一. 1. $\overline{X(t)} = m_x$ $\overline{X(t+\tau)X(t)} = R_x(\tau)$

时间平均 = 集平均.

P71.

2. 积分系统. P104. 均值 → ct

3. ①.证互不相关 $X(t)$. $Y(t) = \dot{X}(t)$

$$E[X(t)\dot{X}(t)] = E\left[X(t)\lim_{\varepsilon\to 0}\frac{X(t+\varepsilon)-X(t)}{\varepsilon}\right] = \lim_{\varepsilon\to 0} E\left[X(t)\frac{X(t+\varepsilon)-X(t)}{\varepsilon}\right]$$

$$= R'(0) = 0$$

高斯: 互不相关 = 独立.

4. $\Delta W_e \cdot \tau_0 = \frac{2\pi}{4}$

$$\Delta W_e = \frac{\int_0^\infty S_Y(w)\,dw}{S_Y(w_0)}$$ 等效通频带 P119

$$= \frac{\int_0^\infty |H(jw)|^2\,dw}{|H(0)|^2}$$

相关时间: $r_x(\tau) = \frac{C_x(\tau)}{C_x(0)} = \frac{R_x(\tau) - m_x^2}{\sigma_x^2}$

P64

$$\tau_0 = \int_0^\infty r(\tau)\,d\tau$$

关系: $\Delta W_e \cdot \tau_0 = \frac{2\pi}{4}$

$\Delta f \cdot \tau_0 = \frac{1}{4}$

5. $\tilde{x}(t) = x(t) + j\hat{x}(t)$

$$G_{\tilde{x}}(w) = 2[G_x(w) + j\hat{G}_x(w)]$$

$$= \begin{cases} 0, & w<0 \\ 2G_x(w), & w=0 \\ 4G_x(w), & w>0 \end{cases}$$

6. 题 6.3.

7. 对于马尔科夫链，k步转移概率满足：

$$P_{ij}^{(m+r)}(n) = \sum_{k\in S} P_{ik}^{(m)}(n)\cdot P_{kj}^{(r)}(n+m)$$

对于马尔可夫链，如果知道其初始分布及转移概率，则它的有限维分布可完全确定。

二. (1). $m_z(t) = E[z(t)] = E\left[\sum_{k=1}^{m} A_k e^{jw_k t}\right] = 0$

$$C(t_1,t_2) = E\{[z(t_1)-m_z][z(t_2)-m_z]^*\} = E[z(t_1)z^*(t_2)]$$

$$= E\left[\sum_{i=1}^{m} A_i e^{jw_i t_1} \cdot \sum_{j=1}^{m} A_j e^{-jw_j t_2}\right]$$

$$= E\left[\sum_{i=1}^{m} A_i^2 e^{jw_i(t_1-t_2)}\right]$$

$$= \sum_{i=1}^{m} e^{jw_i \tau}$$

(2). $\psi^2 = E[z^2(t)] = \sum_{i=1}^{m} e^{jw_i\tau}\Big|_{\tau=0} = m$

(3). $R_z(t, t-\tau) = \sum_{i=1}^{m} e^{jw_i\tau}$ $\quad m_z = 0$

是平稳随机过程

题 2.18

三. 1. $R_X(t, t-\tau) = E\{[U\cos w_0 t + V\sin w_0 t][U\cos w_0(t-\tau) + V\sin w_0(t-\tau)]\}$

$= E[U^2\cos w_0 t\cos w_0(t-\tau) + UV\sin w_0(t-\tau)\cos w_0 t$

$+ UV\sin w_0 t\cos w_0(t-\tau) + V^2\sin w_0 t\sin w_0(t-\tau)]$

$= \cos w_0\tau$

同理 $R_Y(\tau) = \cos w_0\tau$

2. $R_{XY}(t_1, t_2) = E[(U\cos w_0 t_1 + V\sin w_0 t_1)(U\sin w_0 t_2 + V\cos w_0 t_2)]$

$\cancel{= E[U^2\cos w_0 t_1\sin w_0 t_2 + UV\cos w_0 t_1\sin w_0 t_2 + UV\sin w_0 t_1\cos w_0 t_2}$

$\cancel{+ V^2\sin^2 w_0 t_1}$

$= E[U^2\cos w_0 t_1\sin w_0 t_2 + UV\cos w_0 t_1\cos w_0 t_2 + UV\sin w_0 t_1\sin w_0 t_2$

$+ V^2\sin w_0 t_1\cos w_0 t_2]$

$= \sin w_0(t_1 + t_2)$

不是广义~~平~~联合平稳，无法求互功率谱密度

3. $m_X = m_Y = 0$

$$C = \begin{bmatrix} E[X^2(t)] & E[X(t)Y(t)] \\ E[Y(t)X(t)] & E[Y^2(t)] \end{bmatrix} = \begin{bmatrix} 1 & \sin w_0 2t \\ \sin w_0 2t & 1 \end{bmatrix}$$

~~R_{xy}~~ $$P_{XY}(x, y) = \frac{1}{2\pi|C|^{\frac{1}{2}}} \cdot \exp\left\{-\frac{1}{2}[x, y]C^{-1}\begin{bmatrix} x \\ y \end{bmatrix}\right\}.$$

题 3.25

四、1. $R_{Y_1Y_2}(\tau) = R_X(\tau) * h_1(\tau) * h_2(-\tau)$

$S_{Y_1Y_2}(\omega) = S_X(\omega) \cdot H_1(j\omega) \cdot H_2^*(j\omega)$

2. $H_1(j\omega) \cdot H_2^*(j\omega) = 0$. 通频带不重叠.

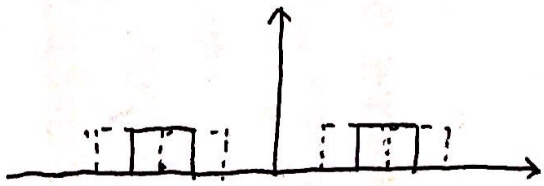
$$\begin{cases} \omega_1 + \frac{B}{2} < \omega_2 - \frac{B}{2} \\ \omega_1 - \frac{B}{2} > \omega_2 + \frac{B}{2} \end{cases} \rightarrow \begin{cases} \omega_2 - \omega_1 < -B \\ \omega_2 - \omega_1 > B \end{cases} \rightarrow |\omega_2 - \omega_1| > B$$

3. $r_{Y_1Y_2} = \dfrac{R_{Y_1Y_2}(0)}{\sqrt{\sigma_{Y_1}^2 \sigma_{Y_2}^2}}$ $\qquad R_X(\tau) = \delta(\tau)$. $\qquad S_X(\omega) = 1$

$S_{Y_1}(\omega) = \begin{cases} 1 & , |\omega \pm \omega_1| < \frac{B}{2} \\ 0 \end{cases}$ $\qquad \sigma_{Y_1}^2 = \frac{1}{2\pi}\int_{-\infty}^{+\infty} S_{Y_1}(\omega)\, d\omega = \frac{1}{2\pi} \cdot 2B = \frac{B}{\pi}$

同理 $\sigma_{Y_2}^2 = \frac{B}{\pi}$

$r_{Y_1Y_2} = \dfrac{R_{Y_1Y_2}(0)}{\frac{B}{\pi}} = \dfrac{1}{2}$ $\qquad \frac{1}{2\pi}\int_{-\infty}^{+\infty} H_1(j\omega)H^*(j\omega)\, d\omega = \frac{B}{2\pi}$



$\int_{-\infty}^{+\infty} H_1(j\omega)H_2^*(j\omega)\, d\omega = B$

$\omega_1 + \frac{B}{2} = \omega_2$

$\omega_1 - \frac{B}{2} = \omega_2$ $\qquad |\omega_2 - \omega_1| = \frac{B}{2}$

扫描全能王 创建

五. 1. $m_W = E[W(t)] = \int_0^t m_x(s)\,ds = 0$

2. $R_W(t_1, t_2) = E\left[\int_0^{t_1}\int_0^{t_2} X(s)X(\lambda)\,ds\,d\lambda\right] = \int_0^{t_1}\int_0^{t_2} E[X(s)X(\lambda)]\,ds\,d\lambda$

$= \int_0^{t_1}\int_0^{t_2} R_x(s, \lambda)\,ds\,d\lambda$ $\qquad R_x(\tau) = \delta(\tau)$

① $t_1 < t_2$ $\qquad R_Y(t_1, t_2) = t_1$

② $t_1 > t_2$ $\qquad R_Y(t_1, t_2) = t_2$

$R_Y(t_1, t_2) = \min(t_1, t_2)$

③. $W(t_1) - W(t_2)$.

$E[W(t_1) - W(t_2)] = 0$

$E\{[W(t_1) - W(t_2)]^2\} = E[W^2(t_1) - 2W(t_2)W(t_1) + W^2(t_1)]$.

$= t_1 + t_2 - 2\min(t_2, t_1)$