System Dynamics and Vibrations

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Chapter 3: Single degree-of-freedom systems
Exercises lot 3

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Exercise 6

In the vibration testing of a structure, an impact hammer with a load cell to measure the impact force is used to cause excitation, as shown in the figure.

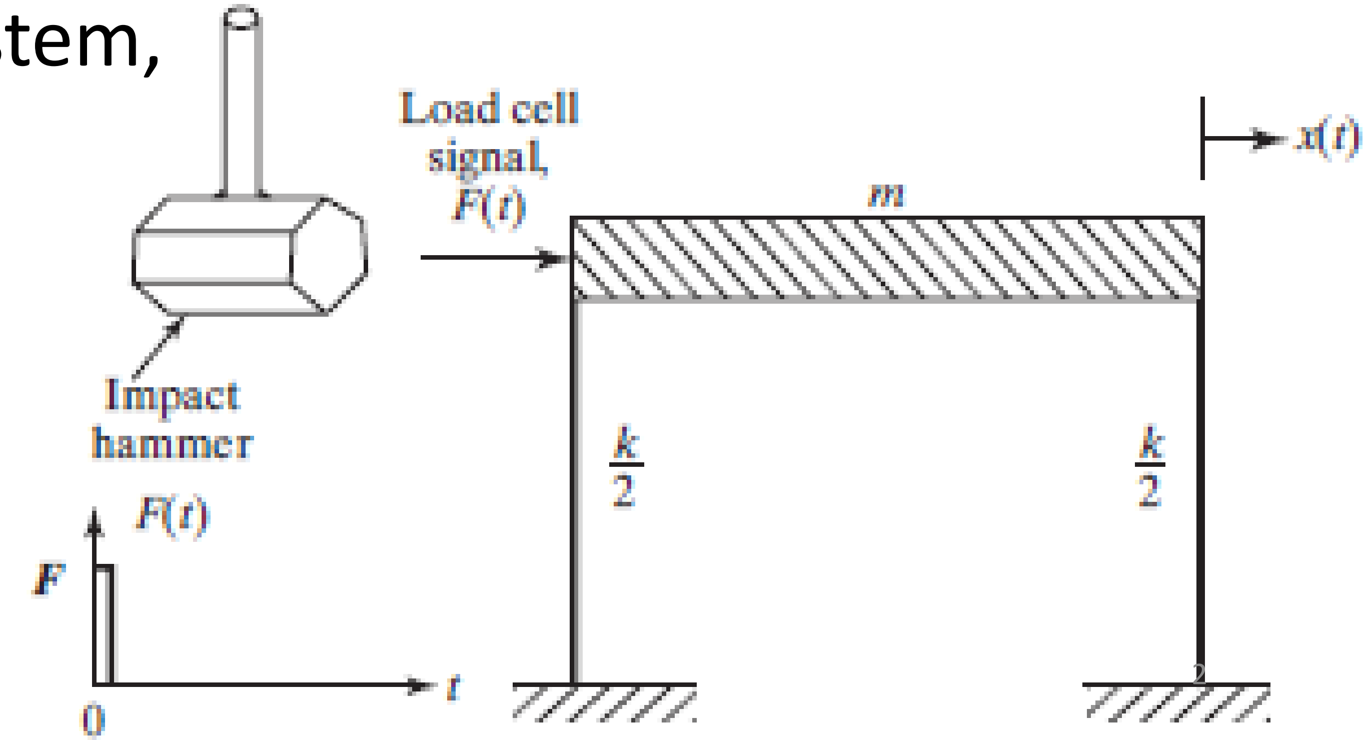
Find the response of the system, assuming:

m = 5 kg

k = 2000 N/m

c = 10 Ns/m

F = 20 Ns



Exercise 6

- **Impulsive response of a SDOF**: $x(t) = \hat{F}\dfrac{1}{m\omega_d}e^{-\varsigma\omega_n t}\sin\omega_d t$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{2000}{5}} = 20\text{rad/s}$$

$$\varsigma = \frac{c}{2m\omega_n} = 0.05$$

$$\omega_d = \sqrt{1-\varsigma^2}\,\omega_n = 19.975\text{rad/s}$$

$$\hat{F} = 20\text{Ns}$$

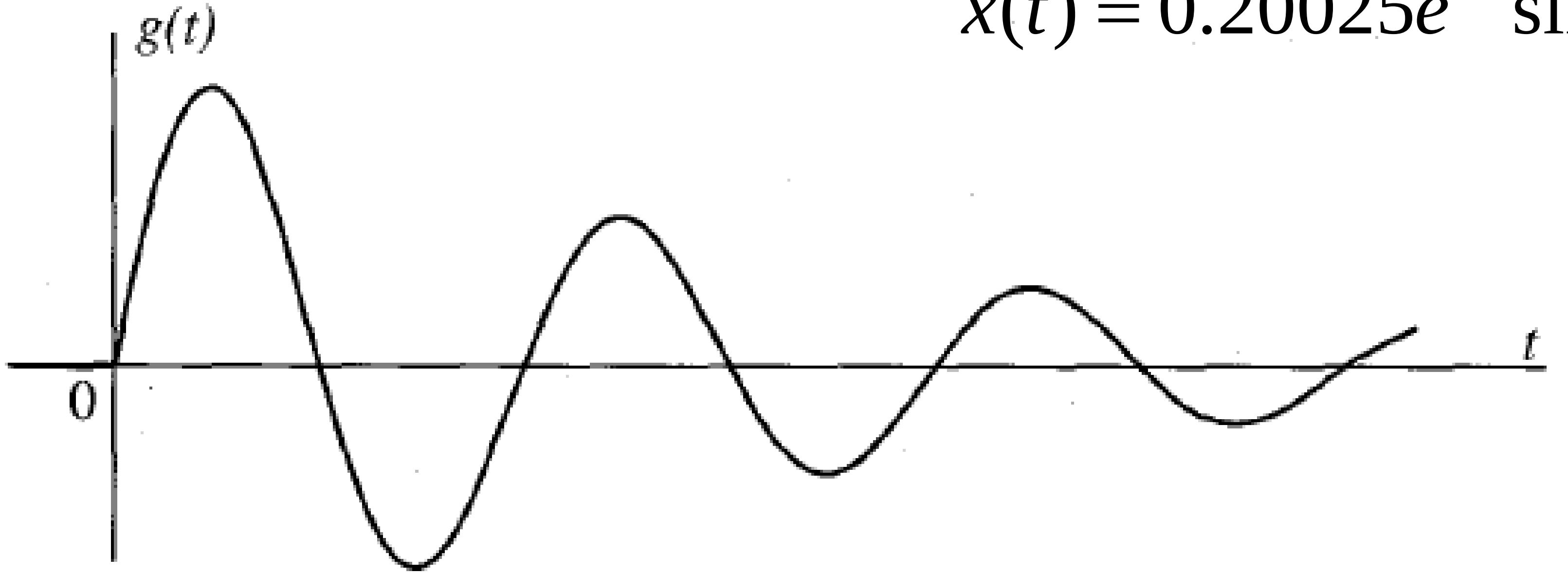
$$x(t) = 0.20025e^{-t}\sin 19.975t$$

The unit impulse. Impulse response.

- **Impulsive response of a SDOF**:

    ➔ underdamped system $\zeta < 1$

$$x(t) = 0.20025e^{-t}\sin 19.975t$$

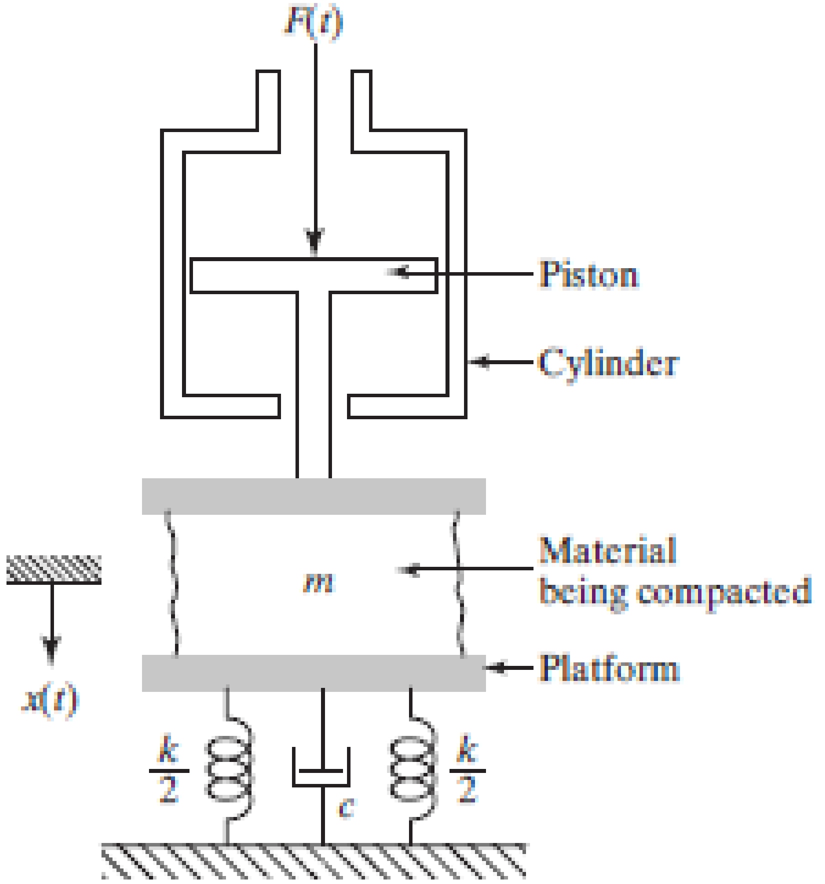


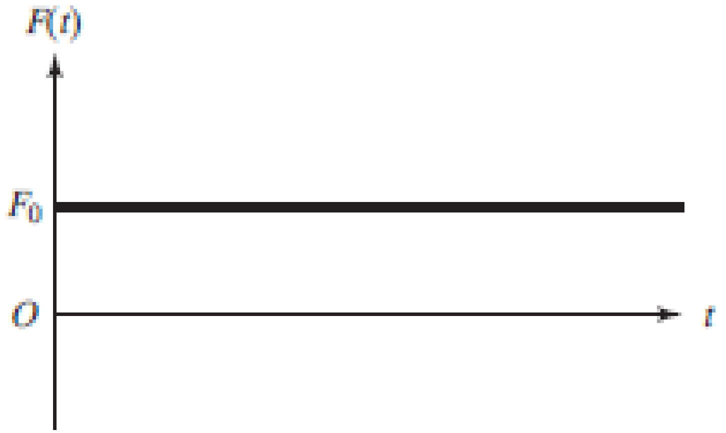
Exercise 7

A compacting machine, modeled as a single-degree-of-freedom system, is shown in the figure. The force acting on the mass *m* due to a sudden application of the pressure can be idealized as a step force. Determine the response of the system for:

- Damped case
- Undamped case

(m includes the masses of the piston, the platform, and the material being compacted)





Exercise 7

- **<u>Step response of a SDOF</u>**:

$$\mathrm{s}(t) = \int_{-\infty}^{t} g(\tau)\,d\tau$$

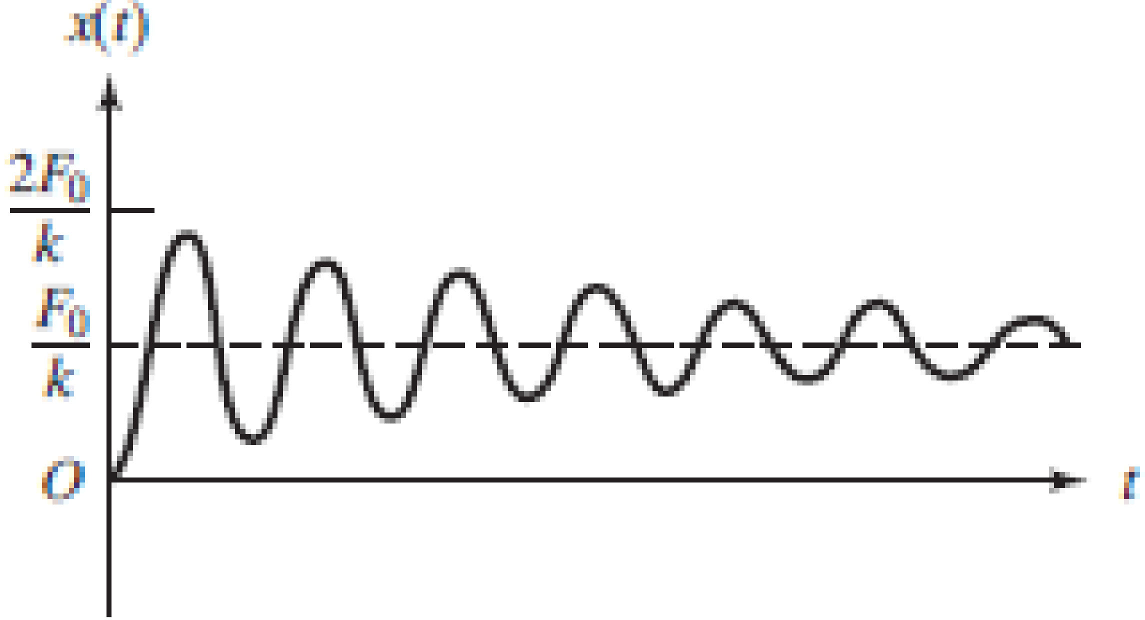
$$g(t) = \frac{F_0}{m\omega_d} e^{-\varsigma\omega_n t} \sin \omega_d t u(t)$$

$$\Rightarrow \quad \mathrm{s}(t) = \frac{F_0}{k}\left[1 - e^{-\varsigma\omega_n t}\left(\cos \omega_d t + \frac{\xi\omega_n}{\omega_d} \sin \omega_d t\right)\right] u(t)$$

Exercise 7

- **<u>Step response of a SDOF</u>**:

$$s(t) = \frac{F_0}{k}\left[1 - e^{-\varsigma\omega_n t}\left(\cos\omega_d t + \frac{\xi\omega_n}{\omega_d}\sin\omega_d t\right)\right]u(t)$$

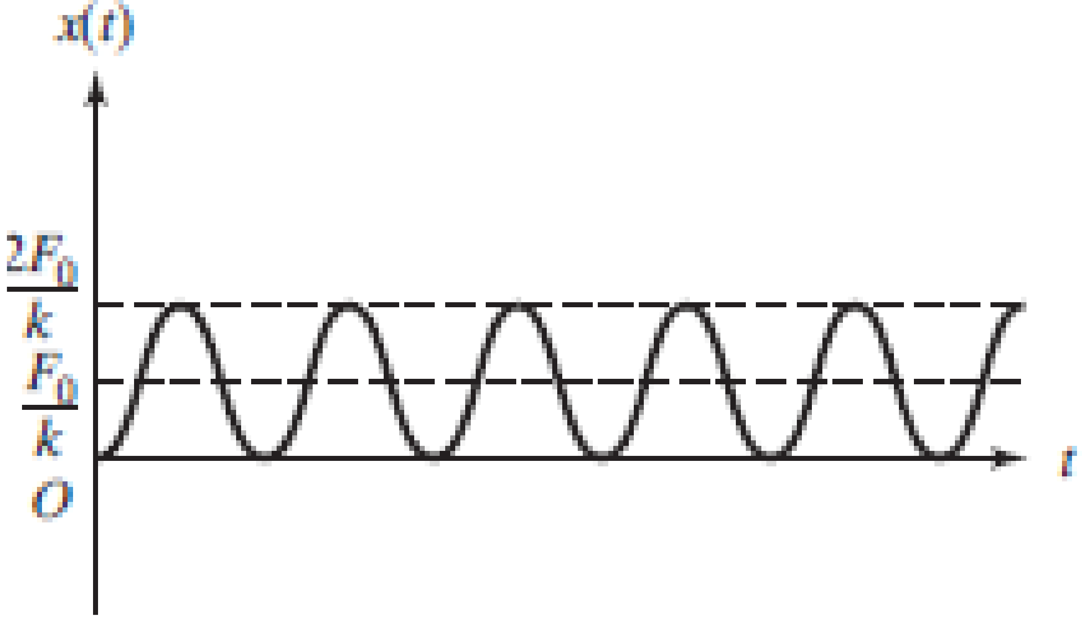


Exercise 7

- **Step response of a SDOF**: undamped case

$$\mathrm{s}(t) = \frac{F_0}{k}\left[1 - e^{-\varsigma\omega_n t}\left(\cos\omega_d t + \frac{\xi\omega_n}{\omega_d}\sin\omega_d t\right)\right]u(t) =$$

$$\frac{F_0}{k}\left(1 - \cos\omega_n t\right)u(t)$$

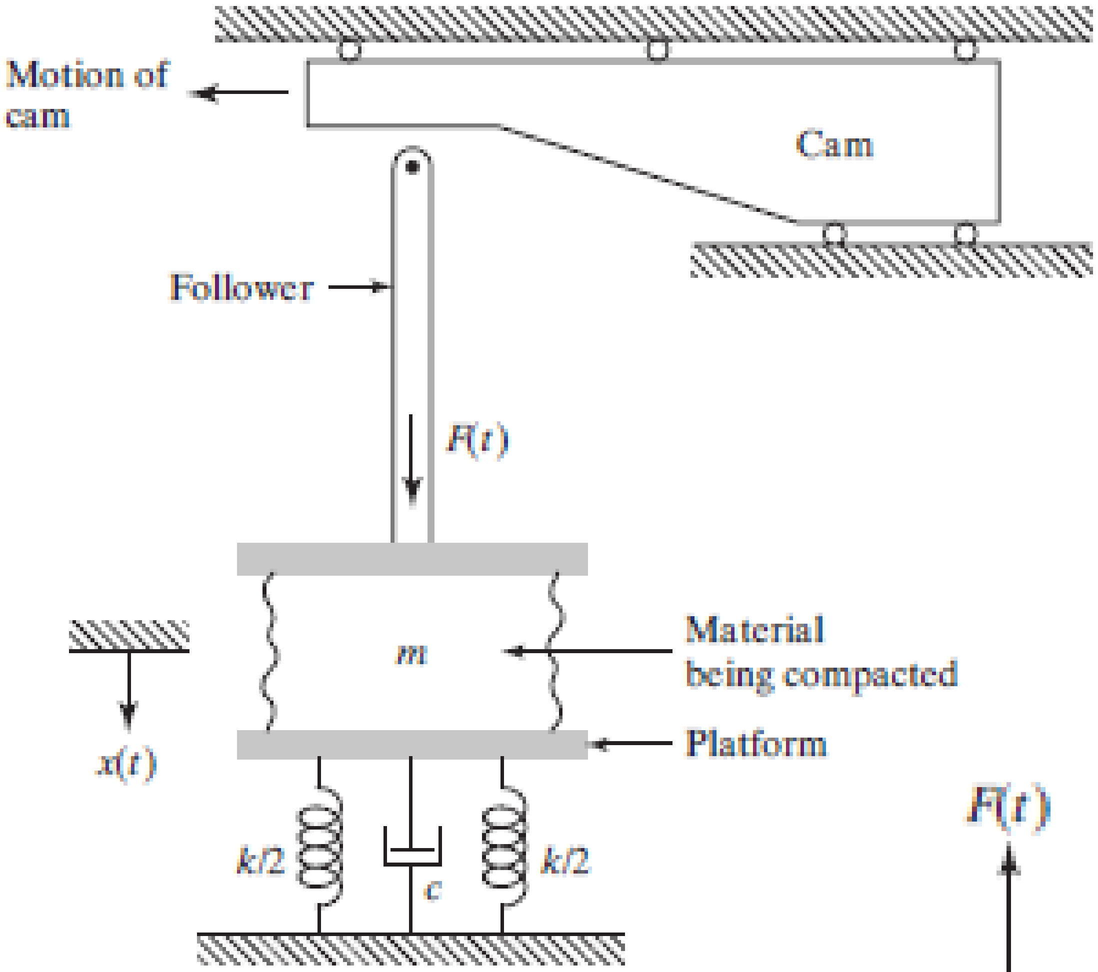


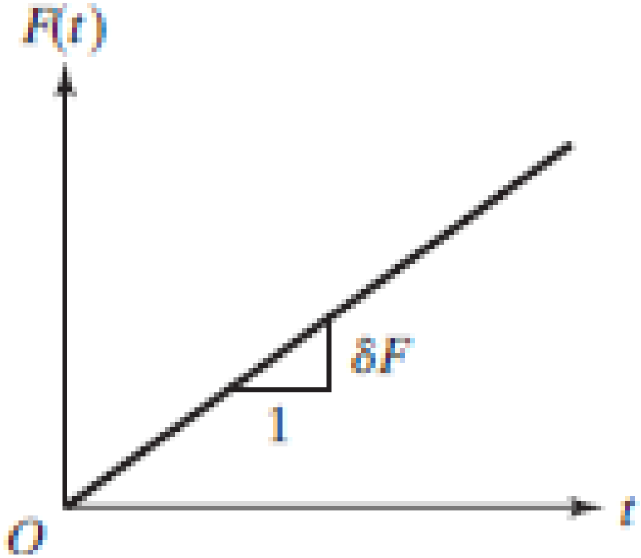
Exercise 8

Determine the response of the compacting machine shown in the figure when a linearly varying force is applied due to the motion of the cam.

Assume c = 0

(undamped case)





The unit ramp function. Ramp response.

- The ramp response: $r(t) = \int_{-\infty}^{t} s(\tau)d\tau$

$$s(t) = \frac{F_0}{k}\left[1 - e^{-\varsigma\omega_n t}\left(\cos\omega_d t + \frac{\xi\omega_n}{\omega_d}\sin\omega_d t\right)\right]u(t)$$

- undamped case:

$$s(t) = \frac{\delta F}{k}\left[1 - \left(\cos\omega_d t\right)\right]u(t)$$

$$r(t) = \int_{-\infty}^{t} s(\tau)d\tau = \frac{\delta F}{k\omega_n}\left(\omega_n t - \sin\omega_n t\right)u(t)$$

The unit ramp function. Ramp response.

$$r(t) = \frac{\delta F}{k\omega_n}\left(\omega_n t - \sin\omega_n t\right)u(t)$$