UNIVERSIDAD DE SAN CARLOS DE GUATEMALA
FACULTAD DE INGENIERÍA
ESCUELA DE CIENCIAS
DEPARTAMENTO DE MATEMÁTICA
PRIMER SEMESTRE 2021

TAREA No. 2

Sección

DESCRIPCIÓN DE CALIFICACIÓN	
Presentación	/10
Ejercicios resueltos	/50
Ejercicio(s) calificado(s)	/40
CALIFICACIÓN TOTAL	/100

Nombre: Roberto Carlos Gómez Donis
Carné: 202000544
Profesor(a): Ing. Carlos Garrido

Fecha: 16/03/2021

Pags. 935-936

Ejercicio 32. #1

$z = x^2+xy+3y^2$ $(3,-1)$ $(2.96,-0.95)$

$z_x = \frac{\partial}{\partial x}(x^2-xy+3y^2) = 2x-y$ $z_x(3,-1) = 2(3)-(-1) = 7$ $dx = 2.96-3 = -0.04$

$z_y = \frac{\partial}{\partial y}(x^2-xy+3y^2) = -x+6y$ $z_y(3,-1) = 6(-1)-3 = -9$ $dy = -0.95-(-1) = 0.05$

$dz = z_x dx + z_y dy = 7(-0.04)+(-9)(0.05) = -0.73$ $\boxed{\Delta z = 14.28-15 = -0.72.}$

$z(3,-1) = 3^2+(3)(-1)+3(-1)^2 = 15.$

$z(2.96,-0.95) = 2.96^2-(2.96)(-0.95)+3(-0.95)^2 = \boxed{14.28}$

Ejercicio 33. #2.

$Area = xy$ $dA = \frac{\partial A}{\partial x}dx + \frac{\partial A}{\partial y}dy$

$x = 30cm$
$y = 24cm$ $\partial A = (1)(y)\,dx + (x)(1)\,dy = ydx+xdy = (24)(0.1)+(30)(0.1) = \boxed{5.4}$

Ejercicio 39.

$\frac{1}{R} = \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3} = \frac{1}{25}+\frac{1}{40}+\frac{1}{50}$

$\Delta R1 = 25(0.005) = 0.125$
$\Delta R2 = 40(0.005) = 0.2$
$\Delta R3 = 50(0.005) = 0.25$

$1 = \frac{17}{200}R$ $R = 1.76$

$-\frac{1}{R^2}\frac{\partial R}{\partial R_1} = \frac{\partial R}{\partial R_1}\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right) = \frac{\partial R}{\partial R_1}\left(\frac{1}{R_1}\right)+\frac{\partial R}{\partial R_1}\left(\frac{1}{R_2}\right)+\left(\frac{\partial R}{\partial R_1}\right)\frac{1}{R_3} = -\frac{1}{R_1^2}+0+0.$

$-\frac{1}{R^2}\frac{\partial R}{\partial R_2} = \frac{\partial R}{\partial R_2}\left(\frac{1}{R_1}\right)+\frac{\partial R}{\partial R_2}\left(\frac{1}{R_2}\right)+\frac{\partial R}{\partial R_3}\left(\frac{1}{R_3}\right) = -\frac{1}{R_2^2}$

$-\frac{1}{R^2}\frac{\partial R}{\partial R_3} = \frac{\partial R}{\partial R_3}\left(\frac{1}{R_1}\right)+\frac{\partial R}{\partial R_3}\left(\frac{1}{R_2}\right)+\frac{\partial R}{\partial R_3}\left(\frac{1}{R_3}\right) = -\frac{1}{R_3^2}$

$\frac{\partial R}{\partial R_1} = \frac{R^2}{R_1^2}$ $\frac{\partial R}{\partial R_2} = \frac{R^2}{R_2^2}$ $\frac{\partial R}{\partial R_3} = \frac{R^2}{R_3^2}$ $\Delta R = \frac{dR}{dR1}\Delta R1 + \frac{dR}{dR2}\Delta R2 + \frac{dR}{\partial R3}\Delta R3$

$\Delta R = \frac{R^2}{R_1^2}\Delta R1 + \frac{R^2}{R2^2}\Delta R2 + \frac{R^2}{R_3^2}\Delta R3 = \frac{11.76^2}{25^2}(0.125)+\frac{11.76^2}{40^2}(0.2)+\frac{11.76^2}{50^2}(0.25)$

$\boxed{\Delta R = 0.059.}$

$z = xy^3 - x^2y \quad x = t^2 - 1 \quad y = t^2 - 1$ Ejercicio 1. #4

$\frac{\partial z}{\partial x} = y^3 - 2xy \quad \frac{\partial x}{\partial t} = 2t \quad \frac{dy}{dt} = 2t$

$\frac{\partial z}{\partial y} = 3xy^2 - x^2 \quad \frac{dz}{dt} = (y^3 - 2xy)2t + (3xy^2 - x^2)(2t)$

$= \boxed{((t^2-1)^3 - 2(t^2+1)(t^2-1))(2t) + (3(t^2+1)(t^2-1)^2 - (t^2+1)^2)(2t)}$

$z = \frac{x-y}{x+2y} \quad x = e^{\pi t} \quad y = e^{-\pi t}$ Ejercicio 2 #5.

$\frac{\partial z}{\partial x} = \frac{(1)(x+2y) - 1(x-y)}{(x+2y)^2} = \frac{3y}{(x+2y)^2} \quad \frac{dx}{dt} = \pi e^{\pi t}$

$\frac{dz}{dy} = \frac{(-1)(x+2y) - 2(x-y)}{(x+2y)^2} = \frac{-3x}{(x+2y)^2} \quad \frac{dy}{dt} = -\pi e^{-\pi t}$

$\frac{dz}{dt} = \left(\frac{3y}{(x+2y)^2}\right)(\pi e^{\pi t}) + \left(-\frac{3x}{(x+2y)^2}\right)(-\pi e^{\pi t}) = \left(\frac{3(e^{-\pi t})}{(e^{\pi t} + 2(e^{-\pi t}))^2}\right)(\pi e^{\pi t}) + \left(-\frac{3(e^{\pi t})}{(e^{\pi t} + 2(e^{-\pi t}))^2}\right)(-\pi e^{-\pi t})$

$\frac{dz}{dt} = \frac{6\pi}{(e^{\pi t} + 2e^{-\pi t})^2}.$

$w = xe^{y/z} \quad x = t^2 \quad y = 1 - t \quad z = 1 + 2t$ Ejercicio 5 #6.

$\frac{\partial w}{\partial x} = e^{y/z} \quad \frac{\partial x}{\partial t} = 2t \quad \frac{dy}{dt} = -1 \quad \frac{dz}{dt} = 2$

$\frac{dw}{dy} = \frac{xe^{y/z}}{z} \quad \frac{dw}{dt} = e^{y/z}(2t) + \frac{xe^{y/z}}{z}(-1) - \frac{xye^{y/z}}{z^2}(2).$

$\frac{dw}{dz} = -\frac{xye^{y/z}}{z^2}$

$\boxed{\frac{dw}{dt} = e^{1-t/1+2t}(2t) + \frac{t^2 e^{1-t/1+2t}}{1+2t}(-1) - \frac{t^2(1-t)e^{1-t/1+2t}}{(1+2t)^2}(2)}$

$z = \tan^{-1}\left(\frac{y}{x}\right) \quad x = e^t \quad y = 1 - e^t$ Ejercicio 6 #7

$\frac{dz}{dx} = \frac{1}{(y/x)^2 + 1}\left(-\frac{y}{x^2}\right) = \frac{-y}{y^2 + x^2} \quad \frac{dx}{dt} = e^t \quad \frac{dz}{dt} = \frac{-y}{y^2+x^2}(e^t) + \left(\frac{x}{y^2+x^2}\right)(-e^t) =$

$\frac{dz}{dy} = \frac{x}{y^2+x^2} \quad \frac{dy}{dt} = -e^t$

$= \boxed{\frac{1-e^t}{(1-e^t)^2 + (e^t)^2} e^t + \frac{e^t}{(1-e^t)^2 + (e^t)^2}(-e^t)}$

$z=(x-y)^5 \quad x=s^2t \cdot y=st^2$ Ejercicio 7

#8

$\frac{\partial z}{\partial x}=5(x-y)^4 \quad \frac{dx}{dt}=s^2 \quad \frac{dx}{ds}=2st$

$\frac{\partial z}{\partial y}=-5(x-y)^4 \quad \frac{dy}{dt}=2st \quad \frac{dy}{ds}=t^2$

$\frac{\partial z}{\partial t}=5(x-y)^4(s^2)-5(x-y)^4(2st)=5(x-y)^4(s^2)+5(x-y)^4(-2st)=\boxed{5(x-y)^4(s^2-2st)}$

$\frac{dz}{ds}=5(x-y)^4(2st)-5(x-y)^4(t^2)=5(x-y)^4(2st)+5(x-y)^4(-t^2)=\boxed{5(x-y)^4(2st-t^2)}$

$z=\tan^{-1}(x^2+y^2) \quad x=s\ln t \quad y=te^s$ Ejercicio 38.

#9

$\frac{dz}{dx}=\frac{2x}{(x^2+y^2)^2+1} \quad \frac{dx}{dt}=\frac{s}{t} \quad \frac{dx}{ds}=\ln t$

$\frac{\partial z}{\partial y}=\frac{2y}{(x^2+y^2)^2+1} \quad \frac{dy}{dt}=es \quad \frac{dy}{ds}=ts^5$

$\frac{\partial z}{\partial t}=\left(\frac{2x}{(x^2+y^2)^2+1}\right)\left(\frac{s}{t}\right)+\left(\frac{2y}{(x^2+y^2)^2+1}\right)(e^s)=\boxed{\frac{2xs+2yte^s}{(x^2+y^2)^2t+t}}$

$\frac{dz}{ds}=\left(\frac{2x}{(x^2+y^2)^2+1}\right)(\ln t)+\left(\frac{2y}{(x^2+y^2)^2+1}\right)(te^s)=\boxed{\frac{2x\ln t+2yte^s}{(x^2+y^2)^2+1}}$

$z=e^r\cos\theta \quad r=st \quad \theta=\sqrt{s^2+t^2}$ Ejercicio 11

#10.

$\frac{\partial z}{\partial r}=e^r\cos\theta \quad \frac{\partial r}{\partial t}=s \quad \frac{\partial r}{ds}=t$

$\frac{\partial z}{\partial\theta}=-e^r\operatorname{sen}\theta \quad \frac{d\theta}{dt}=\frac{t}{\sqrt{s^2+t^2}} \quad \frac{d\theta}{ds}=\frac{s}{\sqrt{s^2+t^2}}$

$\frac{dz}{dt}=(e^r\cos\theta)(s)+(-e^r\operatorname{sen}\theta)\left(\frac{t}{\sqrt{s^2+t^2}}\right)=\boxed{e^r\left(s\cos\theta-\frac{t\operatorname{sen}\theta}{\sqrt{s^2+t^2}}\right)}$

$\frac{dz}{ds}=(e^r\cos\theta)(t)+(-e^r\operatorname{sen}\theta)\left(\frac{s}{\sqrt{s^2+t^2}}\right)=\boxed{e^r\left(t\cos\theta-\frac{s\operatorname{sen}\theta}{\sqrt{s^2+t^2}}\right)}$

Ejercicio 15 #11.

$g(u,v) = f(u^v + \text{sen}\,v, e^u + \cos u)$

$g_u(0,0) \quad g_v(0,0)$

	f	g	fx	fy
(0,0)	3	6	4	8
(1,2)	6	3	2	5

$2 \cdot e^0 + 5 \cdot \cos 0$

$g_u(0,0) = f_x(1,2) x_u(0,0) + f_y(1,2) y_u(0,0)$

$2(1) + 5(1) = \boxed{7}$

$g_v = f_x(1,2)\, x_v(0,0) + f_y(1,2)\, y_v(0,0)$

$2(1) + 5(0) = \boxed{2}$

Ejercicio 23 #12.

$w = xy + yz + zx \quad x = r\cos\theta \quad y = r\,\text{sen}\,\theta \quad z = r\theta \quad \frac{\partial w}{\partial r}, \frac{\partial w}{\partial \theta} \quad r = 2 \quad \theta = \pi/2$

$\frac{\partial w}{\partial x} = y + z \quad \frac{\partial x}{\partial r} = \cos\theta \quad \frac{\partial y}{\partial r} = \text{sen}\,\theta \quad \frac{\partial z}{\partial r} = \theta$

$\frac{\partial w}{\partial y} = x + z \quad \frac{\partial x}{\partial \theta} = -r\,\text{sen}\,\theta \quad \frac{\partial y}{\partial \theta} = r\cos\theta \quad \frac{\partial z}{\partial \theta} = r$

$\frac{\partial w}{\partial z} = y + x$

$\frac{\partial w}{\partial r} = (y+z)(\cos\theta) + (x+z)(\text{sen}\,\theta) + (y+x)(\theta) = (2\,\text{sen}\frac{\pi}{2} + 2\frac{\pi}{2})(\cos\frac{\pi}{2}) + (2\cos\frac{\pi}{2} + 2\frac{\pi}{2})(\text{sen}\frac{\pi}{2}) + (2\cos\frac{\pi}{2} + 2\,\text{sen}\frac{\pi}{2})(\frac{\pi}{2})$

$$\boxed{\frac{\partial w}{\partial r} = 2\pi}$$

$\frac{\partial w}{\partial \theta} = (y+z)(-r\,\text{sen}\,\theta) + (x+z)(r\cos\theta) + (y+x)(r) = (2\,\text{sen}\frac{\pi}{2} + 2\frac{\pi}{2})(-2\,\text{sen}\frac{\pi}{2}) + (2\cos\frac{\pi}{2} + 2\frac{\pi}{2})(2\cos\frac{\pi}{2}) + (2\cos\frac{\pi}{2} + 2\,\text{sen}\frac{\pi}{2})(2)$

$$\boxed{\frac{\partial w}{\partial \theta} = -2\pi.}$$

Ejercicio 25 #13.

$N = \frac{p+q}{p+r} \quad p = u + vw \quad q = v + uw \quad r = w + uv \quad \frac{\partial N}{\partial u}, \frac{\partial N}{\partial v}, \frac{\partial N}{\partial w} \quad u = 2 \quad v = 3 \quad w = 4$

$\frac{\partial N}{\partial p} = \frac{1(p+r) - 1(p+q)}{(p+r)^2} = \frac{-q+r}{(p+r)^2} \quad \frac{\partial p}{\partial u} = 1 \quad \frac{\partial q}{\partial u} = w \quad \frac{\partial r}{\partial u} = v$

$\frac{\partial N}{\partial q} = \frac{1}{p+r} \quad \frac{\partial p}{\partial v} = w \quad \frac{\partial q}{\partial v} = 1 \quad \frac{\partial r}{\partial v} = u$

$\frac{\partial N}{\partial r} = (p+q)\left(\frac{-1}{(p+r)^2}\right) = \frac{-p+q}{(p+r)^2} \quad \frac{\partial p}{\partial w} = v \quad \frac{\partial q}{\partial w} = u \quad \frac{\partial r}{\partial w} = 1$

$$\frac{\partial w}{\partial u} = \left(\frac{-q+r}{(p+r)^2}\right)(1) + \left(\frac{1}{p+r}\right)(w) + \left(\frac{-p+q}{(p+r)^2}\right)(v)$$

$p = 14 \quad r = 10$
$q = 11$

$$\left(\frac{-11+10}{(14+10)^2}\right)(1) + \left(\frac{1}{14+10}\right)(4) + \left(\frac{-14+11}{(14+10)^2}\right)(5) = \boxed{\frac{5}{144}}$$

$$\frac{\partial w}{\partial v} = \left(\frac{-q+r}{(p+r)^2}\right)(w) + \left(\frac{1}{p+r}\right)(1) + \left(-\frac{p+q}{(p+r)^2}\right)(u) = \left(\frac{-11+10}{(14+10)^2}\right)(4) + \left(\frac{1}{14+10}\right)(1) + \left(\frac{14+11}{(14+10)^2}\right)(2) = \boxed{\frac{-5}{96}}$$

$$\frac{\partial w}{\partial w} = \left(\frac{-q+r}{(p+r)^2}\right)(v) + \left(\frac{1}{p+r}\right)(u) + \left(-\frac{p+q}{(p+r)^2}\right)(1) = \left(\frac{-11+10}{(14+10)^2}\right)(3) + \left(\frac{1}{14+10}\right)(2) + \left(\frac{14+11}{(14+10)^2}\right)(1) = \boxed{\frac{5}{144}}$$

Ejercicio 35 #14.

$x = \sqrt{1+t} \quad y = 2 + \frac{t}{3} \quad T_x(2,3) = 4 \quad T_y(2,3) = 3$

$y(3) = 2 + \frac{3}{3} = 3$

$x(3) = \sqrt{1+3} = 2$

$\frac{dx}{dt} = \frac{1}{2\sqrt{1+t}} \quad \frac{dy}{dt} = 0 + \frac{1}{3} = \frac{1}{3}$

$$\frac{dT}{dt} = T_x\left(\frac{dx}{dt}\right) + T_y\left(\frac{dy}{dt}\right) = 4\left(\frac{1}{2\sqrt{1+t}}\right) + 3\left(\frac{1}{3}\right) = \boxed{2}$$

Ejercicio 37 #15

$C = 1449.2 + 4.6T - 0.055T^2 + 0.00029T^3 + 0.016D$

$T(25) = 12 \quad D(25) = 10$
$T(20) = 12.5 \quad D(20) = 7.$

$\frac{\partial C}{\partial T} = 4.6 - 0.11T + 0.00087T^2$

$\frac{\partial C}{\partial D} = 0.016 =$

$\frac{dT}{dt} = \frac{12 - 12.5}{25 - 20} = -0.1$

$\frac{dD}{dt} = \frac{10 - 7}{24 - 20} = 0.75$

$$\frac{dC}{dt}(20) = \frac{\partial C}{\partial T}\left(\frac{dT}{dt}\right) + \frac{\partial C}{\partial D}\left(\frac{dD}{dt}\right)$$

$$= (4.6 - 0.11T + 0.00087(12.5)^2)(-0.1) + 0.016(0.75) = \boxed{-0.32}$$

Ejercicio 39 #16.

a) $V = lwh$

$\frac{\partial V}{\partial l} = wh \quad \frac{\partial V}{\partial h} = lw \quad \frac{dl}{dt} = 2 \quad \frac{dh}{dt} = -3.$

$\frac{\partial V}{\partial w} = lh \quad \frac{dw}{dt} = 2$

$$\frac{dV}{dt} = wh(2) + lh(2) + lw(-3) = (2)(2)(2)(2) + (1)(2)(2) + (1)(2)(-3)$$

$\boxed{6\ m^3/s.}$

b) $A = 2(lw + lh + hw)$

$\frac{\partial A}{\partial l} = 2(w+h+0) = 2(w+h)$

$\frac{\partial A}{\partial w} = 2(l+h)$

$\frac{\partial A}{\partial h} = 2(l+w)$

$\frac{\partial A}{\partial t} = 2(w+h)(2) + 2(l+h)(2) + 2(l+w)(-3) =$

$2(1+2)(2) + 2(1+2)(2) + 2(1+2)(-3) = \boxed{10\ m^2/s}$

c) $d^2 = l^2 + w^2 + h^2 = \sqrt{l^2+w^2+h^2}$

$\frac{\partial d}{\partial l} = \frac{1}{2\sqrt{l^2+w^2+h^2}}(2l)$

$\frac{\partial d}{\partial w} = \frac{w}{\sqrt{l^2+w^2+h^2}}$

$\frac{\partial d}{\partial h} = \frac{h}{\sqrt{l^2+w^2+h^2}}$

$\frac{dd}{dt} = \left(\frac{l}{\sqrt{l^2+w^2+h^2}}\right)(2) + \left(\frac{w}{\sqrt{l^2+w^2+h^2}}\right)(2) + \left(\frac{h}{\sqrt{l^2+w^2+h^2}}\right) - 3$

$= \left(\frac{1}{\sqrt{1+4+4}}\right)(2) + \left(\frac{2}{\sqrt{1+4+4}}\right)(2) + \left(\frac{1}{\sqrt{1+4+4}}\right)(-3) \boxed{0\ m/s}$

Ejercicio 41 #17

$V = \frac{8.31T}{P}$ $P = 20\,KPa$ $T = 320\,K$

$\frac{dP}{dt} = 0.05\,KPa/s$ $\frac{dT}{dt} = 0.15\,K/s$

$\frac{\partial V}{\partial P} = 8.31T(-1P^{-2}) = \frac{-8.31T}{P^2}$

$\frac{\partial V}{\partial T} = \frac{8.31}{P}(1) = \frac{8.31}{P}$

$\frac{dV}{dt} = -\frac{8.31T}{P^2}(0.05) + \frac{8.31}{P}(0.15) = \frac{-8.31(320)}{20^2}(0.05) + \frac{8.31}{20}(0.15) = \boxed{-0.27\ L/s}$

Ejercicio 43 #18

$A = \frac{1}{2}xy\ sen\theta$

$0 = \frac{\partial A}{\partial t} = \frac{\partial A}{\partial x}\frac{dx}{dt} + \frac{\partial A}{\partial y}\frac{dy}{dt} + \frac{\partial A}{\partial \theta}\frac{d\theta}{dt}$

$\frac{\partial A}{\partial x} = \frac{y\,sen\theta}{2}$ $\frac{\partial A}{\partial y} = \frac{x\,sen\theta}{2}$ $\frac{\partial A}{\partial \theta} = \frac{xy\cos\theta}{2}$

$\frac{d\theta}{dt} = \frac{-y\,sen\theta\frac{dx}{dt} + x\,sen\theta\frac{dy}{dt}}{xy\cos\theta} = \frac{30(\frac{1}{2})(3) + 20(\frac{1}{2})(-2)}{30(20)(\sqrt{3}/2)} = \boxed{-0.05}$

Ejercicio 5 #19

$f(x,y) = y\cos(xy)$ $(0,1)$ $\theta = \pi/4$

$f_x(x,y) = -y^2 sen(xy)$ $f_x(0,1) = 0$

$f_y = \cos(xy) - xy\,sen(xy)$ $f_y(0,1) = 1$

$\nabla f(0,1) = \langle 0,1\rangle$

$u = \langle \sqrt{2}/2, \sqrt{2}/2\rangle$

$D_u f(x,y) = \langle 0,1\rangle \langle \sqrt{2}/2, \sqrt{2}/2\rangle$

$= \boxed{\frac{\sqrt{2}}{2}}$

Ejercicio 7 #20

$f(x,y)=\frac{x}{y}$ $P(2,1)$ $u=\langle\frac{3}{5},\frac{4}{5}\rangle$

a) $f_x(x,y)=\frac{1}{y}$

$f_y(x,y)=x(-y^{-2})$

$\boxed{\nabla f(x,y)=\langle\frac{1}{y},-\frac{x}{y^2}\rangle}$

b) $f_x(2,1)=1$

$f_y(2,1)=-2.$

$\boxed{\nabla f(2,1)=\langle 1,-2\rangle}$

c) $D_uf(2,1)=\langle 1,-2\rangle\langle\frac{3}{5},\frac{4}{5}\rangle=\boxed{-1}$

Ejercicio 8 #21

$f(x,y)=x^2\ln y$ $P(3,1)$ $u=\langle\frac{5}{13},\frac{12}{13}\rangle$

a) $f_x=\ln y(2x)$

$f_y=\frac{x^2}{y}$

$\boxed{\nabla f(x,y)=\langle 2x\ln y,\frac{x^2}{y}\rangle}$

b) $f_x(3,1)=0$

$f_y(3,1)=9$

$\boxed{\nabla f(3,1)=\langle 0,9\rangle}$

c) $D_uf=\langle 0,9\rangle\langle 5/13,12/13\rangle=\boxed{\frac{108}{13}}$

Ejercicio 9 #22

$f(x,y,z)=xe^{2yz}$ $P(3,0,2)$ $u=\langle\frac{2}{3},-\frac{2}{3},\frac{1}{3}\rangle$

a) $f_x=e^{2yz}$

$f_y=2xe^{2yz}z$

$f_z=2xye^{2yz}$

$\boxed{\nabla f(x,y,z)=\langle e^{2yz},2xze^{2yz},2xye^{2yz}\rangle}$

b) $f_x(3,0,2)=e^0=1$

$f_y(3,0,2)=2(3)(2)e^0=12$

$f_z(3,0,2)=0.$

$\boxed{\nabla f(3,0,2)=\langle 1,12,0\rangle}$

c) $D_uf(3,0,2)=\langle 1,12,0\rangle\langle\frac{2}{3},-\frac{2}{3},\frac{1}{3}\rangle$

$=\boxed{\frac{-22}{3}}$

Ejercicio 11 #23

$f(x,y)=e^x\operatorname{sen}y$ $(0,\frac{\pi}{3})$ $v=\langle -6,8\rangle$

$f_x(x,y)=e^x\operatorname{sen}y$

$f_y(x,y)=e^x\cos y$

$|v|=\sqrt{6^2+8^2}=10$

$\frac{v}{|v|}=\langle-\frac{6}{10},\frac{8}{10}\rangle$

$D_uf(0,\frac{\pi}{3})=\langle e^0\operatorname{sen}\frac{\pi}{3},e^0\cos\frac{\pi}{3}\rangle\langle-\frac{3}{5},\frac{4}{5}\rangle$

$\boxed{=\frac{4-3\sqrt{3}}{10}}$

Ejercicio 13 #24

$g(s,t)=s\sqrt{t}$ $(2,4)$ $v=\langle 2,-1\rangle$

$g_s=\sqrt{t}$

$g_t=\frac{s}{2\sqrt{t}}$

$|v|=\sqrt{2^2+1^2}=\sqrt{5}$

$u=\langle\frac{2}{\sqrt{5}},-\frac{1}{\sqrt{5}}\rangle$

$D_ug(2,4)=\langle\sqrt{4},\frac{2}{2\sqrt{4}}\rangle\langle\frac{2}{\sqrt{5}},-\frac{1}{\sqrt{5}}\rangle=\boxed{\frac{7\sqrt{5}}{10}}$

Ejercicio 17 #25.

$f(x,y,z) = xe^y + ye^z + ze^x$ $(0,0,0)$

$v = \langle 5, 1, -2 \rangle$

$f_x = e^y + ze^x$

$f_y = xe^y + e^z$

$f_z = ye^z + e^x$

$|v| = \sqrt{5^2+1^2+2^2} = \sqrt{30}$

$u = \left\langle \frac{5}{\sqrt{30}}, \frac{1}{\sqrt{30}}, \frac{-2}{\sqrt{30}} \right\rangle$

$D_u f = \langle e^0, e^0, e^0 \rangle \left\langle \frac{5}{\sqrt{30}}, \frac{1}{\sqrt{30}}, \frac{-2}{\sqrt{30}} \right\rangle$

$= \boxed{\frac{2\sqrt{30}}{15}}$

Ejercicio 21 #26.

$f(x,y) = 4y\sqrt{x}$ $(4,1)$

$f_x = \frac{2y}{\sqrt{x}}$ $f_x(4,1) = 1$

$f_y = 4\sqrt{x}$ $f_y(4,1) = 8$

$\nabla f(4,1) = \boxed{\langle 1, 8 \rangle}$

$|\nabla f(4,1)| = \sqrt{1^2+8^2} = \boxed{\sqrt{65}}$

Ejercicio 23 #27

$f(x,y) = \text{sen}(xy)$ $(1,0)$

$f_x = y\cos(xy)$ $f_x(1,0) = 0$

$f_y = x\cos(xy)$ $f_y(1,0) = 1$

$\boxed{\nabla f(1,0) = \langle 0, 1 \rangle \quad |\nabla f(1,0)| = 1}$

Ejercicio 25 #28

$f(x,y,z) = \frac{x}{y+z}$ $(8,1,3)$

$f_x = \frac{1}{y+z}$ $\nabla f(8,1,3) = \left\langle \frac{1}{4}, -\frac{1}{2}, -\frac{1}{2} \right\rangle$

$f_y = \frac{-x}{(y+z)^2}$ $|\nabla f(8,1,3)| = \sqrt{\frac{1}{4}^2 + \frac{1}{2}^2 + \frac{1}{2}^2} = \frac{3}{4}$

$f_z = \frac{-x}{(y+z)^2}$

$f_x(8,1,3) = \frac{1}{1+3} = \frac{1}{4}$

$f_y(8,1,3) = -\frac{1}{2}$

$f_z(8,1,3) = -\frac{1}{2}$

max $\frac{3}{4}$

dirección $\left\langle \frac{1}{4}, -\frac{1}{2}, -\frac{1}{2} \right\rangle$

Ejercicio #29

$120 = \frac{k}{\sqrt{1^2+2^2+2^2}} = \frac{k}{3}$

$k = 360$ $T = \frac{360}{\sqrt{x^2+y^2+z^2}}$

a) $(1,2,2)$ $(2,1,3)$

$V = (2,1,3) - (1,2,2) = \langle 1, -1, 1 \rangle$

$|V| = \sqrt{1^2+1^2+1^2} = \sqrt{3}$

$u = \left\langle \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right\rangle$

$T_x = \frac{-360x}{(x^2+y^2+z^2)^{3/2}}$

$T_y = \frac{-360y}{(x^2+y^2+z^2)^{3/2}}$

$T_z = \frac{-360z}{(x^2+y^2+z^2)^{3/2}}$

$\nabla T(1,2,2) = \left\langle -\frac{40}{3}, -\frac{80}{3}, -\frac{80}{3} \right\rangle$

$D_u T(1,2,2) = \left\langle -\frac{40}{3}, -\frac{80}{3}, -\frac{80}{3} \right\rangle \left\langle \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right\rangle = \boxed{\frac{-40}{3\sqrt{3}}}$

b) $\nabla T(x,y,z) = \left(\frac{-360x}{(x^2+y^2+z^2)^{3/2}}, \frac{-360y}{(x^2+y^2+z^2)^{3/2}}, \frac{-360z}{(x^2+y^2+z^2)^{3/2}} \right) =$

$m = \frac{360}{(x^2+y^2+z^2)^{3/2}}$ $\quad q = (-x, -y, -z)$

Ejercicio 33 #30.

$V(x,y,z) = 5x^2 - 3xy + xyz$ $\quad P(3,4,5)$ $\quad v = \langle 1, 1, -1 \rangle$

a) $|v| = \sqrt{1^2+1^2+1^2} = \sqrt{3}$

$u \cdot \langle \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \rangle$

$V_x = 10x - 3y + yz$ $\quad \nabla f(x,y,z) = \langle 10x - 3y + yz, -3x + xz, xy \rangle$

$V_y = -3x + xz$

$V_z = xy$ $\quad \nabla f(3,4,5) = \langle 38, 6, 12 \rangle$

$D_u V(3,4,5) = \langle 38, 6, 12 \rangle \cdot \langle \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}} \rangle = \boxed{\frac{32\sqrt{3}}{3}}$

b) $D_u V(3,4,5) = \langle 38, 6, 12 \rangle |u| \cos\theta$

$\boxed{\nabla V(3,4,5) = \langle 38, 6, 12 \rangle}$

c) $\nabla V(3,4,5) = \langle 38, 6, 12 \rangle$

$|\nabla V(3,4,5)| = \sqrt{38^2 + 6^2 + 12^2} = \boxed{2\sqrt{406}}$

Ejercicio 41 #31.

$2(x-2)^2 + (y-1)^2 + (z-3)^2 = 10$ $\quad (3,3,5)$

a) $2(x^2 - 4x + 4) + y^2 - 2y + 1 + z^2 - 6z + 9 - 10 = 0$

$2x^2 - 8x + 8 + y^2 - 2y$

$F_x = 4x - 8$ $\quad F_x(3,3,5) = 4$

$F_y = 2y - 2$ $\quad F_y(3,3,5) = 4$

$F_z = 2z - 6$ $\quad F_z(3,3,5) = 4$

$\nabla F(3,3,5) = \langle 4, 4, 4 \rangle$

$4(x-3) + 4(y-3) + 4(z-5) = 0$

$4x - 12 + 4y - 12 + 4z - 20 = 0$

$4x + 4y + 4z - 44 = 0$

$\boxed{x + y + z = 11}$

b) $\frac{x - x_0}{F_x} = \frac{y - y_0}{F_y} = \frac{z - z_0}{F_z}$

$\boxed{\frac{x-3}{4} = \frac{y-3}{4} = \frac{z-5}{4}}$

Ejercicio 43 #32

$xy^2z^3 = 8 \quad (2,2,1)$

a) $f_x = y^2z^3 \qquad f_x(2,2,1) = 4$

$f_y = 2xyz^3 \qquad f_y(2,2,1) = 8$

$f_z = 3xy^2z^2 \qquad f_z(2,2,1) = 24$

$\nabla f(2,2,1) = \langle 4,8,24 \rangle$

$4(x-2)+8(y-2)\,24(z-1)=0$

$4x+8y+24z-48=0$

$\boxed{x+2y+6z=12}$

b) $\frac{x-x_0}{f_x} = \frac{y-y_0}{f_y} = \frac{z-z_0}{f_z}$

$\boxed{\frac{x-2}{4} = \frac{y-2}{8} = \frac{z-1}{24}}$

Ejercicio 45 #33

$e^{xyz} = x+y+z \quad (0,0,1)$

a) $f_x = 1-yze^{xyz} \qquad f_x(0,0,1) = 1$

$f_y = 1+zxe^{xyz} \qquad f_y(0,0,1) = 1$

$f_z = 1-xye^{xyz} \qquad f_z(0,0,1) = 1$

$(x+y+(z-1))=0$

$\boxed{x+y+z=1}$

b) $\boxed{\frac{x}{-1} = \frac{y}{-1} = \frac{z-1}{-1}}$

Ejercicio 47 #34

$xy+yz+zx=3 \quad (1,1,1)$

$f_x = y+z \qquad f_x(1,1,1) = 2$

$f_y = x+z \qquad f_y = 2$

$f_z = y+x \qquad f_z = 2$

$2(x-1)+2(y-1)+2(z-1)=0$

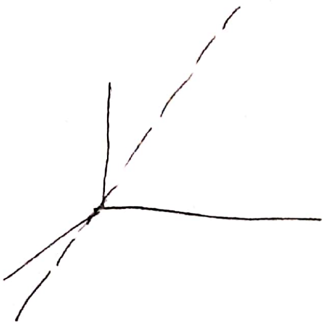
$2x-2+2y-2+2z-2=0$

$2x+2y+2z=6$

$x+y+z=3.$

$\frac{x-1}{2} = \frac{y-1}{2} = \frac{z-1}{2}$

$x=y=z.$



Ejercicio 5 # 35

$f(x,y) = 4 - 2x + 4y - x^2 - 4y^2$

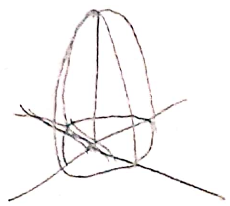
$f_x = -2x - 2$ $\quad -2x - 2 = 0 \quad x = -1$ $\quad (-1, \frac{1}{2}) \rightarrow$ maximo $\quad D = (-2)(-8) - 0^2 = 16$

$f_y = -8y + 4$ $\quad -8y + 4 = 0 \quad y = \frac{1}{2}$

$f_{xx} = -2 \quad f_{xy} = 0$

$f_{yy} = -8$

$f(-1, \frac{1}{2}) = 4 - 2(-1) + 4\frac{1}{2} - 1 - 4(\frac{1}{2})^2 = \boxed{11 \rightarrow \text{maximo local}}$



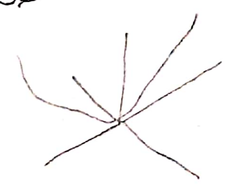
Ejercicio 8 #36

$f(x,y) = ye^x - y$ $\quad f_{yy} = 0$

$f_x(x,y) = ye^x$ $\quad f_{xx} = ye^x$ $\quad y = 0$ $\quad D = -1$

$f_y = e^x - 1$ $\quad f_{xy} = e^x$ $\quad e^x = 1$ $\quad \boxed{f(0,0) = 0 \text{ Punto silla}}$

$x = 0$



Ejercicio 15 #37.

$f(x,y) = e^x \cos y$

$f_x = e^x \cos y$ $\quad e^x \cos y = 0$ $\quad$ No hay puntos críticos

$f_y = -\operatorname{sen} y\, e^x$ $\quad -\operatorname{sen} y\, e^x = 0$

Ejercicio 18 #38

$f(x,y) = x^2 e^{-x} + y^2 e^{-x}$

$f_x = 2xe^{-x} + x^2e^{-x} - y^2 e^{-x}$ $\quad f_y = 2ye^{-x}$ $\quad f_{xy} = -2ye^{-x}$ $\quad xe^{-x}(2-x) = 0$ $\quad (0,0)$

$f_{xx} = 2e^{-x} + x^2e^{-x} + y^2e^{-x}$ $\quad f_{yy} = 2e^{-x}$ $\quad x = 0 \quad x = 2$ $\quad (2,0)$

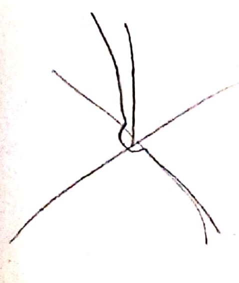
$2ye^{-x} = 0 \quad y = 0$

$D = (2)(2) - 0^2 = 4$

$D = 2e^2 + 4(-e^2) + 4(-e^{-2}) + 4e^{-2} = -2e^{-4}$

$f(0,0) = 0$ min

$f(2,0) = 4$ silla



Ejercicio 39 #39

$f(x,y) = y^2 - 2y\cos x \qquad -1 \le x \le 7$

$f_x = 2y\,\mathrm{sen}\,x$

$f_y = 2y - 2\cos x \qquad (0,1) \qquad (\pi,-1) \qquad y = 1$

$f_y(0,y) = 2y - 2\cos 0 = y = 1 \qquad f_y(\pi,y) = 2y - 2\cos\pi \quad y = -1 \qquad f_y(2\pi,y) = 2y - 2\cos 2\pi$

$(2\pi, 1)$

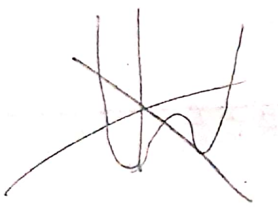
$f_y(\frac{\pi}{2},y) = 2y - 2\cos\frac{\pi}{2} \quad y = 0 \qquad f_y(\frac{3\pi}{2},y) = 2y - 2\cos\frac{3\pi}{2} \quad y = 0$

$(\frac{\pi}{2}, 0) \qquad (\frac{3\pi}{2}, 0)$

	$(0,1)$	$(\frac{\pi}{2},0)$	$(\pi,-1)$	$(\frac{3\pi}{2},0)$	$(2\pi,1)$
$f_{xx} = 2y\cos x$	2	0	2	0	2
$f_{yy} = 2$	2	2	2	2	2
$f_{xy} = 2\,\mathrm{sen}\,x$	0	2	0	-2	0
D	4	-4	4	-4	4
$f(x,y)$	1	0	-1	0	-1

Puntos silla
$f(\frac{\pi}{2},0) = 0$
$f(\frac{3\pi}{2},0) = 0$

Mínimos locales
$f(0,1) = -1$
$f(\pi,-1) = -1$
$f(2\pi,1) = -1$



Ejercicio 43 #40.

$z = \pm\sqrt{x^2+y^2} \qquad (4,2,0)$

$d = \sqrt{(x-4)^2 + (y-2)^2 + (\sqrt{x^2+y^2})^2} = \sqrt{(x-4)^2 + (y-2)^2 + x^2 + y^2}$

$d^2 = x^2 - 8x + 16 + y^2 - 4y + 4 + x^2 + y^2 = f$

$f_x = 2x - 8 + 2x = 4x - 8$

$4x - 8 = 0$
$x = 2$ $\qquad (2,1) \qquad z = \pm\sqrt{2^2+1^2} = \pm\sqrt{5}$

$4y - 4 = 0$
$y = 1$

$\boxed{(x,y,z) \quad (2,1,\pm\sqrt{5})}$

$x+z+y=100$ Ejercicio 45 #41

$z==100-x-y$

$f(x,y)=xy(100-x-y)$

$=100xy-x^2y-y^2x$

$xyz=max$

$D(0,0)=-10000<0$

$D(100,0)=-10{,}000<0$

$D(0,100)=-10000<0$

$D(\frac{100}{3},\frac{100}{3})=\frac{100}{3}>0$

$f_x=100y-2xy-y^2$

$f_y=100x-2xy-x^2$

$100x-2x(0)-x^2$

$x(100-x)$

$x=0$ or $x=100$

$y(100-2x-y)$

$100-2x-y$ $y=0$

$y=100-2x$

$3x^2-100x=x(3x-100)$

$x=\frac{100}{3}$ $y=\frac{100}{3}$

$f_{xx}=-2y$

$f_{xx}(\frac{100}{3},\frac{100}{3})=-\frac{200}{3}<0$

$z=100-\frac{100}{3}-\frac{100}{3}$

$\boxed{z=\frac{100}{3}}\boxed{=x=y}$

$x^2+y^2+z^2=4r^2$ Ejercicio 47 #42

$v=xyz$

$z=\sqrt{4r^2-x^2-y^2}$

$v=xy\sqrt{4r^2-x^2-y^2}$

$v_x=y\left(\sqrt{4r^2-x^2-y^2}+\left(-\frac{x}{\sqrt{4r^2-x^2-y^2}}\right)x\right)$

$v_x=\frac{y(-2x^2-y^2+4r^2)}{\sqrt{4r^2-x^2-y^2}}$

$x^2=\frac{4r^2}{3}$

$x=\frac{2r}{\sqrt{3}}=y=z$

$v_y=\frac{x(-2y^2-x^2+4r^2)}{\sqrt{4r^2-x^2-y^2}}$

$v=xyz$ $v=x^3$ $v=\left(\frac{2r}{\sqrt{3}}\right)^3=\left(\frac{2^3r^3}{\sqrt{3}^3}\right)$

$x=y=z$

$v_x=y(4r^2-2x^2-y^2)\rightarrow 4r^2-2x^2-y^2=0$

$v_y=x(4r^2-2y^2-x^2)\rightarrow 4r^2-2y^2-x^2=0$

$\boxed{v=\frac{8r^3}{3\sqrt{3}}}$

$x+2y+3z=6$ Ejercicio 49 #43

$x=6-2y-3z$

$v=xyz$

$v=(6-2y-3z)yz=6yz-2y^2z-3yz^2$

$y=\frac{-(6-6z)\pm\sqrt{(6-6z)^2-4(-2)(0)}}{2(-2)}=$

$v_y=6z-4yz-3z^2$ $6y-2y^2-6yz=0$

$v_z=6y-2y^2-6yz$

$y=0$ $y=1$

$z=\frac{-(-6)\pm\sqrt{(-6)^2-4(9)(0)}}{2(9)}$

$x+2(1)+3(\frac{2}{3})=6$

$z=0$ $z=\frac{2}{3}$

$x=2$

$v=xyz=2(1)(\frac{2}{3})=\boxed{\frac{4}{3}}$

$V = xyz$ Ejercicio 53 #44

$32000 = xyz$

$z = \frac{32000}{xy}$

$f(x,y,z) = 2xy + 2yz + xz$

$f_x = 2y + z \rightarrow 2y + \frac{32000}{xy}$

$f_y = 2x + 2z \rightarrow 2x + 2\left(\frac{32000}{xy}\right) = 2x + \frac{64000}{xy}$

$2y + \frac{32000}{xy} = 0$

$2x + \frac{64000}{xy} = 0$

$x = \frac{16000}{20^2} = 40$

$x = \frac{16000}{y^2}$

$\left(\frac{16000}{y^2}\right)^2 y = 32000$

$z = \frac{32000}{40 \cdot 20} = 40$

$y = 20$

$\boxed{V = (40 \times 20 \times 40)\,cm}$

$g(x,y) = x^2 + y^2 = 1$ Ejercicio 3 #45

$f_x(x,y) = 2x$ $g_x = 2x$ $2x = 2x\lambda$ $x = 0$

$f_y = -2y$ $g_y = 2y$ $-2y = 2y\lambda$ $\lambda = 1$

$x^2 + y^2 = 1$

$x = 0$

$y^2 = 1$

$y = \pm 1$

$\lambda = 1$

$-2y = 2y$

$y = 0$

$x^2 = 1 \rightarrow x = \pm 1$

$$\boxed{\begin{array}{l} f(0,-1) = -1 \\ f(0,1) = -1 \\ f(-1,0) = 1 \\ f(1,0) = 1 \end{array}}$$

Ejercicio 7 #46

$f(x,y,z) = x^2 + y^2 + z^2$ $x + y + z = 12$

$f_x = 2x$ $g_x = 1$ $\lambda = 2x$

$f_y = 2y$ $g_y = 1$ $\lambda = 2y$

$f_z = 2z$ $g_z = 1$ $\lambda = 2z$

$\lambda = 2x = 2y = 2z$

$x = y = z$

$z + z + z = 12$

$z = 4$

$y + y + 4 = 12$

$y = 4$

$x + 4 + 4 = 12$

$x = 4$

$$\boxed{\begin{array}{l} \max(4,4,4) \\ \min(-4,-4,-4) \end{array}}$$

$f(x,y,z)=xy^2z$ $\quad x^2+y^2+z^2=4$

Ejercicio 9 #47

$\lambda=\frac{y^2z}{2x}$

$f_x=y^2z \quad g_x=2x \quad 2x\lambda=y^2z$

$f_y=2xyz \quad g_y=2y \quad 2y\lambda=2xyz$

$f_z=xy^2 \quad g_z=2z \quad 2z\lambda=xy^2$

$\lambda=xz$

$\lambda=\frac{xy^2}{2z}$

$z^2+2z^2+z^2=4$

$z=\pm1$

$x=\pm y$

$\frac{xy^2}{2z}=\frac{y^2z}{2x}$

$x^2=z^2$

$xz=\frac{y^2z}{2x}$

$2x^2z=y^2z$

$2x^2=y^2$

$1+1+y^2=4$

$y=\pm\sqrt{2}$

$xz=\frac{xy^2}{2z}$

$2z^2=y^2$

$f(1,\pm\sqrt{2},1)=2$

$f(-1,\pm\sqrt{2},-1)=2$

$f(-1,\pm\sqrt{2},1)=-2$

$f(1,\pm\sqrt{2},-1)=-2$

Max	min
$(1,\pm\sqrt{2},1)$	$(-1,\pm\sqrt{2},1)$
$(-1,\pm\sqrt{2},-1)$	$(1,\pm\sqrt{2},1)$

Ejercicio 31 #48

$x+y+z=1 \quad (2,0,-3)$

$d=\sqrt{(x-2)^2+y^2+(z+3)^2}$

$d^2=(x-2)^2+y^2+(z+3)^2$

$f_x=2x-4 \quad g_x=1 \quad \lambda=2x-4$

$f_y=2y \quad g_y=1 \quad \lambda=2y$

$f_z=2z+6 \quad g_z=1 \quad \lambda=2z+6$

$d=\sqrt{(\frac{8}{3}-2)^2+(\frac{2}{3})^2+(-\frac{7}{3}+3)^2}$

$\boxed{d=\frac{2\sqrt{3}}{3}}$

$2x-4=2y=2z+6$

$x-2=y=z+3$

$x+(x-2)+(x-5)=1$

$3x-7=1$

$x=\frac{8}{3}$

$\frac{8}{3}-2=y$

$y=\frac{2}{3}$

$\frac{2}{3}=z+3$

$z=-\frac{7}{3}$

Ejercicio 33 #49

$f_x=2x-8 \quad g_x=2x \quad \lambda 2x=2x-8$

$f_y=2y-4 \quad g_y=2y \quad \lambda 2y=2y-4$

$f_z=2z \quad g_z=-2z \quad -\lambda 2z=2z$

$x=\frac{4}{1-\lambda}$

$y=\frac{2}{1-\lambda}$

$z(1+\lambda)=0$

$x^2+y^2+z^2=0$

$x^2+y^2-0=0$

$x^2+y^2=0$

$2^2+1-0=z^2$

$z^2=5$

$z=\pm\sqrt{5}$

$x=\frac{4}{1-(-1)}=2$

$y=\frac{2}{1-(-1)}=1$

$x=2$

$y=1$

$z=0$

$\boxed{(2,1,\pm\sqrt{5}) \quad a\ (4,2,0)}$

Ejercicio 15 #50

$\int_1^4 \int_0^2 (6x^2y - 2x)\, dy\, dx$

$\int_1^4 [3x^2y^2 - 2xy]_0^2 dx = \int_1^4 12x^2 - 4x\, dx = [4x^3 - 2x^2]_1^4 = \boxed{222}$

Ejercicio 17 #51

$\int_0^1 \int_1^2 (x + e^{-y})\, dx\, dy$

$\int_0^1 [\frac{x^2}{2} + xe^{-y}]_1^2 dy = \int_0^1 \frac{3}{2} - e^{-y} dy = [\frac{3}{2}y - e^{-y}]_0^1 = \boxed{\frac{5}{2} - \frac{1}{e}}$

Ejercicio 19 #52.

$\int_0^2 \int_0^{\pi/2} (x \operatorname{sen} y)\, dy\, dx = \int_0^2 [-x\cos y]_0^{\pi/2} dx = \int_0^2 [x(-\cos \frac{\pi}{2})] - [-x\cos 0]$

$\int_0^2 x\,dx = [\frac{x^2}{2}]_0^2 = \boxed{2}$

Ejercicio 21 #53

$\int_1^4 \int_1^2 (\frac{x}{y} + \frac{y}{x})\, dy\, dx$

$\int_1^4 [x \ln y + \frac{y^2}{2x}]_1^2 dx = \int_1^4 x\ln 2 + \frac{3}{2x}\, dx = [\frac{x^2 \ln 2}{2} + \frac{3}{2}\ln x]_1^4 = \boxed{\frac{21}{2}\ln 2}$

Ejercicio 22 #54

$\int_0^1 \int_0^2 ye^{x-y}\, dx\, dy = \int_0^1 [ye^{x-y}]_0^2 dy = \int_0^1 y(e^{2-y} - e^{-y}) dy = \int_0^1 ye^{-y}(e^2 - 1) dy$

$(e^2-1)\int_0^1 ye^{-y} dy = (e^2-1)[-e^{-y}y - \int -e^{-y} dy]_0^1 = \boxed{e^2 - 2e + \frac{2}{e} - 1}$

Ejercicio #55.

$\int_0^1 \int_1^2 \frac{xe^x}{y}\, dy\, dx$

$\int_0^1 [xe^x(\ln y)]_1^2 dx = \int_0^1 [xe^x \ln 2] - 0 = \ln 2[e^x x - \int e^x dx]_0^1 = \boxed{\ln 2}$

Ejercicio 25 #56

$$\int_0^2 \int_0^{\pi} r\,sen^2\theta\, d\theta\, dr \rightarrow \int_0^2 \int_0^{\pi} r\left(\frac{1-\cos 2\theta}{2}\right) d\theta\, dr$$

$$\int_0^2 \int_0^{\pi} \frac{r}{2}\left(1-\cos 2\theta\right) \rightarrow \int_0^2 \frac{r}{2}\left[\theta - \frac{1}{2} sen 2\theta\right]_0^{\pi} dr$$

$$\int_0^2 \frac{r}{2}\left[\pi - \frac{1}{2} sen 2\pi\right] - [0] dr \rightarrow \int_0^2 \left(\frac{r\pi}{2}\right) dr \rightarrow \left[\frac{\pi r^2}{4}\right]_0^2$$

$$\left[\frac{\pi 2^2}{4}\right] - [0] \rightarrow \frac{\pi 4}{4} = \boxed{\pi}$$

Ejercicio 27 #57

$$\iint_R x\sec^2 y\, dA \qquad R = \{(x,y) \mid 0 \le x \le 2, 0 \le y \le \frac{\pi}{4}\}$$

$$\int_0^2 \int_0^{\frac{\pi}{4}} x\sec^2 y\, dy\, d \rightarrow \int_0^2 x\,[\tan y]_0^{\pi/4} dx \rightarrow \int_0^2 x\left[\tan \frac{\pi}{4}\right] - [\tan 0] dx$$

$$\int_0^2 (x(1) - 0)\, dx \rightarrow [x]_0^2 \rightarrow 2 - 0 = \boxed{2}$$

Ejercicio 31 #58

$$\iint_R x\,sen(x+y)\, dA \qquad R = \{0, \frac{\pi}{6}\} \times \{0, \frac{\pi}{3}\} \rightarrow \int_0^{\frac{\pi}{3}} \int_0^{\frac{\pi}{6}} x\,sen(x+y)\, dx\, dy$$

$u = x \qquad du = dx$

$v = -\cos(x+y) \qquad dv = sen(x+y)\, dx$ $\rightarrow x(-\cos(x+y)) - \int -\cos(x+y)\, dx \rightarrow$

$-x\cos(x+y) + sen(x+y)$

$$\int_0^{\pi/3} \left[-x\cos(x+y) + sen(x+y)\right]_0^{\frac{\pi}{6}} \rightarrow \int_0^{\frac{\pi}{3}} \left[-\frac{\pi}{6}\cos\left(\frac{\pi}{6}+y\right) - sen\left(\frac{\pi}{6}+y\right)\right] -$$

$$- \left[-0\cos(0+y) + sen(x+y)\right]_0^{\pi/6} \rightarrow \int_0^{\pi/3} \left[-\frac{\pi}{6}\cos\left(\frac{\pi}{6}+y\right) - sen\left(\frac{\pi}{6}+y\right) - sen\, y\right) dy$$

$$\left[-\frac{\pi}{6} sen\left(\frac{\pi}{6}+y\right) - \cos\left(\frac{\pi}{6}+y\right) + \cos y\right]_0^{\pi/3}$$

$$\left[-\frac{\pi}{6} sen\left(\frac{\pi}{6}+\frac{\pi}{3}\right) - \cos\left(\frac{\pi}{6}+\frac{\pi}{3}\right) + \cos\frac{\pi}{3}\right] - \left[-\frac{\pi}{6} sen\left(\frac{\pi}{6}+0\right) - \cos\left(\frac{\pi}{6}+0\right) + \cos 0\right]$$

$$\left[-\frac{\pi}{6} + 0 + \frac{1}{2}\right] - \left[-\frac{\pi}{12} - \frac{\sqrt{3}}{2} + 1\right] = -\frac{\pi}{12} + \left(\frac{-1+\sqrt{3}}{2}\right) \qquad \boxed{\frac{\sqrt{3}-1}{2} - \frac{\pi}{12}}$$

Ejercicio 33 #59

$\int_R\int \text{sen}(x-y)\,dA \quad R=\{(x,y)\mid 0\le x\le \frac{\pi}{2}\ y\le \frac{\pi}{2}\}$

$\int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}} \text{sen}(x-y)\,dy\,dx \to$ $u=x-y$ $\quad y=0 \to u=x$

$du=-dy \quad y=\frac{\pi}{2}\to u=x-\frac{\pi}{2}$

$\int_0^{\frac{\pi}{2}}\int^{x-\frac{\pi}{2}} \text{sen}\,u\,du\,dx \to \int_0^{\frac{\pi}{2}} -[-\cos u]_x^{x-\frac{\pi}{2}}\,dx \to$

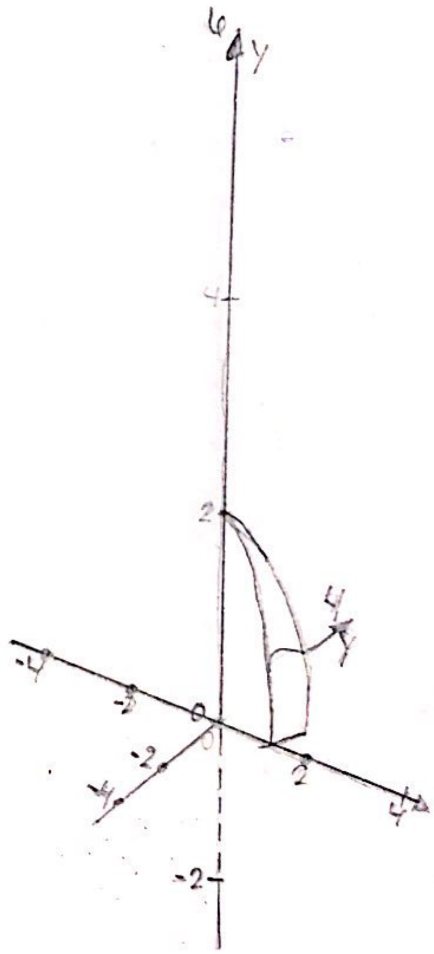
$\int_0^{\frac{\pi}{2}} \cos(x-\frac{\pi}{2})-\cos x\,dx \to \int_0^{\frac{\pi}{2}} \cos x\cos\frac{\pi}{2}+\text{sen}\,x\,\text{sen}\frac{\pi}{2}-\cos x\,dx$

$\int_0^{\frac{\pi}{2}} \text{sen}\,x-\cos x\,dx \to \int_0^{\frac{\pi}{2}}\text{sen}\,x\,dx-\int_0^{\frac{\pi}{2}}\cos x\,dx \to [-\cos x]_0^{\frac{\pi}{2}}-[\text{sen}\,x]_0^{\frac{\pi}{2}} \to$

$[\cos 0-\cos\frac{\pi}{2}]-[\text{sen}\,0-\text{sen}\frac{\pi}{2}] \to 0-(-1)-1=0 \quad \boxed{0}$

Ejercicio 36 #60

$\int_0^1\int_0^1 (2-x^2-y^2)\,dy\,dx \quad R\{(x,4)\mid 0\le x\le 1, 0\le y\le 1\}$



Ejercicio 1 #61

$$\int_1^5 \int_0^x 8x-2y \, dy\, dx$$

$$\int_1^5 (8xy-y^2)_0^x \, dx = \int_1^5 ((8x^2-x^2)-(8x(0)-0^2))dx$$

$$\int_1^5 7x^2 dx = \left(\frac{7x^3}{3}\right)_1^5 = \left(\frac{7(5)^3}{3}\right) - \left(\frac{7(1)^3}{3}\right) = \frac{7(125)}{3} - \frac{7}{3} =$$

$$\frac{875}{3} - \frac{7}{3} = \boxed{\frac{868}{3}}$$

Ejercicio 5 #63

$$\int_0^1 \int_0^y x e^{y^3} dx\, dy$$

$$\int_0^1 e^{-y^3}\left[\frac{x^2}{2}\right]_0^y dy \to \int_0^1 e^{y^3}\left[\frac{y^2}{2}\right] - \left[\frac{0^2}{2}\right] dy \to \int_0 e^{y^3} \frac{y^2}{2} - 0\, dy$$

$$\int_0^1 e^{y^3} \frac{y^2}{2} dy \to u=y^3 \quad \frac{1}{2}\left(\frac{1}{3}\right)\int_0^1 \frac{e^u}{3} du \to \frac{1}{6}[e^u]_0^1 \to \frac{1}{6}[e^1]-[e^0]$$

$$\frac{1}{6}(e-1) = \boxed{\frac{e-1}{6}}$$

Ejercicio 5 # 63

$$\int_0^1 \int_0^{s^2} \cos(s^3)\, dt\, ds \qquad s^3 = x \qquad ds = \frac{dx}{3}$$

$$\int_0^1 [t\cos(s^3)]_0^{s^2} ds = \int_0^1 s^2\cos(s^3)\, ds \quad \int_0^1 a \int_0^1$$

$$= \int_0^1 \cos x \frac{dx}{3} = \frac{1}{3}\int_0^1 \cos dx = \frac{1}{3}(\operatorname{sen} x)_0^1 = \frac{\operatorname{Sen} 1}{3} - \frac{\operatorname{Sen} 0}{3} =$$

$$\frac{\operatorname{Sen} 1}{3} \quad \boxed{\frac{1}{3}\operatorname{sen} 1}$$

Ejercicio 7 #64

$$\iint_D \frac{y}{x^2+1}\,dA \quad D=\{(x,y)\mid 0\le x\le 4,\ 0\le y\le \sqrt{x}\}$$

$$\int_0^4\int_0^{\sqrt{x}} \frac{y}{x^2+1}\,dy\,dx \to \int_0^4 \left[\frac{y^2}{2x^2+2}\right]_0^{\sqrt{x}} dx \to \int_0^4 \left[\frac{(\sqrt{x})^2}{2x^2+2}\right]-\left[\frac{0^2}{2x^2+1}\right]dx$$

$$\int_0^4\int_0^{\sqrt{x}} \frac{y}{2x^2+1}\,dx \to \begin{matrix} u=2x^2+1 \\ du=4x \end{matrix} \to \int_1^{33} \frac{1}{4u}\,du \to \frac{1}{4}\,du \to \frac{1}{4}\left[\ln u\right]_1^{33}$$

$$\to \frac{1}{4}\left[\ln 33\right]-\left[\ln 1\right] \to \frac{\ln 33}{4} - 0 = \boxed{\frac{\ln 33}{4}}$$

Ejercicio 8 #65

$$\iint_D (2x+y)\,dA,\quad D=\{(x,y)\mid 1\le y\le 2,\ y-1\le x\le 1\}$$

$$\int_1^2\int_{y-1}^1 2x+y\,dx\,dy \to \int_1^2 \left[x^2+xy\right]_{y-1}^1 dy \to \int_1^2 \left[1^2+1y\right]-\left[(y-1)^2+(y-1)y\right]dy$$

$$\int_1^2 1+y-y^2+2y-1-y^2+y\,dy \to \int_1^2 4y-2y^2\,dy \to \left[2y^2-\frac{2y^3}{3}\right]_1^2$$

$$\to \left[2(2)^2-\frac{2(2)^3}{3}\right]-\left[2(1)^2-\frac{2(1)^3}{3}\right] \to \left[8-\frac{16}{3}\right]-\left[2-\frac{2}{3}\right] \to$$

$$\frac{8}{3}-\frac{4}{3} = \boxed{\frac{4}{3}}$$

Ejercicio 9 #66

$$\iint_D e^{-y^2}\,dA \quad D=\{(x,y)\mid 0\le y\le 3,\ 0\le x\le y\}$$

$$\int_0^3\int_0^y e^{-y^2}\,dx\,dy \to \int_0^3 \left[xe^{-y^2}\right]_0^y dy \to \int_0^3 \left[ye^{-y^2}\right]-\left[0e^{-y^2}\right]dy$$

$$\int_0^3 ye^{-y^2}\,dy \to \begin{matrix} u=-y^2 \\ du=-2y \end{matrix} \to \int_0^9 -\frac{e^u}{2}\,du \to -\left(-\frac{1}{2}\int_{-9}^0 e^u\,du\right) \to$$

$$\frac{1}{2}\left[e^u\right]_{-9}^0 \to \frac{1}{2}\left[e^0\right]-\left[e^{-9}\right] \to \frac{1}{2}\left(1-\frac{1}{e^9}\right) = \boxed{\frac{e^9-1}{2e^9}}$$