Págs. 853-854

#1 Ejercicio 1

$r(t)=\langle \ln(t+1), \frac{t}{\sqrt{9-t^2}}, 2^t \rangle$

R// $D=\{t \mid t\in(-1,3)\}$

$\hat{\imath}$) $t+1>0$ $(-1,\infty)$
$t>-1$

$\hat{\jmath}$) $9-t^2>0$ $(-\infty,-3)$ ✗, $(-3,3)$ ✓, $(3,\infty)$ ✗
$(3-t)(3+t)>0$

t	$9-t^2$
-4	-7
0	3
4	-7

$\hat{k}$) $2^t=(-\infty,\infty)$

#2 Ejercicio 3

$\lim_{t\to 0}\left(e^{-3t}\hat{\imath}+\frac{t^2}{\operatorname{sen}t^2}\hat{\jmath}+\cos 2t\,\hat{k}\right)=$

Solución

$\langle \lim_{t\to0} e^{-3t}, \lim_{t\to0}\frac{t^2}{\operatorname{sen}^2 t}, \lim_{t\to0}\cos 2t\rangle = \langle 1, \left(\lim_{t\to0}\frac{t}{\operatorname{sen}t}\right)^2, 1\rangle$

$=\langle 1, \left(\lim_{t\to0}\frac{\frac{t}{t}}{\frac{\operatorname{sen}t}{t}}\right)^2, 1\rangle = \langle 1,1,1\rangle$

R// $\langle 1,1,1\rangle$

#3 Ejercicio 4

$\lim_{t\to1}\left(\frac{t^2-t}{t-1}\hat{\imath}+\sqrt{t+8}\,\hat{\jmath}+\frac{\operatorname{sen}\pi t}{\ln t}\hat{k}\right)$

$\langle 1,3,-\pi\rangle$

$\langle \lim_{t\to1}\frac{t^2-t}{t-1}, \lim_{t\to1}\sqrt{t+8}, \lim_{t\to1}\frac{\operatorname{sen}\pi t}{\ln t}\rangle$

R// $\langle 1,3,-\pi\rangle$

$\langle \lim_{t\to1}\frac{t(t-1)}{t-1}, 3, \lim_{t\to1}\frac{\operatorname{sen}\pi t}{\ln t}\rangle$

$\langle 1, 3, \lim_{t\to1}\frac{\pi\cos\pi t}{\frac{1}{t}}\rangle$

#31 #6 Ejercicio 25

(1, 1, 1)

#4

Ejercicio 5

$$\lim_{t\to\infty}\left\langle \frac{1+t^2}{1-t^2},\ \tan^{-1} t,\ \frac{1-e^{-2t}}{t}\right\rangle$$

$$\left\langle \lim_{t\to\infty} \frac{1+t^2}{1-t^2},\ \lim_{t\to\infty} \tan^{-1} t,\ \lim_{t\to\infty} \frac{1-e^{-2t}}{t}\right\rangle$$

$$\left\langle \lim_{t\to\infty} \frac{\frac{1+t^2}{t^2}}{\frac{1-t^2}{t^2}},\ \frac{\pi}{2},\ \frac{1-0}{\infty}\right\rangle$$

R// $\left\langle -1, \frac{\pi}{2}, 0 \right\rangle$

$$\left\langle -1, \frac{\pi}{2}, 0 \right\rangle$$

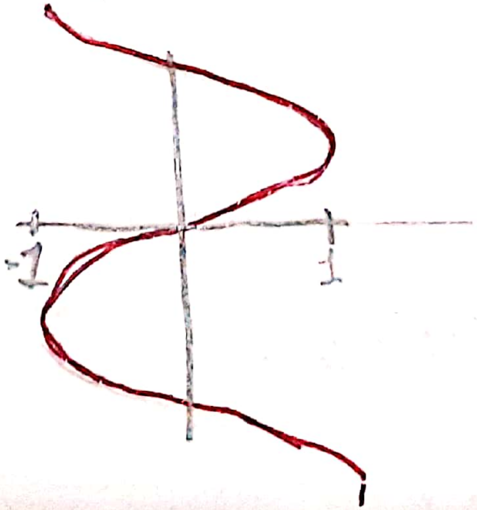
#5

Ejercicio 7

$r(t) = \langle \text{sen}\, t, t \rangle$

$x = \text{sen}\, t \quad y = t$

$x = \text{sen}\, y$



#6

$r = \langle t, 2-t, 2t \rangle$

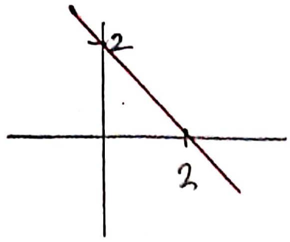
$x = t \quad y = 2 - x \quad z = 2x$

$x = 0 \quad \langle 0, 2, 0 \rangle$

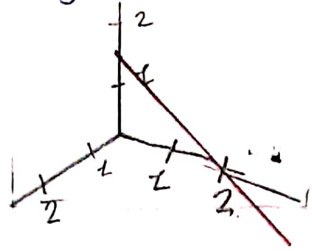
$x = 1 \quad \langle 1, 1, 2 \rangle$

Ejercicio 9

xy



xyz



#7

$r(t) = \langle 3, t, 2-t^2 \rangle$

$x = 3 \quad y = t \quad z = 2 - y^2$

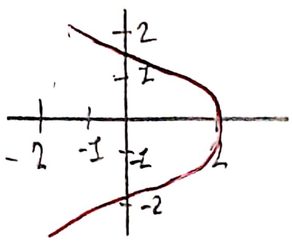
$y = t$

$y = 0 \quad \langle 3, 0, 2 \rangle$

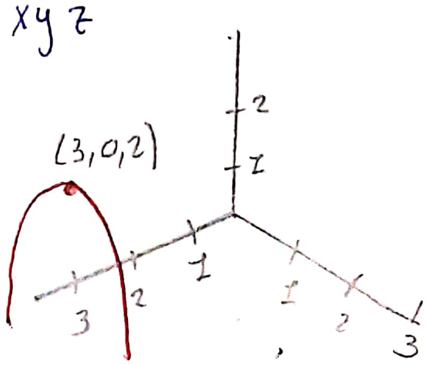
$y = 1 \quad \langle 3, 1, 1 \rangle$

Ejercicio 11

xy



xyz



#8

$r = t^2 i + t^4 j + t^6 k$

$r = (t^2, t^4, t^6)$

$x = t^2 \quad y = x^2 \quad z = x^3$

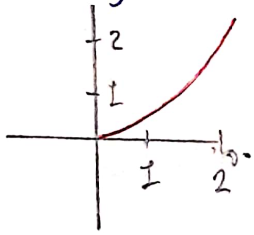
$x = 0 \quad (0, 0, 0)$

$x = 1 \quad (1, 1, 1)$

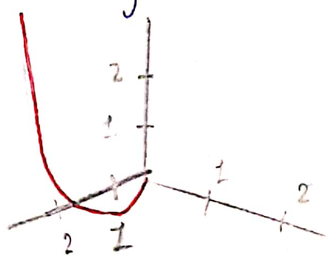
$x = 2 \quad (4, 16, 64)$

Ejercicio 13

xy



xyz



#9.

$x = t\cos t \quad y = t \quad z = t\,\text{sen}\,t$

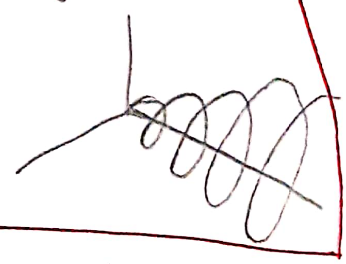
$t \geq 0$

$x = y\cos y$

$z = y\,\text{sen}\,y.$

Ejercicio 21

La grafica 2



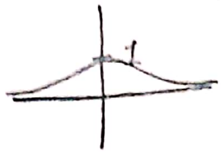
#10

Ejercicio 23

$x = t \quad y = \frac{1}{1+t^2} \quad z = t^2$

$y = \frac{1}{1+x^2} \quad z = x^2$

xy xy



gráfica V

#11

Ejercicio 43

$z = \sqrt{x^2+y^2} \rightarrow$ cono plano $z = 1+y$

$y = t \quad z = 1+t$

$z = \sqrt{x^2+y^2}$

$z^2 = x^2+y^2$

$x = \pm\sqrt{z^2-y^2} = \sqrt{(1+t)^2-(t)^2}$

$x = \pm\sqrt{1+2t}$

$$\boxed{\begin{aligned} r_1(t) &= \langle -\sqrt{1+2t}, t, 1+t \rangle \\ r_2(t) &= \langle \sqrt{1+2t}, t, 1+t \rangle \end{aligned}}$$

#12

Ejercicio 45

$z = x^2 - y^2 \quad x^2+y^2 = 1$

$x = t \quad y = \sqrt{1-t}$

$z = t^2 - \sqrt{1-t^2}$

$$\boxed{r(t) = \langle t, \sqrt{1-t^2}, t^2 - \sqrt{1-t^2} \rangle}$$

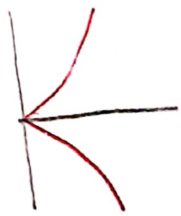
Pags. 860 - 861

#13

Ejercicio 4

$r(t) = \langle t^2, t^3 \rangle \quad t = 1$

a)



b) $x = t^2 \quad y = t^3$

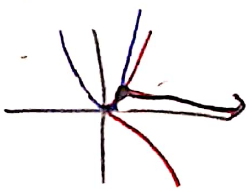
$x = 2t \quad y = 3t^2$

$$\boxed{r'(t) = \langle 2t, 3t^2 \rangle}$$

$r'(1) = \langle 2, 3 \rangle$

b) R// $r'(t) = \langle 2t, 3t^2 \rangle$

c)



$r = (2, 3)$

#14 Ejercicio 5

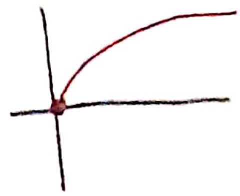
$r(t)=(e^{2t}, e^{t})$ $t=0$

$x' = 2e^{2t}$ $y' = e^{t}$

$r'(t) = (2e^{2t}, e^{t})$

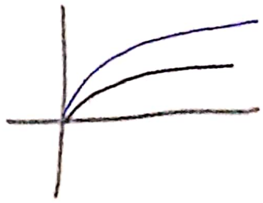
$r'(0) = (2, 1)$

a)



b) R// $r(t) = (2e^{2t}, e^{t})$

c)



• r

• r'

#15 Ejercicio 7

$r(t) = (4 \text{ sen } t, -2\cos t)$ $t = 3\pi/4$

$r'(t) = (4\cos t, 2 \text{ sen } t)$

$r'(t) = (-2\sqrt{2}, \sqrt{2})$

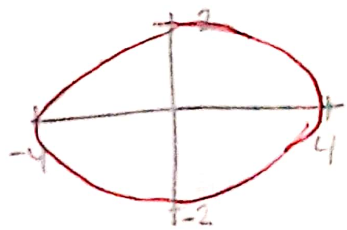
$t=0$ $(0,-2)$

$t = \pi/2$ $(4,0)$

$t = \pi$ $(0, 2)$

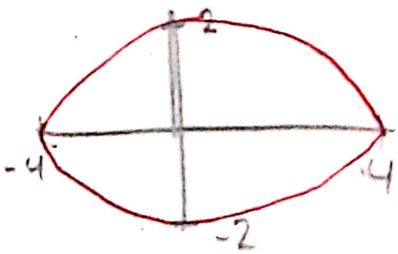
$t = 3\pi/2$ $(-4,0)$

a)



b) $r'(t) = (4\cos t, 2 \text{ sen } t)$

c)



$r'(3\pi/4) = (-2\sqrt{2}, \sqrt{2})$

#16 Ejercicio 8

$r(t) = (\cos t + 1, \operatorname{sen} t - 1)$ $\quad t = -\pi/3$

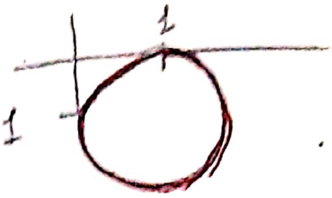
$t = 0$ $\quad (2,-1)$

$t = \pi/2$ $\quad (1,0)$ $\quad r'(t) = (-\operatorname{sen} t, \cos t)$

$t = \pi$ $\quad (0,-1)$ $\quad r'(-\pi/3) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

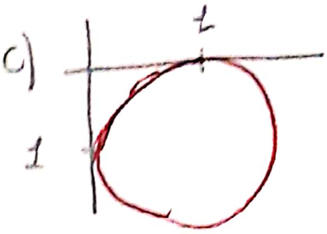
$t = 3\pi/2$ $\quad (1,-2)$

a)



b) $r'(t) = (-\operatorname{sen} t, \cos t)$

c)



$r'(-\pi/3) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

#17 Ejercicio 11

$r(t) = (t^2, \cos t^2, \operatorname{sen}^2 t)$

$x = t^2$ $\quad y = \cos t^2$ $\quad z = \operatorname{sen}^2 t$

$x' = 2t$ $\quad y' = -2t \operatorname{sen} t^2$ $\quad z' = 2 \operatorname{sen} t \cos t$

$z' = \operatorname{sen}(2t)$

R// $r'(t) = (2t, -2t \operatorname{sen} t^2, \operatorname{sen}(2t))$

#18 Ejercicio 13

$r(t) = (t \operatorname{sen} t, e^t \cos t, \operatorname{sen} t \cos t)$

$x = t \operatorname{sen} t$ $\quad y = e^t \cos t$ $\quad z = \operatorname{sen} t \cos t$

$x' = \operatorname{sen} t + \cos t$ $\quad y' = e^t \cos t - \operatorname{sen} t\, e^t$ $\quad z' = \cos^2 t - \operatorname{sen}^2 t$

$y' = e^t(\cos t - \operatorname{sen} t)$ $\quad z' = \cos(2t)$

R// $r'(t) = (\operatorname{sen} t + \cos t, e^t \cos t - e^t \operatorname{sen} t, \cos 2t)$

#19 Ejercicio 17

$r(t) = \langle t^2 - 2t, 1 + 3t, \frac{1}{3}t^3 + \frac{1}{2}t^2$ $t = 2$

$r'(t) = \langle 2t - 2, 3, t^2 + t \rangle$ R// $T(2) = \langle \frac{2}{7}, \frac{3}{7}, \frac{6}{7} \rangle$

$r'(2) = \langle 2, 3, 6 \rangle$

$|r'(2)| = \sqrt{2^2 + 3^2 + 6^2} = \sqrt{49} = 7$

#20 Ejercicio 19

$r(t) = \langle t^3 + 3t, t^2 + 1, 3t + 4 \rangle$ $t = 1$ R// $T(1) = \langle \frac{6}{7}, \frac{2}{7}, \frac{3}{7} \rangle$

$r'(t) = \langle 3t^2 + 3, 2t, 3 \rangle$

$r'(1) = \langle 6, 2, 3 \rangle$

$|r'(1)| = \sqrt{6^2 + 2^2 + 3^2} = \sqrt{49} = 7$

#21 Ejercicio 35

$r(t) = \int_0^1 \langle 6t^2, t, 8t^3 \rangle dt$ R// $r(t) = \langle 2, \frac{1}{2}, 2 \rangle$

$r(t) = \langle \int_0^1 6t^2, \int_0^1 t, \int_0^1 8t^3 \rangle dt$

$r(t) = \langle 2t^3 \Big|_0^1, \frac{t^2}{2} \Big|_0^1, 2t^4 \Big|_0^1 \rangle$

#22 Ejercicio 37

$r(t) = \int_0^1 \langle \frac{1}{t+1}, \frac{1}{t^2+1}, \frac{1}{t^2+1}) dt$

$r(t) = \langle \int_0^1 \frac{1}{t+1}, \int_0^1 \frac{1}{t^2+1}, \int_0^1 \frac{t}{t^2+1} \rangle$

$r(t) = \langle \ln(t+1) \Big|_0^1, \tan^{-1} t \Big|_0^1, \ln(t^2+1) \Big|_0^1 \rangle$

R// $r(t) = \langle \ln 2, \frac{\pi}{4}, \frac{1}{2} \ln 2 \rangle$

#21 Ejercicio 05

#23 Ejercicio 39

$r(t) = \int \langle e^t, 2t, \ln t \rangle$

$u = \ln x \quad dv = dx$

$du = \frac{1}{x} \quad v = x$

$r(t) = \langle \int e^t, \int 2t, \int \ln t \rangle$

R// $r(t) = \langle e^t + c_1, t^2 + c_2, x\ln x - x + c_3 \rangle$

#24 Ejercicio 40

$r(t) = \int \langle \cos \pi t, \operatorname{sen} \pi t, t \rangle dt$

$r(t) = \langle \int \cos \pi t, \int \operatorname{sen} \pi t, \int t \rangle$

R// $r(t) = \langle \frac{1}{\pi} \operatorname{sen} \pi t + c_1, \frac{-\cos \pi t}{\pi} + c_2, \frac{t^2}{2} + c_3 \rangle$

Pags. 868 - 869

#25 Ejercicio 1

$r(t) = \langle 2\cos t, \sqrt{5}\, t, 2\operatorname{sen} t \rangle \qquad -2 \leq t \leq 2.$

$r'(t) = \langle -2\operatorname{sen} t, \sqrt{5}, 2\cos t \rangle$

$|r'(t)| = \sqrt{(-2\operatorname{sen} t)^2 + \sqrt{5}^2 + (2\cos t)^2} = \sqrt{+4\operatorname{sen}^2 t + 5 + 4\cos^2 t} = \sqrt{4(\operatorname{sen}^2 t + \cos^2 t) + 5}$

$|r'(t)| = \sqrt{9} = 3$

$L = \int_{-2}^{2} 3\,dx = 3x \Big|_{-2}^{2} = 12u$

R// 12u.

#26 Ejercicio 2

$r(t) = \langle t^3, 6t, 3t^2 \rangle \qquad 0 \leq t \leq 3.$

$r'(t) = \langle 3t^2, 6, 6t \rangle$

$|r'(t)| = \sqrt{(3t^2)^2 + (6)^2 + (6t)^2} = \sqrt{9t^4 + 36 + 36t^2} = \sqrt{9(t^4 + 4t^2 + 4)} =$

$= 3\sqrt{(t^2+2)^2} = 3t^2 + 6.$

$L = \int_0^3 3t^2 + 6 = t^3 + 6t \Big|_0^3 = 27 + 18 = 45u$

R// 45u

#31 Ejercicio 54

Ejercicio 6

#27

$r(t) = \langle 1, t^2, t^3 \rangle \quad 0 \le t \le 1$

$r'(t) = \langle 0, 2t, 3t^2 \rangle$

$|r'(t)| = \sqrt{(2t)^2 + (3t^2)^2} = \sqrt{4t^2 + 9t^4} = \sqrt{t^2(4+9t^2)} = t\sqrt{4+9t^2}$

$L = \int_0^1 t\sqrt{4+9t^2}\,dt = \frac{1}{18}\int_0^1 \sqrt{u}\,du$ $\qquad u = 4+9t^2$, $du = 18t\,dt$

$\frac{1}{18}\,\frac{2}{3}\left[u^{3/2}\right]\Big|_0^1 = \frac{1}{27}(4+9t^2)^{3/2}\Big|_0^1 =$

$\left[\frac{1}{27}(4+9)^{3/2}\right] - \left[\frac{1}{27}(4)^{3/2}\right]$ $\qquad$ R// 1.440

$\left[\frac{1}{27}(13)^{3/2}\right] - \frac{1}{27}(8)$

$\frac{1}{27}\left[(13)^{3/2} - 8\right] = 1.440$

#28 Ejercicio 9

$r(t) = \langle \cos \pi t, 2t, \operatorname{sen} 2\pi t \rangle$ de $(1,0,0)$ a $(1,4,0)$ $\quad 0 \le t \le 2$

$r'(t) = \langle -\pi \operatorname{sen} t, 2, 2\pi \cos 2\pi t \rangle$

$|r'(t)| = \sqrt{(-\pi \operatorname{sen} t)^2 + 4 + (2\pi \cos 2\pi t)^2} = \sqrt{\pi^2 \operatorname{sen}^2 t + 4 + 4\pi^2 \cos^2(2\pi t)}$

$= \sqrt{4 + \pi^2(\operatorname{sen}^2 t + \cos^2(2\pi t))}$

$L = \int_0^2 \sqrt{4 + \pi^2(\operatorname{sen}^2 t + \cos^2(2\pi t))} = 10.3311$

R// 10.3311

#

#29 Ejercicio 22

$r(t) = \langle t, t, (1+t^2) \rangle$

$r'(t) = \langle 1, 1, 2t \rangle$ $\quad |r'(t)| = \sqrt{1^2+1^2+(2t)^2} = \sqrt{4t^2+2}$

$r''(t) = \langle 0, 0, 2 \rangle$

$r'(t) \times r''(t) = \begin{vmatrix} i & j & k \\ 1 & 1 & 2t \\ 0 & 0 & 2 \end{vmatrix} \quad 2\hat{i} + 2\hat{j} + 0\hat{k}$

$|r'(t) \times r''(t)| = \sqrt{2^2+2^2+0^2} = \sqrt{8}$

$K(t) = \frac{\sqrt{8}}{(4t^2+2)^{3/2}}$ $\quad R// \frac{\sqrt{8}}{(4t^2+2)^{3/2}}$

#30 Ejercicio 23

$r(t) = \langle \sqrt{6}t^2, 2t, 2t^3 \rangle$ $\quad |r'(t)| = \sqrt{(2\sqrt{6}t)^2+2^2+(6t^2)^2}$

$r'(t) = \langle 2\sqrt{6}t, 2, 6t^2 \rangle$ $\quad |r'(t)| = \sqrt{36t^4+24t^2+4}$

$r''(t) = \langle 2\sqrt{6}, 0, 12t \rangle$ $\quad |r'(t)| = \sqrt{(6t^2)^2+2\cdot 6t^2\cdot 2+2^2}$

$|r'(t)| = \sqrt{(6t^2+2)^2} = 6t^2+2 = (6t^2+2)^3$

$r'(t) \times r''(t) = \begin{vmatrix} \hat{i} & \hat{j} & k \\ 2\sqrt{6} & 2 & 6t^2 \\ 2\sqrt{6} & 0 & 12t \end{vmatrix} = \langle 24t, -12\sqrt{6}t^2, -4\sqrt{6} \rangle$

$|r'(t) \times r''(t)| = \sqrt{(24t)^2+(-12\sqrt{6}t^2)^2+(-4\sqrt{6})^2} = \sqrt{(12\sqrt{6}t^2+4\sqrt{6})^2} = 12\sqrt{6}t^2+4\sqrt{6}$

$K(t) = \frac{12\sqrt{6}t^2+4\sqrt{6}}{(6t^2+2)^3} = \frac{4\sqrt{6}(3t^2+1)}{2^3(3t^2+1)^3} = \frac{\sqrt{6}}{2(3t^2+1)^2}$ $\quad R// \frac{\sqrt{6}}{2(3t^2+1)^2}$

#31

Ejercicio 25

Determine la curvatura de $r(t) = \langle t, t^2, t^3 \rangle$ en el punto $(1,1,1)$.

$r'(t) = \langle 1, 2t, 3t^2 \rangle$

$r''(t) = \langle 0, 2, 6t \rangle$

$$|r'(t) \times r''(t)| = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2t & 3t^2 \\ 0 & 2 & 6t \end{vmatrix} = \langle 6t^2, -6t, 2 \rangle$$

$$|r'(t) \times r''(t)| = \sqrt{(6t^2)^2 + (-6t)^2 + 2^2} = \sqrt{36t^4 + 36t^2 + 4}$$

$$|r'(t)| = \sqrt{1 + 4t^2 + 9t^4} = (1 + 4t^2 + 9t^4)^{3/2}$$

$$K(t) = \frac{\sqrt{36t^4 + 36t^2 + 4}}{(1 + 4t^2 + 9t^4)^{3/2}}$$

$$K(1) = \frac{\sqrt{36 + 36 + 4}}{(1 + 4 + 9)^{3/2}} = 1.45$$

#32

Ejercicio 30.

$y = \ln x$ $\quad x \to \infty$

$y' = \frac{1}{x}$

$y'' = -\frac{1}{x^2}$

$$= \frac{\left|\frac{1}{x^2}\right|}{\left[1 + \left(\frac{1}{x}\right)^2\right]^{3/2}} = + \frac{\frac{1}{x^2}}{\left[1 + \frac{1}{x^2}\right]^{3/2}} = x^{-2}\left[1 + \frac{1}{x^2}\right]^{-3/2}$$

$$K'(x) = -2x^{-3}\left[1 + \frac{1}{x^2}\right]^{-3/2} + \frac{3}{2} x^{-2} \left[1 + \frac{1}{x^2}\right]^{-5/2} x^{-3}$$

$$K'(x) = 2x^{-3}\left[1 + x^{-2}\right]^{-3/2} + \frac{3}{2} x^{-5} \left[1 + 1/x^2\right]^{-5/2} = \frac{2}{x^3\left[1 + x^{-2}\right]^{3/2}} + \frac{3}{2x^5\left[1 + x^{-2}\right]^{5/2}}$$

$$= \frac{-2x^2\left(1 + \frac{1}{x^2}\right) + 3}{x^5\left[1 + \frac{1}{x^2}\right]^{5/2}} = \frac{-2x^2 + 1}{x^5\left[1 + \frac{1}{x^2}\right]^{5/2}}$$

$-2x^2 + 1 = 0$

$x^2 = \frac{1}{2}$

$x = \frac{1}{\sqrt{2}}$

$$\lim_{x \to \infty} \frac{1}{x^2[1+1/x^2]^{3/2}} = \frac{1}{\infty} = 0$$

$x = \frac{1}{\sqrt{2}}$

$f\left(\frac{1}{\sqrt{2}}\right) = \ln\left(\sqrt{\frac{1}{2}}\right) = \ln \frac{1}{\sqrt{2}}$

R// El maximo esta en $\left(\frac{1}{\sqrt{2}}, \ln \frac{1}{\sqrt{2}}\right)$ y tiende a 0

#33

Ejercicio 31

$y = e^x$

$y' = e^x$

$y'' = e^x$

$$K(x) = \frac{e^x}{[1+e^{2x}]^{3/2}}$$

$$K'(x) = \frac{e^x[1+e^{2x}]^{3/2} - \frac{3}{2}[1+e^{2x}]^{1/2}(2e^{2x})}{[1+e^{2x}]^3} = \frac{e^x[1+e^{2x}]^{3/2} - 3e^{2x}[1+e^{2x}]^{1/2}}{[1+e^{2x}]^3}$$

$$K'(x) = \frac{(1+e^{2x})^{1/2}\left[e^x(1+e^{2x}) - (3e^{3x})\right]}{[1+e^{2x}]^3} = \frac{e^x + e^{3x} - 3e^{3x}}{[1+e^{2x}]^{5/2}} = \frac{e^x - 2e^{3x}}{[1+e^{2x}]^{5/2}}$$

$$K'(x) = \frac{e^x(1-2e^{2x})}{[1+e^{2x}]^{5/2}}$$

$$\lim_{x \to \infty} \frac{e^x}{[1+e^{2x}]^{3/2}} = 0$$

$1 - 2e^{2x} = 0$

$e^{2x} = \frac{1}{2}$

$2x = \ln \frac{1}{2}$

$x = \frac{1}{2} \ln \frac{1}{2}$

$F\left(\frac{1}{2} \ln \frac{1}{2}\right) = e^{\frac{1}{2} \ln \frac{1}{2}} = \frac{1}{\sqrt{2}}$

R// Maxima curvatura en $\left(\frac{1}{2}\ln\frac{1}{2}, \frac{1}{\sqrt{2}}\right)$ y tiende a 0

#34 Ejercicio 13

$F(x,y)=\sqrt{x-2}+\sqrt{y-1}$

$x-2>0$ $\quad$ $y-1\geq 0$

$x\geq 2$ $\quad$ $y\geq 1$

$$\boxed{D=\{(x,y)\mid x\geq 2,\ y\geq 1\}}$$

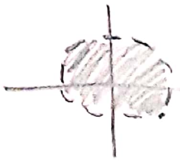
#35 Ejercicio 15.

$F(x,y)=\ln(9-x^2-9y^2)$

$9-x^2-9y^2>0$

$-x^2-9y^2>-9$

$\frac{x^2}{9}+y^2>1$



$\frac{x^2}{9}+y^2=1$

$$\boxed{D=\{(x,y)\mid \frac{x^2}{9}+y^2<1\}}$$

#36. Ejercicio 19

$F(x,y)=\frac{\sqrt{y-x^2}}{1-x^2}$

$1-x^2>0$

$1>x^2$

$\pm 1>x$

$y-x^2\geq 0$

$y\geq x^2$

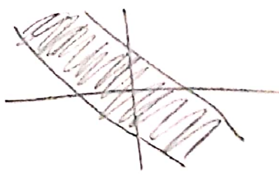
#37 Ejercicio 20.

$F(x,y)=\operatorname{sen}^{-1}(x+y)$

$x+y\geq -1$

$x+y\leq 1$

$y\geq -x-1$ $\quad$ $y\leq 1-x.$



$D=\{(x,y)\mid -1\leq x+y\leq 1$

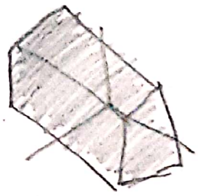
#38 Ejercicio 21

$F(x,y,z)=\sqrt{4-x^2}+\sqrt{9-y^2}+\sqrt{1-z^2}$

$4-x^2\geq 0$ $\quad$ $9-y^2\geq 0$

$x^2\geq 4$ $\quad$ $y^2\geq$

$x\geq 2.$



$D=\{(x,y,z)\mid -2\leq x\leq 2,\ -3\leq y\leq 3$

$-1\leq z\leq 1\}$

#39 Ejercicio 25.

$F(x,y)=10-4x-5y.$

$z+4x+5y=10$

$4x+5(0)=10$

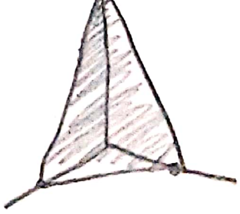
$x=\frac{5}{2}.$

$0+4(0)+5y=10.$

$y=2$

$z+4(0)+5(0)=10$

$z=10$



$$\boxed{4x+5y+z=10}$$

#40 Ejercicio 27

$f(x,y) = \text{sen}\, x$ cilindro

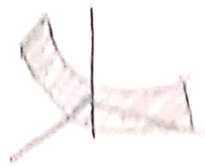
$z = \text{sen}\, x$

$z = \text{sen}$

#41 Ejercicio 30

$f(x,y) = e^{-y}$

$z = e^{-y}$ $z = e^{y}$



#42. Ejercicio 32

a) $f(x,y) = \frac{1}{1+x^2+y^2}$ → creciente, tiende 90 III

b) $f(x,y) = \frac{1}{1+x^2y^2}$ → tiende a 0 nada lo vuelve 0 I

c) $f(x,y) = \ln(x^2+y^2)$ $x^2+y^2>0$ $x^2>-y^2$ I V

d) $f(x,y) = \sqrt{x^2+y^2}$ $x^2+y^2 \geq 0$ $x^2+y^2 = r^2 \cos t$ V

e) $f(x,y) = (x^2-y^2)^2$ $f(x) = y^4$ $f(y) = x^4$ VI

f) $f(x,y) = \cos(xy)$ $f(x) = \cos(0)$ · $f(x) = \cos 0$ II

crece y derecho.

#43. 

Ejercicio 43

#44

$f(x,y) = x^2 - y^2$

$f(x,y) = k$

$k = -2$ $y = \sqrt{x^2+2}$

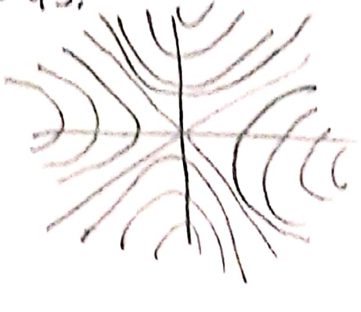
$k = -1$ $y = \sqrt{x^2+1}$

$k = 0$ $y = \sqrt{x^2}$

$k = 1$ $y = \sqrt{x^2-1}$

$k = 2$ $y = \sqrt{x^2-2}$

Ejercicio 45.



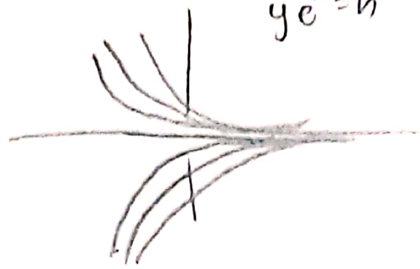
#45 Ejercicio 49.

$f(x,y) = ye^{x}$

$f(x,y) = K$ $\quad y = \frac{K}{e^x}$

$y \cdot e^x = K$

$ye^x = K$



$K = -2 \quad y = \frac{-2}{e^x}$

$K = -1 \quad y = \frac{-1}{e^x}$

$K = 0 \quad y = 0$

#46. Ejercicio 50

$f(x,y) = y - \tan^{-1}x$

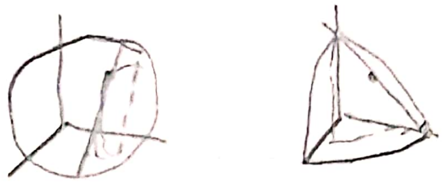
$K = y - \tan^{-1}x$

$K=0 \quad y = \tan^{-1}x$

$K=1 \quad y = 1 + \tan^{-1}x$

#47 Ejercicio 11

$f(x,y) = 16 - 4x^2 - y^2$ $\quad f_x(1,2) \quad f_y(1,2)$



$f_x = -8x \quad (1,2) \to -8(1) = -8 \quad (1,2,8)$

$(1,2) \to -2(2) = -4 \quad (1,2$

#48 Ejercicio. 15

$f_x(x,y) = x^4 + 5xy^3$ $\quad f_x(x,y) = 4x^3 + 5y^3$

$f_y(x,y) = 15xy^2$ $\quad f_y(x,y) = 15xy^2$

#49. Ejercicio 19.

$z = \ln(x + t^2)$

$\frac{dz}{dx} = \frac{1}{x+t^2}$ $\quad \frac{\partial z}{\partial t} = \frac{1}{x+t^2}(2t)$ $\quad \frac{\partial z}{\partial x} = \frac{1}{x+t^2}$ $\quad \frac{\partial z}{\partial t} = \frac{2t}{x+t^2}$

#50. Ejercicio 23.

$f(x,t) = e^{-t}\cos \pi x$

$f_x(x,y) = e^{t}(-\operatorname{sen}\pi x)\pi$ $\quad f_x(x,y) = -\pi e^{-x}$

$f_y = -\cos \pi x e^{-t}$ $\quad f_y(x,y) = -\cos \pi x e^{-t}$

#51 Ejercicio 25.

$z = (2x+3y)^{10}$

$\frac{\partial z}{dx} = 10(2x+3y)^9(2) = 20(2x+3y)^9$ $\quad \frac{\partial z}{\partial x} = 20(2x+3y)^9$

$\frac{\partial z}{\partial y} = 10(2x+3y)^9(3) = 30(2x+3y)^9$ $\quad \boxed{\frac{\partial z}{\partial y} = 30(2x+3y)^9}$

#52

Ejercicio 26.

$x f(x,t) = \sqrt{x} \ln t$

$f(x,t) = \tan^{-1}(x\sqrt{t})$

$f_x = \frac{1}{(x\sqrt{t})^2+1}(\sqrt{t}) = \frac{\sqrt{t}}{x^2t+1}$

$f_y = \frac{1}{(x\sqrt{t})^2+1}\left(\frac{1}{2}x t^{-1/2}\right) = \frac{x}{(x^2t+1)2\sqrt{t}}$

$f_x(x,y) = \frac{\sqrt{t}}{x^2t+1}$

$\boxed{f_y(x,y) = \frac{x}{2\sqrt{t}(x^2t+1)}}$

#53

Ejercicio 33.

$w = \ln(x+2y+3z)$

$\frac{\partial w}{\partial x} = \frac{1}{x+2y+3z}(1)$

$\frac{\partial w}{\partial y} = \frac{1}{x+2y+3z}(2)$

$\frac{\partial w}{\partial y} \cdot \frac{1}{x+2y+3z}(3)$

$\frac{\partial w}{\partial x} = \frac{1}{x+2y+3z}$

$\frac{\partial w}{\partial y} = \frac{2}{x+2y+3z}$

$\frac{\partial w}{\partial z} = \frac{3}{x+2y+3z}$

#54

Ejercicio 43.

$f(x,y,z) = \ln\left(\frac{1-\sqrt{x^2+y^2+z^2}}{1+\sqrt{x^2+y^2+z^2}}\right)$

$f_y(1,2,2)$

$\ln(1-\sqrt{x^2+y^2+z^2}) - \ln(1+\sqrt{x^2+y^2+z^2})$

$f_y(x,y,z) = \frac{1}{1-\sqrt{x^2+y^2+z^2}}\left(-\frac{1}{2}\right)(x^2+y^2+z^2)^{-1/2}(2y)$

$- \frac{1}{1+\sqrt{x^2+y^2+z^2}}\left(\frac{1}{2}\right)(x^2+y^2+z^2)^{-1/2}(2y).$

$f_y = (x,y,z) = \frac{-2y}{2(1-\sqrt{x^2+y^2+z^2})\sqrt{1+4+4}} - \frac{2y}{2(1+\sqrt{x^2+y^2+z^2})(\sqrt{x^2+y^2+z^2})}$

$f_y(x,y,z) = \frac{-2}{(1-\sqrt{1+4+4})\sqrt{1+4+4}} - \frac{2}{(1+\sqrt{1+4+4})(\sqrt{1+4+4})}$

$f_y(x,y,z) = \frac{-2}{(1-3)(3)} - \frac{2}{(1+3)(3)} = \frac{-2}{-6} - \frac{2}{12} = \boxed{\frac{1}{6}}$

#55 $f(x,y) = xy^2 - x^3y$

Ejercicio 45.

$$f_x(x,y) = \lim_{h\to 0} \frac{(x+h)y^2-(x+h)^3y-(xy^2-x^3y)}{h} = \frac{xy^2+hy^2-(x^3+3x^2h+3xh^2+h^3)y-(xy^2-x^3y)}{h}$$

$$= \frac{xy^2+hy^2-x^3y-3x^2hy+3xh^2y+h^3y-xy^2+x^3y}{h} = \frac{hy^2-3x^2hy-3xh^2y}{h} = \frac{h(y^2-3x^2y-y-3xhy+h^2y)}{h}$$

$$= y^2-3x^2y-3x(0)y+(0)^2y)$$

$$f_y(x,y) = \lim_{h\to 0} \frac{x(h+y)^2-x^3(h+y)-(xy^2-x^3y)}{h} = \frac{x(h^2+2hy+y^2)-x^3h-x^3y-xy^2+x^3y}{h}$$

$$= \frac{xh^2+2hy+y^2x-x^3h-x^3y-xy^2+x^3y}{h} = \frac{h(xh+2yx-x^3)}{h} = x(0)+2y\,x-x^3.$$

$f_x(x,y) = y^2-3x^2y$

$f_y(x,y) = 2xy-x^3.$

#56 $f(x,y) = \dfrac{x}{x+y^2}$

Ejercicio 46.

$$f_x(x,y) = \frac{\frac{x+h}{x+h+y^2} - \frac{x}{x+y^2}}{h} = \frac{x+h}{h(x+h+y^2)} - \frac{x}{h(x+y^2)} = \frac{(x+h)(x+y^2)-(x)(x+h+y^2)}{h(x+h+y^2)(x+y^2)}$$

$$= \frac{x^2+xy^2+hx+hy^2-x^2-xh-xy^2}{h(x+h+y^2)(x+y^2)} = \frac{hy^2}{h(x+h+y^2)(x+y^2)} = \frac{y^2}{(x+h+y^2)(x+y^2)} = \frac{y^2}{x^2+2xy^2+y^4}$$

$$= \frac{y^2}{(x+y^2)^2}.$$

$$f_y(x,y) = \frac{\frac{x}{x+(y+h)^2} - \frac{x}{x+y^2}}{h} = \frac{x}{h(x+y^2+2yh+h^2)} - \frac{x}{h(x+y^2)} = \frac{x(x+y^2)-x(x+y^2+2yh+h^2)}{h(x+y^2+2yh+h^2)(x+y^2)}$$

$$= \frac{-2yx}{x^2+2xy^2+y^4}$$

$$\boxed{f_x(x,y) = \frac{y^2}{(x+y^2)^2} \qquad f_y(x,y) = \frac{-2xy}{(x+y^2)^2}.}$$

#61

Ejercicio 47

$x^2+2y^2+3z^2=1$

a) $\frac{\partial x^2}{\partial x}+\frac{\partial 2y^2}{\partial x}+\frac{\partial 3z^2}{\partial x}=\partial$

$2x+0+6zz'=0$ $\quad 2x=-6zz'$

$\frac{\partial z}{\partial x}=\frac{2x}{-6x} \Rightarrow \frac{\partial z}{\partial x}=\frac{x}{-3z}$

b) $0+4y+6z\frac{\partial z}{\partial y}=0$

$4y=-6z\frac{\partial z}{\partial y}$

$\frac{\partial z}{\partial y}=\frac{4y}{6z}$

$\frac{\partial z}{\partial x}=\frac{-2y}{3z}$

#58

Ejercicio 49

$e^z=xyz$

$\boxed{\frac{\partial z}{\partial x}=\frac{yz}{e^z-xy}}$ $\quad \boxed{\frac{\partial z}{\partial y}=\frac{xy}{e^z-xy}}$

#59

Ejercicio 55

$z=\frac{y}{2x+3y}$

$z_y=\frac{2x}{(2x+3y)^2}$ $\quad z_x=\frac{-2y}{(2x+3y)^2}$

$z_{xx}=\frac{8y(2x+3y)}{(2x+3y)^4}$

$z_{yy}=\frac{-12x(2x+3y)}{(2x+3y)^4}$ $=$ $z_{xy}=-4x-6y+12y.$

$\boxed{z_{xx}=\frac{8y}{(2x+y)^3} \quad z_{yy}=\frac{-12x}{(2x+3y)^3} \quad z_{xy}=\frac{-4x+6y}{(2x+3y)^3}=z_{yx}.}$

#60

Ejercicio 57

$v=\operatorname{sen}(s^2-t^2)$

$U_s=2s\cos(s^2-t^2)$

$U_t=-2t\cos(s^2-t^2)$

$Vtt=-(2\cos(s^2-t^2)+4t^2\operatorname{sen}(s^2-t^2)$

$Uss=2\cos(s^2-t^2+2s(-s\operatorname{sen}(s^2-t^2)\cdot 2s.$

$U_st=-2t(-\operatorname{sen}(s^2-t^2))2s$

$\boxed{\begin{array}{l} Uss=2\cos(s^2-t^2)-4s^2\operatorname{sen}(s^2-t^2) \\ Utt=-2\cos(s^2-t^2-4t^2\operatorname{sen}(s^2-t^2) \\ Vst=4st\operatorname{sen}(s^2-t^2)=Vts. \end{array}}$

Ejercicio 59

#61

$u = x^4y^3 - y^4$ $\quad u_x = 4x^3y^3$ $\quad u_y = 3x^4y^2 - 4y^3$

$u_{xy} = 12x^3y^2$ $\quad u_{yx} = 12x^3y^2$

$$\boxed{u_{yx} = u_{xy}}$$

Ejercicio 63

#62

$f(x,y) = x^4y^2 - x^3y$ $\quad f_{xxx}$; f_{xyx}

$f_x = 4x^3y^2 - 3x^2y$ $\quad f_{xy} = 8x^3y - 3x^2$

$f_{xx} = 12x^2y^2 - 6xy$ $\quad f_{xyx} = 24x^2y - 6x$

$f_{xxx} = 24xy^2 - 6y$

$$\boxed{\begin{aligned} f_{xxx} &= 24xy^2 - 6y \\ f_{xyx} &= 24x^2y - 6x \end{aligned}}$$

Ejercicio 64

#63 $f(x,y) = \operatorname{sen}(2x + 5y)$

$f_y = 5\cos(2x + 5y)$

$f_{yx} = -10\operatorname{sen}(2x + 5y)$

$$\boxed{f_{yxy} = -50\cos(2x + 5y)}$$

Ejercicio 65.

#64 $f(x,y) = e^{xyz^2}$

$f_x = yz^2e^{xyz^2}$

$f_{xy} = e^{xyz^2} \cdot z^2 + yz^2 \cdot xz^2 \cdot e^{xyz^2}$

$f_{xyz} = 2z \cdot e^{xyz^2} + 2xyz \cdot e^{xyz^2} + 4yxz^3e^{xyz^2} + xyz^4 \cdot 2xyz e^{xyz^2}$

$f_{xyz} = 2ze^{xyz^2}(1 + 3xyz^2 + x^2y^2z^4)$

Ejercicio 66

#65 $g(r,s,t) = e^r\operatorname{sen}(st)$

$g_r = e^r\operatorname{sen}(st)$ $\quad g_{rs} = te^r\cos(st)$

$g_{rst} = e^r\cos(st) + te^r(-\operatorname{sen}(st))$

$g_{rst} = e^r\cos(st) - ste^r\operatorname{sen}(st)$

Ejercicio 67

#66

$w = \sqrt{u + v^2}$

$w_v = \dfrac{2v}{2\sqrt{u + v^2}} = v(u + v^2)^{-1/2}$ $\quad w_{vu} = -\dfrac{1}{2}v(u + v^2)^{-3/2}$

$w_{vuu} = -\dfrac{1}{2} \cdot \left(\dfrac{-3}{2}\right)v(u + v^2)^{-5/2} = \dfrac{3}{4}v(u + v^2)^{-5/2}$

$$\boxed{\frac{\partial^3 w}{\partial u^2 \partial v} = \frac{3}{4}v(u + v^2)^{-5/2}.}$$

#67

Ejercicio 64

$u = e^{r\theta}\operatorname{sen}\theta$

$\frac{\partial^3 u}{\partial r^2 \partial\theta}$

$\boxed{e^{r\theta} + \theta^2 \operatorname{sen}\theta + e^{r\theta}(2\theta\operatorname{sen}\theta + \theta\cos\theta)}$

$u_r = \theta e^{r\theta}\operatorname{sen}\theta$

$u_{rr} = \theta^2 e^{r\theta}\operatorname{sen}\theta$

$u_{rr\theta} = e^{r\theta}r\theta^2\operatorname{sen}(\theta) + (2\theta\operatorname{sen}\theta + (\cos\theta)\theta^2) e^{r\theta}$

#68 $z = u\sqrt{v-w}$

$\frac{\partial^3 z}{\partial u \partial v \partial w}$

Ejercicio 70

$z_u = \sqrt{v-w}$

$z_{uv} = \frac{1}{2\sqrt{v-w}}$

$z_{uvw} = -\frac{1}{2}\left(\frac{1}{2}(v-w)^{-3/2}(-1)\right) = \frac{1}{4(v-w)^{3/2}}$

$$\boxed{\frac{\partial^3 z}{\partial u \partial v \partial w} = \frac{1}{4(v-w)^{3/2}}}$$

#69

Ejercicio 89

$T\frac{\partial P}{\partial T} \cdot \frac{\partial V}{\partial T} = mR$

$PV = mRT$ $\quad P = \frac{mRT}{V}$ $\quad V = \frac{mRT}{P}$

$\frac{\partial P}{\partial T} = \frac{mR}{V}$ $\quad \frac{\partial V}{\partial T} = \frac{mR}{P}$

$T\frac{\partial P}{\partial T} \cdot \frac{\partial V}{\partial T} = T \cdot \frac{mR}{V} \cdot \frac{mR}{P} = \frac{T(mR)^2}{VP} = \frac{T(mR)^2}{mRT} = \boxed{mR}$