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Ejercicio 16 #1

$\iint_D y^2 e^{xy}\, dA$ $\quad y=x \quad y=4 \quad x=0 \quad D=\{(x,y) \mid 0 \le x \le y,\ 0 \le y \le 4\}$

$\int_0^4 \int_0^y y^2 e^{xy}\, dx\, dy = \int_0^4 y e^{y^2} - y\, dy = \int y e^{y^2} dy - \int y dy = \left[\frac{e^{y^2}}{2} - \frac{y^2}{2}\right]_0^4 = \frac{e^{(4^2)}}{2} - \frac{4^2}{2} - \frac{e^{(0^2)}}{2} - \frac{0^2}{2}$

$= \boxed{\frac{e^{16} - 17}{2}}$

Ejercicio 17 #2

$\iint_D x \cos y\, dA$ $\quad y=0 \quad y=x^2 \quad x=1 \quad D=\{(x,y) \mid 0 \le x \le 1,\ 0 \le y \le x^2\}$

$\int_0^1 \int_0^{x^2} x \cos y\, dy\, dx = \int_0^1 [x \operatorname{sen} y]_0^{x^2} dx = \int_0^1 [x \operatorname{sen} y]_c^{x^2} dx = \int_0^1 x \operatorname{sen} x^2\, dx = \boxed{\frac{1 - \cos 1}{2}}$

Ejercicio 19 #3

$\iint_D y^2\, dA$ $\quad (0,1)(1,2)(4,1) \quad y=1$

para $(0,1)$ y $(1,2)$ $\quad y-1 = \frac{2-1}{1-0}(x-0) \quad y = x+1 \quad x = y-1$

para $(1,2)$ y $(4,1)$ $\quad y-2 = \frac{1-2}{4-1}(x-1) \quad y = \frac{-x}{3} + \frac{7}{3} \quad x = 4-3y$

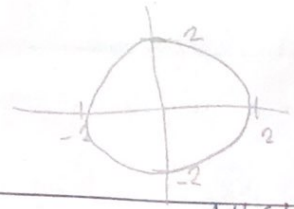
$\iint_D y^2\, dx\, dy = \int_1^2 \int_{y-1}^{7-3y} y^2\, dx\, dy = \int_1^2 (7y^2 - 3y^3 - y^3 + y^2)\, dy = \int_1^2 8y^2 - 4y^3\, dy$

$= \left[\frac{8}{3} y^3 - y^4\right]_1^2 = \left[\frac{64}{3} - 16 - \frac{8}{3} + 1\right] = \boxed{\frac{11}{3}}$

Ejercicio 21 #4

$\iint_D (2x-y)\, dA$ $\quad r=2$ origen

$D = \{(x,y) \mid -2 \le x \le 2,\ -\sqrt{4-x^2} \le y \le \sqrt{4-x^2}\}$

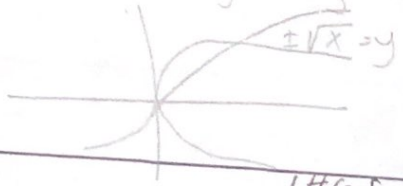


$\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} (2x-y)\, dy\, dx = \int_{-2}^{2} 4x\sqrt{4-x^2}\, dx = \boxed{0}$

Ejercicio 24 #5

$z = 2x + y^2 \quad x = y^2 \quad x = y^3 \quad f(x,y) = 2x + y^2$

$D = \{(x,y) \mid 0 \le x \le 1,\ 0 \le y \le \sqrt{x}\}$



$\iint_D f(x,y)\, dA = \int_0^1 \int_0^{\sqrt{x}} 2x + y^2\, dy\, dx = \int_0^1 \frac{7x\sqrt{x}}{3}\, dx = \boxed{\frac{14}{15}}$

Ejercicio 25 #6

$z = xy \quad (1,1)(4,1)(1,2)$

$\frac{y-2}{1-2} = \frac{x-1}{4-1} \Rightarrow 3(y-2) = x+1$

$x = -3y + 7$

$x = 1$

$D = \{(x,y) \mid 1 \le x \le -3x + 7,\ 1 \le y \le 2\}$

$\int_1^2 \int_1^{-3y+7} xy\, dx\, dy = \int_1^2 \frac{(-3y+7)^2 y}{2} - \frac{1^2 y}{2}\, dy = \int_1^2 \frac{9y^3 - 42y^2 + 49y}{2} - \frac{y}{2}$

$= \frac{1}{2}\int_1^2 9y^3 - 42y^2 + 48y\, dy = 10 - \frac{49}{8} = \boxed{\frac{31}{8}}$

Ejercicio 27 #7

$2x + y + z = 4 \Rightarrow z = 4 - y - 2x \quad (0,0,4);(0,4,0);(2,0,0) \quad (0,0,0)$

$z=0 \quad x=2 \quad y = 4-2x \quad y=0$

$D = \{(x,y,z) \mid 0 \le x \le 2,\ 0 \le y \le 4-2x,\ 0 \le z \le 4-y-2x\}$

$\int_0^2 \int_0^{4-2x} 4 - y - 2x\, dy\, dx = \int_0^2 4(4-2x) - \frac{(4-2x)^2}{2} - 2x(4-2x)\, dx$

$\int_0^2 x^2 - 8x + 8\, dx = \boxed{\frac{16}{3}}$

Ejercicio 28 (#8)

$z = x^2 + 3y^2$ $x=0$ $y=1$ $y=x$ $z=0$

$\int_0^1 \int_x^1 x^2 + 3y^2 \, dy \, dx$

Ejercicio 31 (#9)

$x^2 + y^2 = 1$ $y=2$ $x=0$ $z=0$

$D = \{(x,y) \mid 0 \le x \le 1, 0 \le y \le \sqrt{1-x^2}\}$ $\iint_D f(x,y)\,dA = \int_0^1 \int_0^{\sqrt{1-x^2}} y \, dy \, dx$

$\int_0^1 \frac{1-x^2}{2} \, dx = \frac{x}{2} - \frac{x^3}{6} \Big|_0^1 = \boxed{\frac{1}{3}}$

Ejercicio 39 (#10)

$\int_0^1 \int_0^{1-x} (1-x-y) \, dy \, dx$

$f(x,y) = 1-x-y$

$\boxed{x+y+z=1}$

Ejercicio 45 (#11)

$\int_0^1 \int_x^1 f(x,y) \, dx \, dy = \iint_D f(x,y) \, dA$

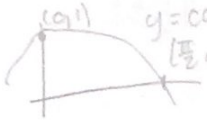
$\boxed{\int_0^1 \int_x^1 f(x,y) \, dy \, dx}$

$D = \{(x,y) \mid 0 \le x \le y, 0 \le y \le 1\}$ $\iint_D f(x,y)\,dA =$

Ejercicio 47 (#12)

$\int_0^{\frac{\pi}{2}} \int_0^{\cos x} f(x,y) \, dy \, dx = \iint_D f(x,y) \, dA \quad D = \{(x,y) \mid 0 \le y \le \cos x, 0 \le x \le \frac{\pi}{2}\}$

$D = \{(x,y) \mid 0 \le x \le \cos^{-1} y, 0 \le y \le 1\}$ $\iint_D f(x,y)\,dA = \boxed{\int_0^1 \int_0^{\cos^{-1} x} f(x,y) \, dx \, dy}$



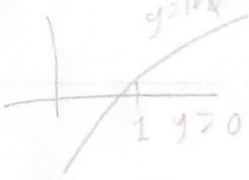
Ejercicio 49 (#13)

$\int_1^2 \int_0^{\ln x} f(x,y) \, dy \, dx \quad R = \{(x,y) \mid 0 \le y \le \ln x, 1 \le x \le 2\}$

$R = \{(x,y) \mid e^y \le x \le 2, 0 \le y \le \ln 2\}$

$\boxed{\int_0^{\ln 2} \int_{e^y}^2 f(x,y) \, dx \, dy}$

$\int_1^2 \int_0^{\ln x} f(x,y) \, dy \, dx$



Ejercicio 51 (#14)

$\int_0^1 \int_{3y}^3 e^{x^2} \, dx \, dy = \iint_D e^{x^2} \, dA \quad D = \{(x,y) \mid 3y \le x \le 3, 0 \le y \le 1\}$ $D = \{(x,y) \mid 0 \le y \le \frac{x}{3}, 0 \le x \le 3\}$

$\iint_D e^{x^2} \, dA = \int_0^3 \int_0^{\frac{x}{3}} e^{x^2} \, dy \, dx = \int_0^3 \frac{x}{3} e^{x^2} \, dx = \frac{e^9}{6} - \frac{e^0}{6} = \boxed{\frac{e^9 - 1}{6}}$

Ejercicio 4 — #15

$\int_0^1 \int_{y/2}^1 y\cos(x^2-1)\,dx\,dy \quad D=\{(x,y) \mid \frac{y}{2} \le x \le 1,\ 0 \le y \le 2\}$

$\int_0^1 \int_0^{2x} y\cos(x^3-1)\,dy\,dx \quad D=\{(x,y) \mid 0 \le y \le 2x,\ 0 \le x \le 1\}$

$= \int_0^1 2x^2\cos(x^3-1)\,dx \Rightarrow \frac{2}{3}\operatorname{sen} 0 - \frac{2}{3}\operatorname{sen}(-1) = \boxed{\frac{2}{3}\operatorname{sen} 1}$

Ejercicio 5c — #16

$\int_0^1 \int_x^1 e^{x/y}\,dy\,dx \quad D=\{(x,y) \mid 0 \le x \le 1,\ x \le y \le 1\}$

$\int_0^1 \int_0^y e^{x/y}\,dx\,dy \quad D=\{(x,y) \mid 0 \le y \le 1,\ 0 \le x \le y\}$

$\int_0^1 y(e-1)\,dy = (e-1)\left[\frac{y^2}{2}\right]_0^1 = \boxed{\frac{e-1}{2}}$

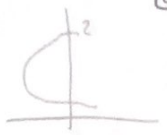
Ejercicio 5 — #17

$\int_{\pi/4}^{3\pi/4} \int_{\frac{1}{2}}^{2} r\,dr\,d\theta = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \left(\frac{1}{2}\right)\left[(2)^2 - \left(\frac{1}{2}\right)^2\right] d\theta = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{3}{2}\,d\theta = \left(\frac{3}{2}\right)\left(\frac{3\pi}{4}\right) - \frac{3}{2}\left(\frac{\pi}{4}\right)$

$= \boxed{\frac{3\pi}{4}}$

Ejercicio 6 — #18

$\int_{\frac{\pi}{2}}^{\pi} \int_0^{2\operatorname{sen}\theta} r\,dr\,d\theta \quad D=\{(r,\theta) \mid 0 \le r \le 2\operatorname{sen}\theta,\ \frac{\pi}{2} \le \theta \le \pi\}$



$r = 2\operatorname{sen}\theta \Rightarrow r^2 = 2r\operatorname{sen}\theta = 2y \Rightarrow x^2+y^2 = 2y \Rightarrow x^2+(y-1)^2 = 1$

$\int_{\frac{\pi}{2}}^{\pi} \int_0^{2\operatorname{sen}\theta} r\,dr\,d\theta = \int_{\pi/2}^{\pi} (2\operatorname{sen}\theta - 0)\,d\theta = \int_{\frac{\pi}{2}}^{\pi} (1-\cos 2\theta)\,d\theta = \theta - \frac{\operatorname{sen} 2\theta}{2}\Big|_{\frac{\pi}{2}}^{\pi} = \boxed{\frac{\pi}{2}}$

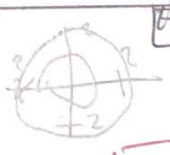
Ejercicio 7 — #19

$\iint_D r^2 y\,dA \quad r=5 \text{ origen} \quad D=\{(r,\theta) \mid 0 \le r \le 5,\ 0 \le \theta \le \pi\} \quad x = r\cos\theta \quad y = r\operatorname{sen}\theta$

$\int_0^{\pi} \int_0^5 (r^2\cos^2\theta)(r\operatorname{sen}\theta)\,r\,dr\,d\theta = \int_0^{\pi} \cos^2\theta\operatorname{sen}\theta\,d\theta \cdot \int_0^5 r^4\,dr = \left[\frac{-\cos^3\theta}{3}\right]_0^{\pi}\left[\frac{r^5}{5}\right]_0^5 = \boxed{\frac{250}{3}}$

Ejercicio 9 — #20

$\iint_R \operatorname{sen}(x^2+y^2)\,dA \quad r=1,3 \quad D=\{(r,\theta) \mid 1 \le r \le 3,\ 0 \le \theta \le \frac{\pi}{2}\}$



$\int_0^{\pi/2} \int_1^3 \operatorname{sen}(r^2)\,r\,dr\,d\theta = \int_0^{\pi/2} d\theta \cdot \int_{-1}^3 \operatorname{sen}(r^2)\,r\,dr\,d\theta = \theta\Big|_0^{\pi/2} \cdot \frac{-\cos r^2}{2}\Big|_1^3 = \boxed{\frac{\pi}{4}(\cos 9 - \cos 1)}$

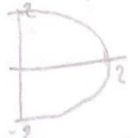
Ejercicio 11 — #21

$\iint_D e^{x^2-y^2}\,dA \quad x=\sqrt{4-y^2} \text{ eje } y \quad -x^2-y^2 = 1^2$

$D=\{(r,\theta) \mid 0 \le r \le 2,\ -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}\}$

$\int_{-\pi/2}^{\pi/2} \int_0^2 e^{-r}\,r\,dr\,d\theta = [\theta]_{-\pi/2}^{\pi/2}\left[\frac{e^{-r}}{e}\right]_0^2$

$= \boxed{\frac{\pi}{2}(1-e^{-4})}$



Ejercicio 14 — #22

$x^2+y^2=4 \quad x^2+y^2=2x \quad D_2=\{(r,\theta) \mid 0 \le \theta \le \frac{\pi}{2},\ 0 \le r \le 2\cos\theta\}$

$D=\{(r,\theta) \mid 0 \le \theta \le \frac{\pi}{2},\ 0 \le r \le 2\} \quad \iint_D x\,dA = \int_0^{\pi/2} \int_0^2 r^2\cos\theta\,dr\,d\theta - \int_0^{\pi/2} \int_0^{2\cos\theta} r^2\cos\theta\,dr\,d\theta$

$= \int_0^{\pi/2} \frac{4}{3}\cos\theta\, d\theta - \int_0^{\pi/2} \frac{3}{3}\cos\theta\, d\theta = \frac{8}{3} - \frac{3}{3}\left[\frac{3}{4}\pi\right] = \boxed{\frac{16-3\pi}{6}}$

Ejercicio 23 #23

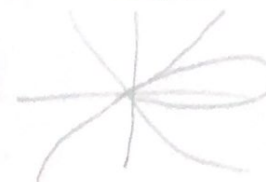
$r = \cos 3\theta$

$\cos 3\theta = 0$

$3\theta = \cos^{-1}(0)$

$\theta = \frac{\cos^{-1} 0}{3} = \theta = -\frac{\pi}{6}, \frac{\pi}{6}$

$D = \{(r,\theta) \mid 0 \le r \le 3\cos\theta, -\frac{\pi}{6} \le \theta \le \frac{\pi}{6}\}$



$\iint_D dA = \int_{-\pi/6}^{\pi/6}\int_0^{3\cos\theta} r\,dr\,d\theta = \int_{-\pi/6}^{\pi/6} \left[\frac{r^2}{2}\right]_0^{3\cos\theta} = \int_{-\pi/6}^{\pi/6} \frac{\cos^2 3\theta}{2} = \boxed{\frac{\pi}{12}}$

Ejercicio 17 #24.

$(x-1)^2+y^2=1 \qquad x^2+y^2=1$

$(x-1)^2+y^2=1$

$(r\cos\theta-1)^2+r^2\operatorname{sen}^2\theta=1$

$(r^2\cos^2\theta-2r\cos\theta+1)+r^2\operatorname{sen}^2\theta=1$

$r^2\cos^2\theta+r^2\operatorname{sen}^2\theta=2r\cos\theta$

$x^2+y^2=2r\cos\theta$

$r^2=2r\cos\theta$

$r=2\cos\theta$

$x^2+y^2=1$

$r^2=\sqrt{1}=1$

$2\cos\theta=1$

$\cos\theta=\frac{1}{2}$

$\theta=\cos^{-1}\frac{1}{2}=-\frac{\pi}{3}, \frac{\pi}{3}$

$D=\{(r,\theta) \mid 1 \le r \le 2\cos\theta, -\frac{\pi}{3} \le \theta \le \frac{\pi}{3}\}$

$\int_{-\pi/3}^{\pi/3}\int_1^{2\cos\theta} r\,dr\,d\theta = \int_{\frac{\pi}{3}}^{2\cos\theta} \cos 2\theta + \frac{d\theta}{2} = \left[\frac{\operatorname{sen}2\theta}{2}+\frac{\theta}{2}\right]_{-\pi/3}^{\pi/3} = \boxed{\frac{\pi}{3}+\frac{\sqrt{3}}{2}}$

Ejercicio 20 #25

$z=\sqrt{x^2+y^2}$

$V=\iint_D \sqrt{x^2+y^2}\,dA$

$D=1 \le r \le 2$

$1 \le x^2+y^2 \le 4$

$1 < r^2 \le 4$

$1 \le r \le 2.$

$\int_0^{2\pi}\int_1^2 r\,dr\,d\theta = \int_0^{2\pi} \frac{7}{3} = \left[\frac{7}{3}\theta\right]_0^{2\pi} = \boxed{\frac{14\pi}{3}}$

Ejercicio 21 #26

$2x+y+z=4$

$z=4-2x-y$

$V=\iint_D 4-2x-y\,dA$

$x^2+y^2 \le 1$

$0 \le r \le 1$

$V=\int_0^{2\pi}\int_0^1 (4-2(r\cos\theta)-r\operatorname{sen}\theta)\,r\,d\theta\,dr = \int_0^{2\pi} 4r-2r^2\cos\theta\, r^2\operatorname{sen}\theta\,dr\,d\theta$

$V=\int_0^{2\pi} 2-\frac{2\cos\theta}{3}-\frac{\operatorname{sen}\theta}{3}\,d\theta = \left[2\theta-\frac{2\operatorname{sen}\theta}{3}+\frac{\cos\theta}{3}\right]_0^{2\pi} = \boxed{4\pi}$

Ejercicio 23 27

$x^2+y^2+z^2=a^2$

$z^2=a^2(x^2+y^2)$

$x^2=a^2-r^2$

$z=\sqrt{a^2-r^2}$

$(0,2\pi)(0,a)$

$f(r,\theta)=\sqrt{a^2-r^2}-(-\sqrt{a^2-r^2})$

$f(r,\theta)=2\sqrt{a^2-r^2}$

$\int_0^{2\pi}\int_0^a (2\sqrt{a^2-r^2})\,r\,dr\,d\theta = \int_0^{2\pi} \frac{2}{3}a^3\,d\theta = \left[\frac{2}{3}a^3 2\right]_0^{2\pi} = \boxed{\frac{4\pi}{3}a^3}$

Ejercicio 23 #28

$z = \sqrt{x^2+y^2}$ $\quad x^2+y^2+z^2 = 1$

$z^2 = 1-(x^2+y^2)$ $\quad f(r,\theta) = \sqrt{1-r^2} - r$ $\quad \int_0^{2\pi}\int_0^{\frac{\sqrt{2}}{2}} (\sqrt{1-r^2}-r)\, r\, dr\, d\theta = \int_0^{2\pi} \frac{-\sqrt{2}}{6} + \frac{1}{3} =$

$z^2 = 1-r$

$z = \sqrt{1-r}$ $\quad r = r$

$z = \sqrt{x^2+y^2}$ $\quad r = r\sqrt{1+r^2}$

$z = \sqrt{1-r^2} = r$ $\quad r^2 = 1-r^2$

$r = \sqrt{\frac{1}{2}}$

$\boxed{\frac{-\sqrt{2}\,\pi + 2\pi}{3}}$

Ejercicio 27 #29

$x^2+y^2 = 4$ $\quad 4x^2+4y^2+z = 64$

$z^2 = 64-(4x^2+4y^2)$ $\quad \rightarrow z = \pm 2\sqrt{16-r^2}$

$z^2 = 64-4r^2$

$z = \sqrt{64-4r^2}$ $\quad f(r,\theta) = 2\sqrt{16-r^2} - (-2\sqrt{16-r^2})$

$f(r,\theta) = 4\sqrt{16-r^2}$

$\int_0^{2\pi}\int_0^2 (4\sqrt{16-r^2})\, r\, dr\, d\theta = \int_0^{2\pi} -32\sqrt{3} + \frac{256}{3}$

$\int_0^{2\pi} -32\sqrt{3} + \frac{256}{3} = \boxed{-64\sqrt{3}\,\pi + \frac{512\pi}{3}}$

Ejercicio 5 #30

$\int_0^{\pi/2}\int_0^{y}\int_0^{x} \cos(x+y+z)\, dz\, dx\, dy = \int_0^{\pi/2}\int_0^{y} \operatorname{sen}(2x+y) - \operatorname{sen}(x+y)\, dx\, dy = \int_0^{\pi/2} -\frac{\cos(3y)}{2} + \cos(y) + \cos(2y)\, dy = \frac{1}{3}$

Ejercicio 7 #31

$\int_0^{\pi}\int_0^{1}\int_0^{\sqrt{1-z^2}} z\operatorname{sen}x\, dy\, dz\, dx = \int_0^{\pi}\int_0^{1} z\operatorname{sen}x\sqrt{1-z^2}\, dz\, dx = \int_0^{\pi} \frac{\operatorname{sen}x}{3} = \frac{2}{3}$

Ejercicio 13 #32

$\iiint_E 6xy\, dv$ $\quad z = 1+x+y$ $\quad xy$ $\quad y = \sqrt{x}$ $\quad y=0$ $\quad y=1$

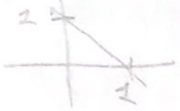
$E = \{(x,y,z) \mid 0 \le z \le 1+x+y,\ 0 \le y \le \sqrt{x},\ 0 \le x \le 1\}$

$\int_0^1\int_0^{\sqrt{x}}\int_0^{1+x+y} 6xy\, dz\, dy\, dx$

$\int_0^1\int_0^{\sqrt{x}} 6xy + 6x^2y + 6xy^2\, dy\, dx = \int_0^1 3x^2 + 3x^3 + 2x^{\frac{7}{2}}\, dx = \frac{63}{28}$

Ejercicio 15 #33

$\iiint_T y^2\, dv$ $\quad (0,0,0)$ $\quad (2,0,0)$ $\quad (0,2,0)$ $\quad (0,0,2)$



$\frac{x}{2} + \frac{y}{2} + \frac{z}{2} = 1 \Rightarrow x+y+z = 2$ $\quad T = \{(x,y,z) \mid 0 \le x \le 2,\ 0 \le y \le 2-x,\ 0 \le z \le 2-x-y\}$

$\int_0^2\int_0^{2-x}\int_0^{2-x-y} y^2\, dz\, dy\, dx = \int_0^2\int_0^{2-x} 2y^2 - xy^2 - y^3\, dy\, dx = \int_0^2 \frac{x^4 - 8x^3 + 24x^2 - 32x + 16}{3} - \frac{(2-x)^2}{4} = \frac{8}{15}$

Ejercicio 17 #34

$\iiint_E x\, dv$ $\quad E \Rightarrow x = 4y^2+4z^2$ $\quad xy$

$D = \{(y,z) \mid y^2+z^2 \le 1\}$

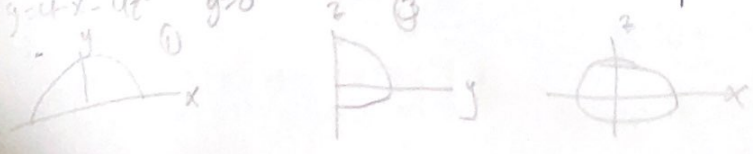
$\iiint_B x\, dv = \iint_D \left[\int_{4y^2+4z^2}^{4} x\, dx\right] = \iint_D \left[\frac{x^2}{2}\right]_{4y^2+4z^2}^{4} =$

$\iint_D 8 - 8(y^2+z^2)^2 = \int_0^{2\pi}\int_0^1 [8-8(1)^2]\, r\, dr\, d\theta = \int_0^{2\pi}\int_0^1 (8-8r^4)\, r\, dr\, d\theta = \int_0^{2\pi} \left[4r^2 - \frac{8r^6}{6}, 0\right]_0^1 = \int_0^{2\pi} 4 - \frac{4}{3}\, d\theta$

$\frac{8}{3}\int_0^{2\pi} d\theta = \frac{8}{3}(2\pi) = \frac{16\pi}{3}$

Ejercicio 29 #35

$y = 4-x^2-4z^2$ $\quad y \ge 0$



#30

① $D=\{(x,y)\mid -2\le x\le 2, 0\le y\le 4-x^2\}$ $D_1=\{(x,y)\mid 0\le y\le 4, -\sqrt{y}\le x\le\sqrt{y}\}$ $E_1=\{(x,y,z)\mid (x,y)\in D_1, 4(x,y)\le z\le 4\}$

② $D_2=\{(x,z)\mid -2\le x\le 2, \frac{x^2-4}{2}\le z\le\frac{x^2-4}{4}\}$ $D_2=\{(y,z)\mid -1\le z\le 1, -\sqrt{4z^2+4}\le x\le\sqrt{4z^2+4}\}$ $G_2=\{(x,y,z)\mid (x,y)\in D_2, u(x,z)\le y\le w(x,z)\}$

③ $D_3=\{(y,z)\mid 0\le y\le 4, -\sqrt{\frac{4-y}{4}}\le z\le\sqrt{\frac{4-y}{4}}\}$ $D_3=\{(y,z)\mid -1\le z\le 1, 0\le y\le 4-4z^2\}$ $E_3=\{(x,y,z)\mid (y,z)\in D_3, u(y,z)\le x\le w(y,z)\}$

① $\int_{-2}^{2}\int_{0}^{4-x^2}\int_{\frac{x^2-4}{4}}^{\frac{4-x^2-y}{4}} f(x,y,z)\,dz\,dy\,dx$

② $\int_{0}^{4}\int_{-\sqrt{4y}}^{\sqrt{4y}}\int_{\frac{x^2-4}{4}}^{\frac{4-x^2-y}{4}} f(x,y,z)\,dz\,dx\,dy$

③ $\int_{-2}^{2}\int_{\frac{x^2-4}{4}}^{\frac{4-x^2}{4}}\int_{0}^{4-x^2-4z^2} f(x,y,z)\,dy\,dz\,dx$

④ $\int_{0}^{1}\int_{-\sqrt{4-4z^2}}^{\sqrt{4-4z^2}}\int_{0}^{4-x^2-4z^2} f(x,y,z)\,dy\,dx\,dz$

⑤ $\int_{0}^{4}\int_{-\frac{\sqrt{4y}}{4}}^{\frac{\sqrt{4y}}{4}}\int_{\sqrt{4z^2+4}}^{\sqrt{4-y-4z^2}} f(x,y,z)\,dx\,dz\,dy$

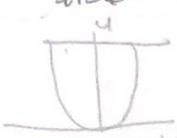
⑥ $\int_{-1}^{1}\int_{0}^{4-4z^2}\int_{\sqrt{4-y-4z^2}}^{\sqrt{4-y-4z^2}} f(x,y,z)\,dx\,dy\,dz$

Ejercicio 31

#30.

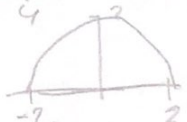
$y=x^2$ $z=0$ $y+2z=4$

$D_1=\{(x,y)\mid -2\le x\le 2, x^2\le y\le 4\}=\{(x,y)\mid 0\le y\le 4, -\sqrt{y}\le x\le\sqrt{y}\}$



$D_2=\{(y,z)\mid 0\le y\le 4, 0\le z\le\frac{4-y}{2}\}$

$D_3=\{(x,z)\mid -2\le x\le 2, 0\le z\le\frac{4-x^2}{2}\}=\{(x,y)\mid 0\le y\le 4, -\sqrt{4-2z}\le x\le\sqrt{4-2z}\}$



$\iiint_E f(x,y,z)\,dV=\int_{-2}^{2}\int_{x^2}^{4}\int_{0}^{2-\frac{y}{2}} f(x,y,z)\,dz\,dy\,dx$ ①

$\int_{0}^{2}\int_{0}^{4-2z}\int_{-\sqrt{y}}^{\sqrt{y}} f(x,y,z)\,dx\,dy\,dz$ ④

$\int_{-\sqrt{4}}^{\sqrt{4}}\int_{0}^{2-\frac{y}{2}} f(x,y,z)\,dz\,dx\,dy$ ②

$\int_{-2}^{2}\int_{0}^{2-\frac{x^2}{2}}\int_{x^2}^{4-2z} f(x,y,z)\,dy\,dz\,dx$ ⑤

$\int_{0}^{4}\int_{0}^{2-y/2}\int_{-\sqrt{y}}^{\sqrt{y}} f(x,y,z)\,dx\,dy\,dz$ ③

$\int_{-2}^{2}\int_{-\sqrt{4-2x}}^{\sqrt{4-2x}}\int_{x^2}^{4-2z} f(x,y,z)\,dy\,dx\,dz$ ⑥

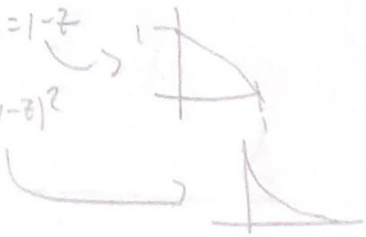
Ejercicio 33

#37

1) $\int_{0}^{1}\int_{\sqrt{x}}^{1}\int_{0}^{1-y} f(x,y,z)\,dz\,dy\,dx$

$x=0$ $z=1-y$ $y=1-z$

$y=0$ $z=1-\sqrt{x}$, $x=(1-z)^2$



2) $\int_{0}^{1}\int_{0}^{y^2}\int_{0}^{1-y} f(x,y,z)\,dz\,dx\,dy$

4) $\int_{0}^{1}\int_{0}^{1-y}\int_{0}^{y^2} f(x,y,z)\,dx\,dz\,dy$

3) $\int_{0}^{1}\int_{0}^{1-z}\int_{0}^{y^2} f(x,y,z)\,dx\,dy\,dz$

5) $\int_{0}^{1}\int_{0}^{(1-z)^2}\int_{\sqrt{x}}^{1-z} f(x,y,z)\,dy\,dx\,dz$

6) $\int_{0}^{1}\int_{0}^{1-\sqrt{x}}\int_{\sqrt{x}}^{1-z} f(x,y,z)\,dy\,dz\,dx$

Ejercicio 14 #38

a) $x^2-x+y^2+z=1$ $\boxed{r^2-r\cos\theta+z^2=1}$

$(x^2+y^2)-x+z^2=1$

b) $z=x^2-y$

$z=(r\cos\theta)^2-(r\,\text{sen}\,\theta)^2=r^2\cos^2\theta-r^2\text{sen}^2\theta$

$z=r^2(\cos^2\theta-\text{sen}^2\theta)=\boxed{r^2\cos(2\theta)}$

Ejercicio 15 #39

$\int_{-\frac{\pi}{2}}^{\pi/2}\int_0^2\int_0^{r^2} r\,dz\,dr\,d\theta=\int_{-\frac{\pi}{2}}^{\pi/2}\int_0^2 r^3\,dr\,d\theta=\int_{-\frac{\pi}{2}}^{\pi/2}4=\boxed{4\pi}$

$z=x^2+y^2$

$z=4$

$0<z<r^2$

$0<z<x^2+y^2$

$-\frac{\pi}{2}\le\theta<\frac{\pi}{2}$

Ejercicio 16. #40.

$\int_0^2\int_0^{2\pi}\int_0^r r\,dz\,d\theta\,dr$ $r=2$ $0\le z\le 2$ $z=r$

$\int_0^2\int_0^{2\pi}\int_0^r r\,dz\,d\theta\,dr=\int_0^2\int_0^{2\pi} zr\,d\theta\,dr=\int_0^2 r^2\theta\Big|_0^{2\pi}=\int_0^2 2\pi r^2=\frac{2}{3}\pi r^3=\boxed{\frac{16}{3}\pi}$

Ejercicio 17 #41

$\iiint_E\sqrt{x^2+y^2}\,dV$ $x^2+y^2=16$ $z=-5$ $z=4$

$x^2+y^2=6$ $0\le\theta\le 2\pi$

$r^2=16$ $-8\le z\le 4$

$r=4$

$0\le r\le 4$

$\int_0^{2\pi}\int_{-5}^4\int_0^4 r^2\,dr\,dz\,d\theta=\int_0^{2\pi}d\theta\int_{-5}^4 dz\int_0^4 r^2\,dr=$

$=\boxed{884\pi}$

Ejercicio 48 #42.

$\iiint_E(x+y+z)\,dr$ $z=4-x^2-y^2$

$z=4-(x^2+y^2)$ $0\le r\le 2$

$z=4-r^2$ $0\le z\le 4-r^2$

$0=4-r^2$ $0\le\theta\le\frac{\pi}{2}$

$r=2$

$\int_0^{\pi/2}\int_0^2\int_0^{4-r^2}(x+y+z)\,dz\,dr\,d\theta=\int_0^{\pi/2}\int_0^2\int_0^{4-r^2}(r\cos\theta+r\,\text{sen}\,\theta+z)\,dz\,dr\,d\theta=$

$=\int_0^{\pi/2}\int_0^2\int_0^{4-r^2}(r^2\cos\theta+r^2\text{sen}\,\theta+z)\,dz\,dr\,d\theta=\int_0^{\pi/2}\int_0^2 r^2(\cos\theta+\text{sen}\,\theta)(4-r^2)+\frac{r}{2}(4-r^2)^2\,dr\,d\theta$

$\int_0^{\pi/2}\int_0^2(4r^2-r^4)(\cos\theta+\text{sen}\,\theta)+(8r-4r^3+r^5)\,dr\,d\theta=\int_0^{\pi/2}\frac{64}{15}(\cos\theta+\text{sen}\,\theta)+\frac{16}{3}\,d\theta$

$\left[\frac{64}{15}(\text{sen}\,\theta-\cos\theta)+\frac{16}{3}\theta\right]_0^{\pi/2}=\boxed{\frac{128}{5}+\frac{8\pi}{3}}$

Ejercicio 21 #43

$\iiint_E z^2\,dV$ $x^2+y^2=1$ $z=0$ $z^2=4x^2+4y^2$

$x^2+y^2=1$ $z^2=4(x^2+y^2)$

$r^2=1$ $z=\sqrt{4r^2}$

$r=1$ $z=2r$

$0\le r\le 1$ $0\le z\le 2r$

$0\le\theta\le 2\pi$ $0\le r\le 1$

$0\le\theta\le 2\pi$

$\int_0^{2\pi}\int_0^1\int_0^{2r} x^2 r\,dz\,dr\,d\theta=\int_0^{2\pi}\int_0^1\int_0^2 r^2\cos^2\theta\,r\,dz\,dr\,d\theta$

$\int_0^{2\pi}\int_0^1\int_0^2 r^3\cos^2\theta\,dz\,dr\,d\theta=\int_0^{2\pi}\int_0^1 2r^4\cos^2\theta\,dr\,d\theta=\int_0^{2\pi}\frac{2}{5}\cos^2\theta\,d\theta$

$=\frac{2}{10}\left[\theta+\frac{\text{sen}\,2\theta}{2}\right]_0^{2\pi}=\boxed{\frac{2\pi}{5}}$

$0\le\phi\le\pi$ $x^2+y^2=1$

Ejercicio 23 #44

$z = \sqrt{x^2+y^2}$ $\quad x^2+y^2+z^2=2$

$z = \sqrt{x^2+y^2}$ $\quad x^2+y^2+z^2=2$

$z = r$ $\quad z = \sqrt{2-r^2}$

$r = \sqrt{2-r^2}$ $\quad 0 \le z \le \sqrt{2-r^2}$

$r^2 = 2-r^2$ $\quad 0 \le \theta \le 2\pi$

$r^2+r^2 = 2$ $\quad 0 \le r \le 1$

$r = 1$

$$\int_0^{2\pi}\int_0^1\int_r^{\sqrt{2-r^2}} r\,dz\,dr\,d\theta = \int_0^{2\pi}\int_0^1 r\sqrt{2-r^2} - r^2\,dr\,d\theta = \int_r^{2\pi}$$

$$\frac{(2\sqrt{2}-1-1)z}{2} = \int_0^{2\pi}\frac{2}{3}(\sqrt{2}-1)\,d\theta = \frac{2}{3}(\sqrt{2}-1)\theta\Big|_0^{2\pi}$$

$$\boxed{\frac{4\pi}{3}(\sqrt{2}-1)}$$

Ejercicio 25 #45

$z = 24-x^2-y^2$ a)

$z = 2\sqrt{x^2+y^2}$

$0 \le r \le 4$

$0 \le z \le 24-r^2$

$0 \le \theta \le 2\pi$

a) $z = z$ $\quad v = \iiint_E dv$

$24-x^2-y^2 = 2\sqrt{x^2+y^2}$

$24-r^2 = 2r$

$2r+r^2 = 24$

$2r+r^2+1 = 24+1$

$\sqrt{(r+1)^2} = \sqrt{25}$

$r+1 = 5$

$r = 4$

$$v = \int_0^{2\pi}\int_0^4 -r^3-2r^2+24r\,dr\,d\theta$$

$$\int_0^{2\pi} d\theta \int_0^4 -r^3-2r^2+24r\,d\theta$$

$$= [\theta]_0^{2\pi}\left[\frac{r^4}{4} - \frac{2r^3}{3} - 12r^2\right]_0^4$$

$$= [2\pi]\left[-\frac{4^4}{4} - \frac{2(4^3)}{3} + 12(4^2)\right] =$$

$$\boxed{\frac{512\pi}{3}}$$

b) $2\sqrt{x^2+y^2} = 24-x^2-y^2$

$2r = 24-r^2$

$r^2+2r = 24$

$r^2+2r+1 = 25$

$\sqrt{(r+1)^2} = \sqrt{25}$

$r+1 = 5$

$r = 4$

$$z = \frac{1}{v}\int_0^{2\pi}\int_0^4 \frac{(24-r^2)^2 r}{2} - \frac{(2r)^2 r}{2}\,dr\,d\theta$$

$$z = \frac{1}{v}\int_0^{2\pi}\int_0^4 \frac{576r - 48r^3 + r^5}{2} - \frac{4r^3}{2}$$

$$\frac{1}{2v}\int_0^{2\pi}\int_0^4 576r - 52r^3 + r^5\,dr\,d\theta =$$

$$\frac{1}{2v}\int_0^{2\pi} 288(4^2) - 13(4^4) + \frac{4^6}{6}\,d\theta =$$

$v = \frac{512\pi}{3}$ $\quad \bar{z} = \frac{588\pi}{3\left(\frac{512\pi}{3}\right)} = \frac{25}{2}$

$$\boxed{(0,0,\tfrac{25}{2})}$$

Ejercicio 24 #46

$$\int_{-2}^{2}\int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}}\int_{\sqrt{x^2+y^2}}^{2} x\,z\,dz\,dx\,dy$$

$-2 \le y \le 2$ $\quad 0 \le \theta \le 2\pi$

$-\sqrt{4-y^2} \le x \le \sqrt{4-y^2}$ $\quad 0 \le r \le 2$

$z = \sqrt{r^2}$

$z = r$

$r \le z \le 2$

$$\int_0^{2\pi}\int_0^2\int_r^2 r^2\cos\theta\, z\,dz\,dr\,d\theta = \int_0^{2\pi}\int_0^2 \frac{r^2}{2}\cos\theta(4-r^2)\,dr\,d\theta$$

$$\frac{1}{2}\int_0^{2\pi}\cos\theta\,d\theta\int_0^2 4r^2-r^4\,dr = \frac{1}{2}[\operatorname{sen}\theta]_0^{2\pi}\left[\frac{4r^3}{3} - \frac{r^5}{5}\right]_0^2$$

$$= \frac{1}{2}(0-0)\left(\frac{4(2^3)}{3} - \frac{2^5}{5}\right) = \boxed{0}$$

Ejercicio 41 #47

a) $x^2+y^2+z^2=9$

$\rho^2 = 3^2$

$\sqrt{\rho^2} = \sqrt{3^2}$

$\boxed{\rho = 3}$

b) $x^2-y^2-z^2=1$

$(\rho\operatorname{sen}\phi\cos\theta)^2 - (\rho\operatorname{sen}\phi\operatorname{sen}\theta)^2 - (\rho\cos\phi)^2 = 1$

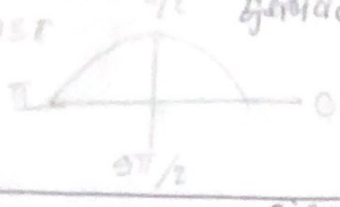
$\rho^2\operatorname{sen}^2\phi\cos^2\theta - \rho^2\operatorname{sen}^2\phi\operatorname{sen}^2\theta - \rho^2\cos^2\theta = 1$

$\rho^2\operatorname{sen}^2\phi(\cos^2\theta - \operatorname{sen}^2\theta) - (\rho\cos\phi)^2 = 1$

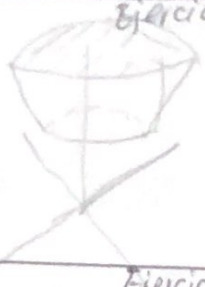
$$\boxed{\rho(\operatorname{sen}^2\phi\cos 2\theta - \cos 2\phi) = 1}$$

Ejercicio 10 #48

Ejercicio 11 #49



Ejercicio 13 #50



Ejercicio 15 #51

Ejercicio 17 #52

Ejercicio 27 #53

Ejercicio 23 #54

Ejercicio 23 #53

$\iiint_E x^2+y^2+z^2 \to E$

$E = \{(\rho,\theta,\phi) \mid 0 \le \rho \le 1,\ 0 \le \theta \le \frac{\pi}{2},\ \rho\cos\phi \le \frac{\pi}{3}\}$

$0 \le \theta \le \frac{\pi}{2}$ $\quad x^2+y^2+z^2 \le 1$

$\rho^2 \le 1$ $\quad 0 \le \phi \le \frac{\pi}{2}$ $\quad \int_0^{\pi/2}\int_0^{\pi/2}\int_0^1 \rho\sin\phi\cos\theta\,\rho^2\sin\phi = \int_0^{\pi/2}\cos\theta\,d\theta \int_0^{\pi/2}\sin\phi\,d\phi \int_0^1 \rho^3\,d\rho$

$\rho \le 1$

$= \int_0^{\pi/2}\cos\theta\,d\theta \int_0^{\pi/3}(1-\cos\phi)\,d\phi \int_0^1 \rho^3\,d\rho = [\sin\theta]_0^{\pi/2}\left[\frac{1}{4}\rho^4\right]_0^1 = [\phi-\sin\phi]_0^{\pi/3}$

$\cdot \left[\frac{1}{4}\rho^4\right]_0^1 - \int_0^1 \rho e^{\rho^2}\,d\rho = \boxed{\frac{\pi}{8}}$

Ejercicio 24 #56

$\rho \le a$ $\quad \phi = \frac{\pi}{6}$ $\quad \phi = \pi/3$

$\frac{\pi}{6} \le \phi \le \frac{\pi}{3}$ $\quad \iiint_E dv = \int_{\pi/6}^{\pi/3}\int_0^{2\pi}\int_0^a \rho^2\sin\phi\,d\rho = \int_{\pi/6}^{\pi/3}\sin\phi\int_0^{2\pi}d\theta\int_0^a \rho^2\,d\rho = [\cos\phi]_{\pi/6}^{\pi/3}[\theta]_0^{2\pi}\left[\frac{\rho^3}{3}\right]_0^a$

$0 \le \rho \le a$

$0 \le \theta \le 2\pi$ $\quad = \left[-\frac{1}{2}+\frac{\sqrt{3}}{2}\right](2\pi)\left[\frac{a^3}{3}\right] = \boxed{\frac{\sqrt{3}-1}{3}\pi a^3}$

Ejercicio #57

a) $\phi = \frac{\pi}{3}$ $\quad \rho = 4\cos\phi$

$0 \le \phi \le \frac{\pi}{3}$ $\quad V = \iiint_E dv = \int_0^{2\pi}\int_0^{\pi/3}\int_0^{4\cos\phi} \rho^2\sin\phi = \int_0^{2\pi}d\theta\int_0^{\pi/3}\sin\phi\left[\frac{\rho^3}{3}\right]_0^{4\cos\phi}$

$0 \le \rho \le 4\cos\phi$

$0 \le \theta \le 2\pi$ $\quad = \frac{128\pi}{3}\int_0^{\pi/3}\sin\phi\cos^3\phi\,d\phi = \frac{128\pi}{3}\left[-\frac{\cos^4\phi}{4}\right]_0^{\pi/3} = \frac{128\pi}{3}\cdot\frac{1}{4}\cdot\frac{5}{16} = \boxed{10\pi}$

Ejercicio 41 #55

$\int_0^1\int_0^{\sqrt{1-x^2}}\int_{\sqrt{x^2+y^2}}^{\sqrt{2-x^2-y^2}} xy\,dz\,dy\,dx$ $\quad 0 \le x \le 1$ $\quad z = \sqrt{2-x^2-y^2}$ $\quad z = \sqrt{x^2+y^2}$

$\int_0^{\pi/4}\int_0^{\pi/2}\int_0^{\sqrt{2}} (\rho\sin\phi\cos\theta)$ $\quad y = 0$ $\quad x^2+y^2 = 1$ $\quad \rho^2 = 2$ $\quad \rho = \sqrt{2}$ $\quad \rho = 0$

$(\rho\cos\phi\,\rho\sin\phi)\rho^2\sin\phi\,d\phi$

$= \int_0^{\pi/4}\int_0^{\pi/2}\int_0^{\sqrt{2}} \rho^4\sin\phi\cos\phi\sin^3\phi = \int_0^{\sqrt{2}}\rho^4 \int_0^{\pi/2}\sin\phi\cos\phi\int_0^{\pi/4}\sin^3\phi = \left[\frac{\rho^5}{5}\right]_0^{\sqrt{2}}\left[\frac{\sin^2\phi}{2}\right]_0^{\pi/2}\left[\frac{\cos^3\phi}{-3}-\cos\phi\right]_0^{\pi/4}$

$\frac{4\sqrt{2}}{5}\cdot\frac{1}{2}\left(\frac{\sqrt{2}}{12}-\frac{\sqrt{2}}{2}-\frac{1}{3}+1\right) = \boxed{\frac{4\sqrt{2}}{15}-\frac{1}{3}}$

Ejercicio 3 #59

$F(x,y) = -\frac{1}{2}\uparrow + (y-x)\uparrow$

$F(x,y) = \langle -\frac{1}{2}, y-x\rangle$

$y = x \quad \langle -\frac{1}{2}, 0\rangle$

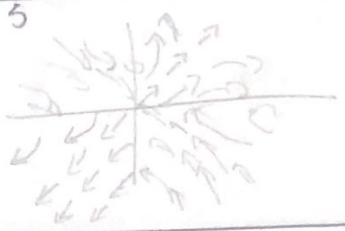
$y-x = \frac{1}{4} \quad \langle -\frac{1}{2}, \frac{1}{4}\rangle$

$y-x = 0$

#60

Ejercicio 5

$F(x,y) = \frac{y\hat{i} + x\hat{j}}{\sqrt{x^2+y^2}} = \left\langle \frac{y}{\sqrt{x^2+y^2}}, \frac{x}{\sqrt{x^2+y^2}} \right\rangle$



#61

Ejercicio 11

$F(x,y) = \langle x, -y \rangle$

x

$x \geq 0$

$x \leq 0$

y

$y > 0 \quad y < 0$

satisface IV.

#62

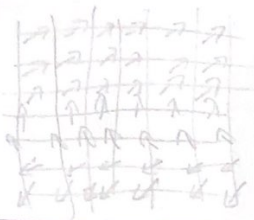
Ejercicio B

$F(x,y) = \langle y, y+2 \rangle$

x

$y = 0$ cbo

$y = 2$ cbe



satisface I

#63

Ejercicio 15

$F(x,y,z) = x\hat{i} + 2y\hat{j} + 3\hat{k}$

x	y	z	F(x,y,z)
1	1	1	⟨1,2,3⟩
0	1	0	⟨1,2,3⟩
1	1	0	⟨1,2,3⟩
2	2	8	⟨1,2,3⟩
2	0	8	⟨1,2,3⟩

satisface IV.

#64

Ejercicio 17

$F(x,y,z) = x\hat{i} + y\hat{j} + 3\hat{k}$

x	y	z	F(x,y,z)
0	0	0	⟨0,0,3⟩
1	1	1	⟨1,1,3⟩
1	1	-1	⟨-1,1,3⟩
1	-1	0	⟨1,-1,3⟩
2	2	2	⟨2,2,3⟩

satisface III

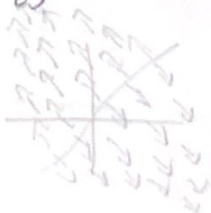
#65

Ejercicio 25

$f(x,y) = \frac{1}{2}(x-y)^2$

$\nabla f = \frac{1}{2} 2(x-y) \frac{\partial}{\partial x}(x-y)\hat{i} + \frac{1}{2} 2(x-y) \frac{\partial}{\partial y}(x-y)\hat{j}$

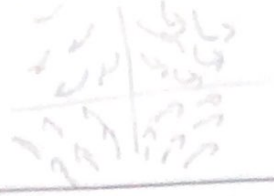
$\nabla f = (x-y)\hat{i} - (x-y)\hat{j}$



Ejercicio 26 #66

$f(x,y) = \frac{1}{2}(x^2+y^2)$

$\nabla f = \frac{1}{2}(2x)\mathbf{i} + \frac{1}{2}(2y)\mathbf{j}$ $\quad \nabla f = x\mathbf{i} + y\mathbf{j}$



Ejercicio 1 #67

$\int_C y\,ds$ c: $x = t^2$ $y = 2t$ $0 \le t \le 3$

$ds = \sqrt{(2t)^2 + 2^2}$ $x'(t) = 2t$

$ds = \sqrt{4t^2+4} = 2\sqrt{t^2+1}$ $y'(t) = 2$

$\int_0^3 2(2t)(2\sqrt{t^2+1})\,dt$

$\int_0^3 4t\sqrt{t^2+1}\,dt = \boxed{\frac{40\sqrt{10}-4}{3}}$

Ejercicio 3 #68

$x^2+y^2=16$ $\int_C xy^4\,ds$

$x = 4\cos t$ $y = 4\sin t$ $\frac{3\pi}{2} \le t \le 2\pi$

$x' = 4\sin t$ $y' = 4\cos t$

$ds = \sqrt{(4\sin t)^2 + (4\cos t)^2} = \sqrt{16\sin^2 t + 16\cos^2 t} = \sqrt{16(\sin^2 t + \cos^2 t)} = \sqrt{16} = 4$

$\int xy^4\,ds = \int_{\frac{3\pi}{2}}^{2\pi} (4\cos t)(4\sin t)^4(4) = \boxed{\frac{8192}{5}}$

Ejercicio 5 #69

$\int_C (x^2y + \sin x)\,dy$ $y = x^2$ $(0,0)$ (π,π^2)

$y = x^2$ $\int_C (x)(x^2) + \sin x)(2x\,dx)$

$dy = 2x\,dx$ $\int_C 2x^3 + 2\sin x\,dx = \int_0^\pi x(2x^4 + 2\sin x)$

$0 \le x \le \pi$ $\left[\frac{x^6}{6} - 2\cos x + 2\sin x\right]_0^\pi = \boxed{\frac{\pi^6}{3} + 2\pi}$

Ejercicio 7 #70

$\int_C (x+2y) + x^2$ $(0,0)$ $(2,1)$ $(2,1)(3,0)$

$y = \frac{x}{2}$ $\quad c_2: y = -1(x-3) = 3-x$ $\quad \int_0^3 (6 - t - t^2) = -\frac{17}{6}$

$x = t$ $y = \frac{t}{2}$ $0 \le t \le 2$ $\quad x = t$ $y = 3-t$ $t=2$ $t=3$

$x' = t$ $y' = \frac{1}{2}$ $\quad x' = 1$ $y' = 1$

$\int_{C_1} (x+2y)(dx + \frac{1}{2}dy) = \int_0^2 (t+2)(1)(\frac{1}{2}) + t^2$ $\quad \int_{C_2} (x+2y) + x^2$

$= \int_0^2 (2t + \frac{t^2}{2}) = \frac{16}{3}$ $\quad \int_2^3 t + 2(3-t) + t^2(-1)$

$\int_C (x+2y)\,dx + x^2 = \frac{16}{3} - \frac{17}{6} = \boxed{\frac{5}{6}}$

Ejercicio 9 #71

$\int_C y^2 y$ $x = \cos t$ $y = \sin t$ $z = t$ $\quad 0 \le t \le \pi/2$

$x' = -\sin t$ $y' = \cos t$ $z' = 1$

$ds = \sqrt{\sin^2 t + \cos^2 t + 1}$ $\quad \int_C x^2 y\,ds = \int_0^{\pi/2} (\cos t)^2(\sin t)\sqrt{2} = \sqrt{2}\int_0^{\pi/2} \cos^2 t \cdot \sin t = -\sqrt{2}\left[\frac{0^3}{3} - \frac{1}{3}\right] = \boxed{\frac{\sqrt{2}}{3}}$

$ds = \sqrt{2}$

Ejercicio 11 #72

$\int_C xe^{yz}\,ds$ $(9,0,0)$ $(1,7,3)$

$d = \langle 1,7,3\rangle$

$r(t) = \langle t, 7t, 3t\rangle$

$x = t \quad y = 7t \quad z = 3t \quad 0 \le t \le 1$

$ds = \sqrt{1^2+7^2+3^2} = \sqrt{59}$

$\int_C xe^{yz}\,ds = \int_0^1 te^{7t^2}\sqrt{59} = \boxed{\frac{\sqrt{59}}{14}\left(e^7-1\right)}$

Ejercicio 15 #73

$\int_C xye^{yz}\,dy \quad C: x=t \quad y=t^2 \quad z=t^3 \quad 0\le t\le 1$

$I = \int_C xye^{yz} = \int_0^1 t(t^2)e^{t^5}2t = 2\int_0^1 t^4e^{t^5} = \frac{2}{5}\int_0^1 e^u\,du = \frac{2}{5}\left[e^u\right]_0^1 = \boxed{\frac{2}{5}[e-1]}$

$u = t^5$

$du = 5t^4\,dt$

$y = t^2$

$dy = 2t\,dt$

Ejercicio 15 #74

$\int_C z^2dx + xy^2 + y^2dz \quad (1,0,0) \quad (4,1,2)$

$v = (4,1,2) - (1,0,0)$

$v = \langle 3,1,2\rangle$

$x = 1+3t \quad y = 0+t \quad z = 2t \quad 0\le t\le 1$

$x' = 3 \quad y' = 1 \quad z' = 2$

$\int_0^1 (2t)^2(3) + (1+3t^2)^2(1) + (t)^2(2) = \int_0^1 (12t^2 + (1+6t+9t^4) + 2t^2 =$

$\int_0^1 23t^2+6t+1 = \left[\frac{23}{3}t^3+3t^2+t\right]_0^1 = \boxed{\frac{35}{3}}$

Ejercicio 23 #75

$\int_C F\cdot dr \quad F(x,y) = \sqrt{x+y}\,\uparrow + \left(\frac{y}{x}\right)\uparrow \quad r(t) = \operatorname{sen}^2t\,\uparrow + \operatorname{sen}t\cos t\,\uparrow \quad \frac{\pi}{6}\le t\le\frac{\pi}{3}$

$r(t) = \operatorname{sen}^2t\,i + \operatorname{sen}t\cos t\,j$

$r'(t) = 2\operatorname{sen}t\cos t\,\uparrow + (\cos^2t - \operatorname{sen}^2t)\uparrow$

$dr = (2\operatorname{sen}t\cos t\,\uparrow + (\cos^2t - \operatorname{sen}^2t)\uparrow$

$F(r(t)) = \sqrt{\operatorname{sen}^2t + \operatorname{sen}t\cos t}\,\uparrow + \frac{\cos t}{\operatorname{sen}t}\uparrow$

$\int_C F\cdot dr = \int_{\pi/6}^{\pi/3}\left(\sqrt{\operatorname{sen}^2t+\operatorname{sen}t\cos t}\right)\uparrow + \frac{\cos t}{\operatorname{sen}t}\uparrow\left(2\operatorname{sen}t\cos t\uparrow + (\cos^2t-\operatorname{sen}^2t)\uparrow\right)$

$\int_{\pi/6}^{\pi/3}(2\operatorname{sen}t\cos t\sqrt{\operatorname{sen}^2t+\operatorname{sen}t\cos t} + \frac{\cos t}{\operatorname{sen}t}\cos^2t - \operatorname{sen}^2t)\,dt = \boxed{0.5424}$

Ejercicio 29 #76

$F(x,y) = x\uparrow + (y+z)\uparrow$

$v(t) = (t-\operatorname{sen}t)\uparrow + (1-\cos t)\uparrow \quad 0\le t\le 2\pi$

$\frac{dv}{dt} = (1-\cos t)\uparrow + \operatorname{sen}t\uparrow \to dr = (1-\cos t)\uparrow + \operatorname{sen}t\uparrow$

$\int_0^{2\pi} \langle t-\operatorname{sen}t, 3-\cos t\rangle\cdot\langle 1-\cos t, \operatorname{sen}t\rangle$

$= \int_0^{2\pi} t - t\cos t - \operatorname{sen}t + \operatorname{sen}t\cos t + 3\operatorname{sen}t - \operatorname{sen}t\cos t$

$\int_0^{2\pi} t - t\cos t + 2\operatorname{sen}t = \left[\frac{t^2}{2} - 2\cos t\right]_0^{2\pi} = \boxed{2\pi^2}$

Ejercicio 3 #77

$F(x,y) = (xy+y^2)\mathbf{i} + (x^2+2xy)\mathbf{j}$

$\frac{\partial P}{\partial y} = x+2y \quad \frac{\partial Q}{\partial x} = 2x+2y$

$x+2y \neq 2x+2y$

No conservativa

Ejercicio 7 #78

$F(x,y) = (ye^x + \operatorname{sen} y)\mathbf{i} + (e^x + x\cos y)\mathbf{j}$

$P \qquad Q$

$\frac{\partial P}{\partial y} = e^x + \cos y \quad \frac{\partial Q}{\partial x} = e^x + \cos y$

$f(x,y) = \int ye^x + \operatorname{sen} y \, dx = ye^x + x\operatorname{sen} y + g(y) \quad \frac{\partial f}{\partial x} = e^x + x\cos y + g'(y) = e^x + x\cos y$

$g'(y) = 0 \Rightarrow g(y) = 0$

$f(x,y) = ye^x + x\operatorname{sen} y + C$

Ejercicio 9 #79.

$F(x,y) = (y^2\cos x + \cos y)\mathbf{i} + (2y\operatorname{sen} x - x\operatorname{sen} y)\mathbf{j}$

$\frac{\partial P}{\partial y} = 2y\cos x - \operatorname{sen} y \quad \frac{\partial Q}{\partial x} = 2y\cos x - \operatorname{sen} y$

$\frac{\partial f}{\partial x} = y^2\cos x + \cos y = y^2 \operatorname{sen} x + x\cos y + g(y)$

$\frac{\partial f}{\partial y} = 2y\operatorname{sen} x - x\operatorname{sen} y + g'(y) = 0 \Rightarrow 2y\operatorname{sen} x - x\operatorname{sen} y + g(y)$

$= y^2\operatorname{sen} x + x\cos y + C.$

Ejercicio 12 #80

$F(x,y) = (3+2xy^2)\mathbf{i} + 2x^2y\mathbf{j} \quad y = \frac{1}{x} \quad (1,1) \quad (4,\frac{1}{4})$

$\frac{\partial P}{\partial y} = 3+2xy^2 = 4xy \quad \frac{\partial Q}{\partial x} = 2x^2y = 4xy$

$f(x,y) = \int 3+2xy^2 \, dx = 3x + x^2y^2 + g(y)$

a) $f = 3x + x^2y^2 + k$

$\frac{\partial f}{\partial y} = 2x^2y + g'(y) = 2x^2y \quad g(y) = 0$

b) $\int_C F \cdot dr = \int_C \nabla f \, dr = f(4,\frac{1}{4}) - f(1,1) = 3(4) + 4^2(\frac{1}{4})^2 + k - 3(1) + (1^2)(1^2) = 9$

Ejercicio 15 #81

$f(x,y,z) = yz\mathbf{i} + xz\mathbf{j} + (xy+2z)\mathbf{k}$ $\quad (1,0,-2) \quad (4,6,5)$

$F_x = yz$

$f(x,y,z) = xyz + g(x,z)$

$f_y = xz \quad g_y(y,z) = 0 \quad g(y,z) = h(z)$

$f(x,y,z) = xyz + h(z)$

$f_z = xy + h'(z)$

$h'(z) = 2z \quad h(z) = z^2 + C$

a) $f(x,y,z) = xyz + z^2$

b) $\int_C F \cdot dr = \int_C \nabla f \, dr = f(4,6,3) - f(1,0,-2) = [4(6)(3) + 3^2 + C)] - [-2 = 47]$

Ejercicio 17 #82

$F(x,y,z) = yze^{xyz}\mathbf{i} + (e^{xyz} + xze^{xyz})\mathbf{j} + xye^{xyz}\mathbf{k}$

$C: r(t) = (t^2+1)\mathbf{i} + (t^2-1)\mathbf{j} + (t^2-2t)\mathbf{k} \quad 0 \le t \le 2$

$F(x,y,z) = yze^{xyz}\mathbf{i} + e^{xyz}\mathbf{j} + xye^{xyz}\mathbf{k}$

$\int M(x) = \int yze^{xyz} \, dx = ye^{xyz}$

$= \int e^{xz} \, dy = ye^{xyz}$

$\int xye^{xyz} = ye^{xyz}$

$F(r(t)) = (t^2-1)e^{(t^2+1)(t^2-2t)} + C$

$F \, dr = [(2^2-1)e^{(2^2+1)(2^2-2(2))} + C] - [(1)e^{(1)(-1)} + C = 3+1 = 4]$