HT: Convection

L11: Free convection heat transfer

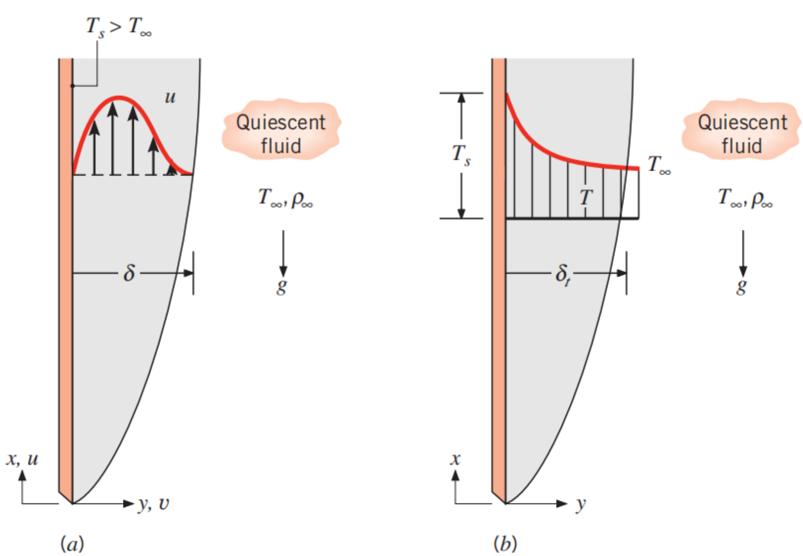
Learning Objectives:

- The principle of the natural convection
- The air flow with hot/cold plate

§ 6-5 Free convection

1.Definition

we consider situations for which there is **no forced velocity**, yet convection currents exist within the fluid. Such situations are referred to as **free or natural convection**, and they originate when a body force acts on a fluid in which there are **density gradients**. The net effect is a buoyancy force, which induces free convection currents. In the most common case, the density gradient is due to a **temperature gradient**, and the body force is due to the gravitational field.



can be classified as:

1. Free boundary flow
2. flow bounded by a surface

- ➢ heat flux is weak, normally less than 100 W/m^2
- ➢ Exist boundary layer, but different with force convection

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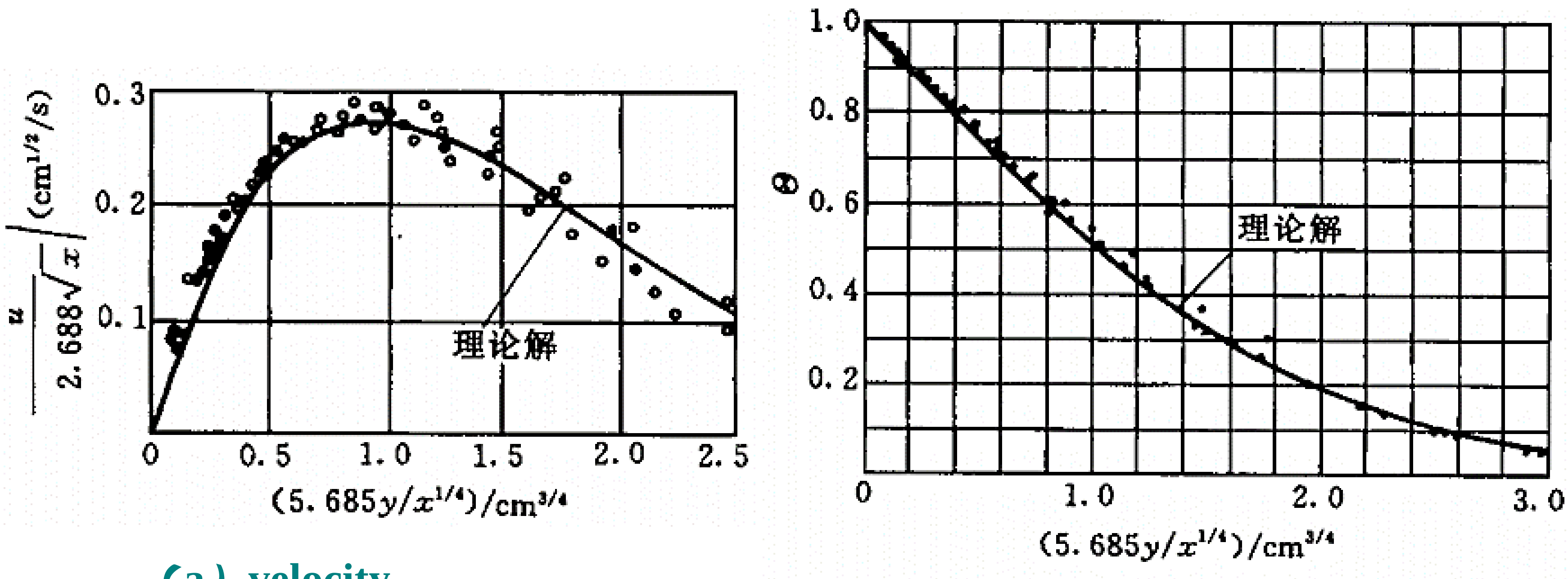

(a) velocity

(b) temperature

- the fluid temperature is equal to the wall temperature at the side of the wall, and gradually decreases in the direction away from the wall until the ambient temperature.

- The velocity of adhesion is zero due to viscosity. At the outer edge of the thin layer, the temperature imbalance and the action disappear, and the velocity is zero. There is a peak in velocity near the middle of the wall.

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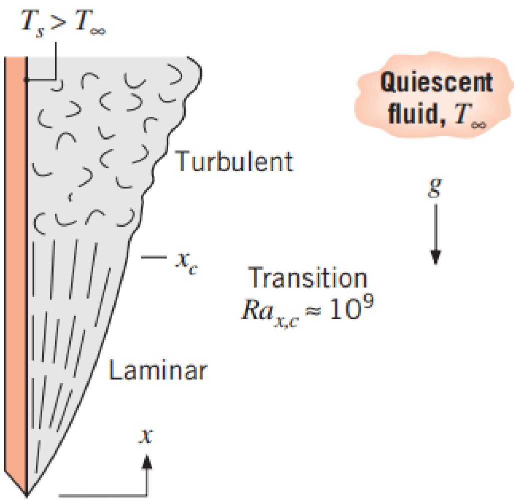

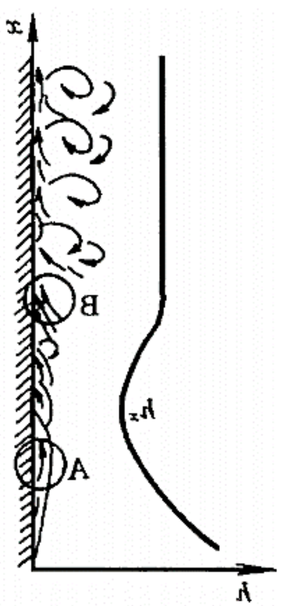

- Natural convection can also be divided into laminar and turbulent region.。
- In laminar, with the thickness increases the h decrease。
- In turbulence, the h also keeps constant。

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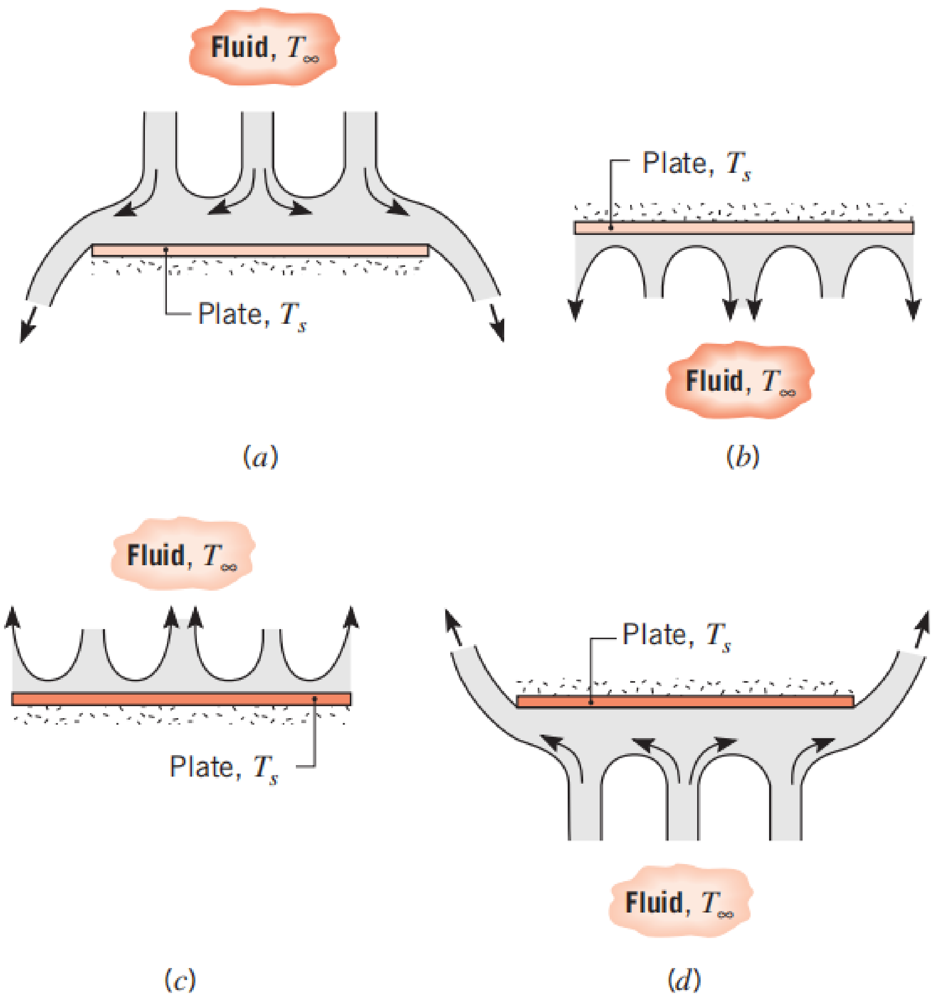
cold

hot



- **hot plate and cold**
- **facing upward or downward**

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2. Governing equation and characteristic number

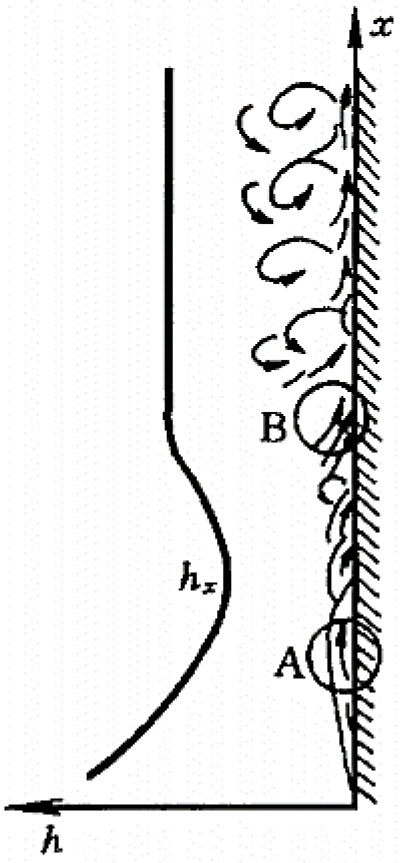
$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -g - \frac{1}{\rho}\frac{dp}{dx} + \nu\frac{\partial^2 u}{\partial y^2}$$

$$\rho c_p[u\frac{\partial(T)}{\partial x} + v\frac{\partial(T)}{\partial y}] = \frac{\partial}{\partial y}\left(\lambda\frac{\partial T}{\partial y}\right)$$

the pressure gradient satisfy:

$$\frac{dp}{dx} = \frac{dp_\infty}{\partial x} = -\rho_\infty g$$



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therefore, we can get:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{g}{\rho}(\rho_\infty-\rho)+\nu\frac{\partial^2 u}{\partial y^2}$$

and we define expansion coefficient

$$\alpha=-\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_p\approx-\frac{1}{\rho}\frac{\rho_\infty-\rho}{T_\infty-T}$$

and the excess temperature

$$\theta=T-T_\infty$$

finally:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=g\alpha\theta+\nu\frac{\partial^2 u}{\partial y^2}$$

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dimensionless:

$$\frac{u_0^2}{l}\left(u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial y^*}\right)=g\alpha\Delta t\Theta^*+\frac{\nu u_0}{l^2}\frac{\partial^2 u^*}{\partial y^{*2}}$$

where:

$$\Theta^*=(t-t_\infty)/(t_w-t_\infty)$$

therefore:

$$\frac{u_0 l}{\nu}\left(u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial y^*}\right)=\frac{g\alpha\Delta t l^2}{\nu u_0}\Theta^*+\frac{\partial^2 u^*}{\partial y^{*2}}$$

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if we define Grashof number like:

$$Gr = \frac{g\alpha\Delta t l^2}{\nu u_0}\frac{u_0 l}{\nu} = \frac{g\alpha\Delta t l^3}{\nu^2}$$

$$Gr = f(Re)$$

The nusselt number is functions of Gr and Pr

$$Nu = f(Gr, \mathrm{Pr})$$

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3.Free boundary flow experimental correlation

(1) uniform wall temperature

$$Nu = C(Gr\mathrm{Pr})^n$$

reference temperature:

$$t_m = \frac{t_w + t_\infty}{2}$$

temperature difference in Gr: $\Delta T = t_w - t_\infty$

reference length:

vertical plate and cylinder: height

horizonal cylinder： outer radius

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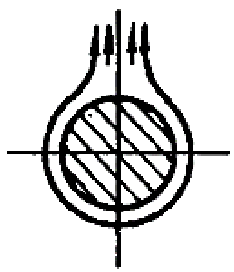
加热表面形状与位置	流动情况示意	流态	系数 C 及指数 n		Gr 数适用范围
			C	n	
竖平板及竖圆柱		层流 过渡 湍流	0.59 0.029 2 0.11	1/4 0.39 1/3	$10^4 \sim 3\times10^9$ $3\times10^9 \sim 2\times10^{10}$ $>2\times10^{10}$
横圆柱		层流 过渡 湍流	0.48 0.044 5 0.10	1/4 0.37 1/3	$10^4 \sim 5.76\times10^8$ $5.76\times10^8 \sim 4.65\times10^9$ $>4.65\times10^9$

vertical cylinder and plate are recognized as same case when:

$$\frac{d}{H} \geq \frac{35}{Gr_H^{1/4}}$$

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For horizonal plate:

hot surface facing upward or cold plate downward:

$$Nu = 0.54(Gr\,\mathrm{Pr})^{1/4} \qquad (10^4 < Gr\,\mathrm{Pr} < 10^7)$$

$$Nu = 0.15(Gr\,\mathrm{Pr})^{1/3} \qquad (10^7 < Gr\,\mathrm{Pr} < 10^{11})$$

hot surface facing upward or cold surface downward :

$$Nu = 0.27(Gr\,\mathrm{Pr})^{1/4} \qquad (10^5 < Gr\,\mathrm{Pr} < 10^{11})$$

Reference temperature: $t_m = (t_w + t_\infty)/2$;

characteristic length: A/P

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(2)uniform heat flux

$$Nu = B(Gr^{*}\mathrm{Pr})^{m}$$

where:

$$Gr^{*} = GrNu = \frac{g\alpha q l^{4}}{\lambda \nu^{2}}$$

mainly applied in electronic devices cooling.

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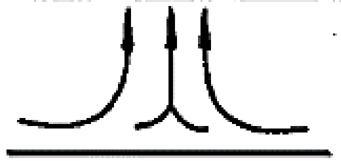
加热表面形状与位置	流动图示	系数 B 及指数 m		Gr^* 数适用范围
		B	m	
水平板热面朝上或冷面朝下		1.076	1/6	$6.37\times10^5-1.12\times10^8$
水平板热面朝下或冷面朝上		0.747	1/6	$6.37\times10^5-1.12\times10^8$

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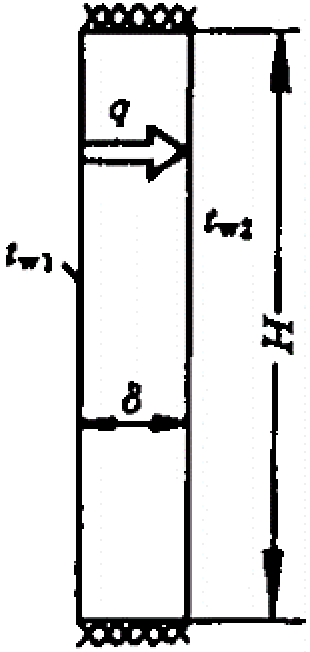
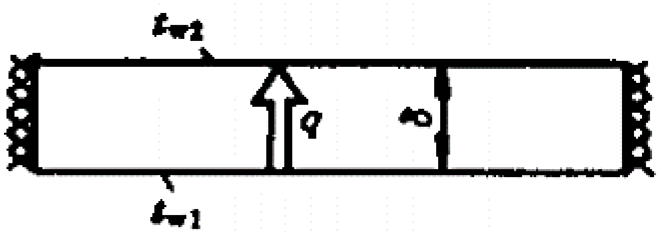
4. flow with bounded surface

only vertical and horizonal enclosure interlayer 。





$(t_{w1} > t_{w2})$

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The flow mainly controled by the charateristic length: δ

$$Gr_\delta = \frac{g\alpha\Delta t\delta^3}{\nu^2}$$

The flow keeps laminar when

vertical interlayer: Gr < 2860

horizonal interlayer: Gr <2430

The aspect ratio influences a lot

$$Nu = C(Gr_\delta \mathrm{Pr})^n \left(\frac{H}{\delta}\right)^m$$

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(1)vertical：

$$Nu = 0.197(Gr_\delta \mathrm{Pr})^{1/4}\left(\frac{H}{\delta}\right)^{-1/9}, (Gr_\delta = 8.6 \times 10^3 \sim 2.9 \times 10^5)$$

$$Nu = 0.073(Gr_\delta \mathrm{Pr})^{1/3}\left(\frac{H}{\delta}\right)^{-1/9}, (Gr_\delta = 2.9 \times 10^5 \sim 1.6 \times 10^7)$$

(2)horizonal：

$$Nu = 0.212(Gr_\delta \mathrm{Pr})^{1/4}, \quad Gr_\delta = 1 \times 10^4 \sim 4.6 \times 10^5$$

$$Nu = 0.061(Gr_\delta \mathrm{Pr})^{1/3}, \quad Gr_\delta > 4.6 \times 10^5$$

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5.combine natural and forced convection

$$\frac{g\alpha\Delta t l^3}{\nu^2}\frac{\nu^2}{u^2 l^2} = \frac{Gr}{\mathrm{Re}^2}$$

the ratio of buyancy force and the inerial force can be recognized by delime viscous effect

$$\frac{Gr}{Re^2} \le 0.01$$ natural convection

$$0.1 \le \frac{Gr}{Re^2} \le 10$$ mixed

$$\frac{Gr}{Re^2} \ge 10$$ force convection

例 4

一水平封闭夹层，其上、下表面的间距为$\delta = 14mm$，夹层内的压力为$1.013 \times 10^5 Pa$空气。设一个表面的温度为90℃，另一表面为30℃，试计算当热表面在冷表面上及在冷表面下两种情形下，通过单位夹层的传热量。

解：

$$\delta = 14mm，\ t_{w1} = 90℃，\ t_{w2} = 30℃，$$
$$t_m = (t_{w1} + t_{w2})/2 = 60℃，\ \alpha = 1/333$$

空气物性$\lambda = 2.9 \times 10^{-2}\, w/(m \cdot k)$，$\upsilon = 18.97 \times 10^{-6}\, m^2/s$，$Pr = 0.696$

（1）热面在上（纯导热）

$$q = \lambda \frac{\Delta t}{\delta} = 0.029 \frac{60}{0.014} = 124.3\, w/m^2$$

（2）热面在下

$$Gr_\delta = \frac{g\alpha \Delta t \delta^3}{\upsilon^2} = \frac{9.80665 \times 60 \times 0.014^3}{333 \times (18.97 \times 10^{-6})^2} = 13473$$

查书P273公式（6-47）$Nu = 0.212(Gr_\delta \mathrm{Pr})^{1/4} = 0.212 \times (13473 \times 0.696)^{0.25} = 0.086$

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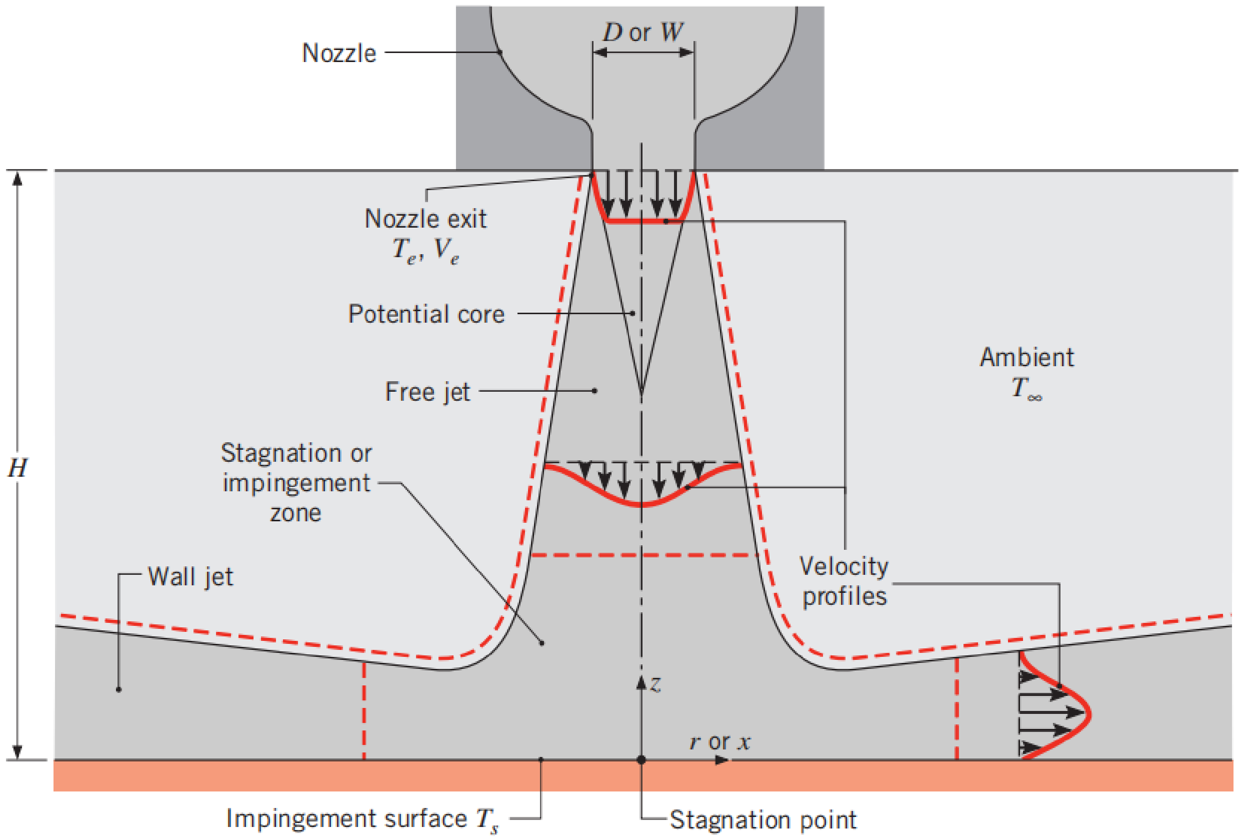
$$h = \frac{\lambda}{\delta} Nu = (0.029/0.014) \times 2.086 = 4.321\, w/(m^2 \cdot k)$$

$$q = h \Delta t = 4.321 \times 60 = 259.3\, w/m^2$$

可见，由于热面在下，自然对流增强了传热量。

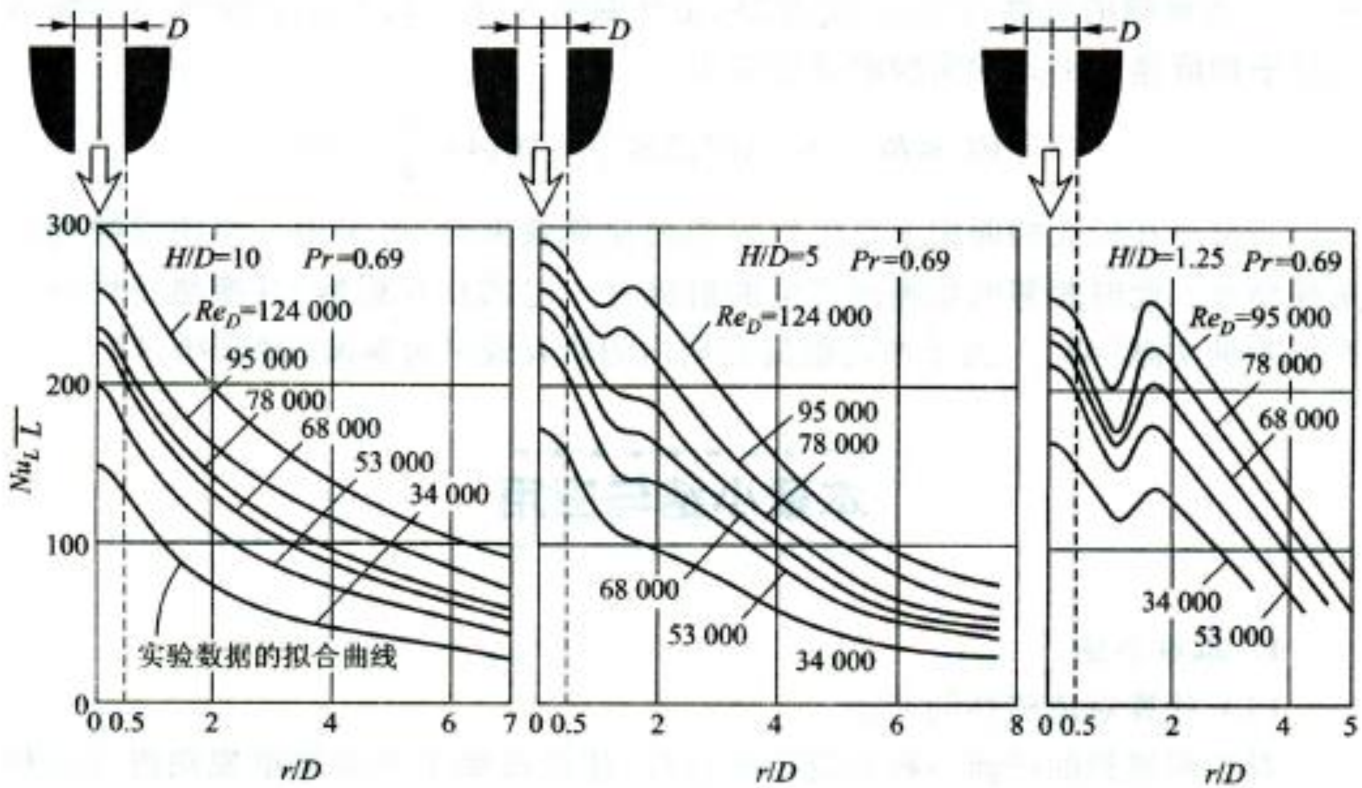
§ 6-6 Impinging jets

A single gas jet or an array of such jets, impinging normally on a surface, may be used to achieve enhanced coefficients for convective heating, cooling, or drying.

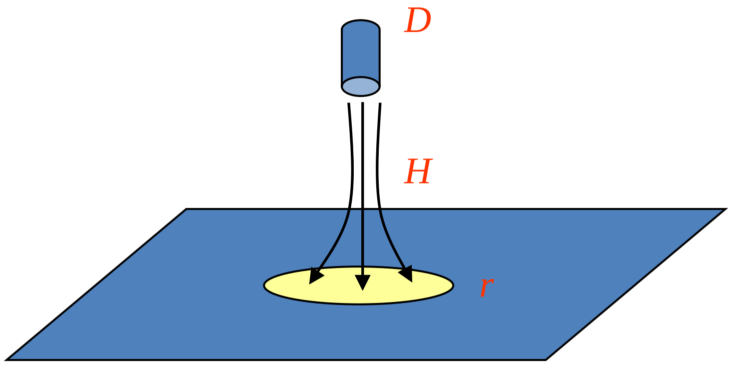


§ 6-6 Impinging jets

with different H/D：



§ 6-6 Impinging jets



single round nozzle

$$\frac{h_m r}{\lambda} = (Nu_r)_m$$

$$= 2\mathrm{Re}^{0.5}\mathrm{Pr}^{0.42}\left(1 + 0.005\mathrm{Re}_D^{0.55}\right)^{0.5} \frac{1 - 1.1\,D/r}{1 + 0.1(H/D - 6)\,D/r}$$

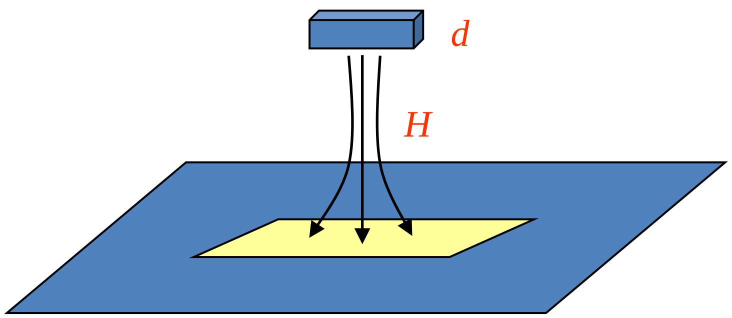
where: $t_m = (t_w + t_\infty)/2$

characteristic length: r or x

characteristic velocity: nozzle average velocity。

experimental conditions:

$$2 \times 10^3 \le \mathrm{Re}_D \le 4 \times 10^5, 2 \le \frac{H}{D} \le 12, 2.5 \le \frac{r}{D} \le 7.5$$



single rectangular nozzle

$$(Nu_b)_m = \frac{3.06}{x/b + H/b + 2.78}\mathrm{Re}_b^m \mathrm{Pr}^{0.42}$$

$$m = 0.695 - \left[\frac{x}{2b} + \left(\frac{H}{2b}\right)^{1.33} + 3.06\right]^{-1}$$

where: $t_m = (t_w + t_\infty)/2$

characteristic length: $2 * d$

characteristic velocity: nozzle average velocity

experimental conditions:

$$3 \times 10^3 \le \mathrm{Re}_b \le 9 \times 10^4, 2 \le \frac{H}{b} \le 10, 4 \le \frac{x}{b} \le 20$$