3-1

解：$\dot{\vec{E}} = \hat{e}_x E_0 e^{-j\beta z}$

满足波动方程 $\nabla^2 \dot{E}_x + \beta^2 \dot{E}_x = 0$

~~$\dot{\vec{E}} = \hat{e}_z E_0 e^{-j\beta z}$~~

且 $\nabla \cdot \hat{e}_x E_x = 0$ ∴可能存在.

$\dot{\vec{E}} = \hat{e}_z E_0 e^{-j\beta z}$

满足波动方程 $\nabla^2 \dot{E}_z + \beta^2 \dot{E}_z = 0$

但不满足 $\nabla \cdot \hat{e}_z \dot{E}_z = 0$ ∴不可能存在.

3-2

解：① 同为电压波节点、终端短路.

$\dot{U}(z) = j2\dot{U}_{i2} \sin\beta z$.

∵同为波节点

∴ $\sin\beta l = 0$ $\beta l = k\lambda$ ($k = 1, 2, 3 \cdots$)

$\beta = \frac{k\lambda}{l}$

$f = \frac{v}{2\lambda}\beta = \frac{k}{2l}v$

② 同为电压波腹点，终端开路.

$\dot{U}(z) = 2\dot{U}_{i2}\cos\beta z$.

$\cos\beta l = 1$

~~$\cos\beta l = 0$. $\beta l = k\lambda$~~

$\beta l = k\lambda$. $\beta = \frac{k\lambda}{l}$

$f = \frac{v}{2\lambda}\beta = \frac{k}{2l}v$ ($k = 1, 2, \cdots$)

③ 终端波节点，始端波腹点.

$\dot{U}(z) = 2j\dot{U}_{i2}\sin\beta z$.

终端波腹点，始端波节点.

$\dot{U}(z) = 2\dot{U}_{i2}\cos\beta z$.

$\beta l = \frac{\lambda}{2} + k\lambda$ ($k = 0, 1, 2, \cdots$)

$\beta = \frac{\frac{\lambda}{2} + k\lambda}{l}$

3-5

证明：由理想介质电磁场基本方程.

$\nabla \times \dot{\vec{E}} = -j\omega\mu\dot{\vec{H}}$

$\nabla \cdot \dot{\vec{E}} = 0$.

代入 $\dot{E}_z = \dot{H}_z = 0$ 到电场旋度方程

得 $\frac{\partial \dot{E}_x}{\partial y} = \frac{\partial \dot{E}_y}{\partial x}$ ①

$\frac{\partial \dot{E}_y}{\partial z} = j\omega\mu \dot{H}_x$ ②

$\frac{\partial \dot{E}_x}{\partial z} = -j\omega\mu \dot{H}_y$ ③

同理得

$\frac{\partial \dot{H}_x}{\partial y} = \frac{\partial \dot{H}_y}{\partial x}$ ④

$\frac{\partial \dot{H}_y}{\partial z} = -j\omega\mu E_x$ ⑤

$\frac{\partial \dot{H}_x}{\partial z} = j\omega\varepsilon E_y$ ⑥

$\dot{E}_x$ 与 $\dot{E}_y$ 可写为 $\dot{E}_x = E_x(x,y)e^{-\gamma z}$

$\dot{E}_y = E_y(x,y)e^{-\gamma z}$

代入 ①-⑥ 得 $\gamma^2 = -\omega^2\mu\varepsilon$

将 $\dot{H}_z = \dot{E}_z = 0$ 代入电场散度方程 可得

$\frac{\partial \dot{E}_x}{\partial x} + \frac{\partial \dot{E}_y}{\partial y} = 0$

对 x 求偏导

$\frac{\partial^2 \dot{E}_x}{\partial x^2} + \frac{\partial^2 \dot{E}_y}{\partial x \partial y} = 0$

代入①式

$\frac{\partial^2 \dot{E}_x}{\partial x^2} + \frac{\partial^2 \dot{E}_y}{\partial y^2} = 0$ 得 $\nabla_T^2 E_x = 0$

同理 $\nabla_T^2 E_y = 0$

分别代入②③

$\nabla_T^2 E_x(x,y) = 0$，$\nabla_T^2 E_y(x,y) = 0$

同理

$\nabla_T^2 H_x(x,y) = 0$ $\nabla_T^2 H_y(x,y) = 0$

~~3-5~~
3-3

解：
$$\begin{cases}\nabla\times\vec{E} = -j\omega\mu\vec{H} \\ \nabla\times\vec{H} = j\omega\varepsilon\vec{E} \\ \nabla\cdot\vec{E} = 0 \\ \nabla\cdot\vec{H} = 0\end{cases}$$

纵横分离
$$\begin{cases}\vec{E} = \hat{i}_z\dot{E}_z + \vec{E}_T \\ \vec{H} = \hat{i}_z\dot{H}_z + \vec{H}_T \\ \nabla = \nabla_z + \nabla_T\end{cases}$$

$\vec{E}$ 与 $\vec{H}$ 可见为
$$\begin{cases}\vec{E}(u,v,z) = \vec{E}(u,v)e^{-j\beta z} \\ \vec{H}(u,v,z) = \vec{H}(u,v)e^{-j\beta z}\end{cases}$$

可导出
$$\begin{cases}\vec{H}_T = \frac{1}{k_c^2}(-j\beta\nabla_T\dot{H}_z - j\omega\varepsilon\hat{i}_z\times\nabla_T\dot{E}_z) \\ \vec{E}_T = \frac{1}{k_c^2}(-j\beta\nabla_T\dot{E}_z + j\omega\mu\hat{i}_z\times\nabla_T\dot{H}_z)\end{cases}$$

TE波：
$$\begin{cases}\vec{H}_T = -\frac{j\beta}{k_c^2}\nabla_T\dot{H}_z \\ \vec{E}_T = -\eta_{TE}\hat{i}_z\times\vec{H}_T \\ \eta_{TE} = \frac{k}{\beta}\eta\end{cases}$$

TM波：
$$\begin{cases}\vec{E}_T = -\frac{j\beta}{k_c^2}\nabla_T\dot{E}_z \\ \vec{H}_T = \frac{1}{\eta_{TM}}\hat{i}_z\times\vec{E}_T\end{cases}$$

3-6

解：① ∵ $\dot{E}_z = 0$

∴ 为TE波.

$$H_x = \frac{-j\beta}{k_c^2}\frac{\partial H_z}{\partial x} = -\frac{j\beta}{k_c}(-A\sin k_c x + B\cos k_c x)$$

$$H_y = -\frac{j\beta}{k_c^2}\frac{\partial H_z}{\partial y} = 0$$

$$E_x = \eta_{TE}H_y = 0$$

$$E_y = -\eta_{TE}H_x = -\eta_{TE}\frac{j\beta}{k_c}(-A\sin k_c x + B\cos k_c x)$$

② ∵金属板为理想导体

∴ ~~切~~ 切向电场为0

$E_y|_{x=0} = 0$　$E_y|_{x=a} = 0$

$E_y = -\eta_{TE}\frac{j\beta}{k_c}\cdot B = 0$　∴ $B = 0$

~~$E_y|_{x=a} = 0$~~

$E_y = \eta_{TE}\frac{j\beta}{k_c}A\sin k_c a = 0$

∴ $k_c = \frac{n\pi}{a}$. $n = 1, 2, 3\cdots$

③ 截止波长 $\lambda_c = \frac{2\pi}{k_c} = \frac{2\pi}{n\pi}a = \frac{2a}{n}$ $(1,2,3\cdots)$