Pag. 1008 - 1009

Ejercicio 16 — #1

$\int_D y^2 e^{xy}\, dA$ $y=x$ $y=4$ $x=0$ $D=\{(x,y) \mid 0 \le x \le y,\ 0 \le y \le 4\}$

$\int_0^4 \int_0^y y^2 e^{xy}\, dx\, dy = \int_0^4 y e^{y^2} - y\, dy = \int y e^{y^2} dy - \int y\, dy = \left[\frac{e^{y^2} - y^2}{2}\right]_0^4 = \frac{e^{(4^2)} - 4^2}{2} - \frac{e^{(0^2)} - 0^2}{2}$

$= \boxed{\frac{e^{16} - 17}{2}}$

Ejercicio 17 — #2

$\iint_D x \cos y\, dA$ $y=0$ $y=x^2$ $x=1$ $D=\{(x,y) \mid 0 \le x \le 1,\ 0 \le y \le x^2\}$

$\int_0^1 \int_0^{x^2} x \cos y\, dy\, dx = \int_0^1 [x \operatorname{sen} y]_0^{x^2} dx = \int_0^1 [x \operatorname{sen} y]_0^{x^2} dx = \int_0^1 x \operatorname{sen} x^2\, dx = \boxed{\frac{1-\cos 1}{2}}$

Ejercicio 19 — #3

$\iint_D y^2\, dA$ $(0,1)(1,2)(4,1)$ $y=1$ para $(0,1)$ y $(1,2)$ $y-1 = \frac{2-1}{1-0}(x-0)$ $y=x+1$

$x = y-1$

para $(1,2)$ y $(4,1)$ $y-2 = \frac{1-2}{4-1}(x-1)$ $y = \frac{-x}{3} + \frac{7}{3}$ $x = 7-3y$

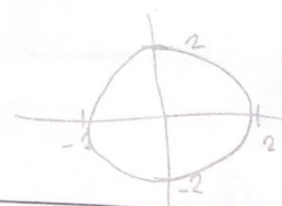
$\iint_D y^2\, dx\, dy = \int_1^2 \int_{y-1}^{7-3y} y^2\, dx\, dy = \int_1^2 (7y^2 - 3y^3 - y^3 + y^2)\, dy = \int_1^2 8y^2 - 4y^3\, dy$

$= \left[\frac{8}{3} y^3 - y^4\right]_1^2 = \left[\frac{64}{3} - 16 - \frac{8}{3} + 1\right] = \boxed{\frac{11}{3}}$

Ejercicio 21 — #4

$\iint_D (2x-y)\, dA$ $r=2$ origen

$D=\{(x,y) \mid -2 \le x \le 2,\ -\sqrt{4-x^2} \le y \le \sqrt{4-x^2}\}$



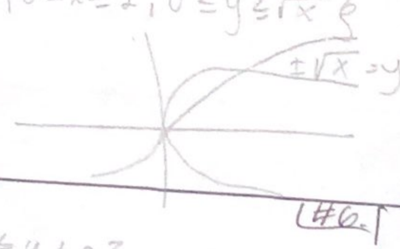
$\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} (2x-y)\, dy\, dx = \int_{-2}^{2} 4x\sqrt{4-x^2}\, dx = \boxed{0}$

Ejercicio 24 — #5

$z = 2x + y^2$ $x=y^2$ $x=y^3$ $f(x,y) = 2x + y^2$

$D=\{(x,y) \mid 0 \le x \le 1,\ 0 \le y \le \sqrt{x}\}$

$\iint_D f(x,y)\, dA = \int_0^1 \int_0^{\sqrt{x}} 2x + y^2\, dy\, dx = \int_0^1 \frac{7x\sqrt{x}}{3}\, dx = \boxed{\frac{14}{15}}$



Ejercicio 25 — #6

$z = xy$ $(1,1)(4,1)(1,2)$ $D=\{(x,y) \mid 1 \le x \le -3x+7,\ 1 \le y \le 2\}$

$\frac{y-2}{1-2} = \frac{x-1}{4-1} \Rightarrow 3(y-2) = -x+1$

$x = -3y+7$

$x = 1$

$\int_1^2 \int_1^{-3y+7} xy\, dx\, dy = \int_1^2 \frac{(-3y+7)^2 y}{2} - \frac{1^2 y}{2}\, dy = \int_1^2 \frac{(9y^2 - 42y + 49)y}{2} - \frac{y}{2}$

$= \frac{1}{2}\int_1^2 9y^3 - 42y^2 + 48y\, dy = 10 - \frac{49}{8} = \boxed{\frac{31}{8}}$

Ejercicio 27 — #7

$2x + y + z = 4 \Rightarrow z = 4 - y - 2x$ $(0,0,0), (0,0,4), (0,4,0), (2,0,0)$ $z=0$ $x=2$ $y=4-2x$ $y=0$

$\int_0^2 \int_0^{4-2x} 4 - y - 2x\, dy\, dx = \int_0^2 4(4-2x) - \frac{(4-2x)^2}{2} - 2x(4-2x)\, dx$ $D=\{(x,y,z) \mid 0 \le x \le 2,\ 0 \le y \le 4-2x,\ 0 \le z \le 4-y-2x\}$ $z = 4 - y - 2x$

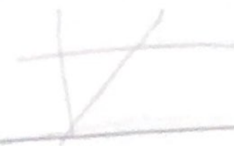
$= \int_0^2 2x^2 - 8x + 8\, dx = \boxed{\frac{16}{3}}$

Ejercicio 48

Ejercicio 28. [#8]

$z = x^2 + 3y^2 \quad x=0 \quad y=1 \quad y=x \quad z=0$

$\int_0^1 \int_x^1 x^2 + 3y^2 \, dy \, dx = \int_0^1 2x^3 + x^2 + 1 \, dx = \boxed{\frac{5}{6}}$



Ejercicio 31 [#9]

$x^2 + y^2 = 1 \quad y = z \quad x = 0 \quad z = 0$

$D = \{(x,y) \mid 0 \le x \le 1, 0 \le y \le \sqrt{1-x^2}\}$ $\iint_D f(x,y)\,dA = \int_0^1 \int_0^{\sqrt{1-x^2}} y \, dy \, dx$

$\int_0^1 \frac{1-x^2}{2} dx = \frac{x}{2} - \frac{x^3}{6} \Big]_0^1 = \boxed{\frac{1}{3}}$

Ejercicio 39 [#10]

$\int_0^1 \int_0^{1-x} (1-x-y) \, dy \, dx$

$f(x,y) = 1-x-y$ $\boxed{x+y+z=1}$

Ejercicio 45 [#11]

$\int_0^1 \int_0^y f(x,y) \, dx \, dy = \iint_D f(x,y)\,dA$

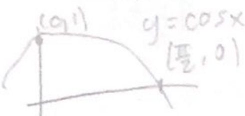
$D = \{(x,y) \mid 0 \le x \le y, 0 \le y \le 1\}$ $\iint_D f(x,y)\,dA =$

$\boxed{\int_0^1 \int_x^1 f(x,y) \, dy \, dx}$

Ejercicio 47 [#12]

$\int_0^{\frac{\pi}{2}} \int_0^{\cos x} f(x,y) \, dy \, dx = \iint_D f(x,y)\,dA \quad D = \{(x,y) \mid 0 \le y \le \cos x, 0 \le x \le \frac{\pi}{2}\}$

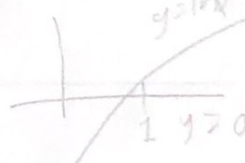
$D = \{(x,y) \mid 0 \le x \le \cos^{-1} y, 0 \le y \le 1\}$ $\iint_D f(x,y)\,dA = \boxed{\int_0^1 \int_0^{\cos^{-1} x} f(x,y) \, dx \, dy}$



Ejercicio 49 [#13]

$\int_1^2 \int_0^{\ln x} f(x,y) \, dy \, dx \quad R \{(x,y) \mid 0 \le y \le \ln x, 1 \le x \le 2\}$

$R = \{(x,y) \mid e^y \le x \le 2, 0 \le y \le \ln 2\}$ $\int_1^2 \int_0^{\ln x} f(x,y) \, dy \, dx$



$\boxed{\int_0^{\ln 2} \int_{e^y}^2 f(x,y) \, dx \, dy}$

Ejercicio 54 [#14]

$\int_0^1 \int_{3y}^3 e^{x^2} dx \, dy = \iint_D e^{x^2} dA \quad D = \{(x,y) \mid 3y \le x \le 3, 0 \le y \le 1\} \quad D = \{(x,y) \mid 0 \le y \le \frac{x}{3}, 0 \le x \le 3\}$

$\iint_D e^{x^2} dA = \int_0^3 \int_0^{\frac{x}{3}} e^{x^2} dy \, dx = \int_0^3 \frac{x}{3} e^{x^2} dx = \frac{e^9}{6} - \frac{e^0}{6} = \boxed{\frac{e^9 - 1}{6}}$

Ejercicio 44 [#15]

$\int_0^2 \int_{\frac{y}{2}}^1 y\cos(x^3-1)\,dx\,dy \qquad D=\{(x,y) \mid \frac{y}{2} \le x \le 1,\ 0 \le y \le 2\}$

$\int_0^1 \int_0^{2x} y\cos(x^3-1)\,dy\,dx \qquad D=\{(x,y) \mid 0 \le y \le 2x,\ 0 \le x \le 1\}$

$= \int_0^1 2x^2\cos(x^3-1)\,dx = \frac{2}{3}\operatorname{sen} 0 - \frac{2}{3}\operatorname{sen}(-1) = \boxed{\frac{2}{3}\operatorname{sen} 1}$

Ejercicio 56 [#16]

$\int_0^1 \int_x^1 e^{x/y}\,dy\,dx \qquad D=\{(x,y) \mid 0 \le x \le 1,\ x \le y \le 1\}$

$\int_0^1 \int_0^y e^{x/y}\,dx\,dy \qquad D=\{(x,y) \mid 0 \le y \le 1,\ 0 \le x \le y\}$

$\int_0^1 y(e-1)\,dy = (e-1)\left[\frac{y^2}{2}\right]_0^1 = \boxed{\frac{e-1}{2}}$

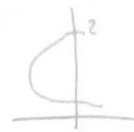
Ejercicio 5 [#17]

$\int_{\pi/4}^{3\pi/4} \int_1^2 r\,dr\,d\theta = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} (\tfrac{1}{2})(2^2) - (\tfrac{1}{2})(1^2)\,d\theta = \int_{\frac{\pi}{4}}^{3\pi/4} \frac{3}{2}\,d\theta = (\tfrac{3}{2})(\tfrac{3\pi}{4}) - \tfrac{3}{2}(\tfrac{\pi}{4})$

$= \boxed{\frac{3\pi}{4}}$

Ejercicio 6 [#18]

$\int_{\pi/2}^{\pi} \int_0^{2\operatorname{sen}\theta} r\,dr\,d\theta \qquad D=\{(r,\theta) \mid 0 \le r \le 2\operatorname{sen}\theta,\ \frac{\pi}{2} \le \theta \le \pi\}$



$r = 2\operatorname{sen}\theta \Rightarrow r^2 = 2r\operatorname{sen}\theta = 2y \Rightarrow x^2+y^2 = 2y \Rightarrow x^2+(y-1)^2 = 1$

$\int_{\frac{\pi}{2}}^{\pi} \int_0^{2\operatorname{sen}\theta} r\,dr\,d\theta = \int_{\pi/2}^{\pi} (2\operatorname{sen}^2\theta - 0)\,d\theta = \int_{\frac{\pi}{2}}^{\pi} (1-\cos 2\theta)\,d\theta = \theta - \frac{\operatorname{sen} 2\theta}{2}\Big|_{\frac{\pi}{2}}^{\pi} = \boxed{\frac{\pi}{2}}$

Ejercicio 7 [#19.]

$\iint_D r^2 y\,dA \qquad r=5$ origen

$D=\{(r,\theta) \mid 0 \le r \le 5,\ 0 \le \theta \le \pi\} \qquad x = r\cos\theta \quad y = r\operatorname{sen}\theta$

$\int_0^{\pi} \int_0^5 (r^2\cos^2\theta)(r\operatorname{sen}\theta)\,r\,dr\,d\theta = \int_0^{\pi} \cos^2\theta \operatorname{sen}\theta\,d\theta \cdot \int_0^5 r^4\,dr = \left[\frac{-\cos^3\theta}{3}\right]_0^{\pi} \left[\frac{r^5}{5}\right]_0^5 = \boxed{\frac{1250}{3}}$

Ejercicio 9 [#20]

$\int\int_R \operatorname{sen}(x^2+y^2)\,dA \qquad r = 1, 3$

$D=\{(r,\theta) \mid 1 \le r \le 3,\ 0 \le \theta \le \frac{\pi}{2}\}$

$\int_0^{\pi/2} \int_1^3 \operatorname{sen}(r^2)\,r\,dr\,d\theta = \int_0^{\pi/2} d\theta \cdot \int_1^3 \operatorname{sen}(r^2)\,r\,dr\,d\theta = \theta\Big|_0^{\pi/2} \cdot \frac{-\cos r^2}{2}\Big|_1^3 = \boxed{-\frac{\pi}{4}(\cos 9 - \cos 1)}$

Ejercicio 11 [#21]

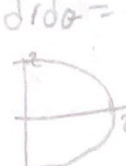
$\iint_D e^{-x^2-y^2}\,dA \qquad x=\sqrt{4-y^2}$ eje y

$\int_{-\pi/2}^{\pi/2} \int_0^2 e^{-r^2} r\,dr\,d\theta = [\theta]_{-\pi/2}^{\pi/2} \left[\frac{-e^{-r^2}}{2}\right]_0^2$

$-x^2-y^2 = r^2$

$D=\{(r,\theta) \mid 0 \le r \le 2,\ -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}\}$

$= \boxed{\frac{\pi}{2}(1-e^{-4})}$



Ejercicio 14 [#22]

$x^2+y^2=4 \qquad x^2+y^2=2x \qquad D_2=\{(r,\theta) \mid 0 \le \theta \le \frac{\pi}{2},\ 0 \le r \le 2\cos\theta\}$

$D=\{(r,\theta) \mid 0 \le \theta \le \frac{\pi}{2},\ 0 \le r \le 2\} \qquad \iint_D x\,dA = \int_0^{\pi/2} \int_0^2 r^2\cos\theta\,dr\,d\theta - \int_0^{\pi/2} \int_0^{2\cos\theta} r^2\cos\theta\,dr\,d\theta$

$= \int_0^{\pi/2} \frac{4}{3}\cos\theta\, d\theta - \int_0^{\pi/2} \frac{2}{3}\cos\theta\, d\theta = \frac{2}{3} - \frac{3}{2}\left[\frac{-3}{4}\pi\right] = \boxed{\frac{16-3\pi}{6}}$

D

D_1

D_2

Ejercicio 23 — #23

$r = \cos 3\theta$

$\cos 3\theta = 0$

$3\theta = \cos^{-1}(0)$

$\theta = \frac{\cos^{-1} 0}{3} = \theta = -\frac{\pi}{6}, \frac{\pi}{6}$

$D = \{(r,\theta) \mid 0 \le r \le 3\cos\theta, -\frac{\pi}{6} \le \theta \le \frac{\pi}{6}\}$

$\iint_D dA = \int_{-\pi/6}^{\pi/6}\int_0^{3\cos 3\theta} r\,dr\,d\theta = \int_{-\pi/6}^{\pi/6}\left[\frac{r^2}{2}\right]_0^{3\cos\theta} = \int_{-\pi/6}^{\pi/6}\frac{\cos^2 3\theta}{2} = \boxed{\frac{\pi}{12}}$

Ejercicio 17 — #24.

$(x-1)^2+y^2=1 \qquad x^2+y^2=1$

$(x-1)^2+y^2=1$

$(r\cos\theta-1)^2+r^2\operatorname{sen}^2\theta=1$

$(r^2\cos^2\theta-2r\cos\theta+1)+r^2\operatorname{sen}^2\theta=1$

$r^2\cos^2\theta+r^2\operatorname{sen}^2\theta=2r\cos\theta$

$x^2+y^2=2r\cos\theta$

$r^2=2r\cos\theta$

$r=2\cos\theta$

$x^2+y^2=1$

$r^2=\sqrt{1}=1$

$2\cos\theta=1$

$\cos\theta=\frac{1}{2}$

$\theta=\cos^{-1}\frac{1}{2}=-\frac{\pi}{3}, \frac{\pi}{3}$

$D=\{(r,\theta) \mid 1 \le r \le 2\cos\theta, -\frac{\pi}{3} \le \theta \le \frac{\pi}{3}\}$

$\int_{-\pi/3}^{\pi/3}\int_1^{2\cos\theta} r\,dr\,d\theta = \int_{-\pi/3}^{\pi/3} \cos 2\theta + \frac{d\theta}{2} = \left[\frac{\operatorname{sen}2\theta}{2}+\frac{\theta}{2}\right]_{-\pi/3}^{\pi/3} = \boxed{\frac{\pi}{3}+\frac{\sqrt{3}}{2}}$

Ejercicio 20 — #25

$z=\sqrt{x^2+y^2}$

$V=\iint_D \sqrt{x^2+y^2}\,dA$

$D = 1 \le r \le 2.$

$1 \le x^2+y^2 \le 4$

$1 < r^2 \le 4$

$1 \le r \le 2.$

$\int_0^{2\pi}\int_1^2 r\,dr\,d\theta = \int_0^{2\pi}\frac{7}{3} = \left[\frac{7}{3}\theta\right]_0^{2\pi} = \boxed{\frac{14\pi}{3}}$

Ejercicio 21 — #26

$2x+y+z=4$

$z=4-2x-y$

$V=\iint_D 4-2x-y\,dA$

$x^2+y^2 \le 1$

$0 \le r \le 1$

$V=\int_0^{2\pi}\int_0^1 (4-2(r\cos\theta)-r\operatorname{sen}\theta)\,r\,d\theta\,dr = \int_0^{2\pi} 4r-2r^2\cos\theta\; r^2\operatorname{sen}\theta\,dr\,d\theta$

$V=\int_0^{2\pi} 2-\frac{2\cos\theta}{3}-\frac{\operatorname{sen}\theta}{3}\,d\theta = \left[2\theta-\frac{2\operatorname{sen}\theta}{3}+\frac{\cos\theta}{3}\right]_0^{2\pi} = \boxed{4\pi}$

Ejercicio 23 — 27

$x^2+y^2+z^2=a^2$

$z^2=a^2-(x^2+y^2)$

$z^2=a^2-r^2$

$z=\sqrt{a^2-r^2}$

$(0,2\pi)(0,a)$

$f(r,\theta)=\sqrt{a^2-r^2}-(-\sqrt{a^2-r^2})$

$f(r,\theta)=2\sqrt{a^2-r^2}$

$\int_0^{2\pi}\int_0^a (2\sqrt{a^2-r^2})\,r\,dr\,d\theta = \int_0^{2\pi}\frac{2}{3}a^3\,d\theta = \left[\frac{2}{3}a^3\theta\right]_0^{2\pi} = \boxed{\frac{4\pi}{3}a^3}$

Ejercicio 25 — #28

$z=\sqrt{x^2+y^2}$ $\quad x^2+y^2+z^2=1$

$z^2=1-(x^2+y^2)$ $\quad f(r,\theta)=\sqrt{1-r^2}-r$ $\quad \int_0^{2\pi}\int_0^{\frac{\sqrt{1}}{2}}(\sqrt{1-r^2}-r)\,r\,dr\,d\theta=\int_0^{2\pi}-\frac{\sqrt{2}}{6}+\frac{1}{3}=$

$z^2=1-r$

$z=\sqrt{1-r}$ $\quad r=r$

$z=\sqrt{r^2+y^2}$ $\quad r=\sqrt{1-r^2}$

$z=\sqrt{r^2}=r$ $\quad r^2=1-r^2$

$r=\sqrt{\frac{1}{2}}$

$$\boxed{\frac{-\sqrt{2}\,\pi+2\pi}{3}}$$

Ejercicio 27 — #29.

$x^2+y^2=4$ $\quad 4x^2+4y^2+z^2=64$

$z^2=64-(4x^2+4y^2) \Rightarrow z=\pm 2\sqrt{16-r^2}$

$z^2=64-4r^2$ $\quad f(r,\theta)=2\sqrt{16-r^2}-(-2\sqrt{16-r^2})$

$z=\sqrt{64-4r^2}$ $\quad f(r,\theta)=4\sqrt{16-r^2}$

$\int_0^{2\pi}\int_0^2(4\sqrt{16-r^2})\,r\,dr\,d\theta=\int_0^{2\pi}-32\sqrt{3}+\frac{256}{3}$

$\int_0^{2\pi}-32\sqrt{3}+\frac{256}{3}=\boxed{-64\sqrt{3}\,\pi+\frac{512\pi}{3}}$

Ejercicio 5 — #30

$\int_0^{\pi/2}\int_0^y\int_0^x \cos(x+y+z)\,dz\,dx\,dy=\int_0^{\pi/2}\int_0^y \operatorname{sen}(2x+y)-\operatorname{sen}(x+y)\,dx\,dy=\int_0^{\pi/2}-\frac{\cos(3y)+\cos(y)}{2}+\cos(2y)\,dy=\boxed{-\frac{1}{3}}$

Ejercicio 7 — #31

$\int_0^{\pi}\int_0^1\int_0^{\sqrt{1-z^2}} z\operatorname{sen}x\,dy\,dz\,dx=\int_0^{\pi}\int_0^1 z\operatorname{sen}x(\sqrt{1-z^2})\,dz\,dx=\int_0^{\pi}\frac{\operatorname{sen}x}{3}=\boxed{\frac{2}{3}}$

Ejercicio 13 — #32

$\iiint_E 6xy\,dv$ $\quad z=1+x+y$ $\quad xy$ $\quad y=\sqrt{x}$, $y=0$, $y=1$

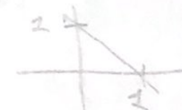
$E=\{(x,y,z)\mid 0\le z\le 1+x+y,\ 0\le y\le\sqrt{x},\ 0\le x\le 1\}$

$6xy\,dv=\int_0^1\int_0^{\sqrt{x}}\int_0^{1+x+y}6xy\,dz\,dy\,dx$

$\int_0^1\int_0^{\sqrt{x}}6xy+6x^2y+6xy^2\,dy\,dx=\int_0^1 3x^2+3x^3+2x^2\sqrt{x}\,dx=\boxed{\frac{65}{28}}$

Ejercicio 15 — #33

$\iiint_T y^2\,dv$ $\quad (0,0,0)\ (2,0,0)\ (0,2,0)\ (0,0,2)$



$\frac{x}{2}+\frac{y}{2}+\frac{z}{2}=1\Rightarrow x+y+z=2$ $\quad T=\{(x,y,z)\mid 0\le x\le 2,\ 0\le y\le 2-x,\ 0\le z\le 2-x-y\}$

$\int_0^2\int_0^{2-x}\int_0^{2-x-y}y^2\,dz\,dy\,dx=\int_0^2\int_0^{2-x}2y^2-xy^2-y^3\,dy\,dx=\int_0^2\frac{x^4-8x^3+24x^2-32x+16}{3}-\frac{(2-x)^4}{4}=\boxed{\frac{8}{15}}$

Ejercicio 17 — #34

$\iiint_E x\,dv$ $\quad E\Rightarrow x=4y^2+4z^2$ $\quad xy$

$D=\{(y,z)\mid y^2+z^2\le 1\}$

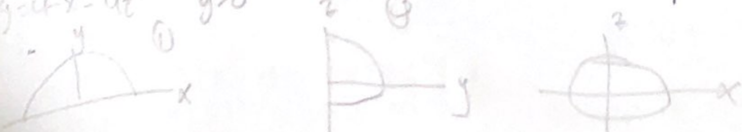
$\iiint_B x\,dv=\iint_D\left[\int_{4y^2+4z^2}^{4}x\,dx\right]=\iint_D\left[\frac{x^2}{2}\right]_{4y^2+4z^2}^{4}=$

$\iint_D 8-8(y^2+z^2)^2=\int_0^{2\pi}\int_0^1[8-8(r^2)^2]\,r\,dr\,d\theta=\int_0^{2\pi}\int_0^1(8-8r^4)\,r\,dr\,d\theta=\int_0^{2\pi}\int_0^1 8r-8r^5\,dr\,d\theta=\int_0^{2\pi}\left[4r^2-\frac{8}{3}r^6\right]_0^1=\int_0^{2\pi}4-\frac{4}{3}d\theta$

$\frac{8}{3}\int_0^{2\pi}d\theta=\frac{8}{3}(2\pi)=\boxed{\frac{16\pi}{3}}$

Ejercicio 29 — #35.

$y=4-x^2-4z^2$ $\quad y\ge 0$



#30

① $D=\{(x,y)\mid -2\le x\le 2, 0\le y\le 4-x^2\}$ $D_1=\{(x,y)\mid 0\le y\le 4, -\sqrt{4-y}\le x\le \sqrt{4-y}\}$ $E_1=\{(x,y,z)\mid (x,y)\in D_1, u_1(x,y)\le z\le u_2(x,y)\}$

② $D_2=\{(x,z)\mid -2\le x\le 2, -\frac{\sqrt{4-x^2}}{2}\le z\le \frac{\sqrt{4-x^2}}{2}\}$ $D_2=\{(x,z)\mid -1\le z\le 1, -\sqrt{4-4z^2}\le x\le \sqrt{4-4z^2}\}$ $E_2=\{(x,y,z)\mid (x,z)\in D_2, u_1(x,z)\le y\le u_2(x,z)\}$

③ $D_3=\{(y,z)\mid 0\le y\le 4, -\sqrt{\frac{4-y}{4}}\le z\le \sqrt{\frac{4-y}{4}}\}$ $D_3=\{(y,z)\mid -1\le z\le 1, 0\le y\le 4-4z^2\}$ $E_3=\{(x,y,z)\mid (y,z)\in D_3, u_1(y,z)\le x\le u_2(y,z)\}$

① $\int_{-2}^{2}\int_0^{4-x^2}\int_{-\frac{\sqrt{4-x^2-y}}{2}}^{\frac{\sqrt{4-x^2-y}}{2}} f(x,y,z)\,dz\,dy\,dx$

④ $\int_{-1}^{1}\int_{-\sqrt{4-4z^2}}^{\sqrt{4-4z^2}}\int_0^{4-x^2-4z^2} f(x,y,z)\,dy\,dx\,dz$

② $\int_0^{1}\int_{-\sqrt{4-y}}^{\sqrt{4-y}}\int_{-\frac{\sqrt{4-x^2-y}}{4}}^{\frac{\sqrt{4-x^2-y}}{4}} f(x,y,z)\,dz\,dx\,dy$

⑤ $\int_0^{4}\int_{-\frac{\sqrt{4-y}}{4}}^{\frac{\sqrt{4-y}}{4}}\int_{-\sqrt{4-y-4z^2}}^{\sqrt{4-y-4z^2}} f(x,y,z)\,dx\,dz\,dy$

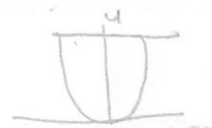
③ $\int_{-2}^{2}\int_{-\frac{\sqrt{4-x^2}}{4}}^{\frac{\sqrt{4-x^2}}{4}}\int_0^{4-x^2-4z^2} f(x,y,z)\,dy\,dz\,dx$

⑥ $\int_{-1}^{1}\int_0^{4-4z^2}\int_{-\sqrt{4-y-4z^2}}^{\sqrt{4-y-4z^2}} f(x,y,z)\,dx\,dy\,dz$

Ejercicio 31

#36.

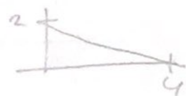
$y=x^2$ $z=0$ $y+2z=4$



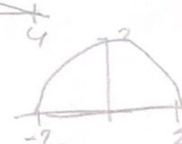
$D_1=\{(x,y)\mid -2\le x\le 2, x^2\le y\le 4\}=\{(x,y)\mid 0\le y\le 4, -\sqrt{y}\le x\le \sqrt{y}\}$

$x=0$ $z=0$ $y=0$ $y+2z=4$

$D_2=\{(y,z)\mid 0\le y\le 4, 0\le z\le 2-\frac{y}{2}\}=\{(y,z)\mid 0\le z\le 2, 0\le y\le 4-2z\}$



$D_3=\{(x,z)\mid -2\le x\le 2, 0\le z\le 2-\frac{x^2}{2}\}=\{(x,z)\mid 0\le z\le 2, -\sqrt{4-2z}\le x\le \sqrt{4-2z}\}$



$\iiint_E f(x,y,z)\,dV=\int_{-2}^{2}\int_{x^2}^{4}\int_0^{2-\frac{y}{2}} f(x,y,z)\,dz\,dy\,dx$ ①

$\int_0^{2}\int_0^{4-2z}\int_{-\sqrt{y}}^{\sqrt{y}} f(x,y,z)\,dx\,dy\,dz$ ④

$\int_0^{4}\int_{-\sqrt{y}}^{\sqrt{y}}\int_0^{2-\frac{y}{2}} f(x,y,z)\,dz\,dx\,dy$ ②

$\int_{-2}^{2}\int_0^{2-\frac{x^2}{2}}\int_{x^2}^{4-2z} f(x,y,z)\,dy\,dz\,dx$ ⑤

$\int_0^{4}\int_0^{2-y/2}\int_{-\sqrt{y}}^{\sqrt{y}} f(x,y,z)\,dx\,dy\,dz$ ③

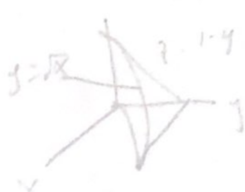
$\int_{-2}^{2}\int_{-\sqrt{4-2x}}^{\sqrt{4-2x}}\int_{x^2}^{4-2z} f(x,y,z)\,dy\,dx\,dz$ ⑥

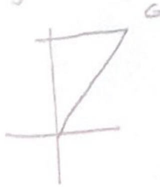
Ejercicio 33

#37

1) $\int_0^1\int_{\sqrt{x}}^{1}\int_0^{1-y} f(x,y,z)\,dz\,dy\,dx$

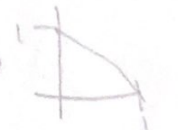
$z=0$ $y=1$ $y=\sqrt{x}$ $x=y^2$

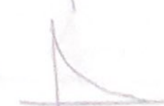




$x=0$ $z=1-y$: $y=1-z$

$y=0$ $z=1-\sqrt{x}$, $x=(1-z)^2$





2) $\int_0^1\int_0^{y^2}\int_0^{1-y} f(x,y,z)\,dz\,dx\,dy$

4) $\int_0^1\int_0^{1-y}\int_0^{y^2} f(x,y,z)\,dx\,dz\,dy$

3) $\int_0^1\int_0^{1-z}\int_0^{y^2} f(x,y,z)\,dx\,dy\,dz$

5) $\int_0^1\int_0^{(1-z)^2}\int_{\sqrt{x}}^{1-z} f(x,y,z)\,dy\,dx\,dz$

6) $\int_0^1\int_0^{1-\sqrt{x}}\int_{\sqrt{x}}^{1-z} f(x,y,z)\,dy\,dz\,dx$

Ejercicio 9 #38

a) $x^2-x+y^2+z^2=1$ $\boxed{r^2-r\cos\theta+z^2=1}$
$(x^2+y^2)-x+z^2=1$

b) $z=x^2-y^2$
$z=(r\cos\theta)^2-(r\,sen\,\theta)^2=r^2\cos^2\theta-r^2sen^2\theta$
$z=r^2(\cos^2\theta-sen^2\theta)=\boxed{r^2\cos(2\theta)}$

Ejercicio 15 #39

$\int_{-\frac{\pi}{2}}^{\pi/2}\int_0^2\int_0^{r^2} r\,dz\,dr\,d\theta=\int_{-\frac{\pi}{2}}^{\pi/2}\int_0^2 r^3\,dr\,d\theta=\int_{-\frac{\pi}{2}}^{\pi/2} 4=\boxed{4\pi}$

$0<z<r^2$
$0<z<x^2+y^2$
$-\frac{\pi}{2}\le\theta<\frac{\pi}{2}$

Ejercicio 16. #40.

$\int_0^2\int_0^{2\pi}\int_0^r r\,dz\,d\theta\,dr$ $r=2$ $0\le z\le 2$ $z=r$

$\int_0^2\int_0^{2\pi}\int_0^r r\,dz\,d\theta\,dr=\int_0^2\int_0^{2\pi} zr\,d\theta\,dr=\int_0^2 r^2\theta\Big|_0^{2\pi}=\int_0^2 2\pi r^2=\frac{2}{3}\pi r^3=\boxed{\frac{16}{3}\pi}$

Ejercicio 17 #41

$\iiint_E\sqrt{x^2+y^2}\,dv$ $x^2+y^2=16$ $z=-5$ $z=4$

$x^2+y^2=16$ $0\le\theta\le 2\pi$
$r^2=16$ $-5\le z\le 4$
$r=4$
$0\le r\le 4$

$\int_0^{2\pi}\int_{-5}^4\int_0^4 r^2\,dr\,dz\,d\theta=\int_0^{2\pi}d\theta\int_{-5}^4 dz\int_0^4 r^2\,dr=$

$=\boxed{384\pi}$

Ejercicio 18 #42.

$\iiint_E(x+y+z)\,dv$ $z=4-x^2-y^2$

$z=4-(x^2+y^2)$ $0\le r\le 2$
$z=4-r^2$ $0\le z\le 4-r^2$
$0=4-r^2$ $0\le\theta\le\frac{\pi}{2}$
$r=2$

$\int_0^{\pi/2}\int_0^2\int_0^{4-r^2}(x+y+z)\,dz\,dr\,d\theta=\int_0^{\pi/2}\int_0^2\int_0^{4-r^2}(r\cos\theta+r\,sen\,\theta+z)r\,dz\,dr\,d\theta=$

$=\int_0^{\pi/2}\int_0^2\int_0^{4-r^2}(r^2\cos\theta+r^2sen\,\theta+zr)\,dz\,dr\,d\theta=\int_0^{\pi/2}\int_0^2 r^2(\cos\theta+sen\,\theta)(4-r^2)+\frac{r}{2}(4-r^2)^2\,dr\,d\theta$

$\int_0^{\pi/2}\int_0^2(4r^2-r^4)(\cos\theta+sen\,\theta)+(8r-4r^3+\frac{r^5}{2})\,dr\,d\theta=\int_0^{\pi/2}\frac{64}{15}(\cos\theta+sen\,\theta)+\frac{16}{3}d\theta$

$\left[\frac{64}{15}(sen\,\theta-\cos\theta)+\frac{16}{3}\theta\right]_0^{\pi/2}=\boxed{\frac{128}{15}+\frac{8\pi}{3}}$

Ejercicio 21 #43

$\iiint_E x^2\,dv$ $x^2+y^2=1$ $z=0$ $z^2=4x^2+4y^2$

$x^2+y^2=1$ $z^2=4(x^2+y^2)$
$r^2=1$ $z=\sqrt{4r^2}$
$r=1$ $z=2r$
$0\le r\le 1$ $0\le z\le 2r$
$0\le\theta\le 2\pi$ $0\le r\le 1$
$0\le\theta\le 2\pi$

$\int_0^{2\pi}\int_0^1\int_0^{2r} x^2 r\,dz\,dr\,d\theta=\int_0^{2\pi}\int_0^1\int_0^{2r} r^2\cos^2\theta\,r\,dz\,dr\,d\theta$

$\int_0^{2\pi}\int_0^1\int_0^{2r} r^3\cos^2\theta\,dz\,dr\,d\theta=\int_0^{2\pi}\int_0^1 2r^4\cos^2\theta\,dr\,d\theta=\int_0^{2\pi}\frac{2}{5}\cos^2\theta$

$=\frac{2}{10}\left[\theta+\frac{sen\,2\theta}{2}\right]_0^{2\pi}=\boxed{\frac{2\pi}{5}}$

$0\le\phi\le\pi$ $x^2+y^2+\ldots$ $5\ldots$ $\ldots 15$

Ejercicio 23 #44

$z = \sqrt{x^2+y^2}$ $\quad x^2+y^2+z^2=2$

$z = \sqrt{x^2+y^2}$ $\quad x^2+y^2+z^2=2$

$z = r$ $\quad z = \sqrt{2-r^2}$

$r = \sqrt{2-r^2}$ $\quad 0 \le z \le \sqrt{2-r^2}$

$r^2 = 2-r^2$ $\quad 0 \le \theta \le 2\pi$

$r^2 + r^2 = 2$ $\quad 0 \le r \le 1$

$r = 1$

$\int_0^{2\pi}\int_0^1\int_r^{\sqrt{2-r^2}} r\,dz\,dr\,d\theta = \int_0^{2\pi}\int_0^1 r\sqrt{2-r^2} - r^2\,dr\,d\theta = \int_0^{2\pi}$

$\frac{(2\sqrt{2}-1-1)}{3} = \int_0^{2\pi}\frac{2}{3}(\sqrt{2}-1)\,d\theta = \frac{2}{3}(\sqrt{2}-1)\theta\Big|_0^{2\pi}$

$\boxed{\frac{4\pi}{3}(\sqrt{2}-1)}$

Ejercicio 25 #45

$z = 24 - x^2 - y^2$ a)

$z = 2\sqrt{x^2+y^2}$

$0 \le r \le 4$

$2r \le z \le 24 - r^2$

$0 \le \theta \le 2\pi$

a) $z = z$ $\quad V = \iiint_E dV$

$24 - x^2 - y^2 = 2\sqrt{x^2+y^2}$

$24 - r^2 = 2r$

$2r + r^2 = 24$

$2r + r^2 + 1 = 24 + 1$

$\sqrt{(r+1)^2} = \sqrt{25}$

$r + 1 = 5$

$r = 4$

$V = \int_0^{2\pi}\int_0^4 -r^3 - 2r^2 + 24r\,dr\,d\theta$

$\int_0^{2\pi} d\theta \int_0^4 -r^3 - 2r^2 + 24r\,dr$

$= [\theta]_0^{2\pi}\left[\frac{r^4}{4} - \frac{2r^3}{3} - 12r^2\right]_0^4$

$= [2\pi]\left[-\frac{4^4}{4} - \frac{2(4^3)}{3} + 12(4^2)\right] =$

$\boxed{\frac{512\pi}{3}.}$

b) $2\sqrt{x^2+y^2} = 24 - x^2 - y^2$

$2r = 24 - r^2$

$r^2 + 2r = 24$

$r^2 + 2r + 1 = 25$

$\sqrt{(r+1)^2} = \sqrt{25}$

$r + 1 = 5$

$r = 4$

$\bar{z} = \frac{1}{V}\int_0^{2\pi}\int_0^4 \frac{(24-r^2)^2 r}{2} - \frac{4r^2}{2} r\,dr\,d\theta$

$\bar{z} = \frac{1}{V}\int_0^{2\pi}\int_0^4 \frac{576r - 48r^3 + r^5}{2} - \frac{4r^3}{2}$

$\frac{1}{2V}\int_0^{2\pi}\int_0^4 576r - 52r^3 + r^5\,dr\,d\theta =$

$\frac{1}{2V}\int_0^{2\pi} 288(4^2) - 13(4^4) + \frac{4^6}{6}\,d\theta =$

$V = \frac{512\pi}{3}$ $\quad \bar{z} = \frac{5888\pi}{3\left(\frac{512\pi}{3}\right)} = \frac{23}{2}$

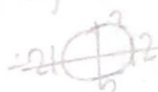
$\boxed{\left(0, 0, \frac{23}{2}\right)}$

Ejercicio 29 #46

$\int_{-2}^{2}\int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}}\int_{\sqrt{x^2+y^2}}^{2} x z\,dz\,dx\,dy$

$-2 \le y \le 2$

$-\sqrt{4-y^2} \le x \le \sqrt{4-y^2}$



$0 \le \theta \le 2\pi$

$0 \le r \le 2$

$z = \sqrt{r^2}$

$z = r$

$r \le z \le 2$

$\int_0^{2\pi}\int_0^2\int_r^2 r^2\cos\theta\, z\,dz\,dr\,d\theta = \int_0^{2\pi}\int_0^2 \frac{r^2}{2}\cos\theta(4-r^2)\,dr\,d\theta$

$\frac{1}{2}\int_0^{2\pi}\cos\theta\,d\theta\int_0^2 4r^2 - r^4\,dr = \frac{1}{2}[\operatorname{sen}\theta]_0^{2\pi}\left[\frac{4r^3}{3} - \frac{r^5}{5}\right]_0^2$

$= \frac{1}{2}(0-0)\left(\frac{4(2^3)}{3} - \frac{2^5}{5}\right) = \boxed{0}$

Ejercicio 41 #47

a) $x^2+y^2+z^2=9$

$\rho^2 = 3^2$

$\sqrt{\rho^2} = \sqrt{3^2}$

$\boxed{\rho = 3.}$

b) $x^2 - y^2 - z^2 = 1$

$(\rho\operatorname{sen}\phi\cos\theta)^2 - (\rho\operatorname{sen}\phi\operatorname{sen}\theta)^2 - (\rho\cos\phi)^2 = 1$

$\rho^2\operatorname{sen}^2\phi\cos^2\theta - \rho^2\operatorname{sen}^2\phi\operatorname{sen}^2\theta - \rho^2\cos^2\phi = 1$

$\rho^2\operatorname{sen}^2\phi(\cos^2\theta - \operatorname{sen}^2\theta) - (\rho\cos\phi)^2 = 1$

$\boxed{\rho(\operatorname{sen}^2\phi\cos 2\theta - \cos^2\phi) = 1}$

Ejercicio 10 #48

$z = x^2 + y^2$

$\rho\cos\phi = \rho^2 sen^2\phi\cos^2\theta + \rho^2 sen^2\phi sen^2\theta$

$\rho\cos\phi = \rho^2 sen^2\phi(\cos^2\theta + sen^2\theta)$

$\rho\cos\phi = \rho^2 sen^2\phi \div \rho sen^2\phi$

$\boxed{\rho = \cot\phi \csc\phi}$

b) $z = x^2 - y^2$

$\rho\cos\phi = \rho^2 sen^2\phi\cos^2\theta - \rho^2 sen^2\phi sen^2\theta$

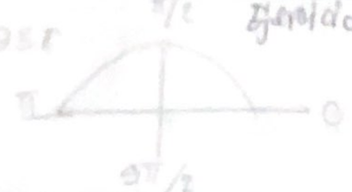
$\rho\cos\phi = \rho^2 sen^2\phi(\cos^2\theta - sen^2\theta)$

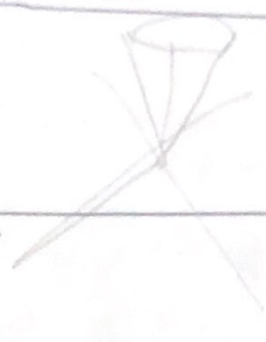
$\rho\cos\phi = \rho^2 sen^2\phi\cos 2\theta \div (\rho sen^2\phi\cos 2\theta)$

$\boxed{\rho = \cot\phi\csc\phi\sec 2\theta}$

Ejercicio 11 #49

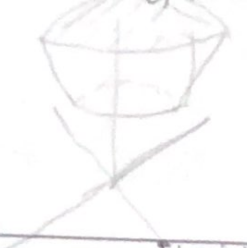
$\rho \leq 1, 0 \leq \phi \leq \frac{\pi}{6}, 0 \leq \theta \leq \pi$





Ejercicio 13 #50

$2 \leq \rho \leq 4 \quad 0 \leq \phi \leq \frac{\pi}{3} \quad 0 \leq \theta \leq \pi$



Ejercicio 15 #51

$z = \sqrt{x^2+y^2} \quad x^2+y^2+z^2 = z$

$z^2 = x^2+y^2$

$2z^2 = x^2+y^2+z^2 = \rho^2$

$z^2 = \rho^2\cos^2\phi$

$\rho^2\cos^2\phi \geq \frac{\rho^2}{2} \Rightarrow \cos^2\phi \geq \frac{1}{2}$

$\cos\phi \geq \frac{1}{\sqrt{2}} \Rightarrow 0 \leq \phi \leq \frac{\pi}{4}$

$\frac{\rho^2}{\rho} = \frac{\rho\cos\phi}{\rho}$

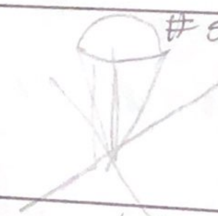
$\rho = \cos\phi \Rightarrow 0 \leq \rho \leq \cos\phi$

$\boxed{0 \leq \rho \leq \cos\phi, \ 0 \leq \phi \leq \frac{\pi}{4}}$

Ejercicio 17 #52

$\int_0^{\pi/6}\int_0^{\pi/2}\int_0^3 \rho^2 sen\phi \, d\rho \, d\phi \, d\theta$

$\int_0^{\pi/6}\int_0^{\pi/2} \frac{3^3}{3} sen\phi) \, d\phi d\theta = \int_0^{\pi/6} (9 \frac{\pi}{2} sen\theta) \, d\theta \ \boxed{\frac{9\pi}{2}\left[\frac{-\sqrt{3}}{2+1}\right]}$



Ejercicio 27 #53

$\iiint_B (x^2+y^2+z^2)^2 dV$

$B = \{(\rho,\theta,\phi) \mid 0 \leq \rho \leq 5, 0 \leq \theta \leq 2\pi, 0 \leq \phi \leq \pi\}$

$\int_0^{\pi}\int_0^{2\pi}\int_0^5 (\rho^2)^2 \rho^2 sen\phi \, d\rho \, d\theta \, d\phi = \int_0^{\pi}\int_0^{2\pi}\int_0^5 \rho^6 sen\phi$

$= \int_0^{\pi} sen\phi \, d\phi \int_0^{2\pi} d\theta \int_0^5 \rho^6 d\rho = [2][2\pi][\frac{5^7}{7}] \ \boxed{\frac{312500\pi}{7}}$

Ejercicio 23 #54

$\iiint_E (x^2+y^2) dV \quad E \ x^2+y^2+z^2 \leq 4 \cdot x^2+y^2+z^2 = 9$

$\rho = 2 \quad \rho = 3 \quad x^2+y^2 = \rho^2 sen^2\phi\cos^2\theta + \rho^2 sen^2\phi sen^2\theta$

$2 \leq \rho \leq 3 \quad x^2+y^2 = \rho^2 sen^2\phi(\cos^2\theta + sen^2\theta)$

$0 \leq \theta \leq 2\pi \quad x^2+y^2 = \rho^2 sen^2\phi$

$0 \leq \phi \leq \pi$

$\int_0^{\pi}\int_0^{2\pi}\int_2^3 (\rho^2 sen^2\phi)\rho^2 sen\phi = \int_0^{\pi}\int_0^{2\pi}\int_2^3 \rho^4 sen^3\phi =$

$\int_0^{\pi}\int_0^{2\pi} [\frac{\rho^5}{5} sen^3\phi]_2^3 = \int_0^{\pi}\int_0^{2\pi} \frac{211}{5} sen^3\phi \, d\theta \, d\phi = \int_0^{\pi} \frac{422\pi}{5} sen^3\phi$

$= \frac{422\pi}{5}\int_0^{\pi} sen\phi(1-\cos^2\phi) = \boxed{\frac{1688\pi}{15}}$

Ejercicio 23 — #53

$\iiint_E e^{(x^2+y^2+z^2)} \to E$

$E = \{(\rho,\theta,\phi) \mid 0 \le \rho \le 1,\ 0 \le \theta \le \frac{\pi}{2},\ 0 \le \phi \le \frac{\pi}{2}\}$

$0 \le \theta \le \frac{\pi}{2}$ $\quad x^2+y^2+z^2 \le 1$

$\rho^2 \le 1$ $\quad 0 \le \phi \le \frac{\pi}{2}$

$\rho \le 1$

$$\int_0^{\pi/2}\int_0^{\pi/2}\int_0^1 \rho\, \text{sen}\,\phi \cos\theta\, e^{\rho^2} \rho^2 \text{sen} = \int_0^{\pi/2} \cos\theta\, d\theta \int_0^{\pi/2} \text{sen}^2\phi \int_0^1 \rho^3 e^{\rho^2}$$

$$= \int_0^{\pi/2} \cos\theta \int_0^{\pi/2} (1-\cos 2\phi)\, d\phi \int_0^1 \rho^3 e^{\rho^2} = [\text{sen}\,\theta]_0^{\pi/2} \left[\tfrac{1}{2}\theta - \tfrac{1}{4}\text{sen}\,2\theta\right]_0^{\pi/2}$$

$$\cdot \left[\tfrac{1}{2}\rho^2 e^{\rho^2}\right]_0^1 - \int_0^1 \rho e^{\rho^2}\, d\rho = \boxed{\frac{\pi}{8}}$$

Ejercicio 27 — #56.

$\rho \le a$ $\quad \phi = \frac{\pi}{6}$ $\quad \phi = \pi/3$

$\frac{\pi}{6} \le \phi \le \frac{\pi}{3}$

$0 \le \rho \le a$

$0 \le \theta \le 2\pi$

$$\iiint_E dv = \int_{\pi/6}^{\pi/3}\int_0^{2\pi}\int_0^a \rho^2 \text{sen}\,\phi\, d\rho = \int_{\pi/6}^{\pi/3} \text{sen}\,\phi \int_0^{2\pi} d\theta \int_0^a \rho^2 d\rho = [\cos\phi]_{\pi/6}^{\pi/3} [\theta]_0^{2\pi} \left[\frac{\rho^3}{3}\right]_0^a$$

$$= \left[-\frac{1}{2}+\frac{\sqrt{3}}{2}\right](2\pi)\left[\frac{a^3}{3}\right] = \boxed{\frac{\sqrt{3}-1}{3}\pi a^3}$$

Ejercicio — #57

a) $\phi = \frac{\pi}{3}$ $\quad \rho = 4\cos\phi$

$0 \le \phi \le \frac{\pi}{3}$

$0 \le \rho \le 4\cos\phi$

$0 \le \theta \le 2\pi$

$$V = \iiint_E dv = \int_0^{2\pi}\int_0^{\pi/3}\int_0^{4\cos\phi} \rho^2 \text{sen}\,\phi = \int_0^{2\pi} d\theta \int_0^{\pi/3} \text{sen}\,\phi \left[\frac{\rho^3}{3}\right]_0^{4\cos\phi}$$

$$= \frac{128\pi}{3}\int_0^{\pi/3} \text{sen}\,\phi \cos^3\phi\, d\phi = \frac{128\pi}{3}\left[-\frac{\cos^4\phi}{4}\right]_0^{\pi/3} = \frac{128\pi}{3} \cdot \frac{1}{4} \cdot \frac{15}{16} = \boxed{10\pi}$$

Ejercicio 41 — #58

$$\int_0^1\int_0^{\sqrt{1-x^2}}\int_{\sqrt{x^2+y^2}}^{\sqrt{2-x^2-y^2}} xy\, dz\, dy\, dx$$

$0 \le x \le 1$

$y = 0$ $\quad z = \sqrt{2-x^2-y^2}$ $\quad z = \sqrt{x^2+y^2}$

$x^2+y^2 = 1$ $\quad \rho^2 = 2$ $\quad \rho = 0$

$r^2 = 1$ $\quad \rho = \sqrt{2}$

$$\int_0^{\pi/4}\int_0^{\pi/2}\int_0^{\sqrt{2}} (\rho\, \text{sen}\,\phi \cos\theta)$$

$(\rho\, \text{sen}\,\phi\, \text{sen}\,\theta)\rho^2 \text{sen}\,\phi$

$$= \int_0^{\pi/4}\int_0^{\pi/2}\int_0^{\sqrt{2}} \rho^4 \text{sen}\,\theta\cos\theta\, \text{sen}^3\phi = \int_0^{\sqrt{2}} \rho^4 \int_0^{\pi/2} \text{sen}\,\theta\cos\theta \int_0^{\pi/4} \text{sen}^3\phi = \left[\frac{\rho^5}{5}\right]_0^{\sqrt{2}} \left[\frac{\text{sen}^2\theta}{2}\right]_0^{\pi/2} \left[\frac{\cos^3\phi}{3} - \cos\phi\right]_0^{\pi/4}$$

$$\frac{4\sqrt{2}}{5} \cdot \frac{1}{2}\left(\frac{\sqrt{2}}{12} - \frac{\sqrt{2}}{2} - \frac{1}{3} + 1\right) = \boxed{\frac{4\sqrt{2}}{15} - \frac{1}{3}}$$

Ejercicio 3 — #59

$F(x,y) = -\frac{1}{2}\hat{\imath} + (y-x)\hat{\jmath}$

$F(x,y) = \langle -\frac{1}{2}, y-x \rangle$

$y = x$ $\langle -\frac{1}{2}, 0 \rangle$

$y - x = \frac{1}{4}$ $\langle -\frac{1}{2}, \frac{1}{4} \rangle$

$y - x = 0$

Ejercicio 5 #60

$F(x,y) = \frac{y\hat{\imath} + x\hat{\jmath}}{\sqrt{x^2+y^2}} = \left\langle \frac{y}{\sqrt{x^2+y^2}}, \frac{x}{\sqrt{x^2+y^2}} \right\rangle$

Ejercicio 11 #61

$F(x,y) = \langle x, -y \rangle$

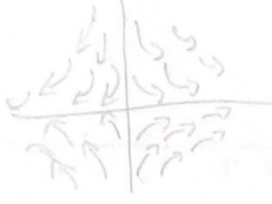
x

$x \geq 0$

$-x \leq 0$

y

$-y > 0 \quad y < 0$



satiface IV

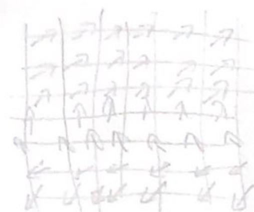
Ejercicio 13 #62

$F(x,y) = \langle y, y+2 \rangle$

x

$y = 0$ cte

$y = 2$ cte



satisface I

Ejercicio 15 #63

$F(x,y,z) = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$

x	y	z	F(x,y,z)
1	1	1	$\langle 1,2,3 \rangle$
0	1	0	$\langle 1,2,3 \rangle$
1	2	0	$\langle 1,2,3 \rangle$
2	2	8	$\langle 1,2,3 \rangle$
2	0	8	$\langle 1,2,3 \rangle$

satisface IV.

Ejercicio 17 #64

$F(x,y,z) = x\hat{\imath} + y\hat{\jmath} + 3\hat{k}$

x	y	z	F(x,y,z)
0	0	0	$\langle 0,0,3 \rangle$
1	1	1	$\langle 1,1,3 \rangle$
-1	1	-1	$\langle -1,1,3 \rangle$
1	-1	0	$\langle 1,-1,3 \rangle$
2	2	2	$\langle 2,2,3 \rangle$

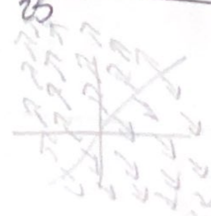
Satisface III

Ejercicio 25 #65.

$f(x,y) = \frac{1}{2}(x-y)^2$

$\nabla f = \frac{1}{2} 2(x-y) \frac{\partial}{\partial x}(x-y)\hat{\imath} + (x-y)\frac{\partial}{\partial y}(x-y)\hat{\jmath}$

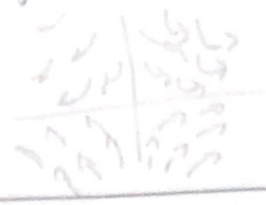
$\boxed{\nabla f = (x-y)\hat{\imath} - (x-y)\hat{\jmath}}$



Ejercicio 2 #66

$f(x,y) = \frac{1}{2}(x^2 - y^2)$

$\nabla f = \frac{1}{2}(2x)\hat{i} + \frac{1}{2}(-2y)\hat{j}$ $\quad \nabla f = x\hat{i} - y\hat{j}$



Ejercicio 1 #67

$\int_c y\,ds \quad c: x = t^2 \quad y = 2t \quad 0 \le t \le 3$

$ds = \sqrt{(2t)^2 + 2^2} \qquad x'(t) = 2t$

$ds = \sqrt{4t^2+4} = 2\sqrt{t^2+1} \qquad y'(t) = 2$

$\int_0^3 (2t)(2\sqrt{t^2+1})$

$\int_0^3 4t\sqrt{t^2+1} = \boxed{\frac{40\sqrt{10}-4}{3}}$

Ejercicio 3 #68

$x^2 + y^2 = 16 \qquad \int_c xy^4\,ds$

$x = 4\cos t \quad y = 4\operatorname{sen} t \quad \frac{-\pi}{2} \le t \le \frac{\pi}{2}$

$x' = -4\operatorname{sen} t \quad y' = 4\cos t$

$ds = \sqrt{(-4\operatorname{sen} t)^2 + (4\cos t)^2} = \sqrt{16\operatorname{sen}^2 t + 16\cos^2 t} = \sqrt{16(\operatorname{sen}^2 t + \cos^2 t)} = \sqrt{16} = 4$

$\int_c xy^4\,ds = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (4\cos t)(4\operatorname{sen} t)^4 (4) = \boxed{\frac{8192}{5}}$

Ejercicio 5 #69

$\int_c (x^2y + \operatorname{sen} x) \cdot y = x^2 \quad (0,0) \quad a\,(\pi, \pi^2)$

$y = x^2 \qquad \int_c (x^2(x^2) + \operatorname{sen} x)(2x\,dx)$

$dy = 2x\,dx$

$0 \le x \le \pi \qquad \int_c 2x^5 + 2x\operatorname{sen} x\,dx = \int_0^\pi x(2x^4 + 2\operatorname{sen} x)$

$\left[\frac{x^6}{3} - 2x\cos x + 2\operatorname{sen} x\right]_0^\pi = \boxed{\frac{\pi^6}{3} + 2\pi}$

Ejercicio 7 #70

$\int_c (x+2y) + x^2 \qquad (0,0)\ (2,1) \quad (2,1)\ (3,0)$

$y = \frac{x}{2}$

$x = t \quad y = \frac{t}{2} \quad 0 \le t \le 2$

$x' = 1 \quad y' = \frac{1}{2}$

$\int_{c_1} (x+2y)\,dx + x^2\,dy = \int_0^2 (t+2)(\frac{t}{2}) + \frac{t^2}{2}$

$= \int_0^2 (2t + \frac{t^2}{2}) = \frac{16}{3}$

$c_2: y = -1(x-3) = 3-x$

$x = t \quad y = 3-t \quad t=2 \quad t=3$

$x' = 1 \quad y' = -1$

$\int_{c_2} (x+2y) + x^2$

$\int_2^3 (t+2(3-t)) + t^2(-1)$

$\int_2^3 (6 - t - t^2) = -\frac{17}{6}$

$\int_c (x+2y)\,dx + x^2 = \frac{16}{3} - \frac{17}{6} = \boxed{\frac{5}{2}}$

Ejercicio 9 #71

$\int_c x^2 y \quad x = \cos t \quad y = \operatorname{sen} t \quad z = t$

$x' = -\operatorname{sen} t \quad y' = \cos t \quad z' = 1 \quad 0 \le t \le \pi/2$

$ds = \sqrt{\operatorname{sen}^2 t + \cos^2 t + 1}$

$ds = \sqrt{2}$

$\int_c x^2 y\,ds = \int_0^{\pi/2} (\cos t)^2 (\operatorname{sen} t)\sqrt{2} = \sqrt{2}\int_0^{\pi/2} \cos^2 t \cdot \operatorname{sen} t = -\sqrt{2}\left[0 - \frac{1}{3}\right] = \boxed{\frac{\sqrt{2}}{3}}$

Ejercicio 11 #72

$\int_C xe^{yz}\,ds$ $(0,0,0)$ $(1,2,3)$

$d = \langle 1,2,3\rangle$

$r(t) = \langle t, 2t, 3t\rangle$

$x=t$ $y=2t$ $z=3t$ $0\le t\le 1$

$ds = \sqrt{1^2+2^2+3^2} = \sqrt{14}$

$\int_C xe^{yz}\,ds = \int_0^1 te^{6t^2}\sqrt{14} = \boxed{\frac{\sqrt{14}}{12}(e^6-1)}$

Ejercicio 13 #73

$\int_C xye^{yz}\,dy$ C: $x=t$ $y=t^2$ $z=t^3$ $0\le t\le 1$ $y=t^2$ $dy=2t\,dt$ $u=t^5$ $du=5t^4dt$

$I = \int_C xye^{yz} = \int_0^1 t(t^2)e^{t^5}2t = 2\int_0^1 t^4e^{t^5} = \frac{2}{5}\int_0^1 e^u\,du = \frac{2}{5}[e^u]_0^1 = \boxed{\frac{2}{5}[e-1]}$

Ejercicio 15 #74

$\int_C z^2dx + x^2dy + y^2dz$ $(1,0,0)$ $(4,1,2)$

$v = (4,1,2)-(1,0,0)$

$v = \langle 3,1,2\rangle$

$x = 1+3t$ $y=0+t$ $z=2t$ $0\le t\le 1$

$x'=3$ $y'=1$ $z'=2$

$\int_0^1 (2t)^2(3)+(1+3t^2)^2(1)+(t)^2(2) = \int_0^1 (12t^2+(1+6t+9t^2))+2t^2 =$

$\int_0^1 23t^2+6t+1 = \left[\frac{23}{3}t^3+3t^2+t\right]_0^1 = \boxed{\frac{35}{3}}$

Ejercicio 23 #75

$\int_C F\cdot dr$ $F(x,y) = \sqrt{x+y}\,\hat{\imath} + \left(\frac{y}{x}\right)\hat{\jmath}$ $r(t) = \text{sen}^2t\,\hat{\imath} + \text{sen}t\cos t\,\hat{\jmath}$ $\frac{\pi}{6}\le t\le\frac{\pi}{3}$

$r(t) = \text{sen}^2t\,\hat{\imath} + \text{sen}t\cos t\,\hat{\jmath}$

$r'(t) = 2\text{sen}t\cos t\,\hat{\imath} + (\cos^2t-\text{sen}^2t)\hat{\jmath}$

$dr = (2\text{sen}t\cos t\,\hat{\imath} + (\cos^2-\text{sen}^2t)\hat{\jmath}$

$F(r(t)) = \sqrt{\text{sen}^2t+\text{sen}t\cos t}\,\hat{\imath} + \frac{\cos t}{\text{sen}t}\hat{\jmath}$

$\int_C F\cdot dr = \int_{\pi/6}^{\pi/3}(\sqrt{\text{sen}^2t+\text{sen}t\cos t}\,\hat{\imath} + \frac{\cos t}{\text{sen}t}\hat{\jmath})\cdot(2\text{sen}t\cos t\,\hat{\imath} + (\cos^2t-\text{sen}^2t)\hat{\jmath})$

$\int_{\pi/6}^{\pi/3}(2\text{sen}t\cos t\sqrt{\text{sen}^2t+\text{sen}t\cos t} + \frac{\cos t}{\text{sen}t}(\cos^2t-\text{sen}^2t))dt = \boxed{0.54243}$

Ejercicio 39 #76

$F(x,y) = x\,\hat{\imath} + (y+2)\hat{\jmath}$

$r(t) = (t-\text{sen}t)\hat{\imath} + (1-\cos t)\hat{\jmath}$ $0\le t\le 2\pi$

$\frac{dr}{dt} = (1-\cos t)\hat{\imath} + \text{sen}t\,\hat{\jmath} \to dr = (1-\cos t)\hat{\imath} + \text{sen}t\,\hat{\jmath}$

$\int_0^{2\pi}\langle t-\text{sen}t, 3-\cos t\rangle\cdot\langle 1-\cos t, \text{sen}t\rangle$

$= \int_0^{2\pi} t - t\cos t - \text{sen}t + \text{sen}t\cos t + 3\text{sen}t - \text{sen}t\cos t$

$\int_0^{2\pi} t - t\cos t + 2\text{sen}t = \left[\frac{t^2}{2} - 2\cos t\right]_0^{2\pi} = \boxed{2\pi^2}$

Ejercicio 3 #77

$F(x,y)=(xy+y^2)\hat{i}+(x^2+2xy)\hat{j}$

$\frac{\partial P}{\partial y}=x+2y \quad \frac{\partial Q}{\partial x}=2x+2y$

$\boxed{x+2y \neq 2x+2y \text{ No conservativa}}$

Ejercicio 7 #78

$F(x,y)=\underbrace{(ye^x+\operatorname{sen} y)}_{P}\hat{i}+\underbrace{(e^x+\cos y\, x)}_{Q}\hat{j}$

$\frac{\partial P}{\partial y}=e^x+\cos y \quad \frac{\partial Q}{\partial x}=e^x+\cos y$

$f(x,y)=\int ye^x+\operatorname{sen} y = ye^x+\operatorname{sen} y\, x+g(y) \quad \frac{\partial f}{\partial y}=e^x+x\cos y+g'(y)=e^x+x\cos y$

$g'(y)=0 \Rightarrow g(y)=0$

$\boxed{f(x,y)=ye^x+\operatorname{sen} y\, x+c}$

Ejercicio 9 #79

$F(x,y)=(y^2\cos x+\cos y)\hat{i}+(2y\operatorname{sen} x-x\operatorname{sen} y)\hat{j}$

$\frac{\partial P}{\partial y}=2y\cos x-\operatorname{sen} y \quad \frac{\partial Q}{\partial x}=2y\cos x-\operatorname{sen} y$

$\int \frac{\partial f}{\partial x}=y^2\cos x+\cos y = y^2\operatorname{sen} x+x\cos y+g(y)$

$\frac{\partial f}{\partial y}=2y\operatorname{sen} x-x\operatorname{sen} y+g'(y)=Q \Rightarrow 2y\operatorname{sen} x-x\operatorname{sen} y+g'(y)$

$\boxed{f=y^2\operatorname{sen} x+x\cos y+c.}$

Ejercicio 12 #80

$F(x,y)=(3+2xy^2)\hat{i}+2x^2y\hat{j} \quad y=\frac{1}{x} \quad (1,1) \quad (4,\frac{1}{4})$

$\frac{\partial P}{\partial y}=3+2xy^2=4xy \quad \frac{\partial Q}{\partial x}=2x^2y=4xy$

$f(x,y)=\int 3+2xy^2=3x+x^2y^2+g(y)$

$\frac{\partial f}{\partial y}=x^2y+g'(y)=2x^2y \quad g(y)=0$

a) $\boxed{f=3x+x^2y^2+k}$

b) $\int_C F\cdot dr=\int_C \nabla f\, dr=f(4,\frac{1}{4})-f(1,1)=3(4)+4^2(\frac{1}{4})^2+k-3(1)+(1^2)(1^2)(k)=\boxed{9}$

Ejercicio 15 #81

$F(x,y,z)=yz\hat{i}+xz\hat{j}+(xy+2z)\hat{k} \quad (1,0,-2) \quad (4,6,3)$

$f_x=yz$

$f(x,y,z)=xyz+g(y,z)$

$f_y=xz \quad g_y(y,z)=0 \quad g(y,z)=h(z)$

$f(x,y,z)=xyz+h(z)$

$f_z=xy+h'(z)$

$f_z=xy+h'(z)=xy+2z$

$h'(z)=2z \quad h(z)=z^2+c$

a) $\boxed{f(x,y,z)=xyz+z^2+C}$

b) $\int_C F\cdot dr=\int_C \nabla f\, dr=f(4,6,3)-f(1,0,-2)=[4(6)(3)+3^2+c]-[-4]$ b) $=\boxed{77}$

Ejercicio 17 #82

$F(x,y,z)=yze^{xz}\hat{i}+e^{xz}\hat{j}+xye^{xz}\hat{k}$

$C: r(t)=(t^2+1)\hat{i}+(t^2-1)\hat{j}+(t^2-2t)\hat{k} \quad 0\leq t\leq 2$

$F(x,y,z)=yze^{xz}\hat{i}+e^{xz}\hat{j}+xye^{xz}$

$\int M\, dx=\int yze^{xz}dx=ye^{xz}$

$=\int e^{xz}dy=ye^{xz}$

$\boxed{\int P\, dz=\int xye^{xz}=ye^{xz}}$

$F(r(t))=(t^2-1)e^{(t^2+1)(t^2-2t)}+C$

$F\cdot dr=[(2^2-1)e^{(2^2+1)(2^2-2(2))}+C]=[3e^0+c-3+1=\boxed{4}$