MATH 328 Fall 2020

Praxis Final Exam

11/27/2020

1) Let $\sigma \in S_n$ be a cycle of length n, and $\tau \in S_n$ one of length $n-1$. Determine $C_{S_n}(\sigma)$ and $C_{S_n}(\tau)$.

2) Let $n \geq 3$ be a natural number.
   (i) Show that a cycle of length $n-1$ is in A_n if and only if n is even.
   (ii) Assume that n is even and let
   $$\alpha_n = (1, 2, 3, \ldots, n-1) \qquad \text{and} \qquad \beta_n = (1, 2, , 3, \ldots, n-2, n).$$
   These are cycles of length $n-1$ and so in A_n by (i). Show that α_n and β_n are not conjugate in A_n, and that the disjoint union
   $$\mathrm{Conj}_{A_n}(\alpha_n) \cup \mathrm{Conj}_{A_n}(\beta_n)$$
   coincides with the set of all cycles of lenght $n-1$ in S_n.

3) Determine all groups G with $|\operatorname{Aut}(G)| = 1$.

4) Let G be a finite group and K, L subgroups which are conjugate to each other. Let further P be a Sylow subgroup of G for some prime divisor of $|G|$. Show that if $yKy^{-1} = K$ and $yLy^{-1} = L$ for all $y \in P$ then there exists $u \in N_G(P)$, such that $uKu^{-1} = L$.

5) Show that groups of order 12 or 196 are not simple.

6) (i) Let $q < p$ be two prime numbers, such that q does not divide $p-1$. Show that a group of order $p \cdot q$ is cyclic.
   (ii) Show that $[G : Z(G)] \neq 15$ for all finite groups G.

ALL ANSWERS HAVE TO BE JUSTIFIED.