Consider a machine with two states

State "1" : Machine is operational

" "0" : " " down (in some failed mode)

- When machine is up, it can fail with time to failure (F) is exponentially distributed with rate p

$$F \sim \exp(p)$$

- When the machine is down, it gets repaired with random repair time

$$R \sim \exp(r)$$

$\Pi \cdot Q = \underline{0}$

$$\begin{tikzcd} u \arrow[r, bend left, "p"] & d \arrow[l, bend left, "r"] \end{tikzcd}$$

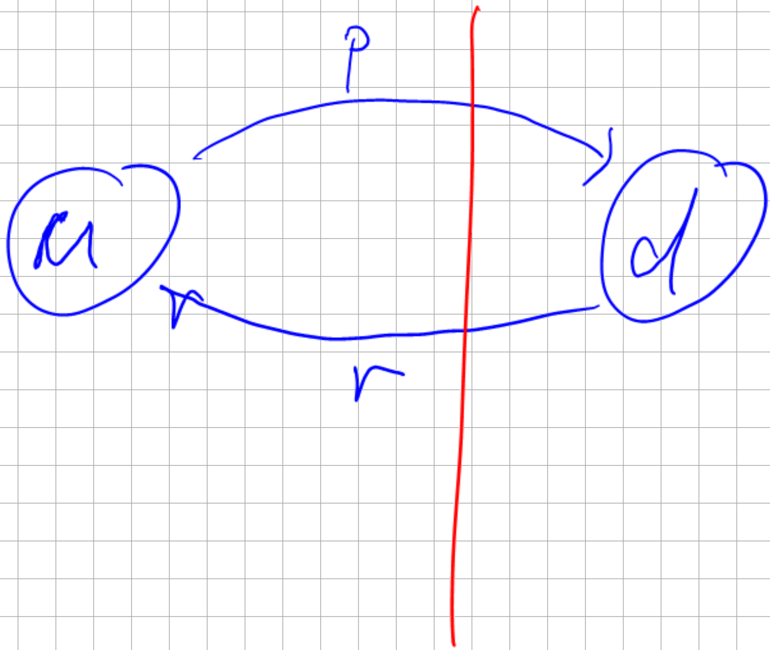
$$Q = \begin{bmatrix} -q_{ud} & q_{ud} \\ q_{du} & -q_{du} \end{bmatrix} = \begin{bmatrix} -p & p \\ r & -r \end{bmatrix}$$

$$\Pi = \begin{pmatrix} \Pi_u & \Pi_d \end{pmatrix} \cdot \begin{pmatrix} -p & p \\ r & -r \end{pmatrix} = \begin{pmatrix} \Pi_u \cdot (-p) + \Pi_d \cdot r & \Pi_u \cdot p + \Pi_d \cdot (-r) \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 \end{pmatrix}$$

$$\Pi_u \cdot (-p) + \Pi_d \cdot r = 0 \iff \Pi_d \cdot r = \Pi_u \cdot p$$

$$\Pi_u \cdot p + \Pi_d \cdot (-r) = 0 \iff \quad \parallel$$



Trans from right to left

$$r \cdot \pi_d$$

Trans from left to right

$$p \cdot \pi_u$$

$$p \cdot \pi_u = r \cdot \pi_d$$

Normalization condition $\quad \pi_u + \pi_d = 1$

$$p \cdot \Pi_u = r \cdot \Pi_d \rightsquigarrow \Pi_u = \frac{r}{p} \cdot \Pi_d$$

$$\Pi_u + \Pi_d = 1$$

$$\frac{r}{p} \cdot \Pi_d + \Pi_d = 1$$

$$\frac{r+p}{p} \cdot \Pi_d = 1$$

$$\Pi_d = \frac{p}{p+r}$$

Prob / fraction of time of the machine being down

$$\rightsquigarrow \quad \Pi_u = \frac{r}{p+r}$$

availability of the machine