MI3 Sección A
Primer Semestre 2021

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MODELADO CON ECUACIONES DIFERENCIALES DE PRIMER ORDEN

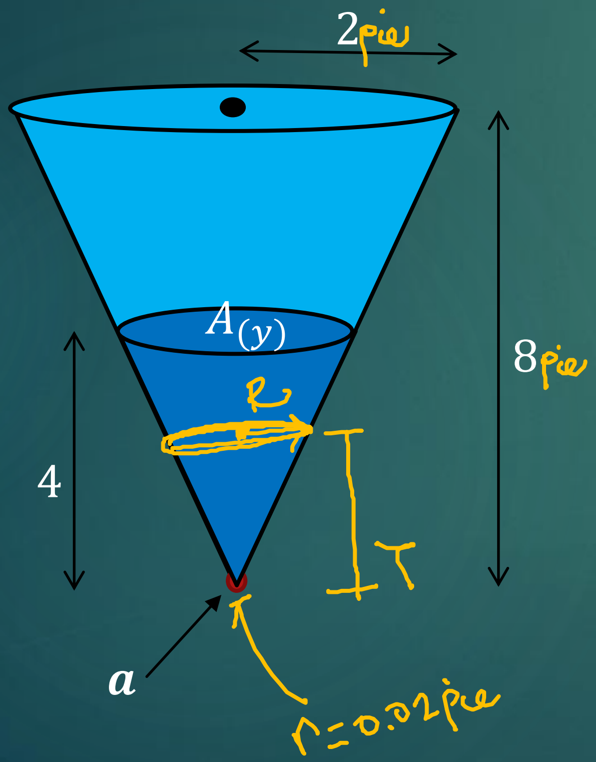
LEY DE TORRICELLI

(DRENADO DE UN DÉPOSITO)

Ejemplo:

Un tanque con forma cónica de 8 pie de altura y 2 pie de radio, se encuentra colocado con su vértice hacia abajo. En el instante en que el tanque se encuentra lleno hasta una altura de 4 pies se abre un agujero de radio 0.02 pies en el fondo. ¿Cuánto tiempo pasará para que se vacíe el tanque?

$t=0 \quad y=4\text{pie}$ $\quad t=? \quad y=\phi$



$y = profundidad\ del\ agua\ en\ el\ tanque$ (pie)

$t = tiempo$ (s)

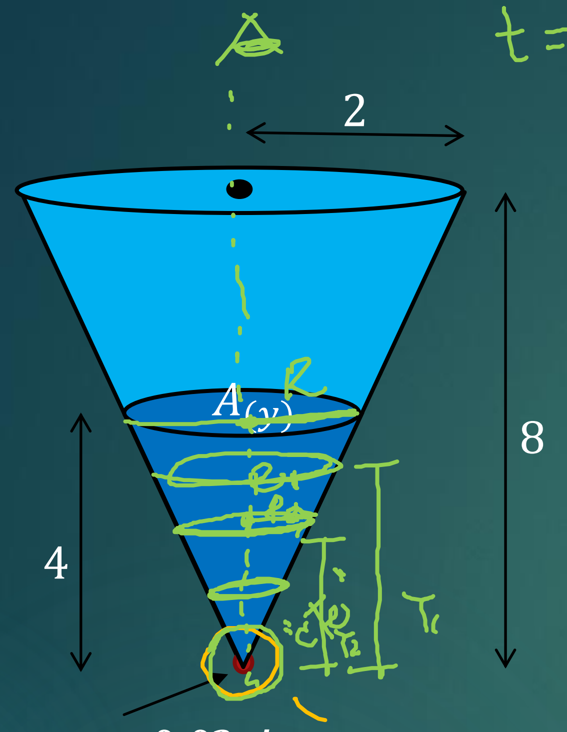
$$A(y)\frac{dy}{dt} = -a\sqrt{2gy}$$

$$g = 32\,{}^{pie}/_{s^2}$$

$$A(y) = ?$$

$$a = ?$$

$t=0, Y=4$ pie

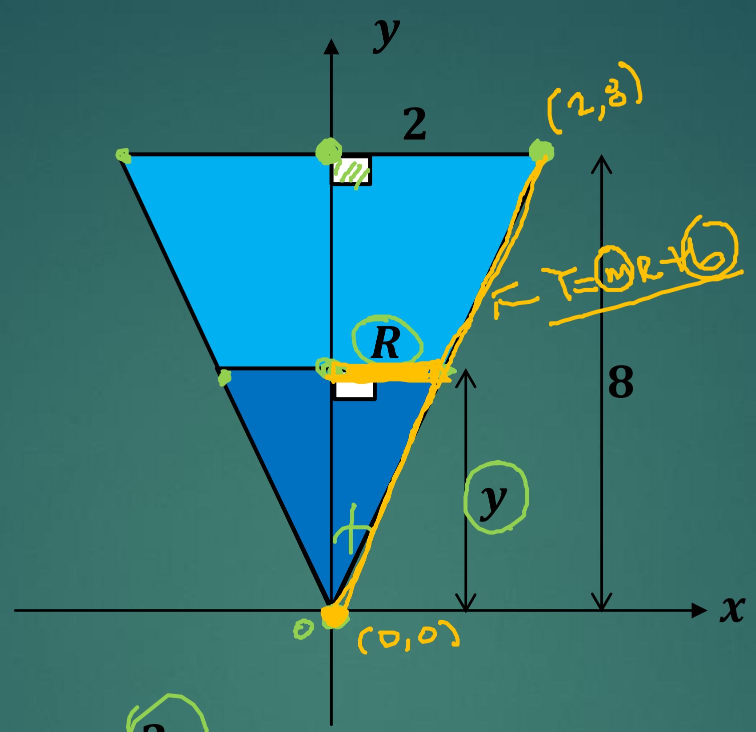


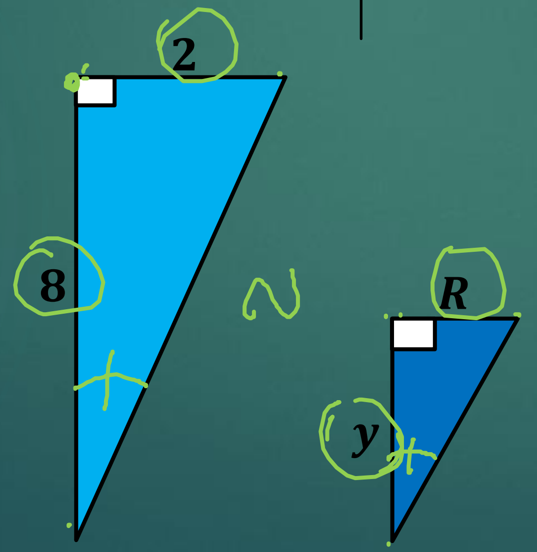
$r = 0.02pie$

$a = \pi r^2$

$a = \pi(0.02)^2$

$$\boxed{a = \frac{\pi}{2500}pie^2}$$





$$\boxed{A(y)\frac{dy}{dt} = -a\sqrt{2gy}}$$

32 pie/s²

$*\ A(y) = \pi R^2$

$a = \pi r^2$

Semejanza

$$\frac{R}{2} = \frac{y}{8}$$

$$\boxed{R = \frac{y}{4}}$$

$$A(y) = \pi R^2$$

$$A(y) = \pi\left(\frac{y}{4}\right)^2$$

$$A(y) = \pi\frac{y^2}{16}$$

$$A(y)\frac{dy}{dt} = -a\sqrt{2gy}$$

$$a = \frac{\pi}{2500}pie^2$$

$$A(y) = \pi\frac{y^2}{16}$$

$$* \quad \left(\cancel{\pi}\frac{y^2}{16}\right)\frac{dy}{dt} = -\left(\frac{\cancel{\pi}}{2500}\right)\sqrt{2(32)y}$$

$$\left(\frac{y^2}{16}\right)\frac{dy}{dt} = -\left(\frac{8}{2500}\right)\sqrt{y}$$

$$\left(\frac{y^2}{\sqrt{y}}\right)dy = -\left[\frac{8(16)}{2500}\right]dt$$

$$\left(\frac{y^2}{y^{1/2}}\right)dy = -\left(\frac{32}{625}\right)dt$$

$$y^{3/2}dy = -\left(\frac{32}{625}\right)dt$$

$$\int y^{3/2}\,dy = -\left(\frac{32}{625}\right)\int dt$$

$$\boxed{\frac{2}{5}y^{5/2} = -\frac{32}{625}t + C}$$

Y(t) = ?

$$t = 0 \qquad y = 4\,pie$$

$$\frac{2}{5}(4)^{5/2} = -\cancel{\frac{32}{625}(0)}^{\,0} + C$$

$$C = \frac{2}{5}(4)^{5/2}$$

$$C = \frac{64}{5}$$

$$\frac{2}{5}y^{5/2} = -\frac{32}{625}t + C$$

$$C = \frac{64}{5}$$

$$\boxed{\frac{2}{5}y^{5/2} = -\frac{32}{625}t + \frac{64}{5}}$$

$t = ?$ $\quad y = 0$

$$\frac{2}{5}(0)^{5/2} = -\frac{32}{625}t + \frac{64}{5}$$

0

$$0 = -\frac{32}{625}t + \frac{64}{5}$$

$$\frac{32}{625}t = \frac{64}{5}$$

$$t = \frac{\frac{64}{5}}{\frac{32}{625}}$$

$$t = 250\ segundos$$

R// *Pasaran 250 segundos para que se vacíe completamente el tanque*

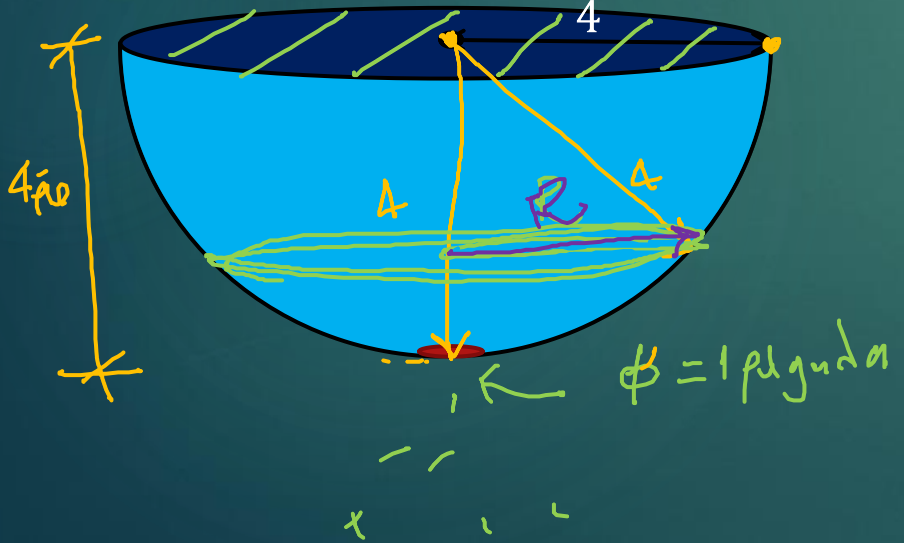
Ejemplo

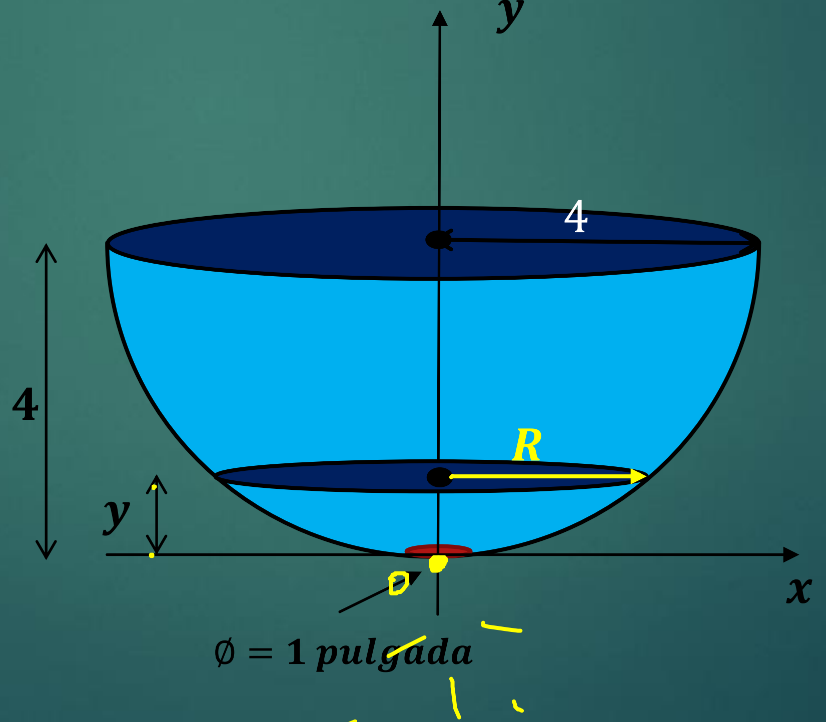
Una tina hemisferica tiene un radio superior de 4 pies y en el instante $t = 0$ esta completamente llena de agua. En ese momento en el fondo de la tina se abre un agujero circular con un diametro de 1 pulgada ? Cuanto tiempo tardará en salir toda el agua del tanque?

$y = profundidad\ del\ agua\ en\ el\ tanque$ (pie)

$t = tiempo$ (s)

$$\boxed{A(y)\frac{dy}{dt} = -a\sqrt{2gy}}$$

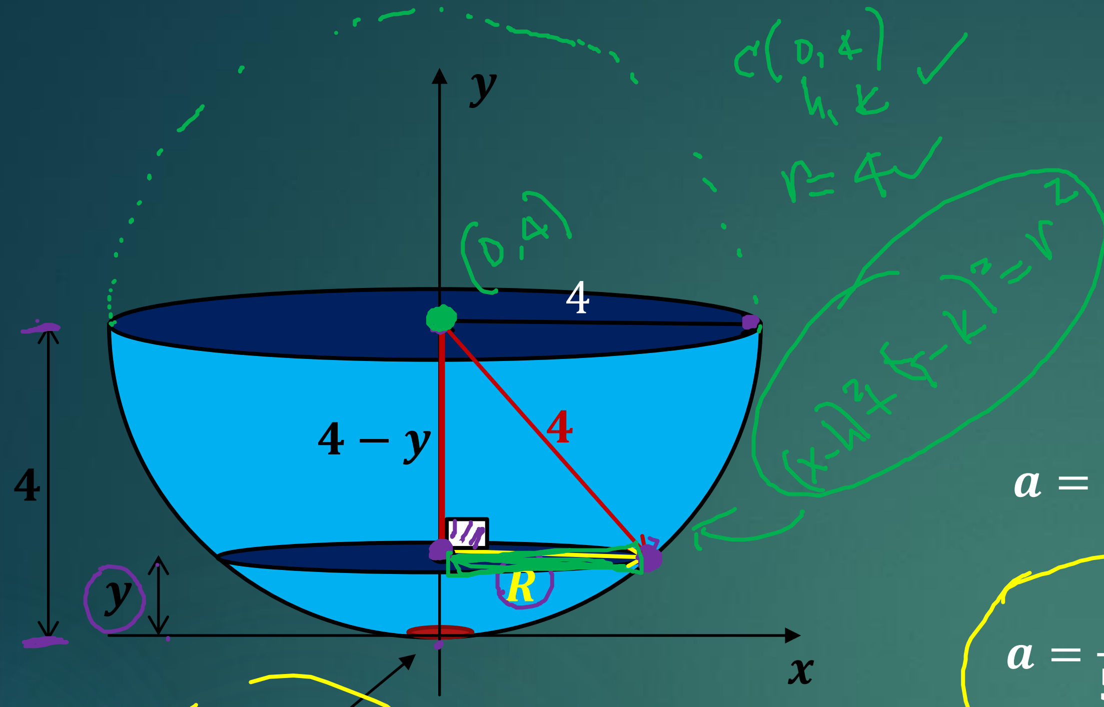




$g = 32\ {}^{pie}/_{s^2}$

$A(y) = ?$

$a = ?$



$$r = \frac{1}{24}\,pie$$

$$\emptyset = 1\,pulgada * \frac{1\,pie}{12\,pulgadas} = \frac{1}{12}\,pie$$

$$r = \frac{\emptyset}{2} = \frac{\frac{1}{12}\,pie}{2} = \frac{1}{24}\,pie$$

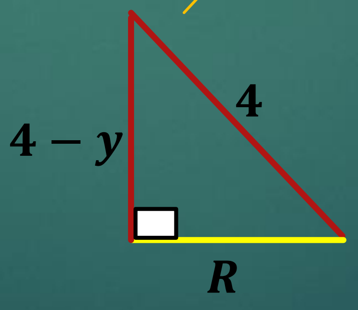
$$A(y)\frac{dy}{dt} = -a\sqrt{2gy}$$

32 pie/s²

$$a = \pi r^2$$

$$a = \pi\left(\frac{1}{24}\right)^2$$

$$a = \frac{\pi}{576}\,pie^2$$



$$A(y) = \pi R^2$$

Pitágoras

$$4^2 = R^2 + (4-y)^2$$

$$\cancel{16} = R^2 + \cancel{16} - 8y + y^2$$

$$0 = R^2 - 8y + y^2$$

$$R^2 = 8y - y^2$$

$$A(y) = \pi\left(8y - y^2\right)$$

$$A(y)\frac{dy}{dt} = -a\sqrt{2gy}$$

$$a = \frac{\pi}{576}pie^2$$

$$A(y) = \pi(8y - y^2)$$

$$\cancel{\pi}(8y - y^2)\frac{dy}{dt} = -\left(\frac{\cancel{\pi}}{576}\right)\sqrt{2(32)y}$$

$$(8y - y^2)\frac{dy}{dt} = -\left(\frac{8}{576}\right)\sqrt{y}$$

$$\frac{(8y - y^2)}{\sqrt{y}}dy = -\left(\frac{1}{72}\right)dt$$

$$\frac{(8y - y^2)}{y^{1/2}}dy = -\left(\frac{1}{72}\right)dt$$

$$\left(8y^{1/2} - y^{3/2}\right)dy = -\left(\frac{1}{72}\right)dt$$

$$\int\left(8y^{1/2} - y^{3/2}\right)dy = -\frac{1}{72}\int dt$$

$$8\int y^{1/2}\,dy - \int y^{3/2}dy = -\frac{1}{72}\int dt$$

$$8\left(\frac{2}{3}\right)y^{3/2} - \frac{2}{5}y^{5/2} = -\frac{1}{72}t + C$$

$$\boxed{\frac{16}{3}y^{3/2} - \frac{2}{5}y^{5/2} = -\frac{1}{72}t + C}$$

$t=0$

$Y = 4$pie

$$\frac{16}{3}y^{3/2} - \frac{2}{5}y^{5/2} = -\frac{1}{72}t + C$$

$t = 0 \qquad y = 4\,pie$

$$\frac{16}{3}(4)^{3/2} - \frac{2}{5}(4)^{5/2} = -\frac{1}{72}(0) + C$$

$$C = \frac{448}{15}$$

$$\boxed{\frac{16}{3}y^{3/2} - \frac{2}{5}y^{5/2} = -\frac{1}{72}t + \frac{448}{15}}$$

$t = ? \qquad y = 0$

$$\frac{16}{3}(0)^{3/2} - \frac{2}{5}(0)^{5/2} = -\frac{1}{72}t + \frac{448}{15}$$

$$0 = -\frac{1}{72}t + \frac{448}{15}$$

$$\frac{1}{72}t = \frac{448}{15}$$

$$t = \frac{\frac{448}{15}}{\frac{1}{72}}$$

$$t = \frac{10752}{5}\,segundos$$

R//

Pasaran 2,150.4 segundos para que se vacíe completamente el tanque

PRUEBA DE CONOCIMIENTO

Tarea

Un tanque tiene la forma de un cubo de 12 pies de arista. Debido a un pequeño orificio situado en el fondo del tanque de 2 pulgadas cuadradas de área, presenta un escape. Si el tanque esta inicialmente lleno hasta las tres cuartas partes de su capacidad, determine:

a) Cuando el tanque estará a la mitad de su capacidad
b) Cuando el tanque estará totalmente vacío

$$\boxed{A(z)\frac{dz}{dt} = -a\sqrt{2gz}}$$