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飞行力学
Flight Mechanics

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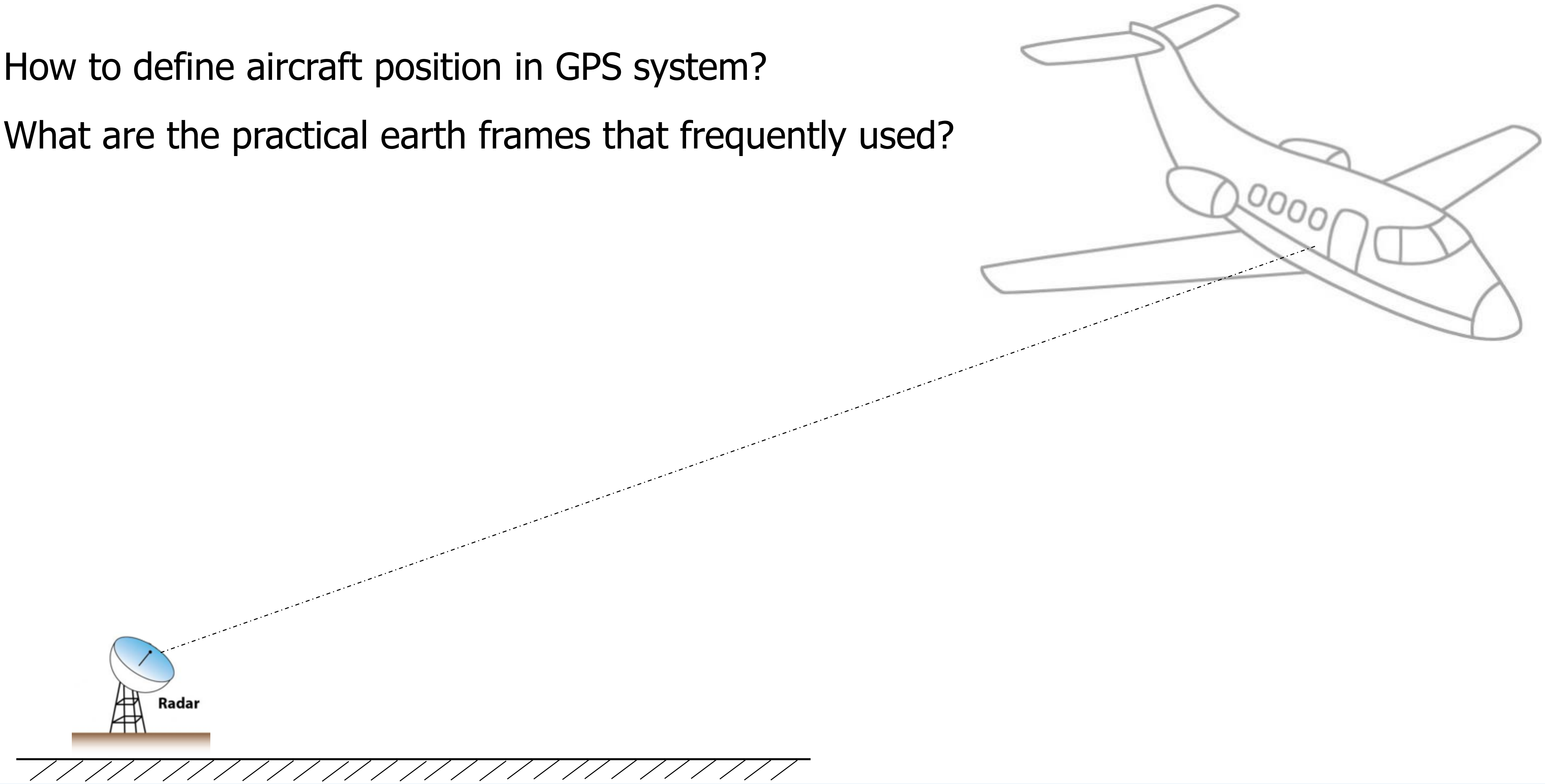
Contents

Questions

How to define aircraft position in GPS system?

What are the practical earth frames that frequently used?



The ECEF Frame

Earth **C**entered **E**arth **F**ixed (地球中心坐标系)

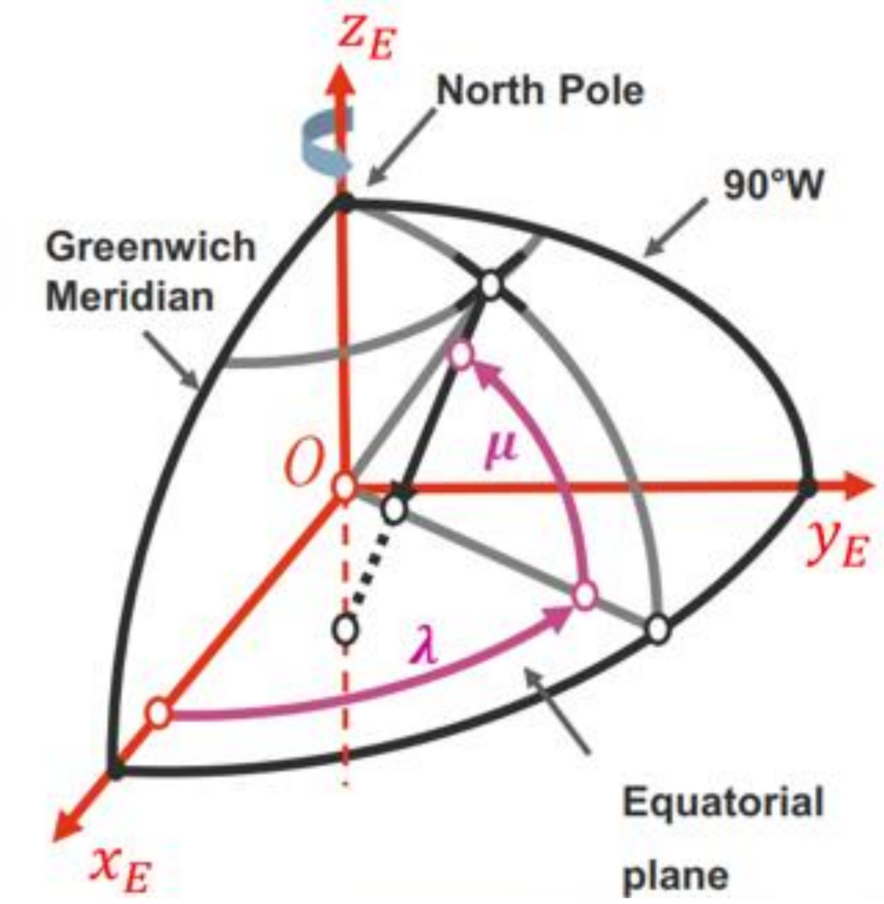
Role: Navigational/positional frame

Origin: Center of Earth

X-Axis: In Equatorial plane, points towards Greenwich Meridian

Y-Axis: In Equatorial plane, forms right-handed coordinate system with x- and z-axis

Z-Axis: Earth rotation Axis



The ECEF Frame

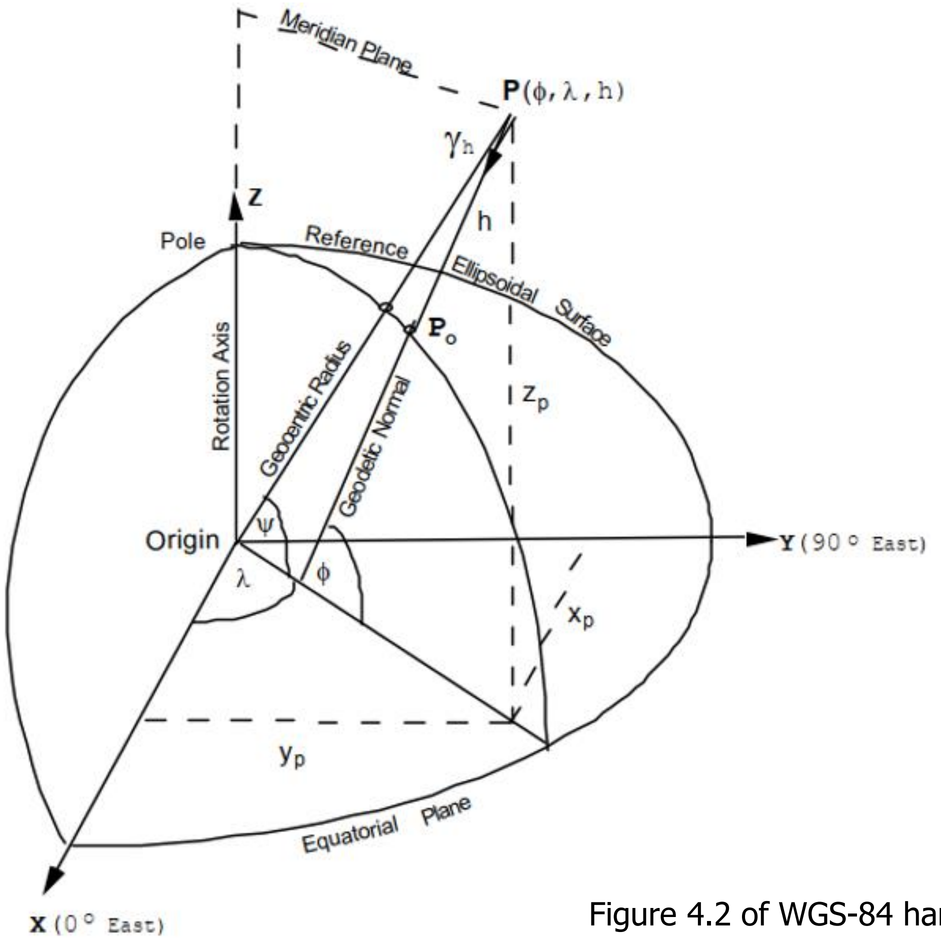


Figure 4.2 of WGS-84 handbook (Jan.2000)

The position of a point in ECEF-frame is either given in cartesian coordinates or **in WSG-84 coordinates**.

The WGS-84 Frame

World **G**eodetic **S**ystem 19**84** (大地坐标系)

WGS-84 are related to the ECEF-frame

Geodetic Longitude: $\boldsymbol{\lambda}$

measured in Equatorial plane, between prime meridian (xz-plane in ECEF-frame) and the meridial plane of point P.

Range: $-\pi \leq \lambda < \pi$

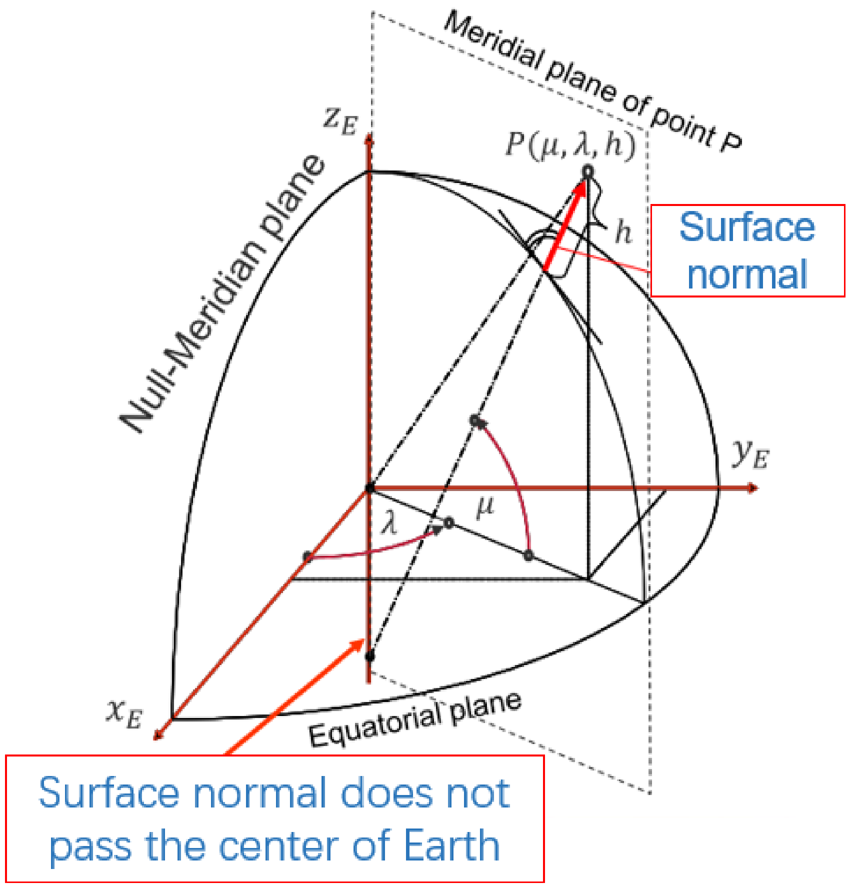
Geodetic Latitude: $\boldsymbol{\mu}$

measured on meridial plane of point P, between Equatorial plane (-plane of ECEF-frame) and the surface normal point.

Range: $-\pi/2 \leq \mu \leq \pi/2$

Geodetic Height: $\boldsymbol{h}$

Height over WGS-84 ellipsoid along surface normal.



The WGS-84 Frame

Transformation of WGS-84 coordinates to cartesian ECEF coordinates

$$\vec{r}_E = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \qquad \vec{r}_{E,WGS84} = \begin{bmatrix} \lambda \\ \mu \\ h \end{bmatrix}$$

Length of semi-major axis: a = 6378137.0 m

Length of semi-minor axis: b = 6356752.3142 m

First eccentricity: e

$$e = \frac{\sqrt{a^2 - b^2}}{a}$$

$$N(\mu) = \frac{a}{\sqrt{1 - e^2 (\sin\mu)^2}}$$

$$x = (N + h) \cdot \cos\mu \cdot \cos\lambda$$

$$y = (N + h) \cdot \cos\mu \cdot \sin\lambda$$

$$z = [N(1 - e^2) + h] \cdot \sin\mu$$

The transformation from cartesian ECEF to WGS-84 frame has to be done iteratively.

The NED Frame

North-East-Down Frame (北-东-地坐标系)

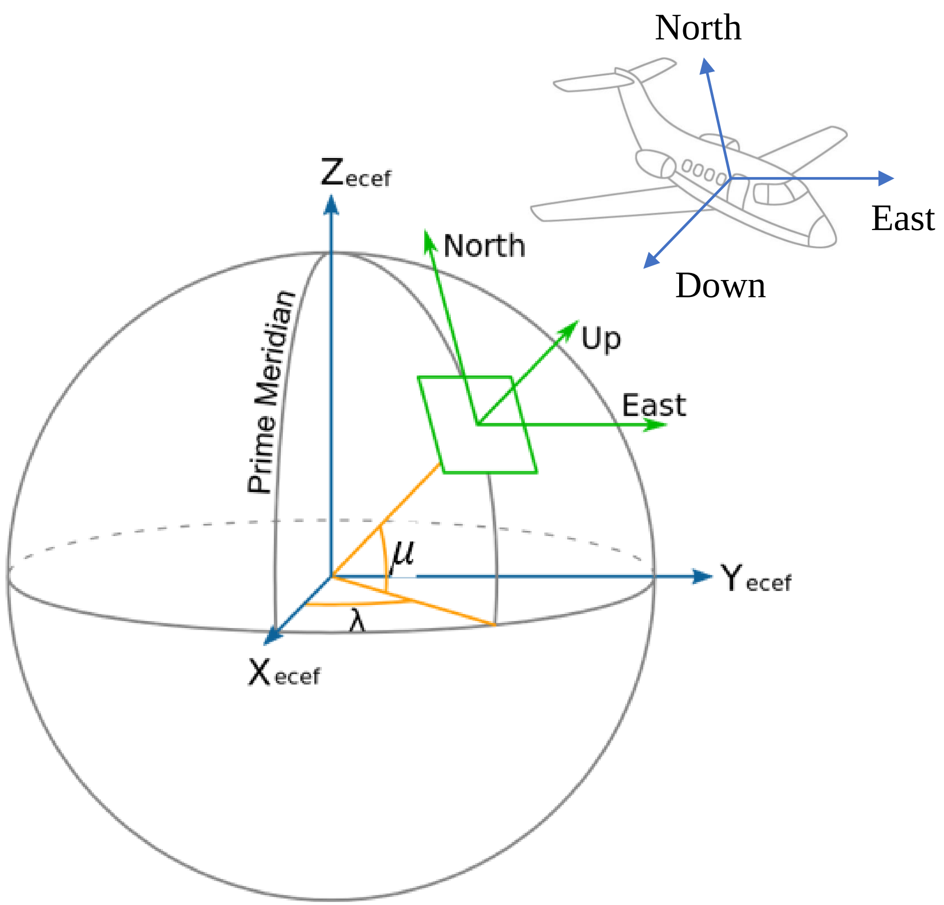
Role: Attitude/Orientation system

Origin: Reference point of aircraft

X-Axis: Parallel to local Geoid surface, pointing toward geographical North Pole

Y-Axis: Parallel to local Geoid surface, pointing towards East to form right-handed system with x-axis and z-axis

Z-Axis: Points downward, perpendicular to local Geoid surface



Summary

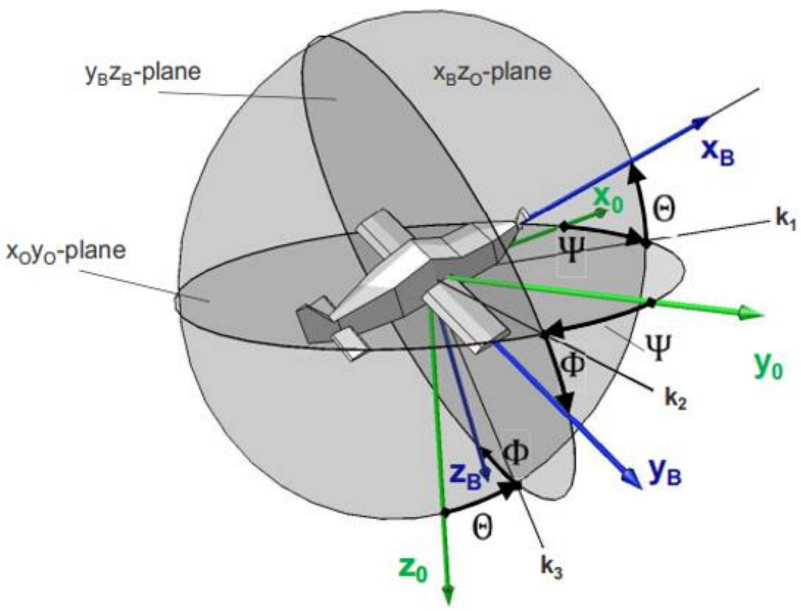
Coordinate Systems/Frames

- ECEF frame (O_E, x_E, y_E, z_E)
- WSG-84 frame (O_E, x_E, y_E, z_E)
- NED frame (O_O, x_O, y_O, z_O)

- **B**ody fixed frame: (O_B, x_B, y_B, z_B)
- **A**erodynamic frame: (O_A, x_A, y_A, z_A)
- **K**inematic frame: (O_K, x_K, y_K, z_K)

Frame Transformation

- From NED to Body-fixed frame



ψ： yaw angle（偏航角）

θ： pitch angle（俯仰角）

ϕ： bank angle（滚转角）

Order of rotation: $\Rightarrow \psi \Rightarrow \theta \Rightarrow \phi$

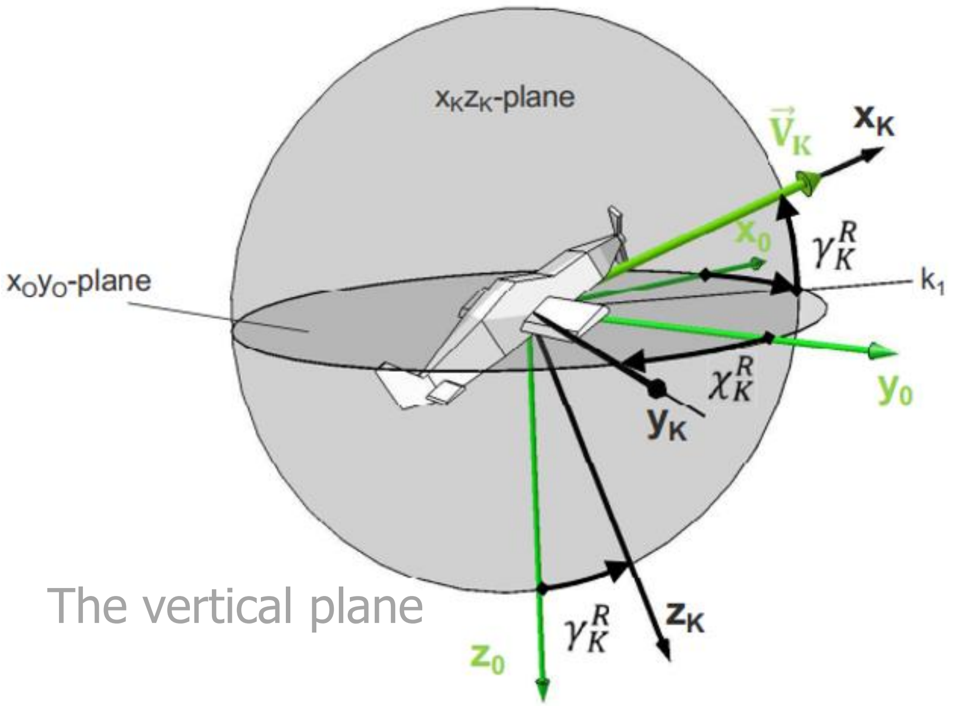
$$L_{BO} = L_x(\phi)L_y(\theta)L_z(\psi) =$$

Error in the Eq. (1.24) of textbook

$$\begin{bmatrix} \cos\psi\cos\theta & \sin\psi\cos\theta & -\sin\theta \\ \cos\psi\sin\theta\sin\phi - \sin\psi\cos\phi & \sin\psi\sin\theta\sin\phi + \cos\psi\cos\phi & \cos\theta\sin\phi \\ \cos\psi\sin\theta\cos\phi + \sin\psi\sin\phi & \sin\psi\sin\theta\cos\phi - \cos\psi\sin\phi & \cos\theta\cos\phi \end{bmatrix}$$

Frame Transformation

- From NED to Kinematic frame



χ： course angle/ flight-path -
（航向角、航迹偏角）

γ： climb angle/flight-path inclination angle
（爬升角、航迹倾角）

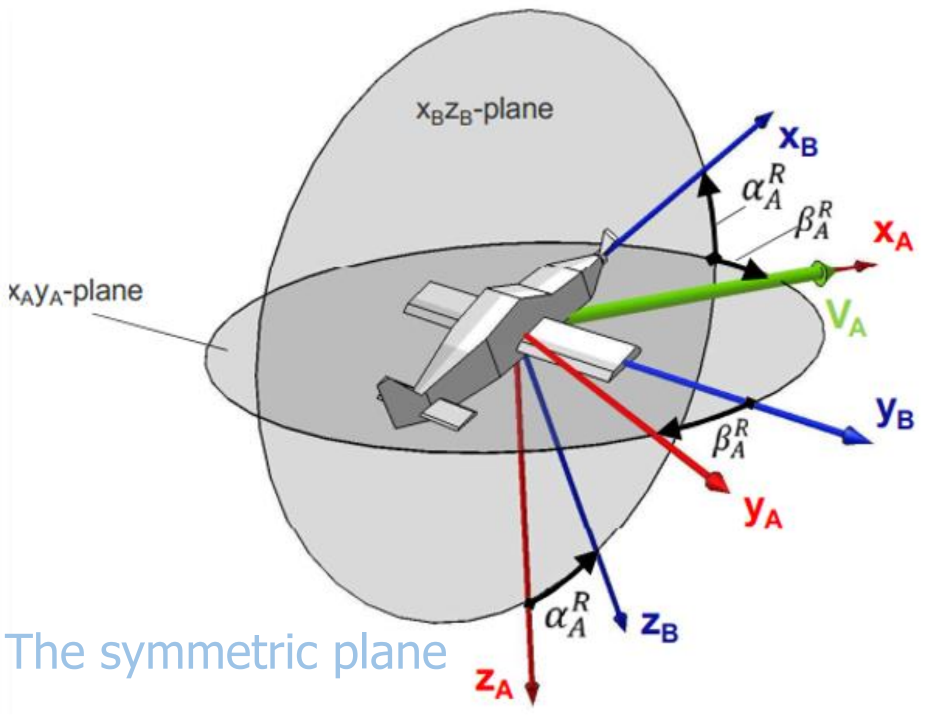
Order of rotation: $\Rightarrow \chi \Rightarrow \gamma$

$$L_{KO} = L_y(\gamma)L_z(\chi) =$$

$$= \begin{bmatrix} \cos\chi\cos\gamma & \sin\chi\cos\gamma & -\sin\gamma \\ -\sin\chi & \cos\chi & 0 \\ \cos\chi\sin\gamma & \sin\chi\sin\gamma & \cos\gamma \end{bmatrix}$$

Frame Transformation

- From Aerodynamic to Body-fixed frame



β： angle of sideslip （侧滑角）
α： angle of attack （攻角）

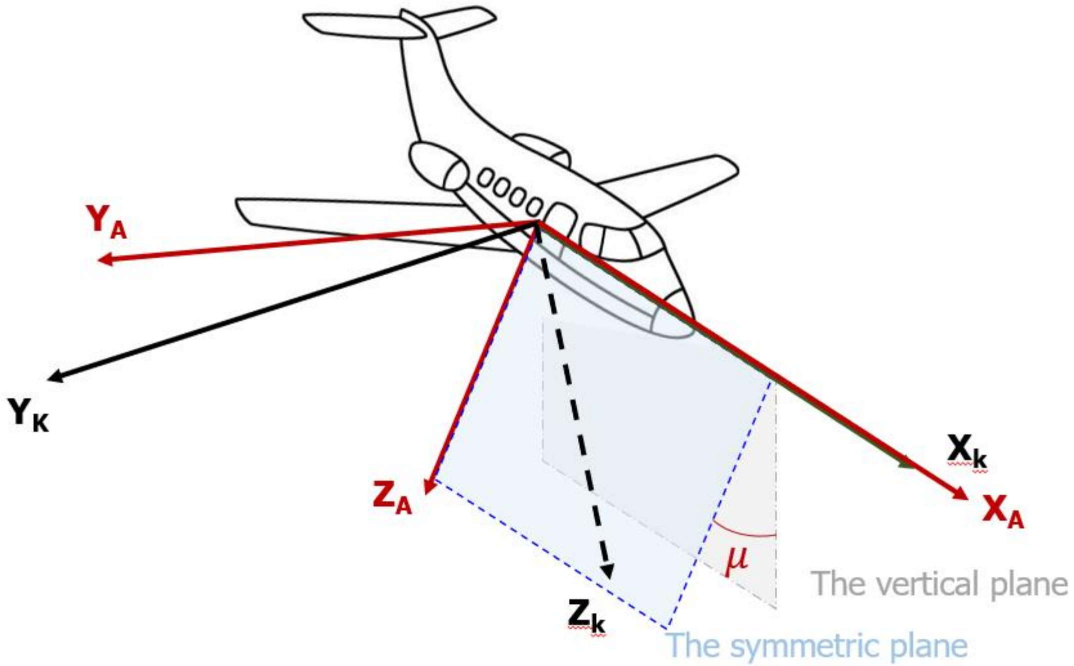
Order of rotation: $\Rightarrow -\beta \Rightarrow \alpha$

$$L_{BA} = L_y(\alpha) L_z(-\beta) =$$

$$\begin{bmatrix} \cos\alpha\cos\beta & -\cos\alpha\sin\beta & -\sin\alpha \\ \sin\beta & \cos\beta & 0 \\ \sin\alpha\cos\beta & -\sin\alpha\sin\beta & \cos\alpha \end{bmatrix}$$

Frame Transformation

- From Kinematic to Aerodynamic frame



μ: bank angle（速度滚转角）

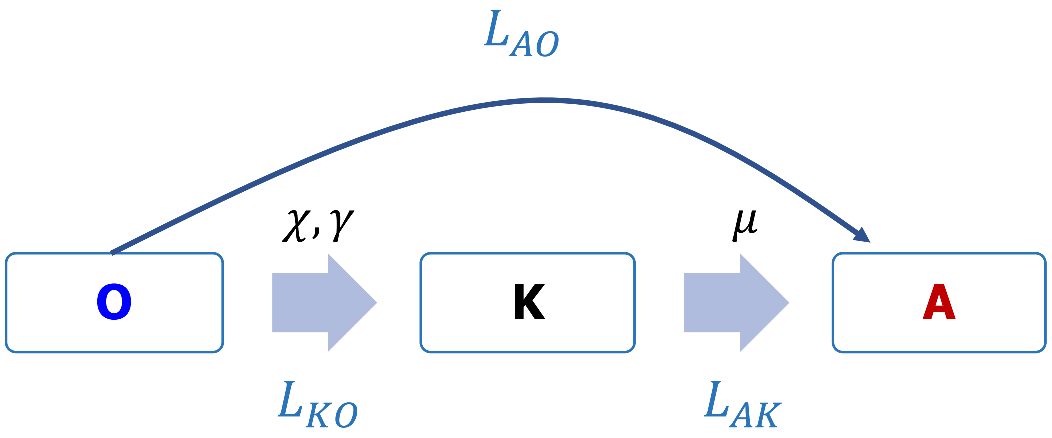
Order of rotation: $\Rightarrow \mu$

$$L_{\mathrm{AK}} = L_x(\mu) =$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\mu & \sin\mu \\ 0 & -\sin\mu & \cos\mu \end{bmatrix}$$

Frame Transformation

- From NED to Aerodynamic frame



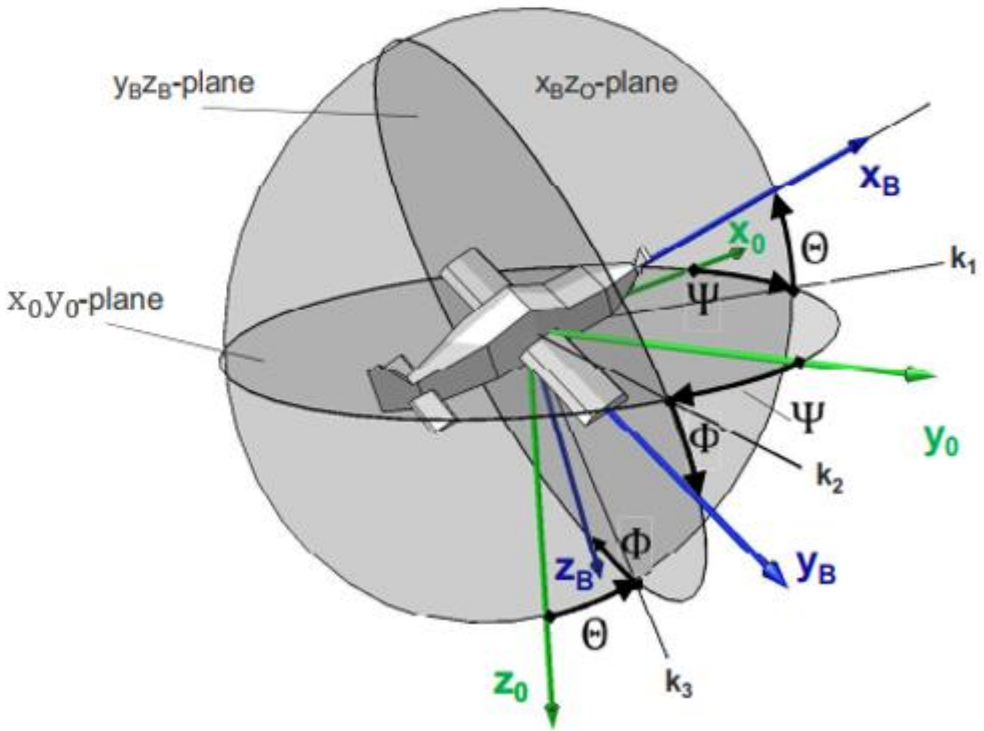
$$L_{\mathrm{AO}} = L_{\mathrm{AK}} L_{\mathrm{KO}} =$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\mu & \sin\mu \\ 0 & -\sin\mu & \cos\mu \end{bmatrix} \begin{bmatrix} \cos\chi\cos\gamma & \sin\chi\cos\gamma & -\sin\gamma \\ -\sin\chi & \cos\chi & 0 \\ \cos\chi\sin\gamma & \sin\chi\sin\gamma & \cos\gamma \end{bmatrix}$$

$$= \begin{bmatrix} \cos\chi\cos\gamma & \sin\chi\cos\gamma & -\sin\gamma \\ \cos\chi\sin\gamma\sin\mu - \sin\chi\cos\mu & \cos\chi\cos\mu + \sin\chi\sin\gamma\sin\mu & \cos\gamma\sin\mu \\ \sin\chi\sin\mu + \cos\chi\sin\gamma\cos\mu & \sin\chi\sin\gamma\cos\mu - \cos\chi\sin\mu & \cos\gamma\cos\mu \end{bmatrix}$$

Frame Transformation

- From Body-fixed to NED frame



χ: course angle（航向角）

γ: climb angle（爬升角）

β: angle of sideslip （侧滑角）

α: angle of attack （攻角）

μ: bank angle（速度滚转角）

$$L_{OB} = L_{BO}^T$$

Frame Transformation

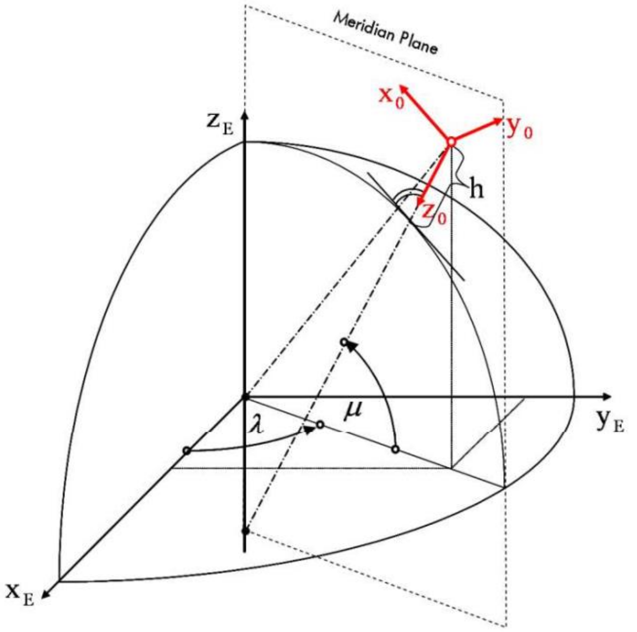
- From ECEF- to NED frame

1: Rotation around axis Z_E by the angle of geodetic longitude λ

$$L_{OE,1}(\lambda) = \begin{bmatrix} \cos\lambda & \sin\lambda & 0 \\ -\sin\lambda & \cos\lambda & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

2: Rotation around axis y_0 by the angle of geodetic latitude $(-\mu - \pi/2)$

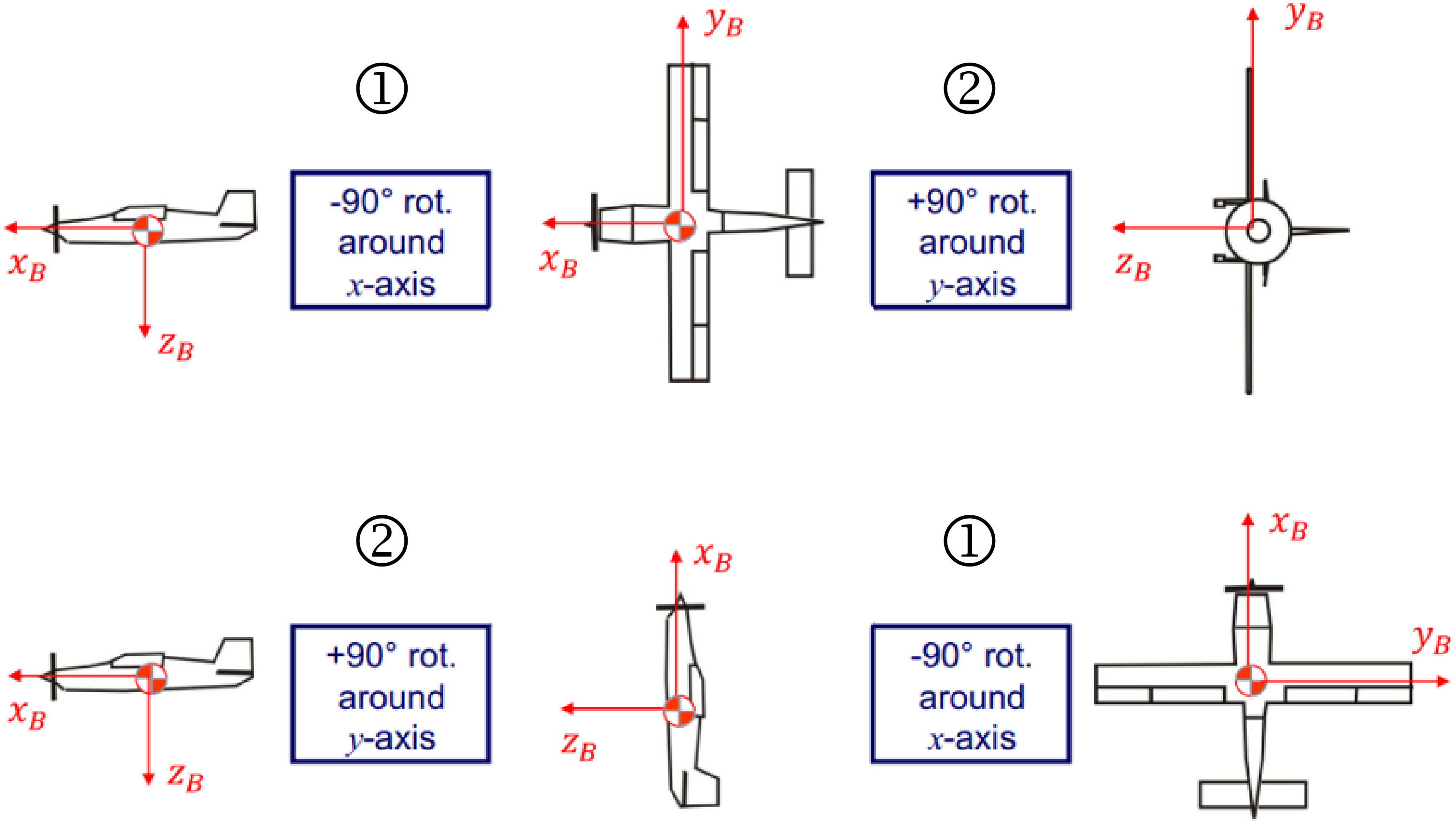
$$L_{OE,2}(\mu) = \begin{bmatrix} \cos(-\mu-\pi/2) & 0 & \sin(-\mu-\pi/2) \\ 0 & 1 & 0 \\ -\sin(-\mu-\pi/2) & 0 & \cos(-\mu-\pi/2) \end{bmatrix}$$



$$L_{OE} = L_{OE,2}(\mu) \cdot L_{OE,1}(\lambda)$$

$$L_{OE} = \begin{bmatrix} -\sin\mu\cos\lambda & -\sin\mu\sin\lambda & \cos\mu \\ -\sin\lambda & \cos\lambda & 0 \\ -\cos\mu\cos\lambda & -\cos\mu\sin\lambda & -\sin\mu \end{bmatrix}$$

Rotation Rule

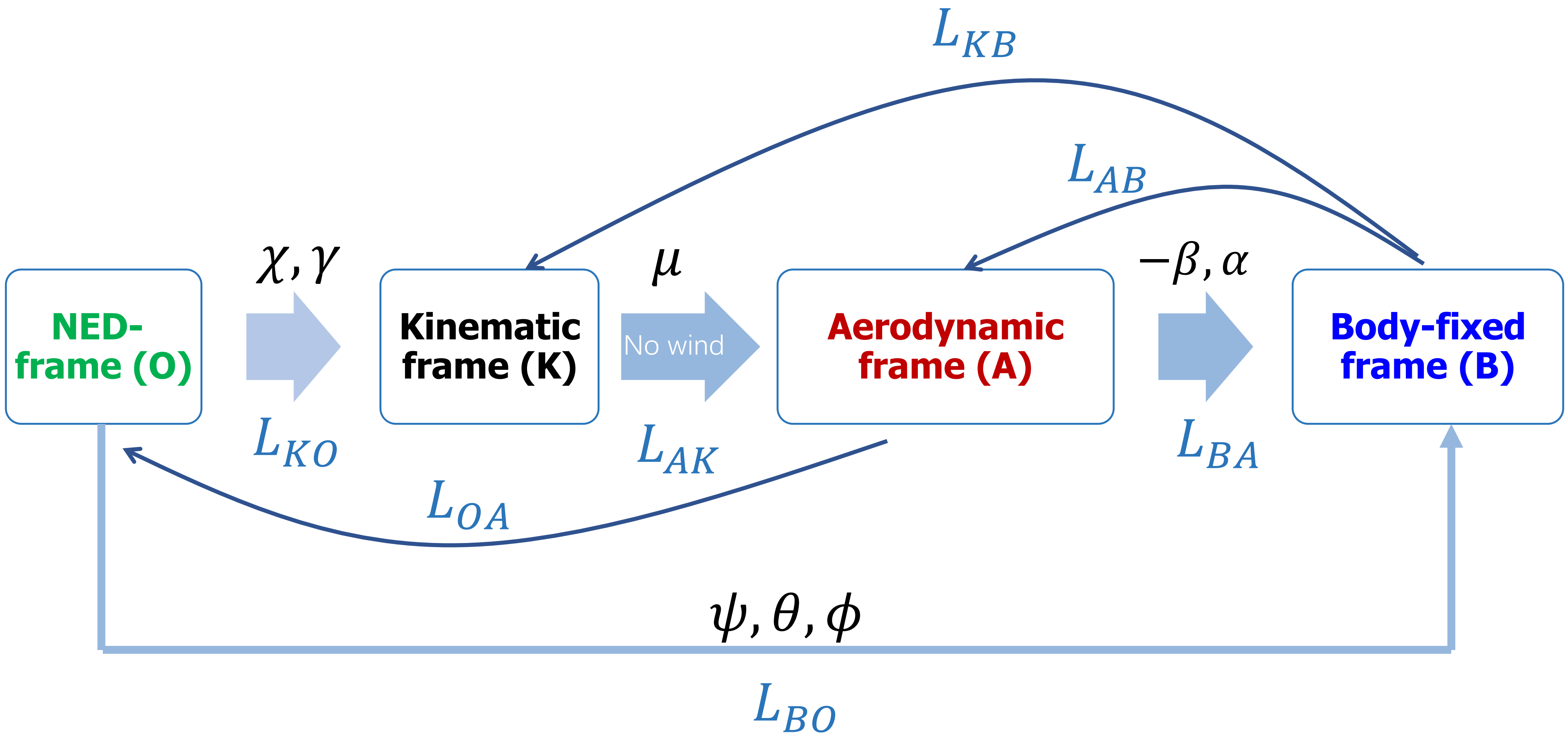


$$1 \rightarrow 2 \neq 2 \rightarrow 1$$

Rotations are not commutative.

Be careful about the sequence!

Summary



L_{KB}
L_{AB}
χ, γ
μ
$-\beta, \alpha$
NED-frame (O)
Kinematic frame (K)
No wind
Aerodynamic frame (A)
Body-fixed frame (B)
L_{KO}
L_{AK}
L_{BA}
L_{OA}
ψ, θ, ϕ
L_{BO}

Equation of Motion

The equation of motion in general coordinate system

$$\begin{cases} m(\dfrac{dV_x}{dt}+V_z\omega_y-V_y\omega_z)=F_x \\ m(\dfrac{dV_y}{dt}+V_x\omega_z-V_z\omega_x)=F_y \\ m(\dfrac{dV_z}{dt}+V_y\omega_x-V_x\omega_y)=F_z \end{cases}$$

Equation of Motion in Kinematic Frame

1) Velocity components

$$\begin{cases} V_x = V \\ V_y = 0 \\ V_z = 0 \end{cases}$$

According to the definition of the kinematic frame, Ox_k is the direction of ground speed.

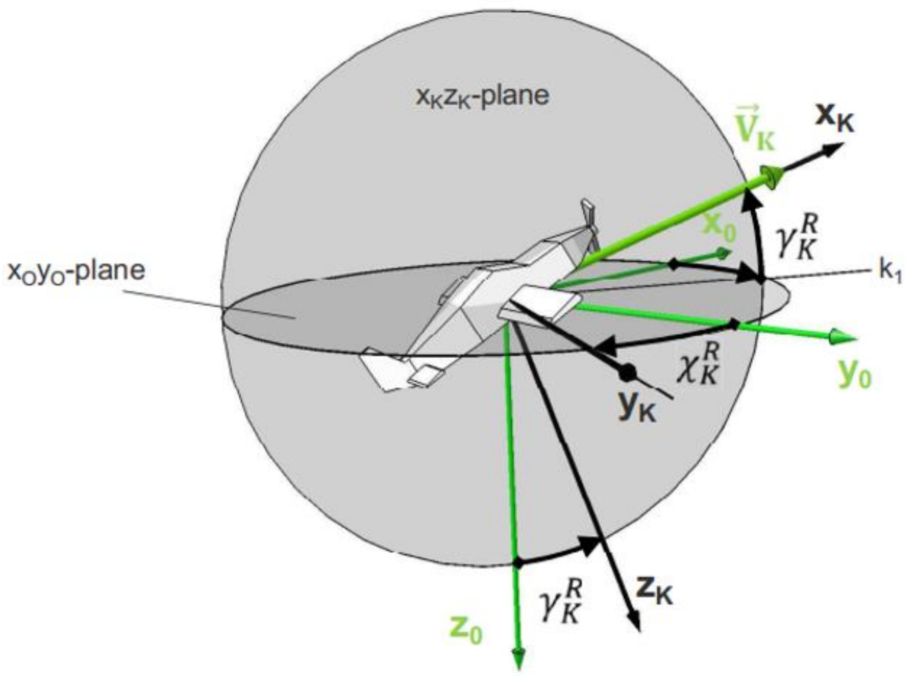
Equation of Motion in Kinematic Frame

2) Rotation rate

First rotate χ around z_o axis (y_o coincides with y_k), then rotate γ around y_k axis.

$$\vec{\omega} = \frac{d\chi}{dt} + \frac{d\gamma}{dt}$$

The rotation vectors should be projected into kinematic frame respectively.

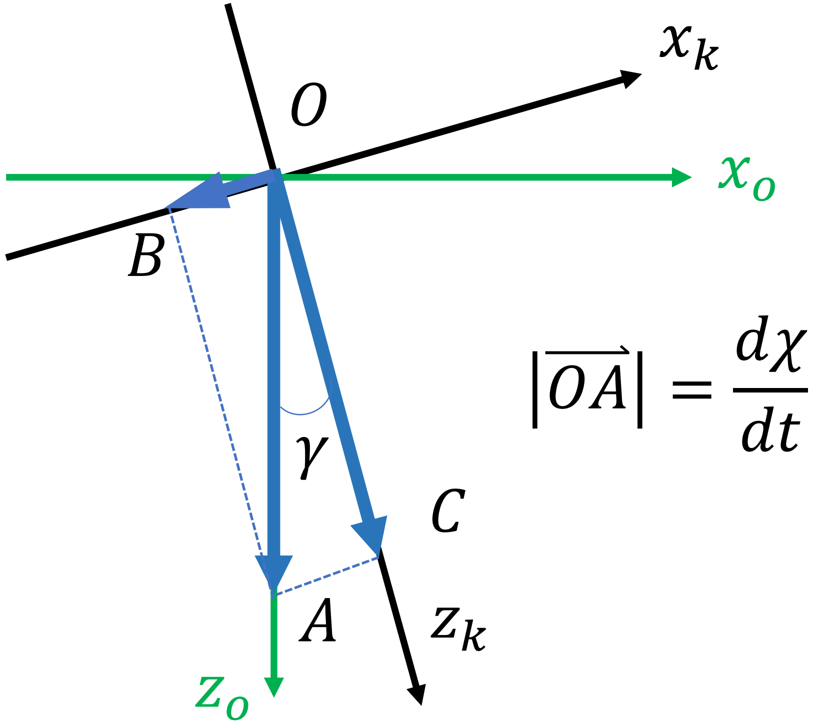


Equation of Motion in Kinematic Frame

2) Rotation rate

$$\left(\frac{d\chi}{dt}\right)_{zo} = -\left(\frac{d\chi}{dt}\sin\gamma\right)_{xk} + \left(\frac{d\chi}{dt}\cos\gamma\right)_{zk}$$

$$\left(\frac{d\gamma}{dt}\right)_{yo} = \left(\frac{d\gamma}{dt}\right)_{yk}$$



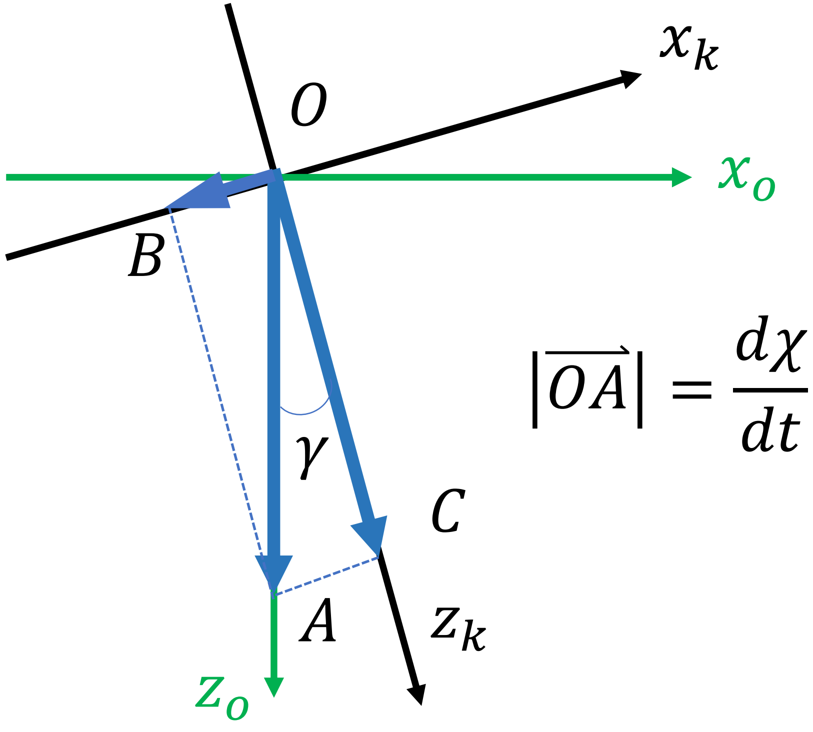
$$|\overrightarrow{OA}| = \frac{d\chi}{dt}$$

Equation of Motion in Kinematic Frame

2) Rotation rate

The rotation vector in the kinematic frame

$$\begin{bmatrix}\omega_x\\ \omega_y\\ \omega_z\end{bmatrix}_K = \begin{bmatrix}-\frac{d\chi}{dt}\sin\gamma\\ 0\\ \frac{d\chi}{dt}\cos\gamma\end{bmatrix} + \begin{bmatrix}0\\ \frac{d\gamma}{dt}\\ 0\end{bmatrix} = \begin{bmatrix}-\frac{d\chi}{dt}\sin\gamma\\ \frac{d\gamma}{dt}\\ \frac{d\chi}{dt}\cos\gamma\end{bmatrix}$$



$$|\overrightarrow{OA}| = \frac{d\chi}{dt}$$

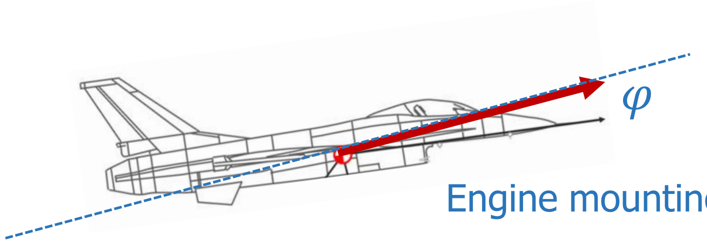
Homework: derive the result with transformation matrix.

Equation of Motion in Kinematic Frame

3) Thrust

First project the thrust into body-fixed frame

$$\begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix}_B = \begin{bmatrix} T\cos\varphi \\ 0 \\ -T\sin\varphi \end{bmatrix}$$



Then project the thrust into kinematic frame by transformation matrix

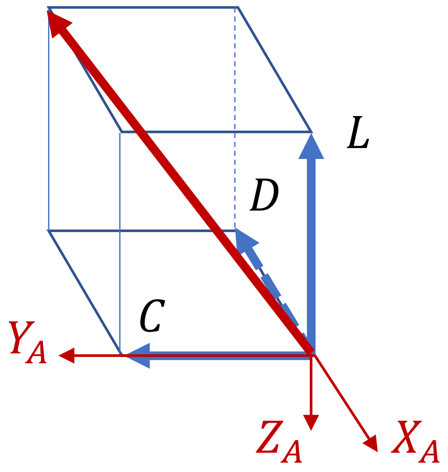
$$\begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix}_K = L_{KB} \begin{bmatrix} T\cos\varphi \\ 0 \\ -T\sin\varphi \end{bmatrix} = T \begin{bmatrix} \cos(\alpha+\varphi)\cos\beta \\ \sin(\alpha+\varphi)\sin\mu - \cos(\alpha+\varphi)\sin\beta\cos\mu \\ -\sin(\alpha+\varphi)\cos\mu - \cos(\alpha+\varphi)\sin\beta\sin\mu \end{bmatrix}$$

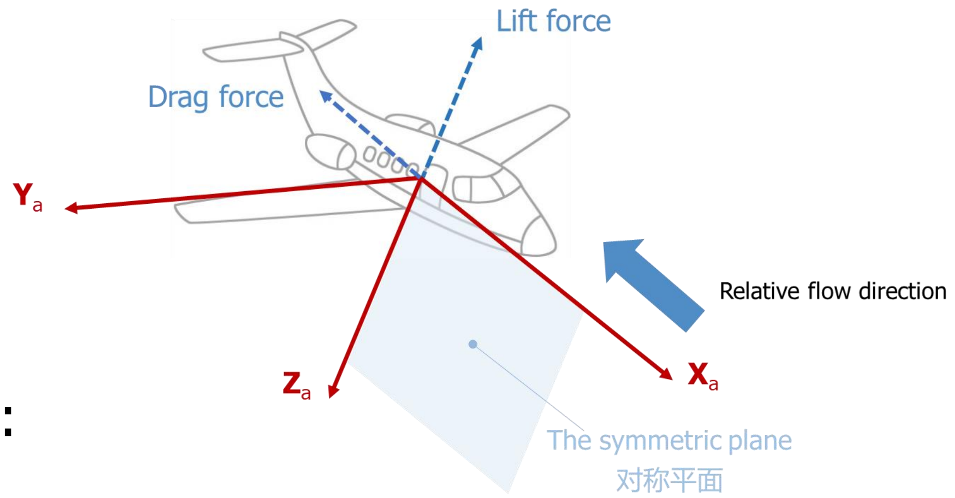
Equation of Motion in Kinematic Frame

4) Aerodynamic forces

In aerodynamic frame:

$$\begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix}_A = \begin{bmatrix} -D \\ C \\ -L \end{bmatrix}$$





Project into kinematic frame by transformation matrix:

$$\begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix}_K = L_{KA} \begin{bmatrix} -D \\ C \\ -L \end{bmatrix} = \begin{bmatrix} -D \\ C\cos\mu + L\sin\mu \\ C\sin\mu - L\cos\mu \end{bmatrix}$$

Equation of Motion in Kinematic Frame

4) Weight

Project from NED frame to kinematic frame by transformation matrix:

$$m\begin{bmatrix} g_x \\ g_y \\ g_z \end{bmatrix}_K = L_{KO} m \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix} = \mathrm{m}\begin{bmatrix} -g\sin\gamma \\ 0 \\ g\cos\gamma \end{bmatrix}$$

Equation of Motion in Kinematic Frame

- The final scalar form of centroid dynamics equation

$$
\begin{cases}
m\dfrac{dV}{dt} = T\cos(\alpha+\varphi)\cos\beta - D - mg\sin\gamma \\
mV\cos\gamma\dfrac{d\chi}{dt} = T[\sin(\alpha+\varphi)\sin\mu - \cos(\alpha+\varphi)\sin\beta\cos\mu] + C\cos\mu + L\sin\mu \\
-mV\dfrac{d\gamma}{dt} = T[-\sin(\alpha+\varphi)cos\mu - \cos(\alpha+\varphi)\sin\beta\sin\mu] + C\sin\mu - Lcos\mu + mg\cos\gamma
\end{cases}
$$

Equation of Motion in Kinematic Frame

- The centroid kinematic equation

First project the velocity obtained from dynamical equation into NED frame

$$\begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix}_o = L_{OK} \begin{bmatrix} V \\ 0 \\ 0 \end{bmatrix} = L_{KO}^T \begin{bmatrix} V \\ 0 \\ 0 \end{bmatrix}$$

Since $\mathrm{d}x_o/dt = V_{x_o}$, we have

$$\boxed{\begin{cases} \dfrac{dx_o}{dt} = V\cos\gamma\cos\chi \\ \dfrac{dy_o}{dt} = V\cos\gamma\sin\chi \\ \dfrac{dz_o}{dt} = -V\sin\gamma \end{cases}}$$

Summary

The parameters

Symbols	Physical meaning	Translation	Notation
V	Speed	速度	Description of aircraft position, speed, direction and attitude. In total 9 unknows.
χ, γ	course and climb angles	航向角和爬升角	
x_O, y_O, z_O	Centroid coordinates	质心坐标	
α, β	Angle of attack, angle of sideslip	攻角和侧滑角	
μ	Bank angle	速度滚转角	
δ_e	Elevator	升降舵偏角	Aircraft control parameters
δ_a	Aileron	副翼偏角	
δ_r	Rudder	方向舵偏角	
δ_T	Engine throttle position	发动机油门位置	
δ_{flap}	Flap	襟翼偏角	

Summary

In total 9 equations, the system is closed.

Expressions	Equations	Translation
$\begin{cases} \frac{dx_o}{dt} = V\cos\gamma\cos\chi \\ \frac{dy_o}{dt} = V\cos\gamma\sin\chi \\ \frac{dz_o}{dt} = -V\sin\gamma \end{cases}$	Kinematic equation	运动学方程
$\begin{cases} m\frac{dV}{dt} = T\cos(\alpha+\varphi)\cos\beta - D - mg\sin\gamma \\ mV\cos\gamma\frac{d\chi}{dt} = T[\sin(\alpha+\varphi)\sin\mu - \cos(\alpha+\varphi)\sin\beta\cos\mu] + C\cos\mu + L\sin\mu \\ -mV\frac{d\gamma}{dt} = T[-\sin(\alpha+\varphi)\cos\mu - \cos(\alpha+\varphi)\sin\beta\sin\mu] + C\sin\mu - L\cos\mu + mg\cos\gamma \end{cases}$	Dynamics equation	动力学方程
$\sum L = 0,\ \sum M = 0\ ,\sum N = 0$	Moment equation	力矩方程

Motion in Vertical Plane

Assumptions:

The symmetry plane and the flight path are in vertical plane.

$\Rightarrow \ \beta = 0$, $\mu = 0$, $\mathrm{d}\chi/dt = 0.$

The dynamic equation can be simplified as

$$m\frac{dV}{dt} = T\cos(\alpha + \varphi) - D - mg\sin\gamma$$

$$-mV\frac{d\gamma}{dt} = -T\sin(\alpha + \varphi) - L + mg\cos\gamma$$

Motion in Vertical Plane

Assumptions:

The symmetry plane and the flight path are in vertical plane.

Angle of attack is small.

$\Rightarrow \beta = 0,\ \mu = 0,\ \mathrm{d}\chi/dt = 0,\ \alpha + \varphi \approx 0.$

The dynamic equation can be further simplified as

$$m\frac{dV}{dt} = T - D - mg\sin\gamma$$

$$-mV\frac{d\gamma}{dt} = -L + mg\cos\gamma$$

Motion in Vertical Plane

Aircraft dives

Aircraft dives

Aircraft climbs

Motion in Vertical Plane

Assumptions:

The symmetry plane and the flight path are in vertical plane.

Angle of attack is small; fly in a straight line.

$\Rightarrow \beta = 0,\ \mu = 0,\ \mathrm{d}\chi/dt = 0,\ \mathrm{d}\gamma/dt = 0,\ \alpha + \varphi \approx 0.$

The dynamic equation becomes

$$m\frac{dV}{dt} = T - D - mg\sin\gamma$$

$$L = mg\cos\gamma$$

Motion in Vertical Plane

Assumptions:

The symmetry plane and the flight path are in vertical plane.

Angle of attack is small; flight in a level, straight line.

$\Rightarrow \beta = 0,\ \mu = 0,\ \mathrm{d}\chi/dt = 0,\ \gamma = 0,\ \alpha + \varphi \approx 0.$

The dynamic equation becomes

$$m\frac{dV}{dt} = T - D$$

$$L = mg$$

Motion in Vertical Plane

Assumptions:

The symmetry plane and the flight path are in vertical plane.

Angle of attack is small; fly with a constant speed in a straight line.

$\Rightarrow \beta = 0,\ \mu = 0,\ \mathrm{d}\chi/dt = 0,\ dV/dt = 0,\ d\gamma/dt = 0,\ \alpha + \varphi \approx 0.$

The dynamic equation becomes

$$T = D + mg\sin\gamma$$

$$L = mg$$

Motion in Vertical Plane

Assumptions:

The symmetry plane and the flight path are in vertical plane.

Angle of attack is small; fly with a constant speed in a level, straight line.

$\Rightarrow \beta = 0,\ \mu = 0,\ \mathrm{d}\chi/dt = 0,\ dV/dt = 0,\ \gamma = 0,\ \alpha + \varphi \approx 0.$

The dynamic equation becomes

$$\boxed{\begin{aligned} T &= D \\ L &= mg \end{aligned}}$$

Motion in Vertical Plane

Assumptions:

The symmetry plane and the flight path are in vertical plane.

The initial azimuth angle is zero $\chi = 0$.

The kinematic equation becomes

$$\frac{dx_o}{dt} = V\cos\gamma\cos\chi$$
$$\frac{dy_o}{dt} = V\cos\gamma\sin\chi$$
$$\frac{dz_o}{dt} = -V\sin\gamma$$

$$\Rightarrow$$

$$\frac{dx_o}{dt} = V\cos\gamma$$
$$\frac{dz_o}{dt} = -V\sin\gamma$$

Motion in Horizontal Plane

Assumptions:

$\gamma = 0$, $(\mathrm{d}\gamma/dt = 0)$

The dynamic equation can be simplified as

$$m\frac{dV}{dt} = T\cos(\alpha+\varphi)\cos\beta - D$$

$$mV\frac{d\chi}{dt} = T[\sin(\alpha+\varphi)\sin\mu - \cos(\alpha+\varphi)\sin\beta\cos\mu] + C\cos\mu + L\sin\mu$$

$$T[\sin(\alpha+\varphi)\cos\mu - \cos(\alpha+\varphi)\sin\beta\sin\mu] + L\cos\mu = C\sin\mu + mg$$

Motion in Horizontal Plane

Assumptions:

The flight path is in horizontal plane. No sideslip, hovering flight.

$\beta = 0$, $\gamma = 0$, $C = 0$

The dynamic equation becomes

$$m\frac{dV}{dt} = T\cos(\alpha+\varphi) - D$$

$$mV\frac{d\chi}{dt} = [T\sin(\alpha+\varphi) + L]\sin\mu$$

$$[T\sin(\alpha+\varphi) + L]\cos\mu = mg$$

Motion in Horizontal Plane

Assumptions:

The flight path is in horizontal plane. No sideslip, constant speed hovering flight.

$\beta = 0$, $\gamma = 0$, $C = 0$, $dV/dt = 0$

The dynamic equation becomes

$$T\cos(\alpha+\varphi) = D$$

$$mV\frac{d\chi}{dt} = [T\sin(\alpha+\varphi)+L]\sin\mu$$

$$[T\sin(\alpha+\varphi)+L]\cos\mu = mg$$

Motion in Horizontal Plane

Assumptions:

The flight path is in horizontal plane. No sideslip, constant speed hovering flight.

Angle of attack is small.

$\beta = 0$, $\gamma = 0$, $\mathrm{C} = 0$, $\mathrm{d}V/dt = 0$, $(\alpha + \varphi) \approx 0$

The dynamic equation becomes

$$T = D$$

$$mV\frac{d\chi}{dt} = L\sin\mu$$

$$L\cos\mu = mg$$

Motion in Horizontal Plane

Assumptions:

The flight path is in horizontal plane.

$dz_o/dt = 0$, $\gamma = 0$

The kinematic equation becomes

$$\frac{dx_o}{dt} = V\cos\gamma\cos\chi$$
$$\frac{dy_o}{dt} = V\cos\gamma\sin\chi$$
$$\frac{dz_o}{dt} = -V\sin\gamma$$

$$\Rightarrow$$

$$\frac{dx_o}{dt} = V\cos\chi$$
$$\frac{dy_o}{dt} = V\sin\chi$$

Summary of Chapter 1

- The Lift, Drag and Thrust
- Common Coordinate Frames
- Frame Transformation
- Transformation Matrices and Angles
- Equation of Motion in General Form
- Equation of Motion in Special Cases

Homework

1. Prove that 3D transformation matrix has the same properties as 2D case.
2. Derive the transformation matrix from the kinematic frame to the body-fixed frame. (page 32 of Textbook, 1.9)
3. Derive the three angular velocity components in kinematic frame $\left(\omega_x, \omega_y, \omega_y\right)_K^T$ using transformation matrix.

Homework

4. An aircraft is flying with constant velocity. The ground-station would like to calculate the velocity, the kinematic course angle χ_K and the kinematic climb angle γ_K of the aircraft based on the position and GPS-height of the aircraft at $t = 0s$ and $t = 160s$. All available data is presented in the tables below:

t1 = 0 s

Geodetic longitude	λ_1	116.353792 deg
Geodetic latitude	μ_1	39.98766 deg
GPS-height	h_1	1500 m

t1 = 160 s

Geodetic longitude	λ_2	116.276079 deg
Geodetic latitude	μ_2	40.16096 deg
GPS-height	h_2	1620 m

Homework

...You can assume the earth is flat and non-rotating. The following steps will guide you through the calculation of desired quantities:

a) Calculate the position vectors of both points and the velocity vector in the cartesian ECEF system.

b) Calculate the velocity vector of the aircraft at point 1 in NED-frame. (hint: use L_{OE} in page 15 of this lecture)

c) Calculate the angles χ_K and γ_K, in point 1 (at t_1) based on the velocity vector in NED-frame.

d) Give an easy way of checking the results of exercise c) using the results of exercise b) ?

e) If you were given the task of designing an aircraft to carry as many students as possible from point 1 to point 2, which type of wing and engine will you choose? Please state your reasons.

Useful reference:

1. https://www.cnblogs.com/ethanda/p/10325109.html
2. https://www.mathworks.com/help/map/ref/geodetic2ned.html

Due Friday, March 18