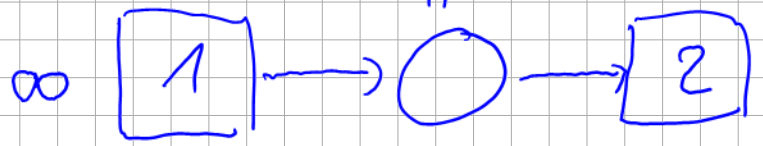
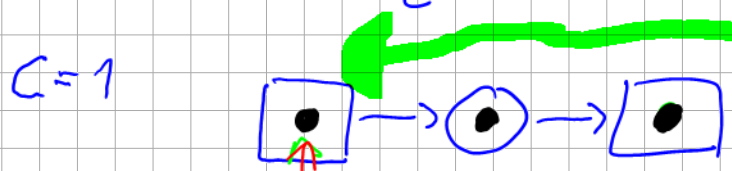
Two machine flow line with reliable machines making discrete parts

buffer





Part already processed by machine 1

⇒ machine 1 is blocked

Part not yet processed by machine 1

first machine never starved

second " " blocked

first machine can be blocked

second " can be starved

blocking after service (BAS)

machine 1 processing time $T_1 \sim exp(\mu_1)$

$\Rightarrow E\{T_1\} = \frac{1}{\mu_1}$

machine 2 processing time $T_2 \sim exp(\mu_2)$

$\Rightarrow E\{T_2\} = \frac{1}{\mu_2}$

We have two possible events

Event 1: Machine 1 completes processing of a part, event occurs with rate μ_1

Event 2: Machine 2 completes processing of a part, event occurs with rate μ_2

Analysis: KPI TH, Inventory ? (in expectation)

→ we need state probabilities

→ we need a state description !

→ How can we describe the state of the system at any moment in time ??!

- We cannot have a state with a non-empty buffer and a starved machine 2 !

- We cannot have a state with a non-full buffer and a blocked machine 1 !

↑ simultaneously !!

State description

$s = s(n)$ with $n = 0, \dots, C+2$

n = number of parts that have already been processed by machine 1, but by machine 2 !

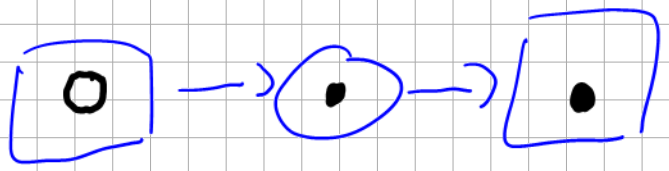
Example $C = 2$ (buffer capacity)

Case 1: $n = C+2 = 4$

m_1 is blocked

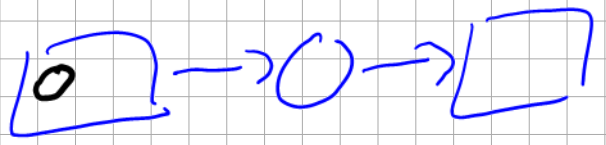
Case 2: $n = 3$

Case 3: $n = 2$



Case 4: $n = 1$

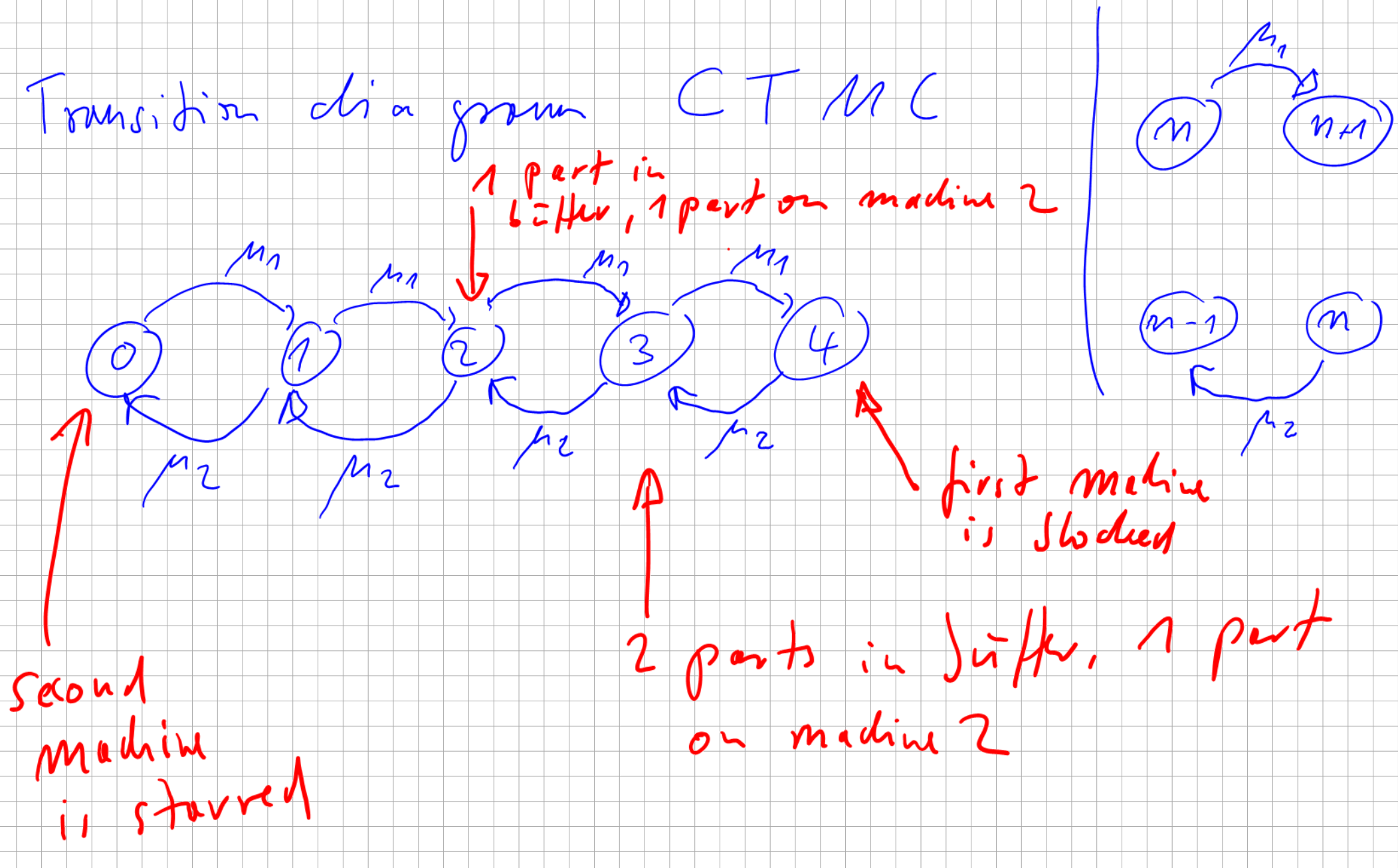
Case 5: $n = 0$



machine 2 is starved

Cont. with $C = 2 \Rightarrow n = 0, \dots, C+2 = N$

Transition diagram CTMC



So for $C=2$, we have states $n=0,\dots,4$

We are interested in state probabilities

$\overline{\Pi}_0$, $\overline{\Pi}_1$, $\overline{\Pi}_2$, $\overline{\Pi}_3$ and $\overline{\Pi}_4$

Let us assume that we already know $\overline{\Pi}_0 \dots \overline{\Pi}_4$!!

Can we determine the throughput TH of the line in this case (i.e., knowing $\overline{\Pi}_0 \dots \overline{\Pi}_4$)?

TH_1 throughput through machine 1?

TH_2 throughput through machine 2?

{ Throughput is the long-term number of completed parts per time unit }

Homework:

a) Determine TH_1 in terms of TH_2, i.e. other words: How does TH_1 relate to TH_2?

b) Give a formula to compute TH_1 in terms of $\Pi_0, \Pi_1 \dots, \Pi_4, \mu_1, \mu_2$ C as needed!!

c) Give a similar formula for TH_2...

d) Give a formula to determine the average number $\bar{m}$ of parts already processed by machine 1 and still in the system, i.e., not yet processed by machine 2!

(average inventory)