Problems 3

09/28/2020

1) Let G, H be finite groups. Show that the product group $G \times H$ defined in Section 2.19 of the Lecture Notes is cyclic if and only if
   (a) both G and H are cyclic, and
   (b) the integers $|G|$ and $|H|$ are coprime (*i.e.* have greatest common divisor 1).

2) Let G be a group and $\alpha : G \longrightarrow H$ be a homomorphism of groups with H abelian. Show that α factors via $G/[G,G]$, *i.e.* there exists a homomorphism $\beta : G/[G,G] \longrightarrow H$, such that $\alpha = \beta \circ q$, where $q : G \longrightarrow G/[G,G]$ is the quotient homomorphism.

3) Show that every automorphism of S_3 is an inner automorphism.

4) Show that every semidirect product is an internal semidirect product.

5) Let G be a group. Show that G is abelian if and only if the map $G \longrightarrow G$, $g \mapsto g^2$, is a homomorphism of groups. Show further that this map is an isomorphism if G is a finite abelian group of odd order.

6) Let G be a group and $H \supseteq [G,G]$ a subgroup of G. Show that H is normal and G/H is abelian.

7) Show that if G_1 and G_2 are isomorphic groups then $\mathrm{Aut}(G_1)$ and $\mathrm{Aut}(G_2)$ are also isomorphic as groups. Give an example of two groups, which have isomorphic automorphism groups, but which are not isomorphic to each other as groups.