Multi-Agent Path Optimization using Calculus of Variations

Course Project
MATH 146 — Methods of Applied Mathematics

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Abstract

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1 List of Symbols

N	number of robots
M	number of circular obstacles
x_i	initial position of the i-th robot
y_i	target position of the i-th robot
p_i	path of the i-th robot
r	radius or repulsive distance of robots
c_j	center of the j-th circular obstacle
R	radius of circular obstacles
F	objective function
L	Lagrangian function
g	barrier function

2 Introduction

3 Theory

3.1 Problem Definition

Add description of problem here.

Let $p_i \in C^1([0,1])^2$ be the vector valued function corresponding to the path of the i-th robot. Then we have that $p_i(0) = x_i$ and $p_i(1) = y_i$. The length of the path is defined as

$$l_i = \int_0^1 \left\| p_i'(t) \right\| \, dt. \tag{1}$$

3.2 Objective Function

We wish to minimize the total length of all robot paths. Define F as the total path length

$$F(P) = \sum_{i=1}^{N} l_i = \int_0^1 \sum_{i=1}^{N} \left\| p_i'(t) \right\| \, dt, \tag{2}$$

where $P = [p_1, \ldots, p_N]^\intercal$ is the path ensemble.

We will modify the objective function in order to represent the no-collision and obstacle constraints. As an example, consider the obstacle constraint. In order to check whether or not a robot collides with an obstacle, a function g can be defined such that it evaluates to infinity if the distance between the robot and the center of the obstacle is less than R and evaluates to 0 when not. However, g would be discontinuous and forbid the use of calculus of variations techniques.

A barrier function $g_{d,\mu}$ is a continuous function that goes to infinity as it approaches a "barrier" value d. A parameter μ can be varied in order for g to approach the discontinuous form. In this case, we wish to devise a function that goes to infinity when approached from above to act as a repulsor. In this project, we will consider two types of barrier functions

1. Log function:

$$g_{d,\mu}(x) = \frac{-1}{\mu} \log(x - d), \quad g'_{d,\mu}(x) = \frac{-1}{\mu(x - d)}. \tag{3}$$

2. Inverse function:

$$g_{d,\mu}(x) = \frac{1}{\mu(x-d)}, \quad g'_{d,\mu}(x) = \frac{-1}{\mu(x-d)^2}. \tag{4}$$

Using this barrier function we can rewrite F as

$$F(P) = \int_0^1 \left[\sum_{i=1}^{N} \|p'_i(t)\| + \sum_{i,j=1;i\neq j}^{N} g_{r,\mu_1}\left(\|p_i(t) - p_j(t)\|\right) + \sum_{i=1}^{N}\sum_{j=1}^{M} g_{R,\mu_2}\left(\|p_i(t) - c_j\|\right) \right] dt, \tag{5}$$

and the optimization problem becomes an unconstrained optimization problem.

3.3 First-Order Necessary Conditions

To derive first-order necessary condition, we first identify the Lagrangian L,

$$L(Y,Z) = \sum_{i=1}^{N} \|z_i\| + \sum_{i,j=1;i\neq j}^{N} g_{r,\mu_1}\left(\|y_i - y_j\|\right) + \sum_{i=1}^{N}\sum_{j=1}^{M} g_{R,\mu_2}\left(\|y_i - c_j\|\right). \tag{6}$$

Applying the Euler–Lagrange equation,

$$L_{p'_{i,x}}(t) = \frac{p'_{i,x}(t)}{\|p'_i(t)\|} \quad \Longrightarrow \quad \frac{d}{dt}\left[L_{p'_{i,x}}(t)\right] = \frac{p''_{i,x}(t)}{\|p'_i(t)\|} - \frac{p'_{i,x}(t)\big(p'_i(t)\cdot p''_i(t)\big)}{\|p'_i(t)\|^3} \tag{7}$$

$$\begin{aligned} L_{p_{i,x}}(t) = &\sum_{i,j=1;i\neq j}^{N} g'_{r,\mu_1}\left(\|p_i(t) - p_j(t)\|\right) \frac{p_{i,x}(t) - p_{j,x}(t)}{\|p_{i,x}(t) - p_{j,x}(t)\|} \\ &+ \sum_{j=1}^{M} g'_{R,\mu_2}\left(\|p_i(t) - p_j(t)\|\right) \frac{p_{i,x}(t) - c_{j,x}}{\|p_i(t) - c_j\|}. \end{aligned} \tag{8}$$

Setting Equations (7) and (8) equal to each other, the differential equation formed will be used to find the optimal path ensemble.

4 Cases

5 Results

6 Discussion

7 Conclusion

8 Acknowledgements

Appendices

A MATLAB Code