HW4

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1 Problem 1

From the python code, I obtained the following: for 20 step trapezoidal, the integration is 4.4267; for 10 step, the result is 4.5066. Using the equation 5.28, I got the theoretical error for 20 steps is 0.0266. While, the difference between trapezoidal method and actual integral is 0.0267. Compare those two result, the difference is very small. They are not exactly the same because the error calculated using estimation is based on the assumption of

$$I = I_2 + ch^2$$

, where I is the actual integration result, I_2 is the result obtained using trapezoidal method. This assumption assumes that there exist a constant c that the error is proportional to step size squared h^2, which may not be exact.

2 Problem 2

2.1 (a)

we have

$$V(a) = \frac{1}{2}mx'^2 + V(x)$$

Using separation of variable

$$\frac{dx}{dt} = \sqrt{\frac{2(V(a) - V(x))}{m}} \tag{1}$$

$$dt = \sqrt{\frac{m}{2}}\frac{1}{\sqrt{V(a) - V(x)}} \tag{2}$$

$$\int_0^{\frac{T}{4}} dt = \sqrt{\frac{m}{2}}\int_0^a \frac{1}{\sqrt{V(a) - V(x)}}dx \tag{3}$$

$$T = \sqrt{8m}\int_0^a \frac{1}{\sqrt{V(a) - V(x)}}dx \tag{4}$$

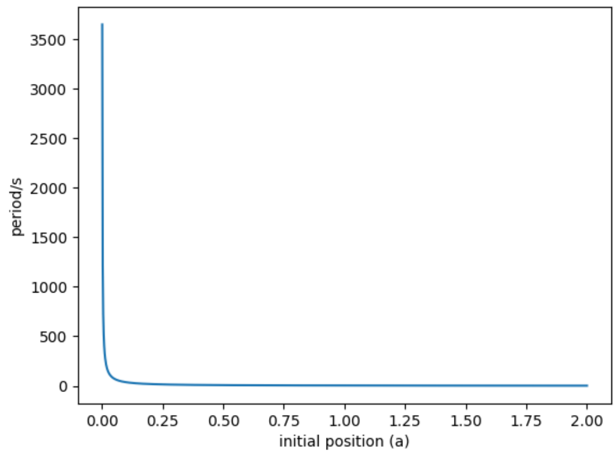

Figure 1: Assume potential is $V(x) = x^4$, integrate using Gaussian quadrature with N = 20 from a=0 to 2, m = 1 to find the period. One can see that the period increases as a decreases. Blow up at 0.

2.2 (b)

if $V(x) = x^4$, from the python code, integrate using Gaussian quadrature with N = 20 from a=0 to 2, the result I got looks like figure 1. While running the code, I found that if I include 0 into the calculation, it will have runtime error, probably because the function blow up at 0. So I compute the period beginning from a = 0.001.

2.3 (c)

The period indeed diverge as amplitude goes to zero. I think the reason is that unlike harmonic oscillators, the potential goes like $V(x) = x^4$, which means that around 0 amplitude, the potential is "flatter", or the derivative of potential is nearly zero, so if one release the ball at very amplitude very close to 0, the potential energy is so small that it takes infinity amount of time in order to gain some kinetic energy to move. Also, the force is proportional to the derivative of the potential, so if potential is very flat, the force is nearly zero, which keeps the ball at stationary. By contrast, if the amplitude is high, the potential energy is high and the derivative of potential is large, the ball will gain the speed in short time and thus the period decreases.

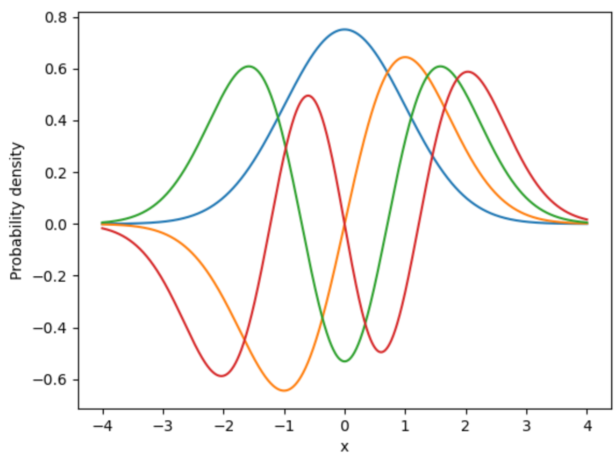

Figure 2: wave function for one-dimensional quantum harmonic oscillator for n = 1,2,3,4. x-axis from -4 to 4

3 Problem 3

3.1 (a)

Figure 2 shows the wave function for one-dimensional quantum harmonic oscillator for n = 1,2,3,4. While I doing this problem, I found there may be a mistake in the question where Hermite polynomials should be

$$H_{n+1} = 2xH_n(x) - 2(n-1)H_{n-1}(x)$$

. This occurs to me when I did the last question for this problem where I found 1.88 as the integration answer instead of 2.3 proposed by the question, so I looked up the internet to figure out where's wrong. All my answers for this problem is based on the correct Hermite polynomials that I found on internet.

3.2 (b)

Figure 3 shows the wave function for n = 30

3.3 (c)

Using Gaussian quadrature on 100 point with n = 5 to find the uncertainty, I get $\sqrt{\langle x^2 \rangle} = 2.3452078799117153$. In order to find this value with 100 points, one need to carefully choose the interval of evaluation, since if the interval is too small, some of the wave function may not be considered into the integration; if the interval is too large, only few points will pass through the location where

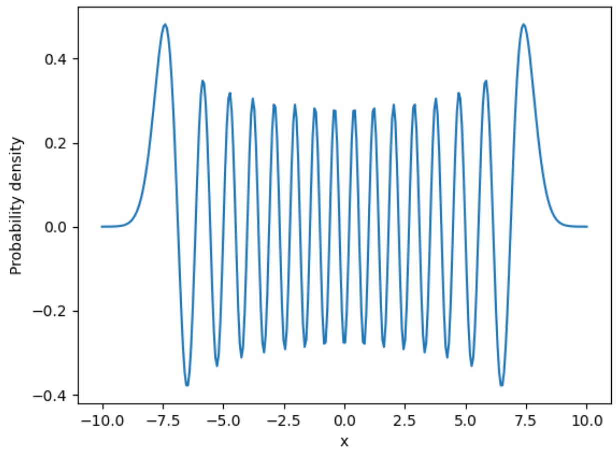

Figure 3: wave function for n = 30, I used the correct Hermite polynomials found on internet

wave-function has noticeable values. Thus, in order to determine a good region, I first plotted the wave function for n = 5 and then see at which position, the wave function starts to vanish. I found that point to be around 8, so I set the interval at (-10, 10).

3.4 (d)

using scipy, I was able to get $\sqrt{\langle x^2 \rangle} = 2.3452078799117144$. This number is very close to the previous one, which made me confident about the approximation result is very close to the actual result.