

# MI3 Sección A

## Primer Semestre 2021

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# CLASE

## 24/03/2021

# ECUACIONES DIFERENCIALES LINEALES DE ORDEN SUPERIOR

# ECUACIONES DIFERENCIALES NO HOMOGENEAS

# MÉTODO COEFICIENTES INDETERMINADOS

Sin resolver completamente, encuentre  $y_c$  y plantee  $y_p$

10.  $y^{(4)} + y''' = (1 - x^2)e^{-x}$

Funcion complementaria  $\zeta_c$

Ecuacion homogenea asociada

$$y^{(4)} + y''' = 0$$

Ecuacion caracteristica

$$r^4 + r^3 = 0$$

$$r^3(r + 1) = 0$$

$$r_c = 0 \text{ multiplicidad } 3, -1$$

$$y_c = c_1 + c_2x + c_3x^2 + c_4e^{-x}$$

$\underbrace{r=0}_{r=0} \underbrace{r=0}_{r=0} \underbrace{r=-1}_{r=-1}$

Solucion particular  $\zeta_p$

$$g(x) = (1 - x^2)e^{-x}$$

$$r_p = -1 \text{ multiplicidad } 3$$

$$y_p = (Ax + Bx^2 + Cx^3)e^{-x}$$

$$y_p = Axe^{-x} + Bx^2e^{-x} + Cx^3e^{-x}$$

$r_p = -1$  mult. 3  
 $\zeta_p = Ax e^{-x} + Bx^2 e^{-x} + Cx^3 e^{-x}$

Resolver

$$y'' + y = 4x + 10\operatorname{sen}x; \quad y(\pi) = 0, \quad y'(\pi) = 2$$

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Funcion complementaria  $\gamma_c$

$$y(x) = y_c + y_p$$

Ecuacion homogenea asociada

$$y'' + y = 0$$

Solución particular  $(\gamma_p)$

$$g(x) = 4x + 10\operatorname{sen}x$$

Ecuacion caracteristica

$$r^2 + 1 = 0$$

$$r^2 = -1$$

$$r_c = \pm i$$

$$\alpha = 0 \quad \beta = 1$$

$$y_p = A + Bx + x(C\cos x + E\operatorname{sen}x)$$

$$y_p = A + Bx + Cx\cos x + Ex\operatorname{sen}x$$

$$y_c = c_1\cos x + c_2\operatorname{sen}x$$

Solución particular

$$y_p = (A + Bx + Cx\cos x + Ex\sin x) \rightarrow Y_p = \cancel{A} + Bx + Cx\cos x + Ex\sin x$$

$$y'_p = B - Cx\sin x + C\cos x + Ex\cos x + E\sin x$$

$$y''_p = -(Cx\cos x + C\sin x) - C\sin x - Ex\sin x + E\cos x + E\cos x$$

$$y''_p = -Cx\cos x - C\sin x - C\sin x - Ex\sin x + E\cos x + E\cos x$$

$$y''_p = -Cx\cos x - 2C\sin x - Ex\sin x + 2E\cos x$$

$$y'' + y = 4x + 10\sin x$$

$$[-Cx\cos x - 2C\sin x - Ex\sin x + 2E\cos x] + [A + Bx + Cx\cos x + Ex\sin x] = 4x + 10\sin x$$

$$-2C\sin x + 2E\cos x + A + Bx = 4x + 10\sin x$$

$x$ :

$$B = 4$$

$\sin x$ :

$$-2C = 10$$

$$C = -5$$

$\cos x$ :

$$2E = 0$$

$$E = 0$$

$$A = 0$$

$$y_p = 0 + 4x - 5x\cos x + (0)x\sin x$$

$$y_p = 4x - 5x\cos x$$

$$Y_p = 4x - 5x\cos x$$

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$$Y(x) = Y_c + Y_p$$



Solución particular

$$y_p = 4x - 5x\cos x$$

Funcion complementaria

$$y_c = c_1\cos x + c_2\sen x$$

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$$y(x) = y_c + y_p$$

$$y(x) = \underbrace{c_1\cos x + c_2\sen x}_{y_c} + \underbrace{4x - 5x\cos x}_{y_p}$$

Sol general

$$y(\pi) = 0 \quad (\pi, 0)$$

$$y(x) = c_1\cos x + c_2\sen x + 4x - 5x\cos x$$

$$0 = c_1\cos(\pi) + c_2\sen(\pi) + 4(\pi) - 5(\pi)\cos(\pi)$$

$$0 = -c_1 + 4(\pi) + 5(\pi)$$

$$\boxed{c_1 = 9\pi} \checkmark$$

$$y'(\pi) = 2 \quad (\pi, 2)$$

$$y'(x) = -c_1\sen x + c_2\cos x + 4 + 5x\sen x - 5\cos x$$

$$2 = -c_1\sen(\pi) + c_2\cos(\pi) + 4 + 5(\pi)\sen(\pi) - 5\cos(\pi)$$

$$2 = -c_2 + 4 + 5$$

$$\boxed{c_2 = 7} \checkmark$$

$$y(x) = \underbrace{y_c}_{y_c} + \underbrace{y_p}_{y_p}$$

$$y(x) = 9\pi\cos x + 7\sen x + 4x - 5x\cos x$$

# PRUEBA DE CONOCIMIENTO

$$Y(x) = Y_c + Y_p$$

Determine las soluciones generales de las siguientes Ecuaciones Diferenciales.

$$1) y'' + 7y' = 24$$

$$2) 4y'' - 2y' - 2y = e^{2x}$$

$$3) 25y'' + 10y' + y = 1 - 4e^{2x}$$

Método coeficientes indeterminados  
por superposición !!!

$$① Y'' + 7Y' = \textcircled{24}_{g(x)}$$

$Y_c:$

$$Y'' + 7Y' = 0$$

ec. característica:

$$r^2 + 7r = 0$$

$$r(r+7) = 0$$

$$r_c = \textcircled{0, -7}$$

$$Y_c = C_1 + C_2 e^{-7x}$$

$Y_p:$

$$\textcircled{r_p = 0}$$

$$Y_p = AX$$

$$Y_p' =$$

$$Y_p'' =$$

$$Y_p'' + 7Y_p' = 24$$

# SOLUCION

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1)  $y'' + 7y' = 24$

$$y(x) = c_1 + c_2 e^{-7x} + \frac{24}{7}x$$

2)  $4y'' - 2y' - 2y = e^{2x}$

$$y(x) = c_1 e^x + c_2 e^{-\frac{1}{2}x} + \frac{1}{10} e^{2x}$$

3)  $25y'' + 10y' + y = 1 - 4e^{2x}$

$$y(x) = c_1 e^{\left(-\frac{1}{5}\right)x} + c_2 x e^{\left(-\frac{1}{5}\right)x} + 1 - \frac{4}{121} e^{2x}$$

T9 y T10

Fecha de entrega  
viernes 26/03

T11

Fecha de entrega  
martes 6 de abril

26/03 → 04/04

clases → lunes 05/04