北京航空航天大学
BEIHANG UNIVERSITY

飞行力学
Flight Mechanics

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Chapter 3

- Maneuverability and agility

  Overload during maneuvering of aircraft

  Maneuverability at the vertical plane

  Maneuverability at the horizontal plane

  Maneuverability at 3D space

  Analysis of the overall maneuverability

Contents

- Introduction
- Equation of motion
- Maneuverability at the vertical plane
- Maneuverability at the horizontal plane
- Turning performance
- Examples

Introduction

Question

- Why maneuverability is important?
- How to fly a turn?

Introduction

Some definitions

Maneuverability（机动性）：Maneuverability is defined as the ability to change the speed and flight direction of an airplane within a certain time.

Agility（敏捷性）：Agility is a measure of how quickly the aircraft can be maneuvered. It relates to minimizing the time required to perform some tasks or to achieve a desired aircraft state. The simplest definition of agility is the ability to move quickly in any direction or to perform a specific task.

Equation of motion

Equation of motion in Kinematic Frame

$$\begin{cases} m\dfrac{dV}{dt} = T\cos(\alpha+\varphi)\cos\beta - D - mg\sin\gamma \\ mV\cos\gamma\dfrac{d\chi}{dt} = T[\sin(\alpha+\varphi)\sin\mu - \cos(\alpha+\varphi)\sin\beta\cos\mu] + C\cos\mu + L\sin\mu \\ -mV\dfrac{d\gamma}{dt} = T[-\sin(\alpha+\varphi)\cos\mu - \cos(\alpha+\varphi)\sin\beta\sin\mu] + C\sin\mu - L\cos\mu + mg\cos\gamma \end{cases}$$

Assume: $C = 0,\ \beta = 0,\ \alpha + \varphi \approx 0, \Rightarrow ?$

Equation of motion

Equation of motion in Kinematic Frame

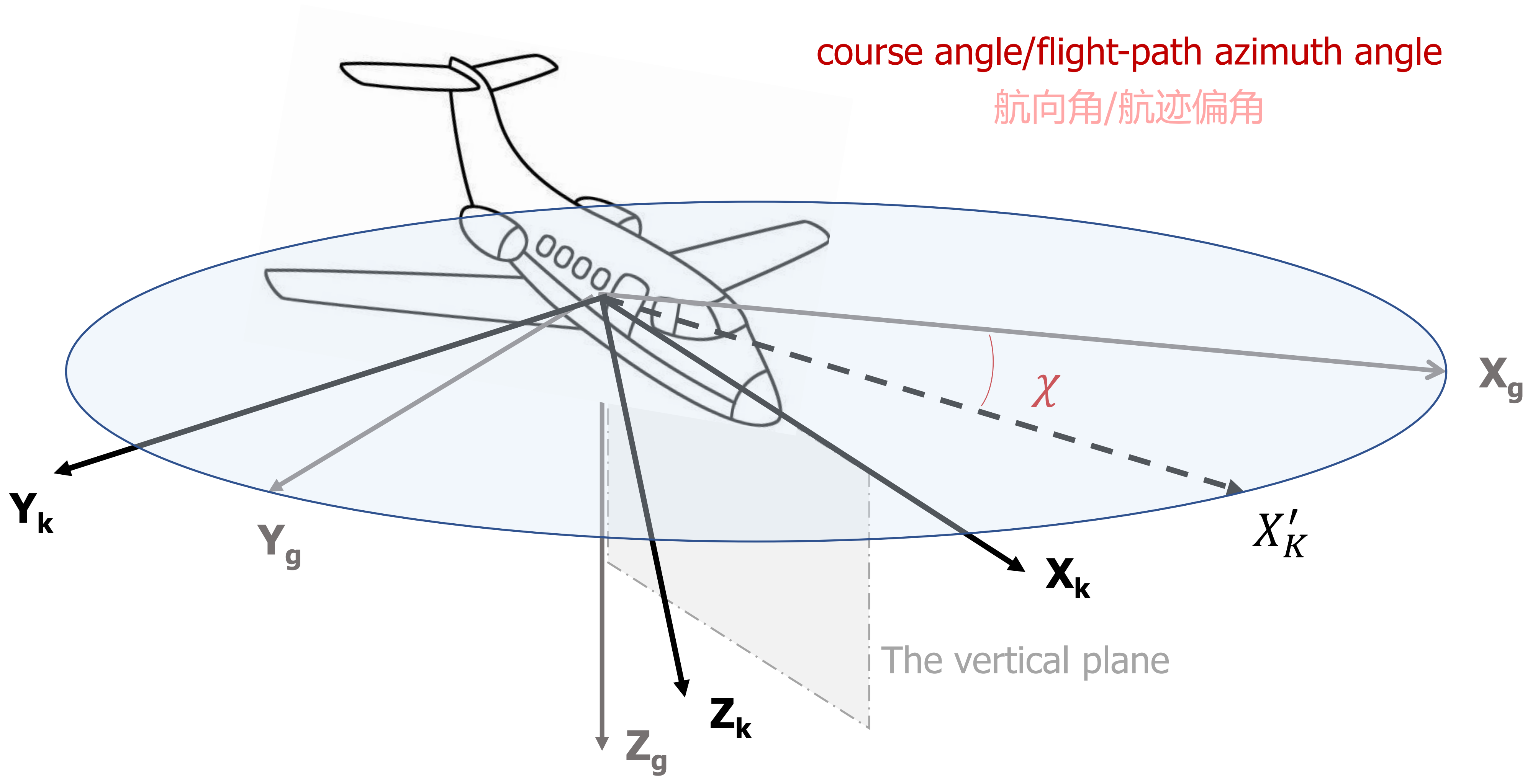
$$m\frac{dV}{dt} = T - D - mg\sin\gamma$$

$$mV\cos\gamma\frac{d\chi}{dt} = L\sin\mu$$

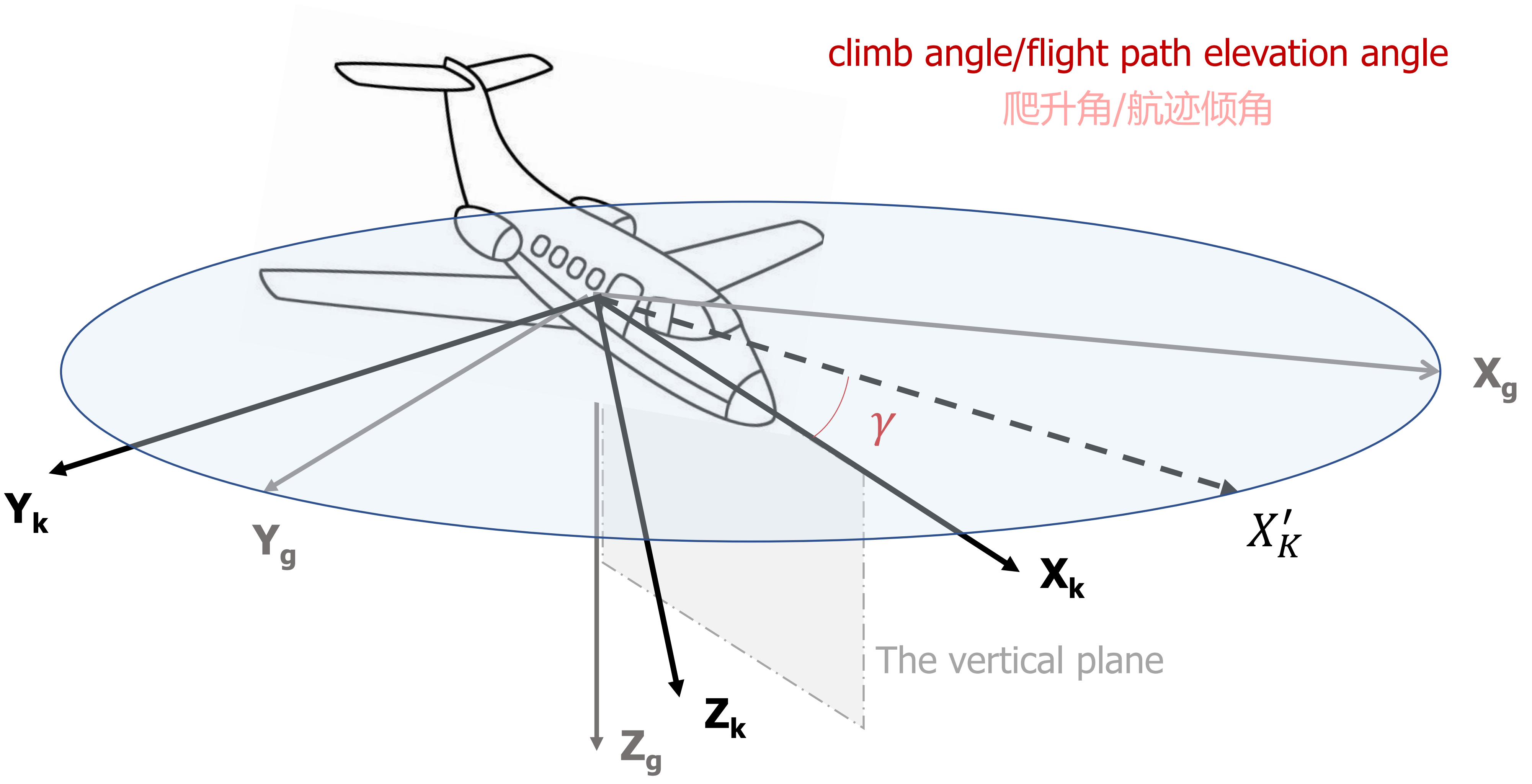
$$-mV\frac{d\gamma}{dt} = -L\cos\mu + mg\cos\gamma$$

Assumptions: $C = 0,\ \beta = 0,\ \alpha + \varphi \approx 0.$

Review - χ


course angle/flight-path azimuth angle
航向角/航迹偏角
χ
X_g
X'_K
Y_k
Y_g
X_k
Z_k
Z_g
The vertical plane

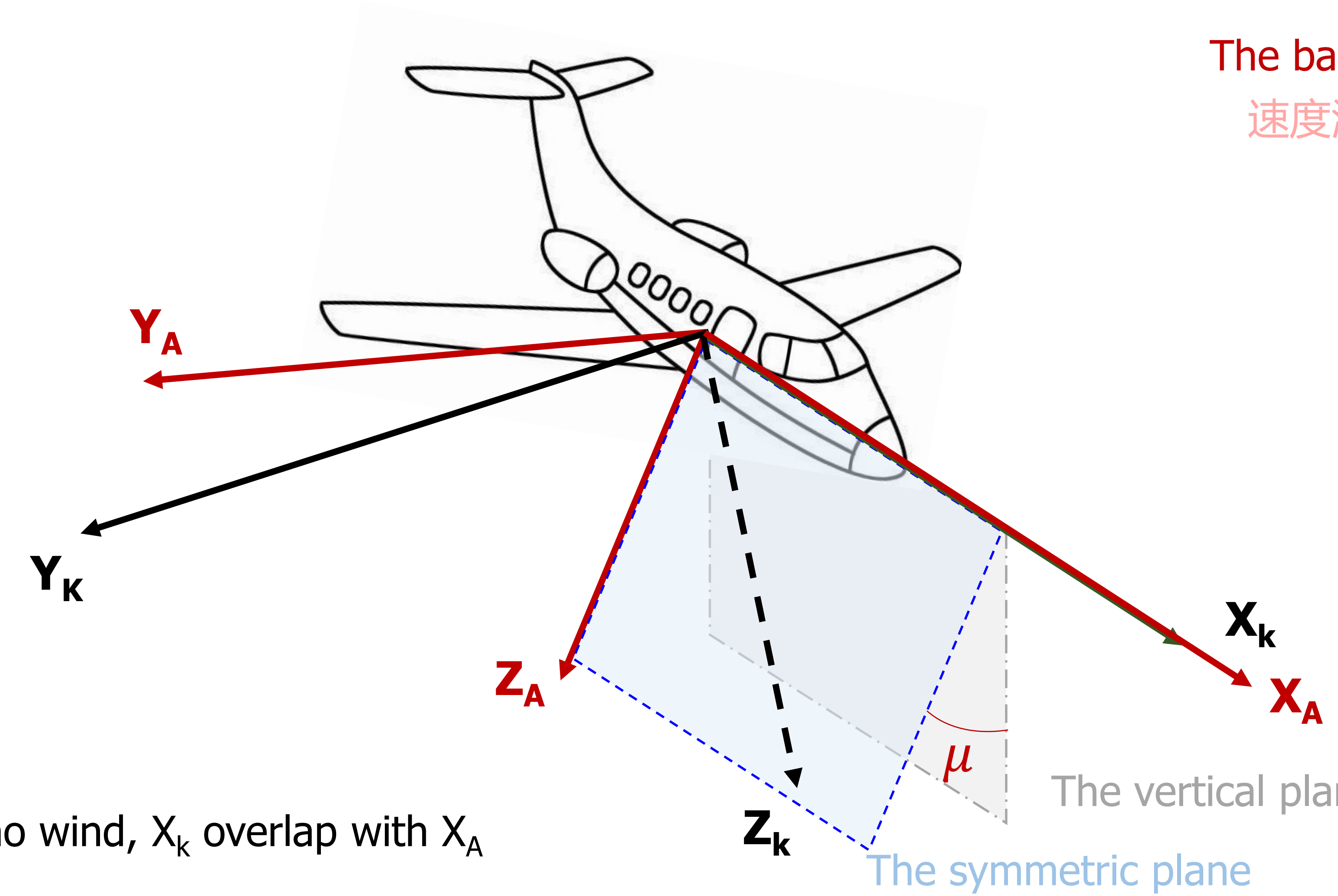
Review - γ



Review - μ

The bank angle

速度滚转角



Assume no wind, X_k overlap with X_A

Equation of motion

Overload（过载）

The ratio of external forces $(\vec{T} + \vec{A})$ to the weight of the aircraft.

$$\vec{n} = \frac{\vec{T} + \vec{A}}{W}$$

Projection to Orthogonal Coordinate System

$$\vec{n} = n_x\vec{i} + n_y\vec{j} + n_z\vec{k}$$

Equation of motion

Overload（过载）

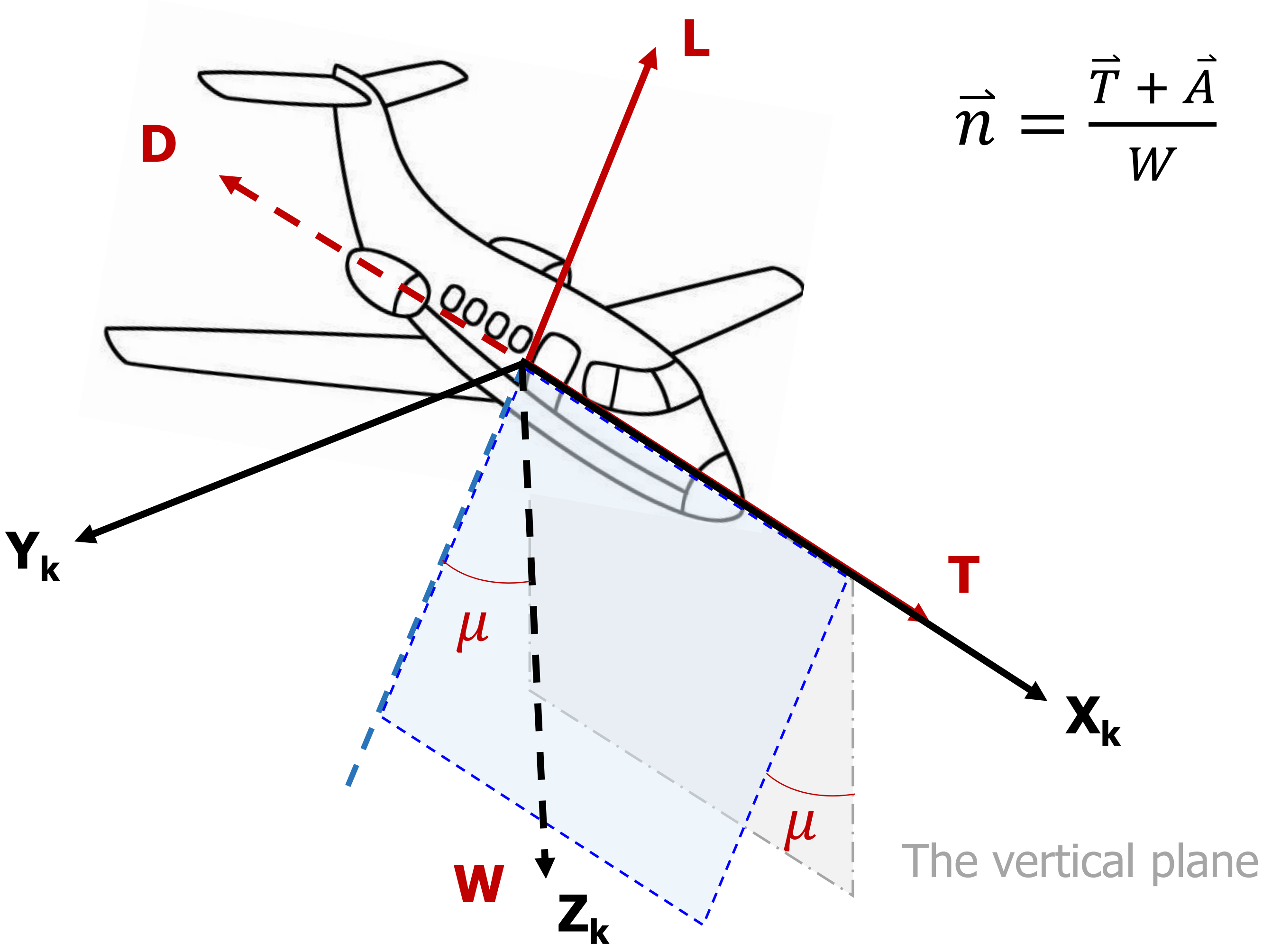
$$\vec{n} = \frac{\vec{T} + \vec{A}}{W}$$

$$\vec{n} = n_x\vec{i} + n_y\vec{j} + n_z\vec{k}$$

$n_x = ?$

$n_y = ?$

$n_z = ?$



Equation of motion

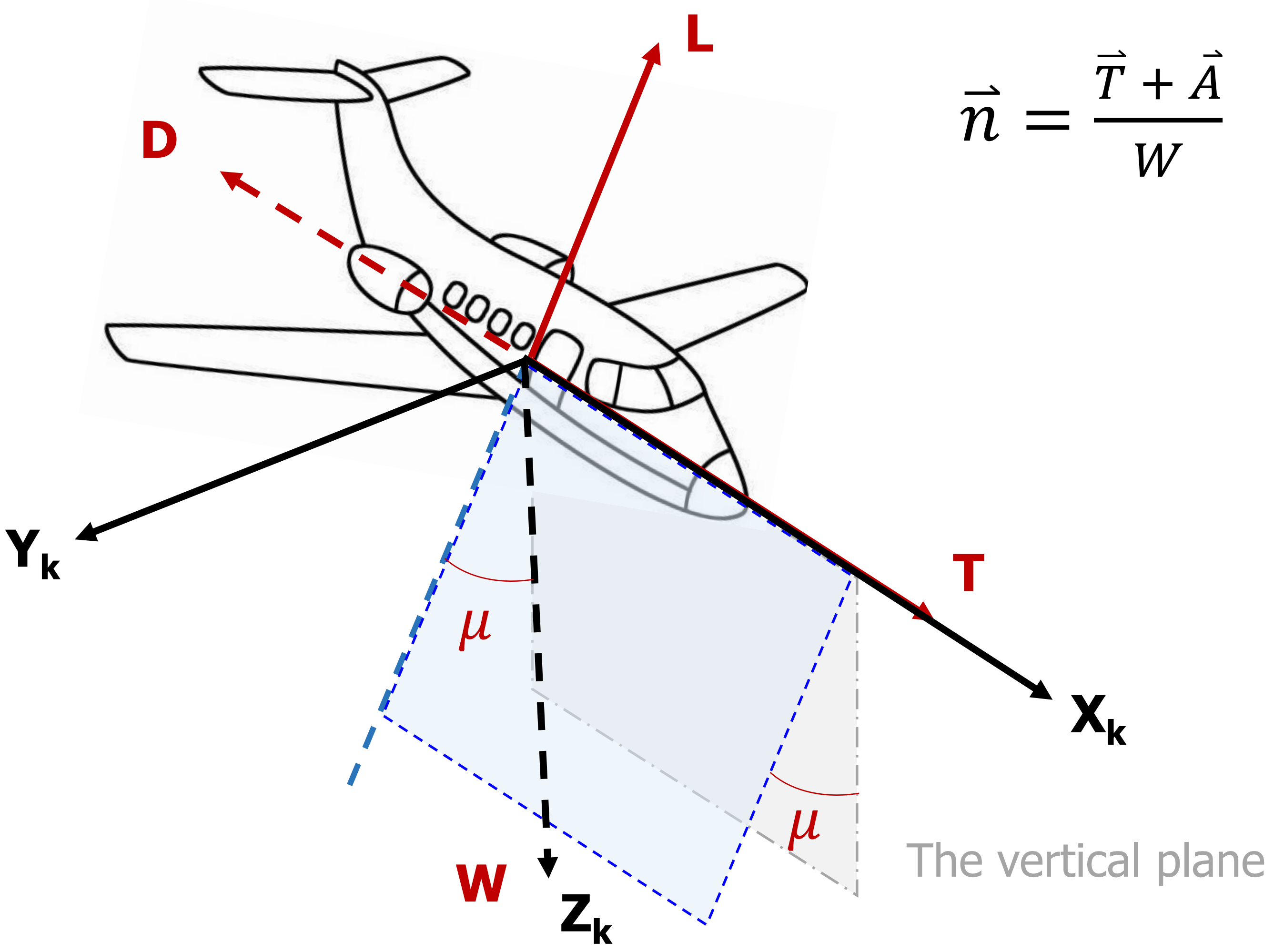
Overload（过载）

$$\vec{n} = n_x\vec{i} + n_y\vec{j} + n_z\vec{k}$$

$$n_x = \frac{T - D}{W}$$

$$n_y = \frac{L\sin\mu}{W}$$

$$n_z = \frac{L\cos\mu}{W}$$



$$\vec{n} = \frac{\vec{T} + \vec{A}}{W}$$

Equation of motion

Relation of motion to overload

$$n_x = \frac{T - D}{W}$$

$$n_y = \frac{L \sin \mu}{W}$$

$$n_z = \frac{L \cos \mu}{W}$$

$$m\frac{dV}{dt} = T - D - mg \sin \gamma$$

$$mV \cos \gamma \frac{d\chi}{dt} = L \sin \mu$$

$$-mV\frac{d\gamma}{dt} = -L \cos \mu + mg \cos \gamma$$

$$\frac{dV}{dt} = g(n_x - \sin \gamma)$$

$$V \cos \gamma \frac{d\chi}{dt} = g n_y$$

$$V\frac{d\gamma}{dt} = g(n_z - \cos \gamma)$$

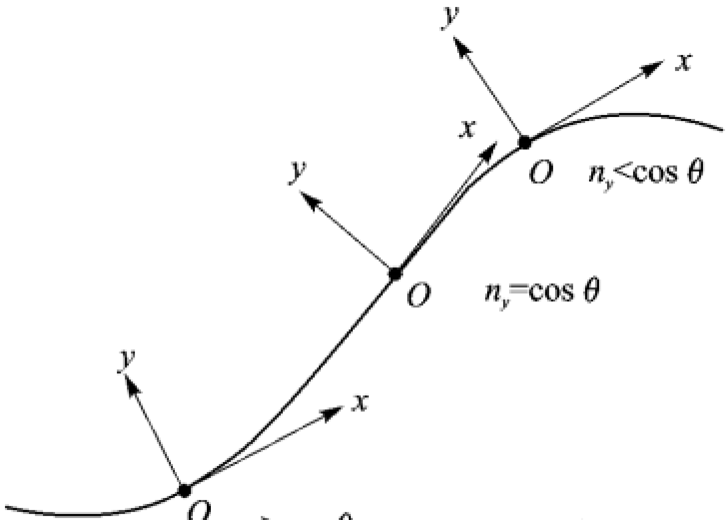
Equation of motion

$$\begin{cases} \dfrac{dV}{dt} = g(n_x - \sin\gamma) & \Rightarrow \begin{cases} n_x = \sin\gamma & \text{steady straight line flight} \\ n_x < \sin\gamma & \text{decelerated flight} \\ n_x > \sin\gamma & \text{accelerated flight} \end{cases} \\ V\cos\gamma \dfrac{d\chi}{dt} = g n_y & \Rightarrow \begin{cases} n_y = 0 \Rightarrow d\chi/dt = 0 & \text{horizontal straight line flight} \\ n_y < 0 \Rightarrow d\chi/dt < 0 & \text{left turn} \\ n_y > 0 \Rightarrow d\chi/dt > 0 & \text{right turn} \end{cases} \\ \text{Horizontal plane} & \\ V\dfrac{d\gamma}{dt} = g(n_z - \cos\gamma) & \Rightarrow \begin{cases} n_z = \cos\gamma \Rightarrow d\gamma/dt = 0 & \text{straight line climb} \\ n_z < \cos\gamma \Rightarrow d\gamma/dt < 0 & \text{bend downwards} \\ n_z > \cos\gamma \Rightarrow d\gamma/dt > 0 & \text{bend upwards} \end{cases} \\ \text{Vertical plane} & \end{cases}$$

Equation of motion

Vertical plane

$$n_y = 0$$



(a) 垂直面

$$V\frac{d\gamma}{dt} = g(n_z - \cos\gamma) = \mathrm{g}(n_n - \cos\gamma)$$

Curvature（曲率）

$$K_V = \frac{d\gamma}{dt} = \frac{g}{V}(n_n - \cos\gamma)$$

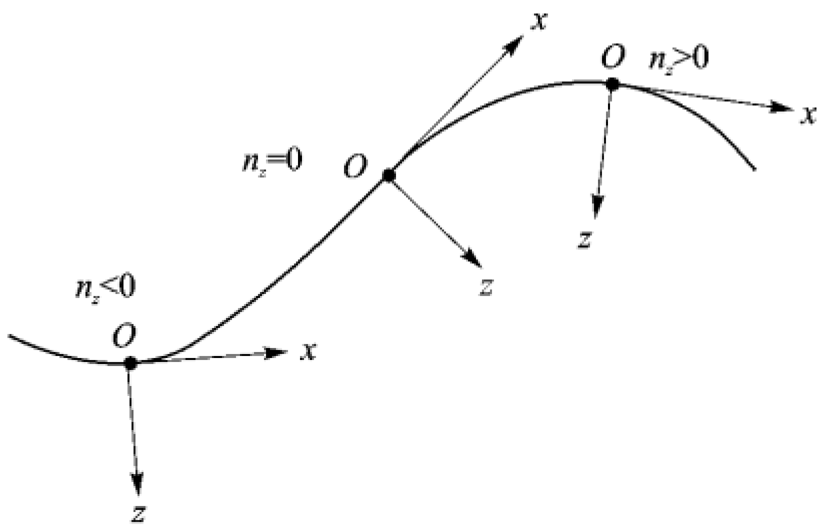
Radius of curvature（曲率半径）

$$R_V = \frac{ds}{d\gamma} = \frac{V^2}{g(n_n - \cos\gamma)}$$

Equation of motion

Horizontal plane

$$\boxed{n_z = 0,\ \cos\gamma = 1}$$



(b) 水平面

$$V\frac{d\chi}{dt} = gn_y = g\sqrt{n_n^2 - 1}$$

Turning rate（转弯速率） $$\Omega = \frac{d\chi}{dt} = \frac{g}{V}\sqrt{n_n^2 - 1}$$

Turning radius（转弯半径） $$R_h = \frac{ds}{d\chi} = \frac{V^2}{g\sqrt{n_n^2 - 1}}$$

Equation of motion

$$n_x = \frac{T - D}{W}$$

$$n_y = \frac{L \sin\mu}{W}$$

$$n_z = \frac{L \cos\mu}{W}$$

Load factor

$$n_n = \sqrt{n_y^2 + n_z^2} \quad \text{(法向过载)}$$

$$= \sqrt{\left(\frac{L \sin\mu}{W}\right)^2 + \left(\frac{L \cos\mu}{W}\right)^2} = \frac{L}{W}$$

$n_n = L/W$ is also called load factor. For simplicity, replace n_n with n

Equation of motion

Overload of the pilot （驾驶员过载）

$$\vec{a} = \frac{\vec{T} + \vec{A} + \vec{W}}{m} = \vec{n}g + \vec{g}$$

$$= \frac{\vec{F} + m_{pilot}\vec{g}}{m_{pilot}} = \frac{\vec{F}}{W_{pilot}}g + \vec{g}$$

$$\Rightarrow \vec{F} = \vec{n}W_{pilot}$$

Support force of the seat
（座椅支持力）

Equation of motion

Restrictions of the overload（过载限制）

- Pilot Physiological Limits.

$8g -$ 5 ~ 10 s

$5g -$ 20 ~ 30 s

- Structural Limits.

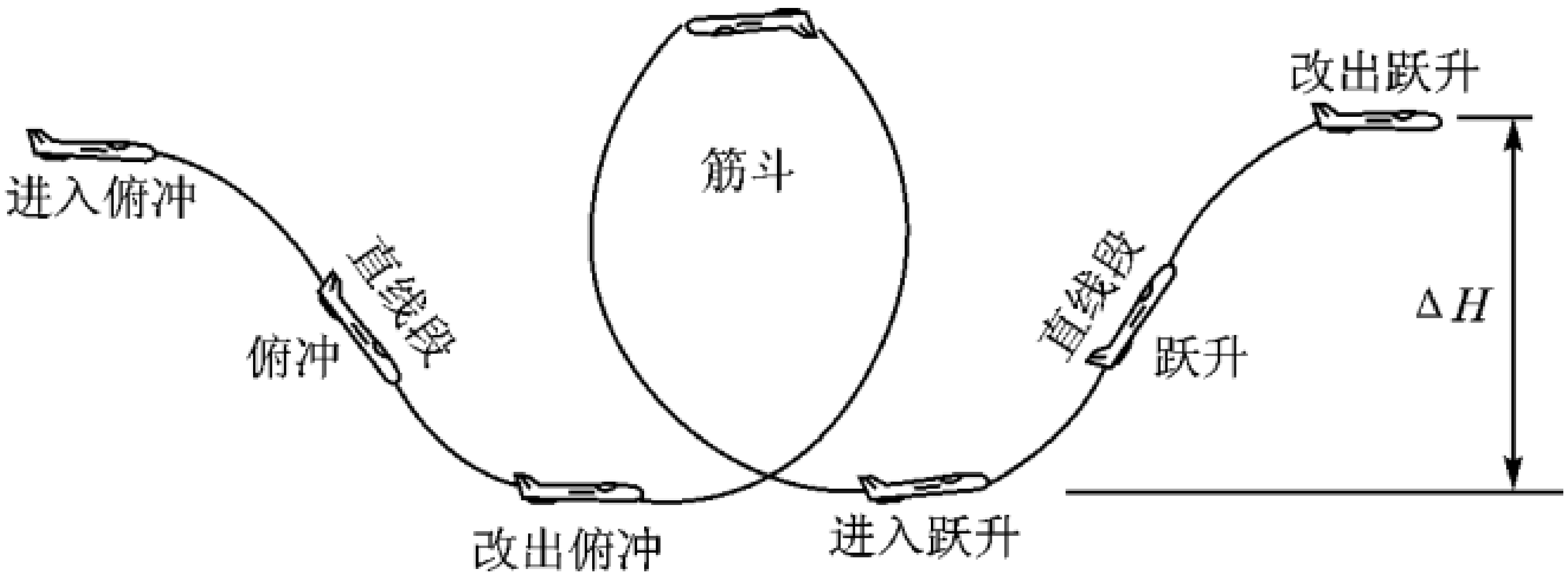
Fighter jet – 8~9 g

Bomber – 2.5~3.5 g

Civil airliner – < 2 g

- Meter operational limit.

Maneuvering in the vertical plane



Maneuvering flight action of aircraft in the vertical plane

Maneuvering in the vertical plane

Straight line acceleration and deceleration

$$\frac{dV}{dt} = \frac{g}{W}(T - D) = \frac{\Delta T}{W} g = n_x\ g$$

$$L = W$$

Maneuvering in the vertical plane

Straight line acceleration and deceleration

Acceleration index:

$$\begin{cases} \dfrac{dV}{dt} = n_x g \\ L = W \end{cases} \quad \Rightarrow t = \frac{1}{g}\int_{V_0}^{V_1} \frac{dV}{n_x} = \frac{W}{g}\int_{V_0}^{V_1} \frac{dV}{\Delta T}$$

Maneuvering in the vertical plane

Jump（跃升）: kinetic energy $\Rightarrow$ potential energy

Dynamic equation:

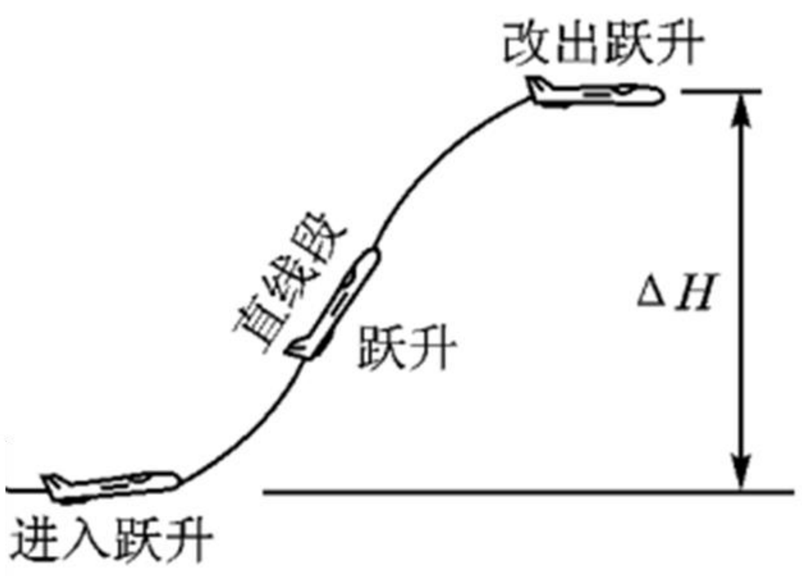
$$m\frac{dV}{dt} = T - D - mg\sin\gamma$$

$$mV\frac{d\gamma}{dt} = L - mg\cos\gamma$$

Maneuvering in the vertical plane

Jump（跃升）: kinetic energy ⇒ potential energy

Estimate the height: ΔH



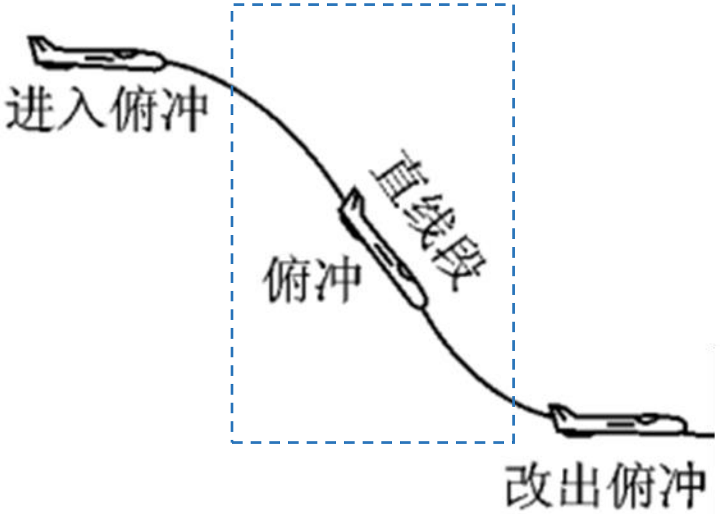
Maneuvering in the vertical plane

Dive (俯冲): potential energy ⇒ kinetic energy

Straight line dive

$$\frac{dV}{dt} = g\frac{T - D - W\sin\gamma}{W}$$

$$L = W\cos\gamma$$



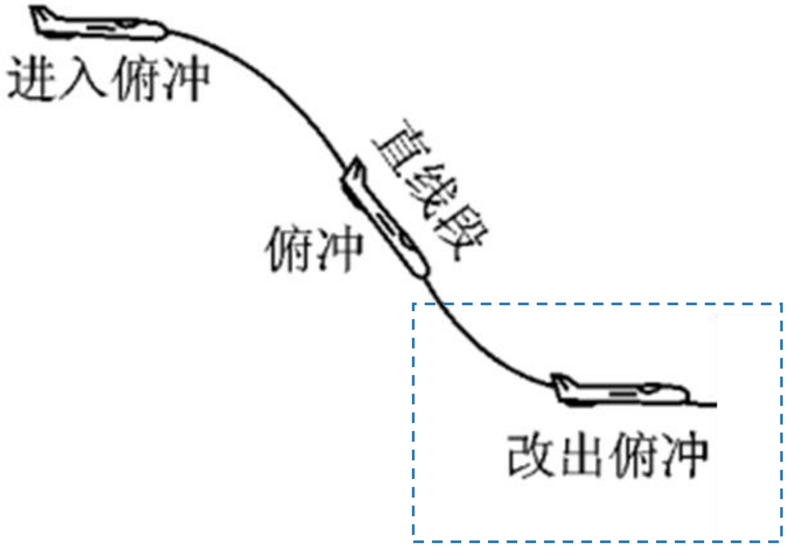
Maneuvering in the vertical plane

Dive (俯冲): potential energy ⇒ kinetic energy

Recovery dive

$$\begin{cases} \dfrac{dV}{dt} = -g\sin\gamma \\ \dfrac{d\gamma}{dt} = \dfrac{g}{V}(n_z - \cos\gamma) \end{cases}$$

$$\Rightarrow V = V_1 \frac{n_z - \cos\gamma_1}{n_z - 1}$$



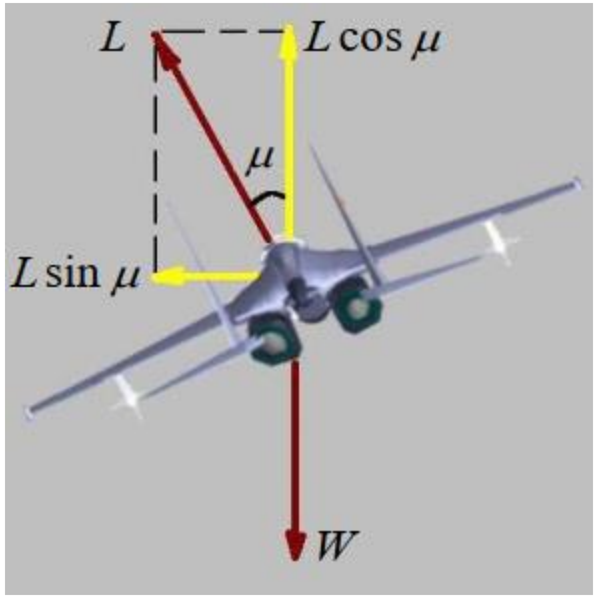
Maneuvering in the horizontal plane

The ability to change speed direction

Turn （转弯） : Maneuverability in horizontal plane with direction change.

Hovering （盘旋） Turn continuously more than 360 degree .

Normal Hovering （正常盘旋） : no sideslip, steady turn.



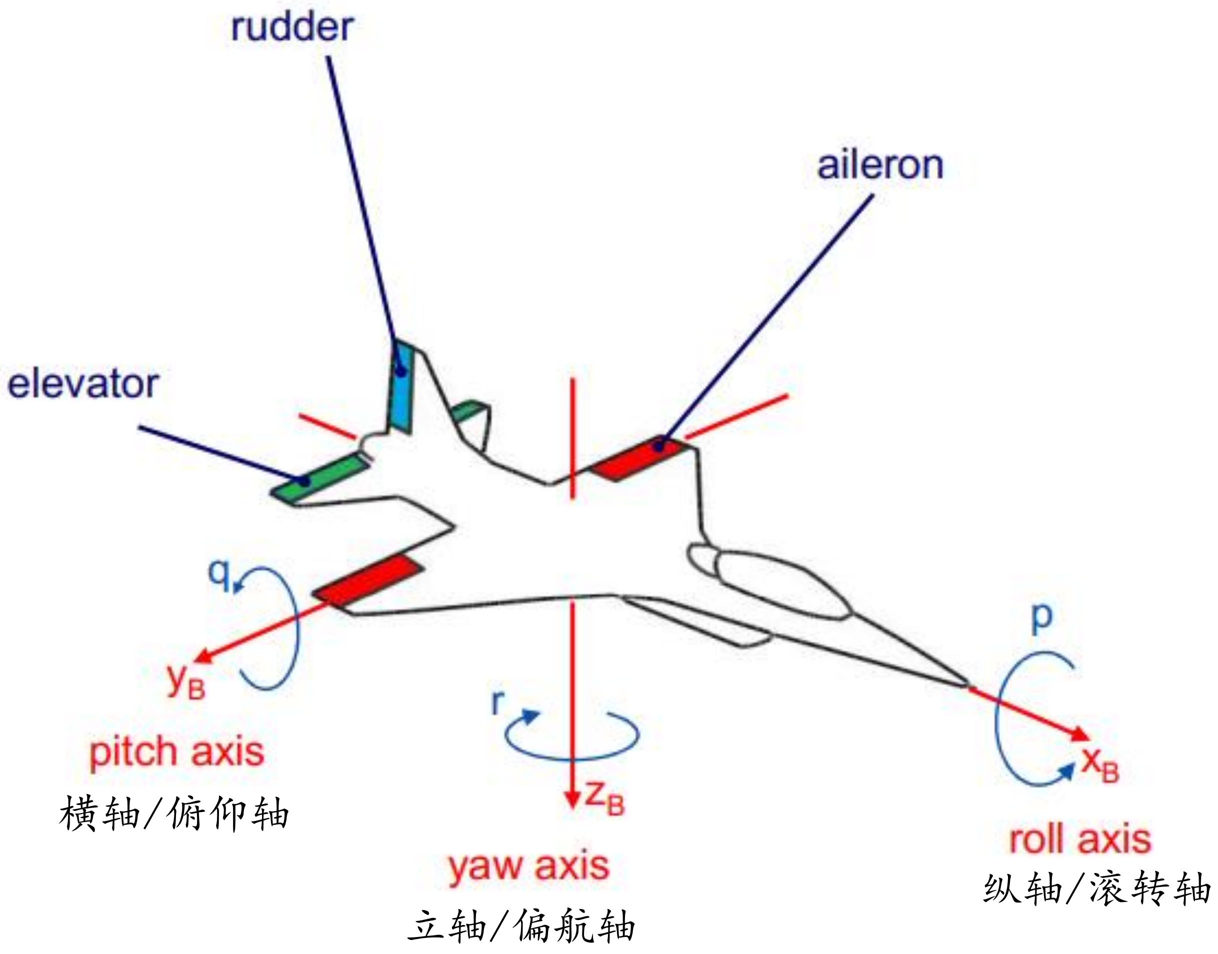
Normal hovering Index:

$\boldsymbol{r}$: Normal hovering radius.

$\boldsymbol{t_{2\pi}}$: Time for normal hovering 360 degree.

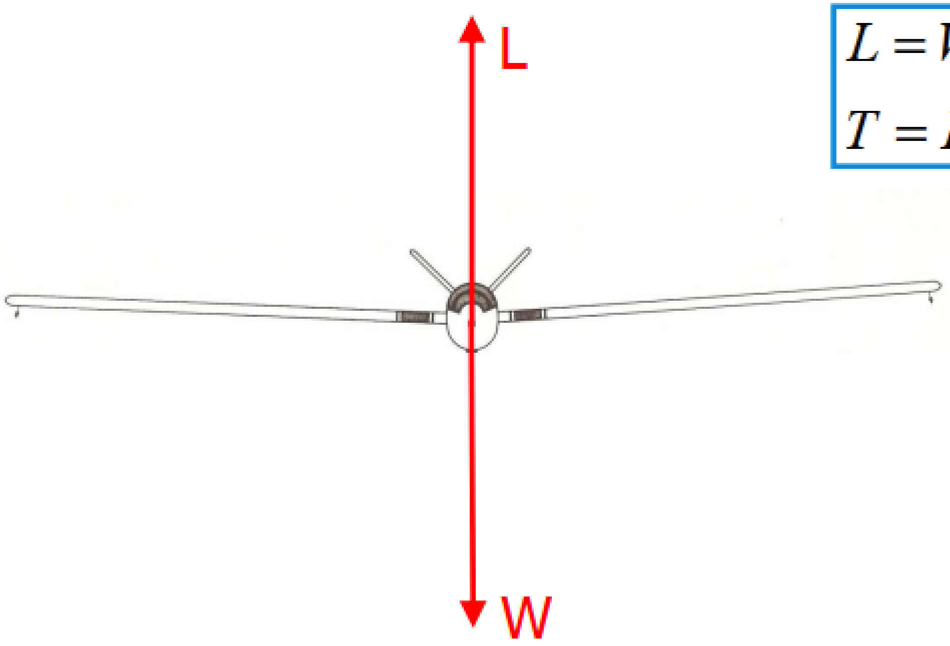
$\dot{\chi}$: Angular velocity of normal hovering.

How to fly a turn?



How to fly a turn

Steady horizontal flight



$$\boxed{\begin{aligned} L &= W \\ T &= D \end{aligned}}$$

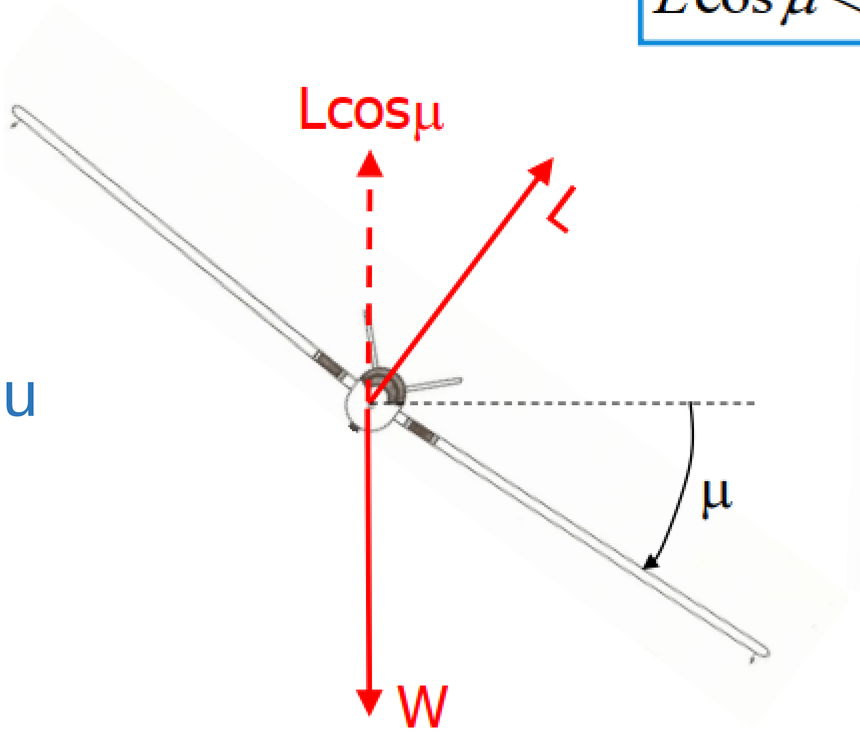
How to fly a turn

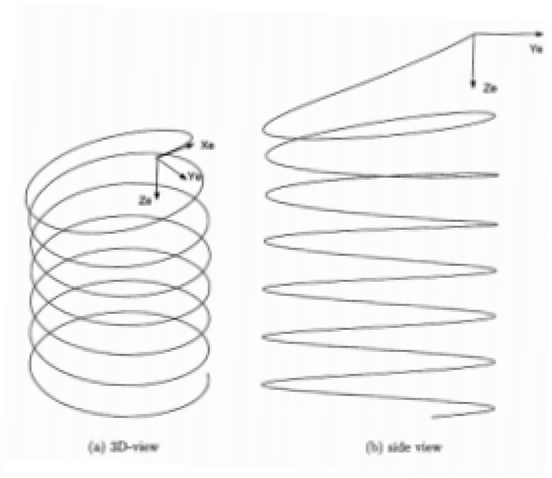
Roll the aircraft

$$\boxed{L\cos\mu < W}$$

What will happen if you

only roll the aircraft?

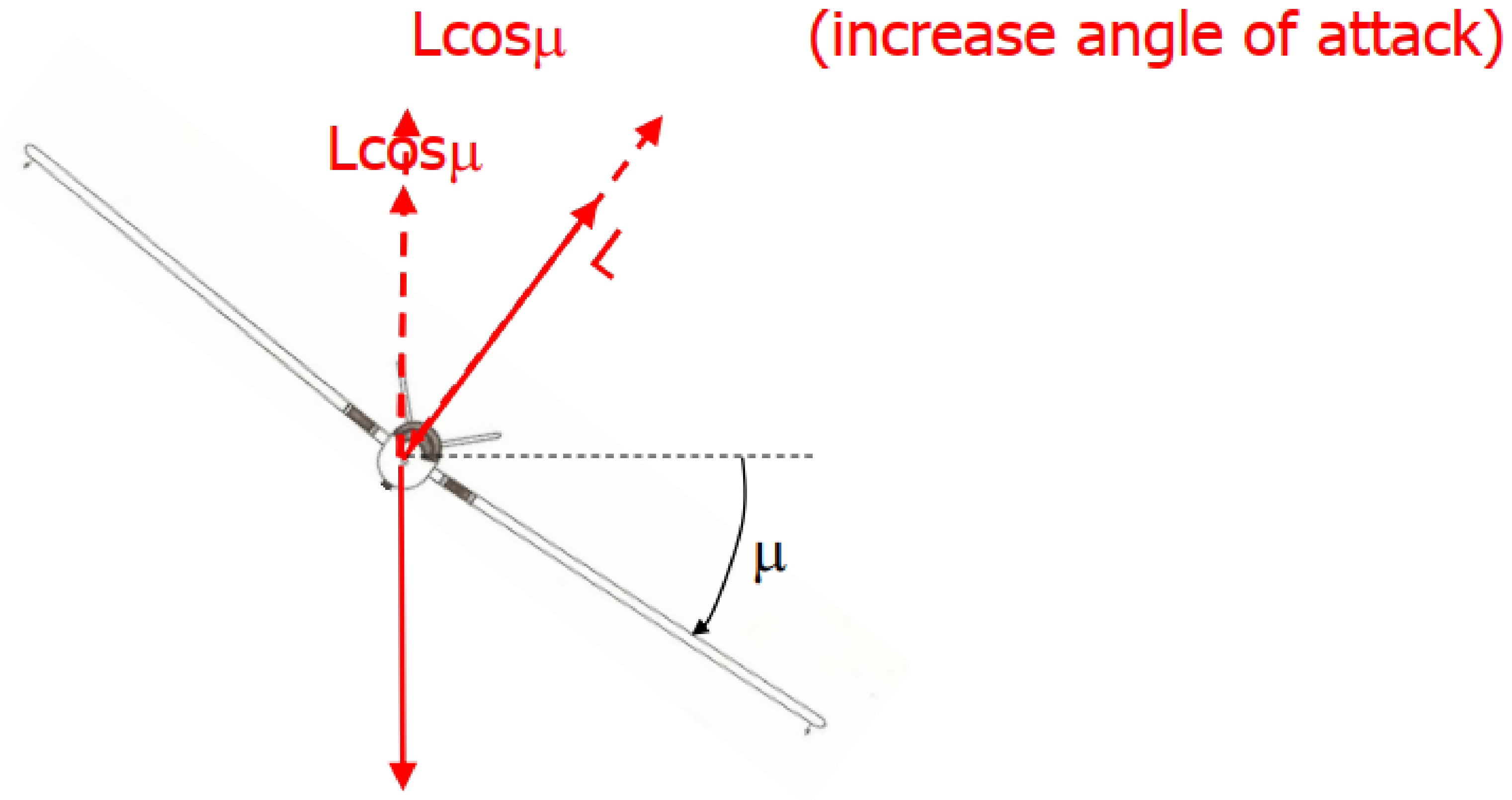




How to fly a turn

Roll the aircraft

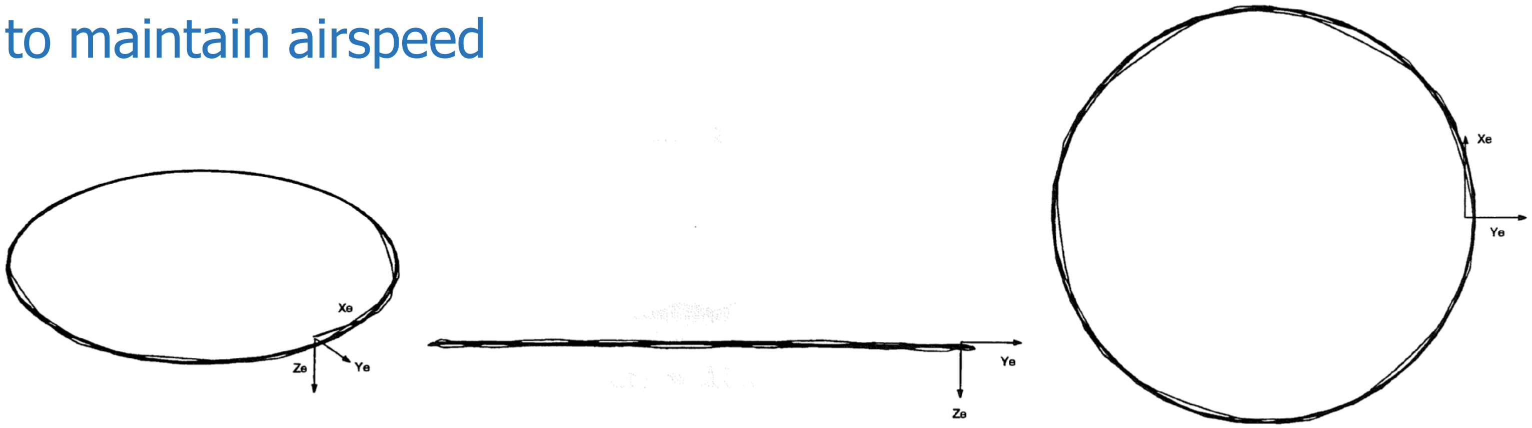
What will happen if you increase angle of attack to balance the weight?



How to fly a turn

Perform a steady turn

- Roll the aircraft to start turning
- Nose up to maintain altitude
- Increase the thrust to maintain airspeed

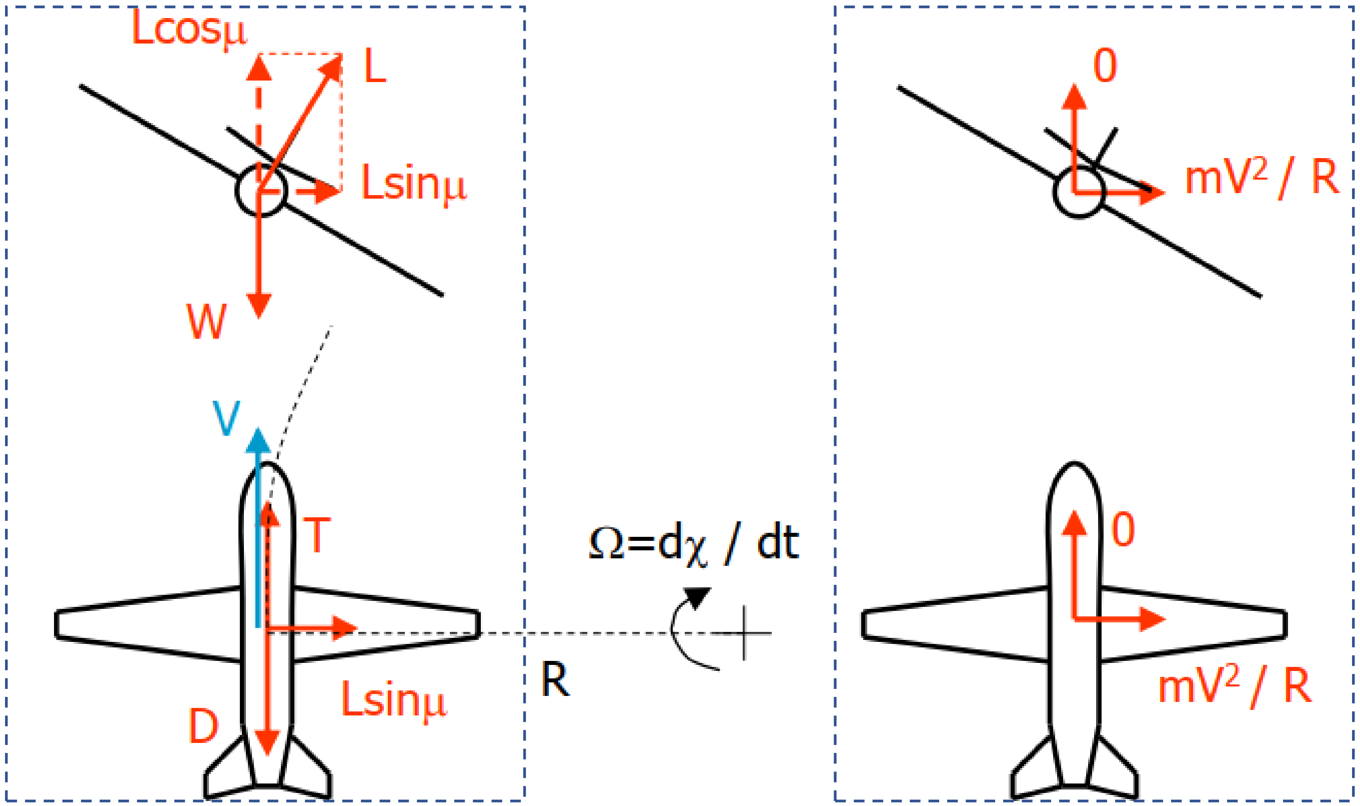


(a) 3D-view (b) side view (c) top view

Equation of motion

Horizontal steady turn （水平定常盘旋）

NED frame



Kinematic frame

Maneuvering in the horizontal plane

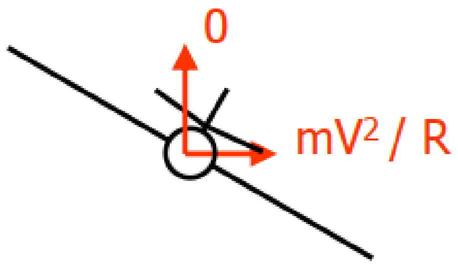
The ability to change speed direction

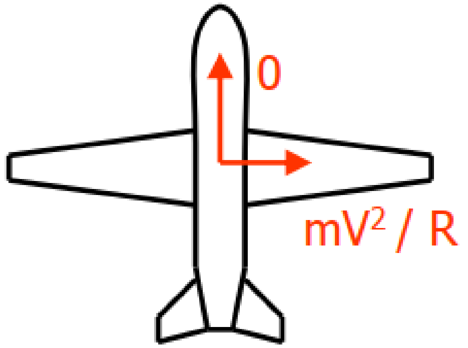
Dynamic equation:

$$T\cos(\alpha+\varphi)=D$$

$$[T\sin(\alpha+\varphi)+L]\cos\mu=mg$$

$$mV\frac{d\chi}{dt}=[T\sin(\alpha+\varphi)+L]\sin\mu$$



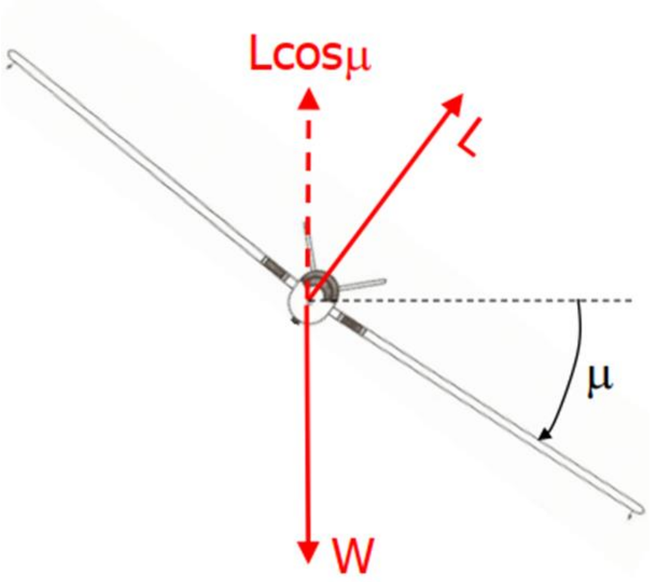


Maneuvering in the horizontal plane

Load factor and performance diagram

Load factor:

$$n \equiv \frac{L}{W} \qquad \Rightarrow L = nW$$



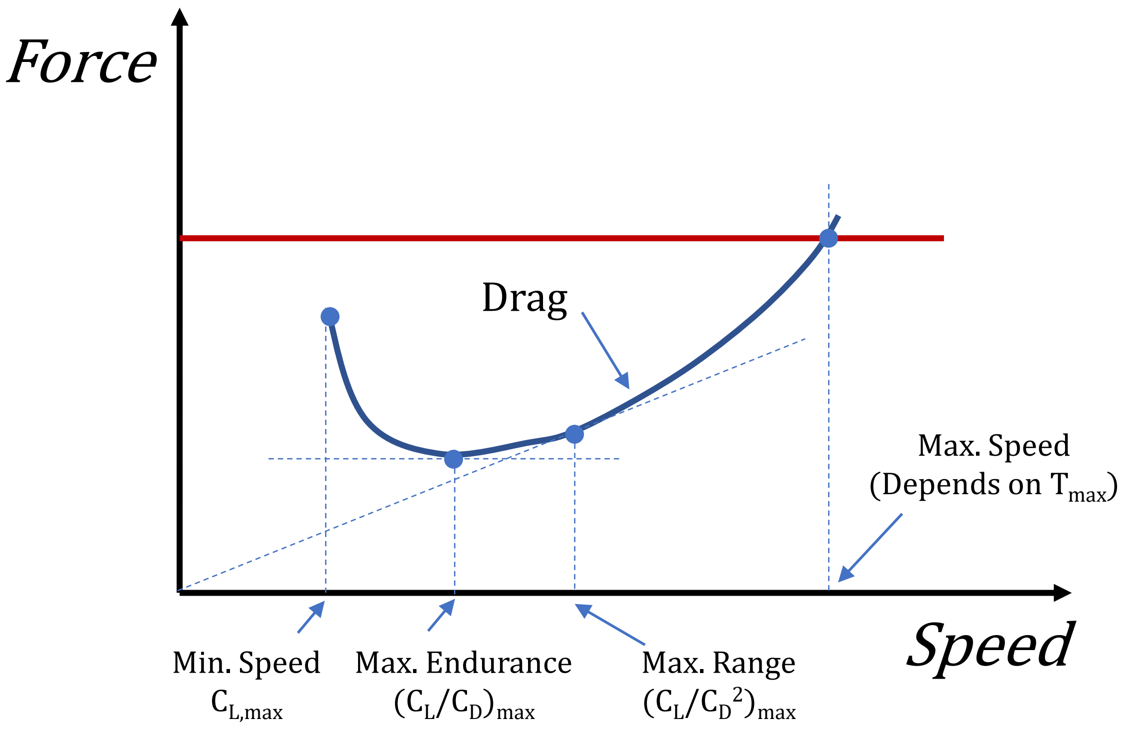
Load factor during steady horizontal turn

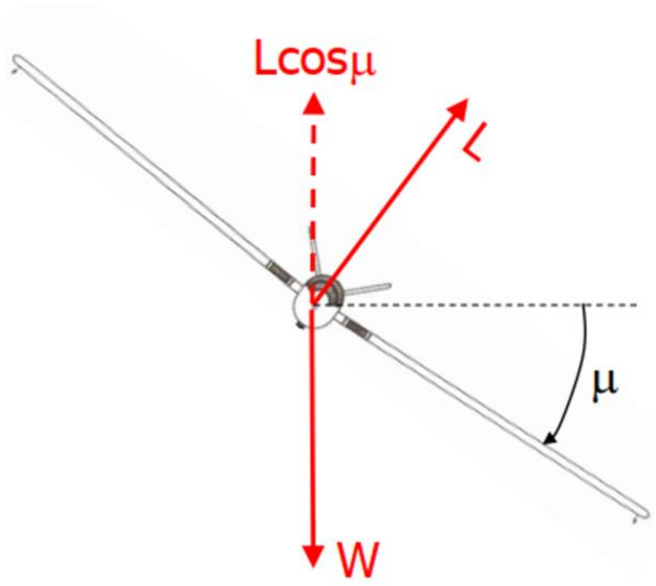
$$W = L\cos\mu$$

$$n = \frac{L}{W} = \frac{L}{L\cos\mu} = \frac{1}{\cos\mu}$$

Maneuvering in the horizontal plane

Load factor and performance diagram

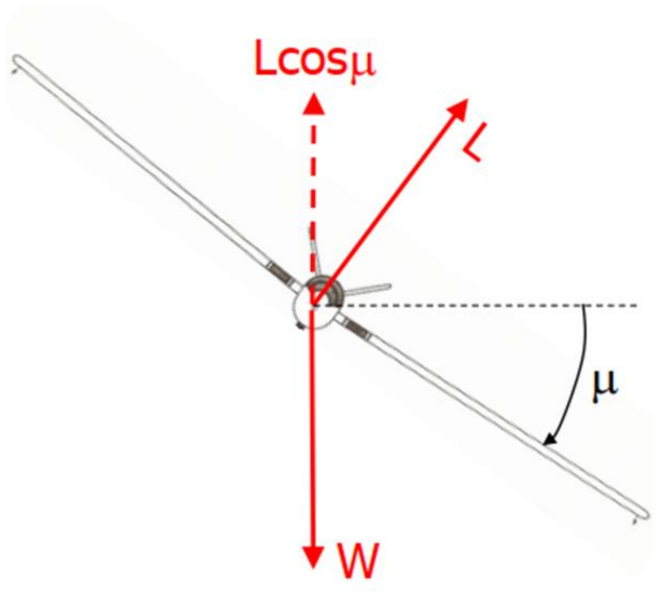




Maneuvering in the horizontal plane

Load factor and performance diagram

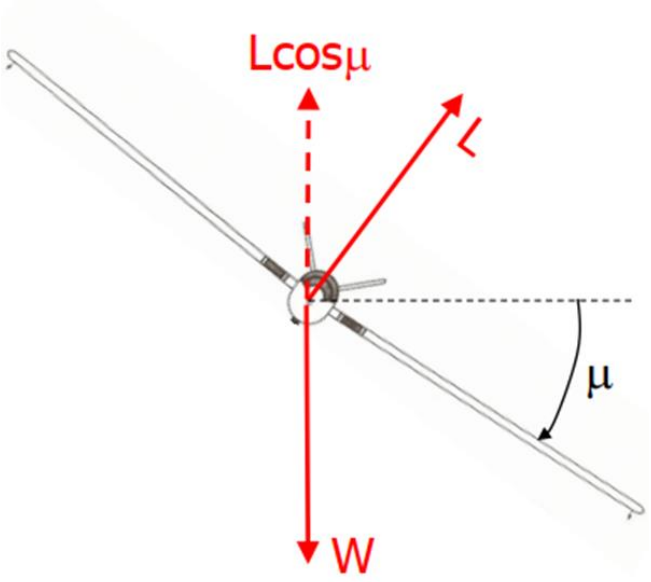
What will happen to
performance diagram in a turn?



Maneuvering in the horizontal plane

Load factor and performance diagram

What will happen to
performance diagram in a turn?



V, D, P_r are functions of lift coefficient and load factor.

(In symmetric flight, they only depend on the lift coefficient)

Maneuvering in the horizontal plane

Load factor and performance diagram

Airspeed
$$V = \sqrt{\frac{nW}{S}\frac{2}{\rho}\frac{1}{C_L}} \propto \sqrt{n}$$

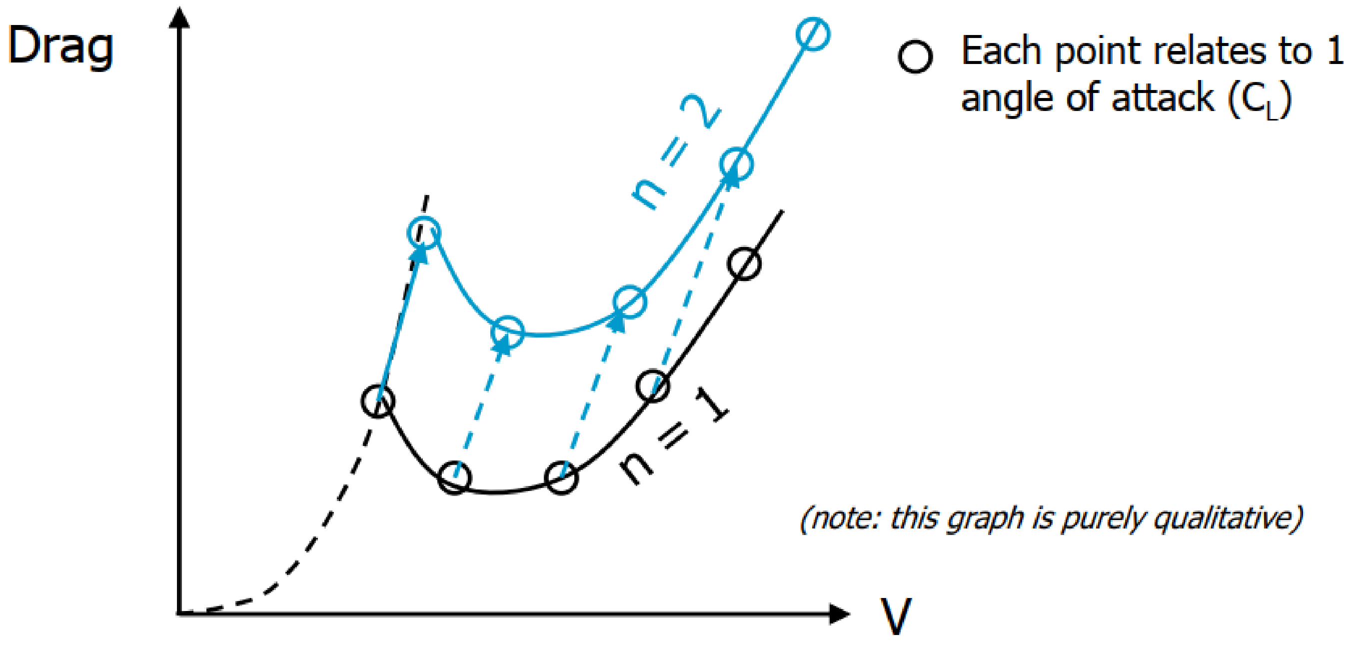
Aerodynamics drag
$$D = \frac{C_D}{C_L} nW \propto n$$

Power required
$$P_r = DV \propto n\sqrt{n}$$

Maneuvering in the horizontal plane

Load factor and performance diagram

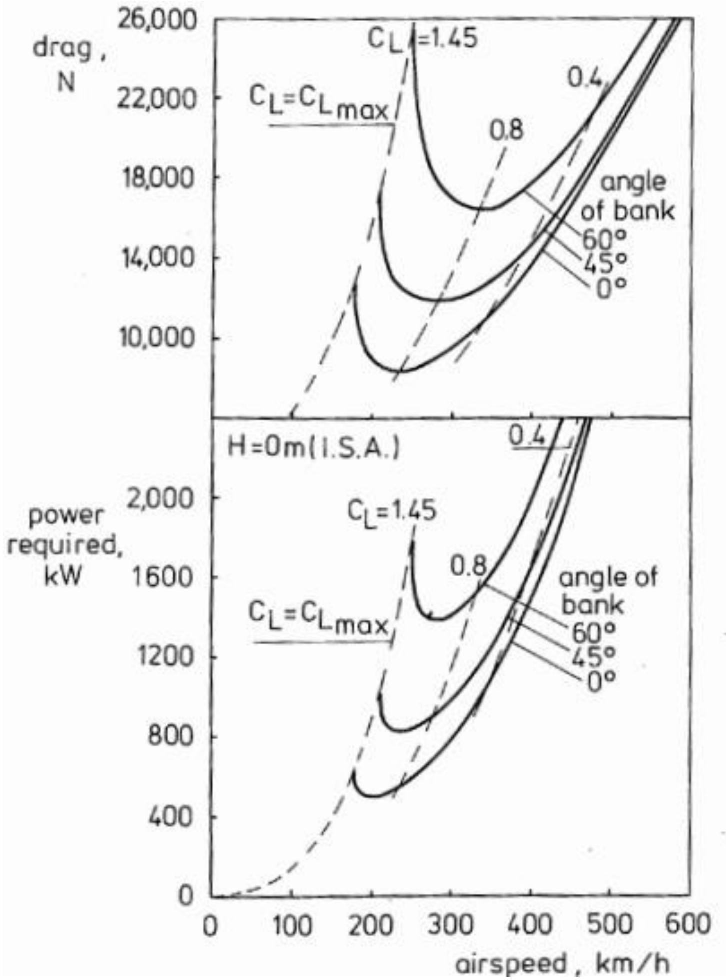
$$D = \frac{C_D}{C_L} nW \propto n$$



Maneuvering in the horizontal plane

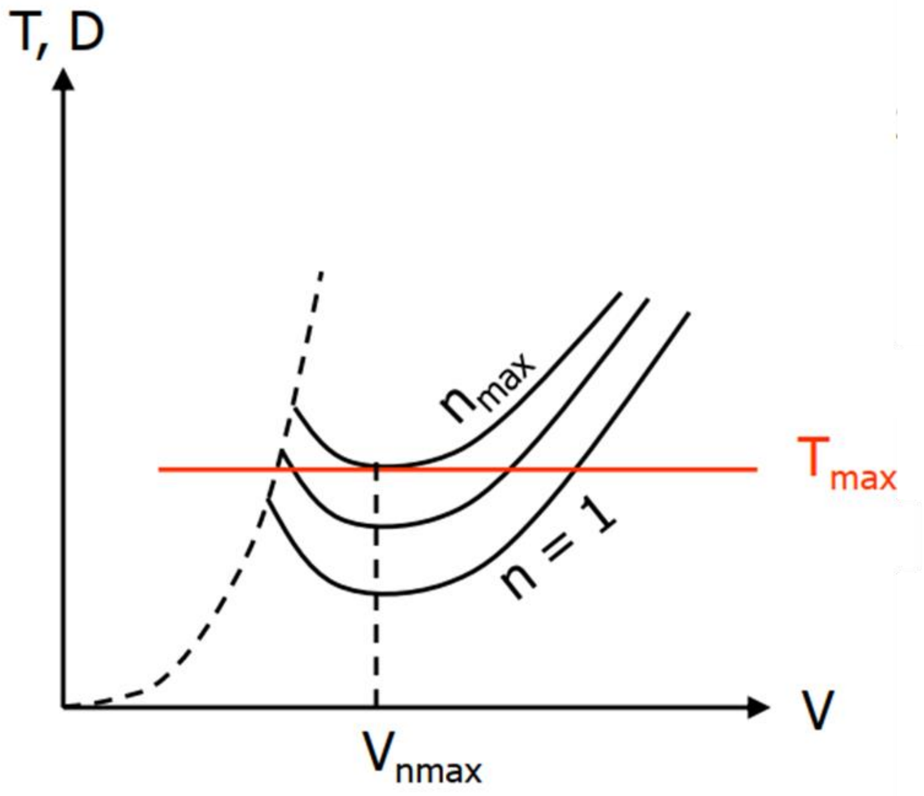
$$D = \frac{C_D}{C_L} nW \propto n$$

$$P_r = DV \propto n\sqrt{n}$$



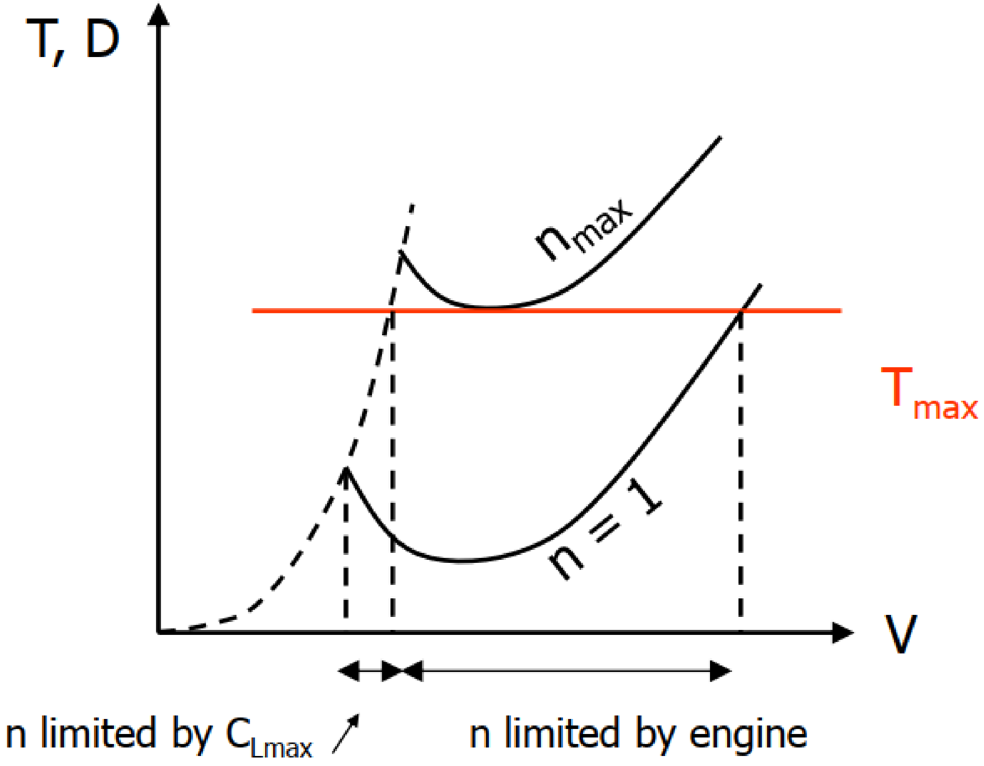
Maneuvering in the horizontal plane

- V_{min} increases when n increases
- V_{min} first aerodynamically limited then thrust limited
- V_{max} decreases when n increases
- At n_{max} , $V_{min} = V_{max}$



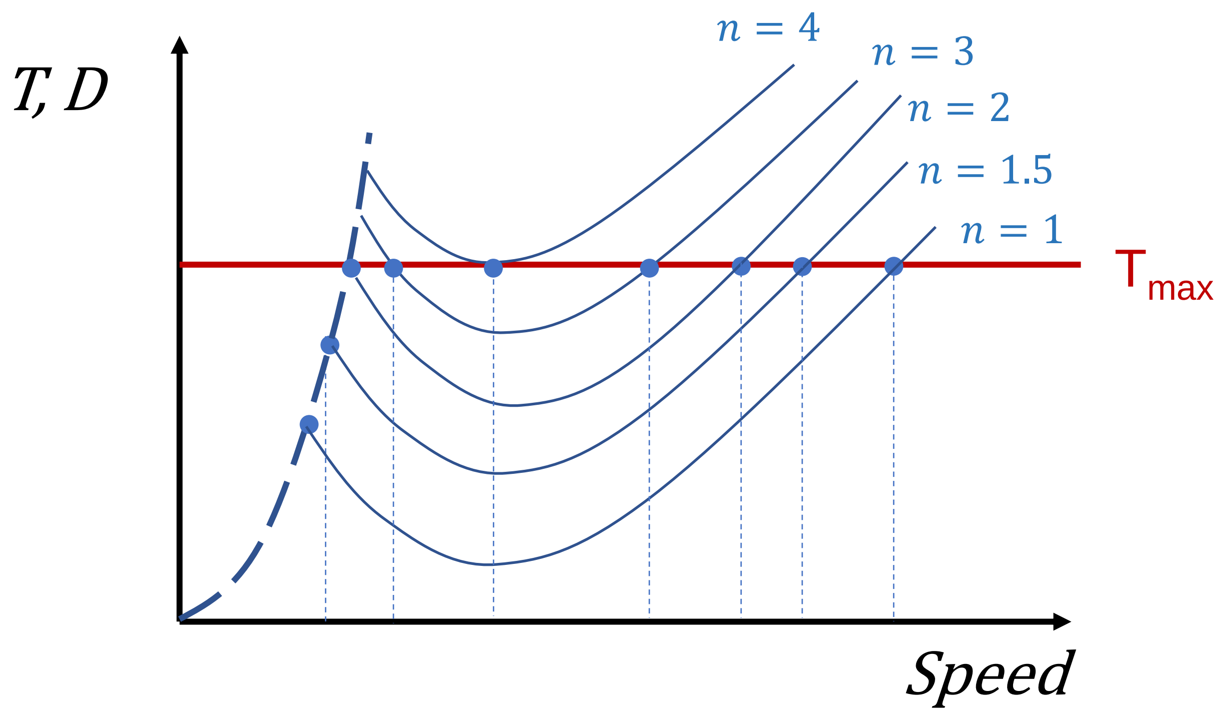
Maneuvering in the horizontal plane

Speed range



Maneuvering in the horizontal plane

Diagram for load factor

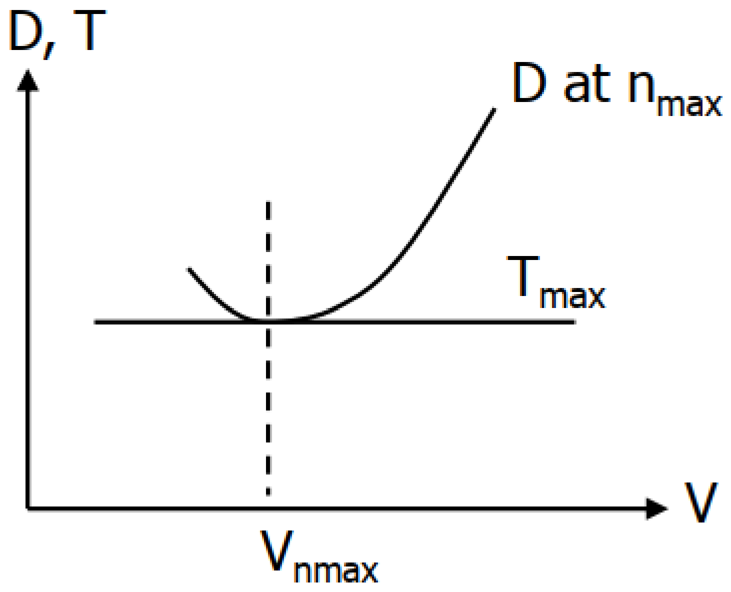


Textbook p. 103~104

(极限盘旋)

Example

Steepest turn ($n \Rightarrow n_{max}$)

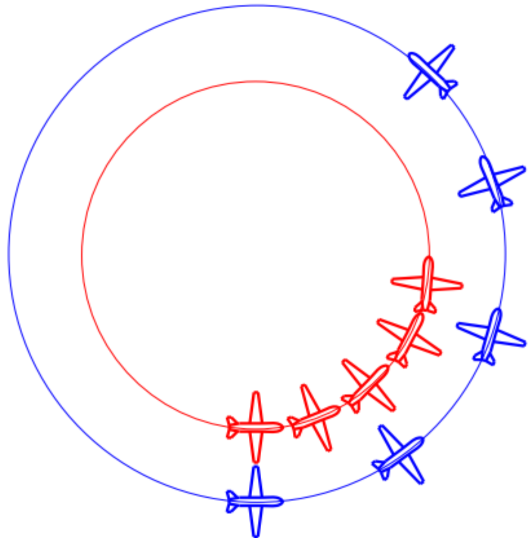


Maneuvering in the horizontal plane

Fastest turn

Time to complete a full circle

$$t_{2\pi} = \frac{2\pi R}{V} = \frac{2\pi V}{g\sqrt{n^2-1}}$$



Smaller Radius doesn't mean faster.

Maneuvering in the horizontal plane

Unsteady hovering (非定常盘旋)

$$\begin{cases} \dfrac{dV}{dt} = \dfrac{g}{W}(T_a - D) \Rightarrow dt = \dfrac{W}{g}\dfrac{dV}{T_a - D} \Rightarrow t = \displaystyle\int_{V_0}^{V} \dfrac{W}{g}\dfrac{dV}{T_a - D} \\ L\cos\mu = W \;\Rightarrow \cos\mu = \dfrac{W}{L} = \dfrac{1}{n_n} \Rightarrow \sin\mu = \dfrac{\sqrt{n_n^2 - 1}}{n_n} \\ \dfrac{d\chi}{dt} = \dfrac{g}{V}\dfrac{L}{W}\sin\mu \Rightarrow d\chi = \dfrac{g\sqrt{n_n^2 - 1}}{V}dt = \dfrac{W}{V}\dfrac{\sqrt{n_n^2 - 1}}{T - D}dV \end{cases}$$

Textbook p. 113~114

(非定常盘旋)

$(n_n \Leftrightarrow n)$

Maneuverability analysis

Energy maneuverability (能量机动性)

The ability of changing kinetic energy or potential energy.

$$E_s = \frac{1}{2g}V^2 + H$$

$$\frac{dE_s}{dt} = \frac{V}{g}\frac{dV}{dt} + \frac{dH}{dt} = \frac{(T-D)}{W}\mathrm{V} = \frac{\Delta TV}{W} = \mathrm{RC}^*$$

$$\mathrm{RC}^* = \text{Specific Excess Power (SEP)}$$

SEP reflects the aircraft's ability to change energy under the condition of (V, H, n)

Review - RC

Rate of climb for unsteady climb

Steady rate of climb

$$RC = \frac{dH}{dt} = \frac{\Delta TV}{W}\frac{1}{1+\frac{1}{2g}\frac{dV^2}{dH}} = RC^*\chi$$

$$\chi = \frac{1}{1+\frac{1}{2g}\frac{dV^2}{dH}} = \frac{1}{1+\frac{V}{g}\frac{dV}{dH}}$$

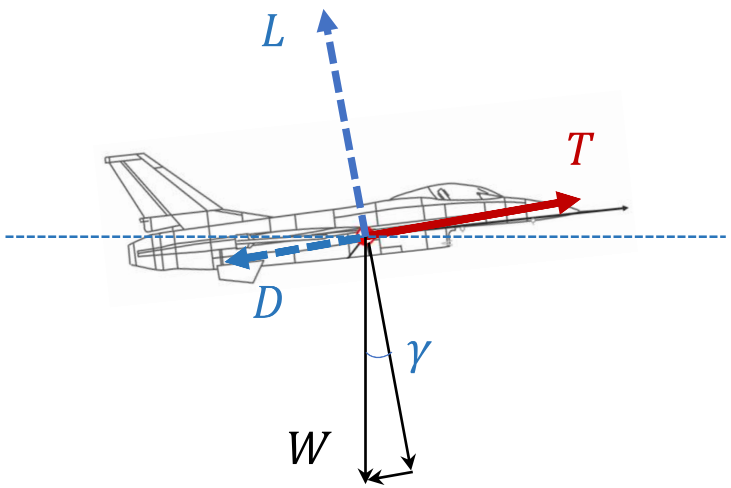
Correction factor

Review - RC

Rate of climb for steady climb

$$T = D + W \sin\gamma$$

$$\Rightarrow \sin\gamma = \frac{T - D}{W} = n_x$$



Maneuverability analysis

Energy maneuverability

1) V = constant $\Leftrightarrow n_x = \sin\gamma$

$$RC = RC^* \Rightarrow \frac{dE_s}{dt} = \frac{dH}{dt} = RC$$

2) H = constant $\Leftrightarrow \sin\gamma = 0$

$$\frac{dE_s}{ds} = \frac{dE_s}{dt} \Big/ \frac{ds}{dt} = \frac{T-D}{W} = n_x$$

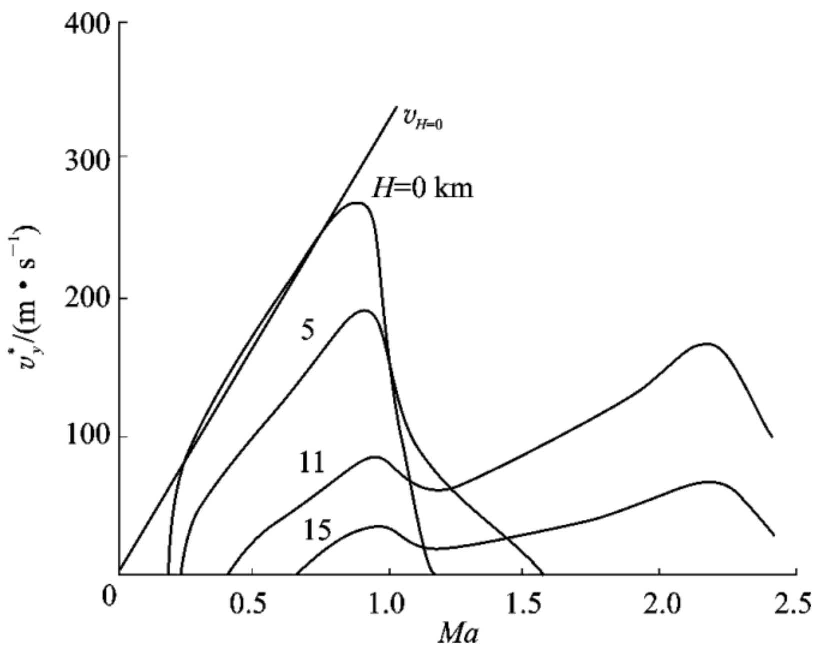
$$\frac{dV}{dt} = g(n_x - \sin\gamma)$$

$$\boxed{\begin{gathered}\frac{dE_s}{dt} = \frac{V}{g}\frac{dV}{dt} + \frac{dH}{dt} = \mathrm{RC}^* \\ RC = RC^* \Big/ \left(1 + \frac{V}{g}\frac{dV}{dH}\right)\end{gathered}}$$

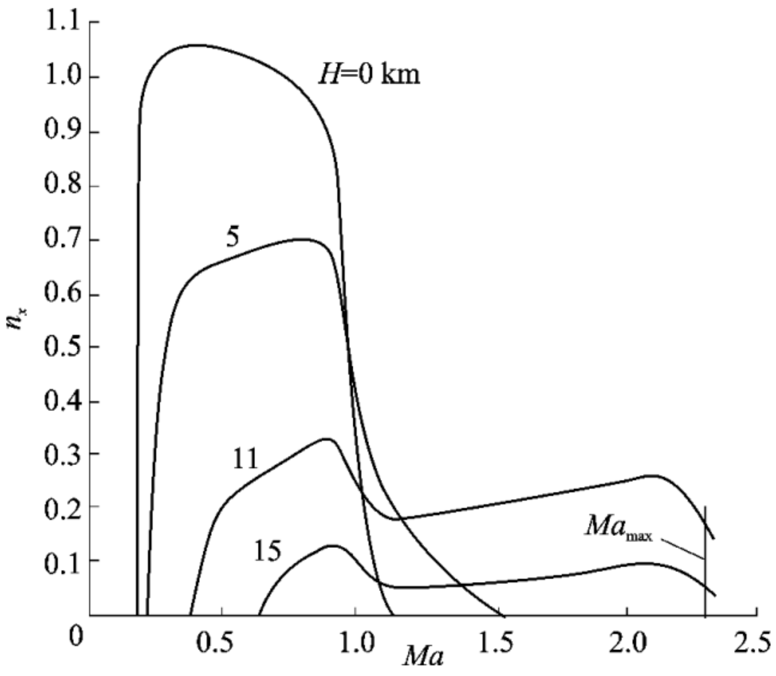
Maneuverability analysis

Energy maneuverability

Conclusions?



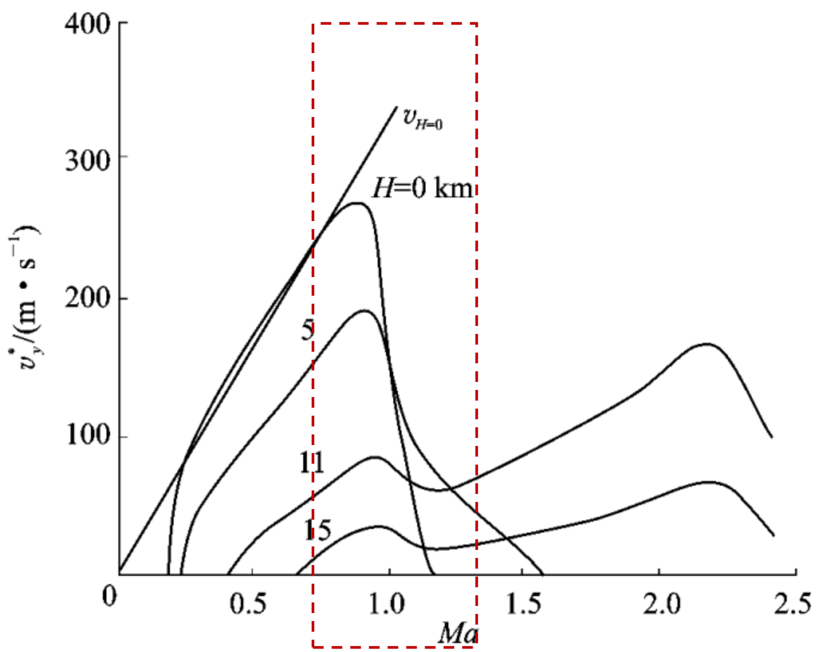
1) SEP or RC* as a function of Ma



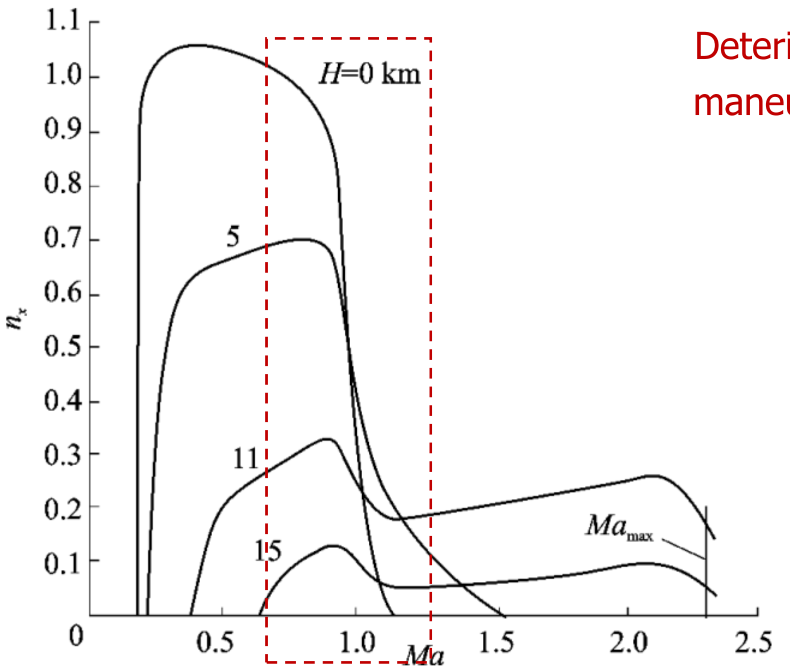
2) n_x as a function of Ma

Maneuverability analysis

Energy maneuverability



1) SEP or RC* diagram under n=1



2) n_x diagram under n=1

Deterioration of maneuverability

Maneuverability analysis

Steady or limit angular speed

Vertical plane $$\frac{d\gamma}{dt} = \frac{g}{V}(n - \cos\gamma)$$

Horizontal plane $$\frac{d\chi}{dt} = \frac{g}{V}\sqrt{n^2 - 1}$$

Equation of motion

$$\frac{\dot{\gamma}}{\dot{\chi}} = \frac{n - \cos\gamma}{\sqrt{n^2 - 1}}$$

ratio of vertical angular speed to horizontal speed

Maneuverability analysis

Steady or limit angular speed

$$\frac{\dot{\gamma}}{\dot{\chi}} = \frac{n - \cos\gamma}{\sqrt{n^2 - 1}}$$

$\gamma = 0^{\circ}, n > 1$

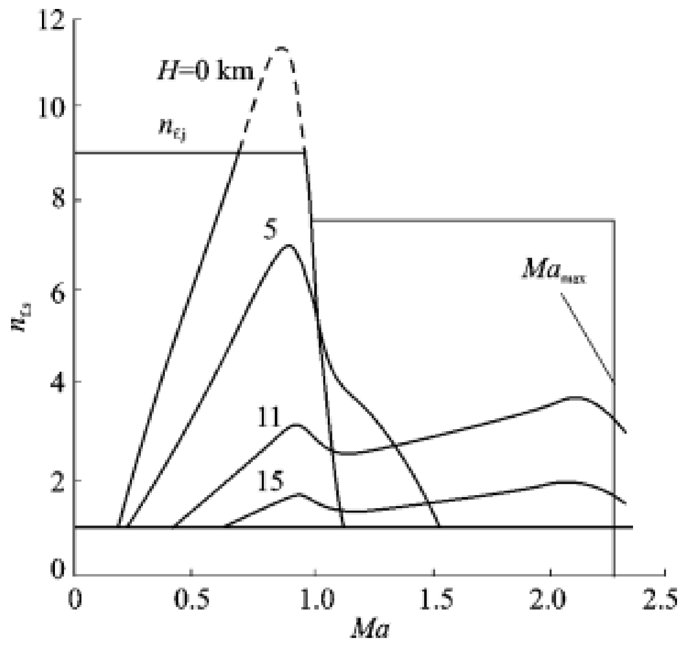
$$\frac{\dot{\gamma}}{\dot{\chi}} = \sqrt{\frac{n-1}{n+1}} < 1$$

$\gamma = 180^{\circ}, n > 1$

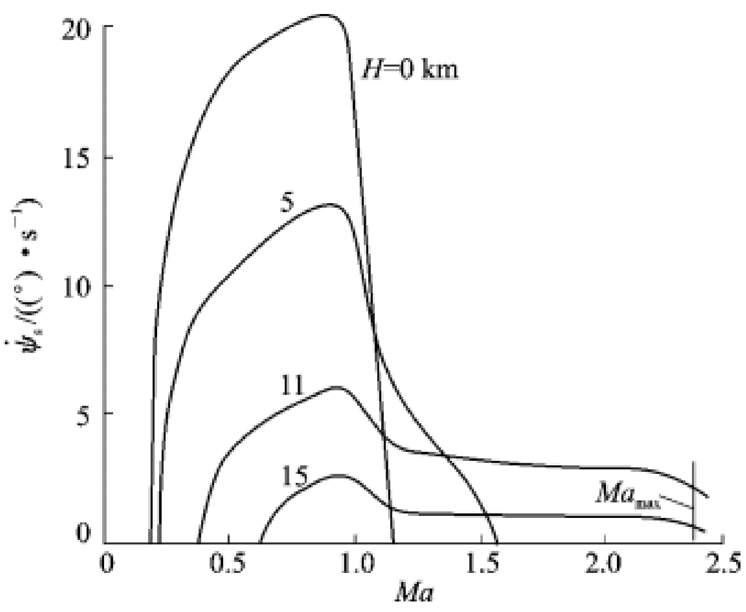
$$\frac{\dot{\gamma}}{\dot{\chi}} = \sqrt{\frac{n+1}{n-1}} > 1$$

Maneuverability analysis

Steady or limit angular speed



1) Load factor diagram for horizontal hovering at n_x=0



2) Angular speed for horizontal steady hovering

Maneuverability analysis

Steady or limit turning radius

Vertical plane $R_V = \frac{V}{\dot{\gamma}} = \frac{V^2}{g(n - \cos\gamma)}$

Horizontal plane $R_h = \frac{V}{\dot{\chi}} = \frac{V^2}{g\sqrt{n^2 - 1}}$

Equation of motion

$$\frac{R_V}{R_h} = \frac{\sqrt{n^2 - 1}}{n - \cos\gamma}$$

ratio of turning radius to horizontal turning radius

Maneuverability analysis

Steady or limit turning radius

$$\boxed{\frac{R_V}{R_h} = \frac{\sqrt{n^2 - 1}}{b - \cos\gamma}}$$

$\gamma = 0^\circ, n > 1$

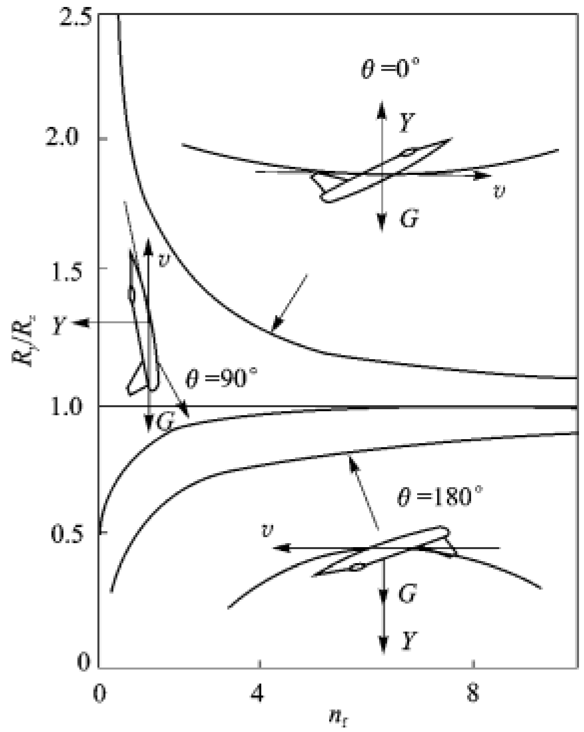
$$\frac{R_V}{R_h} = \sqrt{\frac{n+1}{n-1}} > 1$$

$\gamma = 180^\circ, n > 1$

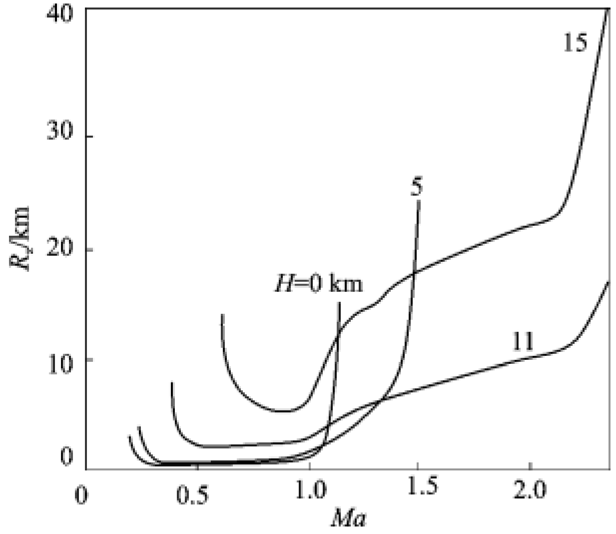
$$\frac{R_V}{R_h} = \sqrt{\frac{n-1}{n+1}} < 1$$

Maneuverability analysis

Steady or limit turning radius



1) R_V/R_h as a function of n



2) Horizontal turning radius R_h as a function of H and Ma

Maneuverability analysis

Minimum turn radius

$$R_h = \frac{ds}{d\chi} = \frac{V^2}{g\sqrt{n^2 - 1}}$$

Maneuverability analysis

Minimum turn radius

$$R_h = \frac{V^2}{g\sqrt{n^2 - 1}}$$

- When n is constant, R decreases when V decreases.
- When V is constant, R decreases as n increases.

Example

Ma = 2.5

$\mu = 30°$

$R_h \approx 121\ km$

Example

An aircraft is performing horizontal steady turn

$$V = 80\ m/s \qquad t_{2\pi} = 120\ s$$

- calculate the load factor n and bank angle μ

Example

An aircraft is flying at 7 km and has the following data

$T_{max} = 8.67\ [kN]$

$W = 6\ [kN]$

$C_{D0} = 0.021$

$\lambda_e = 7$

$S = 30\ [m^2]$

$\rho = 0.59\ [kg/m^3]$

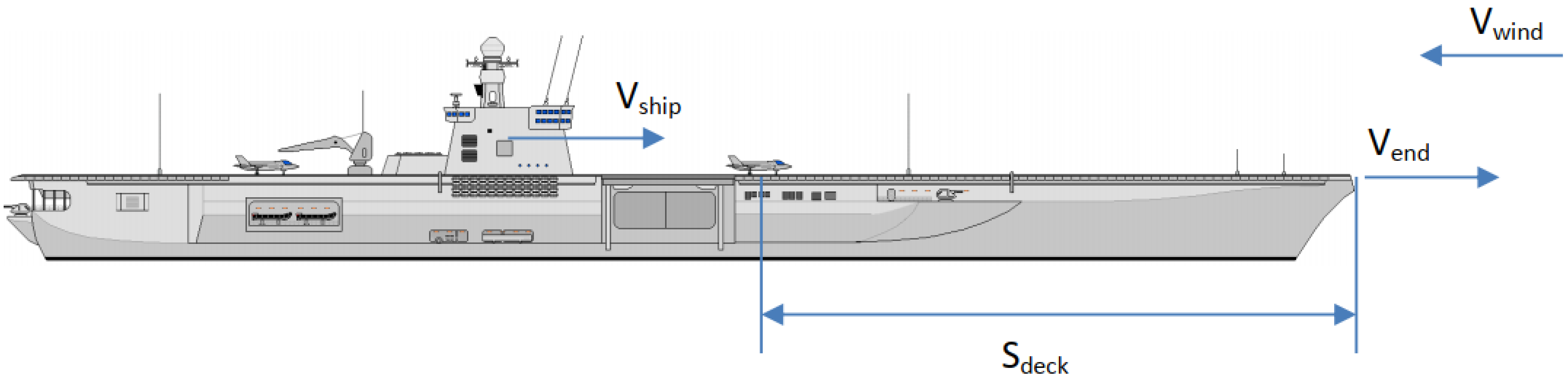
- calculate maximum load factor n_{max} and the corresponding airspeed

Homework

1. Analyze the impact of thrust to weight ratio T/W and wind load W/S on the take-off and landing performance of the aircraft. *(Textbook, Q.2-4, p.90)*
2. An aircraft has the weight W = 85000 N, wing area S = 32 m², C_{Lmax} = 1.5, $C_D = 0.04 + 0.0833\ C_L^2$ and $n_{n,max}$ = 6. What the thrust required T_R for completing 90 degree turn at horizontal plane within 6 s? *(Textbook, Q.3-6, p.135)*
3. An aircraft has the weight of W = 58.8 kN, wing area S = 28 m². It performs horizontal straight line flight with V= 250 m/s at the altitude of H = 6 km. Assume the aircraft starts to accelerate with the acceleration of dV/dt = 5 m/s². What is the corresponding Thrust available T_a ? *(Textbook, Q.3-3, p.134)*
   (The drag polar is given as $C_D = 0.0144 + 0.08C_L^2$)

Homework

4. Aircraft carriers make use of catapult systems to launch aircraft from the limited distance available on the deck. During the launch, maximum thrust is also applied by the aircraft. In general, the ship will have a forward speed into the direction of the wind (as indicated in the picture) to improve the take-off performance.



Homework

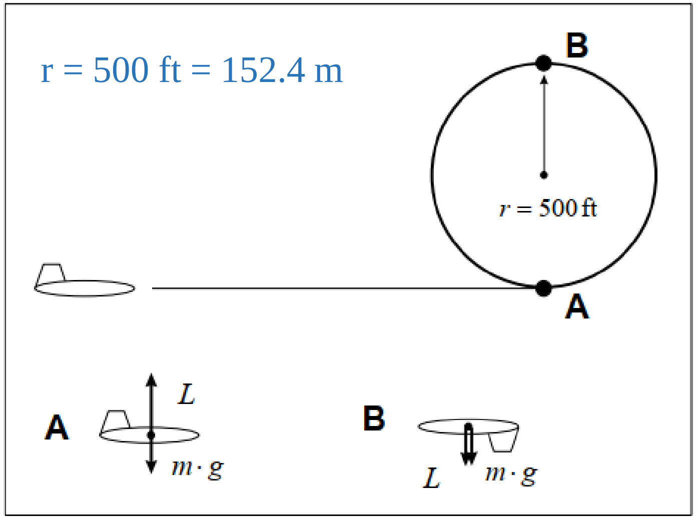
a) Draw a clear free body diagram (FBD) and kinetic diagram (KD) in which all the relevant forces, accelerations, angles and velocities are indicated.

b) Derive the equations of motion for the aircraft during the acceleration over the ship deck

c) Derive an expression for the ground run distance s_{deck} in terms of a mean acceleration and the speed at the moment the aircraft leaves the deck (V_{end}). Clearly indicate if the velocity in the equation is expressed relative to the air or relative to the ship.

Homework

5. An aerobatic airplane flies a perfect circular looping. The pilot reads an altitude of 5000 ft at the highest point of the looping and an altitude of 4000 ft a the lowest point of the looping. The looping is flown at such a speed that the lowest load factor in the looping is n = 0. Assume that aircraft speed is nearly constant in the looping.



a) During which part of the looping does the airplane experience the highest load factor?

b) What is the aircraft's speed in the looping?

c) Calculate the highest load factor n during the looping.