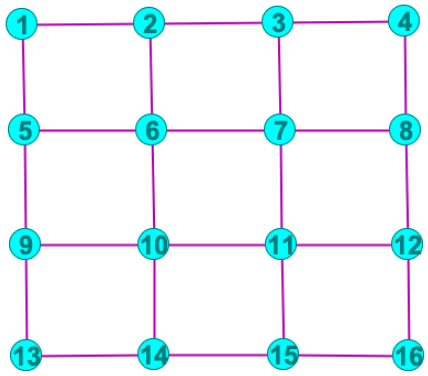
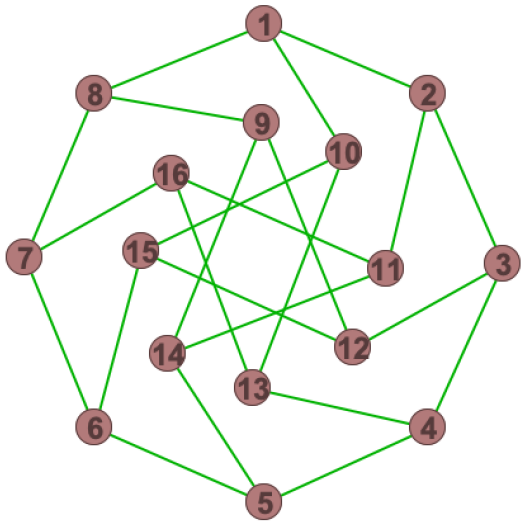
Math 322
Homework Problem Set 6

Problem 1. For each of the graphs below, determine whether it has a one-factor and/or a one-factorisation. Justify your answer fully, and in the case that the graph does have a one-factor (or a one-factorisation), describe such a one factor (or such a one-factorisation).



Graph H_1



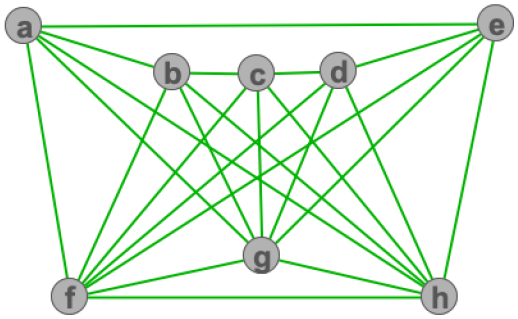
Graph H_2

Problem 2. (a) Let n, m be positive integers, and consider the complete bipartite graph $K_{n,m}$ on $n+m$ vertices. Show that every subdivision H of $K_{n,m}$ has chromatic number $\chi(H) = 2$ or $\chi(H) = 3$.

(b) Is there a subdivision of K_5 that has chromatic number > 5? Justify your answer fully.

(c) Is there a subdivision of K_5 that has chromatic number $= 2$? Justify your answer fully.

Problem 3. We saw in class that, for every graph G, we have that $\chi(G) \geqslant \omega(G)$. The following example shows that this inequality can be strict sometimes.

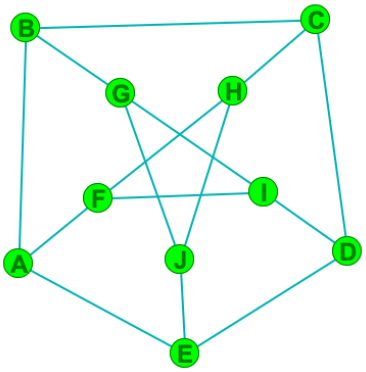


Graph G_0

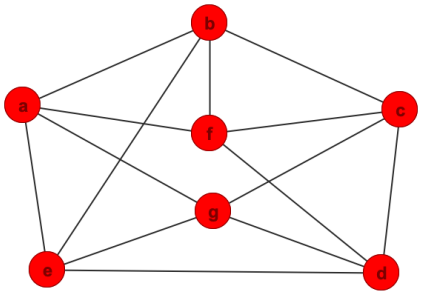
(a) Show that $\omega(G_0) = 5$.

(b) Show that $\chi(G_0) = 6$.

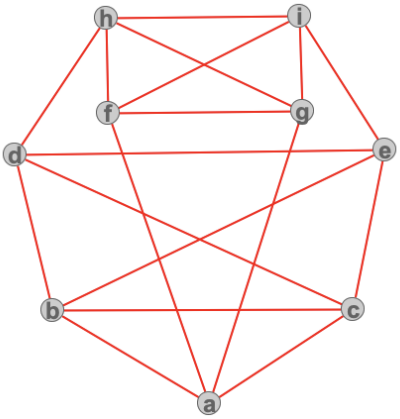
Problem 4. None of the following five graphs is planar, so, according to Kuratowski's theorem, each of them must have a Kuratowski subgraph. Find such a subgraph, and explain how we can get it from subdividing $K_{3,3}$ or K_5.



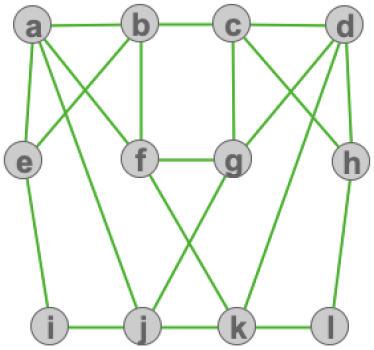
Graph G_1



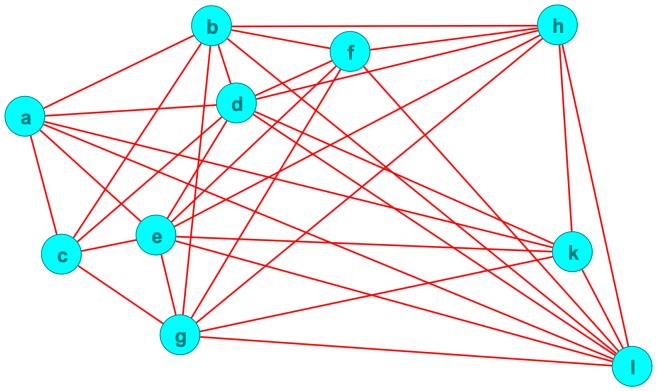
Graph G_2



Graph G_3



Graph G_4



Graph G_5