第2-1次作业

数学作业纸

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2-2

解：$\omega = 2\pi f = \pi \times 10^9$

$$\gamma = \sqrt{(R_0 + j\omega L_0)(G_0 + j\omega C_0)}$$

$$= \sqrt{(5+j200\pi)(0.01+j0.3\pi)}$$

$$\approx 0.226 + j24.335 (m^{-1})$$

$$Z_0 = \sqrt{\frac{R_0 + j\omega L_0}{G_0 + j\omega C_0}}$$

$$= \sqrt{(5+j200\pi)(0.01+j0.3\pi)}$$

$$\approx 25.82 + j0.03 \ \Omega$$

在输线无耗时

$$\gamma = \sqrt{j\omega L_0 \cdot j\omega C_0}$$

$$= j\omega\sqrt{L_0 C_0} = j24.3 \ m^{-1}$$

$$Z_0 = \sqrt{\frac{L_0}{C_0}} = 25.8\,\Omega.$$

2-3

解：时域：

$$\begin{cases} \frac{\partial u(z,t)}{\partial t} = -L_0 \cdot \frac{\partial i(z,t)}{\partial t} \\ \frac{\partial i(z,t)}{\partial t} = -C_0 \cdot \frac{\partial u(z,t)}{\partial t} \end{cases}$$

频域：

$$\begin{cases} \frac{d^2 \dot{U}(z)}{dz^2} - \gamma^2 \dot{U}(z) = 0 \\ \frac{d^2 \dot{I}(z)}{dz^2} - \gamma^2 \dot{I}(z) = 0 \end{cases}$$

解得：

$$\dot{U}(z) = A_1 e^{-j24.3z} + A_2 e^{j24.3z}.$$

$$\dot{I}(z) = \frac{1}{25.8}\left(A_1 e^{-j24.3z} - A_2 e^{j24.3z}\right)$$

2-4.

解：由KVL：

~~$u(z+\frac{\Delta z}{2},t) - u$~~

$$u(z,t) = i(z,t)\cdot\frac{R_0\Delta z}{2} + L_0\frac{\Delta z}{2}\cdot\frac{\partial i(z,t)}{\partial t} + u(z+\frac{\Delta z}{2},t)$$

由KCL：

$$i(z,t) = G_0\Delta z \cdot u(z+\frac{\Delta z}{2},t) + C_0\Delta z \cdot \frac{\partial u(z+\frac{\Delta z}{2},t)}{\partial t} + i(z+\Delta z,t)$$

整理后.

$$\begin{cases} u(z+\frac{\Delta z}{2},t) - u(z,t) = -i(z,t)\cdot\frac{R_0\Delta z}{2} - L_0\frac{\Delta z}{2}\cdot\frac{\partial i(z,t)}{\partial t} \\ i(z+\Delta z,t) - i(z,t) = -G_0\Delta z\cdot u(z+\frac{\Delta z}{2},t) - C_0\Delta z\cdot\frac{\partial u(z+\frac{\Delta z}{2},t)}{\partial t} \end{cases}$$

应用泰勒公式

$$\begin{cases} i(z+\Delta z,t) = i(z,t) + \frac{\partial i(z,t)}{\partial z}\cdot\Delta z + \cdots \\ u(z+\frac{\Delta z}{2},t) = u(z,t) + \frac{\partial u(z,t)}{\partial z}\cdot\frac{\Delta z}{2} + \cdots \\ \frac{\partial u(z+\frac{\Delta z}{2},t)}{\partial t} = \frac{\partial u(z,t)}{\partial t} + \frac{\partial^2 u(z,t)}{\partial z}\cdot\frac{\Delta z}{2} + \cdots \end{cases}$$

代入后得

$$\begin{cases} \frac{\partial u(z,t)}{\partial z} = -R_0 i(z,t) - L_0\frac{\partial i(z,t)}{\partial t} \\ \frac{\partial i(z,t)}{\partial z} = -G_0\cdot u(z,t) - C_0\cdot\frac{\partial u(z,t)}{\partial t} \end{cases}$$