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[1]

$$\iint_D x\cos y\, dA = \int_0^1\int_0^{x^2} x\cos y\, dy\, dx$$

$u = x^2$
$du = 2x$
$\frac{du}{2} = x$

$$\int_0^1 [x \operatorname{sen} y]_0^{x^2} dx = \int_0^1 x \operatorname{sen} x^2\, dx = \frac{-\cos x^2}{2}\Big]_0^1$$

$$\boxed{\frac{1-\cos 1}{2}}$$

[2]

$$\iint_D xy^2\, dA = \int_{-1}^{1}\int_0^{\sqrt{1-y^2}} xy^2\, dx\, dy$$

$$= \int_{-1}^{1} \frac{x^2y^2}{2}\Big|_0^{\sqrt{1-y^2}} = \int_{-1}^{1} \frac{(\sqrt{1-y^2})^2 y^2}{2} dy = \int_{-1}^{1}\frac{(1-y^2)y^2}{2} dy = \int_{-1}^{1} \frac{y^2-y^4}{2} = \frac{y^3}{6} - \frac{y^5}{10}\Big|_{-1}^{1}$$

$$\left(\frac{1}{6}-\frac{1}{10}\right) - \left(-\frac{1}{6}+\frac{1}{10}\right) = \boxed{\frac{2}{15}}$$

[3]

$$\iint_D y^2\, dA$$

$(0,1)\ (1,2)\ (4,1)$
$x\ y \quad x_1\ y_1 \quad x_2\ y_2$

$m_1 = \frac{2-1}{1-0} = 1$

$y - 2 = x - 1$
$y = x + 1$
$x = y - 1$

$m_2 = \frac{2-1}{1-4} = -\frac{1}{3}$

$y - 2 = -\frac{1}{3}(x-1)$
$y - 2 = -\frac{1}{3}x + \frac{1}{3}$
$y = -\frac{1}{3}x + \frac{7}{3}$
$3\left(\frac{7}{3} - y\right) = x$

$$\int_1^2\int_{y-1}^{7-3y} y^2\, dx\, dy$$

$$\int_{y-1}^{7-3y} y^2\, dx = xy^2\Big]_{y-1}^{7-3y} = y^2(7-3y-y+1) = y^2(8-4y) = 8y^2-4y^3$$

$$\int_1^2 8y^2-4y^3\, dy = \frac{8y^3}{3} - \frac{4y^4}{4}\Big|_1^2 = \boxed{\frac{11}{3}}$$