Science Olympiad
Boyceville Invitational
December 7, 2024

Astronomy C Walkthrough

In this walkthrough, we will go over Sections B (JS9), C (Deep-Sky Objects), and D (Astrophysics). References to various sources (e.g. online material and textbooks) are included to guide the reader towards resources to learn the concepts more in depth. We hope readers find it useful.

Section A: General Knowledge

This section consists of a mix of multiple choice and free-response questions about general astronomy concepts. Each question is worth 2 points, for a total of 40 points.

1. A main-sequence star of $2\,M_\odot$ will eventually evolve to become which of the following?
   - A. Red dwarf
   - **B. Red giant**
   - C. Red supergiant
   - D. Brown dwarf
2. At what point does a star leave the main sequence?
   - A. When it initiates hydrogen burning.
   - **B. When it runs out of hydrogen fuel.**
   - C. When it initiates helium burning.
   - D. When it runs out of helium fuel.
3. A pre-main sequence star typically has a spectral class that is ____ than it will be when the star reaches the main sequence.
   - A. Bluer
   - **B. Redder**
   - C. Brighter
   - D. Dimmer

Two stars, A and B, have apparent magnitudes $m_A = 5$ and $m_B = 8$.

4. Which of these stars appears brighter from Earth?
   - **A. Star A**
   - B. Star B
   - C. They have the same brightness.
   - D. Not enough information.
5. Which of these stars is intrinsically brighter?
   - A. Star A
   - B. Star B
   - C. They have the same brightness.
   - **D. Not enough information.**

An H–R diagram is shown in Image 1.

6. Order these points by increasing temperature (coldest object first). **ABDC**
7. All of these points fall roughly on the evolutionary track of a $1\,M_\odot$ star. Arrange these points in order of the lifetime of this star. **BADC**
8. At which one of these points on this track would the $1\,M_\odot$ star be shedding its envelope? **D**

Consider the object in Image 4, which was taken in the optical band.

9. What is the term for the dark region indicated in this image?
   - **A. Absorption nebula**
   - B. Diffuse nebula
   - C. Emission nebula
   - D. Reflection nebula
10. This object primarily obscures light from which of the following regions of the EM spectrum?
    - A. Radio
    - B. Microwave
    - C. Infrared
    - **D. Optical**
11. Obscuring light in this band implies that the dust particles in the nebula are (on order) how large?
    - **A. 500 nm**
    - B. 100 µm
    - C. 50 mm
    - D. 10 m

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12. A protostar that forms with a mass of less than ________ is likely to become a brown dwarf.
    A. $0.008\,M_\odot$
    B. $0.08\,M_\odot$
    C. $0.8\,M_\odot$
    D. $8\,M_\odot$

13. What key process in stars are objects below this mass unable to complete? **Hydrogen fusion**

14. A very young brown dwarf primarily generates energy through which of the following reactions?
    A. p–p chain hydrogen fusion
    B. CNO cycle hydrogen fusion
    C. Deuterium fusion
    D. Helium fusion

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15. Which of the following planets is most easily detected using the radial-velocity method?
    A. Neptune-like
    B. Hot Jupiter
    C. Terrestrial
    D. Sub-Neptune

16. List two key properties of this planet type that make it easier to detect with radial velocity.

    High mass and small orbit distance

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17. The mass of a Super-Earth planet falls in what mass range?
    A. Less massive than Earth
    B. More massive than Earth, less massive than ice giants
    C. More massive than ice giants, less massive than Jupiter
    D. More massive than Jupiter

18. We can get a general idea of the elemental composition of an exoplanet based on the elemental composition of its star. Why would the composition of these objects be linked?

    The exoplanet and its star form from the same material.

Taken by JWST, Image 5 displays an intense process of stellar evolution.

19. What is the term for the object in this image?
    A. Circumstellar disk
    B. Relativistic jet
    C. Stellar wind
    D. Herbig–Haro object

20. Briefly (1–2 sentences) describe the process occurring at the "ends" of the objects (indicated by arrows) that cause them to emit light.

    High velocity ejecta hit dense pockets of interstellar medium and form shocks. The shocked material is ionized and excited which then produces emissions.

Section B: JS9

This section consists of a lab using the JS9 imaging software. Unless otherwise specified, each question is worth 2 points, for a total of 15 points.

On the provided laptop, JS9 should be open, showing **a white dot in the middle of a black screen**. If you do not see this, or need the file re-opened, raise your hand.

For questions 1-3, do not add a region to perform this analysis!
This object dominates the image, so adding regions will be time-consuming and unnecessary.

1. Run `[Analysis > Server-side Analysis: Energy Spectrum]`.

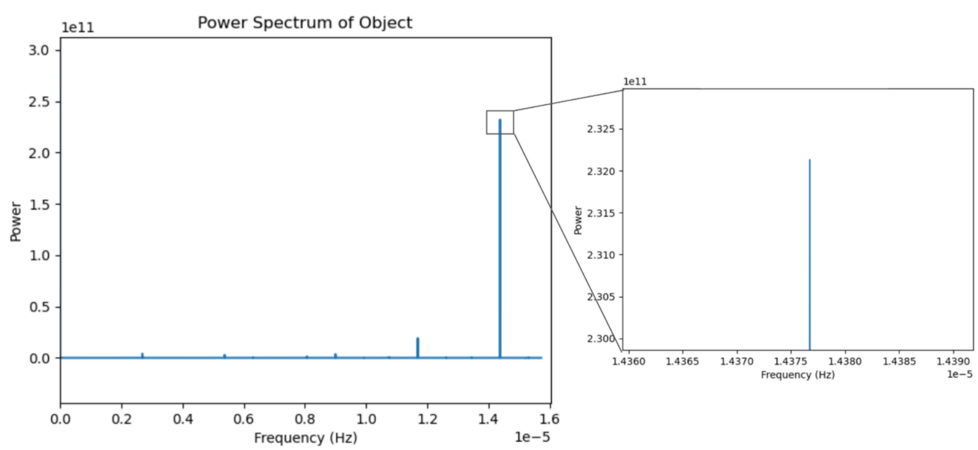
   What major spectral features does this object exhibit? Briefly describe these, and give the wavelengths of any peaks.

2. The lowest energy (farthest left) spectral line is a Neon line, and has a natural width of 0.24 eV. By what factor has this line been broadened?
   (Note: The number that we get will be off, because of the reality of analyzing this raw spectral data. However, it won't be a bad estimate.)

3. This object is a protostar. What is the most likely reason for the broadening of these lines?

This object also has an interesting light curve!

4. The power spectrum of this object is given below. Estimate this object's period, in hours. Be careful with the x–axis—notice that it is scaled by 1×10^{-5}.
   (Hint: Remember the power spectrum x-axis is a frequency, so take its reciprocal to get the period.)



5. What is the exposure of this image, in hours? How does the object's period compare with the exposure?
   (Hint: The FITS header beckons. . .)

6. [3 pts] Encircle the bright central point in a region, with `[Regions > circle]`. Use `[Analysis > Server-side Analysis: Light Curve]` to generate this object's light curve.

   Roughly sketch the resulting waveform. Label the y-axis with the amplitude of any major peaks, and the background.

7. List one mechanism that can result in periodic emission from a protostar.

Solution: You can find a video walkthrough of the JS9 lab here!

There are a ton of great material available for you to get comfortable with JS9. The main JS9 site has guided tutorials and past JS9 labs along with their solutions. Another useful resource is the community-made Scioly.org wiki page for JS9.

This NOIRLab article discusses line broadening from accretion disks. It goes over line broadening in the context of accretion disks around supermassive black holes, but a lot of the principles are similar for protostar accretion disks.

Section C: Deep-Sky Objects

This section consists of a mix of multiple choice and free-response questions about this year's DSOs. Unless otherwise specified, each question is worth 2 points, for a total of 45 points.

Match the following ten (10) statements with the corresponding deep-sky object in the list below. Each choice may be used once, more than once, or not at all.

A. Orion Nebula	D. LTT 9779b	F. TOI-270d	I. Kepler-62
B. HD 80606b	E. K2-18b	G. WD 1856+534	J. AU Microscopii
C. WASP-121b		H. 55 Cancri	

1. This Messier object contains an open cluster, notable for its four young OB stars. **A**
2. A red dwarf with two confirmed Neptune-like planets detected by TESS. **J**
3. Hubble detected a stratosphere (i.e. an atmospheric layer with a temperature inversion) in this ultra-hot Jupiter. **C**
4. This binary system is located just 41 light-years away, with its primary star named after the astronomer who placed the Sun at the center of the universe. **H**
5. JWST detected methane, carbon dioxide, and water vapor in the atmosphere of this planet, which resides in a system with two other confirmed planets. **F**
6. A planetary system with five confirmed exoplanets with the innermost one being a super-Earth. **H or I**
7. A highly eccentric gas giant in the constellation Ursa Major. **B**
8. Image 2 depicts the spectra of this object. **E**
9. This object is part of a triple star system. **G**
10. A high albedo planet with a G-type main sequence host star. **D**

Solution: When collecting information about the deep-sky objects, it's handy to make a concise "fact sheet" for each one. That way, you can quickly reference them during the exam!

Some useful pieces of information to include:

1. object name(s);
2. object type (e.g. emission nebula, hot Jupiter, A-type star, white dwarf);
3. a few images with added info (e.g. wavelength(s), imaging telescope(s));
4. distance (and redshift, if applicable);
5. key facts (i.e. What makes this object special? Is it a prototype for a class of objects? Does it have a unique property? Did it lead to a surprising result? If it's a planet, how was it discovered?);
6. constellation;
7. coordinates (i.e. right ascension (RA) and declination (Dec));
8. other quantitative values (e.g. magnitude, mass, temperature, no. of planets).

11. The central region of 30 Doradus is shown in Image 6.
    (a) What process formed the cavity in the bottom left of the image?
    (b) [3 pts] Is the blue star in the cavity younger or older than the stars in the colored regions. Explain your answer.
    (c) What instrument produced this image?

Solution: 30 Doradus is one of the two nebulae in the object list this year and capture the "Star Formation" portion of this year's topic focus.

(a) High-mass (8+ times the mass of the Sun) stars release extremely energetic stellar winds that push gas and dust outwards.

(b) We'd expect the blue star to be older. Its stellar wind has had enough time to clear out the material it formed in. The stars in the colored regions are newly forming protostars still shrouded in dust.

(c) The Near-Infrared Camera (NIRCam) on JWST was used to produce this image, taken from this ESA Webb article.

A few sources about 30 Doradus for futher reading: HubbleSite article (primer), Chandra article (primer), and review paper (advanced).

12. WASP-17b is a gas giant tidally locked to its host star.
    (a) Explain what it means to be "tidally locked." Give an example of this occurring in the Solar System.
    (b) What type of silicate was discovered in its atmosphere?
    (c) What observational technique was used to make this discovery?

Solution:

(a) For an orbiting body to be tidally locked, its rotation rate is the same as its orbital rate—completing a full rotation for every orbit. There are many examples of tidal locking in our Solar System: Moon, Phobos, Deimos, Pluto/Charon, etc. Since tidal locking is for an exact 1:1 resonance, Mercury is not an example of it. Mercury is in a 3:2 resonance, rotating three times for every two orbits about the Sun.

This phenomenon occurs due to tidal forces exerted by the primary on the secondary (e.g. Earth–Moon). Read more about it in this NASA article.

(b) Quartz. This JPL article goes over the discovery.

(c) Transmission spectroscopy.

This Webb article introduces the different types of spectroscopy. A more advanced discussion of all types of exoplanet spectroscopy can be found in this article and this talk (slides).

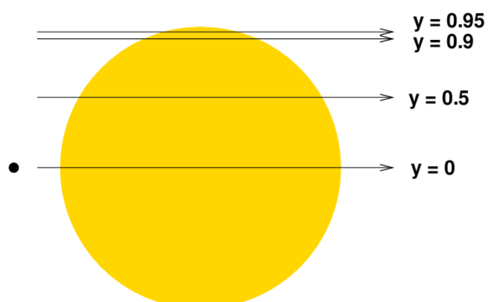
13. Image 3 shows two light curves.
    (a) Name the wavelengths these two curves are observed in.
    (b) What type of event is occurring in this light curve?
    (c) A typical simplification in the analysis of these curves leads to the bottom of the curve being flat. Give two possible reasons why we don't observe this.

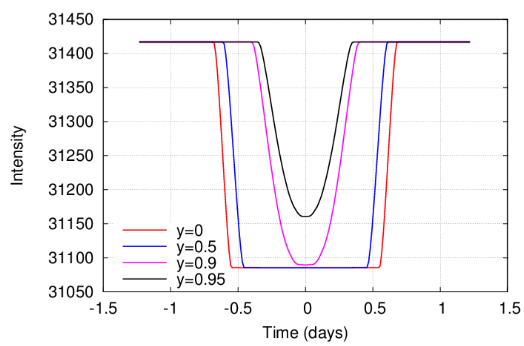
Solution: This question follows WD 1856+534, a white dwarf with a transiting gas giant.

(a) The left light curve is imaged at 0.48 µm which is in visible light range of 400–700 nm. The right light curve is imaged at 4.5 µm which is longer than visible and less than the 1 mm upper bound of infrared light. HyperPhysics goes over each regime of the electromagnetic spectrum.

(b) A planet is passing in front of a star causing its light to dim. This event is called a transit—a key method for planet detection.

(c) Limb darkening is one possible reason. This MinutePhysics video explains how it works. Another possible reason is that this is a grazing transit, where the planet passes only partially in front of its host star's disk. See the figures below, taken from here, for an Earth-size planet passing in front of a Sun-like star at different impact factors y.





14. Image 7 highlights a star found in the southern hemisphere, located less than 11 light-years from Earth.
    (a) Identify this star.
    (b) A planet was detected orbiting about this star. What method was used to do so?
    (c) What property of the star impacted the validity of the exoplanet's detection?

Solution:

(a) This star is Epsilon Eridani. The image is taken from its Wikipedia article.

(b) Epsilon Eridani b was discovered using the radial-velocity method/Doppler spectroscopy.

(c) Since Epsilon Eridani is a young star, its strong magnetic activity causes photosphere fluctuations, affecting Doppler measurements and compromising the integrity of the exoplanet detection (paper).

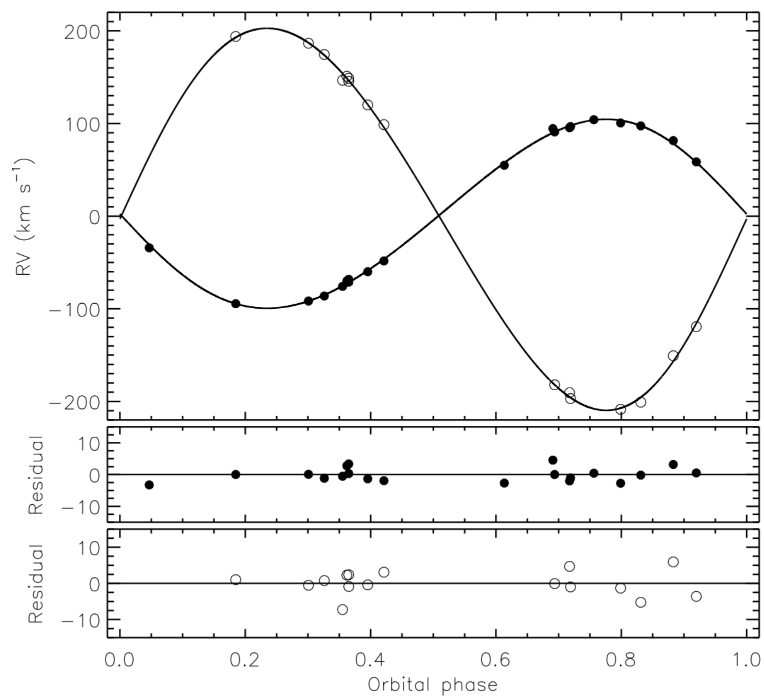
Section D: Astrophysics

This section consists of astrophysics calculations and free-response questions. Points are shown for each sub-question, for a total of 40 points. Numerical answers must be provided to 3 significant figures. Please show your work: no work, no points. Partial credit may be awarded for correct work.

Conversions and constants you may find helpful:

$1\,\text{au} = 1.496 \times 10^{11}\,\text{m}$	$1\,\text{R}_\odot = 6.957 \times 10^{8}\,\text{m}$	$G = 6.674 \times 10^{-11}\,\text{N}\,\text{m}^2\,\text{kg}^{-2}$
$1\,\text{ly} = 9.461 \times 10^{15}\,\text{m}$	$1\,\text{M}_\odot = 1.989 \times 10^{30}\,\text{kg}$	$b = 2.898 \times 10^{-3}\,\text{m}\,\text{K}$
$1\,\text{pc} = 3.086 \times 10^{16}\,\text{m}$	$M_\odot = +4.74$ (Abs. mag.)	$\sigma = 5.670 \times 10^{-8}\,\text{W}\,\text{m}^{-2}\,\text{K}^{-4}$

1. **Binary Stars.** You come across a binary star system containing two main-sequence stars: **A** and **B**. As a generally lucky astronomer, you assume the system is approximately edge-on and measure the radial-velocity of the system over some time, as seen below. Assume star **A** is more massive than star **B**, and all orbits are circular.



(a) [2 pts] This system has a parallax of 0.001″; how far away is it, in light-years?

(b) [2 pts] Is the binary system moving relative to the observer? Why or why not?

(c) [2 pts] What does our assumption—that the system is viewed edge-on—allow us to conclude?

(d) [3 pts] Given the period of the stars is 3 days, find the radius of each star's orbit from the center of mass of the system, in meters.

(e) [3 pts] Find the mass of the entire binary system, in solar masses.

(f) [2 pts] Find the mass of star **A**, M_A, and star **B**, M_B, individually in solar masses.

Solution: Even though binary stars are not a focus of this year's rules, the ideas used by the radial-velocity detection method are one and the same: what it means to view a system edge-on, Kepler's third law, and conservation of angular momentum.

(a) This question tests your understanding of parallax. Using the parallax equation, we find:

$$d = \frac{1}{p} = \frac{1}{0.001''} = 1000\,\mathrm{pc}$$

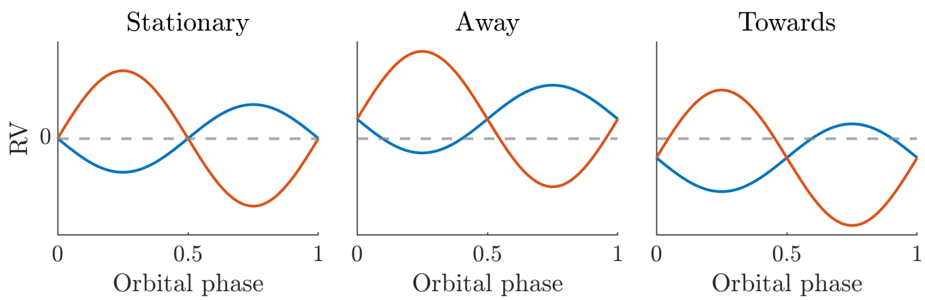
where d is the distance and p is the parallax angle. We want to find the distance in light-years, so we use the given conversions to get

$$d = 1000\,\mathrm{pc} \times \frac{3.086 \times 10^{16}\,\mathrm{m\,pc^{-1}}}{9.461 \times 10^{15}\,\mathrm{m\,ly^{-1}}} = \boxed{3260\,\mathrm{ly.}}$$

It's important to give the answer in the specified units! Many teams forgot to do this last conversion.

(b) $\boxed{\text{No,}}$ the motion of the system is stationary. By conservation of *linear* momentum, the barycenter (center of mass of the system) moves at a constant velocity. The radial (towards or away from the observer) component of the system's motion can found by looking for when the two stars have the *same* radial velocity—when the curves "meet"—which is $0\,\mathrm{km\,s^{-1}}$. Another way of finding this radial component is by taking the "average" radial velocity of one of the stars which we can see is also at $0\,\mathrm{km\,s^{-1}}$.

The figures below show how the curves would look if it was moving away (center) or towards (right) the observer instead. Remember that a positive RV corresponds to an object moving *away*.



(c) If the system is viewed edge-on, we are able to conclude the maximum radial velocity of each star is its true velocity.

(d) Since we know the true velocity of each star and they follow circular orbits, we can use the period P to determine their radii:

$$vP = 2\pi r \implies \begin{cases} r_A = \frac{1}{2\pi} \times (100 \times 10^3\,\mathrm{m\,s^{-1}}) \times \left(3\,\mathrm{d} \times \dfrac{86\,400\,\mathrm{s}}{1\,\mathrm{d}}\right) = \boxed{4.13 \times 10^9\,\mathrm{m},} \\ r_B = \frac{1}{2\pi} \times (200 \times 10^3\,\mathrm{m\,s^{-1}}) \times \left(3\,\mathrm{d} \times \dfrac{86\,400\,\mathrm{s}}{1\,\mathrm{d}}\right) = \boxed{8.25 \times 10^9\,\mathrm{m.}} \end{cases}$$

By convention, the primary (star) in the system is the brighter one and the secondary (star) is the dimmer one. Since both stars are on the main sequence, the more massive one is brighter. And from the ratio of maximum velocities, we know that the one moving slower is the more massive one, so we assign it to star A.

(e) We use Kepler's third law and the sum of the radii we found above to find the system mass:

$$M_A + M_B = \frac{a^3}{P^2} = \frac{(r_A + r_B)^3}{P^2} = \frac{\left(12.38 \times 10^9\,\mathrm{m} \times \dfrac{1\,\mathrm{au}}{1.496 \times 10^{11}\,\mathrm{m}}\right)^3}{\left(3\,\mathrm{d} \times \dfrac{1\,\mathrm{yr}}{365.25\,\mathrm{d}}\right)^2} = \boxed{8.40\,\mathrm{M}_\odot.}$$

Note that we used the "au–yr–$\mathrm{M}_\odot$" form of Kepler's third law. We could have also converted everything into base SI units and included the $(G/4\pi^2)$ factor.

(f) For binary systems in circular systems, the conservation of angular momentum leads us to some nice relations:

$$\frac{M_A}{M_B} = \frac{r_B}{r_A} = \frac{v_B}{v_A}.$$

Since the velocity of star A and B are in a 1:2 ratio, we know that their masses must be in a 2:1 ratio. We can set up a system of two equations with our two unknowns, M_A and M_B, and solve it:

$$\begin{aligned} M_A &= 2M_B, \\ M_A + M_B &= 8.40\,\mathrm{M}_\odot, \end{aligned} \quad \Longrightarrow \quad \begin{aligned} &\boxed{M_A = 5.60\,\mathrm{M}_\odot,} \\ &\boxed{M_B = 2.80\,\mathrm{M}_\odot.} \end{aligned}$$

"The Masses of Stars" from Astronomy Notes goes over this process of stellar mass determination.

2. **A Little Shifty.** Continuing with the same scenario as the previous question, after more careful observation, you estimate that one of the stars has a surface peak wavelength emission at 300 nm.

   (a) [2 pts] What is the surface temperature of the star, in Kelvin?

   (b) [2 pts] Identify the spectral type and subclass of this star. *(Hint: It's a letter, then a number.)*

   (c) [1 pt] Which star is more likely to have this surface temperature?
   (If you couldn't derive the masses of the stars, assume $M_A = 6\,M_\odot$ and $M_B = 2\,M_\odot$.)

   (d) [2 pts] What parts of the orbital phase in the radial-velocity curve (from question 1) should we observe the stars to get the most accurate surface temperature estimates?

   (e) [3 pts] Another astronomer makes an observation and finds the star has a peak wavelength 0.04 nm less than the original 300 nm, which is your (perfectly accurate) measurement. What was the radial velocity (in $\mathrm{km\,s^{-1}}$) of the star at this time? Is it moving towards or away from Earth?

   (f) [2 pts] Assuming your measurement was made at the optimal time, how long after your observation (in days) was their observation made? As a reminder, the total orbital period is 3 days.
   (Note: There are multiple valid answers, but you need only list one of them.)

Solution:

(a) Using Wien's law, we get:

$$T_{\text{eff}} = \frac{b}{\lambda_{\text{peak}}} = \frac{2.898 \times 10^6\,\text{nm K}}{300\,\text{nm}} = \boxed{9660\,\text{K.}}$$

See §3.2 of *A Student's Guide to the Mathematics of Astronomy* (2013) written by Daniel Fleisch and Julia Kregenow (hereafter referred to as GMA).

(b) We use our temperature from above and compare it to a spectral type table to find that it is an $\boxed{\text{A-type star with subclass 0}}$ (or 1).

Learn about spectral types from this UNL article. A bit more detail can be found here and some tables can be found here and here.

This question just asks for the spectral classification, but we also know that its luminosity class is V; the previous question specified that stars A and B are main sequence stars. So the full classification is A0V.

(Rabbit Hole: When astronomers began collecting stellar spectra, they came up with a whole host of classification schemes. They iterated on each others' schemes, ultimately leading to the Morgan–Keenan (MK) system we use now—a "two-dimensional" system combining the Harvard spectral classification and luminosity classes. A detailed history can be found here[1].)

(c) Stars on the main sequence are strongly ordered: luminosity, mass, and temperature all being correlated. Wikipedia has a table of sample parameters for stars along main sequence. Using this, we find that $\boxed{\text{star B}}$ most closely matches the A0 stellar class.

(d) The accuracy of our surface temperature estimates depends on the accuracy of our measured peak wavelength, which may be shifted by the Doppler effect. To minimize this effect, we would like to

[1] Ch. 1 of *Stellar Spectral Classification* (2009) by Richard O. Gray and Christopher J. Corbally (ISBN: 9780691125114)

observe the stars when their $\boxed{\text{radial velocities are zero}}$. Or equivalently, (1) when they're at the closest or furthest points from the observer or (2) $\phi = 0, 0.5, 1$.

Learn about the Doppler effect here and some of its applications here.

(e) Using the Doppler effect equation (§3.3 in GMA), we find

$$\frac{v_r}{c} = \frac{\Delta\lambda}{\lambda_0} \implies v_r = \frac{-0.04\,\mathrm{nm}}{300\,\mathrm{nm}} \times (3 \times 10^5\,\mathrm{km\,s^{-1}}) = \boxed{-40\,\mathrm{km\,s^{-1}},}$$

where v_r is the radial velocity, c is the speed of light, $\Delta\lambda$ is the change in the wavelength, and λ_0 is the original wavelength. Since v_r is negative, the star is $\boxed{\text{moving towards Earth.}}$

(f) For this question, we are looking for the orbital phase when star B has a radial velocity of $-40\,\mathrm{km\,s^{-1}}$. Consulting the graph or solving a trigonometric equation, we find

$$v_B \sin(2\pi\phi) = v_r \implies \phi = 0.532 \text{ or } 0.968.$$

From part (d), we could've made our "optimal" measurements at $\phi = 0, 0.5, 1$, so $\Delta\phi = 0.032$, $0.468, 0.532,$ and 0.968 which is equal to $\boxed{0.10,\ 1.40,\ 1.60,\ \text{and}\ 2.90\,\mathrm{d}.}$

3. **Resolution!** A planet that was previously thought to have orbited a single host star is found to be orbiting a very compact set of binary stars.

   (a) [2 pts] If the planet was discovered using direct imaging, what common tool would have been used to blot out the central stars?

   (b) [3 pts] If the diameter of your space telescope is 10 meters and you are observing the system at a wavelength of 10 micrometers, what is the limiting angular resolution of your telescope (by diffraction), in radians?

   (c) [2 pts] The wavelength used by the telescope in part (b) is often used for direct imaging. Why?

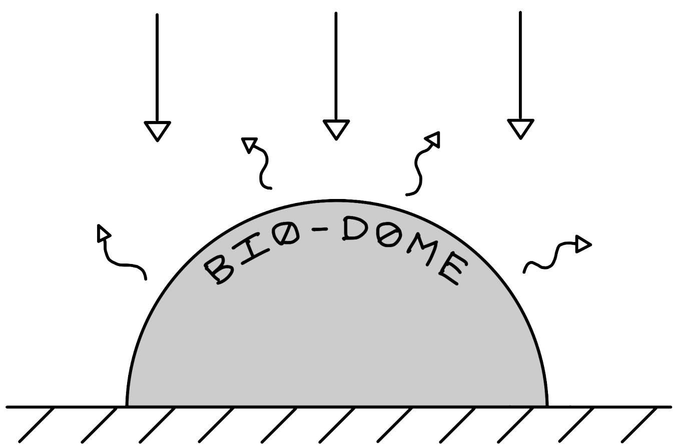
   (d) [3 pts] The stars have an absolute magnitude of +1 and +4, respectively. When viewed together, what is their combined absolute magnitude?

   (e) [4 pts] In the far future, astro-neers land and settle on this planet. They find it orbits at a distance of 7 au and is tidally locked to its host stars. More importantly, it lacks an atmosphere!

   So, they construct a bio-dome 2 km in diameter at the planet's substellar point. The shell of the bio-dome is designed to replicate the thermal properties of Earth, having a bond albedo of 0.3 and an emissivity of 0.9. For simplicity, we'll assume the bio-dome is an opaque hemisphere.

   (Note: Use +4.74 *for the absolute magnitude of the Sun.)*

   What is the temperature in the bio-dome, in Celsius? Is it habitable?



Solution:

(a) $\boxed{\text{A coronagraph}}$ is a telescope attachment that blocks out stars or other bright sources in order to focus on dimmer objects near them.

(b) Using the Rayleigh criterion for a circular aperture (equation (4.13) in §4.3 of GMA), we compute

$$1.22 \times \frac{\lambda}{d} = \sin\theta_R \approx \theta_R = 1.22 \times \frac{10 \times 10^{-6}\,\mathrm{m}}{10\,\mathrm{m}} = \boxed{1.22 \times 10^{-6}\,\mathrm{rad},}$$

where λ is target wavelength to be observed, d is the diameter of the aperture, and θ_R is the angular resolution (in radians). Notice that since θ_R is so small, we can approximate $\sin\theta_R \approx \theta_R$.

(c) Planets are $\boxed{\text{primarily bright in infrared}}$ and, more importantly, the $\boxed{\text{brightness ratio}}$ between the planet and its star is highest for infrared.

This video and this article introduce the direct imaging method. To get a sense of how difficult it would be to use direct imaging to detect Jupiter from Proxima Centauri b—the nearest exoplanet—check out questions C9–14 from the 2024 Solon Astronomy exam.

(d) To find the combined magnitude, we can't just add the magnitudes together! We need to first convert their absolute magnitudes into luminosities, add them together, then convert it back. (§5.3 in GMA describes the magnitude system in more detail.)

The luminosity–magnitude relation is

$$M - M_\odot = -2.5\log_{10}\left[\frac{L}{\mathrm{L}_\odot}\right],$$

where $\mathrm{L}_\odot$ represents 1 solar luminosity and $M_\odot$ is the absolute magnitude of the Sun.

Plugging in and solving, we find the luminosities of the two stars

$$\begin{aligned} +1 - 4.74 &= -2.5\log_{10}\left[\frac{L_A}{\mathrm{L}_\odot}\right] \\ +4 - 4.74 &= -2.5\log_{10}\left[\frac{L_B}{\mathrm{L}_\odot}\right] \end{aligned} \quad\Longrightarrow\quad \begin{cases} L_A = 31.33\,\mathrm{L}_\odot, \\ L_B = 1.98\,\mathrm{L}_\odot. \end{cases}$$

Then summing and converting back:

$$M_{\mathrm{tot}} = M_\odot - 2.5\log_{10}\left[\frac{L_A + L_B}{\mathrm{L}_\odot}\right] = \boxed{+0.933.}$$

(Aside: The magnitude system depends on a zero point, which in this case we take to be our Sun, but it technically isn't necessary since we're combining magnitudes to get another magnitude. The key relation we need is that 5 steps in magnitude correspond to a 100 times increase in luminosity.)

(e) To find the temperature in the bio-dome, we balance the heat/energy absorbed from the host stars $\dot{Q}_{\mathrm{in}}$ and the heat/energy emitted out from the bio-dome $\dot{Q}_{\mathrm{out}}$.

Since the binary stars are very compact, we can treat them together as a point source that has a luminosity of $33.3\,\mathrm{L}_\odot$ (from part (d)) that radiated evenly outwards, following the inverse square law (§5.2 in GMA). The amount of energy received per unit area (flux F) at a distance of 7 au is

$$F = \frac{L_A + L_B}{4\pi d^2} = \frac{33.3\,\mathrm{L_\odot} \times \dfrac{3.828 \times 10^{26}\,\mathrm{W}}{1\,\mathrm{L_\odot}}}{4\pi \times \left(7\,\mathrm{au} \times \dfrac{1.496 \times 10^{11}\,\mathrm{m}}{1\,\mathrm{au}}\right)^2} = 925\,\mathrm{W\,m^{-2}}.$$

(Aside: The solar constant is about 1.5 times higher, at $1361\,\mathrm{W\,m^{-2}}$. On the other hand, F is higher than the solar irradiance at Mars of $586\,\mathrm{W\,m^{-2}}$ (source).)

The bio-dome is located at the planet's substellar point which means the stars are directly overhead. So the bio-dome has a cross-sectional area of $\pi D^2/4$ (Why not $\pi D^2/2$?), where D is the diameter, absorbing 70 % of the incident light (due to a bond albedo $\alpha = 0.3$), and thus receives heat at a rate of

$$\dot{Q}_{\mathrm{in}} = F \times \frac{\pi}{4} D^2 \times (1 - \alpha) = \frac{0.7\pi}{4} \times 925\,\mathrm{W\,m^{-2}} \times (2000\,\mathrm{m})^2 = 2.03 \times 10^9\,\mathrm{W}.$$

This incident energy needs to go somewhere. The bio-dome sheds its excess heat in the form of blackbody radiation (§3.2 of GMA). Using the Stefan–Boltzmann law—making sure to factor in the bio-dome's emissivity $\varepsilon = 0.9$—we get

$$\dot{Q}_{\mathrm{out}} = \varepsilon\sigma T_{\mathrm{eq}}^4 \times \frac{\pi}{2} D^2 = \frac{0.9\pi}{2} \times \left(5.670 \times 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}\right) \times (2000\,\mathrm{m})^2 \times T_{\mathrm{eq}}^4.$$

Finally, we set $\dot{Q}_{\mathrm{in}} = \dot{Q}_{\mathrm{out}}$ and solve for $T_{\mathrm{eq}} = 282\,\mathrm{K} = \boxed{9.08\,^\circ\mathrm{C}.}$

Since T_{eq} in the temperature range for liquid water 0–100 °C, the bio-dome is $\boxed{\text{habitable!}}$

This question slightly differs from the "standard" planetary equilibrium temperature calculation as the bio-dome only radiates its energy along a hemisphere, rather than a full sphere.

(Aside: Dark concrete and stone is an opaque material that matches the thermal properties of the bio-dome (source). Thermal engineers, especially those working on systems in space, have to carefully select materials with thermal properties that best align with the environment it is subject to. A bio-dome subject to the solar irradiance at Venus might need to be painted over in white to minimize the ratio of absorptivity and emissivity!)