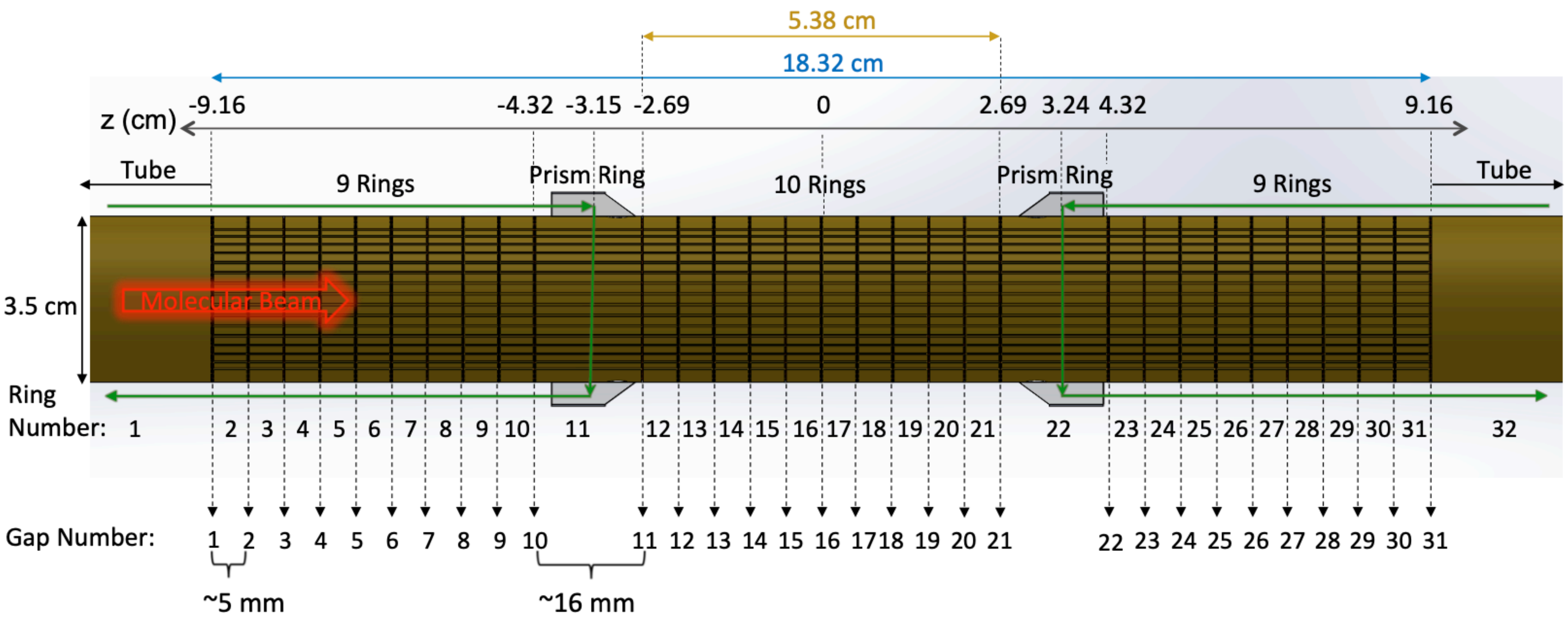
Error Signal Simulation

Unipolar $\mathscr{E}_{nr}$ Pulse Centered at Gap 22 and 2nd Depletion Laser Detuning Offset Simulation

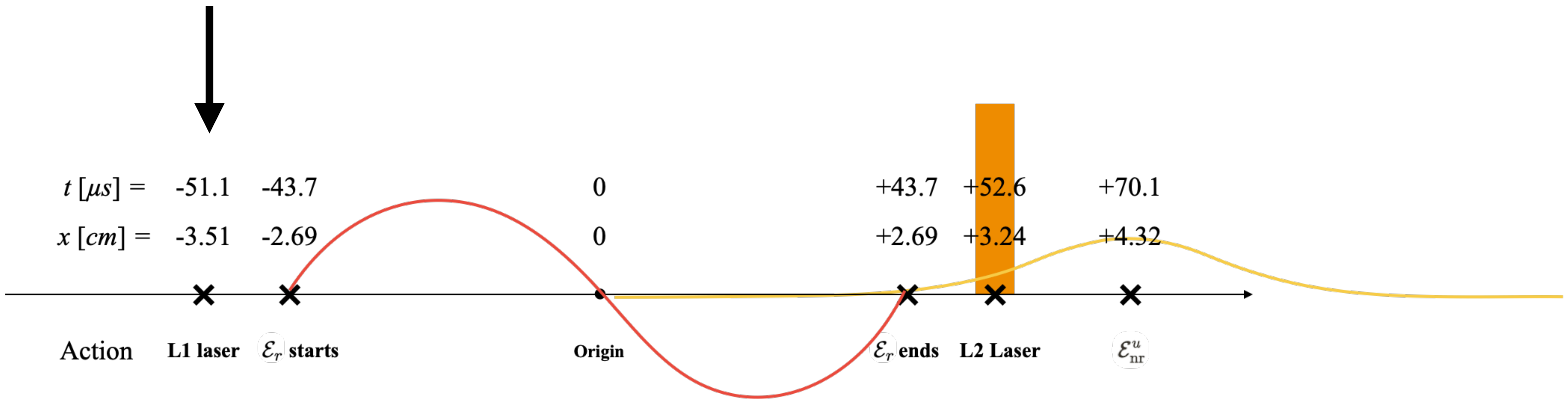
The Problem

When determine the relationship between W and systematic errors, we observed a shift in W proportional to the **product** of **non-reversing field** strength at the center of Gap 22 and **laser detuning**.

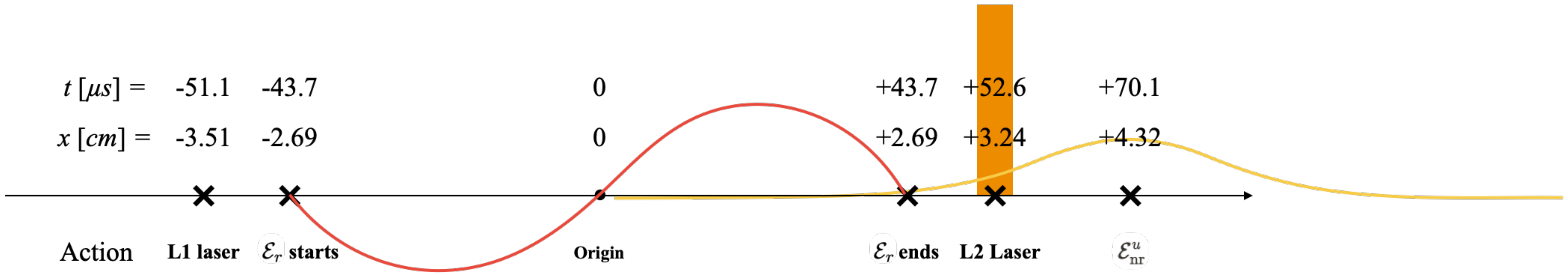


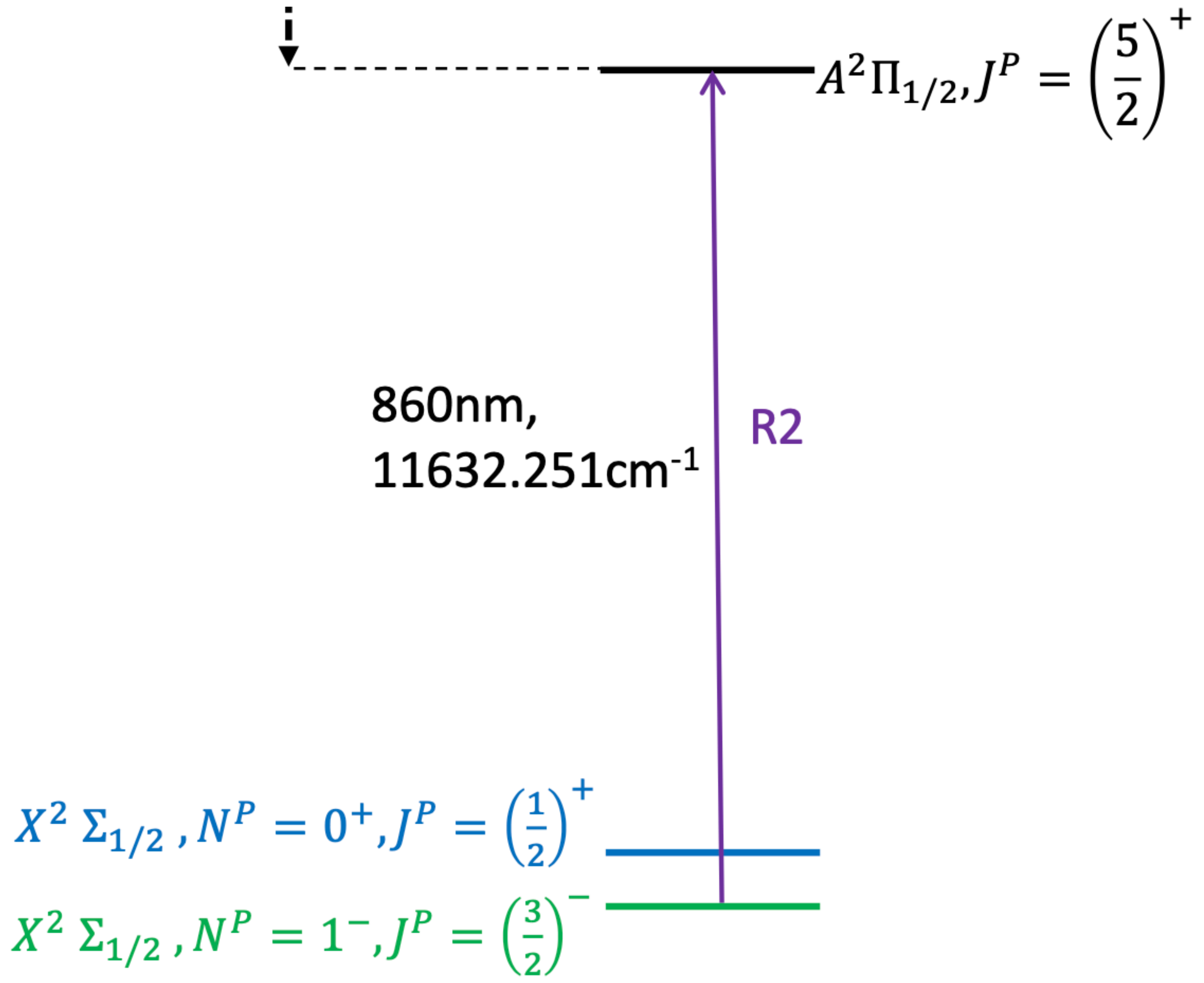
The Problem

Start



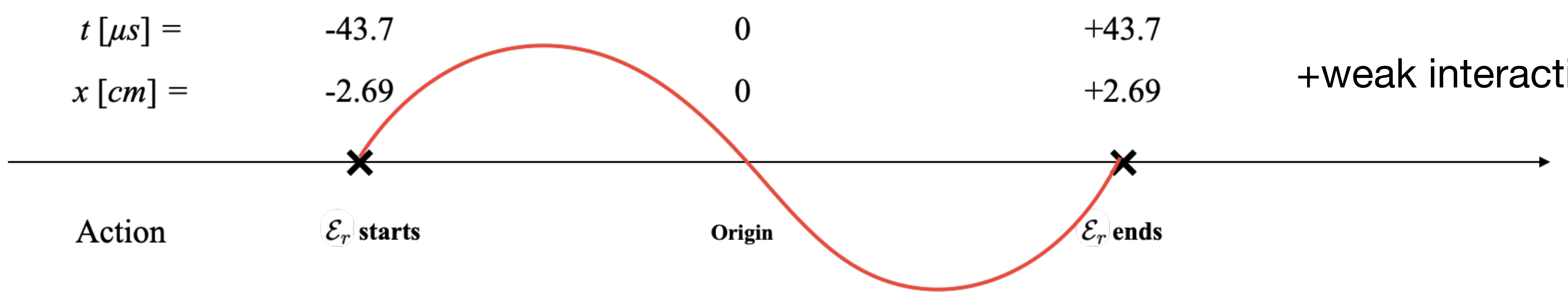
Reverse the $\mathscr{E}$ field





Should see a difference in the excited state population when Stark field reversed, if $W \neq 0$

Test Simulation (2-level)



+weak interaction

The Hamiltonian for the near-degenerate states

$$\mathscr{H}_{\pm} = \begin{pmatrix} 0 & iW + d\cdot\mathscr{E} \\ -iW + d\cdot\mathscr{E} & \Delta \end{pmatrix}, \qquad d = \langle 1 | -e\mathbf{r} | 2 \rangle$$

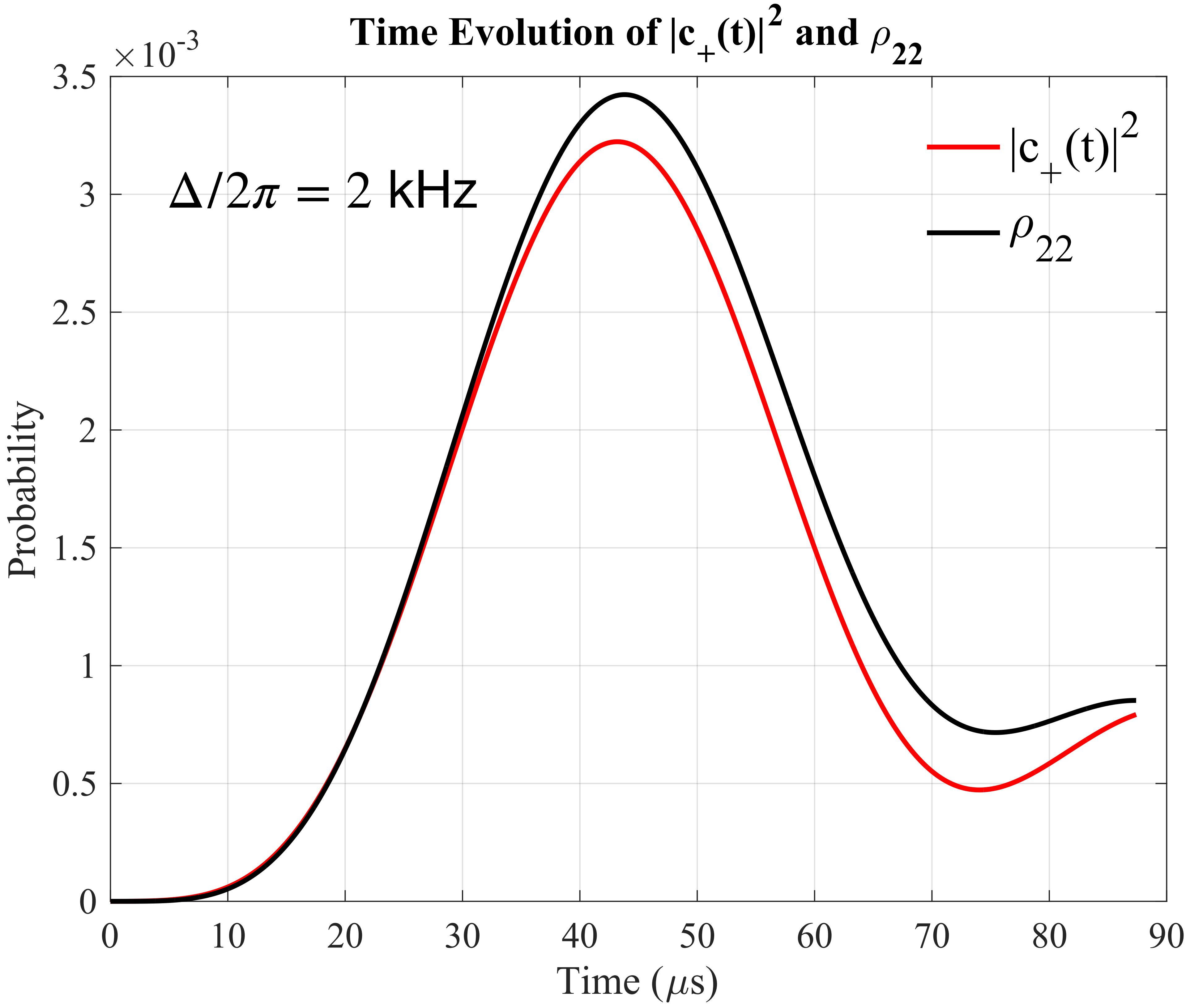
Use optical Bloch equation to get the full dynamics (no relaxation terms)

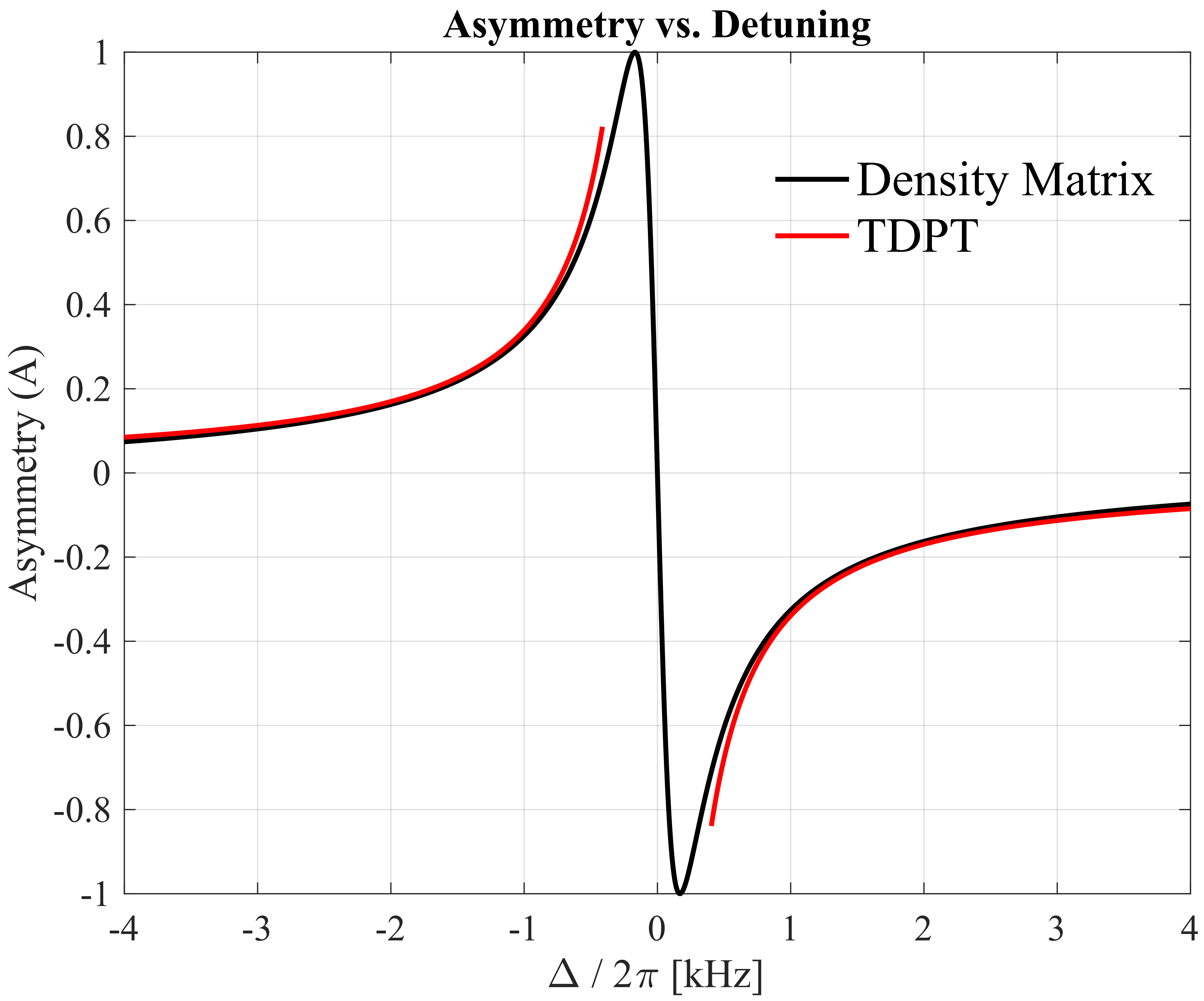
$$\frac{d\rho}{dt} = -i[\mathscr{H}_{\pm}, \rho]$$

Then use Matlab ode45 ODE solver to simulate the dynamics

Test Result (Analytic vs TDPT)

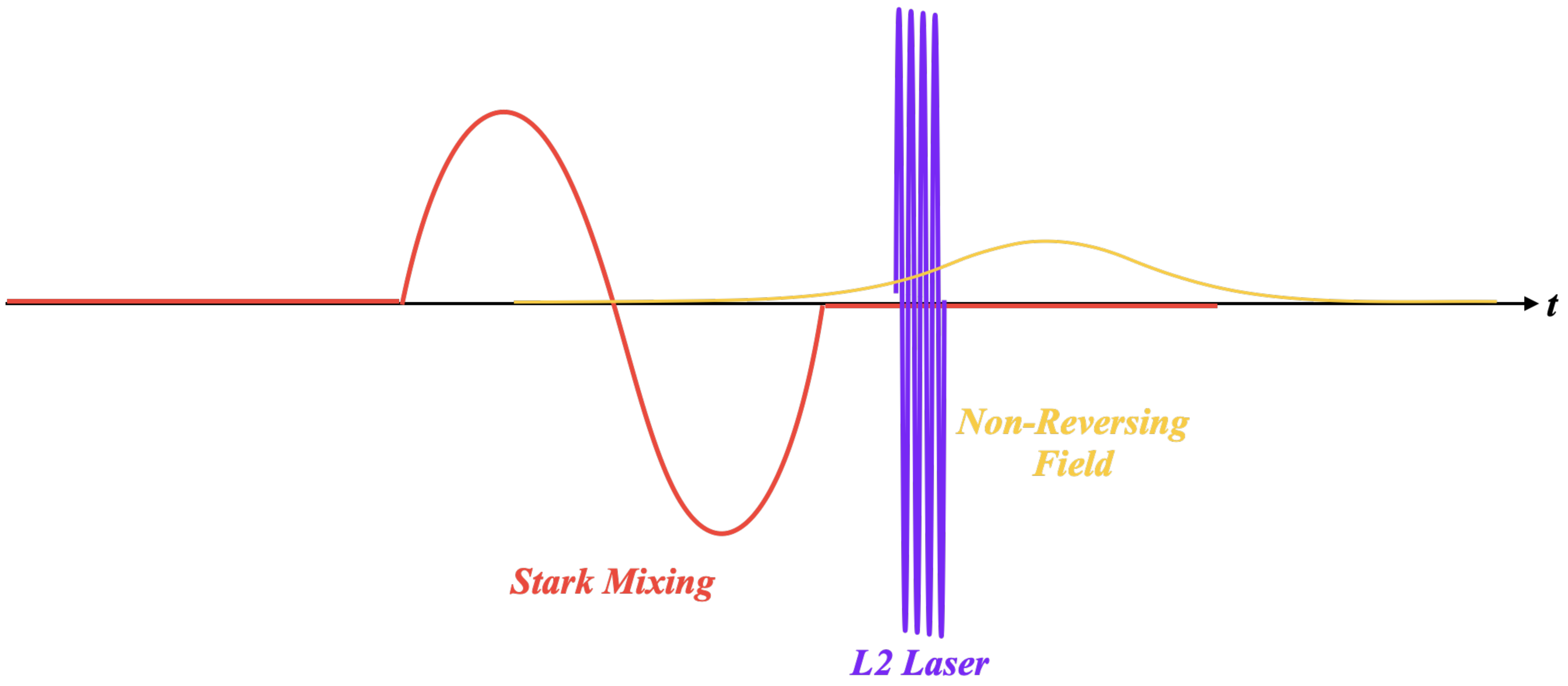
2 level (near degeneracy) system under 2π $\mathscr{E}$-field excitation (total time is a period)





$W/2\pi = 5$ Hz
$\mathscr{E}_0 = 0.1$ V/cm
$\omega/2\pi = 11.4$ kHz
$d/2\pi = -3360$ Hz/(V/cm)
$T_e = 87.4\ \mu s$

Move on to 3 level system



$$\mathscr{H} = \begin{pmatrix} 0 & iW + d_{12}\cdot\mathscr{E}(t) & d_{13}\cdot\mathscr{E}(t) \\ -iW + d_{12}\cdot\mathscr{E}(t) & \Delta & d_{23}\cdot\mathscr{E}(t) \\ d_{13}\cdot\mathscr{E}(t) & d_{23}\cdot\mathscr{E}(t) & \Delta_{13} \end{pmatrix}$$

$\Delta \rightarrow$ two ground states detuning

$\Delta_{13} \rightarrow$ energy diff. between |1> & |3>

$d_{12} = \langle 1 \,|\, -e\mathbf{r} \,|\, 2 \rangle$

Move on to 3 level system

$$\mathscr{H} = \begin{pmatrix} 0 & iW + d_{12} \cdot \mathscr{E}(t) & d_{13} \cdot \mathscr{E}(t) \\ -iW + d_{12} \cdot \mathscr{E}(t) & \Delta & 0 \\ d_{13} \cdot \mathscr{E}(t) & 0 & \Delta_{13} \end{pmatrix}$$

where $\mathscr{E}(t) = \mathscr{E}_{stark}(t) + \mathscr{E}^{u}_{nr}(t) + \mathscr{E}_{L2}(t)$

$$\begin{cases} \mathscr{E}_{stark} = \mathscr{E}_0 \sin(\omega t), \quad -43.7\mu s < t < 43.7\mu s \\ \mathscr{E}^{u}_{nr} = \mathscr{E}^{u}_{nr0}\, \text{sech}\left(\frac{v(t-t_0)}{\sigma_u}\right) \\ \mathscr{E}_{L2} = \mathscr{E}_{L20} \cos\left(\omega_{L2} t\right), \quad 51.85\mu s < t < 53.35\mu s \end{cases}$$

Move Into Rotating Frame

Because the high oscillating term makes the computation time extremely long, we transform into a rotating frame.

$$U(t) = \text{diag}(1,\ 1, e^{-i\omega_{L2}t})$$

$\omega \rightarrow$ Stark field frequency

$\omega_{13} \rightarrow$ L2 laser frequency

Transformed Hamiltonian

$$\mathscr{H}'_{\pm}(t) = U^{\dagger}(t)H(t)U(t) - iU^{\dagger}(t)\frac{dU(t)}{dt}\,.$$

$$\mathscr{H}'_{\pm}(t) = \begin{bmatrix} 0 & iW + d_{12}\cdot\mathscr{E}(t) & d_{13}\cdot\mathscr{E}(t)e^{-i\omega_{L2}t} \\ -iW + d_{12}\cdot\mathscr{E}(t) & \Delta & 0 \\ d_{13}\cdot\mathscr{E}(t)e^{i\omega_{L2}t} & 0 & \Delta_{13} - \omega_{L2} \end{bmatrix}.$$

RWA

1. $$\mathscr{E}(t) \cdot e^{-i\omega_{L2}t} = (\mathscr{E}_{stark}(t) + \mathscr{E}^{u}_{nr}(t) + \mathscr{E}_{L20}(\frac{e^{i\omega_{L2}t} + e^{-i\omega_{L2}t}}{2})) \cdot e^{-i\omega_{L2}t}$$
$$\approx (\mathscr{E}_{stark}(t) + \mathscr{E}^{u}_{nr}(t)) \cdot e^{-i\omega_{L2}t} + \frac{\mathscr{E}_{L20}}{2}$$
$$\approx \frac{\mathscr{E}_{L20}}{2}$$

2. $$d_{12} \cdot \mathscr{E}(t) \approx d_{12} \cdot (\mathscr{E}_{stark}(t) + \mathscr{E}^{u}_{nr}(t)) = d_{12} \cdot \mathscr{E}_{nL2}(t)$$

Let $W = 0$

$$\mathscr{H}'_{\pm}(t) \approx \begin{bmatrix} 0 & d_{12} \cdot \mathscr{E}_{nL2}(t) & \frac{\Omega_{13}(t)}{2} \\ d_{12} \cdot \mathscr{E}_{nL2}(t) & \Delta & 0 \\ \frac{\Omega_{13}(t)}{2} & 0 & \Delta_{13} - \omega_{L2} \end{bmatrix}. \qquad \Omega_{13}(t) = d_{13} \cdot \mathscr{E}_{L20}(t)$$

Find $$\Omega_{13} = d_{13} \cdot \mathscr{E}_{L2} = \sqrt{\frac{3I\Gamma\lambda^3}{4\pi^2 c}}$$

The laser cross-section profile is a ellipse with a long axis 11.5mm and a short axis of 0.92mm. Area $A \approx 8.31mm^2$.

Plug in $I = P/A = 8mW/8.31mm^2$ $\lambda = 860nm$ $\Gamma = 2.7MHz$

From $I = \frac{1}{2}\epsilon_0 c \mathscr{E}_{L2}^2$, we can derive

$\mathscr{E}_{L2} = 8.514$V/cm

gives $\Omega_{13} = 9.67 \times 10^7 rad/s$, and $d_{13} \approx 1.136 \times 10^5$ rad/(s·V/m) .

We also need to consider the decay of the excited state:

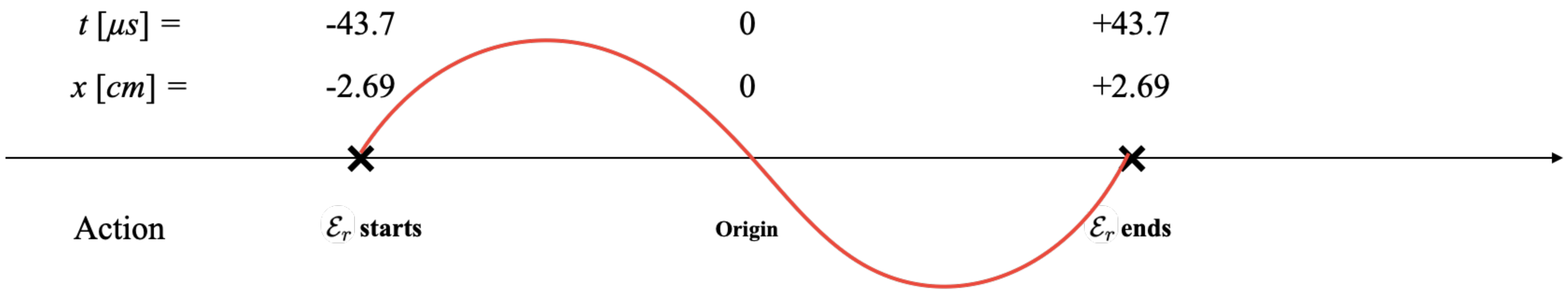
$$\frac{d\rho}{dt} = -\, i[\mathscr{H}'_{\pm}, \rho] + \mathscr{L}(\rho)$$

where the dissipator

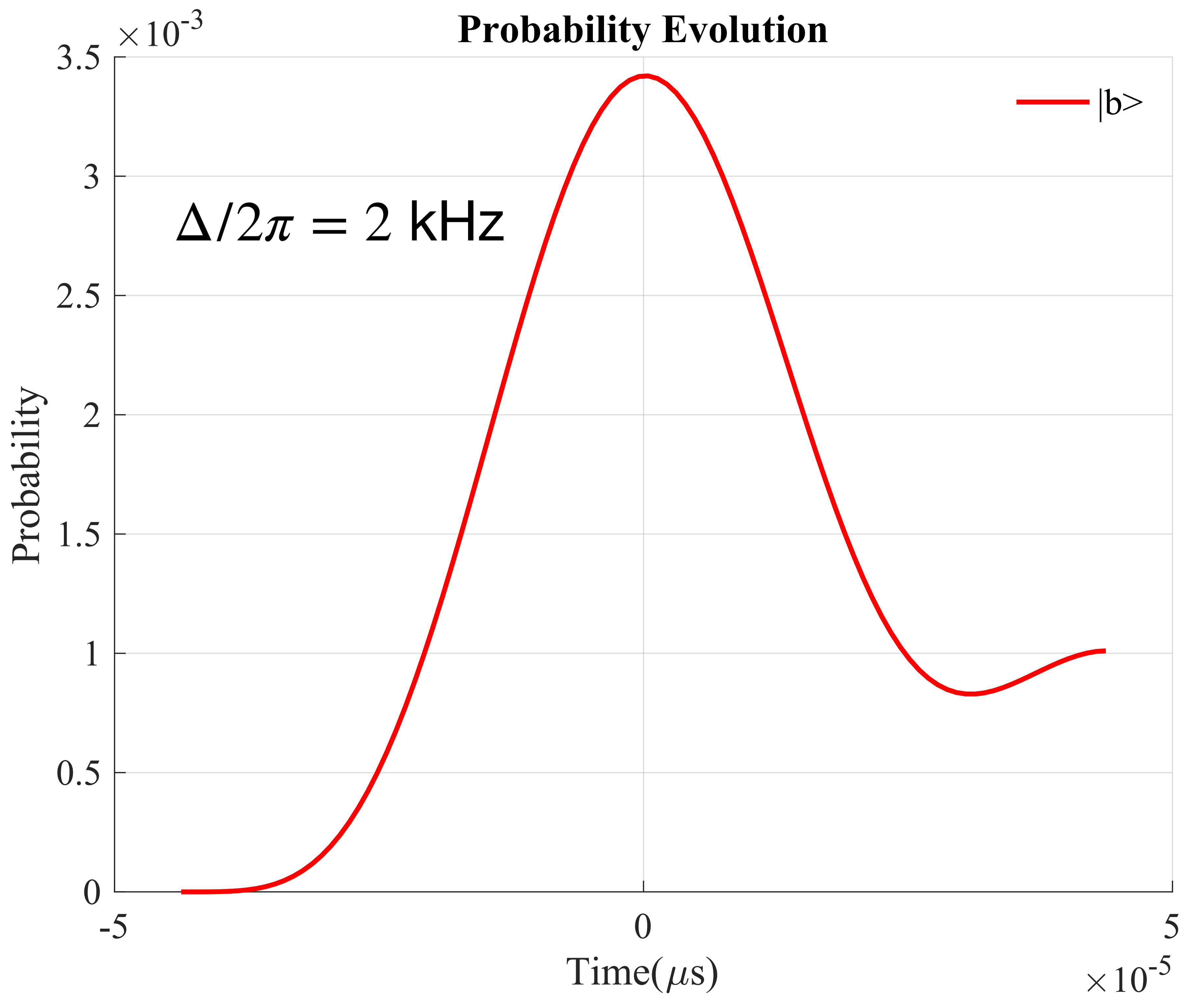
$$\mathscr{L}(\rho) = \begin{pmatrix} 0 & 0 & -\frac{\Gamma}{2}\rho_{13} \\ 0 & 0 & -\frac{\Gamma}{2}\rho_{23} \\ -\frac{\Gamma}{2}\rho_{31} & -\frac{\Gamma}{2}\rho_{32} & -\Gamma\rho_{33} \end{pmatrix}.$$

$\Gamma =$ decay constant, so that $\left\{ \frac{\mathrm{d}}{\mathrm{d}t}\rho_{33} \right\}_{rel} = -\,\Gamma\rho_{33}$

Test Simulation (3-levels)

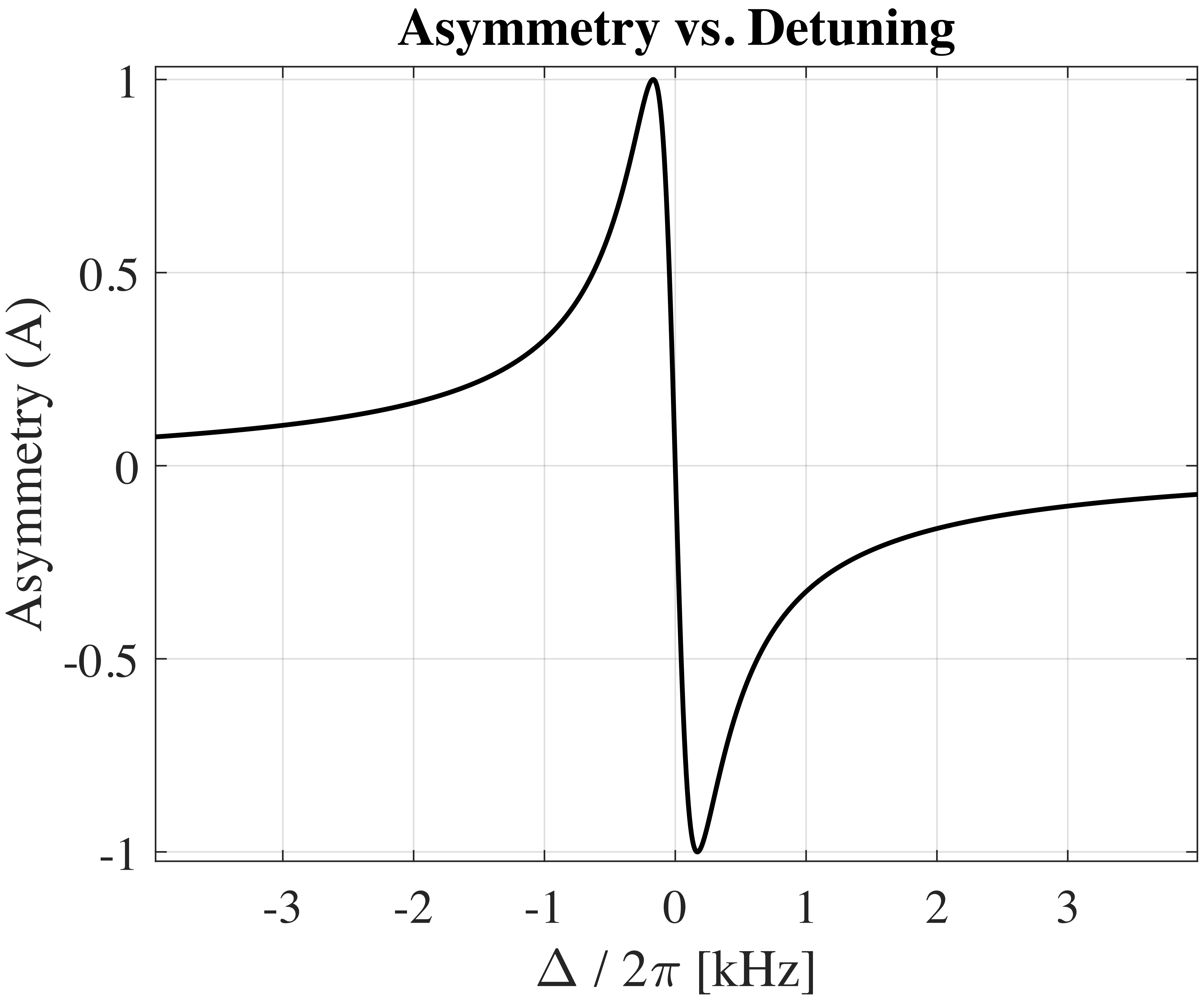


Deliberately **remove all coupling** between third state to other states in the system, i.e. $d_{13} = d_{23} = 0$, and **consider only the stark field**, so it is completely equivalent to the two level system we just simulated

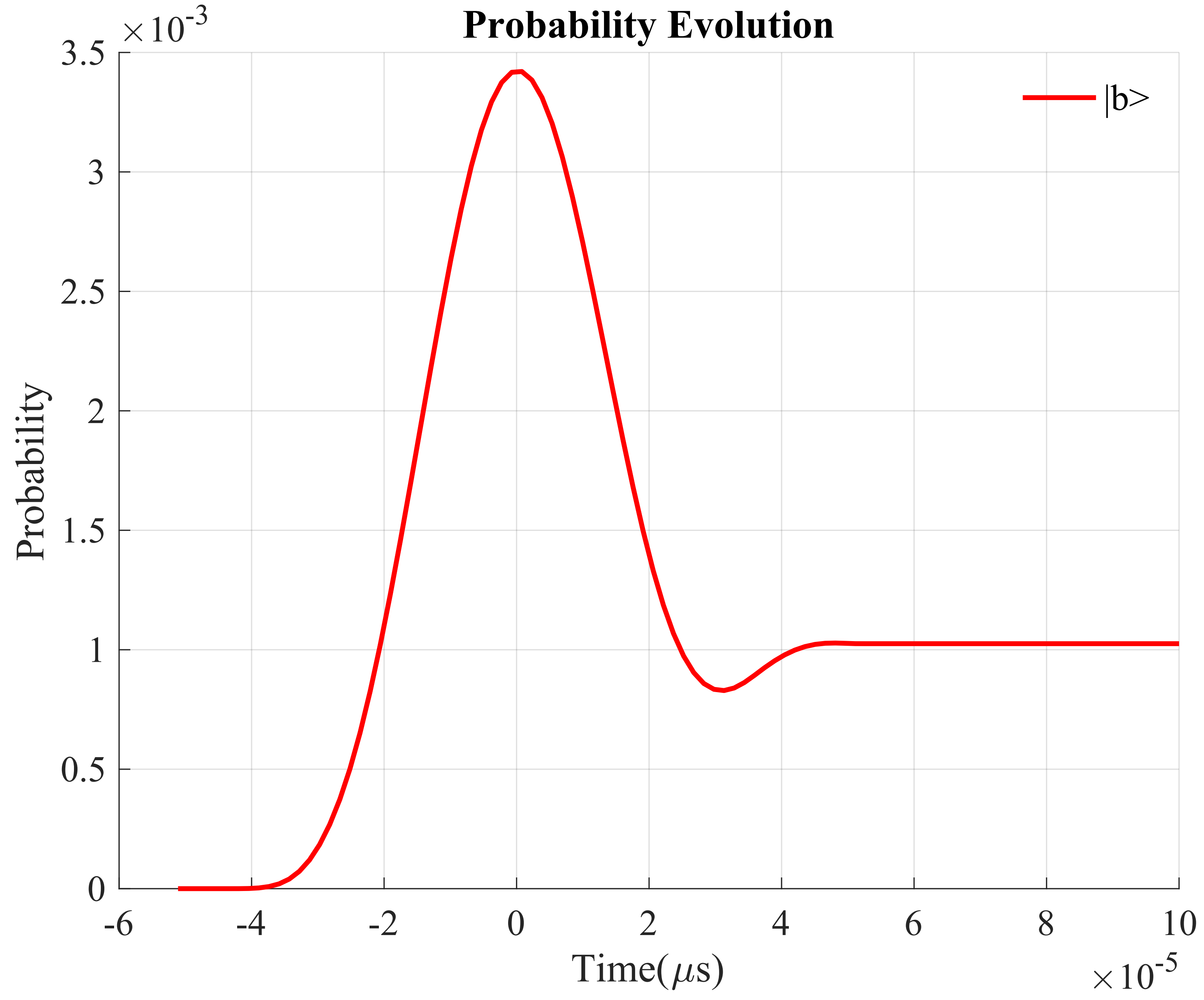


Test Simulation

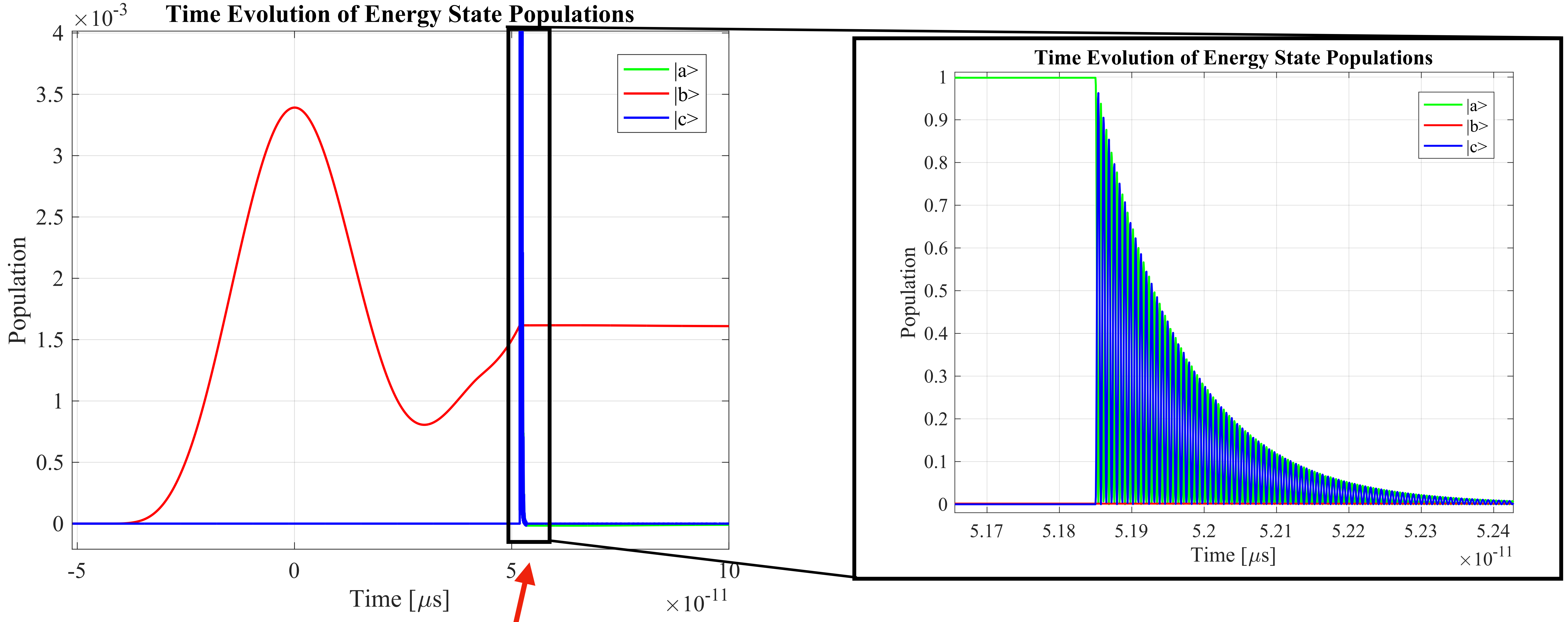
Exactly the same result as in 2-level system



Only Stark field



Including all fields



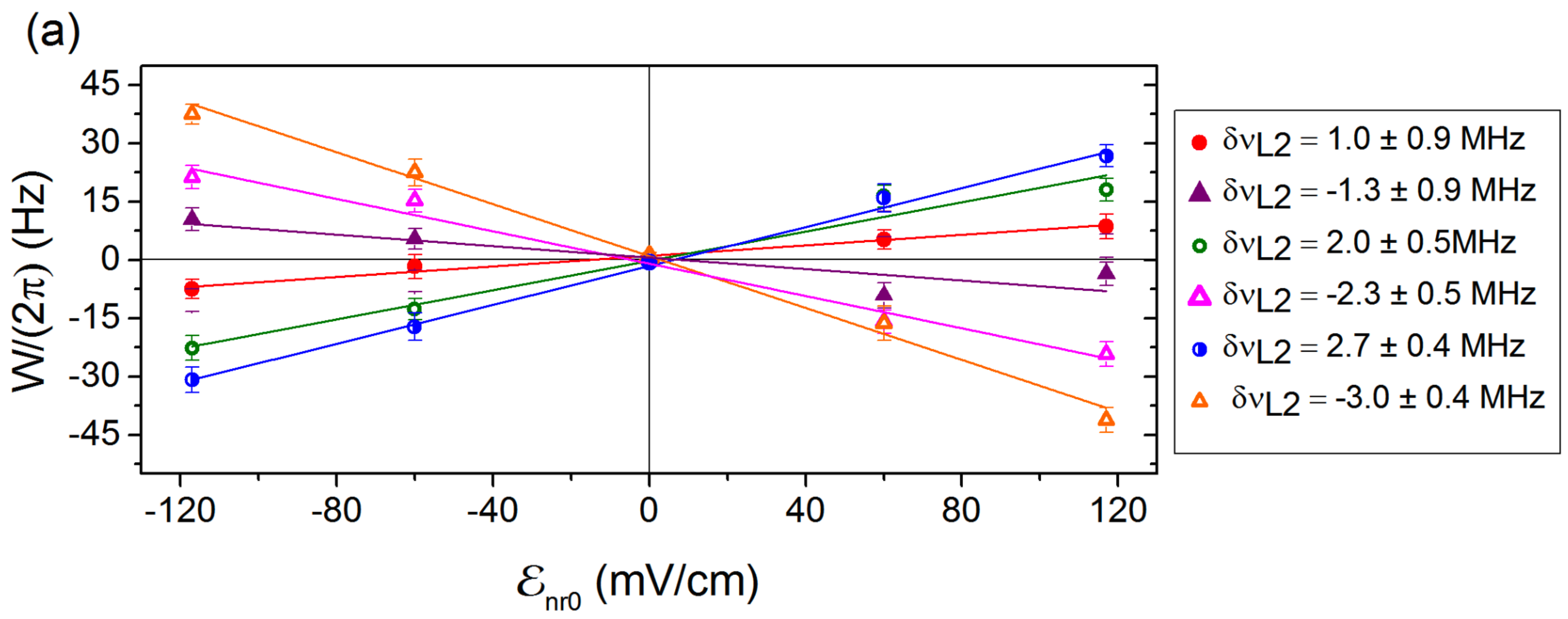
Depletion Laser Applied

$$\boxed{-51.1 < t < 100\mu s}$$

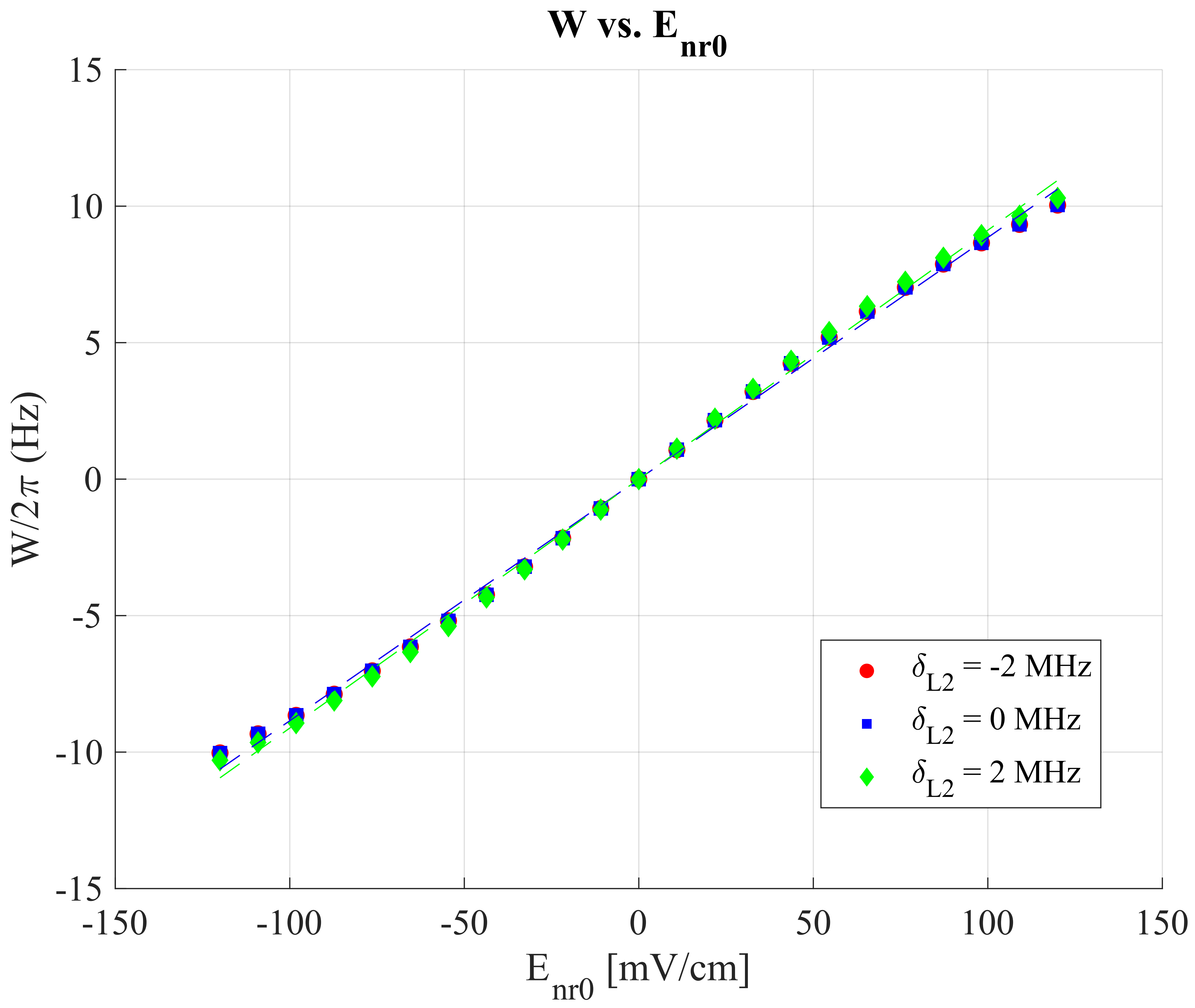
$$\begin{cases} \delta\nu_{L2} = 3\,MHz \\ d_{13} = 10^5\,rad/s \\ d_{23} = 0\,rad/s \\ E_{0,stark} = 0.1\,V/cm \\ E_{0,nr} = -0.12\,V/cm \\ E_{0,L2} = 85.14\,V/cm \\ \Gamma = 2.7\times 10^6 \times 2\pi\,rad/s \\ \sigma_u = 0.76\,cm \end{cases}$$

$\mathcal{E}_{nr0}$ vs. W

Expected:

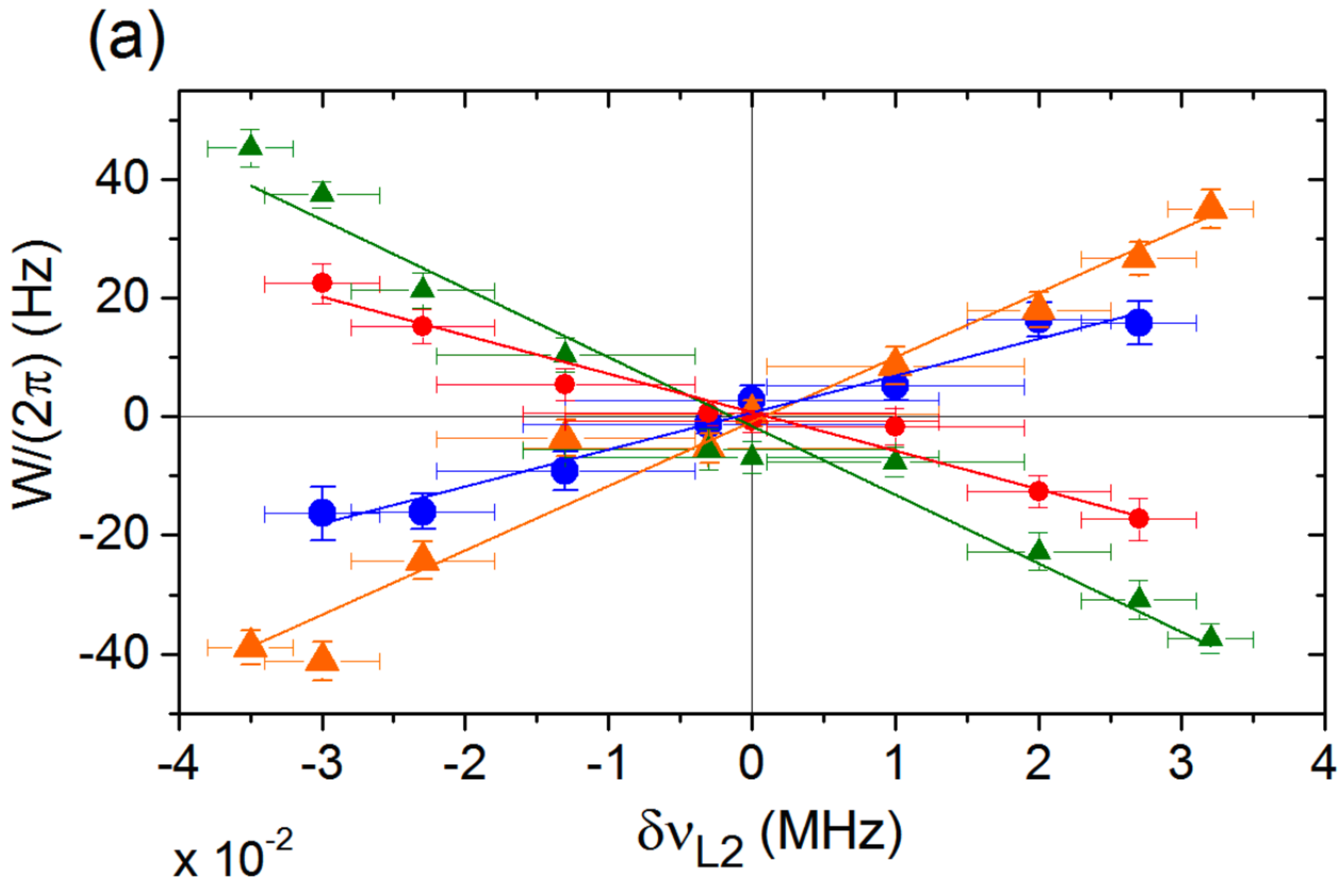


From Simulation:

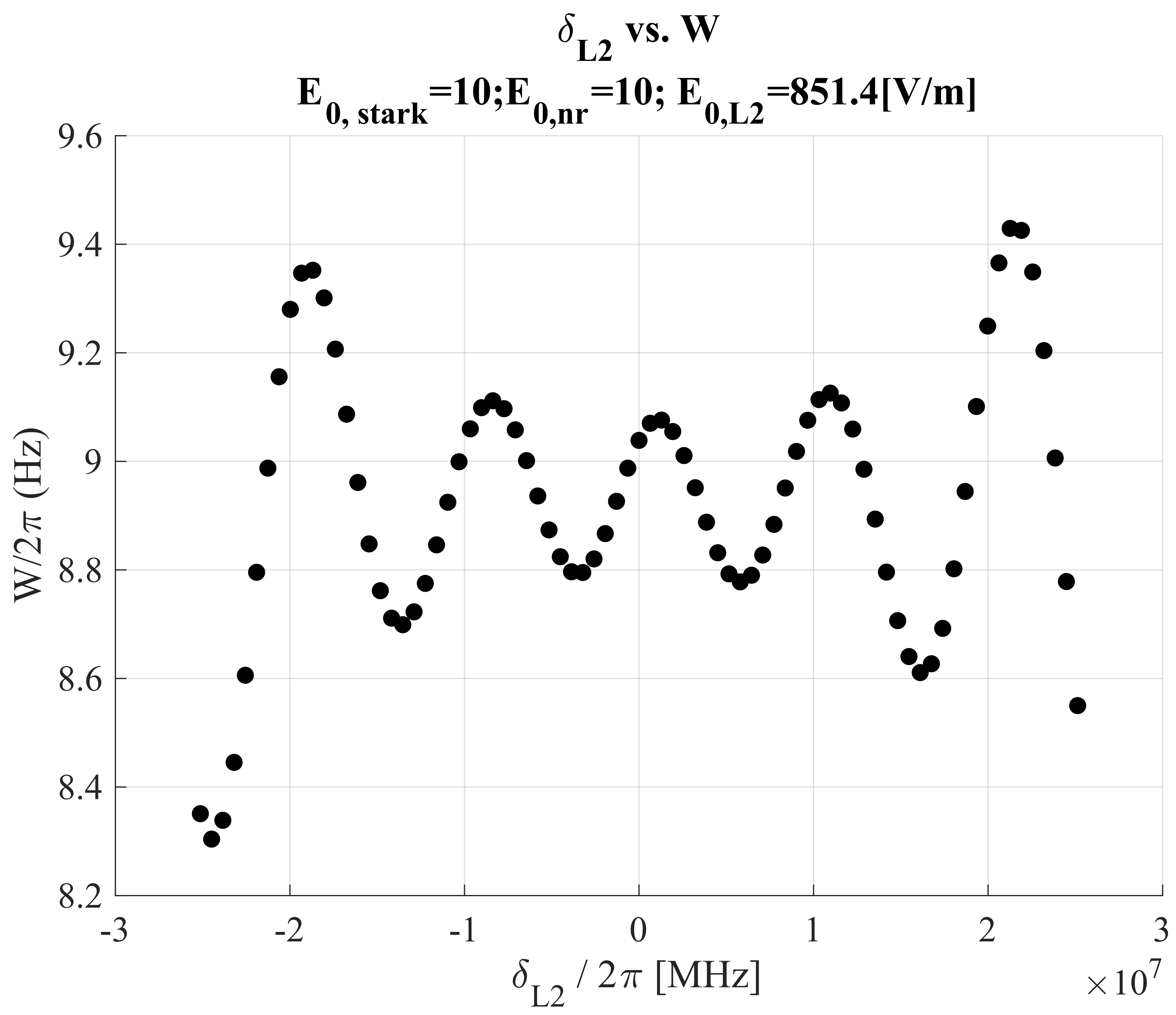


δ_{L2} vs. W

Expected:

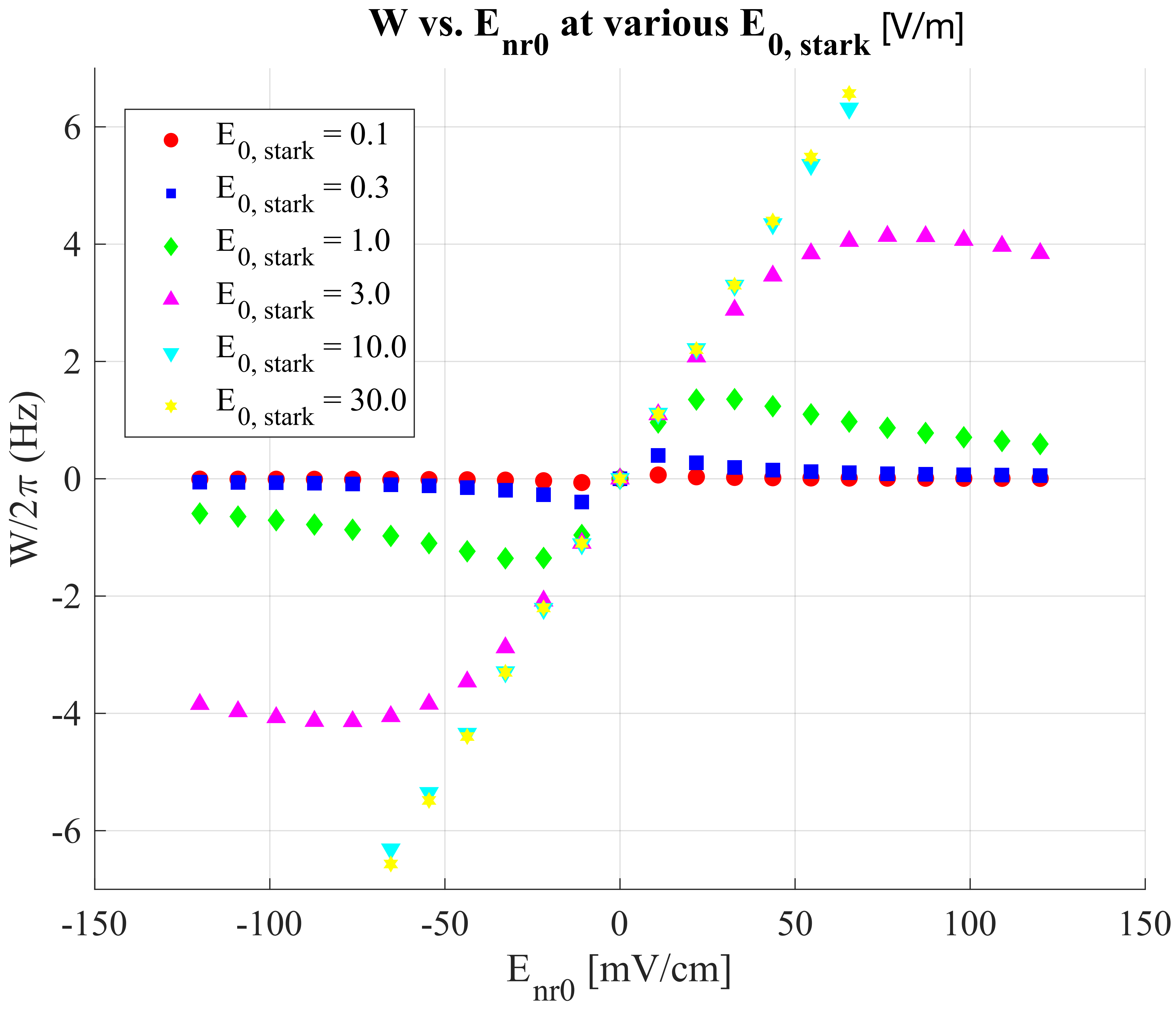


From Simulation:



Observations

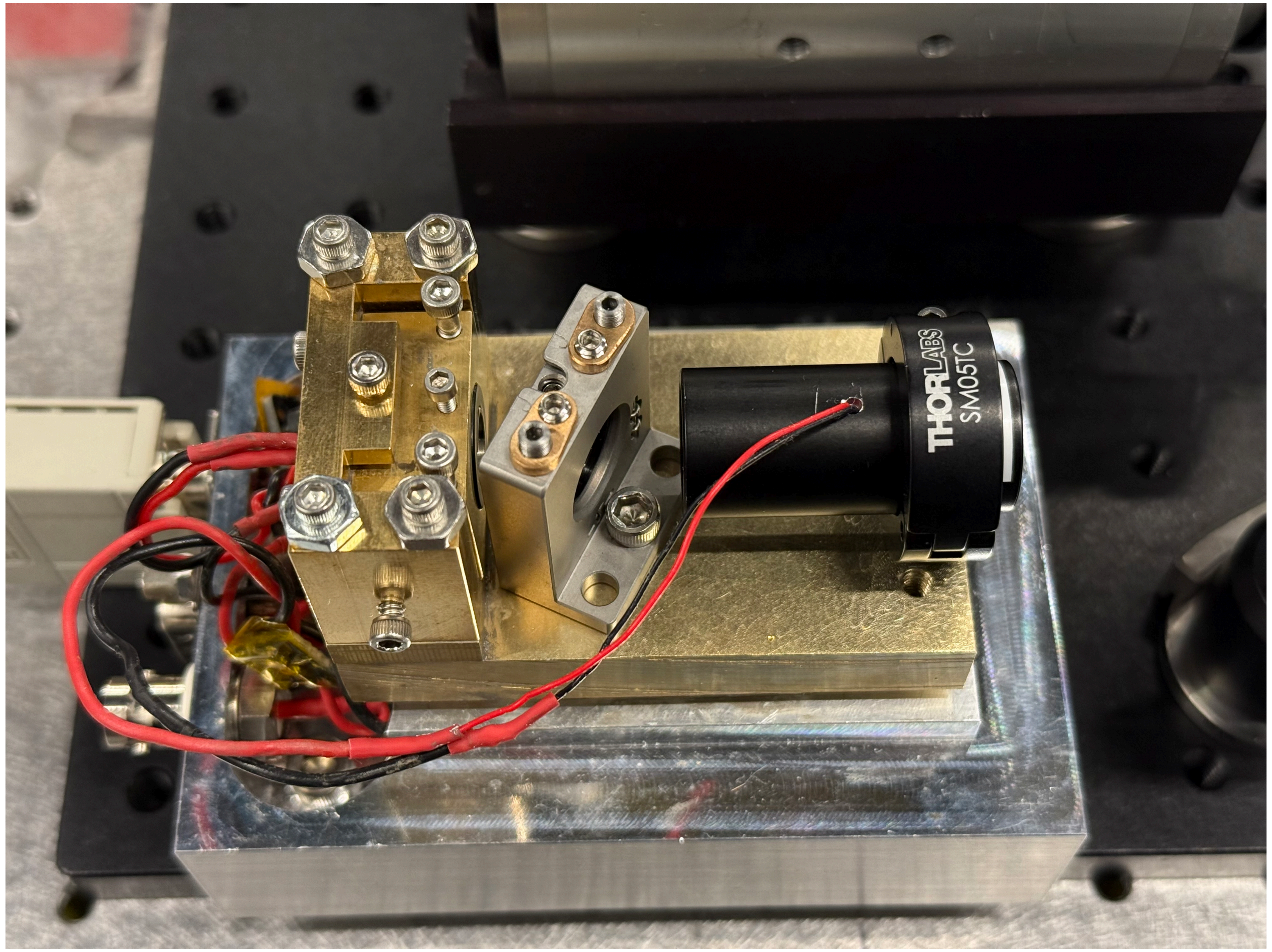
1. if $\mathscr{E}_{stark} = 0$, then $W = 0$.
2. $W \neq 0$ even when $\mathscr{E}_{L2} = 0$.

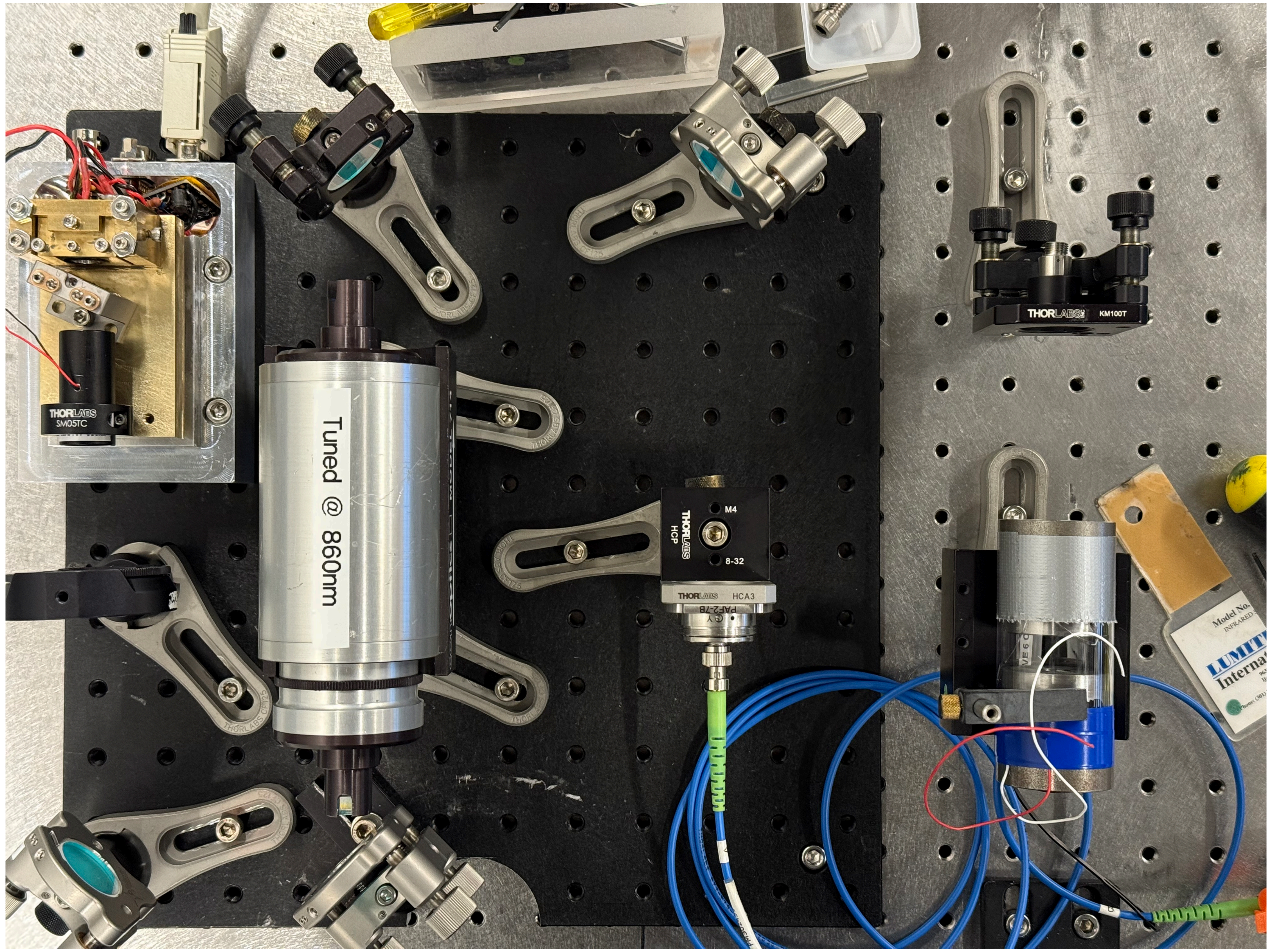


All The Constants and Variables

Name	Notation	Value	unit
Molecule Beam Velocity	v	616	m/s
Dipole Matrix Element between opposite parity ground states	d_{12}	$-3360 \times 2\pi$	Hz/(V/cm)
Excitation Length	L_e	5.38	cm
Sine Wave Field Amplitude	$\mathcal{E}_0$	(~ 1.0)	V/cm
Non-reversing Field Amplitude (unipolar)	$\mathcal{E}_{\mathrm{nr}0}$	(~ 0.42)	mV/cm
Non-reversing Field Width (unipolar)	σ_u	0.76	cm
2nd Depletion Laser Detuning Offset	$\delta\nu_{L2}$	$(\sim \pm 3)$	MHz
B-field Required For The Zeeman Shift	$\mathcal{B}_0$	(~ 1)	T
B-field Dependent Detuning From Degeneracy	Δ/ω	$(\sim 2 * 10^2)$	Hz
Energy Between Excited State and Ground	Δ_{13}/ω_{13}	348.69	THz
NSD-PV Weak Matrix Element	W	$(5 \times 2\pi)$	Rad/s
Depletion Laser Diameter	d_{L2}	0.92	mm
Depletion Laser Frequency	ν_{L2}	348.69	THz
Depletion Laser Power	$\mathcal{P}_{\mathrm{L2}}$	8	mW
Depletion Laser Amplitude	$\mathcal{E}_{\mathrm{L2}}$	8.514	V/cm
Excited Level Decay rate	γ_c	$2.7 \times 10^6 \times 2\pi$	Rad/s

ECDL update

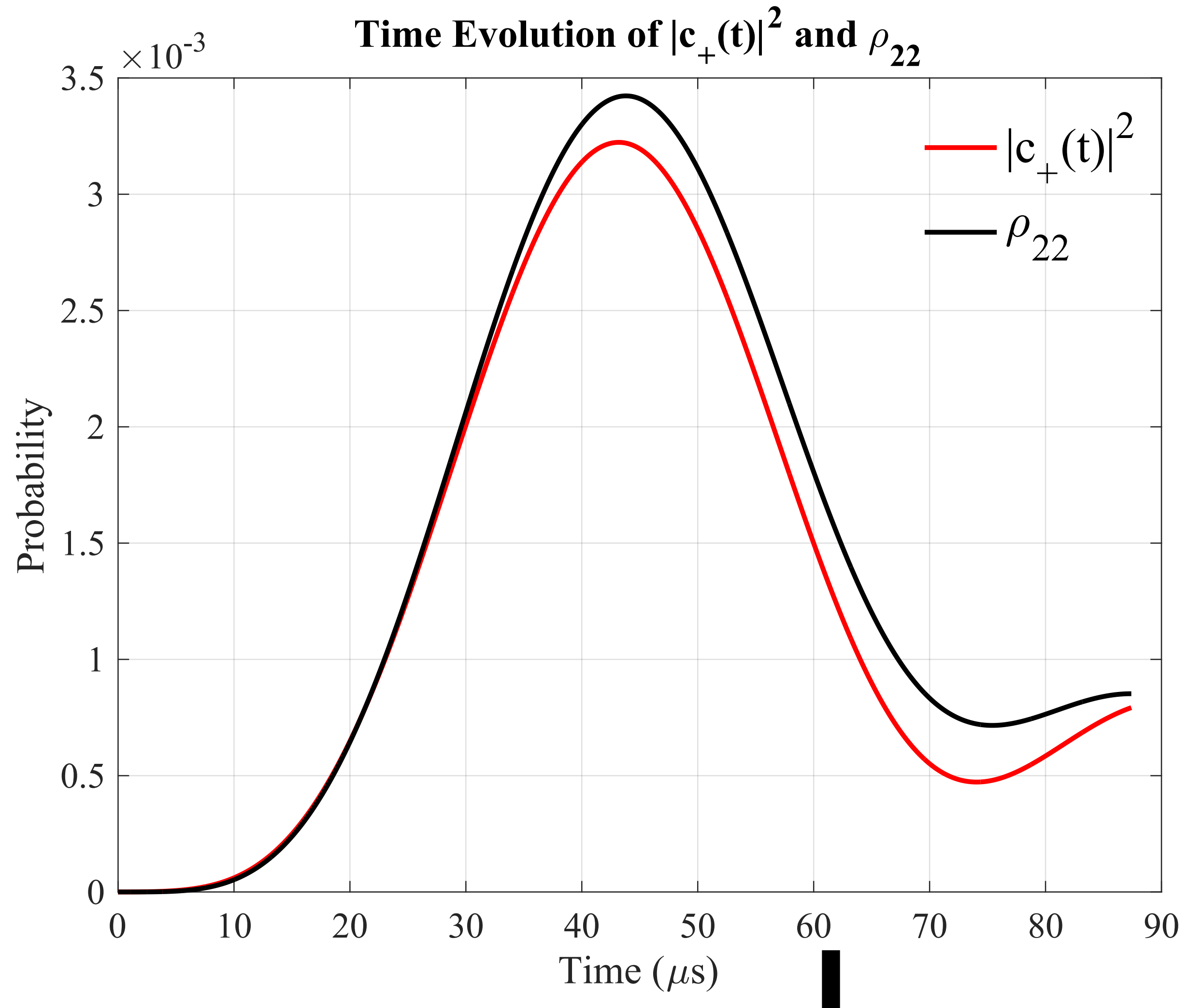


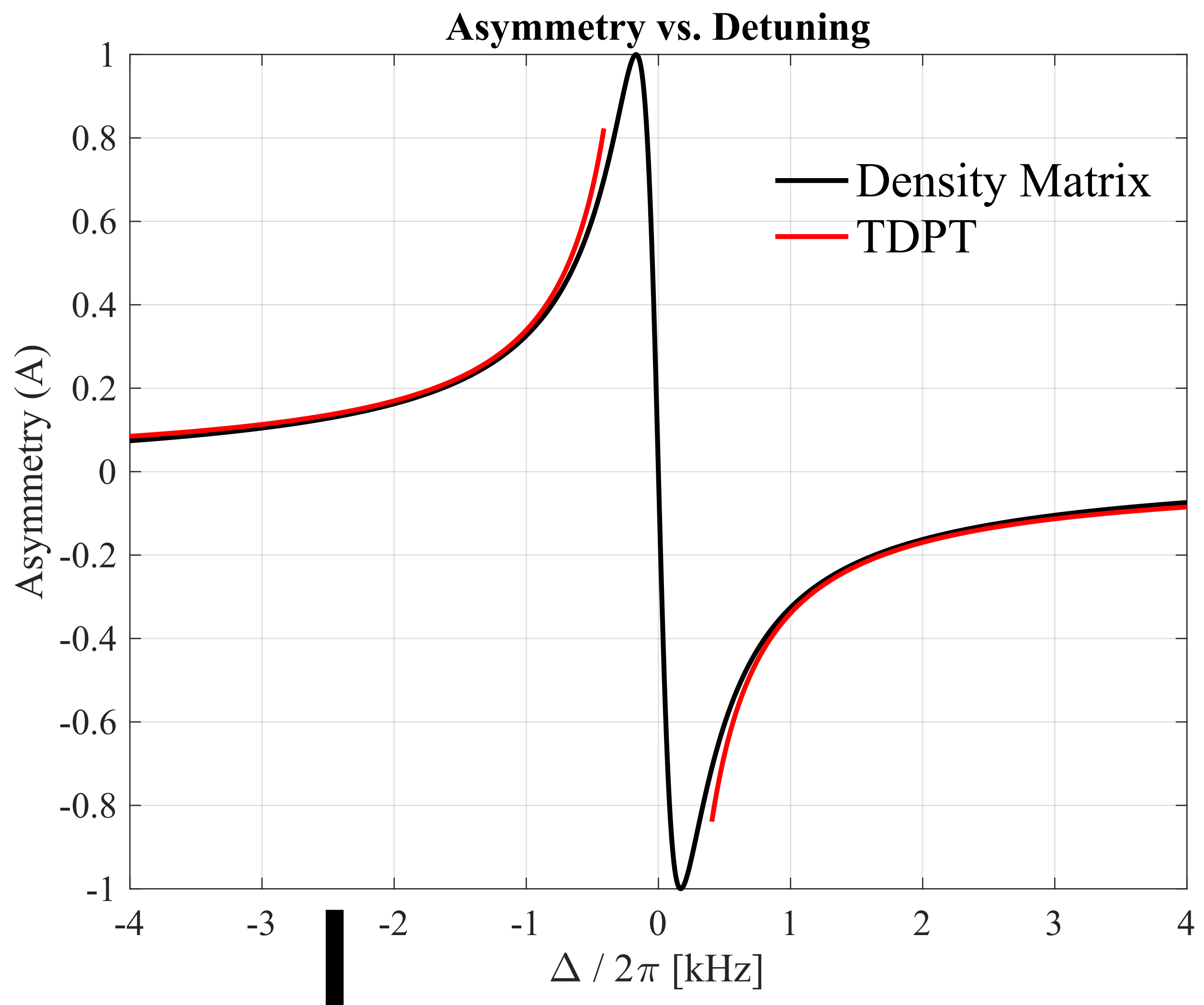


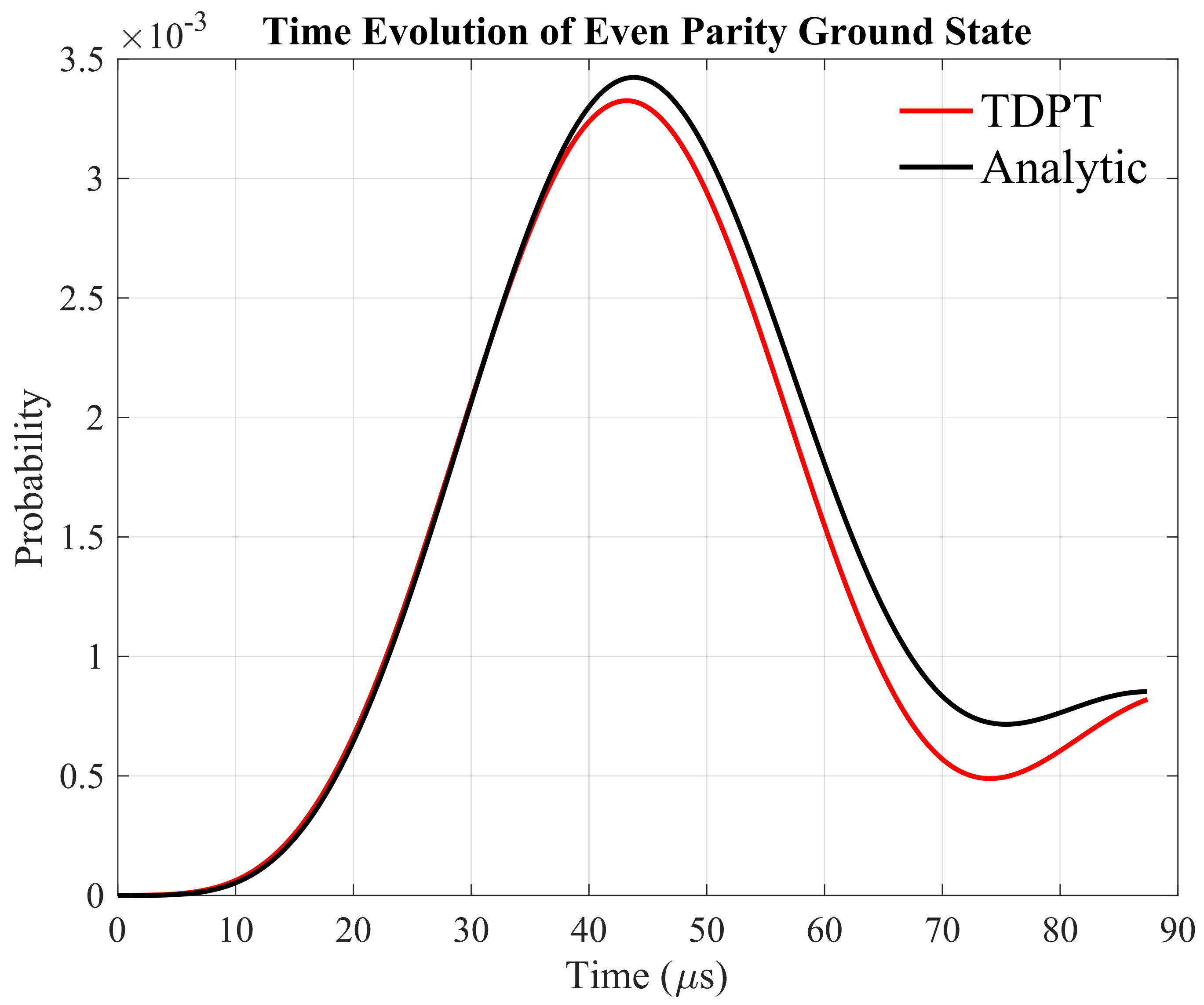
To do list

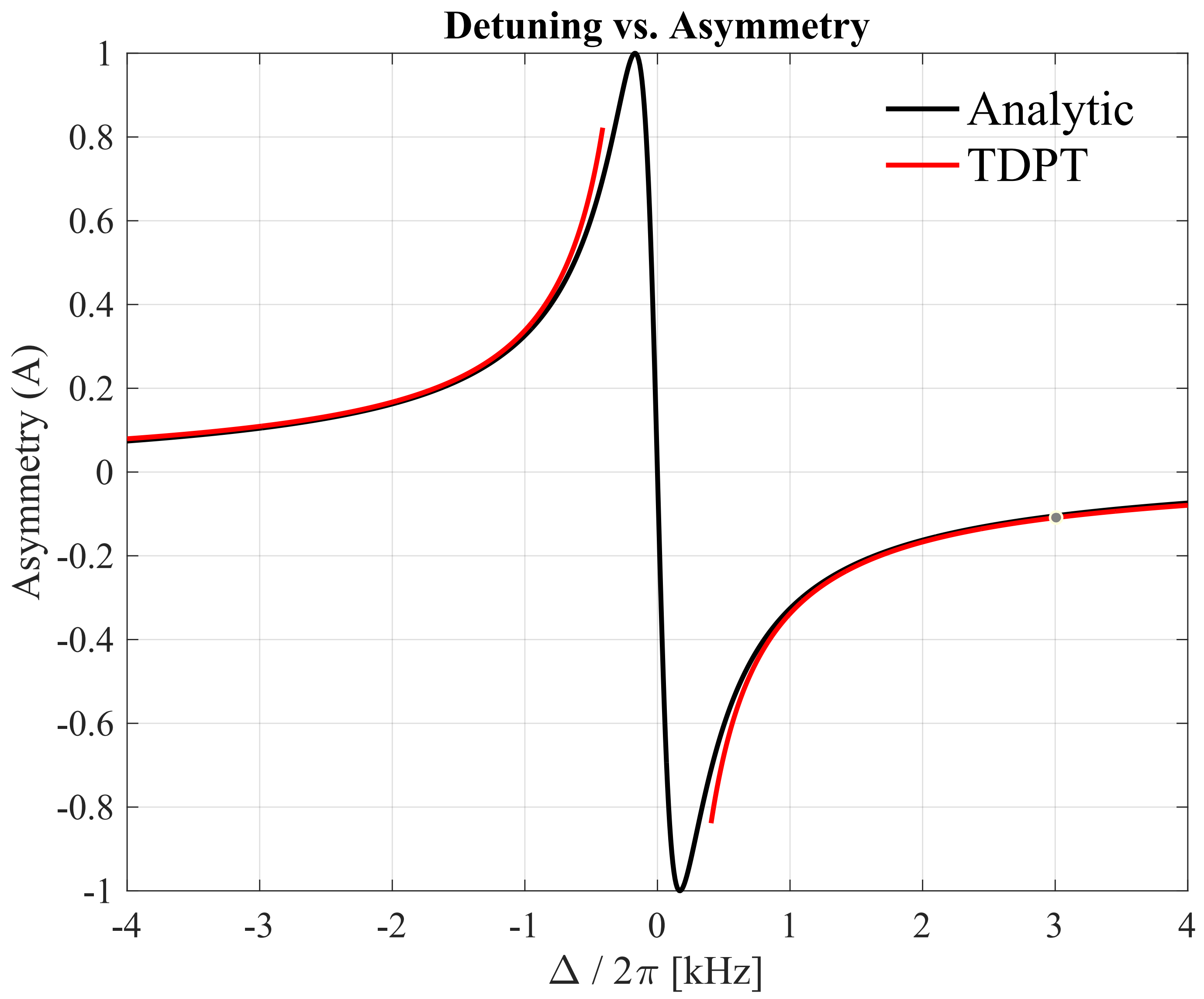
1. Build a new Fabry-Perot cavity for 860nm
2. New Filter from Alluxa / Semrock (currently asking)
3. Buy some optical fibers for 860nm

More accurate TDPT result 2025/1/28





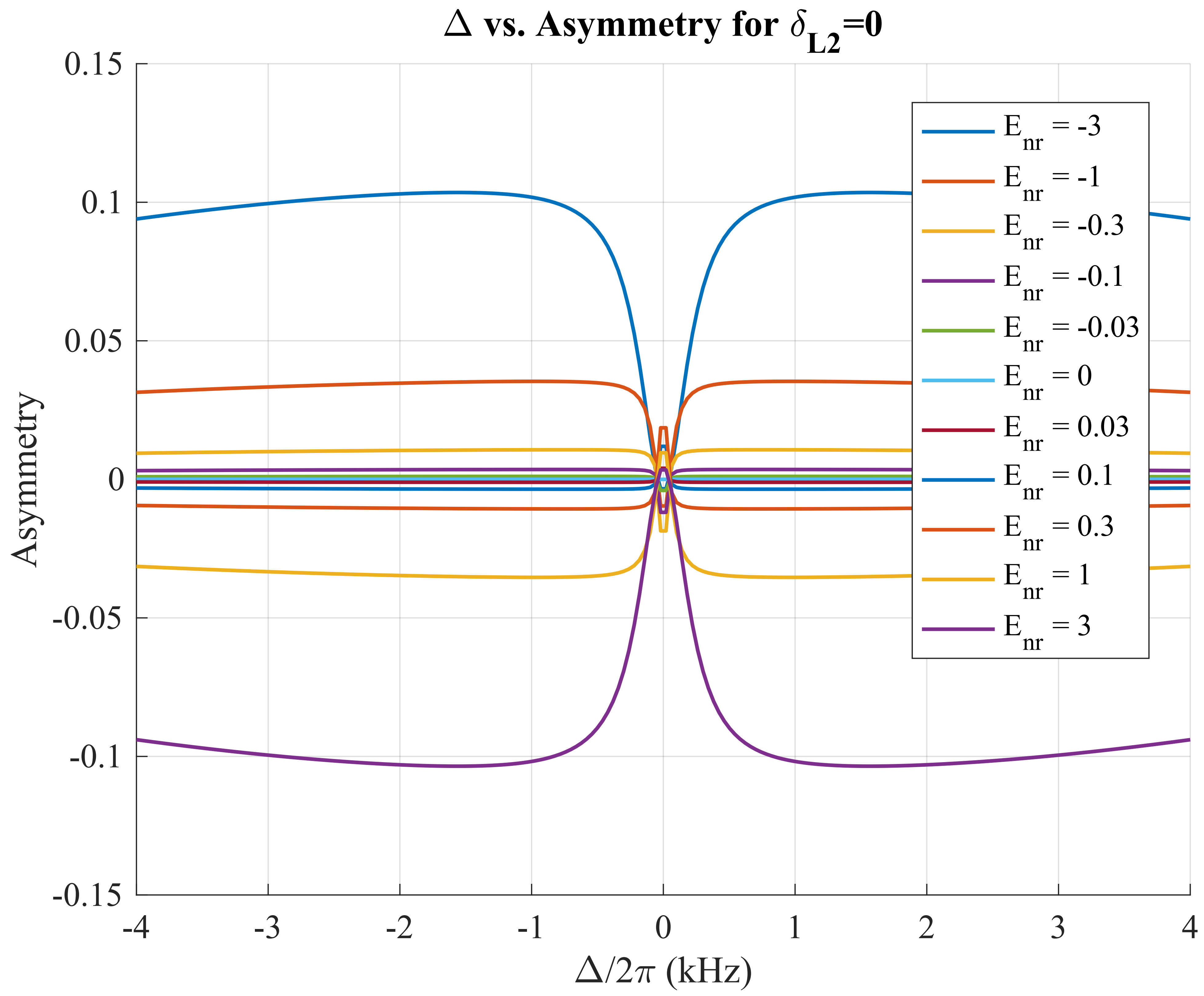


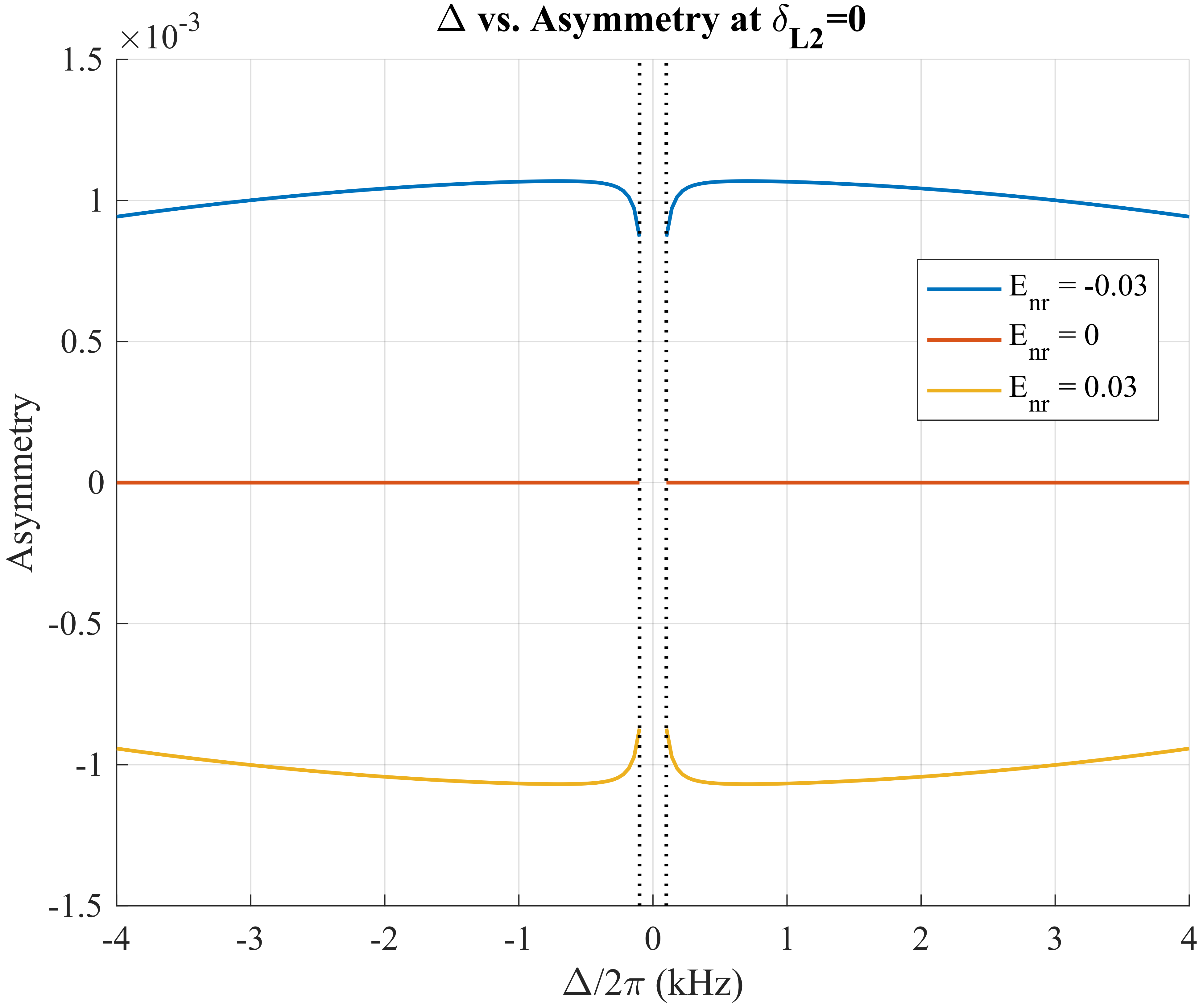


More accurate TDPT result.
(doesn't changed much)

Δ vs. Asymmetry

Asymmetry is even in Δ

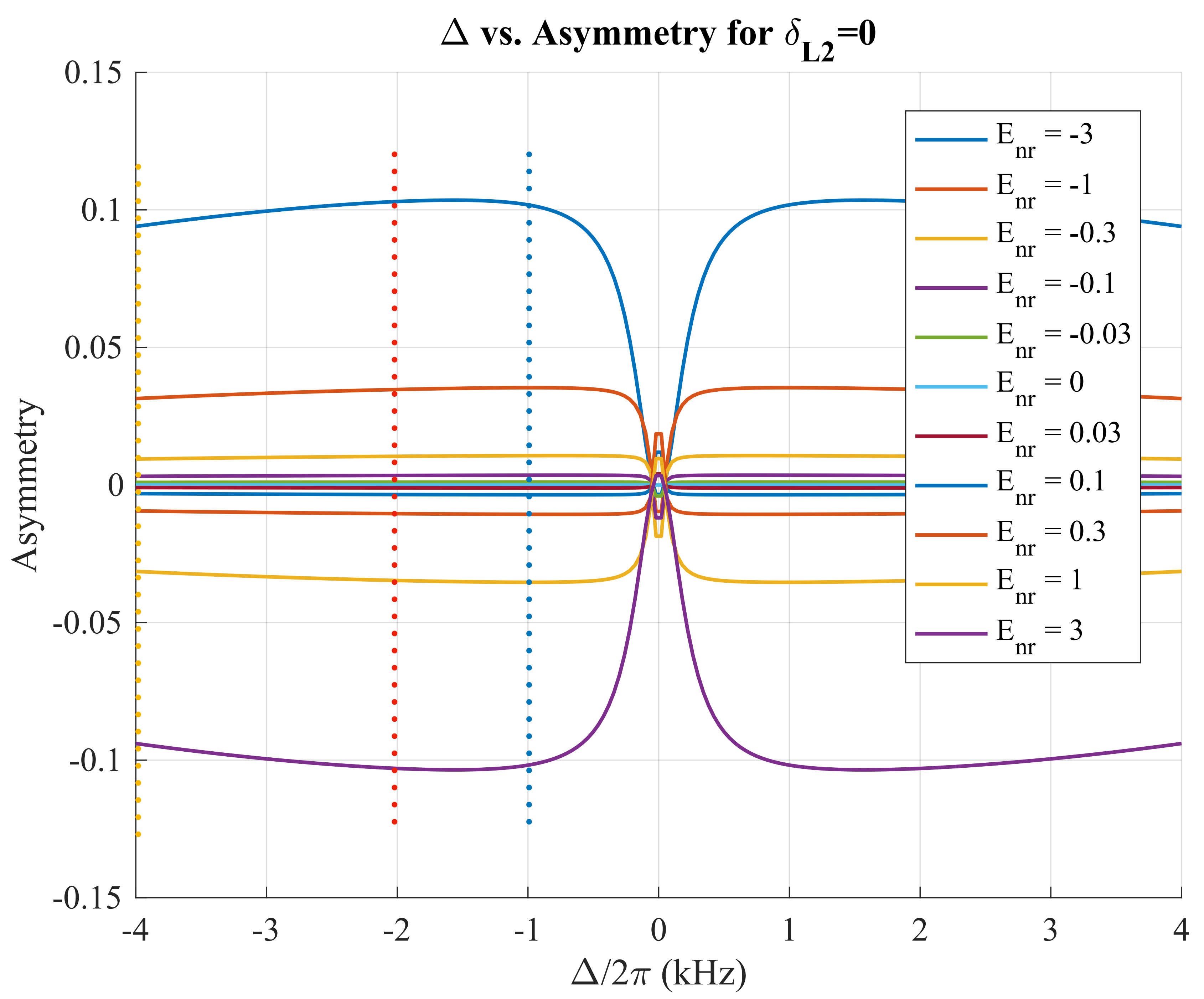


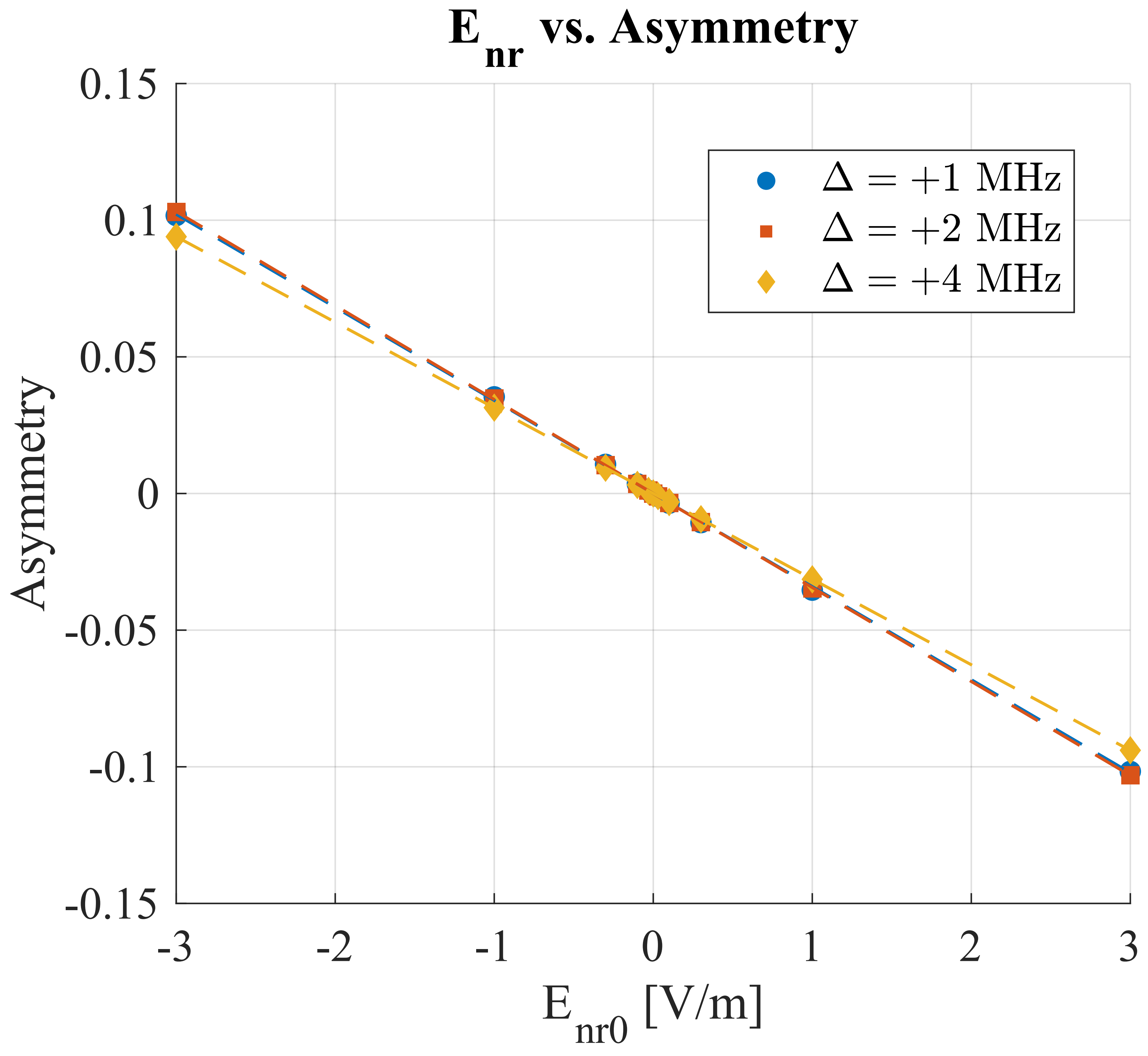


Asymmetry offset seems to be proportional to the non-reversing field strength

L2 Laser detuning is set to 0

The Asymmetry Shift by Non-zero $\mathscr{E}_{nr}$





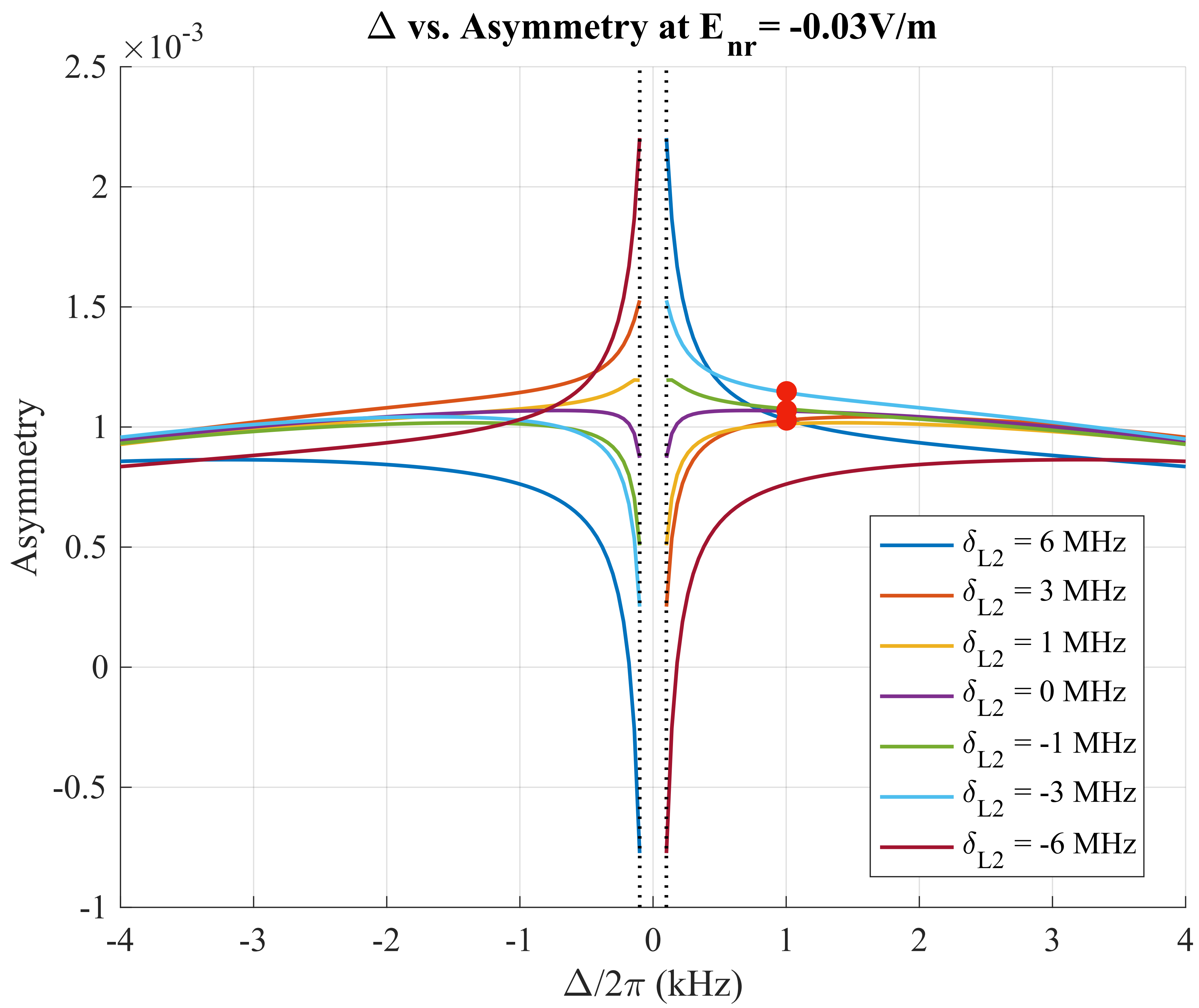
Can be modeled using a linear function: $y = ax + b, a \approx -0.034$

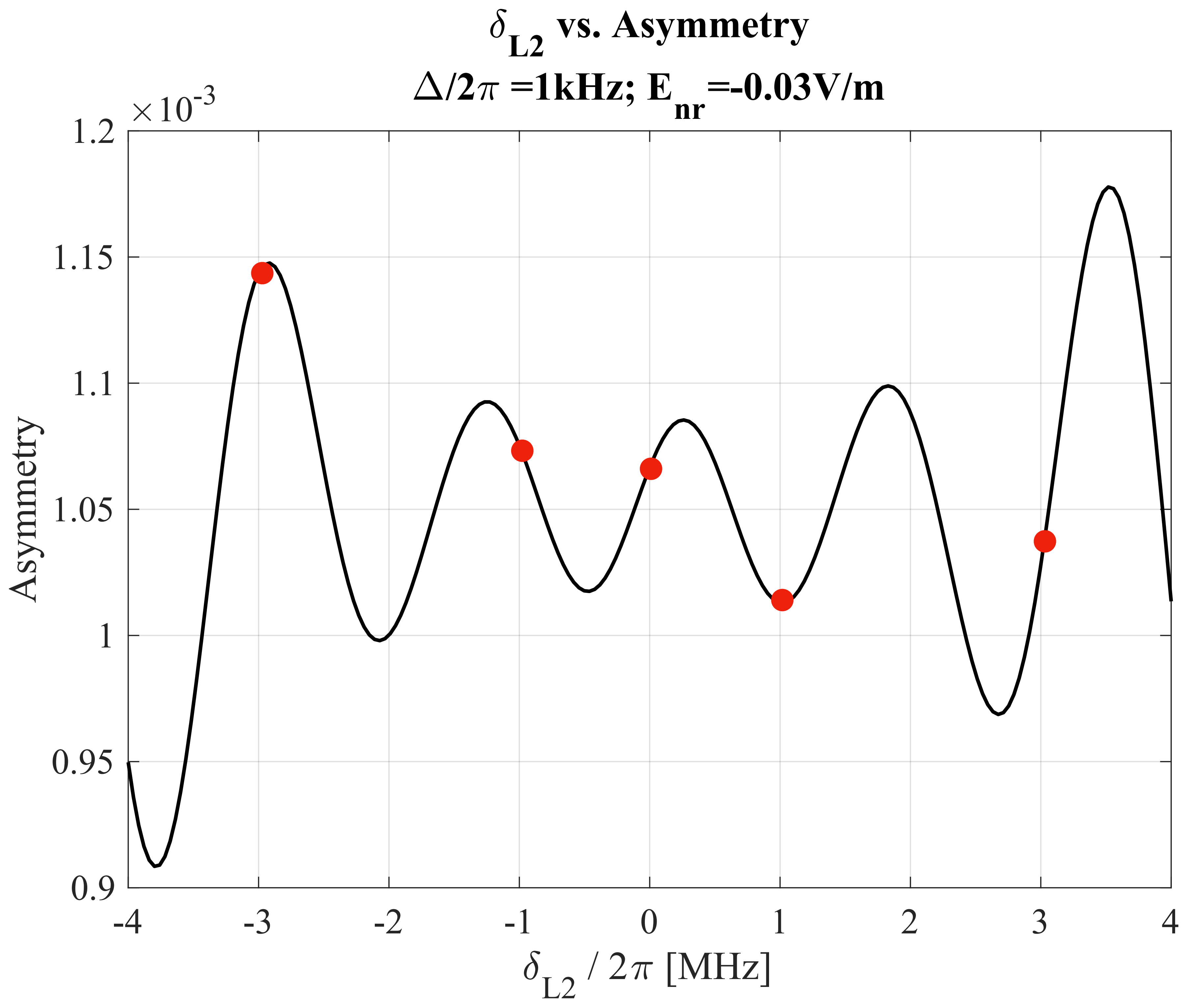
Δ (MHz)	a	b
1	-0.0341	~0
2	-0.0344	~0
4	-0.0313	~0

"a" is not strongly dependent on Δ (1~4MHz) for $\mathscr{E}_{nr} < 30mV/cm$

Scan δ_{L2} (Something weird)

when L2 detuning is not 0, $\mathscr{A}(\Delta)$ is no longer even, but wiggles around when scanning δ_{L2}





A More Precise Model for Asymmetry

I previously used the wrong formula to extract W: $\mathscr{A} = 2\frac{W}{\Delta}\frac{\omega}{d\mathscr{E}_0}$, which is obviously incorrect.

The right fitting function to use is:

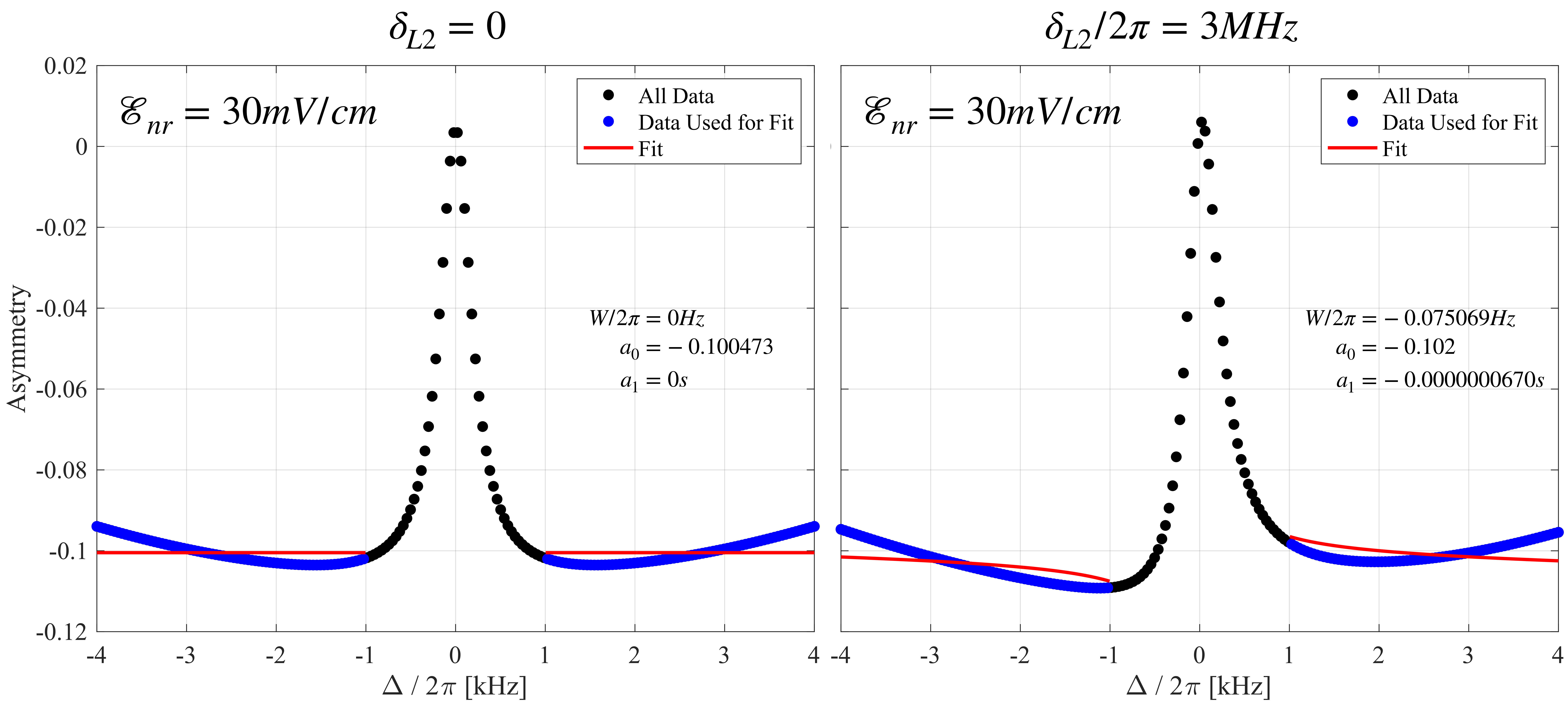
$$\mathscr{A}_{\text{fit}} = W\mathscr{A}(\Delta) + a_0 + a_1 \cdot \Delta$$

$$\mathscr{A}(\Delta) = \frac{2W}{\Delta} \cdot \frac{\omega_{stark}^2 - \Delta^2}{d_{12}E_0\omega_{stark}} \cdot \frac{\sin(\frac{\Delta}{2}(T_e + T_{f1} + T_{f2}))}{\sin(\frac{\Delta}{2}T_e)} \cdot \cos(\frac{\Delta}{2}(T_{f1} - T_{f2}))$$

where W, a_0, a_1 are three parameters to be optimized. I used least square fit to model the relationship between asymmetry and detuning Δ, while excluding data points in range [-1kHz, 1kHz].

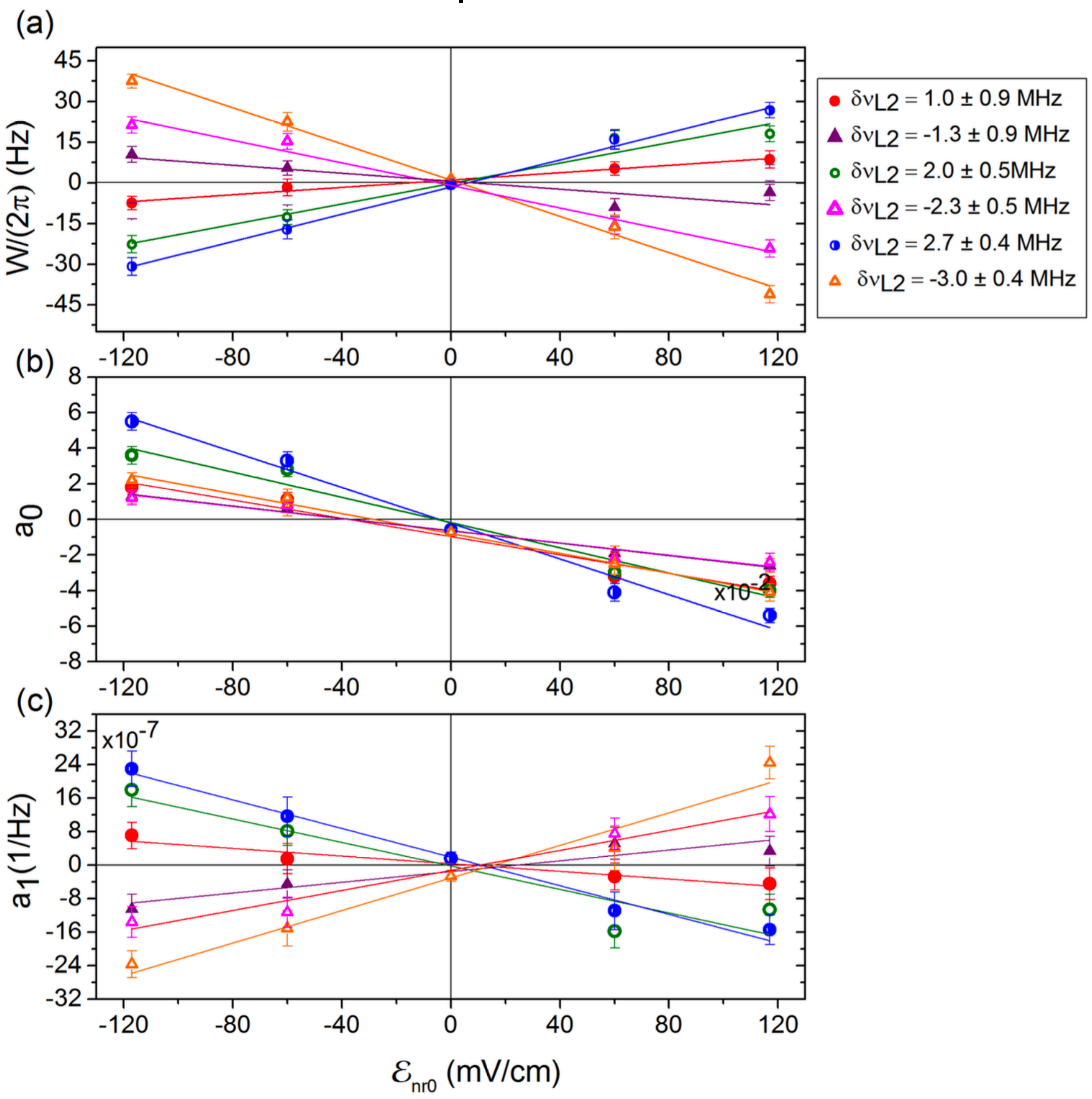
*An **error function** is defined as the sum of squared differences between the experimental data (asymmetry_fit) and the model function (fit_function). MATLAB's **fminsearch** function is used to minimize this error and determine the best-fit values for* `W`*,* `a_0`*, and* `a_1`*.*

Examples

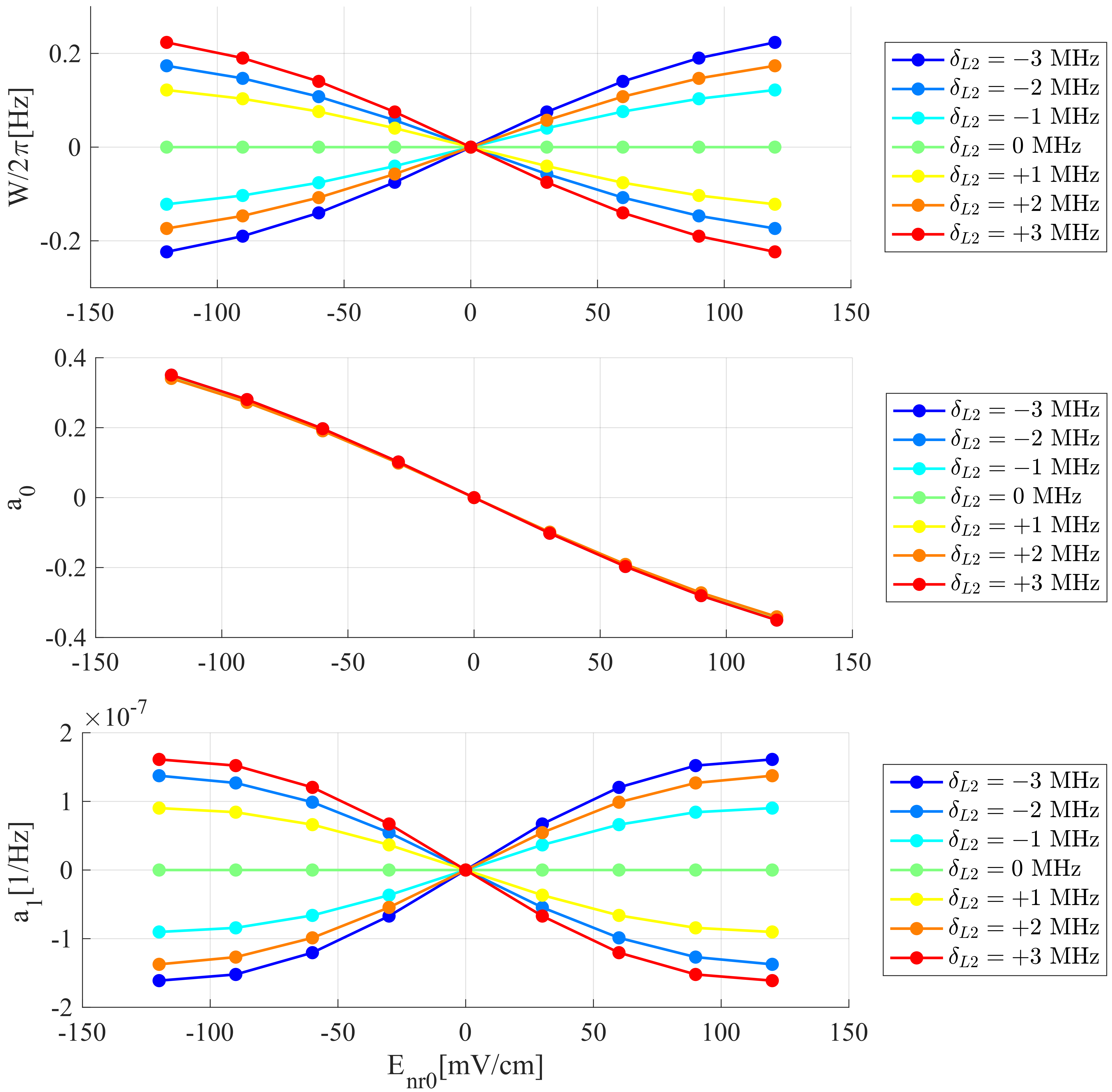


$\delta_{L2} = 0$
$\delta_{L2}/2\pi = 3MHz$
$\mathscr{E}_{nr} = 30mV/cm$
All Data
Data Used for Fit
Fit
$W/2\pi = 0Hz$
$a_0 = -0.100473$
$a_1 = 0s$
$\mathscr{E}_{nr} = 30mV/cm$
All Data
Data Used for Fit
Fit
$W/2\pi = -0.075069Hz$
$a_0 = -0.102$
$a_1 = -0.0000000670s$
Asymmetry
0.02
0
-0.02
-0.04
-0.06
-0.08
-0.1
-0.12
-4 -3 -2 -1 0 1 2 3 4
Δ / 2π [kHz]
-4 -3 -2 -1 0 1 2 3 4
Δ / 2π [kHz]

Experiment



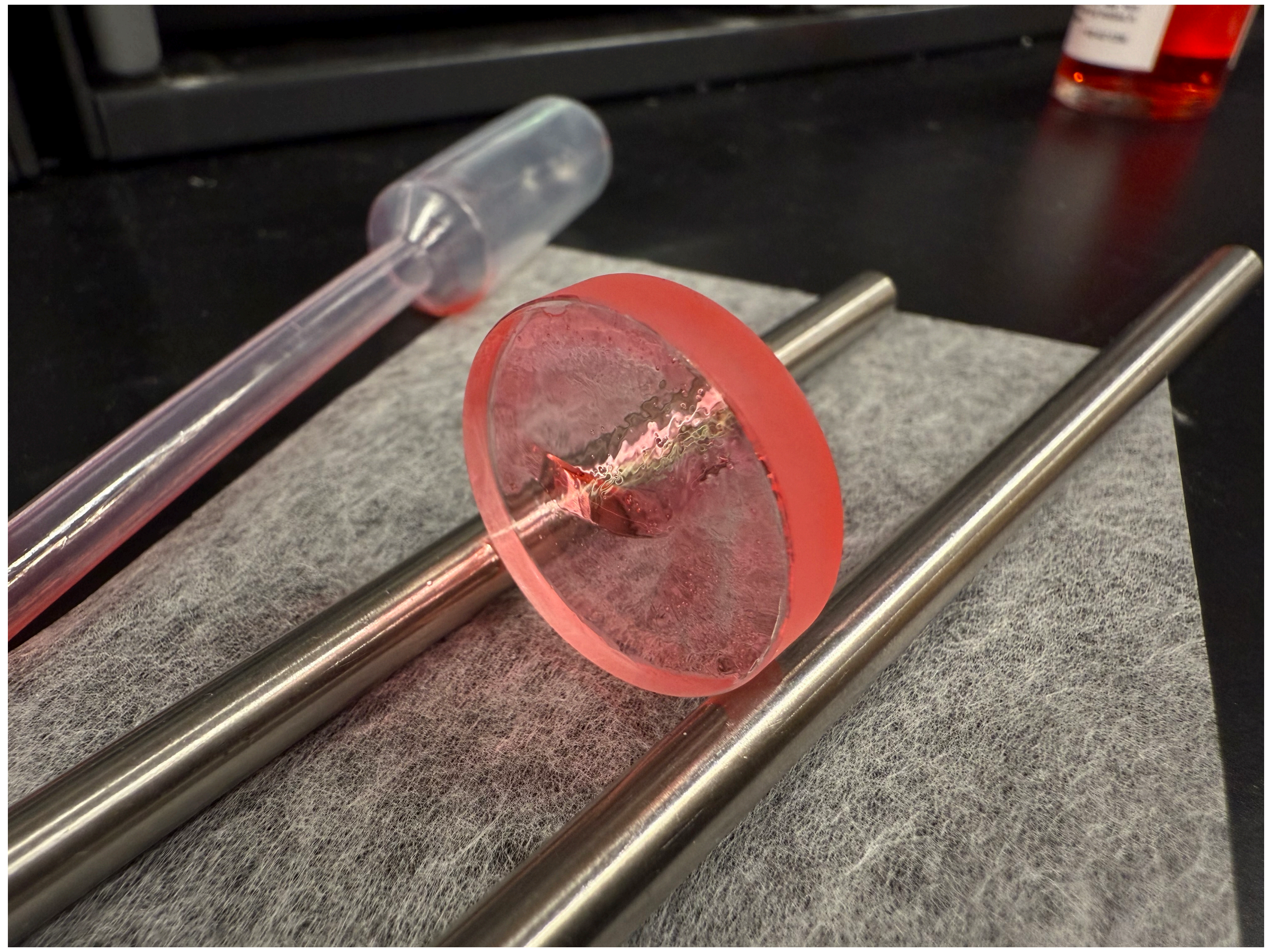
Simulation

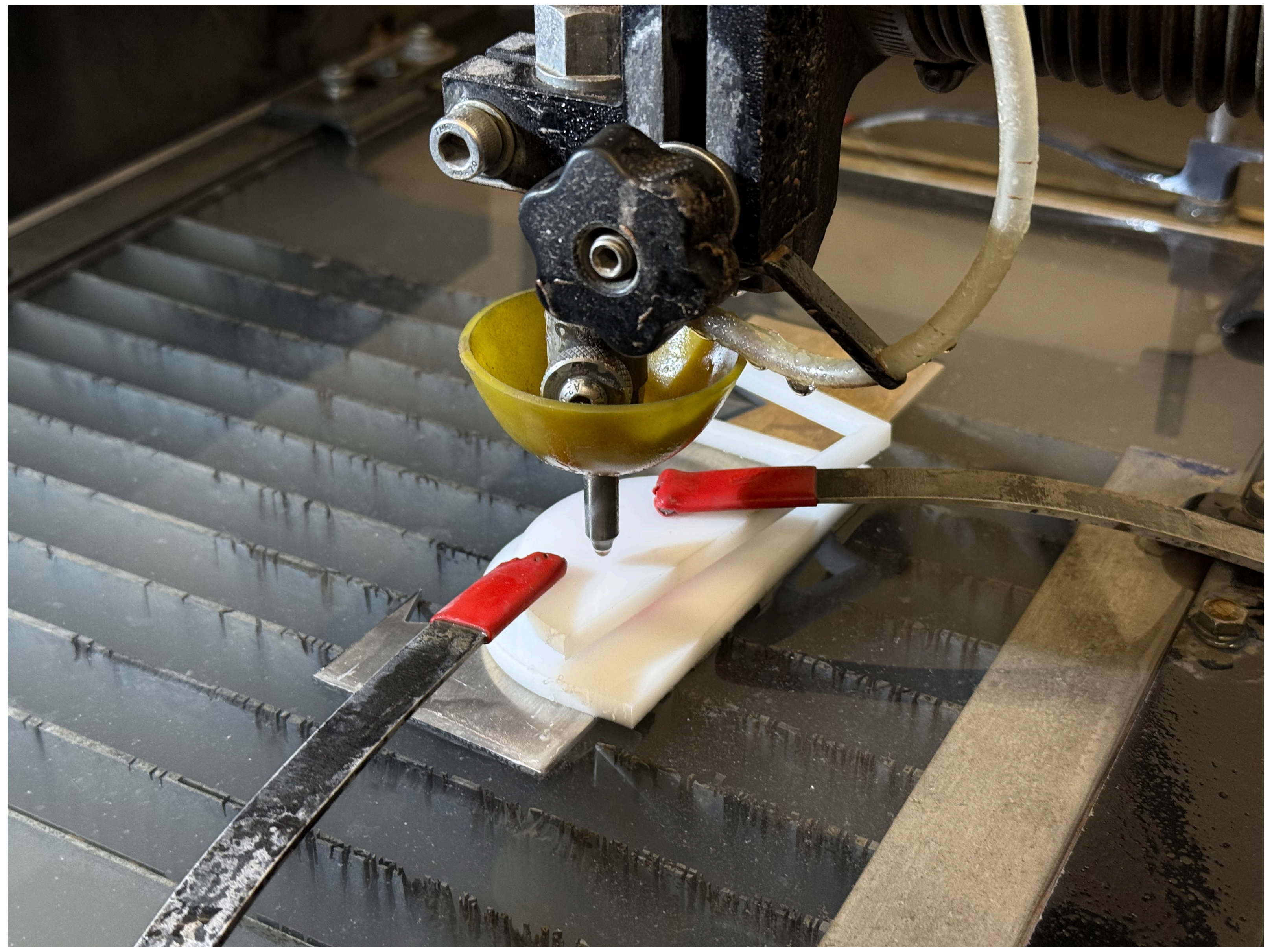


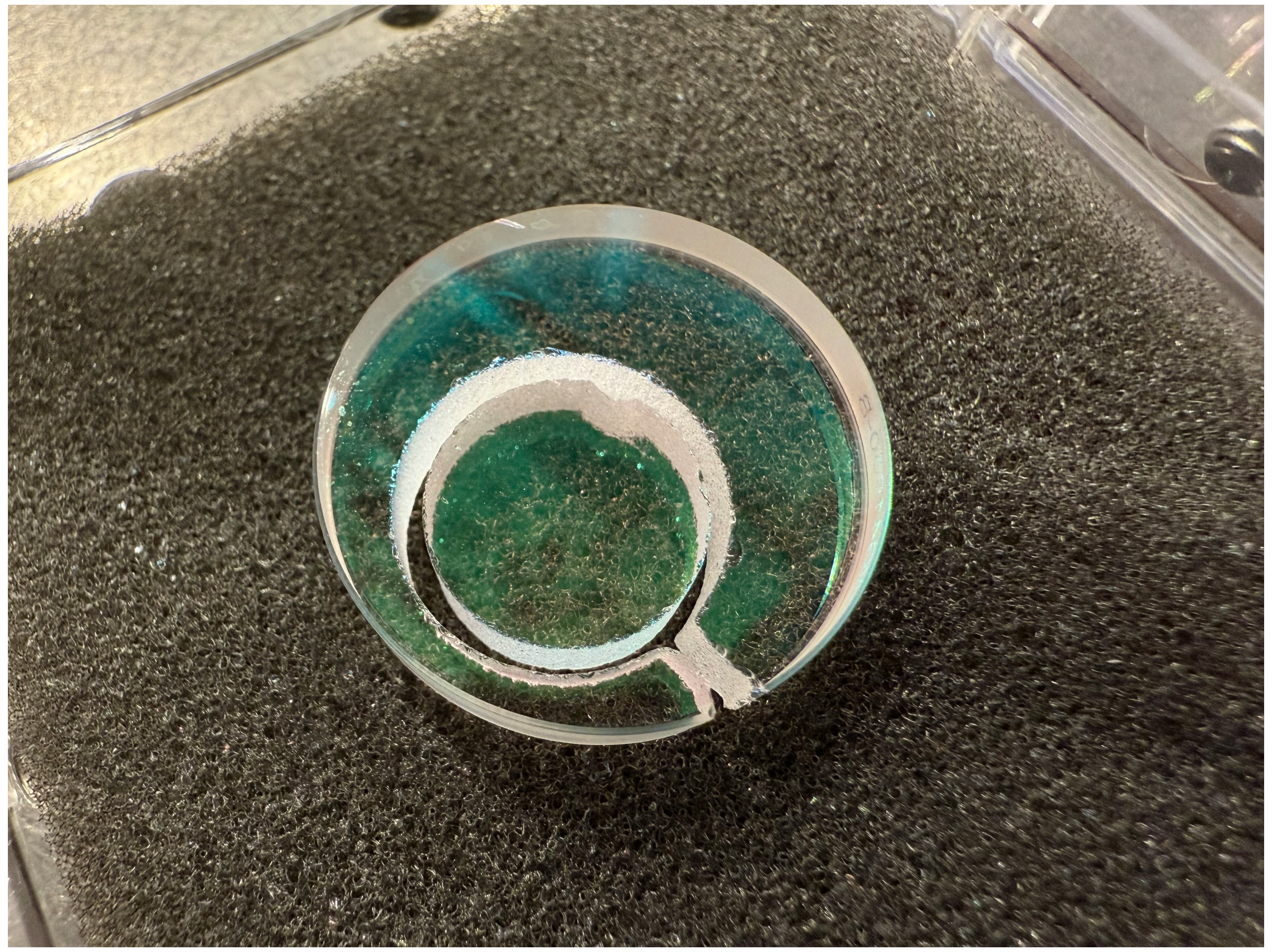
scale, slope is not right

2025.2.5 update

ECDL Update





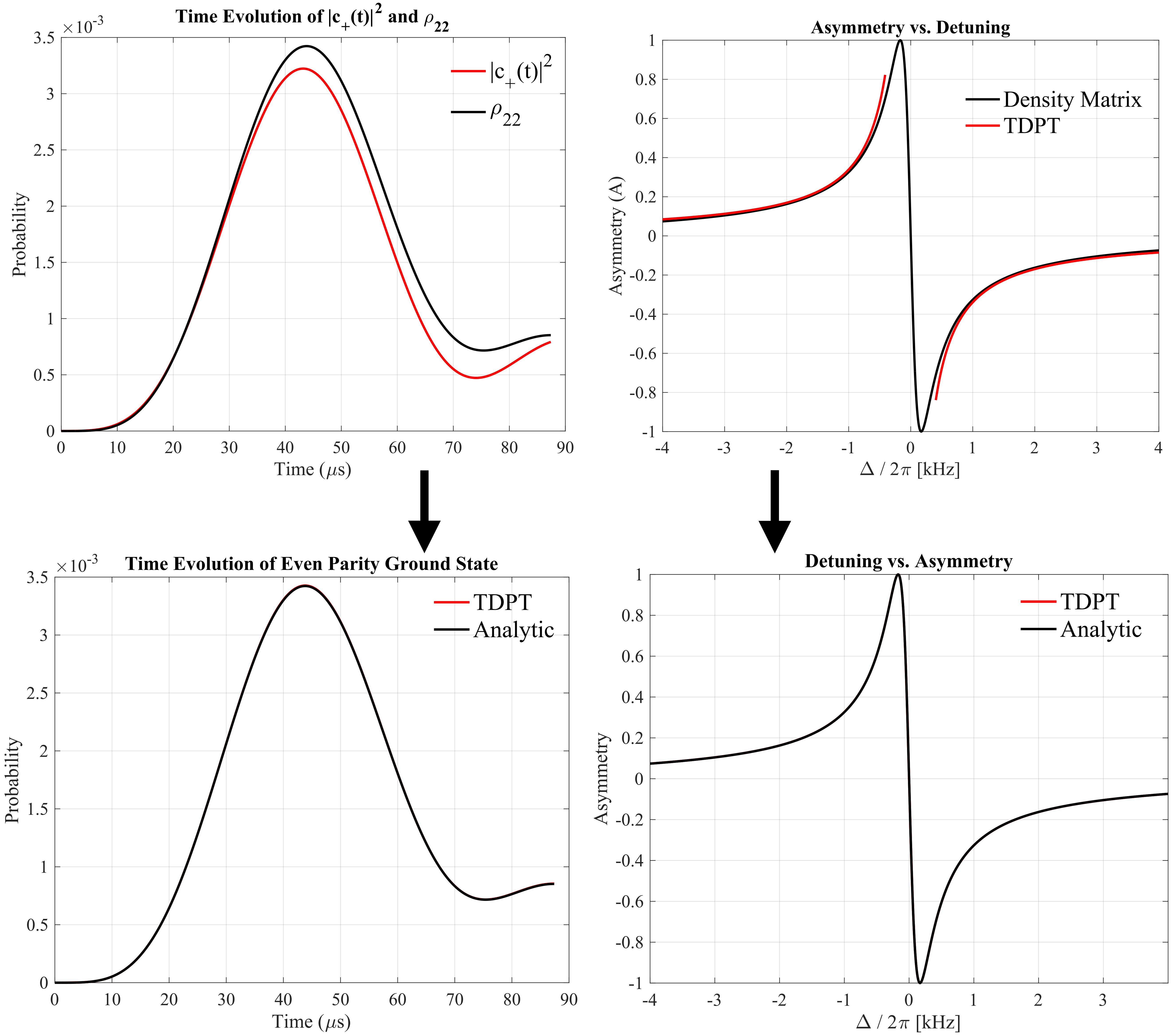


Fused silica window, no cracks after the cut (Ø 12.5mm), little rough around the edge.

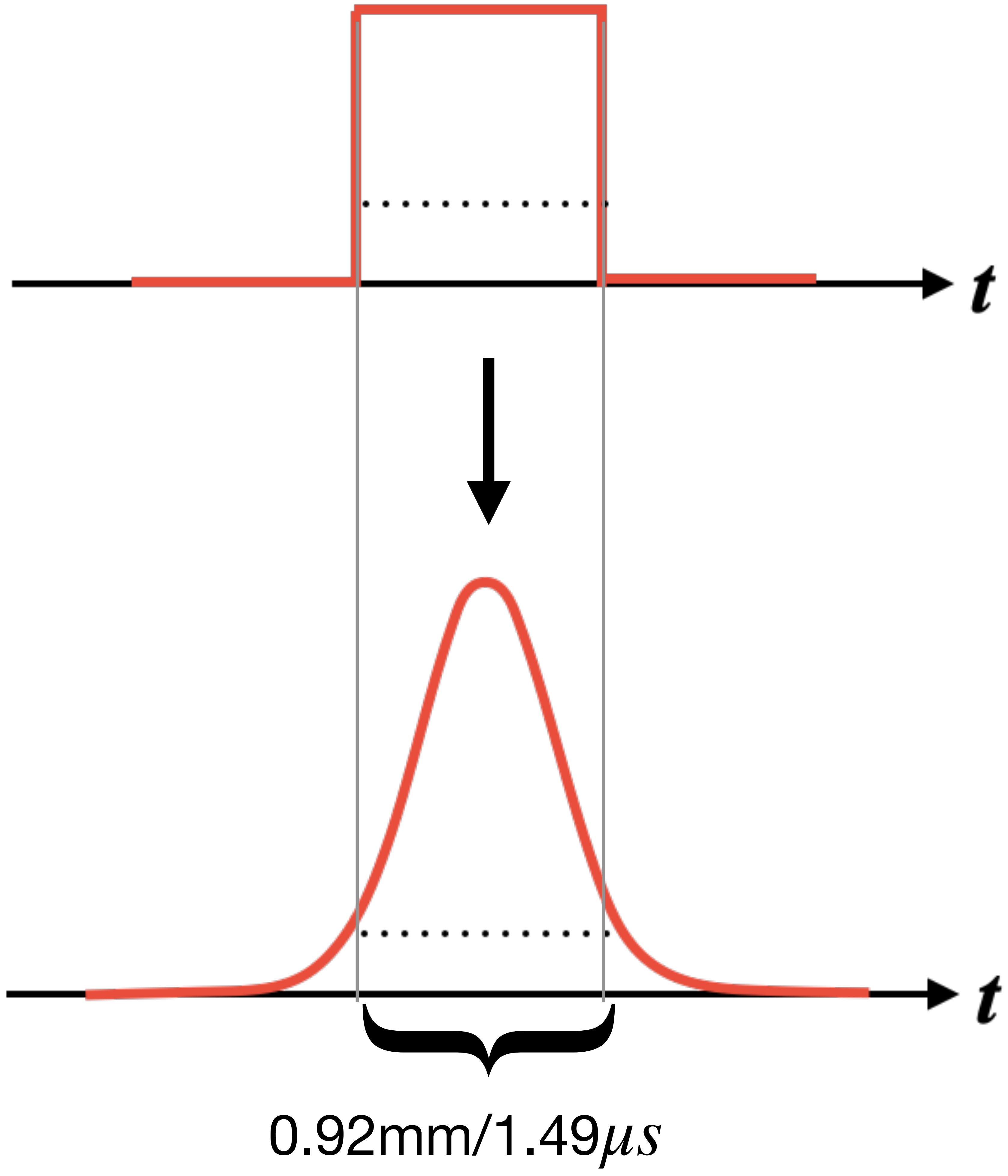
No transmission loss at 860 nm. Fit the rotational mount. Quote sent to Sam!

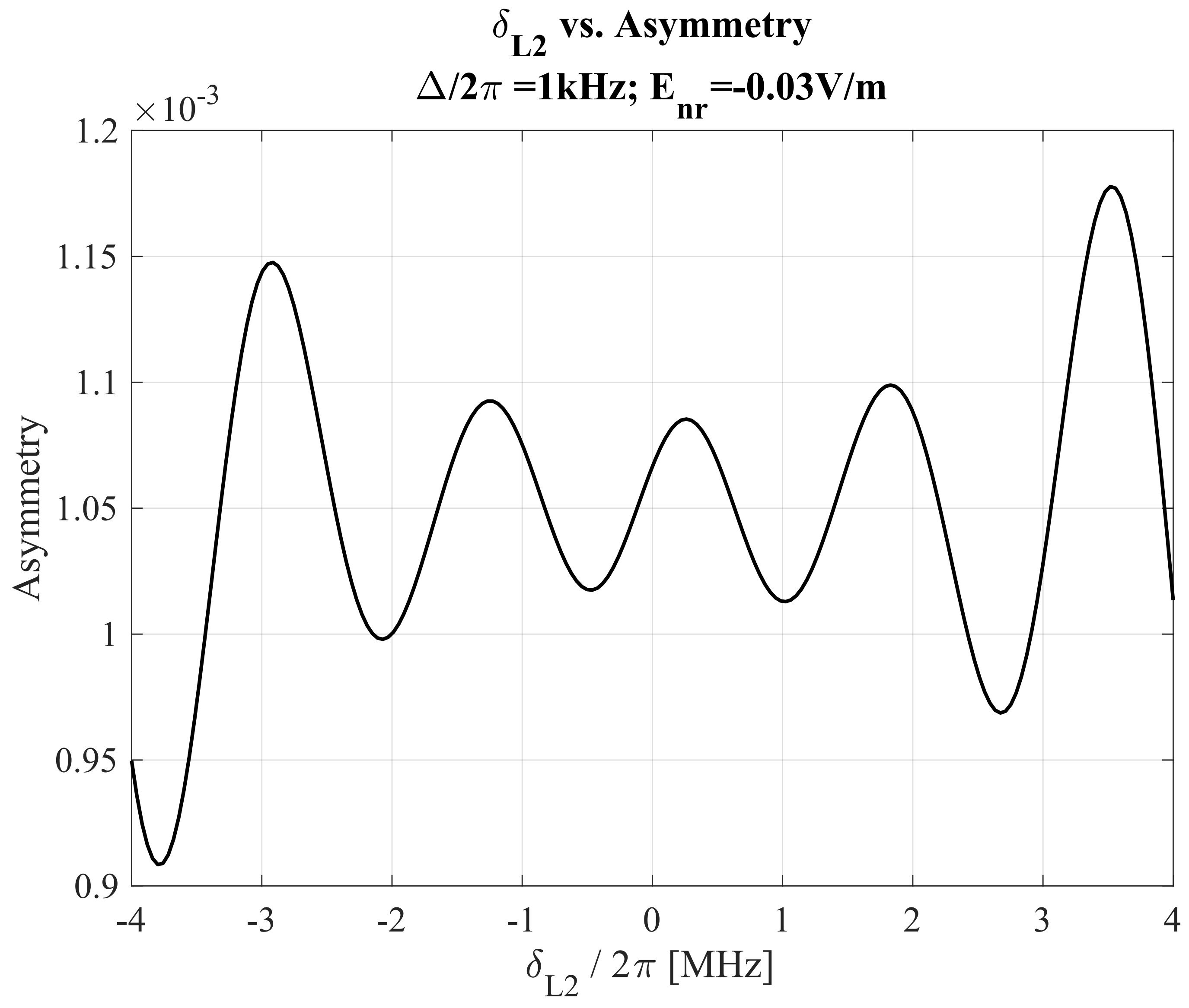
More accurate 1st order TDPT result

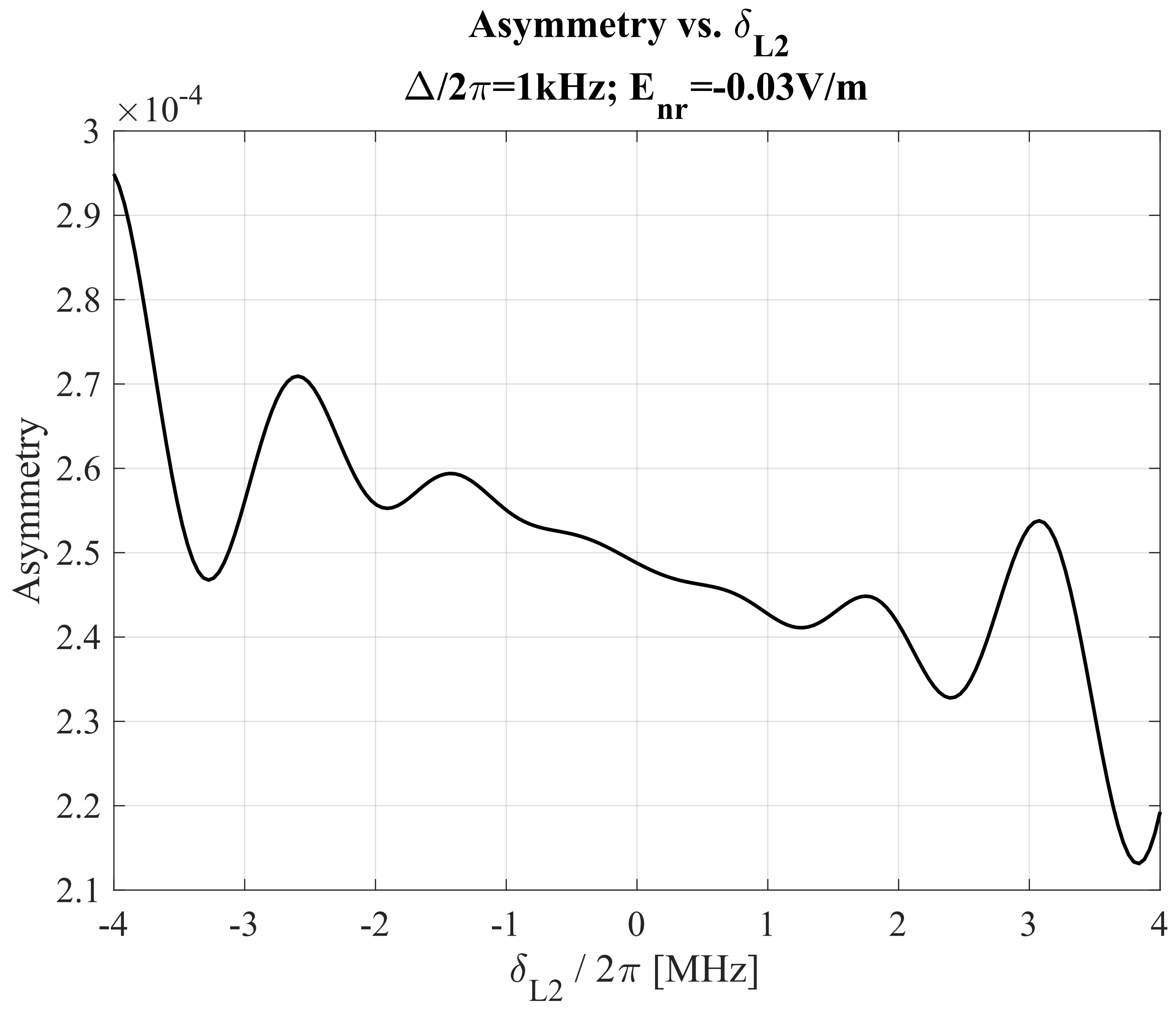
Integrate $\dot{c}_{+}^{(1)}(t) = -ie^{-i\Delta t}(iW + d \cdot \mathscr{E})$

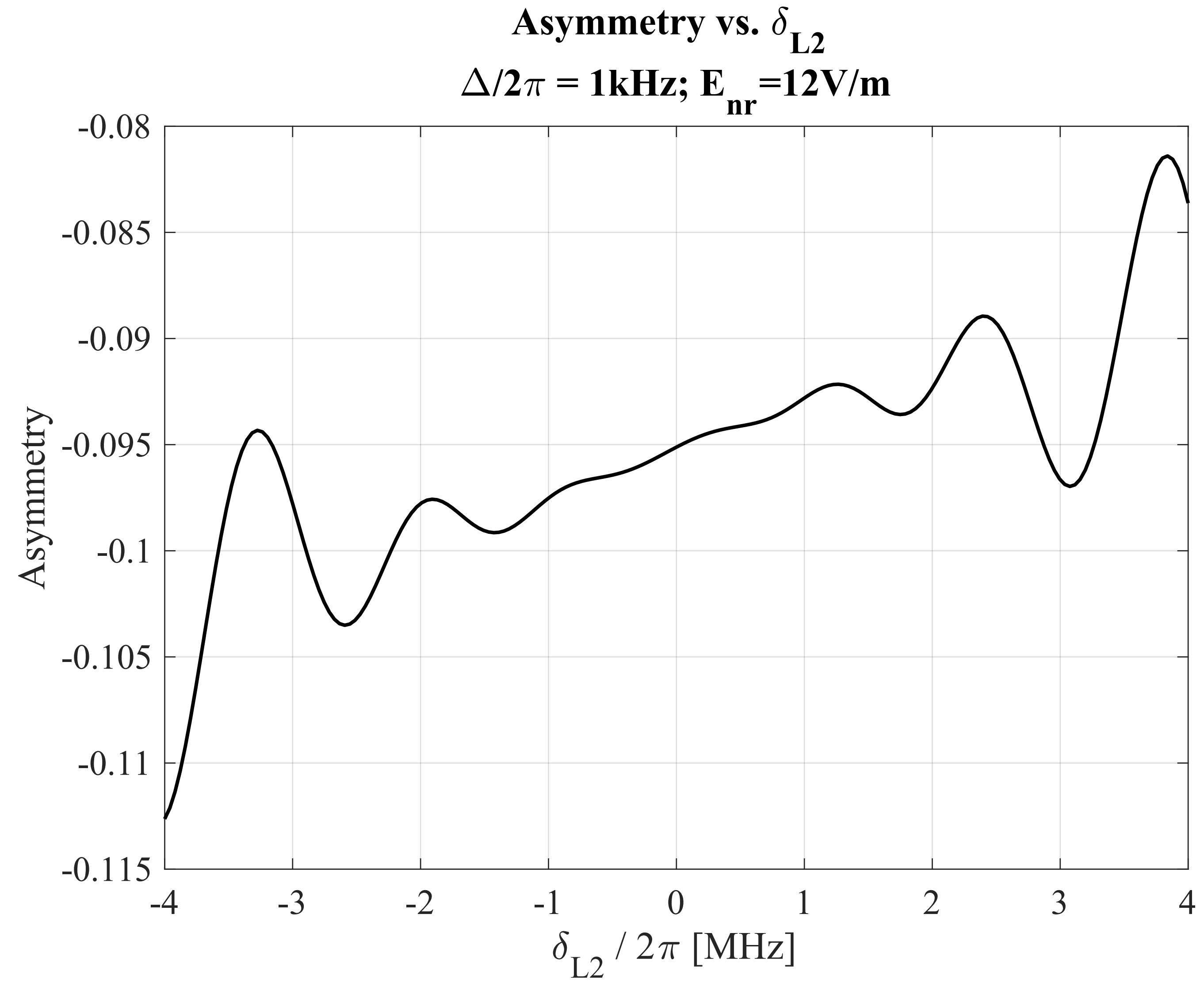


Change Laser to a Gaussian Profile

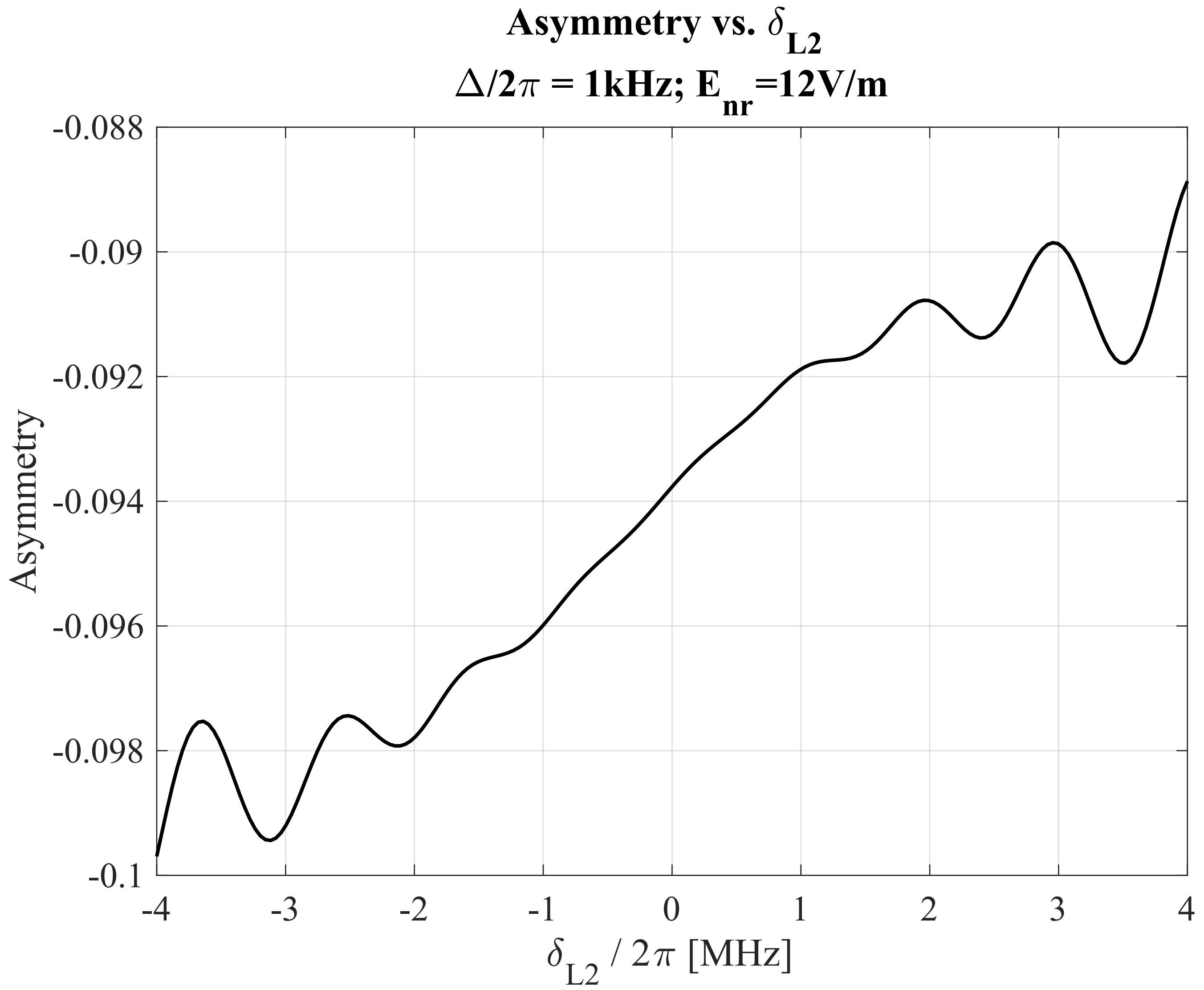




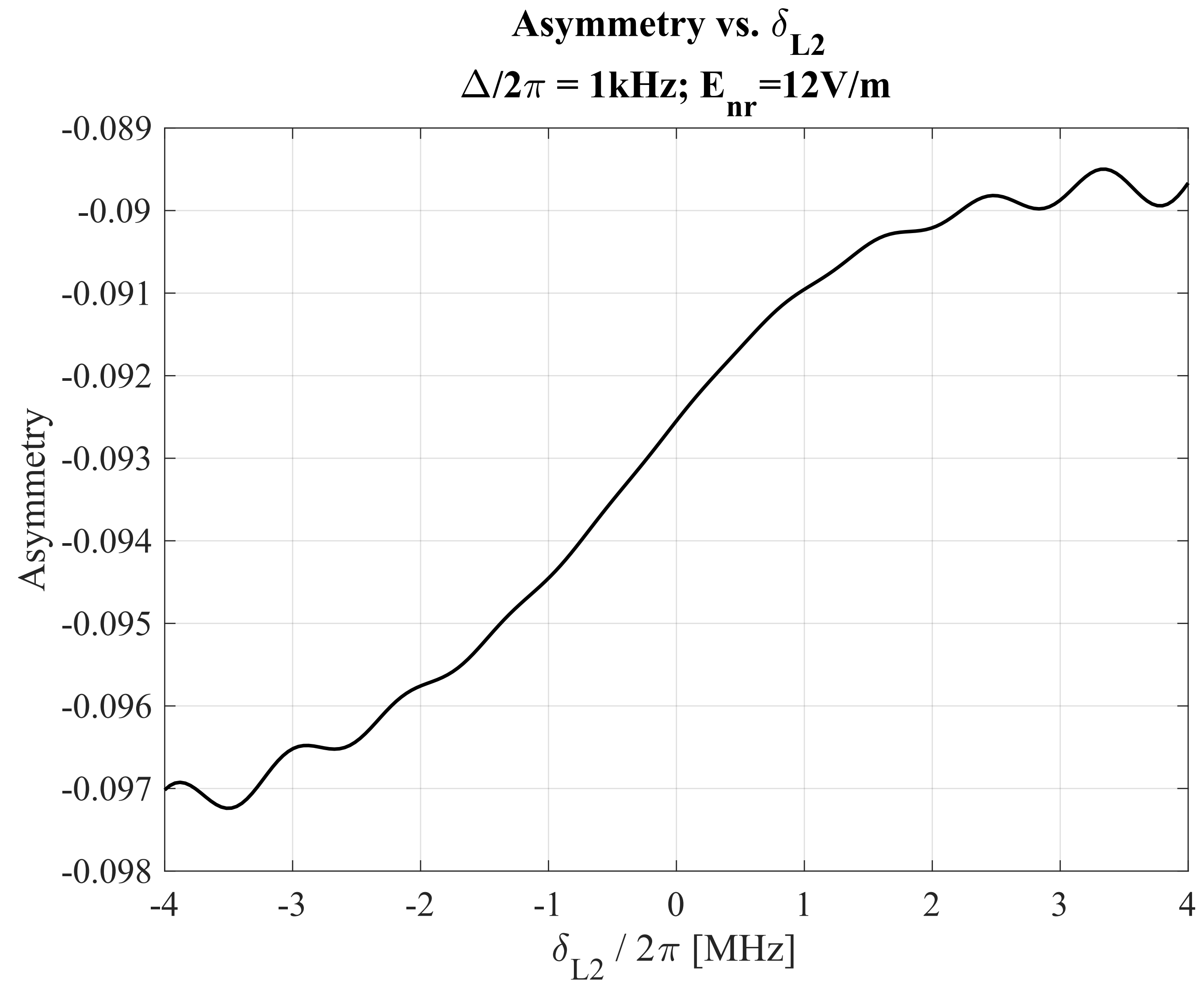




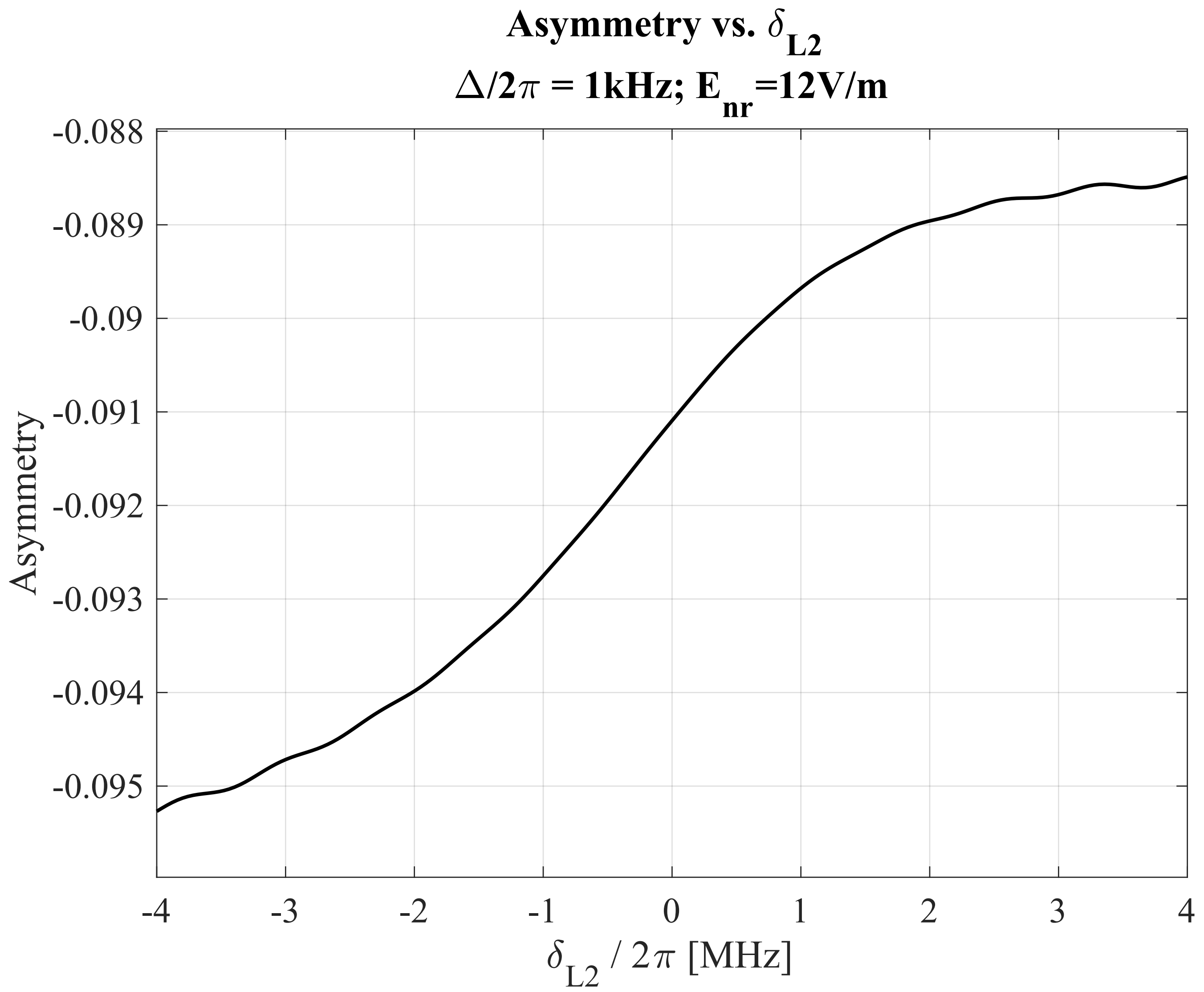
$\mathscr{E}_{L2} = 851.4V/m$



$2 \times \mathscr{E}_{L2}$



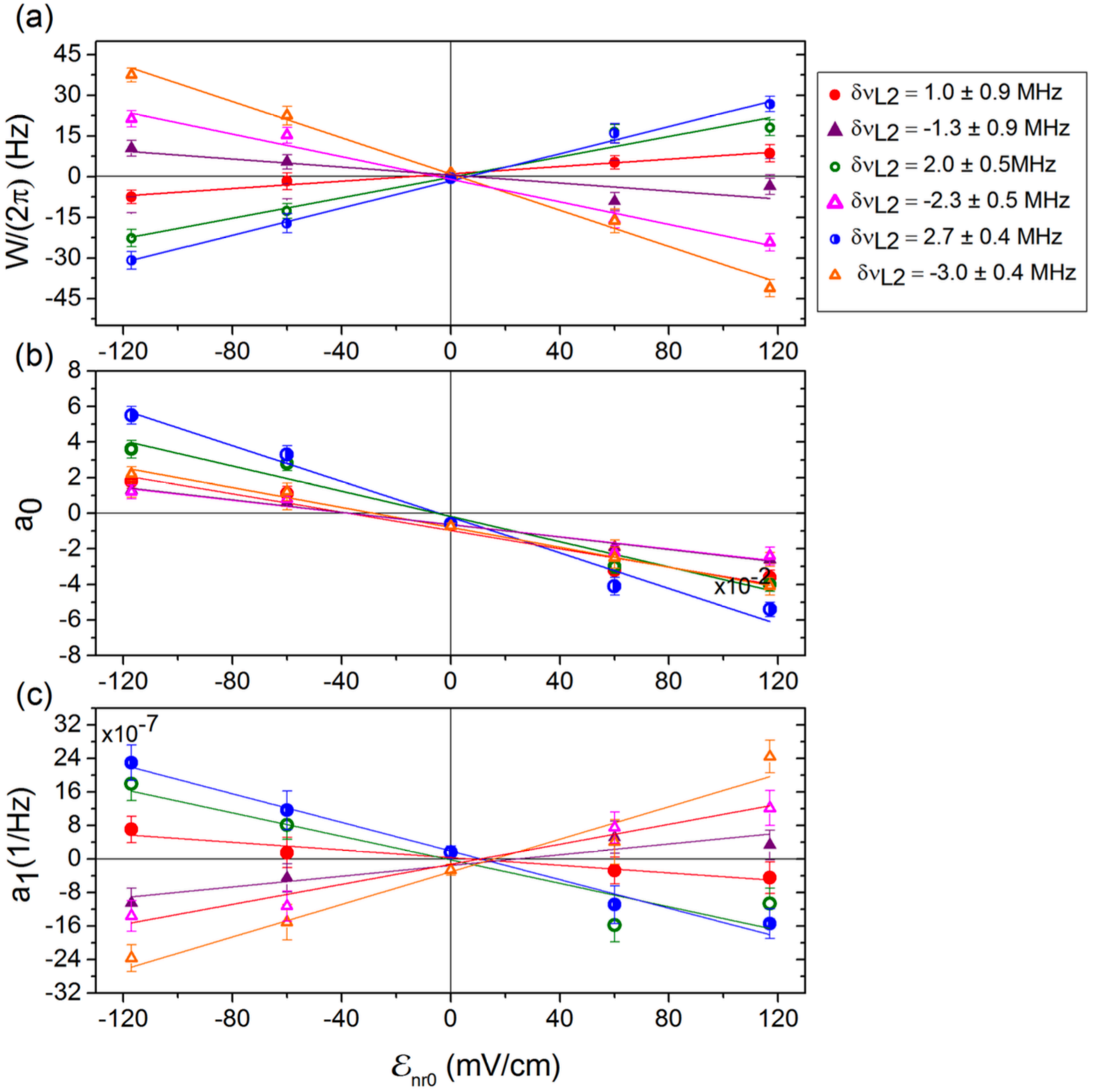
$4 \times \mathscr{E}_{L2}$

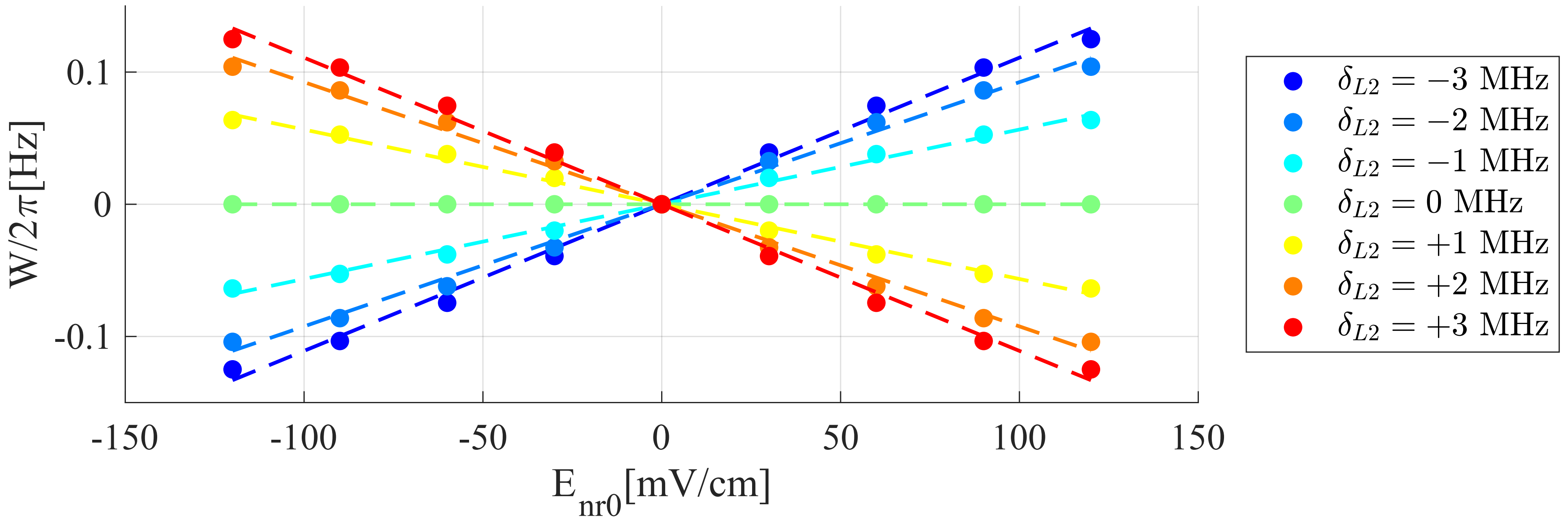


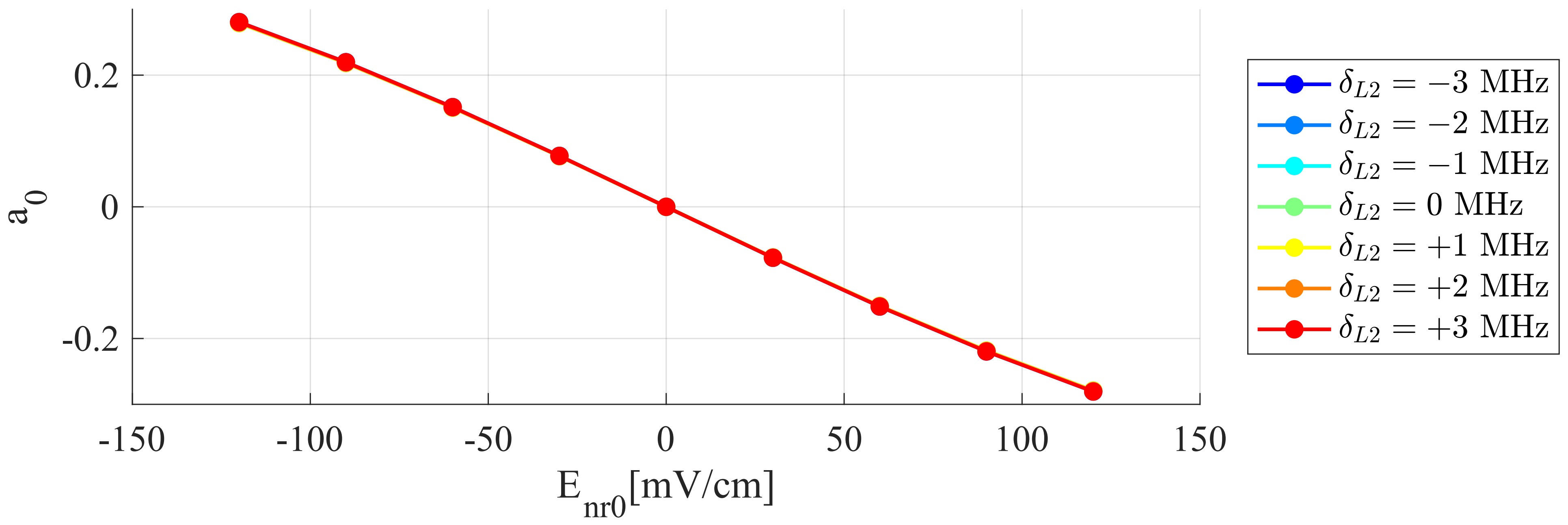
$10 \times \mathscr{E}_{L2}$

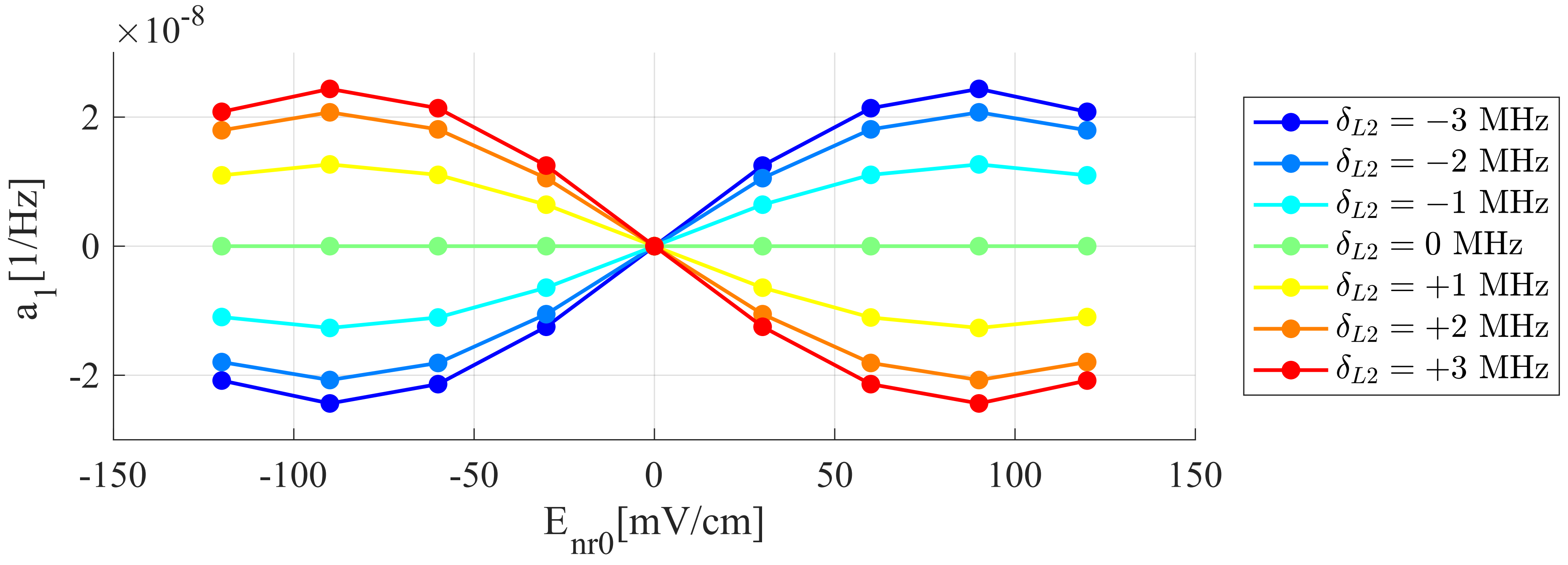
Increase Laser intensity will reduce the wiggle behavior

slightly changing $\mathscr{E}_{nr}, \Delta, \delta_{L2}$ won't change the shape much (so no averaging effect)









Deliberately make the L2 amplitude 10 times larger, to reduce the wiggle behavior

$$\delta_{L2} = \omega - \omega_0$$

Issues: 1. scale is not right. 2. as δ_{L2} increases, slope decreases, which is opposite to what found in experiment (definition of δ_{L2}?). 3. The slope is not changing linearly with δ_{L2}. 4. a_1 graph is not linear.

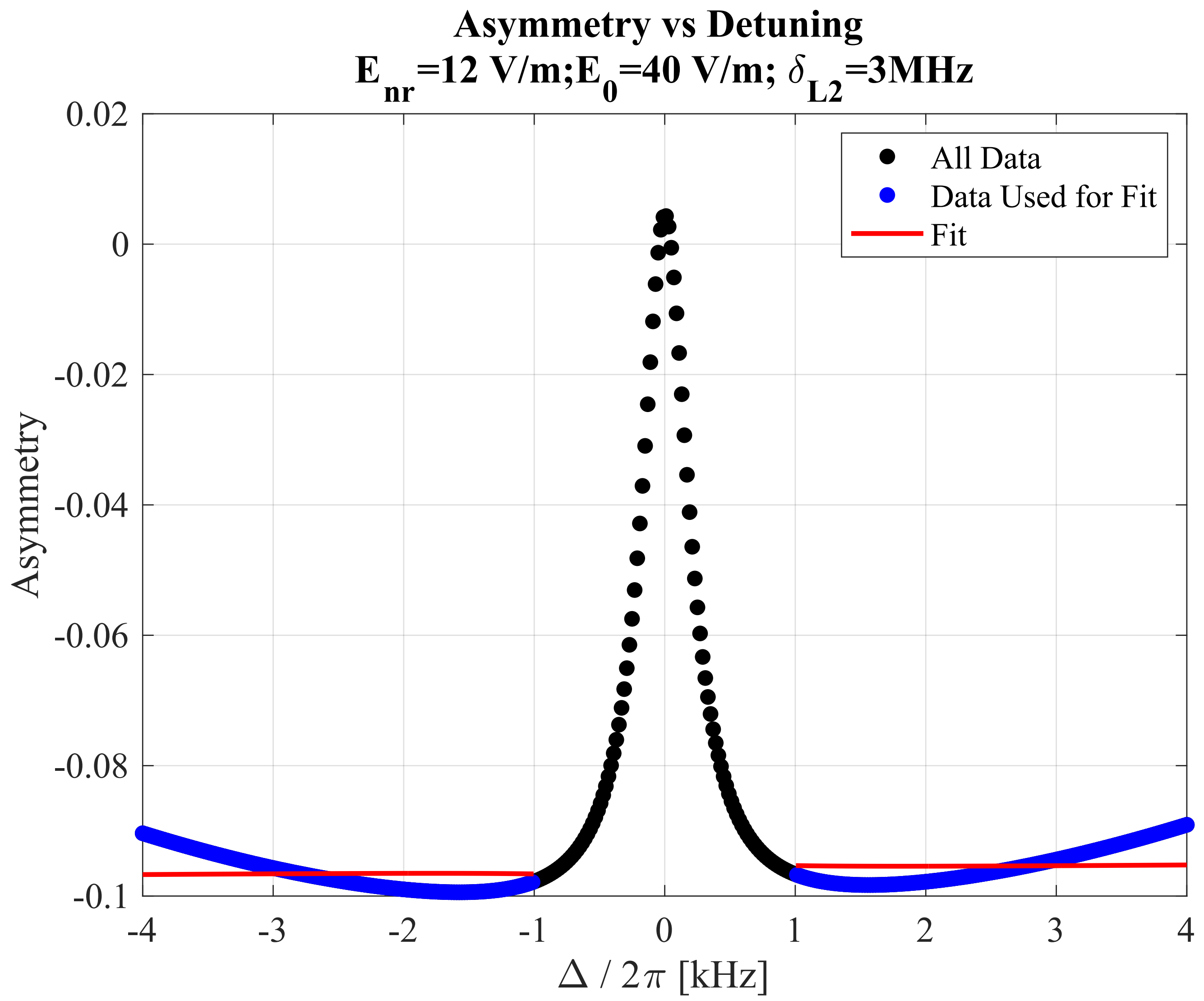
Why W is way smaller in simulation?

W extracted from simulation (~$0.1 \times 2\pi$ rad/s) is way smaller than the experiment (~$40 \times 2\pi$ rad/s)

$40 \times 2\pi$ rad/s is huge even compare to the actual W = $5 \times 2\pi$ rad/s for ^{137}BaF. We should see a clear asymmetry pattern even when we set W = 0 in H, which is not the case.

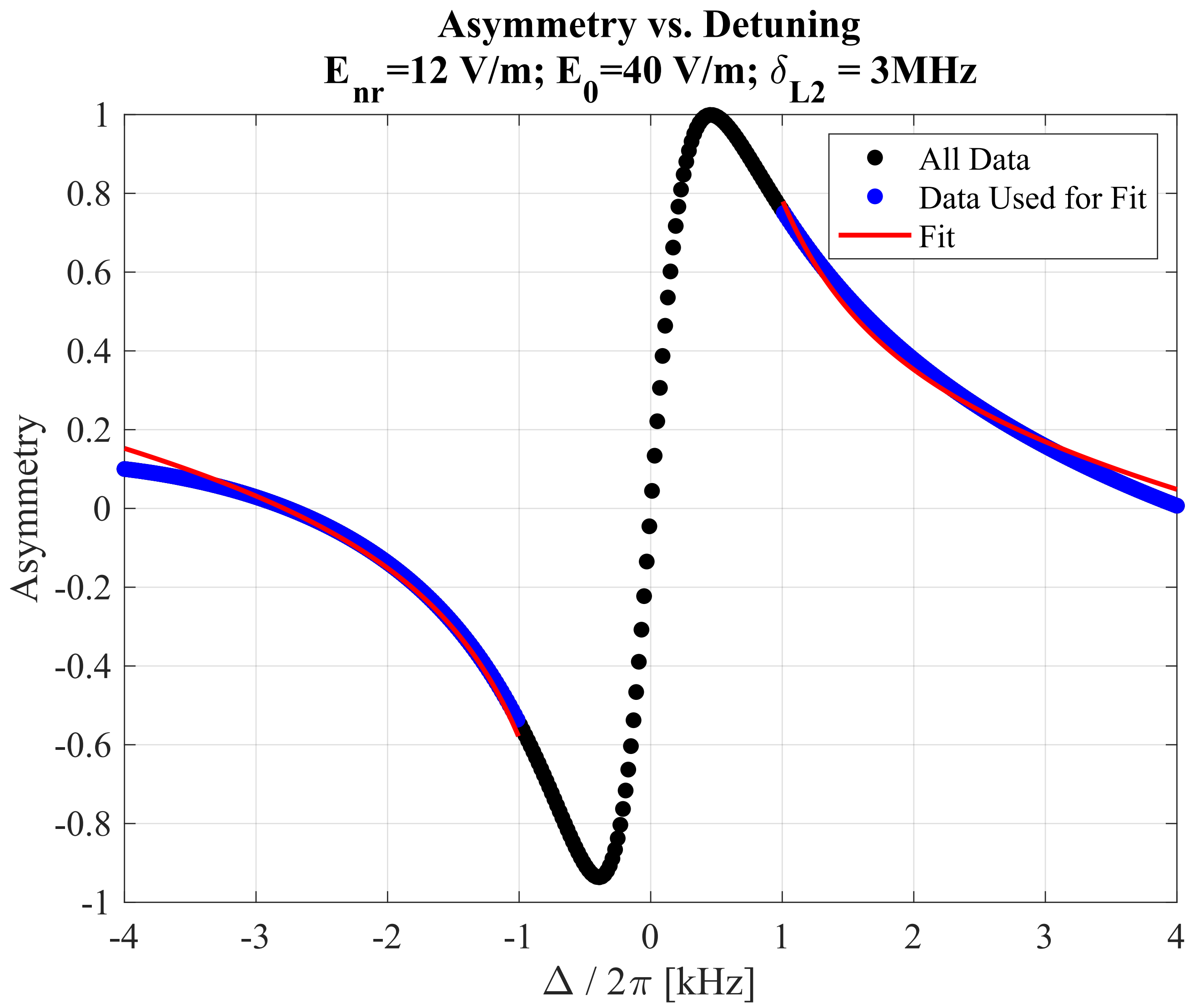
I remember seeing a clear asymmetry function like curve when I turn down the laser intensity; W gets "approximately" larger when the population of odd parity ground state after depletion is larger

With calculated L2 intensity (correct)



$$W/2\pi = -2.413 \times 10^{-2} Hz$$

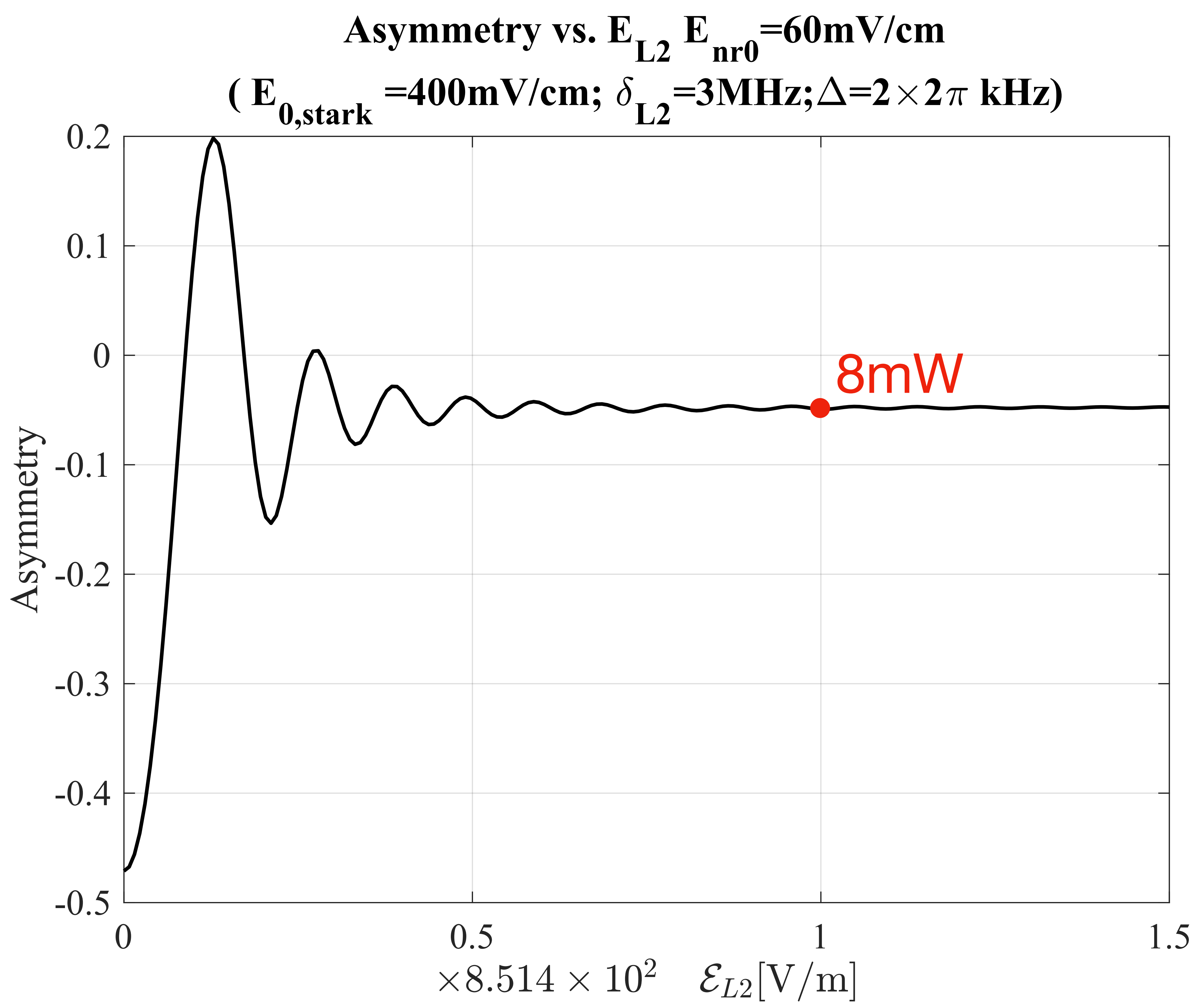
0.13 x L2 intensity (turned down)



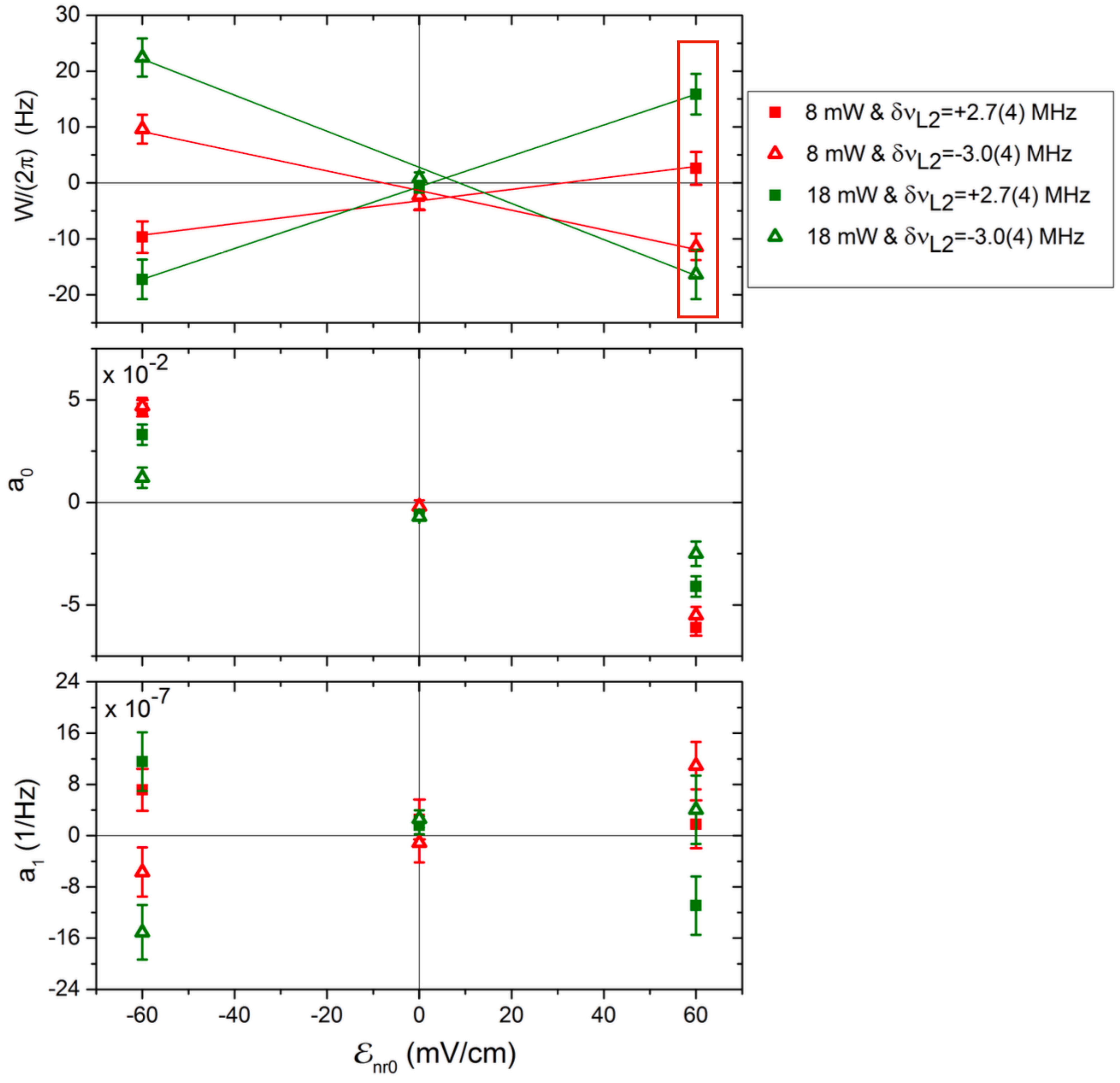
$$W/2\pi = -3.68 Hz$$

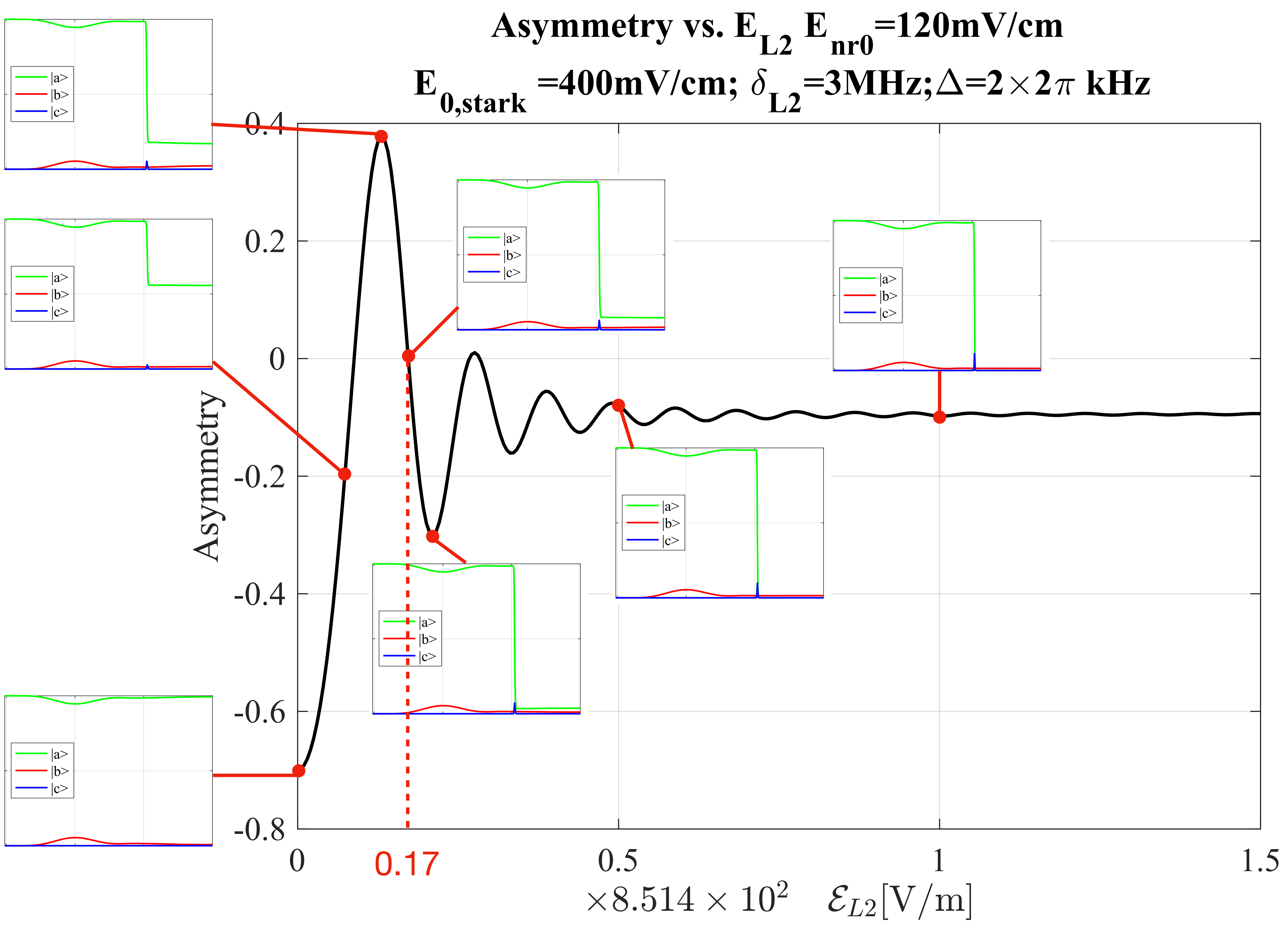
Asymmetry vs. $\mathscr{E}_{L2}$

Simulation says that Asymmetry $\propto W$ doesn't depend on $\mathscr{E}_{L2}$ around 8mW



from experiment, $W \propto \mathscr{E}_{L2}$





"for the 2nd parity state projection laser we have typically 6~12% population remaining after optical pumping.

Effective laser intensity is around 17% of the calculate intensity?

However, if we turn down the laser intensity, the wiggle behavior comes back!

2025.2.12 update

Last time

One possible reason for the small W in the simulation is due to the **laser intensity too strong**. *(reduce the power would increase W significantly.)*

Fig 7.8 in thesis shows the OPR of the L2 laser is approximately **8%** when detuning is 0. *(meaning there are still 8% odd parity ground states at the time of the detection after depletion, which is a significant difference from my simulation of ~0% (perfect depletion)).*

$$R_2(\pi) = \frac{S(\pi, o)}{S(0,c)}$$

More odd parity ground state left behind essentially allows more interaction between the observation state via $\mathscr{E}_{nr}$, thus produce larger error signal.

Dave suggested that the 8% in the experiment might be caused by “rain down”, where atoms in higher energy states created during the ablation stage constantly decaying down to the lower levels.

However, some evidence still makes me believe the laser in the experiment wasn’t very effective in depletion.

2nd Laser OPR vs. δ_{L2}

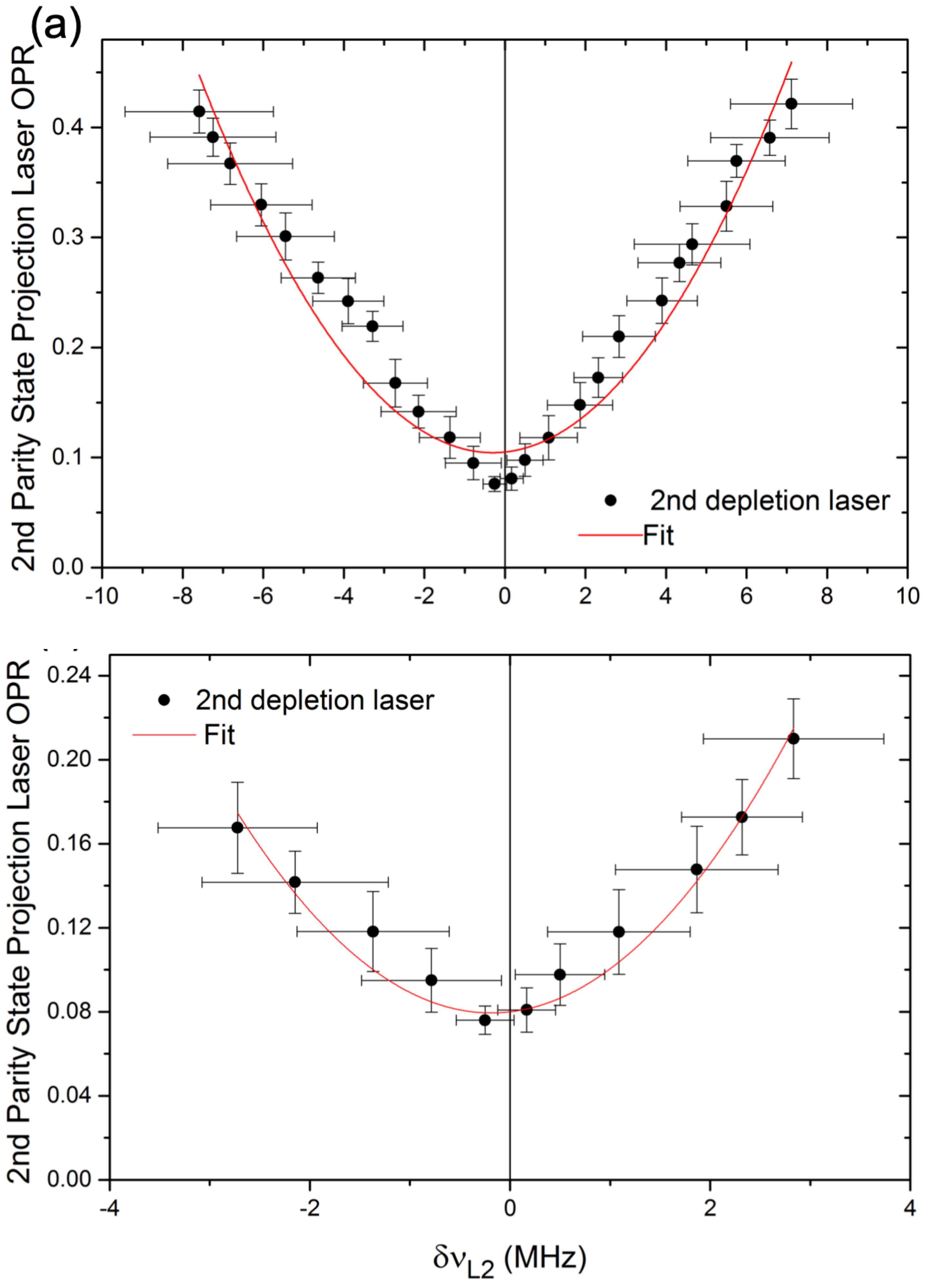
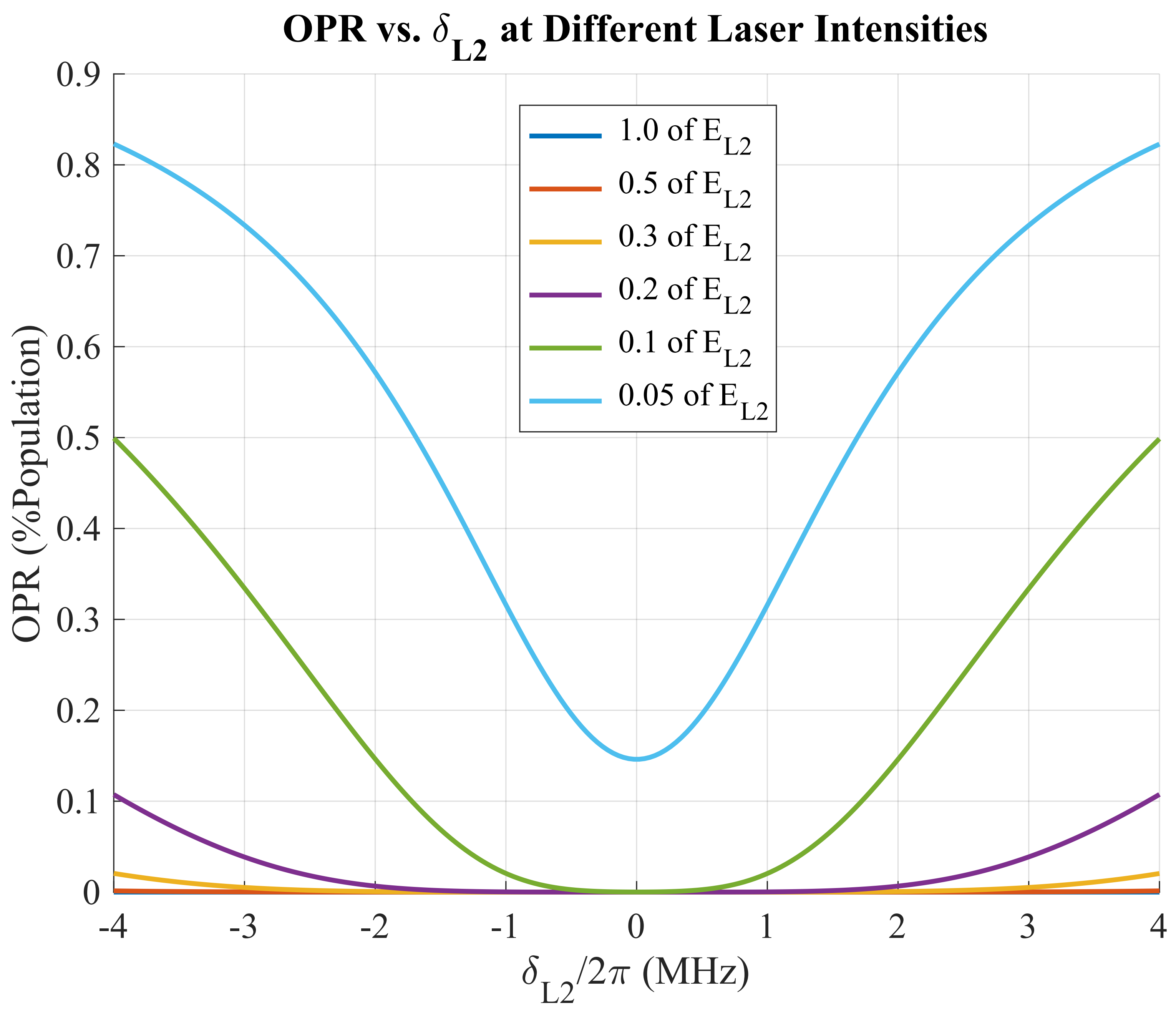


Figure 7.8

From experiment, 2nd laser OPR has a near quadratic relationship with δ_{L2}. (If the laser “oversaturated”, then OPR shouldn’t depend strongly on slight detuning. Should be near constant if caused by “rain down”.)

Scan δ_{L2} at different intensity (in unit of theoretical 8mW power)

y: % of odd parity ground state population after depletion.

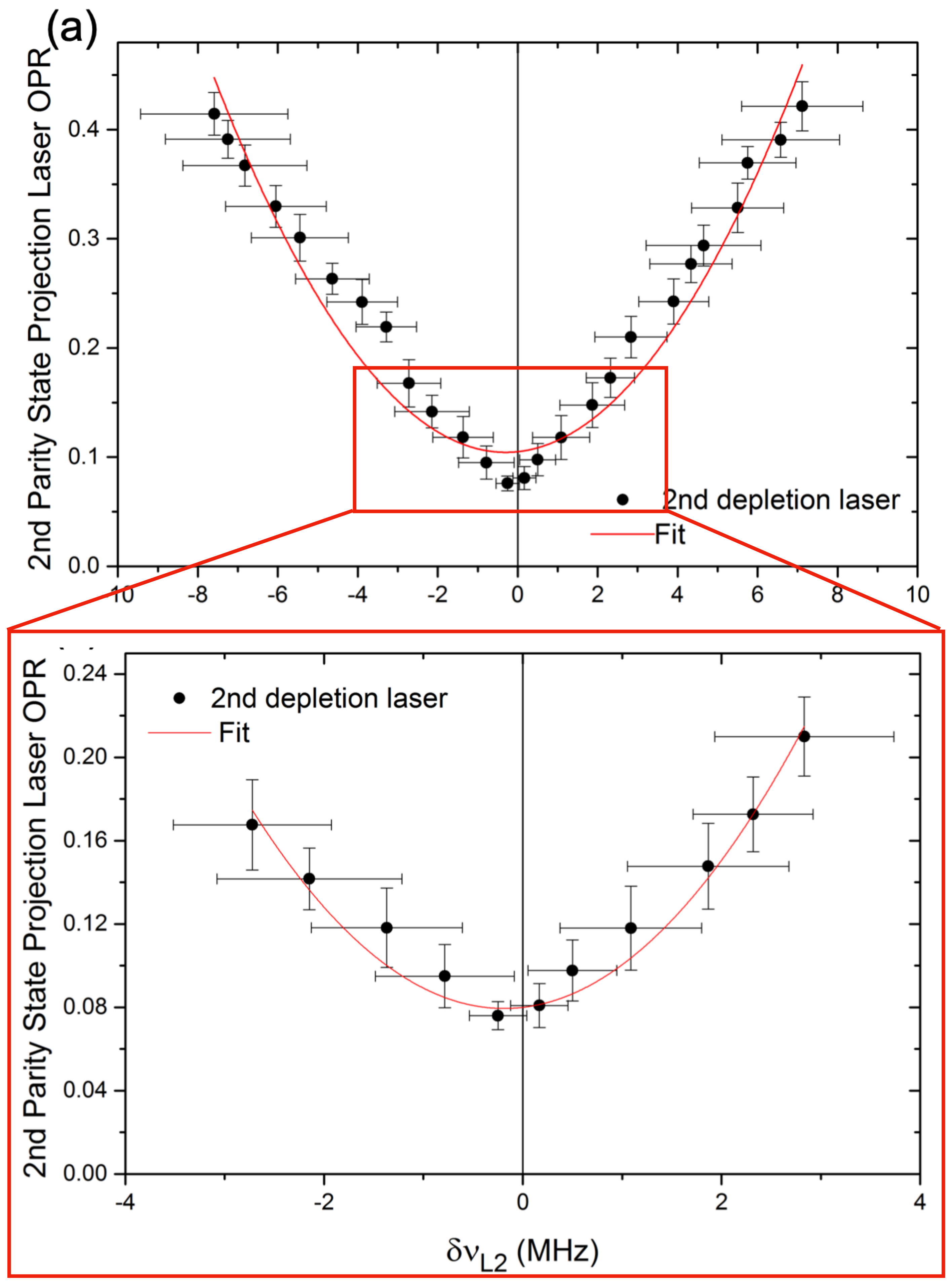


2nd Laser OPR vs. δ_{L2}

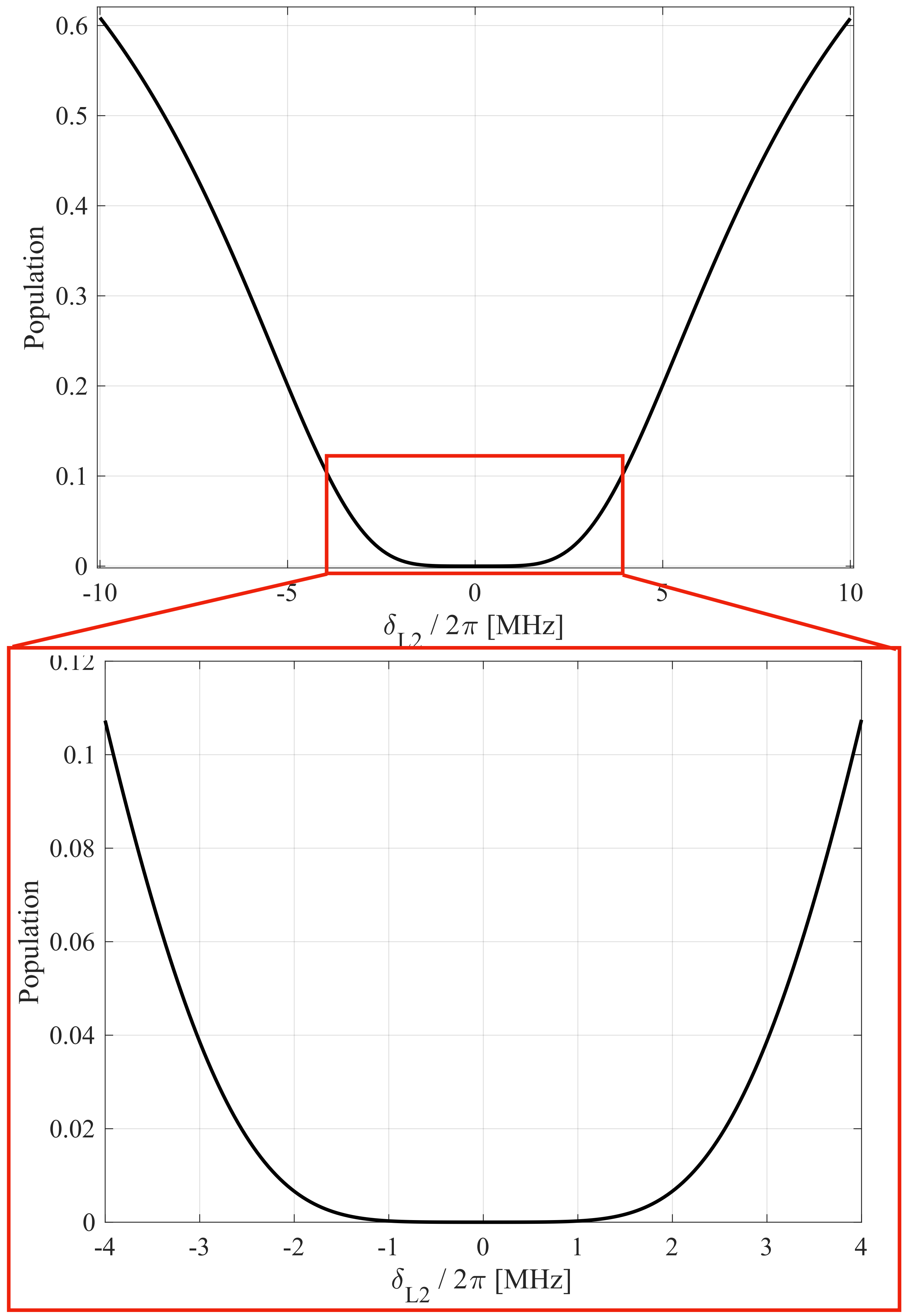
The simulation resembles to the experiment result when laser amplitude is turned at ~20% of the theoretical value, with a constant offset

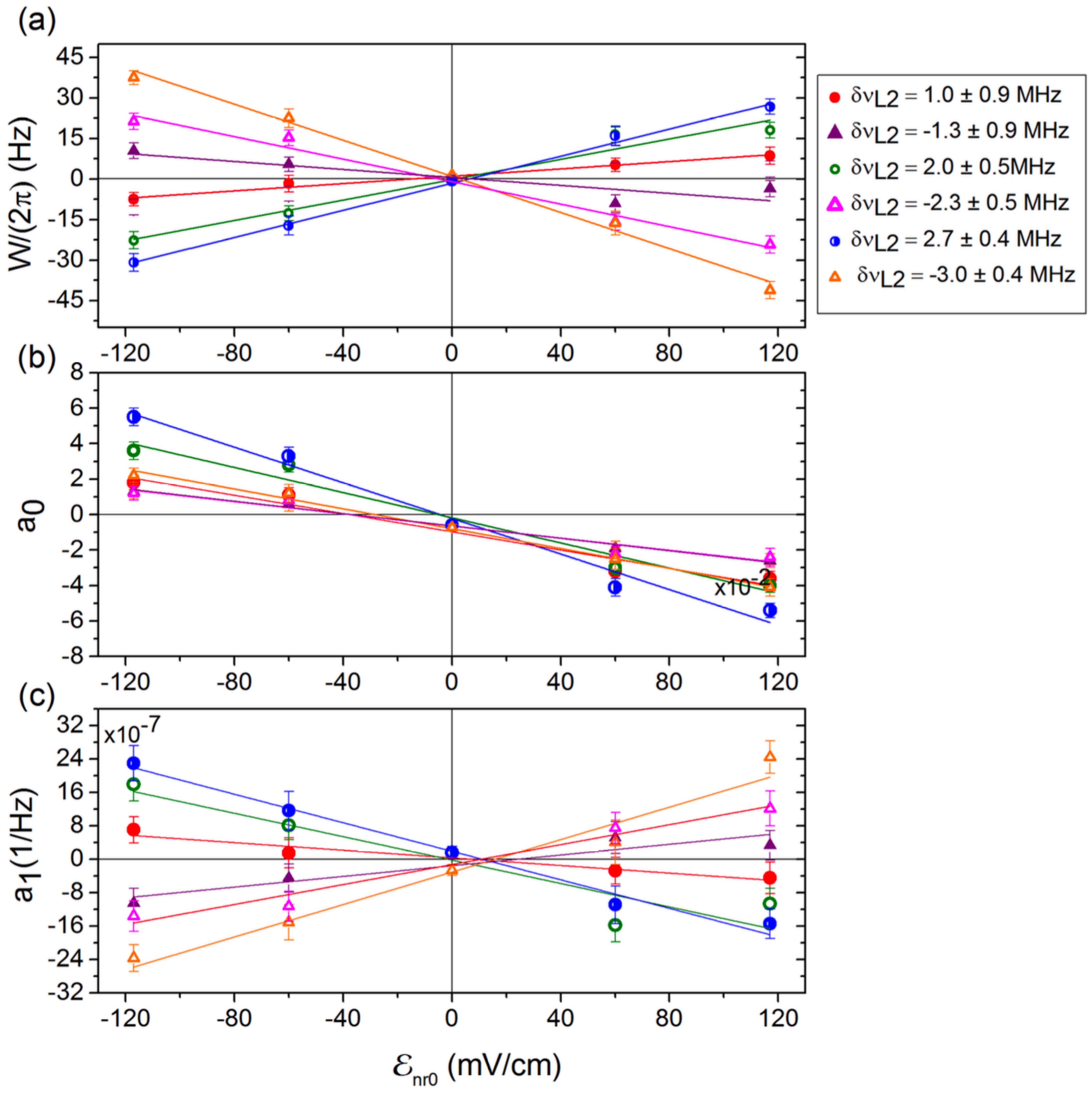
To make sure if this fix the small W issue without impose other issues, I reproduce Fig 7.9 again, among others

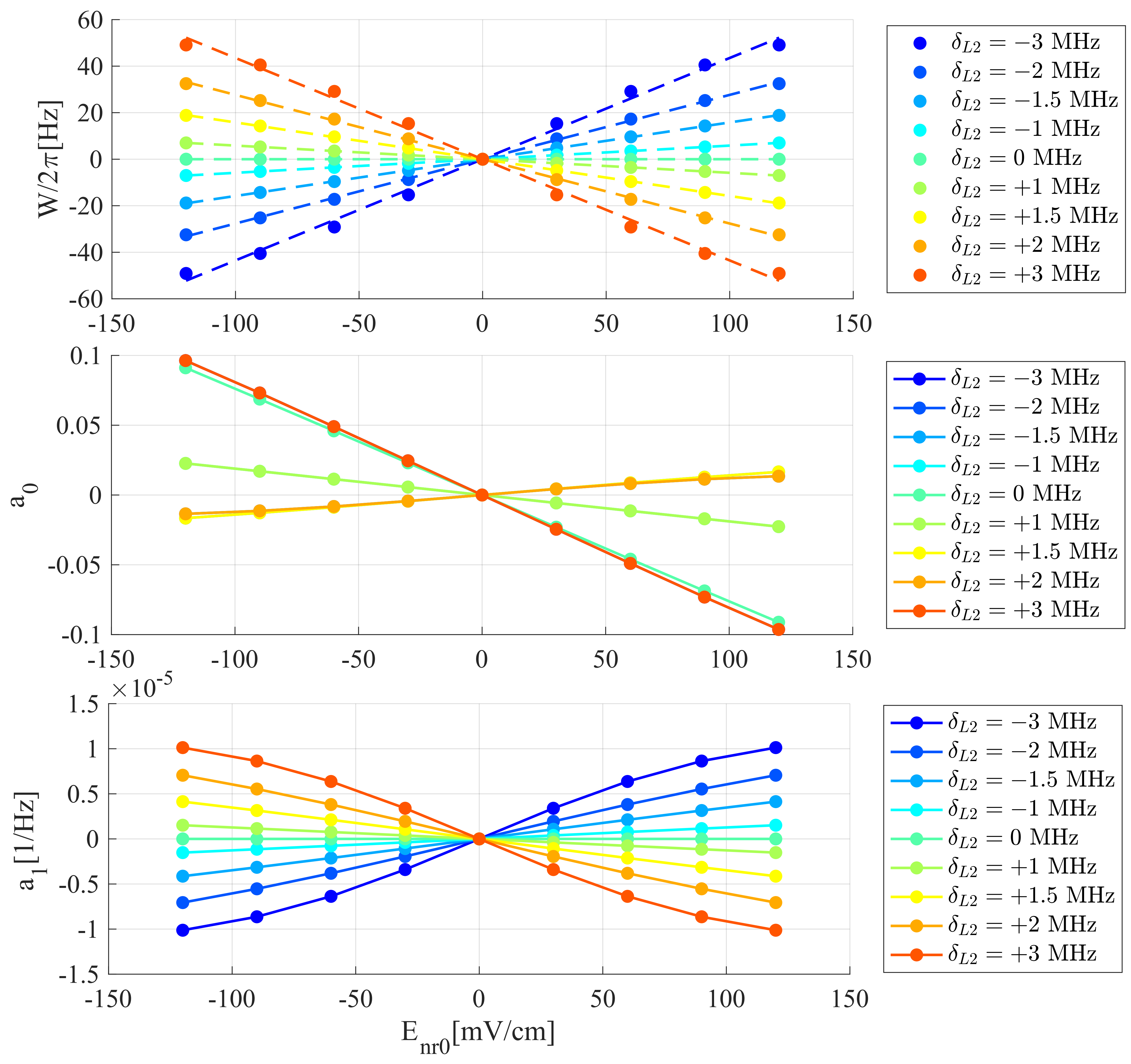
Experiment



Simulation







with 11% theoretical laser amplitude $\mathscr{E}_{L2}$

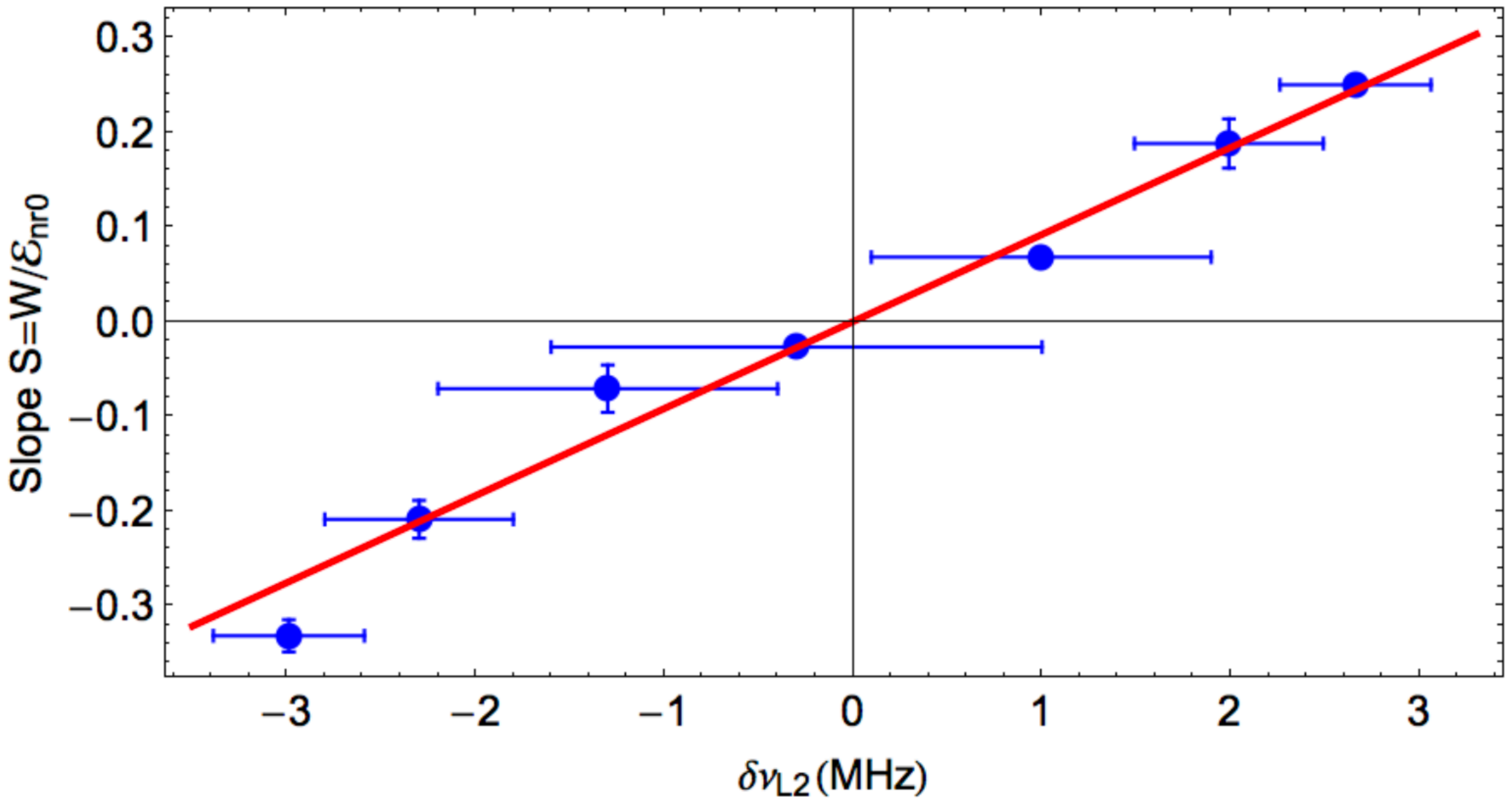
exhibit similar magnitude, similar trends

only a small range of laser amplitude that could achieve this result ($\pm 2\,\%$).

Slope vs. δ_{L2}

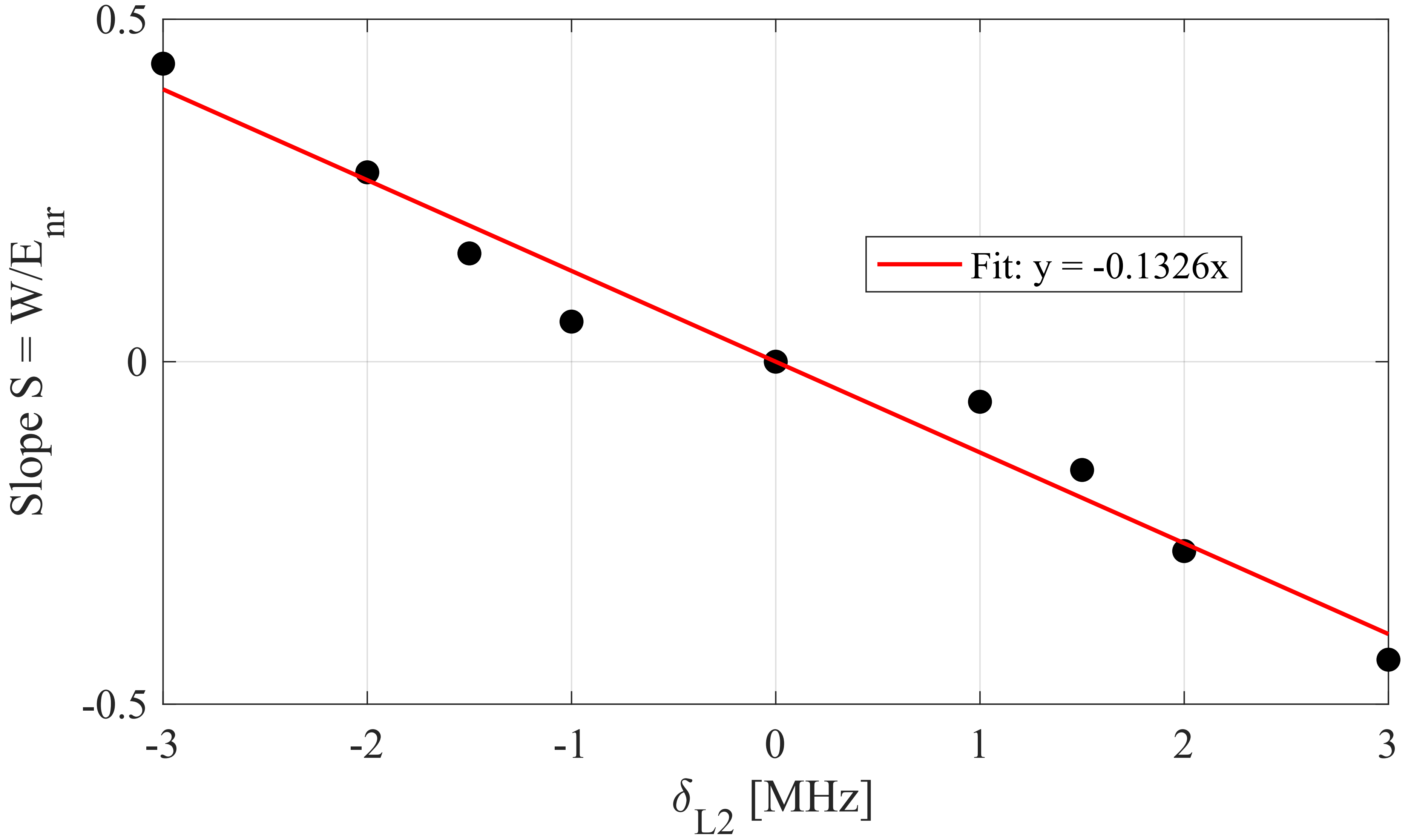
$$S(\delta_{L2}) = C_W \delta_{L2}$$

Experiment



$C_W = 0.09 \pm 0.01$ [Hz/MHz(mV/cm)]

Simulation



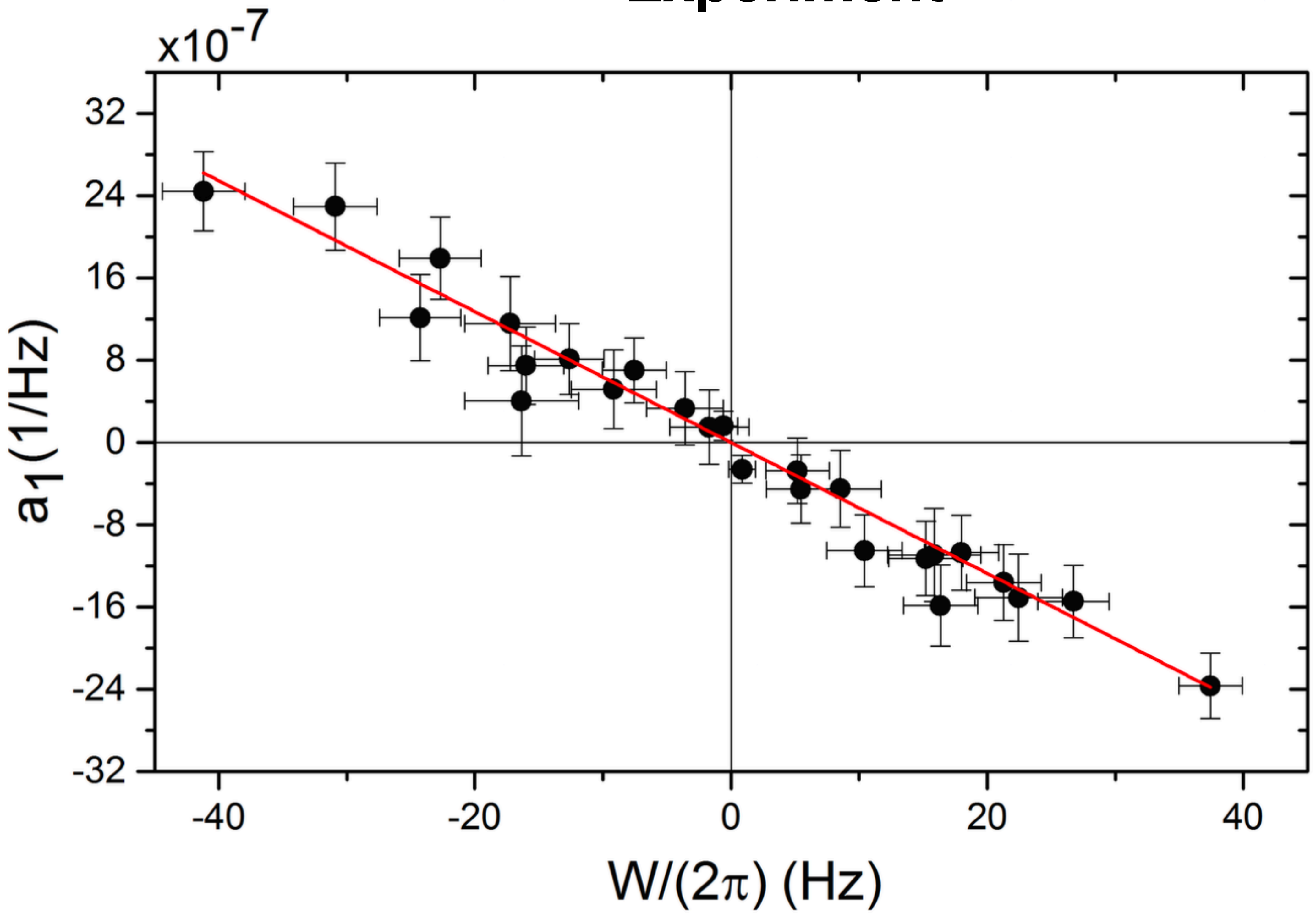
$C_W \approx -0.1326$ [Hz/MHz(mV/cm)]

opposite sign can be explained by different δ_{L2} definition

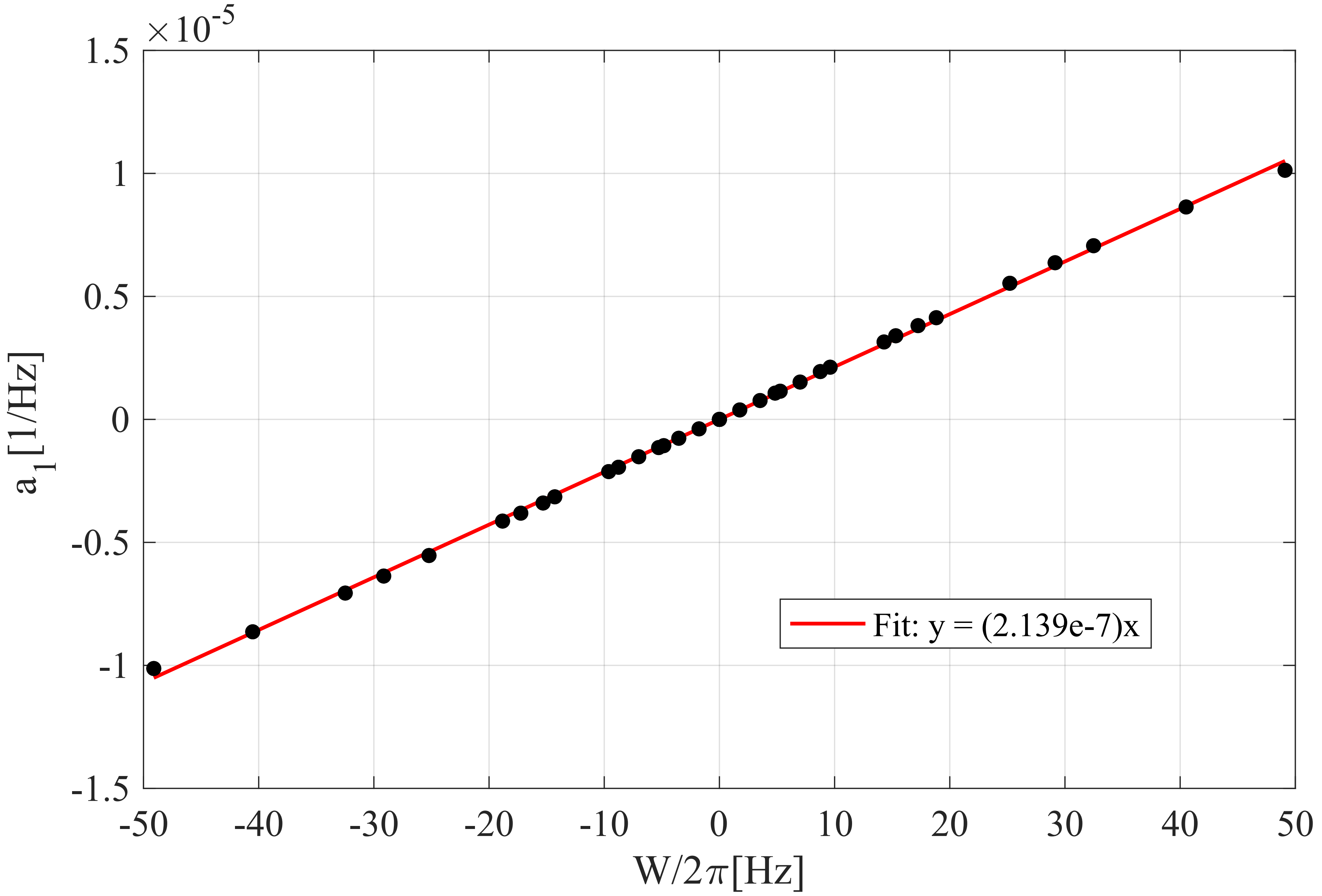
a_1 vs. $W/2\pi$

$a_1 = b_1 W$

Experiment



Simulation



$b_1 = -6.4 \times 10^{-8}[1/Hz^2]$

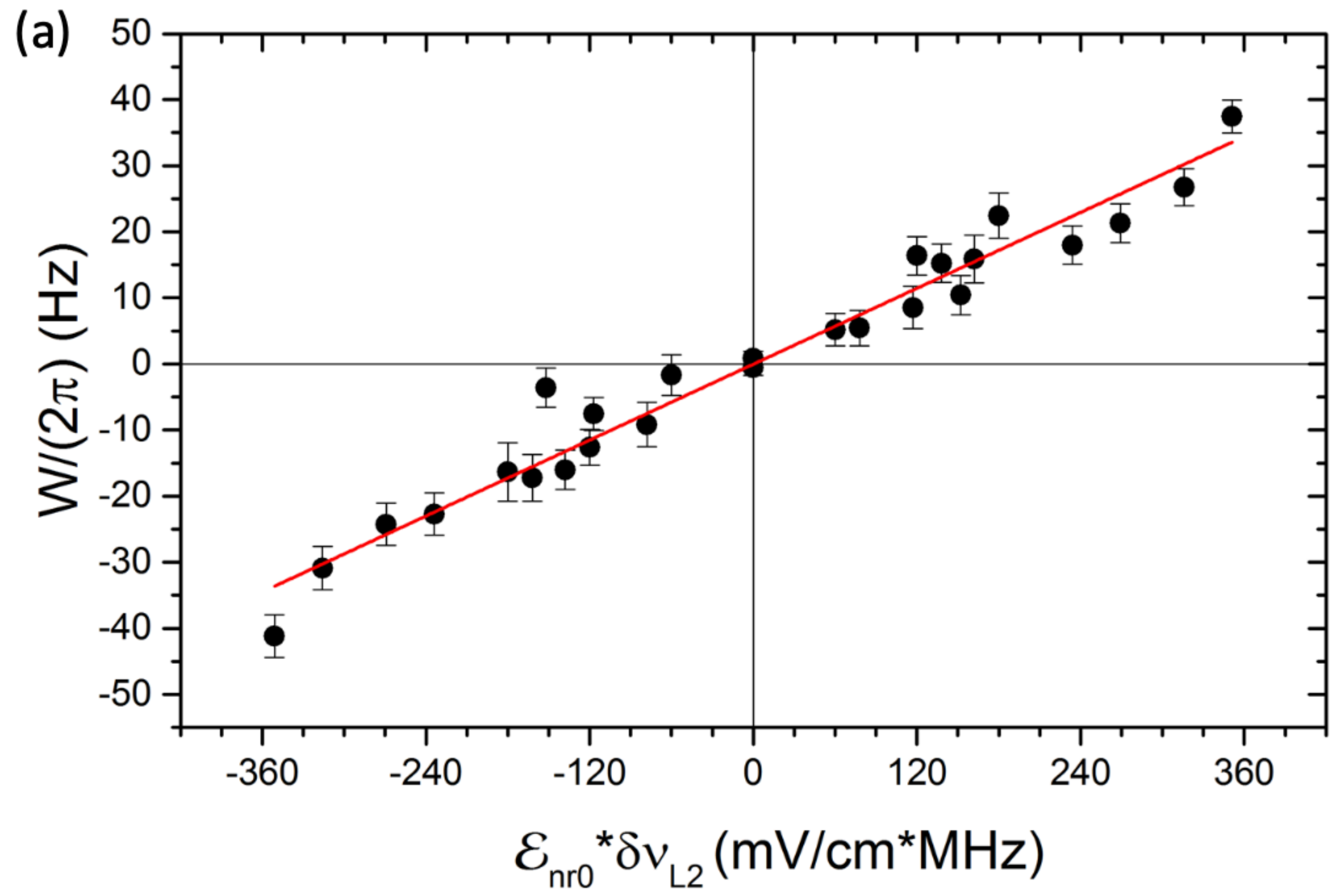
$b_1 = 21.39 \times 10^{-8}[1/Hz^2]$

opposite sign **cannot** be explained by different δ_{L2} definition

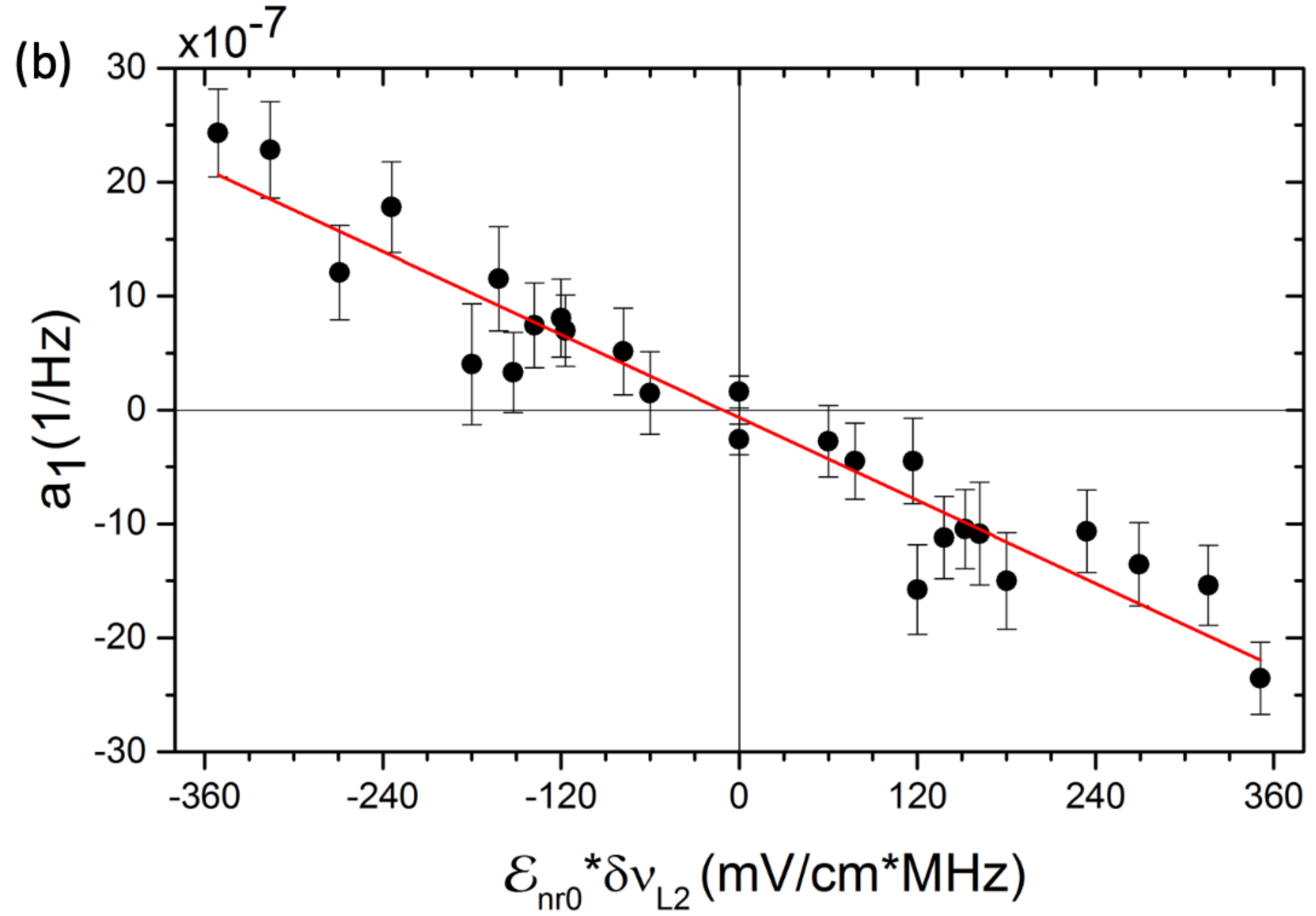
$$W = C_W * (\mathscr{E}_{nr0} * \delta_{L2})$$
$$a_1 = C_{a_1} * (\mathscr{E}_{nr0} * \delta_{L2})$$

$W/2\pi, a_1$ vs. $\mathscr{E}_{nr0} * \delta_{L2}$

Experiment



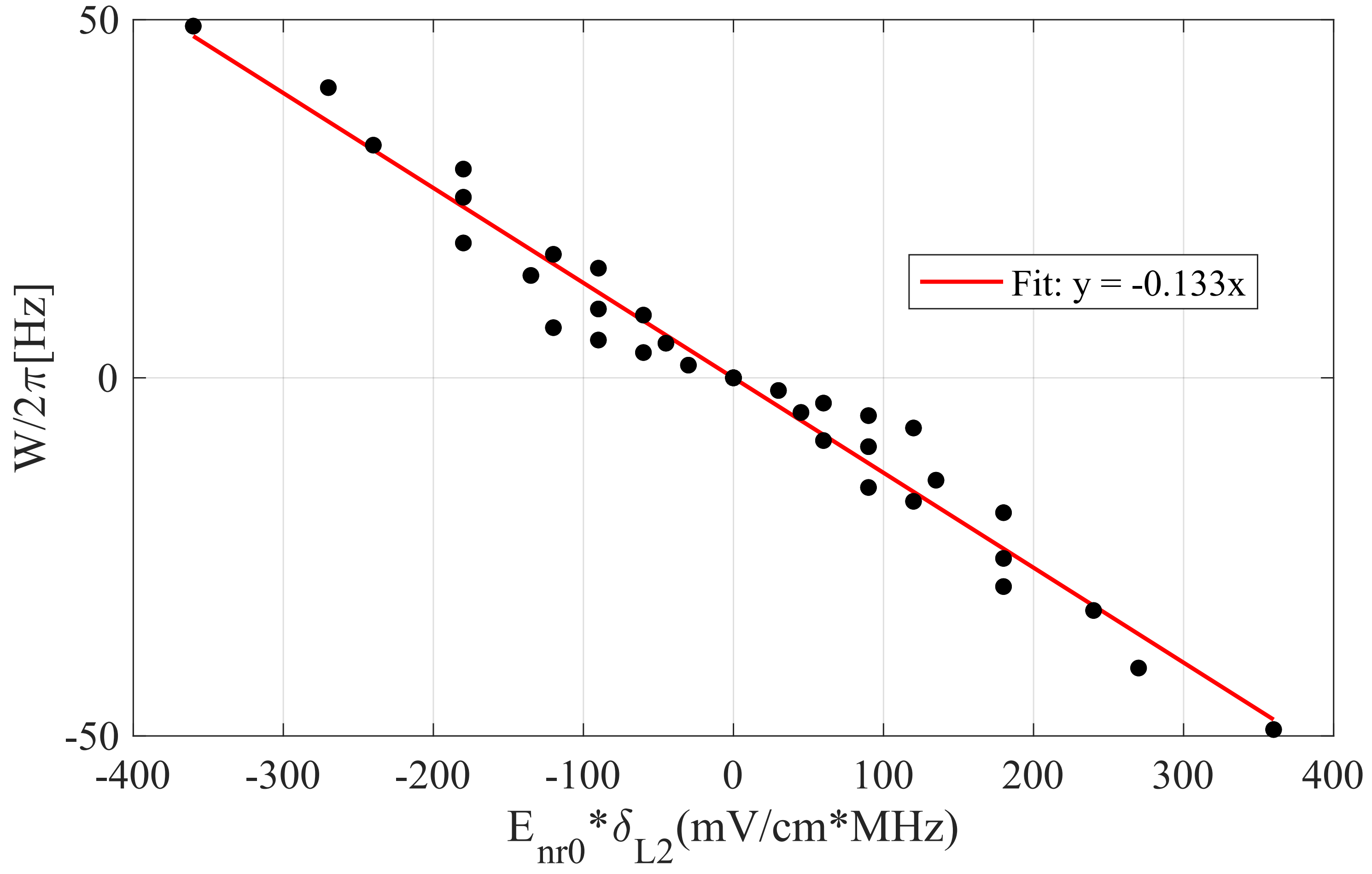
$C_W = 0.095$ [Hz/(MHz*mV/cm)]



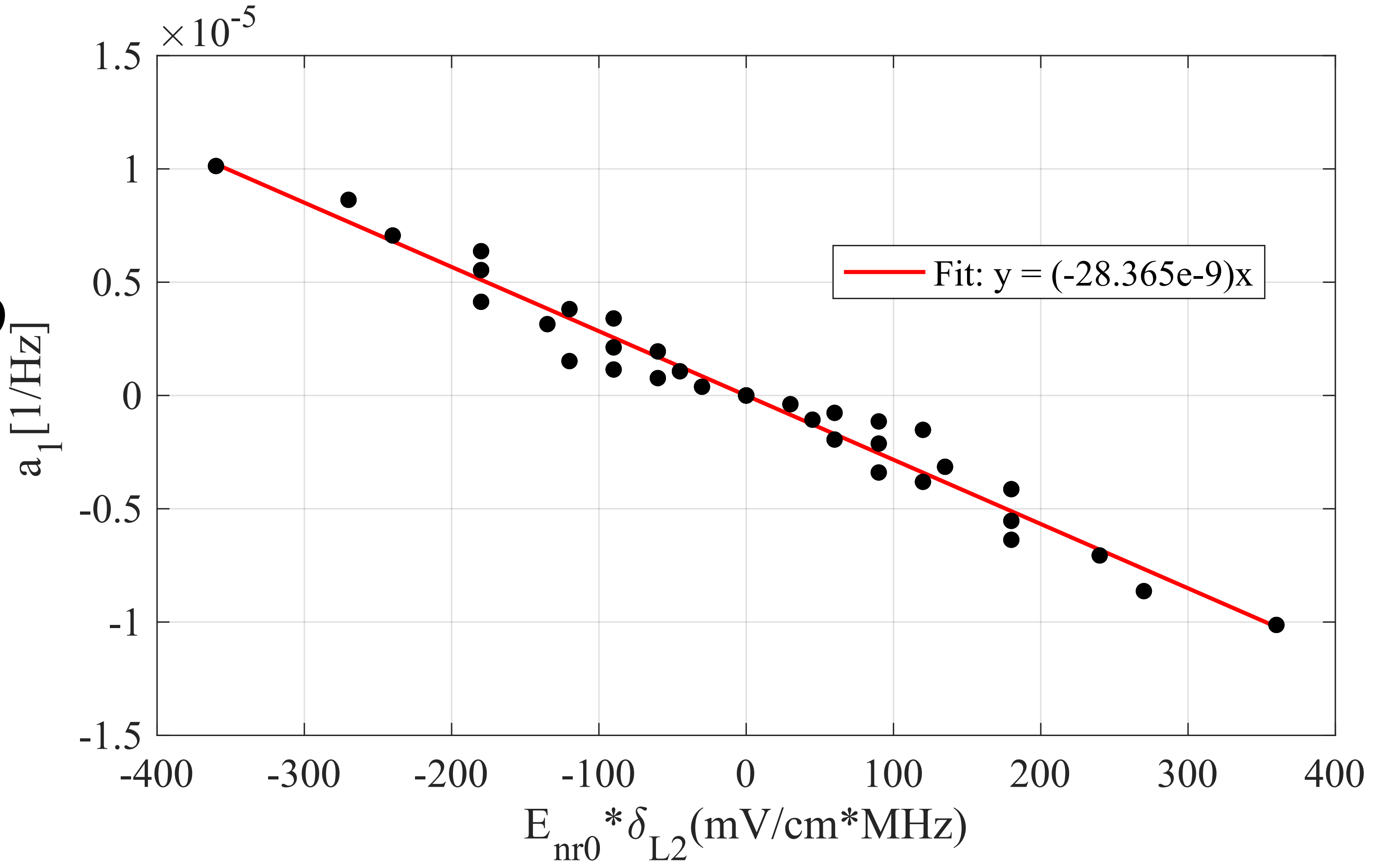
$C_{a_1} = -6.1 \times 10^{-9}$ [1/(Hz*MHz*mV/cm)]

Simulation

$C_W = -0.133$ [Hz/(MHz*mV/cm)]



$C_{a_1} = -28.365 \times 10^{-9}$ [1/(Hz*MHz*mV/cm)]



$$\frac{d\rho}{dt} = -i[\mathscr{H}'_{\pm}, \rho] + \mathscr{L}(\rho)$$

$$\mathscr{L}(\rho) = \begin{pmatrix} 0 & 0 & -\frac{\Gamma}{2}\rho_{13} \\ 0 & 0 & -\frac{\Gamma}{2}\rho_{23} \\ -\frac{\Gamma}{2}\rho_{31} & -\frac{\Gamma}{2}\rho_{32} & -\Gamma\rho_{33} \end{pmatrix}.$$

should I put $\Gamma = 2\pi \times 2.7$MHz or $\Gamma = 2.7$MHz?