HW5

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1 Problem 1

1.1 (a)

Here is the figure output from the code where each line represents a different a value. See figure 1.

1.2 (b)

To find the extreme point of a function, calculate the derivative of the function and find when this derivative is equals to 0.

$$\begin{aligned} \frac{dx}{dt} &= (a-1)x^{a-2}e^{-x} - e^{-x}x^{a-1} && (1) \\ &= 0 && (2) \\ & && (3) \end{aligned}$$

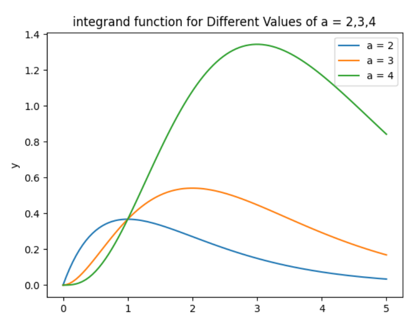

Figure 1: the integrand function evaluated from x=0 to 5. each curve represents a different a value.

thus,

$$
\begin{align}
(a-1)x^{a-2}e^{-x} &= e^{-x}x^{a-1} \tag{4}\\
(a-1)x^{a-2} &= x^{a-1} \tag{5}\\
(a-1) &= x \tag{6}\\
\tag{7}
\end{align}
$$

1.3 (c)

when x = c, $z = \frac{c}{c+c} = \frac{1}{2}$. Therefore, to make the peak of the function at $z = \frac{1}{2}$, c should equal to the x value when the original function is the maximum, which is just $c = a - 1$ from last question.

1.4 (d)

since

$$x^{a-1} = e^{(a-1)lnx}$$

plug into the original equation we have

$$x^{a-1}e^{-x} = e^{(a-1)lnx}e^{-x} = e^{(a-1)lnx-x}$$

this expression will suffer less from the overflow or underflow because when x goes to infinity, neither x or lnx will be too small, their difference will be smaller than the difference between x^{a-1} and e^{-x}.

1.5 (e)

replacing all x with $z = \frac{x}{a-1+x}$, equivalently, $x = \frac{z(a-1)}{1-z}$ we have the following:

$$e^{(a-1)lnx-x} = e^{(a-1)ln(\frac{z(a-1)}{1-z})-\frac{z(a-1)}{1-z}}$$

the integral thus became

$$\Gamma(a) = \int_0^1 \frac{(a-1)(e^{(a-1)ln(\frac{z(a-1)}{1-z})-\frac{z(a-1)}{1-z}})}{(1-z)^2}dz$$

Using code, I was able to get the result for $\Gamma(\frac{3}{2}) = 0.8862269613087213$.

1.6 (f)

using code, I got $\Gamma(3) = 2.0000000000000657$, very close to $2! = 2$; $\Gamma(6)$= 120.0, very close to $5! = 120$; $\Gamma(10)$= 362879.9999999, very close to $9! = 362880$.

2 Problem 2

2.1 (a)

Figure 2 shows the plotted signal data.

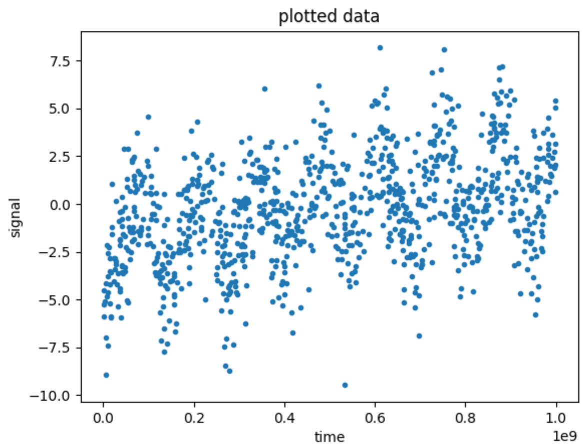

Figure 2: plotted signal

2.2 (b)

Figure 3 shows the best third-order polynomial fit in time to the signal.

2.3 (c)

From figure 3, one can see that the residue is not so even, in other word, the residue itself resembles the characteristics of the original data, which implies a poor fitting. A good fitting should looks like a Gaussian distribution from the mean value, but in this case, it's not.

2.4 (d)

I tested several different orders of polynomials, here I plotted few of them including 3,5,8,10,20,30,50,100, 1000,10000th order. If we define "reasonable polynomial" as whether the design matrix has a viable condition number, then, in my case, condition number only become a real number outputted by computer for polynomial higher than 25 order, thus this 25th order maybe the lowest order that is a "good" explanation of the data. The condition number in this case is usually very large though. For 25th order, the condition number is 1.3993e+16.

2.5 (e)

For N = 20, I was able to get a very good fitting using harmonic sequence with increasing frequency. The condition number in this case is 1.869, which is much smaller than before. Figure 13 shows the result. For N=10, the condition number is 1.669, Figure 14 shows the result. Those illustration indicates a better fit by using harmoic sequence.

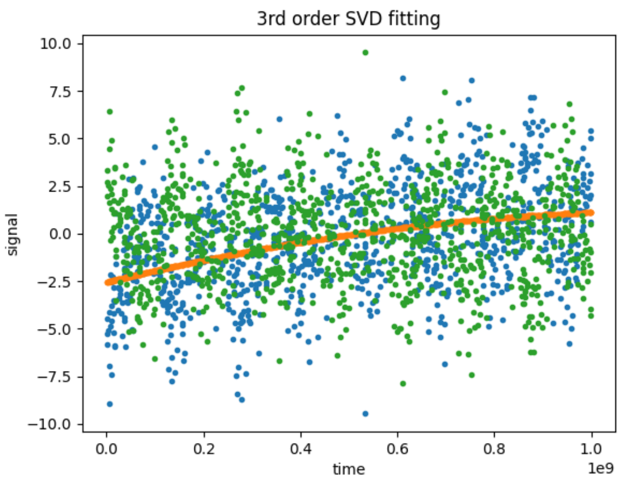

Figure 3: 3rd order SVD of the data, the residue is included as green dots

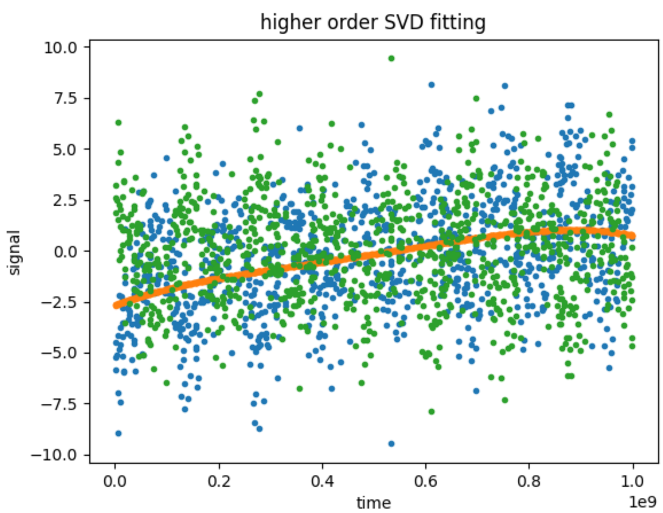

Figure 4: 5th order SVD, Orange curve is the fitting, blue is original signal, green is the residual

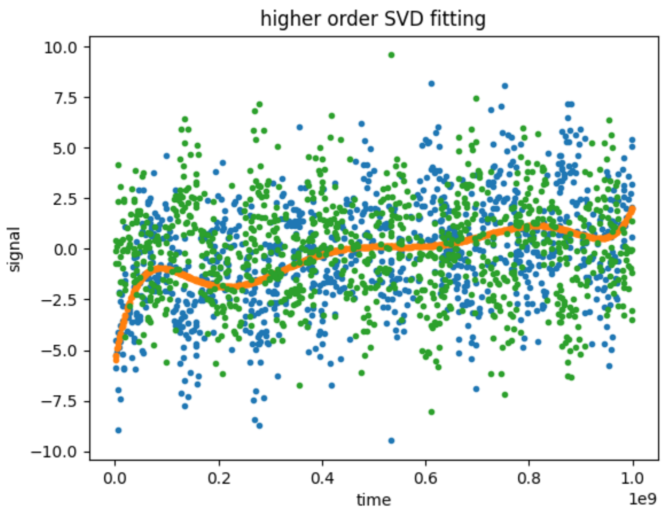

Figure 5: 8th order SVD, Orange curve is the fitting, blue is original signal, green is the residual

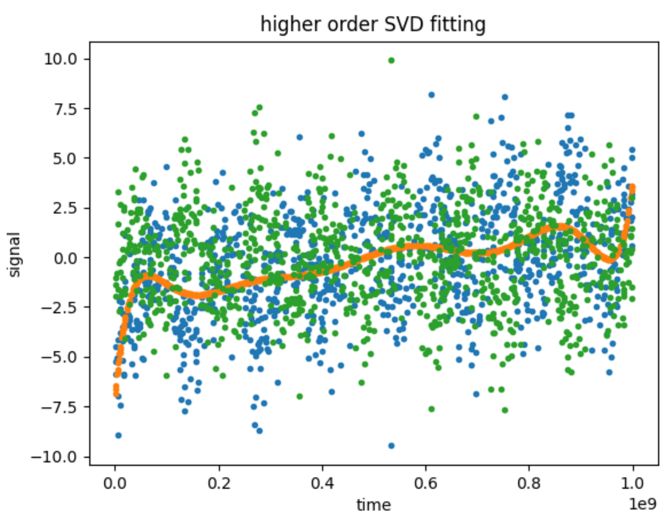

Figure 6: 10th order SVD, Orange curve is the fitting, blue is original signal, green is the residual

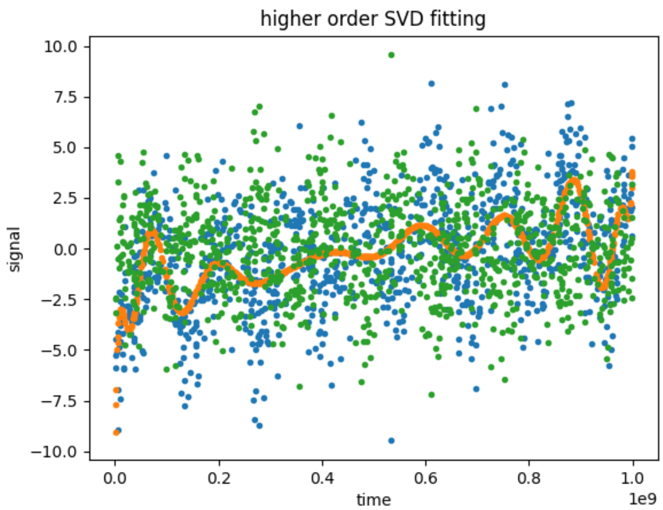

Figure 7: 20th order SVD, Orange curve is the fitting, blue is original signal, green is the residual

Figure 8: 30th order SVD, Orange curve is the fitting, blue is original signal, green is the residual

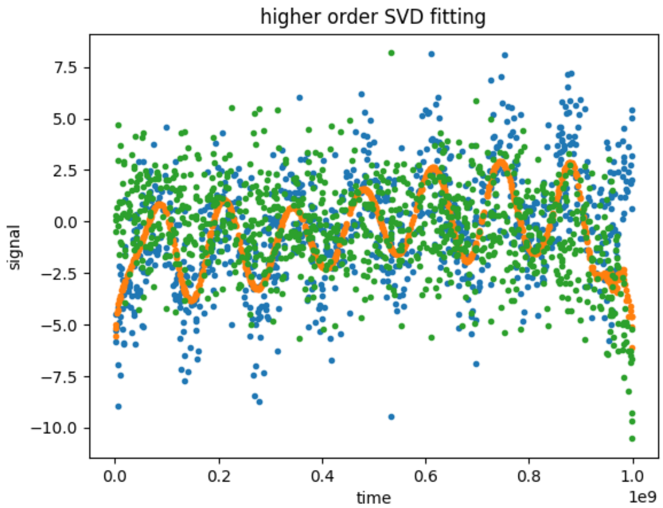

Figure 9: 50th order SVD, Orange curve is the fitting, blue is original signal, green is the residual

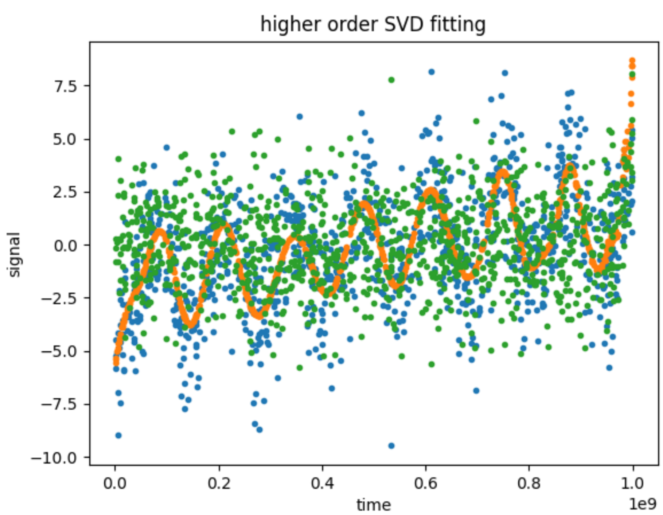

Figure 10: 100th order SVD, Orange curve is the fitting, blue is original signal, green is the residual

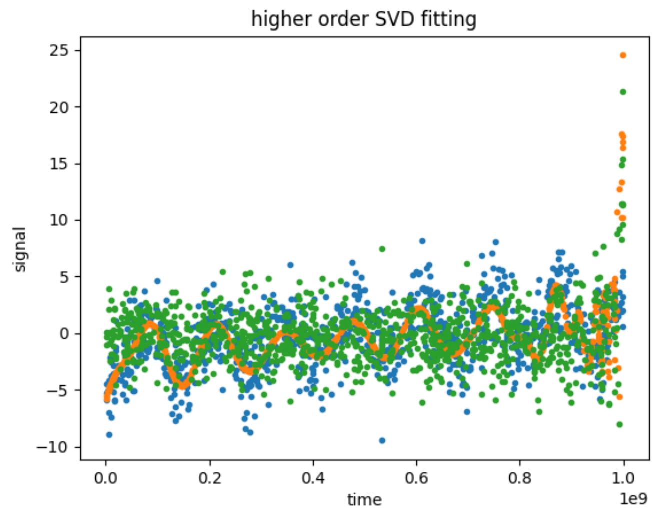

Figure 11: 1000th order SVD, Orange curve is the fitting, blue is original signal, green is the residual

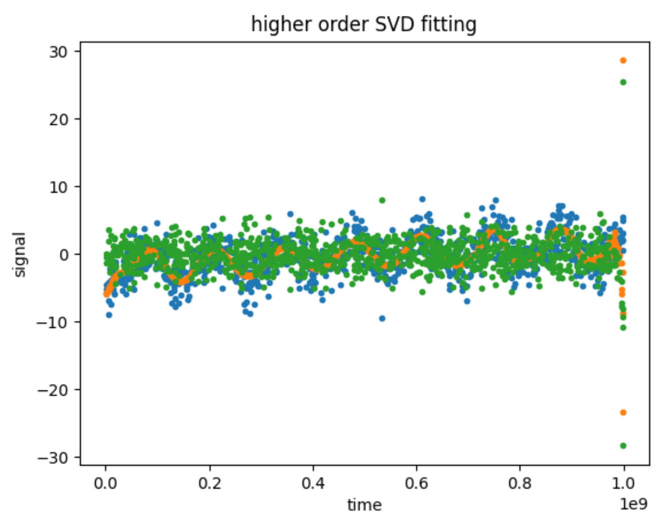

Figure 12: 10000th order SVD, Orange curve is the fitting, blue is original signal, green is the residual

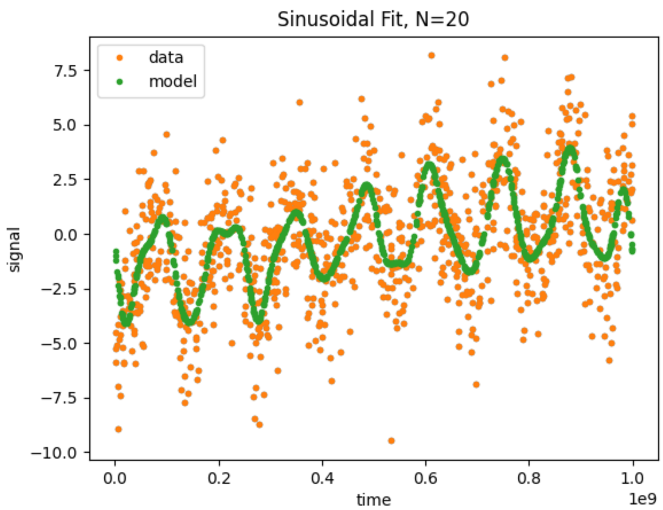

Figure 13: Sinusoidal fit, N = 20, condition number is 1.869, indicates that the fit is much better

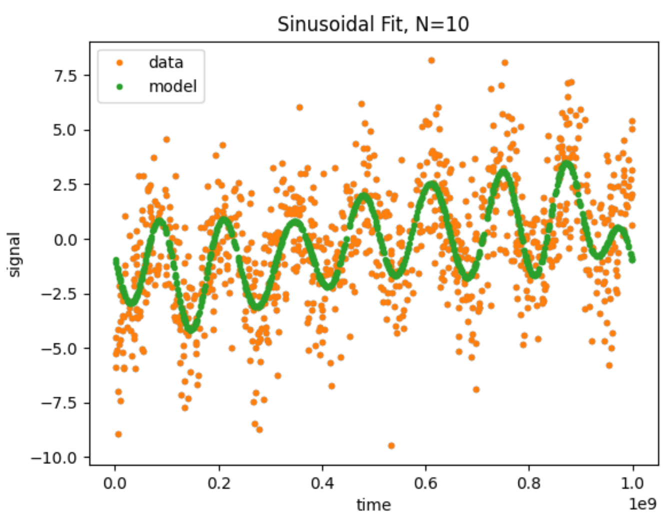

Figure 14: Sinusoidal fit, N = 20, condition number is 1.869, indicates that the fit is much better

3 Github

username: robertXi6

link: https://github.com/robertXi6/phys-ua210