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3.6

解：(1) $R_E(\tau)=E\{[Y(t)-X(t)][Y(t-\tau)-X(t-\tau)]\}$
$=R_Y(\tau)+R_X(\tau)-R_{XY}(\tau)-R_{YX}(\tau)$

$S_E(\omega)=\mathcal{F}[R_E(\tau)]$
$=S_Y(\omega)+S_X(\omega)-S_{XY}(\omega)-S_{YX}(\omega)$
$=(|H(j\omega)|^2+1)S_X(\omega)-H(j\omega)S_X(\omega)-H^*(j\omega)S_X(\omega)$

(2) $Y(j\omega)=X(j\omega)\dfrac{\frac{1}{j\omega C}}{\frac{1}{j\omega C}+R}$

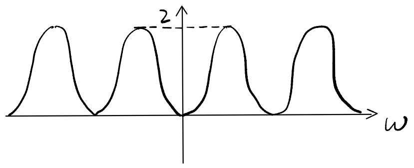
$H(j\omega)=\dfrac{\alpha}{\alpha+j\omega}$ $\alpha=\dfrac{1}{RC}$

$S_X(\omega)=\dfrac{\sin^2(\frac{\omega}{2})}{(\frac{\omega}{2})^2}$

$S_E(\omega)=|H(j\omega)-1|^2S_X(\omega)=\dfrac{4\sin^2(\frac{\omega}{2})}{\alpha^2+\omega^2}$

3.7

证明：(1) $Y(t)=X(t)-X(t-\tau)$
$R_Y(\tau)=2R_X(\tau)-R_X(\tau-T)-R_X(\tau+T)$
$G_Y(\omega)=2G_X(\omega)-G_X(\omega)(e^{j\omega T}+e^{-j\omega T})$
$|H(j\omega)|^2=2(1-\cos\omega T)$
$\omega=2\pi f$. 得证！



(2) 当 $f\ll\frac{1}{T}$ 时，$\cos x=1-\frac{x^2}{2}+o(x^4)$
$S_Y(f)=2(1-(1-\frac{(2\pi fT)^2}{2}))S_X(f)$
$=4\pi^2f^2T^2S_X(f)$. 得证！
当 $f\ll\frac{1}{T}$ 时，$\frac{S_Y(f)}{S_X(f)}\propto f^2$

3.9

(1) 证明：设 $X(t)$ 为白噪声过程，$R_X(\tau)=\delta(\tau)$
$R_{YX}(\tau)=E[Y(t)X(t-\tau)]$
$=\frac{1}{T}\int_{t-T}^{t}E\{X(\alpha)X(t-\tau)\}d\alpha$
$=\frac{1}{T}\int_{t-T}^{t}\delta(\alpha-t+T)d\alpha$
令 $u=\alpha-t+\tau$
$=\frac{1}{T}\int_{t-\tau}^{\tau}\delta(u)du$

$=\begin{cases}\frac{1}{T} & 0\le\tau\le T\\ 0 & 其他\end{cases}$

又因 $X(t)$ 为白噪声，故 $h(\tau)=R_{YX}(\tau)=\begin{cases}\frac{1}{T} & 0\le\tau\le T\\ 0 & 其他\end{cases}$

(2) 解：$h(t)=\frac{1}{T}[u(t)-u(t-T)]$
$H(j\omega)=\int_{-\infty}^{+\infty}h(t)e^{-j\omega t}dt$
$=\frac{1}{T}\int_0^T e^{-j\omega t}dt$
$=\dfrac{\sin(\frac{\omega T}{2})}{\frac{\omega T}{2}}e^{-j\frac{\omega T}{2}}$
$\therefore S_Y(\omega)=|H(j\omega)|^2S_X(\omega)$
$=\dfrac{\sin^2(\frac{\omega T}{2})}{(\frac{\omega T}{2})^2}S_X(\omega)$

3.16

解：$R_Y(t_1,t_2)=R_X(t_1,t_2)*[h(t_1)*h(t_2)]$

$R_X(t_1,t_2)=E[X(t_1)X(t_2)]$

3.17

解：$H(j\omega)=\frac{j\omega-\alpha}{j\omega+\beta}=1-(\alpha+\beta)\frac{1}{j\omega+\beta}$

$h(\tau)=\delta(\tau)-(\alpha+\beta)e^{-\beta\tau}u(\tau)$

$R_{XY}(\tau)=R_X(\tau)*h(-\tau)=\int_{-\infty}^{+\infty}R_X(\tau-t)h(-t)dt$

$=\int_{-\infty}^{+\infty}e^{-\alpha|\tau-t|}[\delta(t)-(\alpha+\beta)e^{\beta t}u(-t)]dt$

$=e^{-\alpha|\tau|}-(\alpha+\beta)\int_{-\infty}^{0}e^{\beta t}e^{-\alpha|\tau-t|}dt$

$=\begin{cases}(1+\frac{\alpha+\beta}{\alpha+\beta})e^{\alpha\tau}, & \tau<0\\ e^{-\alpha\tau}+\frac{\alpha+\beta}{\alpha-\beta}(e^{-\beta\tau}-e^{-\alpha\tau})+\frac{\alpha+\beta}{\alpha+\beta}e^{-\beta\tau}, & \tau\geq0\end{cases}$

3.21

解：(1) $R_Y(\tau)=E[Y(t)Y(t-\tau)]$

$=E[X(t-a)X(t-a-\tau)]$

$=R_X(\tau)$

$R_{XY}(\tau)=E[X(t)Y(t-\tau)]$

$=E[X(t)X(t-a-\tau)]$

$=R_X(a+\tau)$

(2) 合理

3.25

解：(1) $R_{Y_1Y_2}(\tau)=\int_{-\infty}^{+\infty}R_{Y_1X}(\tau+u)h_2(u)du$

$R_{Y_1X}(\tau)=\int_{-\infty}^{+\infty}R_X(\tau-\alpha)h_1(\alpha)d\alpha$

$\therefore R_{Y_1Y_2}(\tau)=R_X(\tau)*h_1(\tau)*h_2(\tau)$

$S_{Y_1Y_2}(\omega)=S_X(\omega)*H_1(j\omega)*H_2^*(j\omega)$

(2) 解：当 $R_{Y_1Y_2}(\tau)=0$ 时，Y_1, Y_2 不相关

即 $h_1(\tau)*h_2(\tau)=0$ 或 $H_1(j\omega)\cdot H_2^*(j\omega)=0$、