System Dynamics and Vibrations

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Chapter 3: Single degree-of-freedom systems
Exercises

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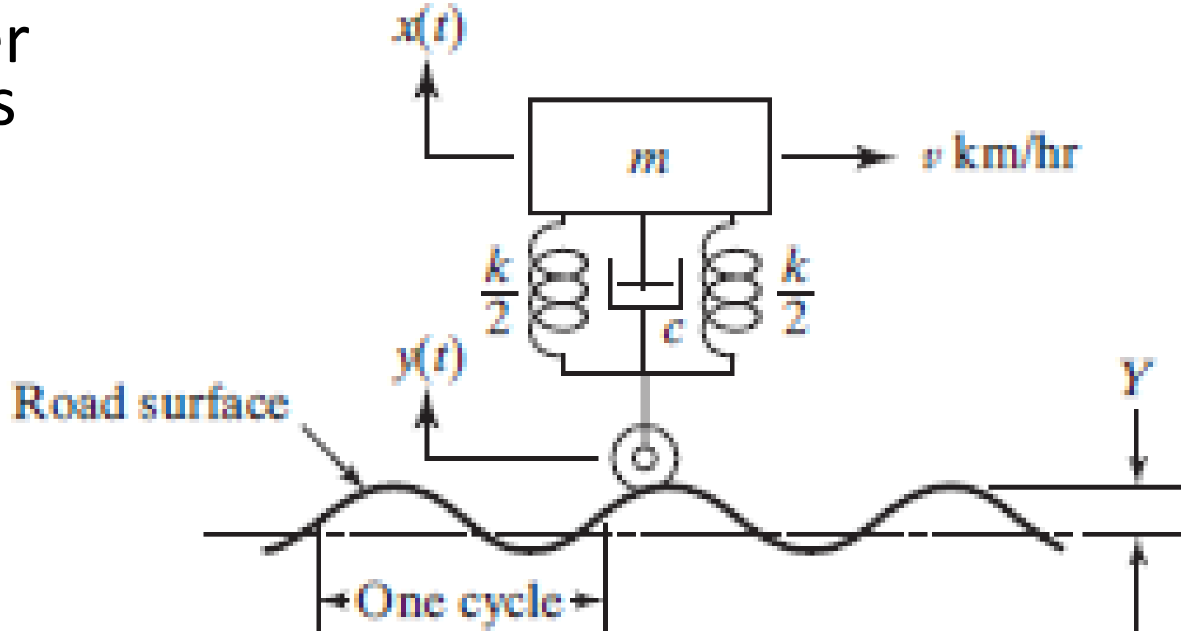
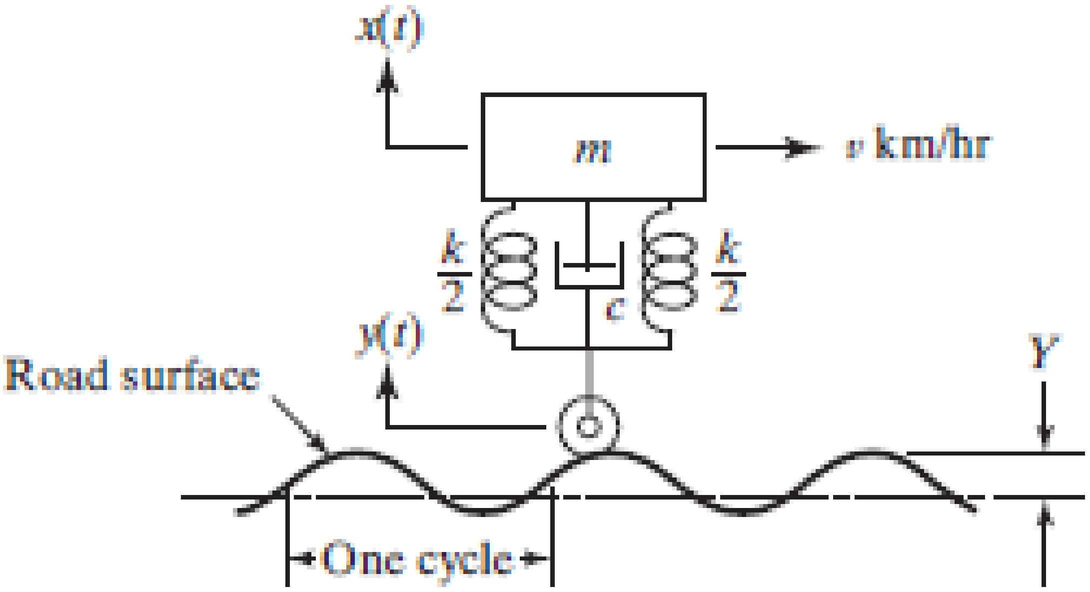
Exercise 4

The figure shows a simple model of a motor vehicle that can vibrate in the vertical direction while traveling over a rough road. The vehicle has a mass of 1200 kg. The suspension system has a spring constant of 400 kN/m and a damping ratio of $\zeta = 0.5$.

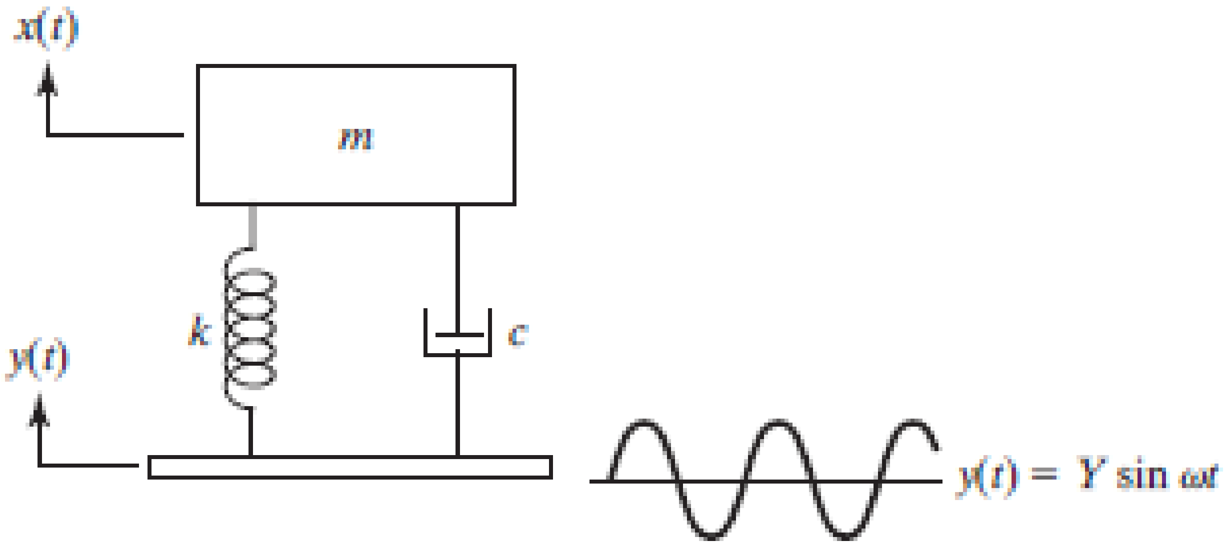
If the vehicle speed is 20 km/hr, determine the displacement amplitude of the vehicle. The road surface varies sinusoidally with an amplitude of $A = 0.05$ m and a wavelength of 6 m.



Exercise 4



x(t)
m
v km/hr
k/2
c
k/2
y(t)
Road surface
Y
One cycle



x(t)
m
k
c
y(t)
y(t) = Y sin ωt

Exercise 4

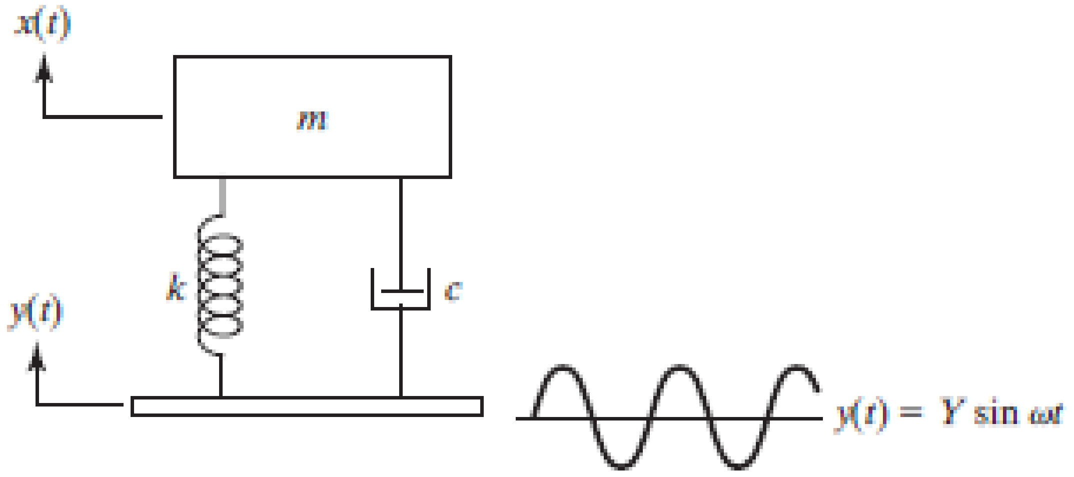
Base excitation: frequency?

➔ related to the speed and the wavelength $f = \dfrac{v}{\lambda}$

$$\omega = 2\pi f = 2\pi \left(\frac{v \times 1000}{3600} \right) \frac{1}{6} = 5.82 \text{rad/s}$$

$$\omega_n = \sqrt{\frac{k}{m}} = 18.26 \text{rad/s}$$

$$\frac{\omega}{\omega_n} = 0.32$$

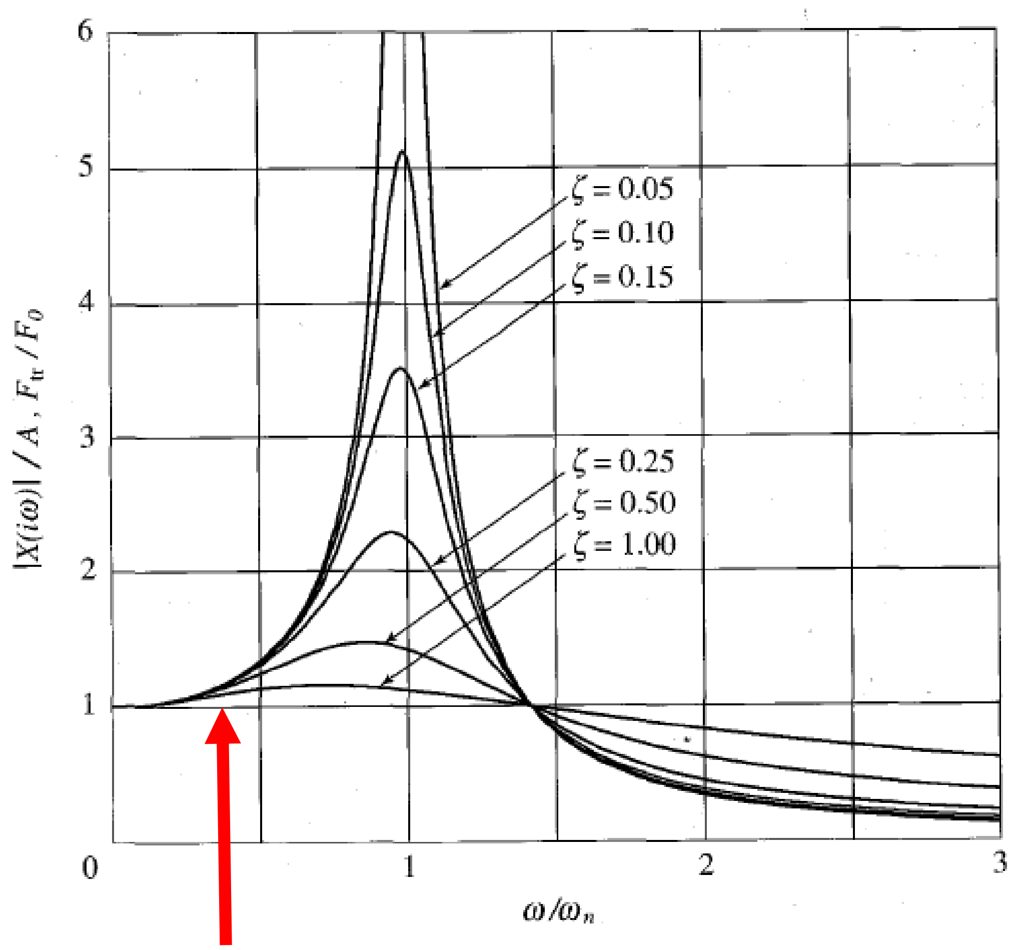


Exercise 4

The displacement amplitude is:

$$|X(i\omega)| = \left[1 + \left(\frac{2\varsigma\omega}{\omega_n}\right)^2\right]^{1/2} |G(i\omega)| A = 0.055\text{m}$$

$$|G(i\omega)| = \frac{1}{\left\{\left[1-(\omega/\omega_n)^2\right]^2 + \left(2\varsigma(\omega/\omega_n)^2\right)\right\}^{1/2}}$$



$$\frac{\omega}{\omega_n} = 0.32 \Rightarrow \frac{|X(i\omega|}{A} = \frac{0.055}{0.050} > 1$$

Exercise 5

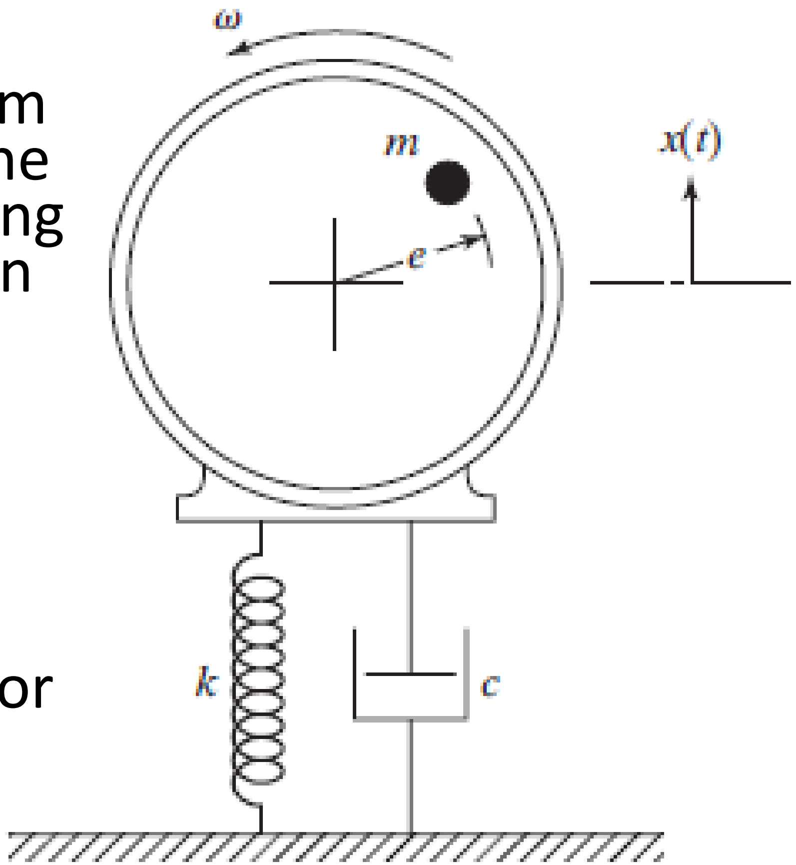
An electric motor of mass *M*, mounted on an elastic foundation, is found to vibrate with a deflection of 0.15 m at resonance. It is known that the unbalanced mass of the motor is 8% of the mass of the rotor due to manufacturing tolerances used, and the damping ratio of the foundation is $\zeta = 0.025$. Determine the following:

a. the eccentricity or radial location of the unbalanced mass (*e*),

b. the peak deflection of the motor when the frequency ratio varies from resonance, and

c. the additional mass to be added uniformly to the motor if the de flection of the motor at resonance is to be reduced to 0.1 m.

Assume that the eccentric mass remains unaltered when the additional mass is added to the motor



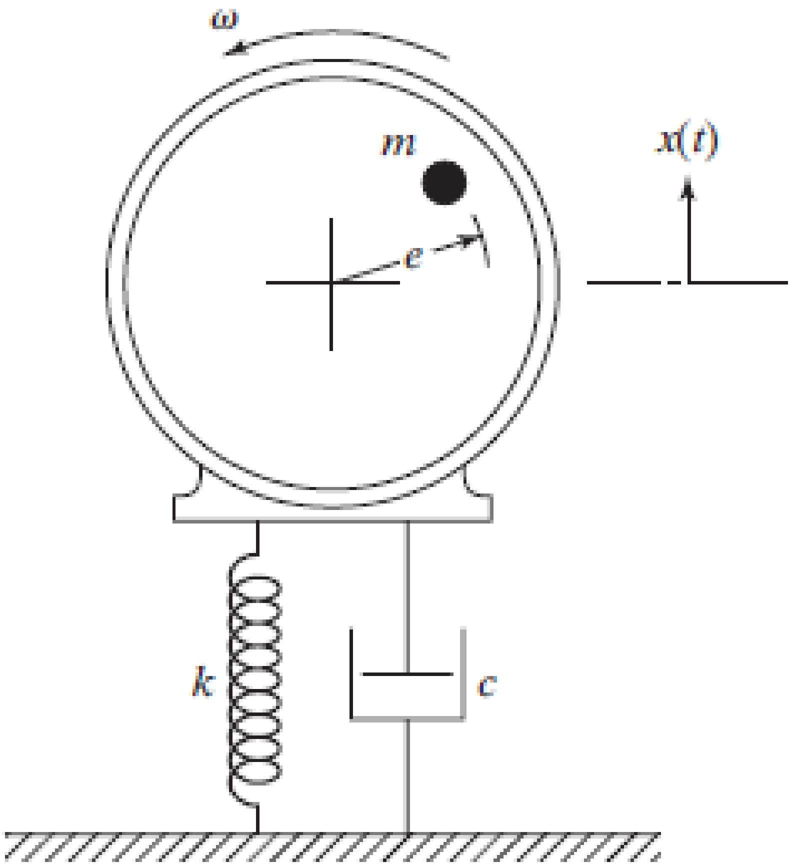
Exercise 5

Deflection at resonance: $\omega = \omega_n$

$$\frac{M\left|X\right|}{me} = \left(\frac{\omega}{\omega_n}\right)^2 \left|G(i\omega)\right| = \frac{1}{2\varsigma} = 20$$

$$\left|G(i\omega)\right| = \frac{1}{\left\{\left[1-\left(\omega/\omega_n\right)^2\right]^2 + \left(2\varsigma\left(\omega/\omega_n\right)^2\right)\right\}^{1/2}} = \frac{1}{2\varsigma}$$

$$e = \frac{MX}{20m} = \frac{M(0.15)}{20(0.08M)} = 0.094\text{m}$$

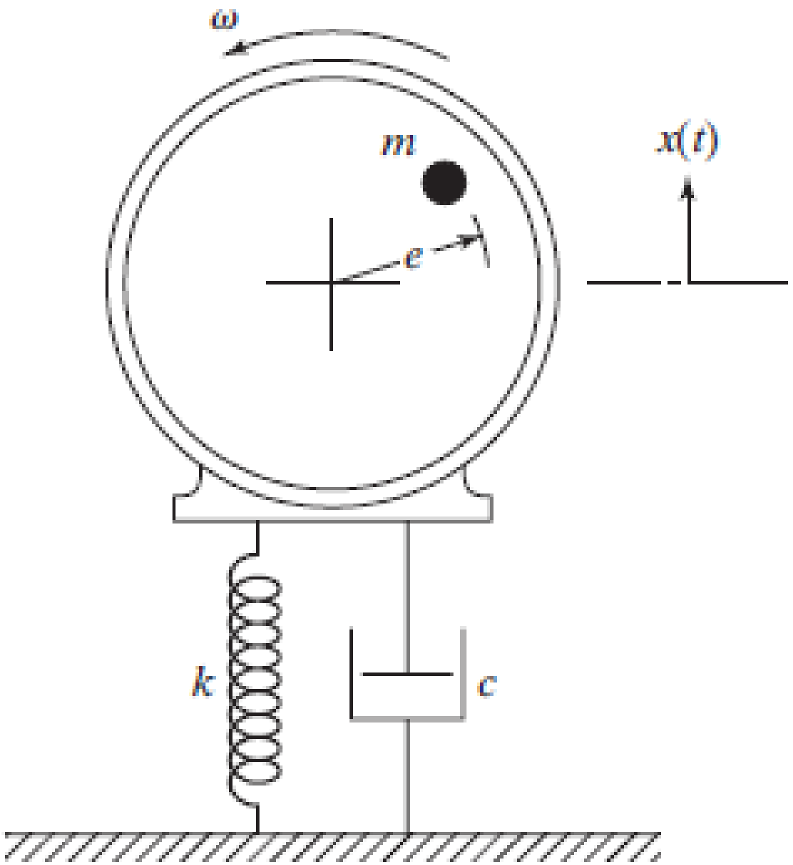


Exercise 5

Peak deflection:

$$\left(\frac{M|X|}{me}\right)_{\max} = \frac{1}{2\varsigma\sqrt{1-\varsigma^2}} = 20.0063$$

$$X_{\max} = 0.150\text{m}$$



Exercise 5

Additional mass to be added:

$$\frac{M|X|}{me} = \frac{1}{2\varsigma} = 20$$

$$\frac{(M + M_a)(0.1)}{(0.08M)(0.09375)} = 20$$

$$M_a = 0.5M$$