HW7

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1 Problem 1

For the first problem, we explored the Brent's one-dimensional minimization method, which utilizes quadratic optimization until specific conditions are violated. These conditions include whether the parabolic step extends beyond the bracketing interval or if the parabolic step is larger than the step before the last one. If either of these conditions is satisfied, the method switches back to using the golden section method.

Here in figure 1, I plotted the actual function

$$f(x) = (x - 0.3)^2 e^x$$

along with the steps takes for finding the minimum x value using Brent's one-dimensional minimization method. For the initial condition, I choose

$$a = 0, c = 1$$

and b is just the point at golden ratio, which is

$$b = a + 0.382(c - a)$$

The result of this minimization is also compared with the built in method in scipy. I designed the method so that it will stop when the difference between the actual value and the minimization result is smaller than 10^{-10}. Using the Brent's one-dimensional minimization method I wrote, the minimum is 0.30000000000387. Number of iterations to get this is 48. The difference from actual minimum is 3.873e-12; The minimum obtained using scipy is 0.3000000000237, which undergoes only 12 iterations, the difference from actual minimum is 2.373e-11. This drastic difference in number of iterations indicates that scipy is more efficient in finding the minimum.

2 Problem 2

For this problem, we use logistic regression to interpret a set of data. The logistic regression function looks like:

$$p(x) = \frac{1}{1 + e^{-(\beta_0 + x\beta_1)}}$$

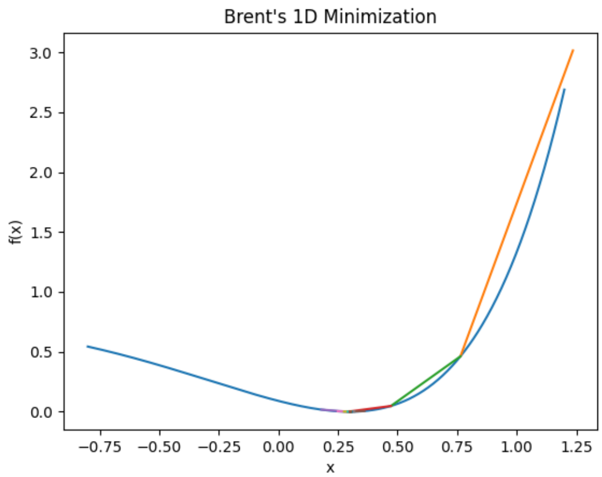

Figure 1: Brent's 1D minimization. The blue curve is the actual function; those straight lines with different colors each represents a step taken using Brent's minimization

Here I present the optimized betas: $\beta_0 = -5.620; \beta_1 = 0.109$. Below is the figure of the optimized logistic model versus the data.

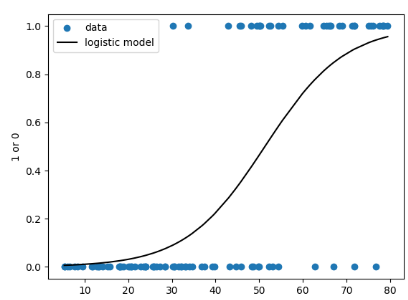

Figure 2: logistic model versus actual data

Here we can see that the logistic model is actually a pretty good representation of the data, since at the middle of the curve, the density of 1 and 0 are similar.

3 Github

username: robertXi6 link: https://github.com/robertXi6/phys-ua210