Problems 6

11/20/2020

1) Let G be a finite group and H a proper subgroup of G, *i.e.* $H \neq G$. Show that
$$\bigcup_{x \in G} xHx^{-1} \neq G\,.$$

2) Let G be a group of order p^2q, where p, q are different prime numbers. Let P be a p-, and Q a q-Sylow subgroup of G. Show that G is an internal semidirect product of P and Q.

3) Let P be a p-group for some prime p and $A \subseteq P$ a normal subgroup of order p. Show that $A \subseteq Z(P)$.

4) Determine all Sylow subgroups of S_4 and of A_4.

5) Let X be a finite set with at least 2 elements and G a finite group acting on X. Show that if this action has only one orbit then there exists $h \in G$, such that $h.x \neq x$ for all $x \in X$.

6) Let G be a group and H a subgroup of G of finite index n. Show that $g^n \in H$ for all $g \in Z(G)$.