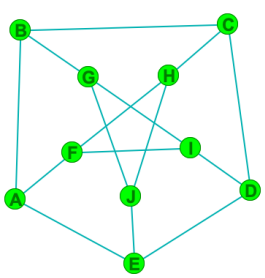
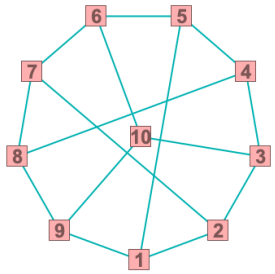
Math 322
Homework Problem Set 4

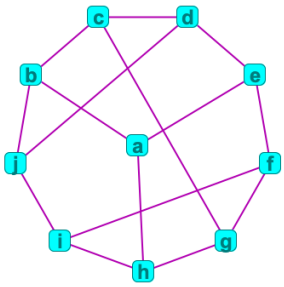
Problem 1. Two of the graphs in the image below are isomorphic.



Graph G_1



Graph G_2

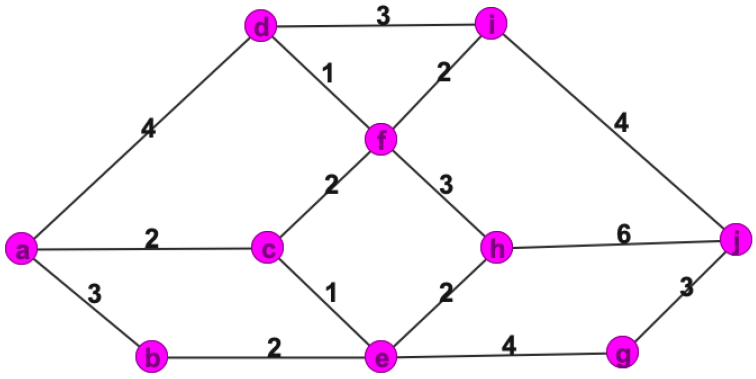


Graph G_3

(a) Determine which two graphs are isomorphic, and give an appropriate graph isomorphism that confirms this.

(b) Explain why the remaining graph is not isomorphic to the other two.

Problem 2. Consider the following weighted graph G_0:



(a) Using Dijkstra's algorithm, find the shortest distance from vertex a to every other vertex of G_0. Show all your work (that is, how you proceed at each stage of the algorithm). You **don't need** to also find paths of shortest length in this part of the problem.

(b) By relying, if you want to, on your work in part (a), find all paths of shortest length from a to j.

Problem 3. In Lecture 18 we saw that, if a connected graph G of size $\geqslant 3$ is Eulerian, then its line graph $L(G)$ is Hamiltonian.

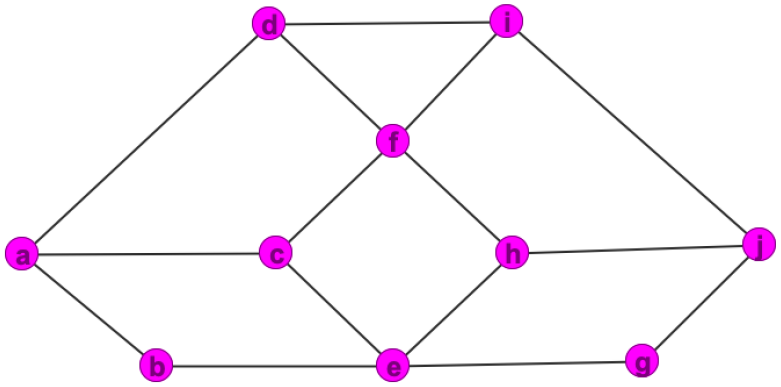
Show that the converse is not always true. That is, find a connected graph H of size $\geqslant 3$ such that

- its line graph $L(H)$ is Hamiltonian,
- but H is **not** Eulerian.

Confirm that your example has the above properties.

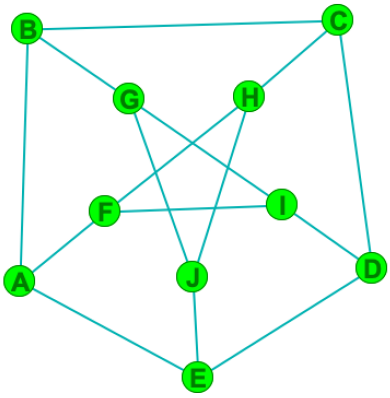
Problem 4. Let d be a positive integer $\geqslant 2$. Prove the following statement: for every connected d-regular graph G, its line graph $L(G)$ is Eulerian.

Problem 5. (a) Consider the graph G_0 from Problem 2:

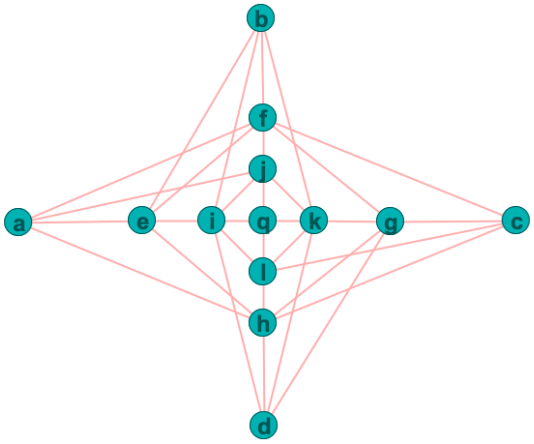


Show that G_0 has a Hamilton path. Moreover, show that G_0 is **not** Hamiltonian.

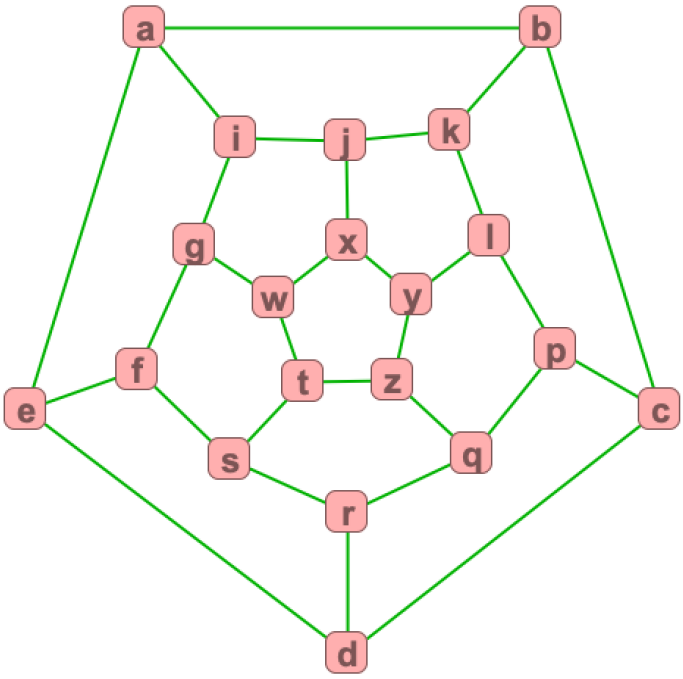
(b) For each of the graphs on the next page, determine whether it is Hamiltonian or not. If it is, find a Hamilton cycle. Otherwise, explain why no such cycle exists.



Graph G_1



Graph H_1



Graph H_2