convection heat transfer in a pipe

1. The introduciton of the flow and heat transfer in a pipe

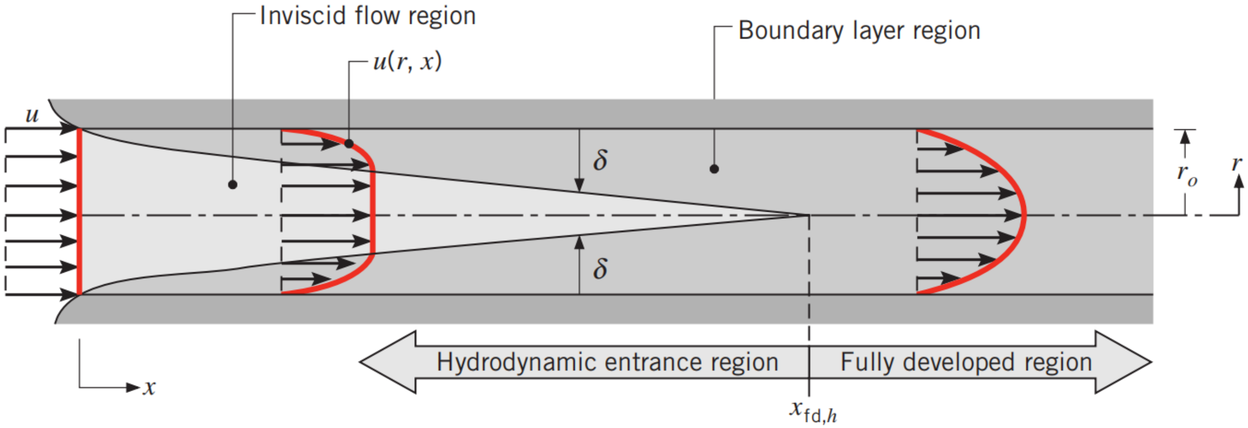
(1) flow state

- **Laminar:** $Re < 2300$
- **Transition:** $2300 < Re < 10000$
- **Developed:** $10000 < Re$

§ 6-3 Forced convection heat transfer in a pipe

(2) Entrance Region and Fully developed region

Flow:



The length of the entrance:

laminar: $l/d \approx 0.05Re$

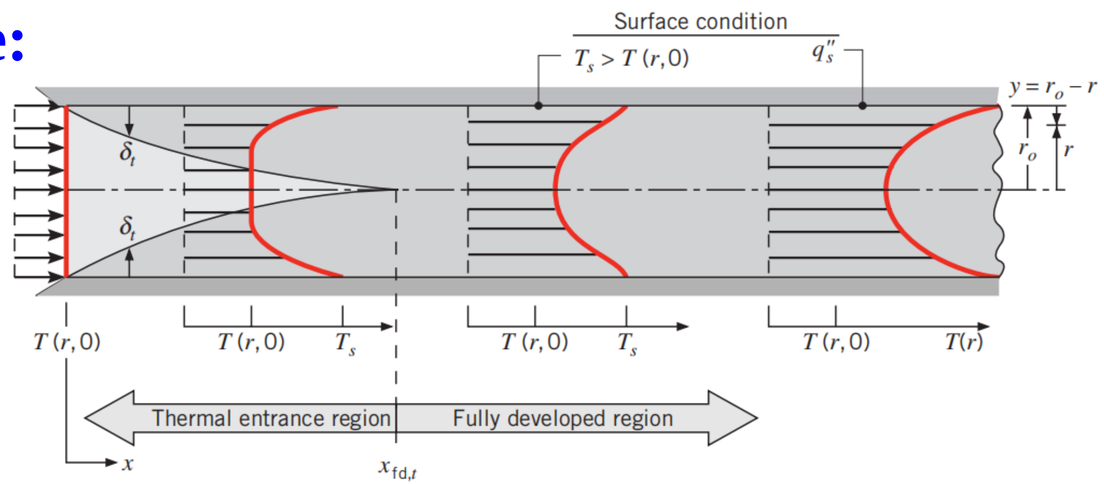
Turbulence: $l/d \approx 60$

the thickness of the velocity boundary layer equals to the radius and the velocity profile does not change in fully developed region

§ 6-3 Forced convection heat transfer in a pipe

(2) Entrance Region and Fully developed region

temperature:



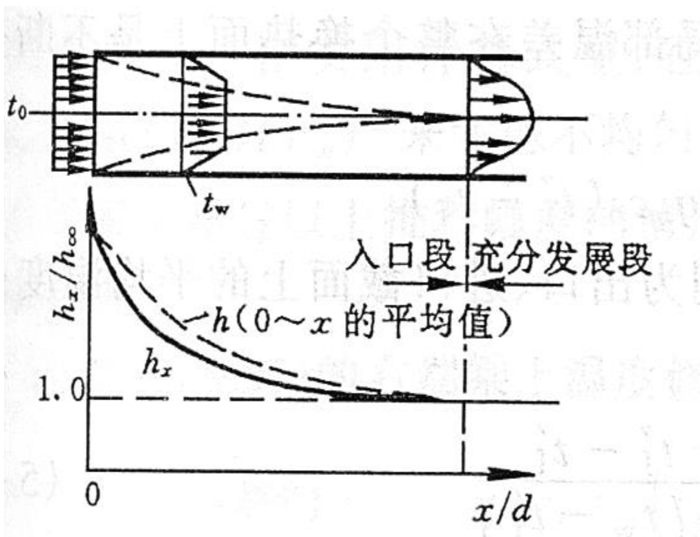
The length of the entrance:

laminar: $l/d \approx 0.05RePr$

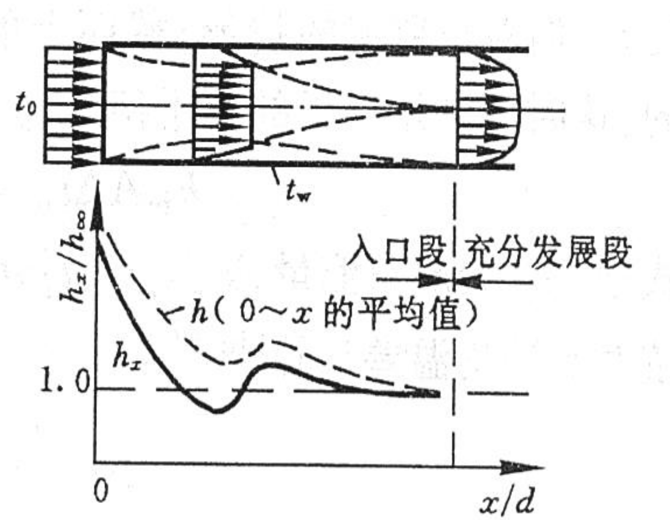
Turbulence: $l/d \approx 60$

With the thickness of the thermal boundary layer increases, the temperature gradient decreases and the convective heat transfer coefficient *h* reduces.

(2) Entrance Region and Fully developed region



laminar



turbulence

h in entrance region is larger which can be used in practical application in industry.

§ 6-3 Forced convection heat transfer in a pipe

2. Effects on forced convection heat transfer in a pipe

$$Q = hA\Delta T$$

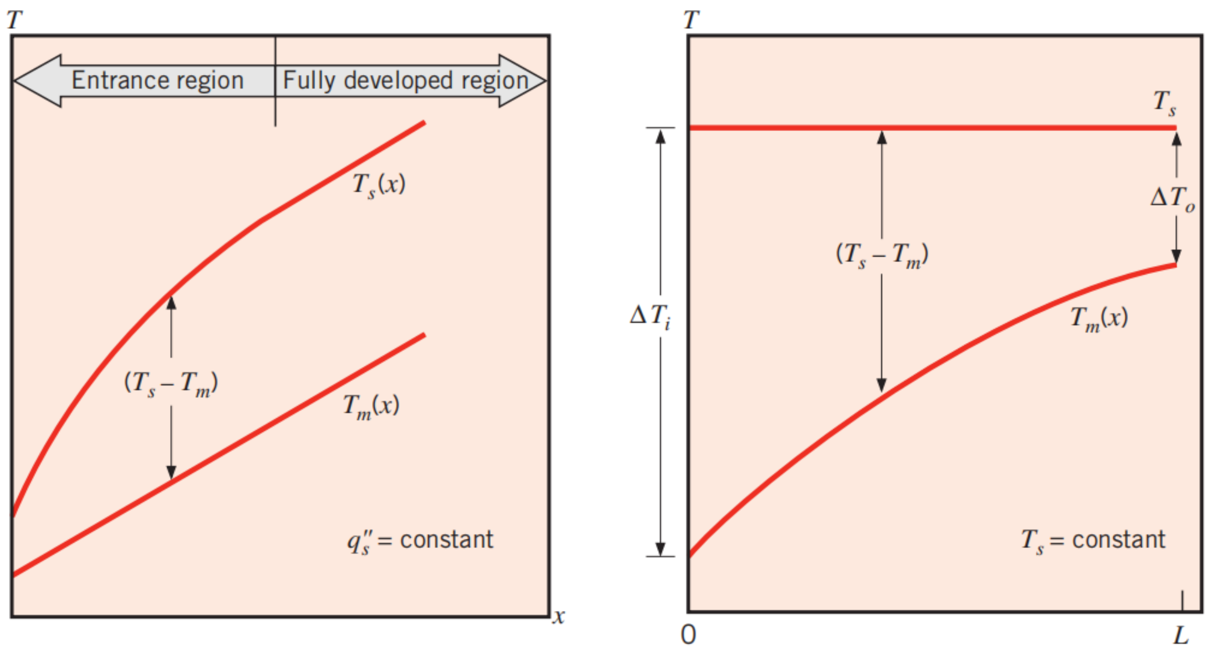
(1) Thermal boundary conditions:

- Constant heat flux： The wall surface is heated by evenly wound electric heating wire。
- Constant temperature： steam condensation heat or liquid boiling cooling

Turbulence: the differences of ***h*** is small, can be neglected

Laminar: significant differences and need detailed discussion

§ 6-3 Forced convection heat transfer in a pipe



(a) (b)

➤ Constant heat flux： in fully developed region, the temperature difference between wall and fluid is constant.

➤ Constant temperature： the wall temperature is fixed and the fluid temperature increases along the axial direction.

§ 6-3 Forced convection heat transfer in a pipe

Reference temperature(velocity):

velocity: the average velocity of the cross-section。

temperature: the average temperature of the cross-section or the average temperature of the inlet and outlet.

Temperature difference:

- Constant heat flux： T_w-T_f
- Constant temperature：

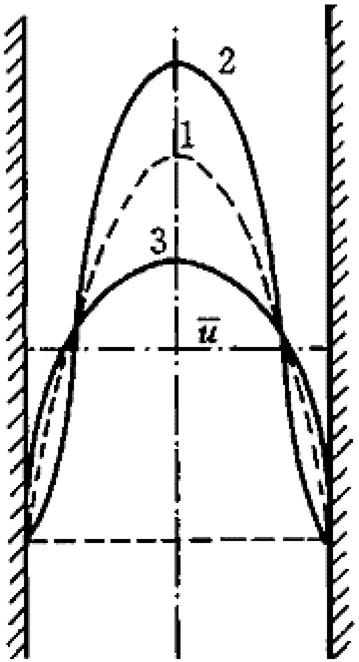
$$\Delta T_{\mathrm{lm}} \equiv \frac{\Delta T_o - \Delta T_i}{\ln\left(\Delta T_o / \Delta T_i\right)}$$

§ 6-3 Forced convection heat transfer in a pipe

2. Effects on forced convection heat transfer in a pipe

(2) The flow is heated or cooled:

- With large temperature difference, the thermal properties varies, and results in different heat transfer.
- For liquid： viscosity
- For gas: viscosity and density

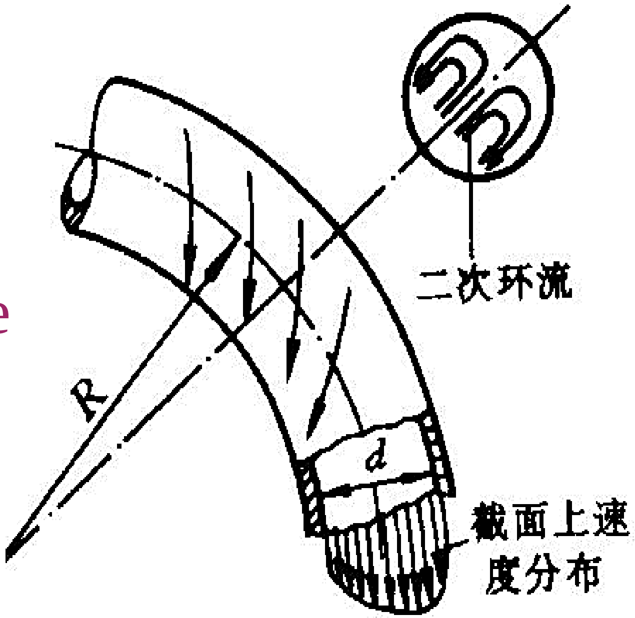


§ 6-3 Forced convection heat transfer in a pipe

2. Effects on forced convection heat transfer in a pipe

(3) The curvature of the flow channel

Secondary swirls due to the centrifugal force

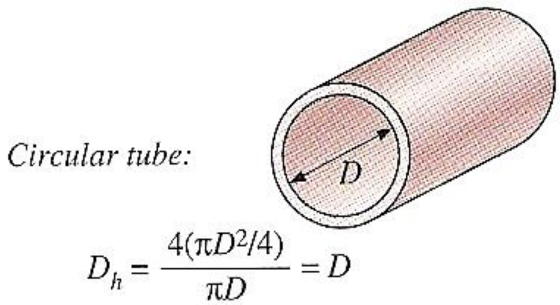


§ 6-3 Forced convection heat transfer in a pipe

3.Empirical heat transfer correlation formula in a pipe flow

3.1 Turbulence:

(a)Dittus-boelter formula（1930）



$$\boxed{Nu_f = 0.023 \mathrm{Re}_f^{0.8} \mathrm{Pr}_f^{\,n}}$$

★ **Suitable for：**

$$\begin{cases} n = \begin{cases} 0.4 & t_w > t_f \\ 0.3 & t_w < t_f \end{cases} \\ 0.7 \le \mathrm{Pr}_f \le 120, \quad \mathrm{Re}_f = 10^4 - 1.2\times 10^5, \\ l/d \ge 60 \end{cases}$$

- Ref Tem： The average temperature of the inlet and outlet； Ref Length: inner diameter.
- **Normally it can apply for cases with moderate temperature difference**

Gas: $\Delta t = t_w - t_f < 50°C$ **water:** $\Delta t < 30°C$ **Oil:** $\Delta t < 10°C$

§ 6-3 Forced convection heat transfer in a pipe

The thermal properties modification:

（1）迪贝斯－贝尔特修正公式

$$Nu_f = 0.023\mathrm{Re}_f^{0.8}\mathrm{Pr}_f^n c_t$$

When heating gas， $c_t = \left(\frac{T_f}{T_w}\right)^{0.5}$

When cooling gas， $c_t = 1.0$

liquid $c_t = \left(\frac{\eta_f}{\eta_w}\right)^m$ $\begin{cases} m = 0.11 & \textbf{heated} \\ m = 0.25 & \textbf{cooled} \end{cases}$

§ 6-3 Forced convection heat transfer in a pipe

(2）采用齐德－泰特公式：

$$Nu_f = 0.027\mathrm{Re}_f^{0.8}\mathrm{Pr}_f^{1/3}\left(\frac{\eta_f}{\eta_w}\right)^{0.14}$$

Suitable for：

$$l/d \geq 60,$$

$$\mathrm{Pr}_f = 0.7\sim16700,$$

$$\mathrm{Re}_f \geq 10^4$$

§ 6-3 Forced convection heat transfer in a pipe

(3) 采用米海耶夫公式：

$$Nu_f = 0.021\mathrm{Re}_f^{0.8}\mathrm{Pr}_f^{0.43}\left(\frac{\mathrm{Pr}_f}{\mathrm{Pr}_w}\right)^{0.25}$$

Suitable for：

$l/d \geq 50$,

$\mathrm{Pr}_f = 0.6 \sim 700$,

$\mathrm{Re_f} \geq 10^4 \sim 1.75 \times 10^6$

§ 6-3 Forced convection heat transfer in a pipe

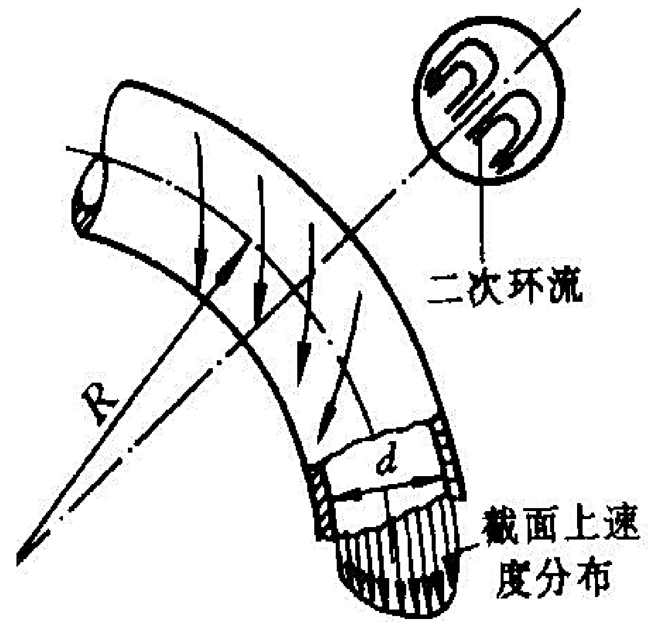
entrance effect

$$c_l = 1 + \left(\frac{d}{l}\right)^{0.7}$$

curvature effect

Gas: $$c_r = 1 + 10.3\left(\frac{d}{R}\right)^{3}$$

Liquid: $$c_r = 1 + 1.77\frac{d}{R}$$



§ 6-3 Forced convection heat transfer in a pipe

Non-circular tube:

$$\boxed{d_e = \frac{4A_c}{p}}$$

Hydraulic radius

§ 6-3 Forced convection heat transfer in a pipe

Gnielinaki公式（1976年）

$$\boxed{Nu_f = \frac{(f/8)(\mathrm{Re}-1000)\,\mathrm{Pr}}{1+12.7\sqrt{f/8}\,(\mathrm{Pr}^{2/3}-1)}\left[1+\left(\frac{d}{l}\right)^{2/3}\right]c_t}$$

- **liquid，** $c_t = \left(\frac{\mathrm{Pr}_f}{\mathrm{Pr}_w}\right)^{0.01}, \quad \frac{\mathrm{Pr}_f}{\mathrm{Pr}_w} = 0.05-20$
- **gas，** $c_t = \left(\frac{T_f}{T_w}\right)^{0.45}, \quad \frac{T_f}{T_w} = 0.5-1.5$
- f **drag coefficient：** $f = (1.82\lg \mathrm{Re} - 1.64)^{-2}$
- **suitable：** $\mathrm{Re}_f = 2300-10^6, \ \mathrm{Pr}_f = 0.6-10^5$

§ 6-3 Forced convection heat transfer in a pipe

For liquid metal:

（1）fixed heat flux：

$$\boxed{Nu_f = 4.82 + 0.0185 Pe_f{}^{0.827}}$$

- suitable： $\mathrm{Re}_f = 3.6\times10^3 - 9.05\times10^5,\ Pe_f = 10^2 - 10^4$

（2）fixed temperature：

$$\boxed{Nu_f = 5.0 + 0.025 Pe_f{}^{0.8}}$$

- 适用条件： $\mathrm{Re}_f = 3.6\times10^3 - 9.05\times10^5,\ Pe_f > 100$

§ 6-3 Forced convection heat transfer in a pipe

3.2 Laminar:

- **Thermal boundary conditions**
- **Shape of the cross-section**
- **Normally the heat transfer occurs in entrance region**
- **The Nu is independent on Re in fully developed region**

（a）heat transfer in entrance region：

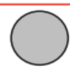
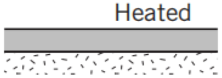
$$\boxed{Nu_f = 1.86\left(\frac{\mathrm{Re}_f\,\mathrm{Pr}_f}{l/d}\right)^{1/3}\left(\frac{\eta_f}{\eta_w}\right)^{0.14}}$$

★适用条件：

$$\begin{cases} \mathrm{Pr}_f = 0.48-16700,\ \dfrac{\eta_f}{\eta_w} = 0.0044-9.75, \\ \left(\dfrac{\mathrm{Re}_f\,\mathrm{Pr}_f}{l/d}\right)^{1/3}\left(\dfrac{\eta_f}{\eta_w}\right)^{0.14} = 2 \end{cases}$$

§ 6-3 Forced convection heat transfer in a pipe

3.2 Laminar: （b）heat transfer in fully developed region：

Cross Section	$\frac{b}{a}$	$Nu_D \equiv \frac{hD_h}{k}$ (Uniform q_s'')	(Uniform T_s)	fRe_{D_h}
	—	4.36	3.66	64
	1.0	3.61	2.98	57
	1.43	3.73	3.08	59
	2.0	4.12	3.39	62
	3.0	4.79	3.96	69
	4.0	5.33	4.44	73
	8.0	6.49	5.60	82
	∞	8.23	7.54	96
Heated  Insulated	∞	5.39	4.86	96
	—	3.11	2.49	53

§ 6-3 Forced convection heat transfer in a pipe

4. Micro/Nano scale heat transfer:

$$Kn = \frac{\lambda}{l}$$

λ: the gas molecule mean free path

l: characteristic length of the channel

$Kn \leq 0.001$	
$0.001 \leq Kn \leq 0.1$	**Flip boundary &temperature jump**
$0.1 \leq Kn \leq 10$	**Transition region**
$Kn \geq 10$	**Molecular dynamcis**

例　2

由内外管组成的套管式开水器，环形空间流经初温为30°C，流量为0.857kg/s的水，内管中水蒸气凝结放热使内管外壁温维持在100°C，外管绝热。内管外径为40mm，外管内径为60mm。试确定将水加热到50°C所需套管长度，并计算管子出口处的局部热流密度。

解：定性温度
$$t_f=\frac{30+50}{2}=40℃$$

由此查出水的物性
$$\begin{cases}\lambda=0.635\ w/(m\cdot k)\\ \eta=653.3\times10^{-6}kg/(m\cdot s)\\ \Pr=4.31\\ c_p=4174\,J/(kg\cdot k)\end{cases}$$

当量直径
$$de=D-d=0.06-0.04=0.02m$$

$$\mathrm{Re}_f=\frac{u\cdot de}{\upsilon}=\frac{G}{\frac{\pi}{4}(D^2-d^2)\rho}\cdot\frac{de}{\upsilon}=\frac{4Gde}{\pi\eta(D^2-d^2)}$$
$$=\frac{4\times0.857\times0.02}{3.1416(0.06^2-0.04^2)\times653.3\times10^{-6}}$$
$$\approx16702$$

§ 6-3 Forced convection heat transfer in a pipe

水以壁温$t_w = 100°C$作为定性温度的动力粘度$\eta_W = 282.5 \times 10^{-6} kg/(m \cdot s)$

由于壁温与流体平均温度的温差100-40=60>30℃，同时流体是被加热，因此选用齐德-泰特公式，

$$
\begin{aligned}
Nu_f &= 0.027 \mathrm{Re}_f^{0.8} \mathrm{Pr}_f^{1/3} \left(\frac{\eta_f}{\eta_w} \right)^{0.14} \\
&= 0.027 \times 16702^{0.8} \times 4.31^{0.333} \times (653.3/282.5)^{0.14} \\
&= 118
\end{aligned}
$$

$$h = Nu_f \left(\frac{\lambda}{de} \right) = 116 \times (0.635/0.02) = 3747\, w/(m^2 \cdot k)$$

由热平衡方程：$c_p G(t'' - t') = Ah(t_w - t_f) = \pi dl \cdot h(t_w - t_f)$

$$l = \frac{c_p G(t'' - t')}{\pi d(t_w - t_f)h} = \frac{4174 \times 0.857(50 - 30)}{3.1416 \times 0.04(100 - 40) \times 3747} = 2.53m$$

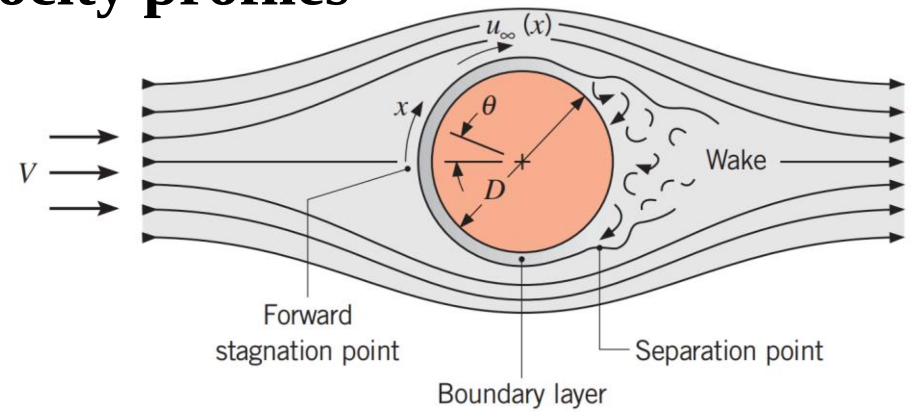
$$q = h\Delta t = 3747 \times (100 - 50) = 187350\, w/m^2$$

把水加热到50℃需要2.53m长套管，在管子出口截面处的局部热流密度是187.35kw/m^2。

§ 6-4 Forced convection heat transfer across a pipe

1.Cross a single pipe

Velocity profiles

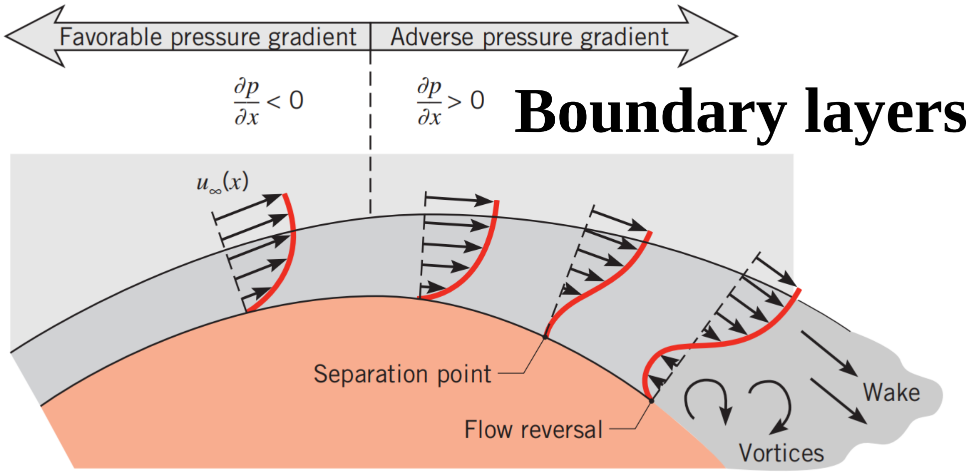


Re < 10, no separation;

10 <Re< $1.5*10^5$; Laminar, θ=80-85 degree;

Re >$1.5*10^5$; Turbulence, θ=140 degree

Highly affected by boundary layer, flow state and separation



Boundary layers

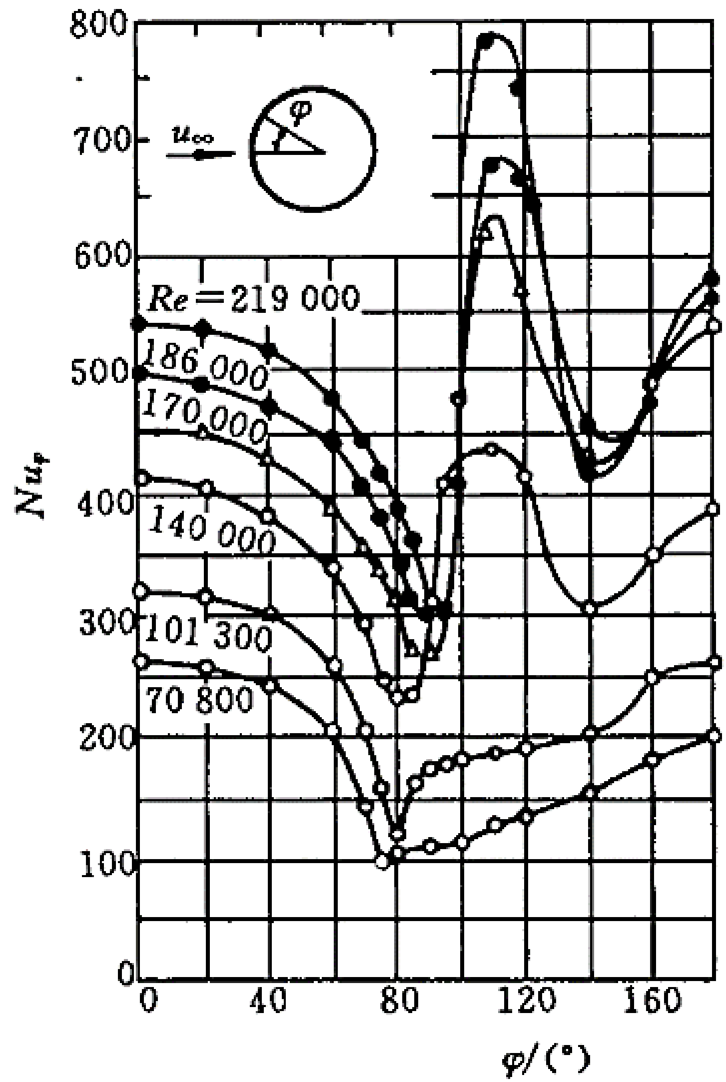
§ 6-4 Forced convection heat transfer across a pipe

Nu decrease since the thickness of boundary layer increase.

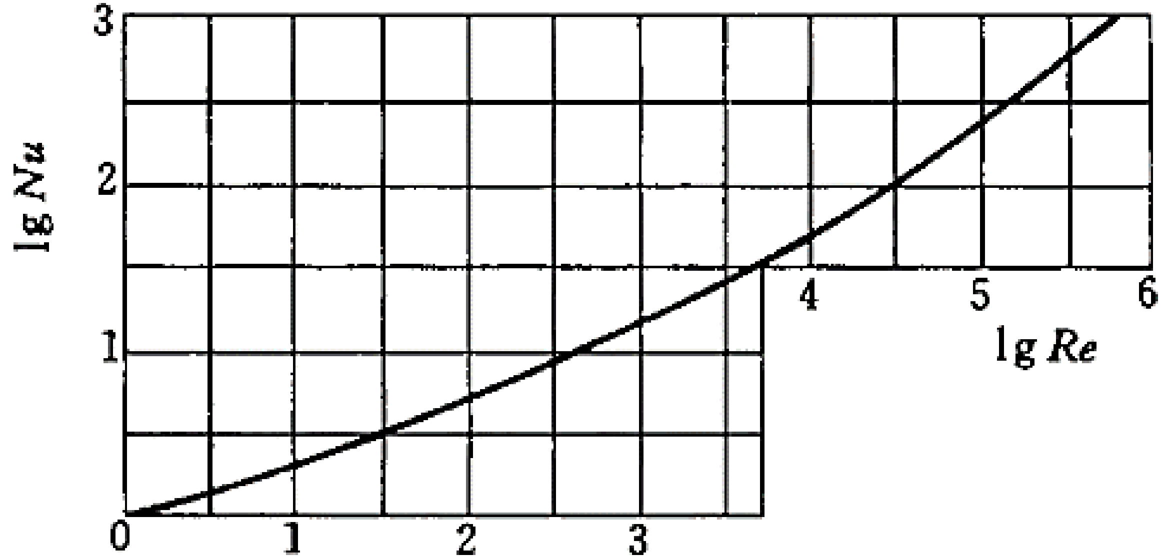
Nu firstly increase back since the transition from laminar to turbulence

Nu decrease since the thickness of boundary layer increase.

Nu increase again due to the flow seperation。



The average Nu:



§ 6-4 Forced convection heat transfer across a pipe

$$Nu = C\mathrm{Re}^n \mathrm{Pr}^{1/3}$$

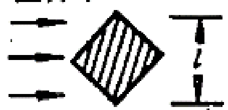
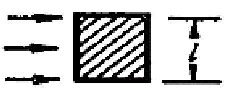
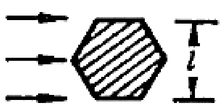
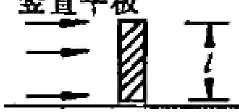
C and n

Re	*C*	*n*
0.4~4	0.989	0.330
4~40	0.911	0.385
40~4 000	0.683	0.466
4 000~40 000	0.193	0.618
40 000~400 000	0.026 6	0.805

§ 6-4 Forced convection heat transfer across a pipe

For non-circular structure we also can use but with different C and n values:

C and n

	Re	C	n
正方形 	$5\times10^3\sim10^5$	0.246	0.588
	$5\times10^3\sim10^5$	0.102	0.675
正六边形 	$5\times10^3\sim1.95\times10^4$ $1.95\times10^4\sim10^5$	0.160 0.038 5	0.638 0.782
	$5\times10^3\sim10^5$	0.153	0.638
竖直平板 	$4\times10^3\sim1.5\times10^4$	0.228	0.731

§ 6-4 Forced convection heat transfer across a pipe

$$Nu = 0.3 + \frac{0.62\,\mathrm{Re}^{1/2}\,\mathrm{Pr}^{1/3}}{[1 + (0.4/\mathrm{Pr})^{2/3}]^{1/4}}\left[1 + \left(\frac{\mathrm{Re}}{282000}\right)^{5/8}\right]^{4/5}$$

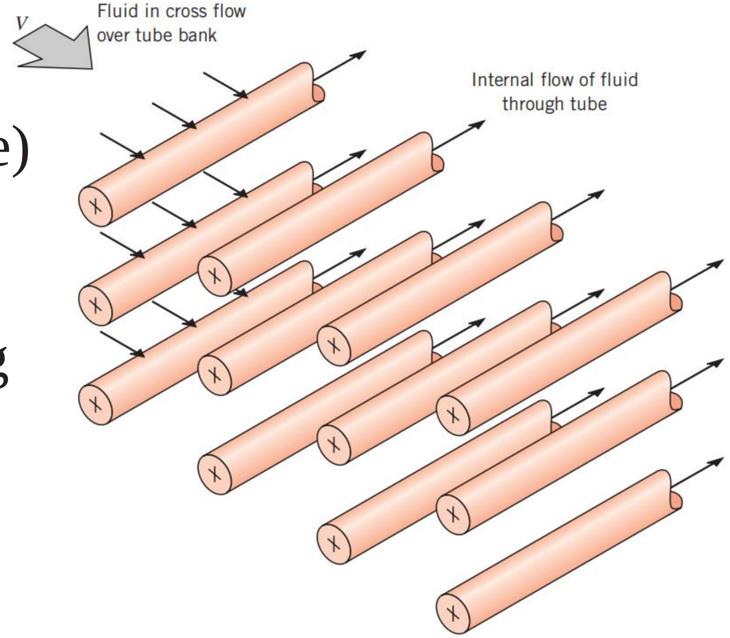
Ref Tem: $(t_w + t_\infty)/2,$

Suitable: $\mathrm{Re}\,\mathrm{Pr} > 0.2$

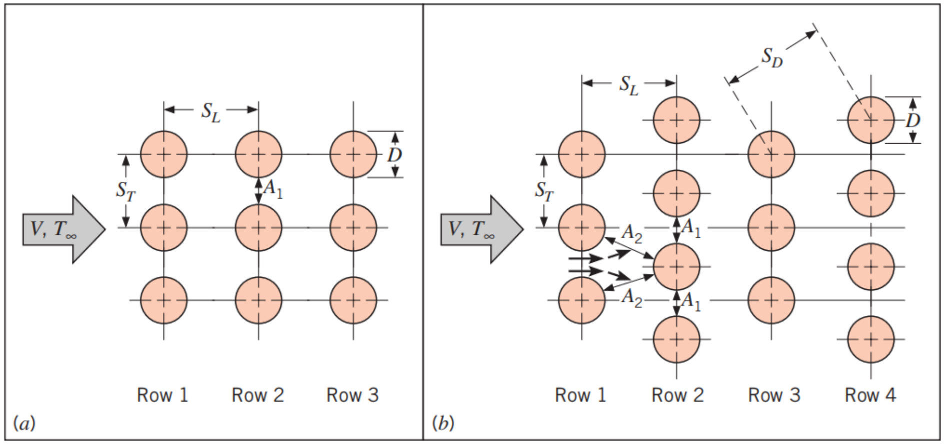
§ 6-4 Forced convection heat transfer across a pipe

2.Across banks of tubes

Heat transfer to or from a bank (or bundle) of tubes in cross flow is relevant to numerous industrial applications, such as steam generation in a boiler or air cooling in the coil of an air conditioner

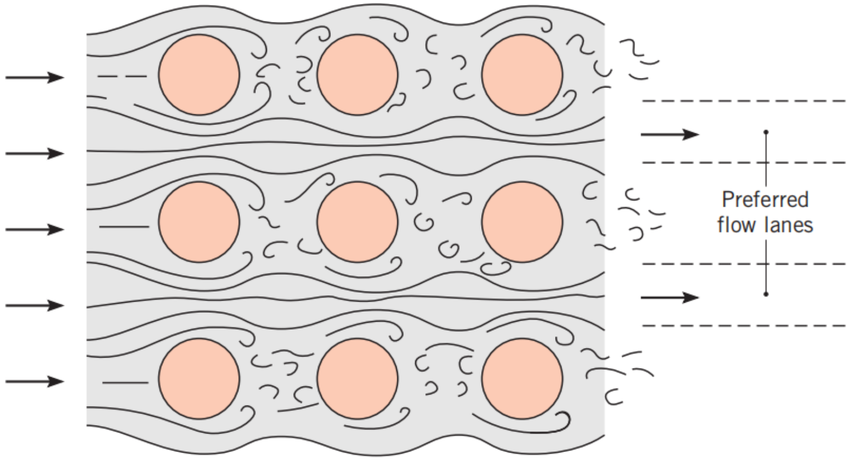


The heat transfer is sensitive with Re、Pr and the configuration including the numbers, distance etc.

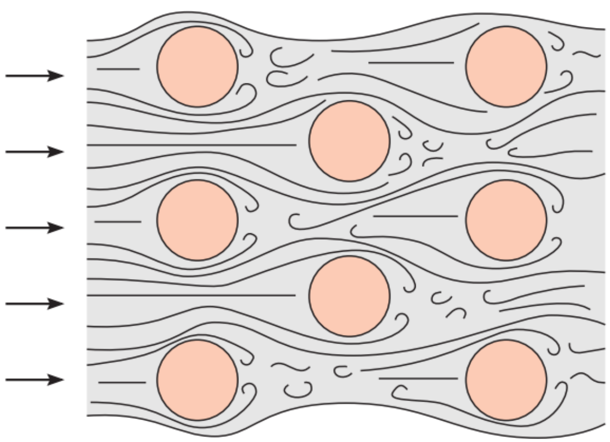


aligned staggered

§ 6-4 Forced convection heat transfer across a pipe



(a)



(b)

§ 6-4 Forced convection heat transfer across a pipe

When the tube rows is more than 10 (20), the influence of the flow is relevant equilibrated and the perturbation due to the former tube could be neglected. Therefore, we give the average heat transfer coefficient for the entire tube bank:

$$Nu = C\,\mathrm{Re}^{m}$$

§ 6-4 Forced convection heat transfer across a pipe

C and m

s_2/d \ s_1/d	1.25		1.5		2		3	
	C	m	C	m	C	m	C	m
顺排								
1.25	0.348	0.592	0.275	0.608	0.100	0.704	0.063 3	0.752
1.5	0.367	0.586	0.250	0.620	0.101	0.702	0.067 8	0.744
2	0.418	0.570	0.299	0.602	0.229	0.632	0.198	0.648
3	0.290	0.601	0.357	0.584	0.374	0.581	0.286	0.608
叉排								
0.6							0.213	0.636
0.9					0.446	0.571	0.401	0.581
1			0.497	0.558				
1.125					0.478	0.565	0.518	0.560
1.25	0.518	0.556	0.505	0.554	0.519	0.556	0.522	0.562
1.5	0.451	0.568	0.460	0.562	0.452	0.568	0.488	0.568
2	0.404	0.572	0.416	0.568	0.482	0.556	0.449	0.570
3	0.310	0.592	0.356	0.580	0.440	0.562	0.421	0.574

§ 6-4 Forced convection heat transfer across a pipe

If there are 20 or fewer rows of tubes, the average heat transfer coefficient is typically reduced, and a correction factor may be applied such that

总排数	1	2	3	4	5	6	7	8	9	10
顺　排	0.64	0.80	0.87	0.90	0.92	0.94	0.96	0.98	0.99	1.0
叉　排	0.68	0.75	0.83	0.89	0.92	0.95	0.97	0.98	0.99	1.0

§ 6-4 Forced convection heat transfer across a pipe

茹卡乌斯卡斯对流体外掠管束传热总结出一套在很宽的 Pr 数变化范围内更便于使用的公式。

式中：定性温度为进出口流体平均流速；Pr_w 按管束的平均壁温确定；Re 数中的流速取管束中最小截面的平均流速；特征长度为管子外径。

实验验证范围：Pr = 0.6 – 500。

流体横掠顺排管束平均表面传热系数计算关联式（≥16排）

关联式	适用 Re 数范围	
$Nu_f=0.9Re_f^{0.4}Pr_f^{0.36}(Pr_f/Pr_w)^{0.25}$	$1\sim10^2$	(5-75a)
$Nu_f=0.52Re_f^{0.5}Pr_f^{0.36}(Pr_f/Pr_w)^{0.25}$	$10^2\sim10^3$	(5-75b)
$Nu_f=0.27Re_f^{0.63}Pr_f^{0.36}(Pr_f/Pr_w)^{0.25}$	$10^3\sim2\times10^5$	(5-75c)
$Nu_f=0.033Re_f^{0.8}Pr_f^{0.36}(Pr_f/Pr_w)^{0.25}$	$2\times10^5\sim2\times10^6$	(5-75d)

§ 6-4 Forced convection heat transfer across a pipe

流体横掠叉排管束平均表面传热系数计算关联式（≥16排）

关 联 式	适用 Re 数范围	
$Nu_f=1.04Re_f^{0.4}Pr_f^{0.36}(Pr_f/Pr_w)^{0.25}$	$1\sim5\times10^2$	(5-76a)
$Nu_f=0.71Re_f^{0.5}Pr_f^{0.36}(Pr_f/Pr_w)^{0.25}$	$5\times10^2\sim10^3$	(5-76b)
$Nu_f=0.35\left(\frac{s_1}{s_2}\right)^{0.2}Re_f^{0.6}Pr_f^{0.36}(Pr_f/Pr_w)^{0.25},\ \frac{s_1}{s_2}\leqslant2$	$10^3\sim2\times10^5$	(5-76c)
$=0.40Re_f^{0.6}Pr_f^{0.36}(Pr_f/Pr_w)^{0.25},\ \frac{s_1}{s_2}>2$	$10^3\sim2\times10^5$	(5-76d)
$Nu_f=0.031\left(\frac{s_1}{s_2}\right)^{0.2}Re_f^{0.8}Pr_f^{0.36}(Pr_f/Pr_w)^{0.25}$	$2\times10^5\sim2\times10^6$	(5-76e)

茹卡乌斯卡斯公式的管排修正系数 ε_n

总排数	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
顺排，$Re>10^3$	0.700	0.800	0.865	0.910	0.928	0.942	0.954	0.965	0.972	0.978	0.983	0.987	0.990	0.992	0.994
叉排 $10^2<Re<10^3$	0.832	0.874	0.914	0.939	0.955	0.963	0.970	0.976	0.980	0.984	0.987	0.990	0.993	0.996	0.999
$Re>10^3$	0.619	0.758	0.840	0.897	0.923	0.942	0.954	0.965	0.971	0.977	0.982	0.986	0.990	0.994	0.997

例 3

温度为t_f=35℃的空气横向吹过一组平均表面温度为65℃的圆形截面直肋，在流动方向上肋片交叉排列，$s_1/d = s_2/d = 2$，d=10mm，排数大于10，最小截面处的空气流速为3.8m/s，肋片导热系数为98$w/(m \cdot k)$，肋根温度维持定值。为有效的利用金属，规定肋片的mH值不应大于1.5，试计算此肋片应为多高？

解：采用气流外掠管束的计算公式来计算肋束与气流间的对流传热。

定性温度：$t_m = \frac{t_w+t_f}{2} = \frac{35+65}{2} = 50℃$

空气物性$\begin{cases} \lambda = 0.0283\, w/(m \cdot k) \\ \nu = 17.95 \times 10^{-6}\, m^2/s \end{cases}$

$$\mathrm{Re} = \frac{u \cdot d}{\upsilon} = \frac{3.8 \times 0.01}{17.95 \times 10^{-6}} = 2117$$

由叉排$s_1/d = s_2/d = 2$，查表得，c=0.482,m=0.556

$$Nu = c\mathrm{Re}^m = 0.482 \times 2117^{0.556} = 34.05$$

$$h = Nu\frac{\lambda}{d} = 34.05\frac{0.0283}{0.01} = 96.4\, w/(m^2 \cdot k)$$

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$$m = \sqrt{\frac{4h}{\lambda d}} = \sqrt{\frac{4 \times 96.4}{98 \times 0.01}} = 19.84$$

$$H = 1.5/m = 1.5/19.84 = 0.0756m$$

肋片高度应为0.0756m。