Response to Reviewers

The reviewer comments are in plain text and our response is in italic font.

Reviewer #3:

The authors use MuMax3 micromagnetic simulations to explore spin Hall nano-oscillators (SHNOs) where the effective ferromagnet is a metal(Py)/insulator(LAFO) hybrid. The first experimental demonstration by H. Ren et al. Nat. Commun. 2023 (ref. 21) was an important result. In particular, they model how the key functional parameters (e.g. threshold current and oscillation amplitude) of these hybrid oscillators depend on LAFO properties: thickness, perpendicular magnetic anisotropy, and saturation magnetization Ms. In my opinion, the results are timely and will motivate further experimental studies to optimize this new type of SHNO. Journal of Applied Physics seems like a good home for this body of work, and I support publication with the following concerns addressed.

Major Concerns to be addressed:

1. The authors should state their estimated exchange length in Py and LAFO. They use a 5x5x5 nm3 cell size. The exchange length of Py is commonly experimentally reported to be on the order of 5 nm. Concerning micromagnetic simulations, uMAG standard problem #5 (https://www.ctcms.nist.gov/~rdm/std5/spec5.xhtml) lists that Py should have an exchange length of ~5.7 nm. The micromagnetic cell size should be chosen to be below the exchange length. M. Najafi et al J. Appl. Phys. 105, 113914 (2009) shows that a cell size of 2x2x2 nm3 is appropriate for Py. Larger cell sizes lead to non-negligible errors. The authors should benchmark a few of their key results to confirm whether their cell size is too large.

We thank the referee for this comment. Indeed, having a cell size smaller than the exchange length leads to more accurate simulations at the expense of simulation time. Our need to simulate large volumes in reasonable times led to our choice of 5x5x5 nm^3 cell sizes. *A 2.5x2.5x2.5 nm cell size for Py is adequate for doing accurate micromagnetic simulations of Py, according to M. Najafi et al.* Appl. Phys. 105, 113914 (2009)*. Thus, to ensure that our 5x5x5 nm^3 cell size gives reasonable results, we ran a comparison of 2.5x2.5x2.5 nm^3 cell size with the following parameters: $K_{u, LAFO}$= -31.7 kJ/m^3 and $M_{s, LAFO}$ = 0.17 T. The simulations were conducted with an in-plane field μ_0H_{ext} = 0.08T at an angle φ = 70°.*

Figures R1 and R2 compare the time evolution of M_z and corresponding FFT amplitudes. We performed FFT on the steady-state spin dynamics, marked by the yellow broken lines in Fig. R1. In Fig. R2, we determined that the peak FFT frequency and amplitude to be 6.30 GHz, 32.2 +/- 0.6 for 2.5x2.5x2.5 nm^3 cell sizes, and 6.27 GHz, 27.0 +/-0.5 for 5x5x5 nm^3 cell sizes by fitting the FFT to a Lorentzian function. We find that the oscillation frequencies are close to the same, while there is a 20% discrepancy in the FFT amplitude. We therefore believe 5x5x5 nm^3 adequately describes the hybrid SHNO response.

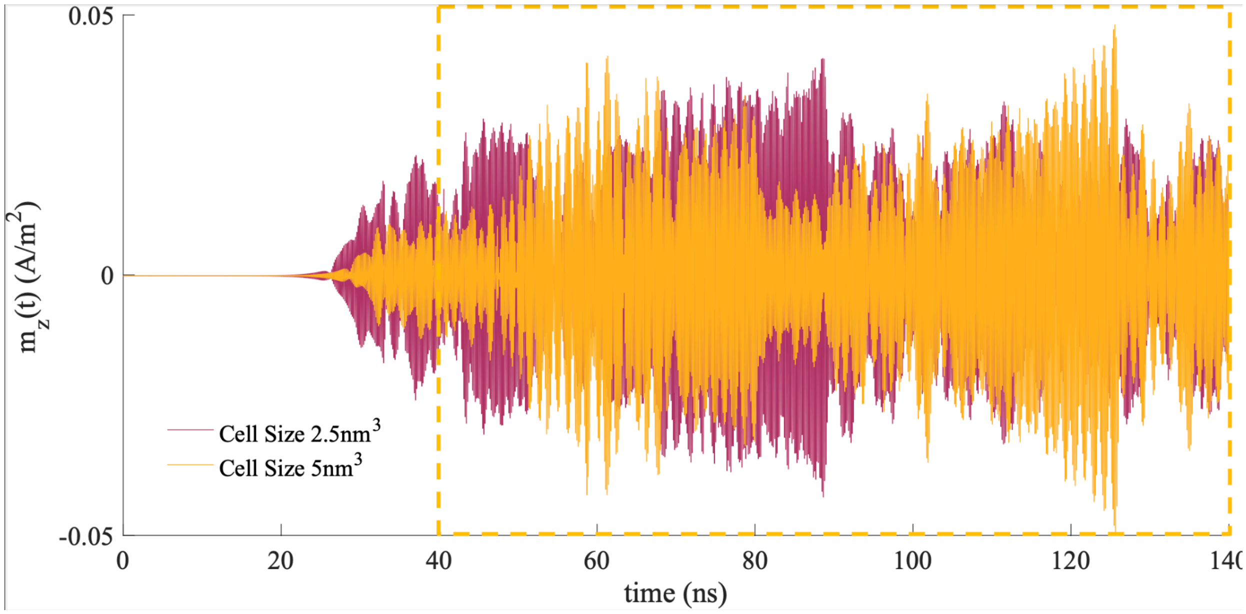

Figure R1: LAFO10/Py5/Pt5 spatial averaged $m_z(t)$ for 2.5x2.5x2.5 nm^3 and 5x5x5 nm^3 cell sizes. Parameters: $K_{u,\ LAF\ O}$= -31.7 kJ/m^3 and $M_{s,\ LAFO}$ = 0.17 T. This simulation was conducted with an in-plane field of μ_0H_{ext} = 0.08T at an angle of φ = 70°.

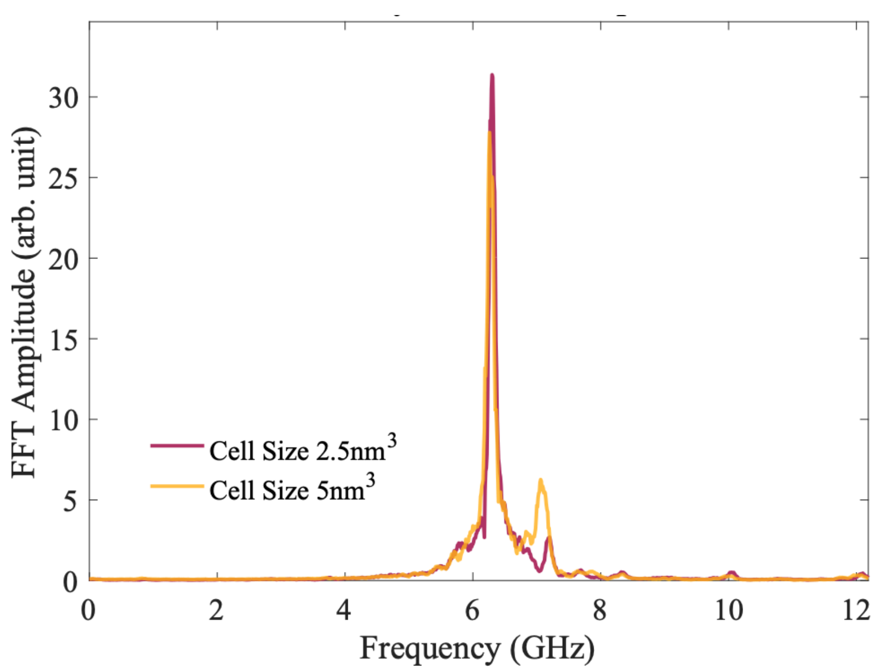

Figure R2: LAFO10/Py5/Pt5 FFT amplitudes for 2.5x2.5x2.5 nm^3 and 5x5x5 nm^3 cell sizes. Parameters: $K_{u,\ LAF\ O}$= -31.7 kJ/m^3 and $M_{s,\ LAFO}$ = 0.17 T. This simulation was conducted with an in-plane field of μ_0H_{ext} = 0.08T at angle of φ = 70°.

2. Pg. 3. "The Oersted field generated from Je is negligible and not included in our study." I disagree with this statement. The Oersted field generated by the current densities shown in Fig.

2 running through the Pt layer would be approximately between 5 and 12 mT (50-120 Oe). [One can crudely treat Pt as an infinite plane with this current density or, more accurately, as a current carrying ribbon, for example, as teated by N, Gauthier American Journal of Physics 56, 819 (1998)] While this may seem small, it is about 10% of the applied field. The Oersted field from a positive spin Hall angle material like Pt will be opposite to the externally applied field in the anti-damping configuration needed to operate the SHNO. Therefore, in the injection region, the effective field is lower than the region under the leads. A lower field means a lower frequency of oscillation. Lower frequency means lower energy. Therefore, the Oersted field, even if somewhat small compared to the applied field, creates a potential well for spin-wave modes that can lead to enhanced confinement of spin-waves to the injection region. The discussion on pg. 7-8 notes the importance of potential wells to confine the spin-waves. The Oersted field should modify this confinement and thus should not be assumed to be negligible.

We performed a COMSOL analysis and found the Oersted field generated from an applied electrical current (1.3 mA) has a maximum z-component of 5.4 mT at the edges of the Py nanowire. Our simulation studies study parametric dependent cases under an external field of 80 mT. The Oersted field is thus less than 10% of the external field, which will thus only have a minor effect on the spin dynamics. We have modified our manuscript to address this point.

3. Pg 3. "To simulate auto-oscillation excitation, we apply a charge current density Je to the central region of the Py nanowire because this is where Je flows through in the device as the remaining Py region is covered by electrical contacts." It is implied by the relation Js = ThetaSH*Je that the Je is the current density in the Pt layer and NOT the device level current density. The Py layer will shunt current that does not lead to spin Hall torque from Pt. It needs to be clarified whether the authors simulate a current applied to the whole device or the Pt layer alone.

We have edited the manuscript to indicate that the Pt layer is the source of spin current that is then injected in the active (400 x 400 nm^2) region of the Py layer. The current density we indicate is thus the current density in the Pt layer. We now state this in the manuscript.

4. Pg. 3. The authors use a criterion to judge entering the auto-oscillatory state of increased average torque within 150 ns of simulation. Have the authors determined the system reaches steady state dynamics? They should. Are transient dynamics included in PSD and FFT amplitudes? They should not be. These details should be explained more carefully. It would be nice to see at least one time trace of Mx, My, and Mz above the threshold current.

We thank the referee for this comment. Indeed, we have checked that the system reaches steady-state dynamics when there is an increase in the average torque. Figures R3 and R4 plot the time evolution of M_x, M_y, M_z, and the average max torque with parameters $K_{u,\,LAFO}$ = -50 and -25 kJ/m^3, and $M_{s,\,LAFO}$ = 0.0942 T. The simulations were conducted with an in-plane $\mu_0 H_{ext}$ = 0.08T at φ = 70°. We perform FFT analysis on the data in the time window enclosed by the yellow broken lines. As shown in Figs. R3 and R4, the region enclosed by the yellow broken line denote the data we used to perform the FFT operation and PSD amplitude calculations. So the *transient dynamics are not included in the calculation. We have edited the manuscript to clarify this point.*

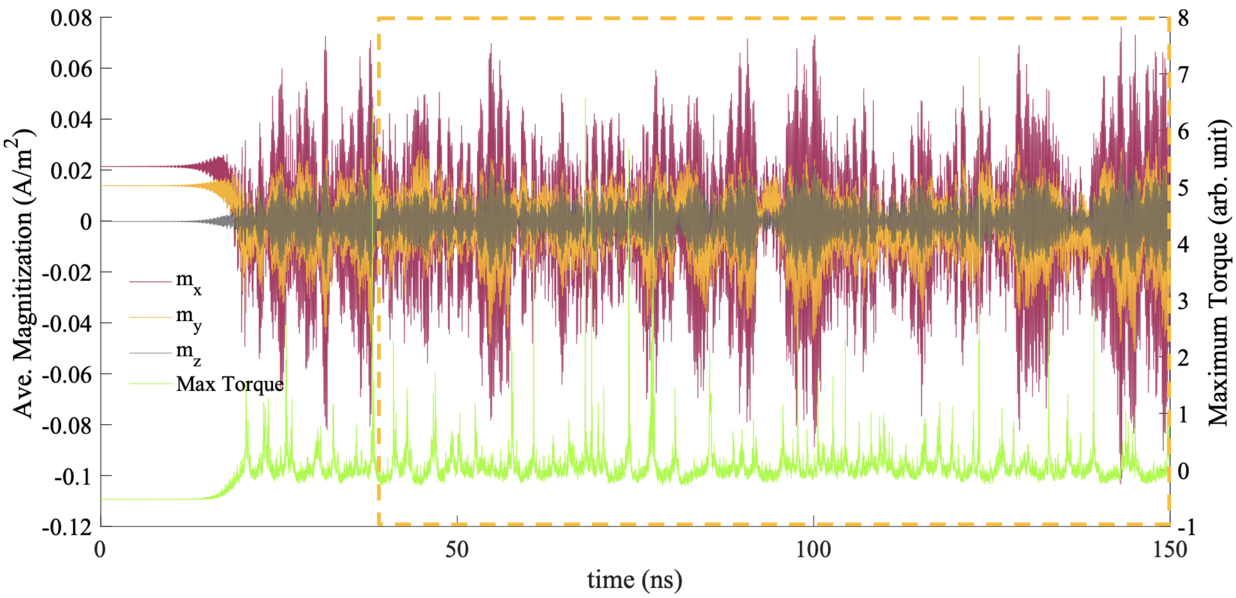

Figure R3: LAFO10/Py5/Pt5 spatial averaged m_x, m_y, m_z, and max torque as a function of time. Parameters: $K_{u,\ LAF\ O}$= -50 kJ/m^3 and $M_{s,\ LAFO}$ = 0.0942 T. This simulation was conducted with an in-plane field of $\mu_0 H_{ext}$ = 0.08T at angle of φ = 70°.

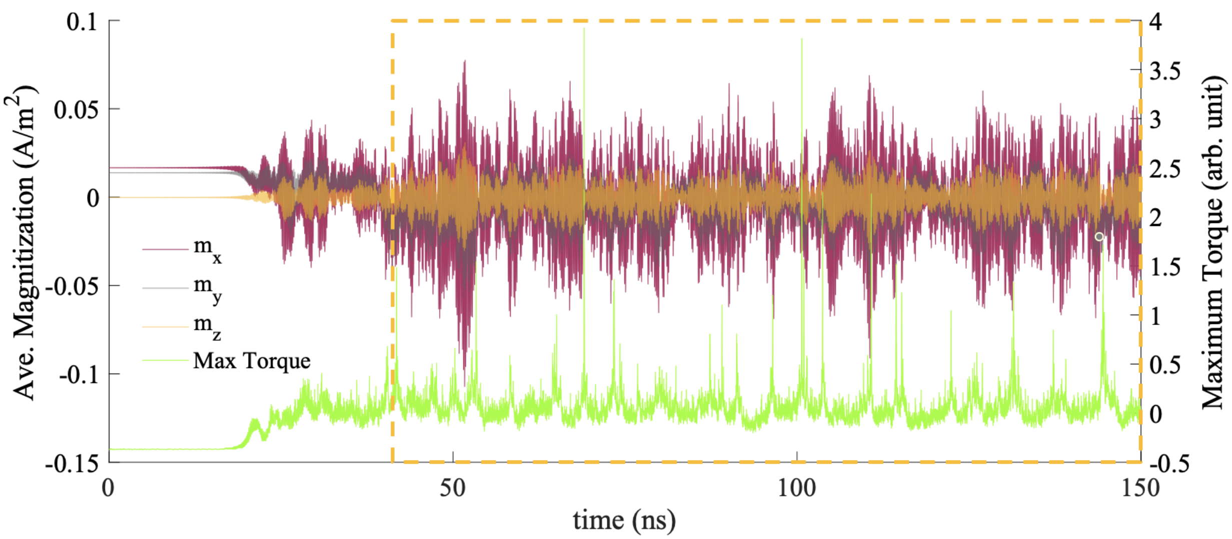

Figure R4: LAFO10/Py5/Pt5 spatial averaged m_z and max torque as a function of time. Parameters: $K_{u,\ LAF\ O}$= -25 kJ/m^3 and $M_{s,\ LAFO}$ = 0.0942 T. This simulation was conducted with an in-plane field of $\mu_0 H_{ext}$ = 0.08T at angle of φ = 70°.

5. Pg 5-6 and Fig. 3(a). What is the value of Ms and Ku? Has the thickness dependence been studied near and away from the peaks in Fig. 3 b and c [a few values of Ms and Ku]? The results of Fig. 3 b and c are quite nice, showing that Meff,LAFO is the key parameter to tune to optimize the SHNO. To me, this is the key point of the paper. Perhaps nothing happens with thickness, but it would be a shame to miss something by not sampling enough parameter space.

We appreciate this comment. In Fig. 3(a), the following parameters for LAFO are used: $\mu_0 M_s$ = *0.0942 T and* $K_u = -31.7$ *kJ/m*^{3}*. We have included the parameters in the updated manuscript. We did not study the thickness dependence near the peaks in Fig. 3(b). This would be interesting to examine but is beyond the scope of this paper.*

Minor concerns to address:

1. Pg. 3. "To reduce the spin-wave reflection at the boundary, we set an exponentially increased damping approaching the boundaries. Periodic boundary conditions are implemented in the x direction to properly model the demagnetization field." The language is very loose. In which layers and which boundaries are periodic boundary conditions and absorbing boundary conditions used?

We thank the reviewer for this comment. We have made edits in our manuscript to clarify the exponentially increased damping is set in all layers and that the periodic boundary condition only applies to the direction transverse to the electrical current direction.

2. Pg 3. "To simulate auto-oscillation excitation, we apply a charge current density Je to the central region of the Py nanowire because this is where Je flows through in the device as the remaining Py region is covered by electrical contacts." The authors should state the length of the injection region. It can be inferred from Figure 1(a).

We have included the dimension of the spin current injection in the revised manuscript.

3. Pg 4. Fig. 1 (b). It would be nice to include scale bars and some sort of indication of the injection region. Are the modes extending under the leads, etc?

Figure 1(b) has been updated to include a marker indicating the region of spin current injection.

4. Fig. 4. Perhaps consider adding guides to the eye joining the data points. Alternatively, use colors with greater perceptual contrast or both.

Figure 4 has been updated to have more contrast between different data points.

5. Pg 7. PSD is not defined. One can infer power spectral density, but it should be stated.

We have made edits to specify the definition of PSD.

6. Pg 7. The gyromagnetic ratio is negative, i.e., spin and magnetization point in opposite directions. Please state the absolute or modulus of the gyromagnetic ratio.

We have made corresponding edits in the manuscript.

7. Pg 8 and Fig 6. Please confirm the color and shape descriptions match. I found this confusing.

We adjusted the colors of different data points in Fig. 6 to make them easier to see. The corresponding description in the main text has also been updated to match the color and shapes in the figure.

Reviewer #4:

This paper presents a study of the spin Hall nano-oscillator, comprising Py and LAFO, through an analysis of the results obtained from micromagnetics simulations. The system is identical to that described in their previous paper, published in Nature Communications (2023), 14:1406. The authors examined the influence of LAFO thickness, perpendicular magnetic anisotropy (Ku), and saturation magnetization (Ms) on the threshold current and output power. It was found that the threshold current is not significantly influenced by these parameters, whereas the output power exhibits a markedly nonlinear dependence on Ku and Ms. Additionally, the region of Ku where propagating spin wave modes are excited was identified. The results are both intriguing and pertinent. The paper will be published after consideration of the comments below.

1. The use of symbols may be confusing at times, as the subscripts indicating the material type are not always included. It is recommended that the authors explicitly indicate the subscripts, for example Ku_LAFO and Ms_LAFO.

We thank the reviewer for the suggestion. We have specified LAFO and Py for K_u and M_s in our revised manuscript.

2. The definition of M_eff should be provided at the first instance of its appearance.

We have added the definition of M_{eff} to our revised manuscript.

3. It would be beneficial to provide the exchange length of Py and LAFO.

We have made edits to include the information in our manuscript. The exchange length is 5.3 nm for Py and 12-31 nm for LAFO when $\mu_o M_{s,\ LAFO}$ ranges from 0.1 to 0.25 T.

4. It is essential to provide detailed information regarding the material parameters, including the exchange constant and the Gilbert damping constant of LAFO.

The exchange constant of LAFO is 4 pJ/m. The Gilbert damping of LAFO is 0.001. We have included the information in the revised manuscript.

5. The authors claimed that the system reaches a steady state at each time step. The term "time step" is not clearly defined. In the majority of micromagnetics simulations, the time step should be approximately 1 ps or less.

The main text has been revised to include that we use time steps of 1 ps.

6. It would be beneficial to include the Cartesian coordinates (x-y) in Fig. 1(b) for clarity.

Figure 1(b) has been edited to indicate the coordinates (x-y).

7. The authors should provide a detailed explanation of the relationship between the resistance and the averaged magnetization, which may be in Py, as well as the definition of the output power.

We have added an explanation to relate the oscillation angle to the output microwave power via the oscillating resistance in an experimental setup. We further use this relation to establish our method of inferring the output microwave power by the simulated FFT amplitude of the Py magnetization.

8. The meaning of the sentence "As Ku becomes less negative ... M_eff,LAFO decreases" is unclear.

*We have made edits to this sentence to reduce the confusion***.**

9. Please explain the rationale behind assuming an angle of magnetic filed, phi, of 70 degrees. Is this the angle that maximizes the output power? Have calculations been performed for other angles?

Signal detection in an experimental setup utilizes the AMR effect to generate the electrical signal. The AMR signal is largest when the magnetization oscillation is around an angle of the magnetization of 45 degrees to the current. However, the spin-torque effects (the largest anti-damping torques) occur when the magnetization is at an angle of 90 degrees to the current. Therefore, we choose an angle of 70 degrees, i.e., an angle between these two angles. This angle is also consistent with our previous experimental studies (H. Ren et al., Nat. Commun. 2023 (ref. 21)*).*

10. Could you explain the physics behind the peak is determined by M_eff,LAFO?

The amplitude is maximum when the resonance frequency of the Py layer and the LAFO layer nearly coincide. The effective magnetization of LAFO determines the resonance frequency of the auto-oscillation mode. We have addressed this in Discussion Section C. in the revised manuscript.

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11. Captions of Fig. 3 are not consistent with the figures. The figures do not show the output power but show the FFT amplitude.

We use this FFT amplitude as an indication of the device's actual output power. We have included this explanation in the revised text.

12. It would be beneficial to provide a definition of the FFT amplitude. It may be reasonably assumed that the authors performed an FFT on the z component of the averaged magnetization of Py.

We thank the reviewer for the suggestion. We performed the FFT on the time evolution of the spatially averaged steady-state magnetization along the z direction. We have specified this in our manuscript.