Problems 1

09/09/2020

1) Let G be a finite group and $g \in G$. Show that $g^m = e$ for some positive integer m.

2) Let X be a set with at least three elements. Show that $\mathrm{Bij}(X)$ is a non abelian group.

3) Let X be a finite non empty set equipped with an operation
$$X \times X \longrightarrow X\,,\ (x,y) \longmapsto x \cdot y\,,$$
which is associative, *i.e.* $x \cdot (y \cdot z) = (x \cdot y) \cdot z$ for all x, y, and z in X. Show that X with this operation is a group if and only if the following two maps are injective for all $x \in X$:
$$l_x\,:\,X \longrightarrow X\,,\ y \longmapsto x \cdot y$$
and
$$r_x\,:\,X \longrightarrow X\,,\ y \longmapsto y \cdot x\,.$$
Is this also true if X is not finite?

4) Show that the following subset of the real 2×2-matrizes is an abelian group with respect to the matrix multiplication:
$$\left\{ \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} \,\middle|\, a \in \mathbb{R} \right\}.$$

5) Prove that a group with ≤ 4 elements is commutative.