System Dynamics and Vibrations

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Chapter 3: Single degree-of-freedom systems
Exercises

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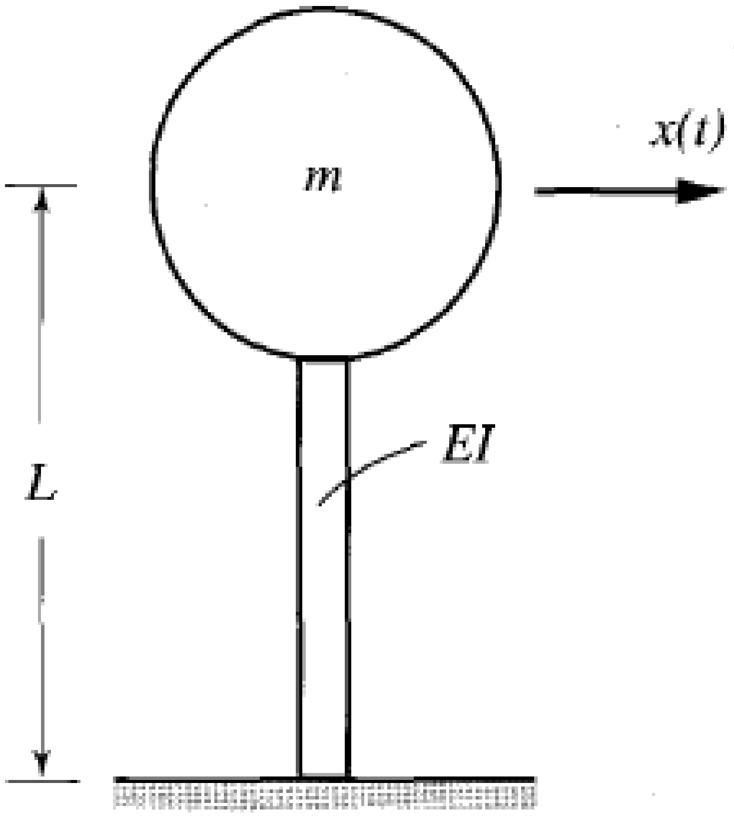
Exercise 1

The column of the water tank shown in the figure is 100 m high and is made of reinforced concrete with a tubular cross section of inner diameter 2.7 m and outer diameter 3.3 m. The tank weighs 250 tons when filled with water. By neglecting the mass of the column and assuming the Young s modulus of reinforced concrete as 30 GPa, determine the following:

a. the natural frequency and the natural time period of transverse vibration of the water tank.

b. the vibration response of the water tank due to an initial transverse displacement of 25 cm.

c. the maximum values of the velocity and acceleration experienced by the water tank.



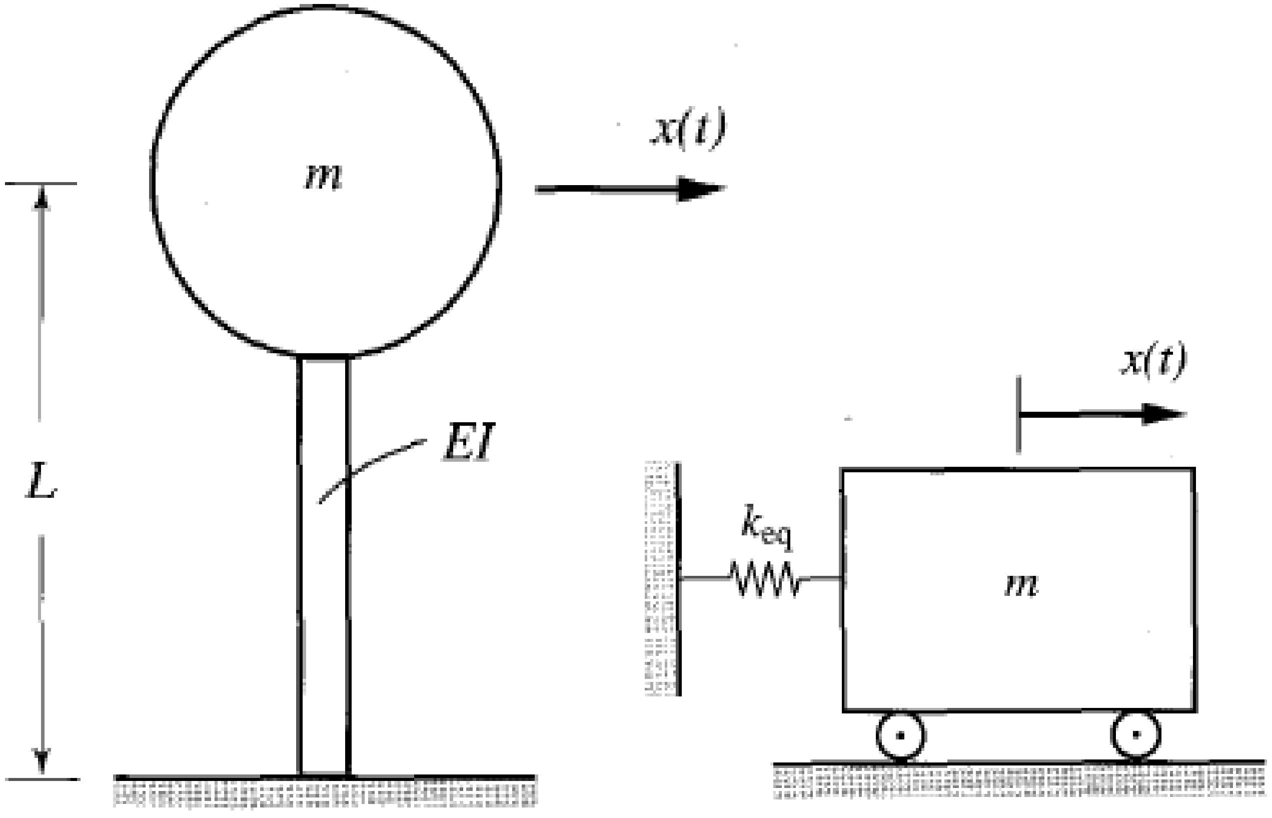
Undamped SDOF system

Assumption: the tank and water act as a rigid body and the support column is a massless uniform cantilever beam of bending stiffness EI.

$$k_{eq} = \frac{3EI}{L^3}$$

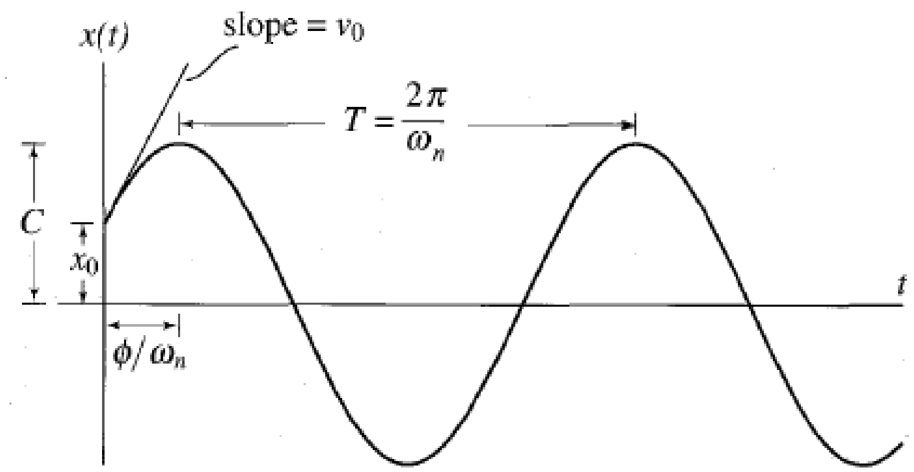
$$\omega_n = \sqrt{k_{eq}/m}$$

$$T = \frac{2\pi}{\omega_n}$$



Undamped SDOF system

$$x(t) = C\cos\left(\omega_n t - \phi\right) = x_0 \cos\omega_n t + \frac{v_0}{\omega_n}\sin\omega_n t$$



Initial conditions:

$$x(0) = x_0 = 0.25\text{m}$$
$$\dot{x}(0) = v_0 = 0$$

$$C = \sqrt{x_0^2 + \left(\frac{v_0}{\omega_n}\right)^2} = x_0 = 0.25\text{m}$$

$$\phi = \tan^{-1}\frac{v_0}{x_0\omega_n} = \tan^{-1} 0 = 0$$

$$x(t) = x_0 \cos\omega_n t$$

Undamped SDOF system

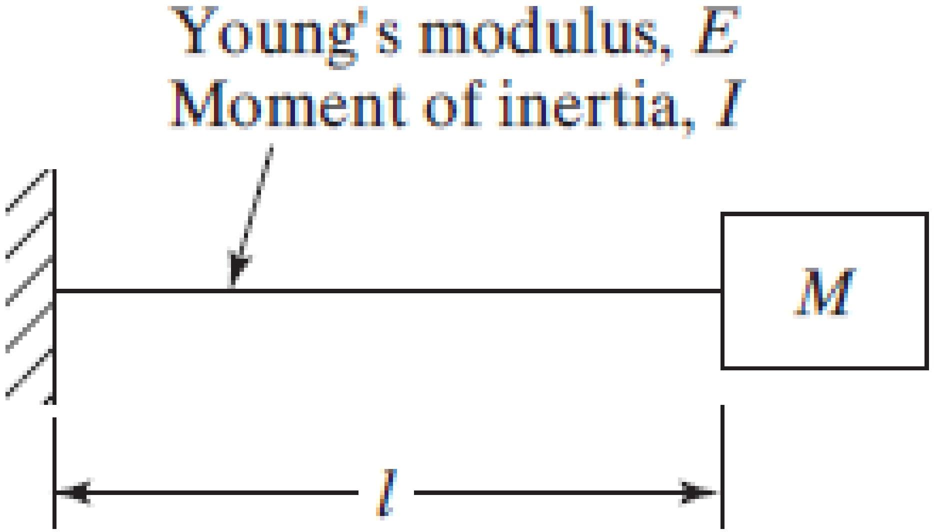
$$x(t) = x_0 \cos \omega_n t$$

$$\dot{x}(t) = -\omega_n x_0 \sin \omega_n t$$

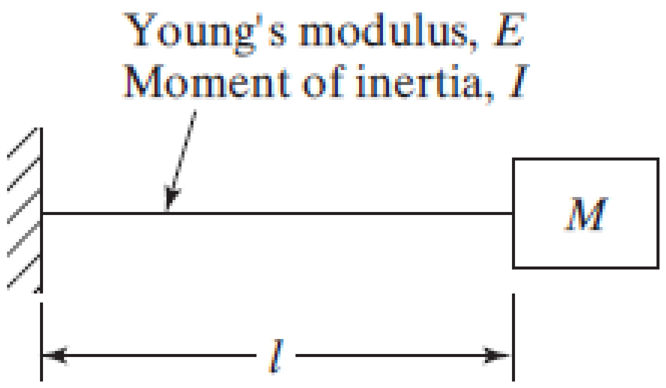
$$\ddot{x}(t) = -\omega_n^2 x_0 \cos \omega_n t$$

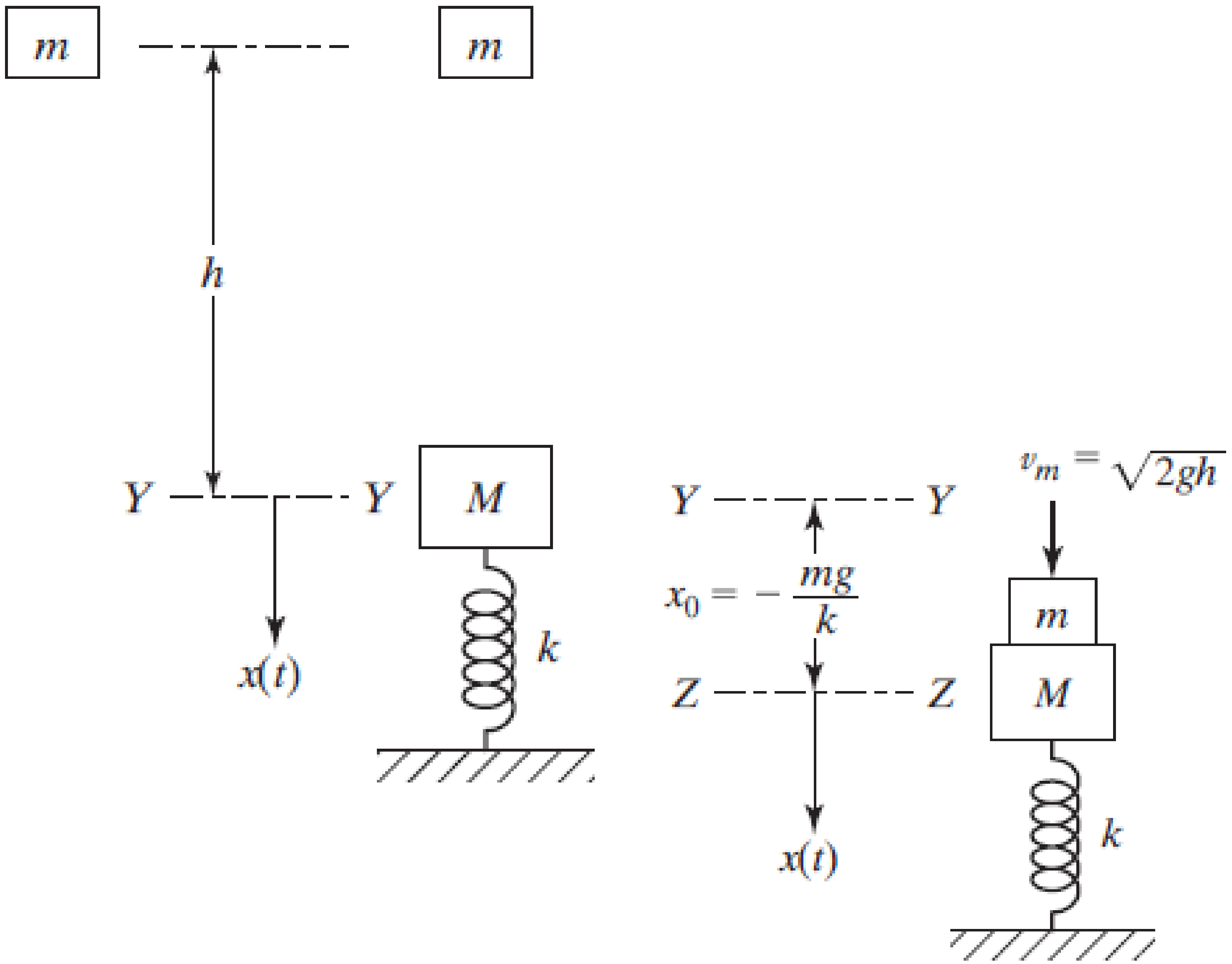
Exercise 2

A cantilever beam carries a mass M at the free end as shown in the figure. A mass m falls from a height h onto the mass M and adheres to it without rebounding. Determine the resulting transverse vibration of the beam.



Exercise 2


Young's modulus, E
Moment of inertia, I
M
l


m
m
h
Y
Y
M
x(t)
k
Y
Y
$v_m = \sqrt{2gh}$
$x_0 = -\frac{mg}{k}$
m
Z
Z
M
x(t)
k

Undamped SDOF system

$$x(t) = C\cos\left(\omega_n t - \phi\right) = x_0 \cos\omega_n t + \frac{v_0}{\omega_n}\sin\omega_n t$$

$$C = \sqrt{x_0^2 + \left(\frac{v_0}{\omega_n}\right)^2}$$

$$\phi = \tan^{-1}\frac{v_0}{x_0\omega_n}$$

Initial conditions:

$$x(0) = x_0 = \frac{mg}{k}$$

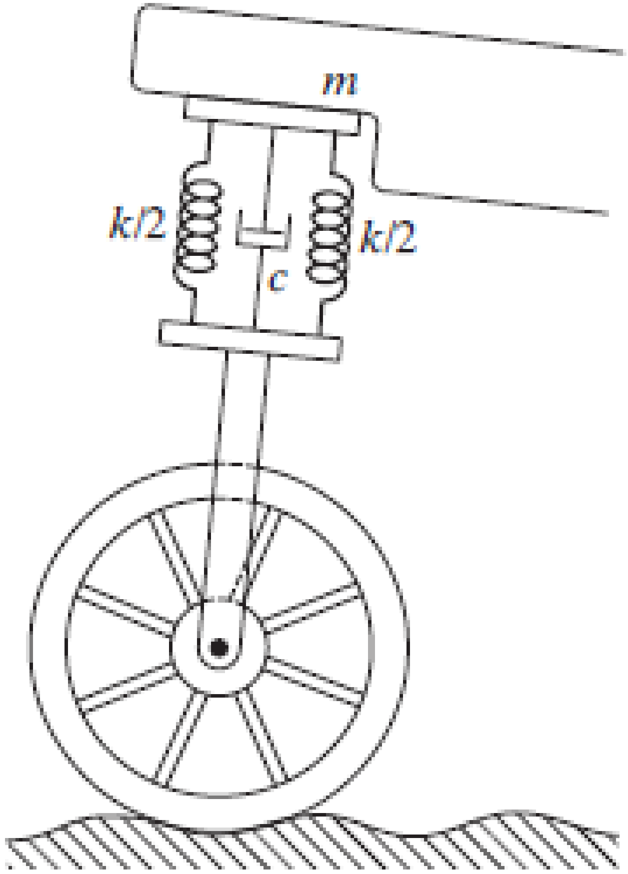
$$\dot{x}(0) = v_0 = \left(\frac{m}{m+M}\right)v_m = \left(\frac{m}{m+M}\right)\sqrt{2gh}$$

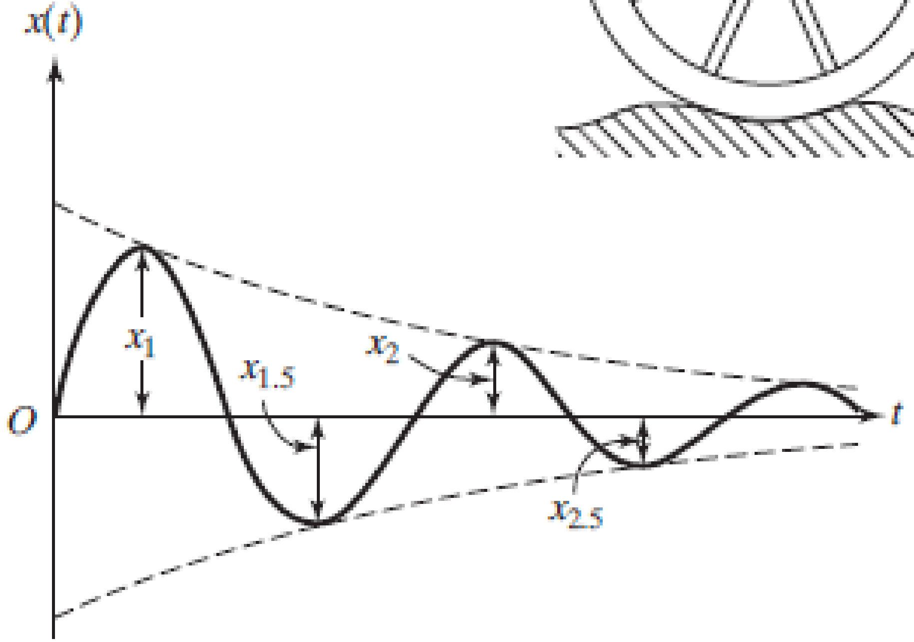
$$k_{eq} = \frac{3EI}{L^3}$$

$$\omega_n = \sqrt{\frac{k_{eq}}{(m+M)}}$$

Exercise 3

An underdamped shock absorber is to be designed for a motorcycle of mass 200 kg. When the shock absorber is subjected to an initial vertical velocity due to a road bump, the resulting displacement-time curve is to be as indicated in the figure. Find the necessary stiffness and damping constants of the shock absorber if the damped period of vibration is to be 2 s and the amplitude is to be reduced to one-fourth in one half cycle ($x_{1.5} = x_1 / 4$).



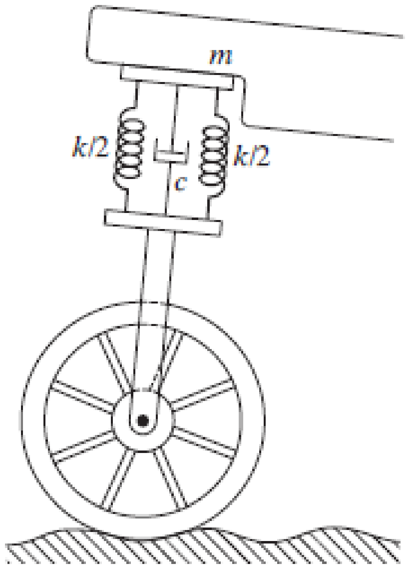


Exercise 3

$$x_{1.5} = x_1/4$$
$$x_2 = x_{1.5}/4 = 16$$

$$\delta = \ln\left(\frac{x_1}{x_2}\right) = \ln(16) = 2.7726 = \frac{2\pi\varsigma}{\sqrt{1-\varsigma^2}}$$

$$\varsigma = 0.4037$$



$$T = 2 = \frac{2\pi}{\omega_d} = \frac{2\pi}{\omega_n\sqrt{1-\varsigma^2}}$$

$$\omega_n = 3.4338\text{rad/s}$$

$$c_c = 2m\omega_n = 1373.54\text{Ns/m}$$
$$\text{c}=\varsigma\text{c}_c = 554.4981\text{Ns/m}$$
$$k = m\omega_n^2 = 2358.2652\text{N/m}$$