$\textcircled{1}$ (1a)

$$u_t = \operatorname{div}(|\nabla u|^{p-2}\nabla u)$$

$$\boxed{u(\underline{x},t) = t^{-\beta} w\left(\frac{|x|}{t^\alpha}\right)} \qquad \boxed{\alpha,\beta>0} \qquad \xi = \frac{r}{t^\alpha}$$

$$u_t = -\beta t^{-\beta-1} w(\xi) - \alpha t^{-\beta} w'(\xi)\frac{r}{t^{\alpha+1}} = -\beta t^{-\beta-1} w(\xi) - \alpha t^{-\beta-1} w'(\xi)\,\xi$$

$$u_t = t^{-\beta-1}\left[-\beta w(\xi) - \alpha \xi w'(\xi)\right]$$

$$\boxed{u_t = -t^{-\beta-1}\left[\beta w(\xi) + \alpha \xi w'(\xi)\right]} \qquad r = |\underline{x}|_2$$

$$u_{x_j} = t^{-\beta}\cdot w'\left(\frac{r}{t^\alpha}\right)\cdot\frac{1}{t^\alpha}\frac{x_j}{r} \qquad \nabla u = t^{-\beta-\alpha} w'(\xi)\frac{x_j}{r}$$

$$|\nabla u|_2 = t^{-\alpha-\beta}(-w'(\xi))$$

$$|\nabla u|_2^{p-2}\nabla u = t^{-(\alpha+\beta)(p-2)}(-w'(\xi))^{p-2}\, t^{-(\alpha+\beta)} w'(\xi)\frac{x_j}{r}$$

$$= -t^{-(\alpha+\beta)(p-1)}(-w'(\xi))^{p-1}\frac{x_j}{r}$$

$$\boxed{\xi = \frac{r}{t^\alpha}}$$

$$\operatorname{div}(|\nabla u|_2^{p-2}\nabla u) = -t^{-(\alpha+\beta)(p-1)}\cdot\sum_{j=1}^{n}\frac{\partial}{\partial x_j}\left[(-w'(\xi))^{p-1}\cdot\frac{x_j}{r}\right]$$

$$= -t^{-(\alpha+\beta)(p-1)}\sum_{j=1}^{n}\Bigg[-(p-1)(-w'(\xi))^{p-2}\frac{x_j}{r}w''(\xi)\cdot\frac{1}{t^\alpha}\frac{x_j}{r} +$$

$$+ (-w'(\xi))^{p-1}\frac{1}{r} - (-w'(\xi))^{p-1} x_j\frac{1}{r^2}\frac{x_j}{r}\Bigg]$$

$$= -t^{-(\alpha+\beta)(p-1)}\Bigg[-(p-1)(-w'(\xi))^{p-2}\frac{1}{t^\alpha}w''(\xi) +$$

$$(-w'(\xi))^{p-1}\frac{n}{r} - (-w'(\xi))^{p-1}\cdot t^{-\alpha}\frac{1}{r/t^\alpha}\Bigg]$$

$$\boxed{\operatorname{div}(|\nabla u|_2^{p-2}\nabla u) = -t^{-(\alpha+\beta)(p-1)}\Big[-(p-1)(-w'(\xi))^{p-2}\cdot w''(\xi)\cdot t^{-\alpha} + (n-1)(-w'(\xi))^{p-1}\xi^{-1}t^{-\alpha}\Big]}$$

$$p\alpha + (p-2)\beta = 1$$
$$\beta = n\alpha \quad \Rightarrow \quad [p + n(p-2)]\alpha = 1$$

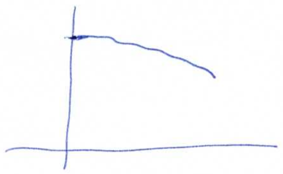
ie: $\alpha = \dfrac{1}{p + n(p-2)}$

Moreover: $\beta = \dfrac{n}{p + n(p-2)}$

$$\frac{d}{d\xi}\left[\xi^{n-1} (-w'(\xi))^{p-1} \right] = \frac{d}{d\xi}\left[\alpha \xi^{n} \cdot w(\xi) \right]$$

giving

$$\xi^{n-1} \cdot (-w'(\xi))^{p-1} = \alpha \cdot \xi^{n} \cdot w(\xi) + \underset{\uparrow \text{ constant}}{\gamma}$$



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$\xi = 0: \quad w'(\xi) = 0$

this gives $\gamma = 0$

$$\xi^{n-1} (-w'(\xi))^{p-1} = \alpha \xi^{n} w(\xi)$$

ie:

$$-w'(\xi) = \alpha^{\frac{1}{p-1}} \xi^{\frac{1}{p-1}} w(\xi)^{\frac{1}{p-1}}$$

ie:

$$w(\xi)^{-\frac{1}{p-1}} \cdot w'(\xi) = -\alpha^{\frac{1}{p-1}} \xi^{\frac{1}{p-1}}$$

"

$$\frac{d}{d\xi}\left[\frac{p-1}{p-2} w(\xi)^{\frac{p-2}{p-1}} \right] = \frac{d}{d\xi}\left[\frac{p-1}{p} \xi^{\frac{p}{p-1}} \right] \cdot \left(-\alpha^{\frac{1}{p-1}}\right)$$

We get

$$\frac{p-1}{p-2} w(\xi)^{\frac{p-2}{p-1}} = \underset{\uparrow \text{ constant}}{\tilde{C}} - \alpha^{\frac{1}{p-1}} \xi^{\frac{p}{p-1}} \cdot \frac{p-1}{p}$$

or

$$w(\xi)^{\frac{p-2}{p-1}} = C - \alpha^{\frac{1}{p-1}} \cdot \frac{p-2}{p} \cdot \xi^{\frac{p}{p-1}}$$

ie:

$$w(\xi) = \left(C - \alpha^{\frac{1}{p-1}} \frac{p-2}{p} \xi^{\frac{p}{p-1}} \right)^{\frac{p-1}{p-2}}.$$

Since

$$R(t) = C^{\frac{p-1}{p}} \cdot \alpha^{-\frac{1}{p}} \cdot \left(\frac{p}{p-2}\right)^{\frac{p-1}{p}} \cdot t^{\alpha},$$

this gives:

$$\boxed{\cancel{R(t) = \left[\overset{\text{solution mass}}{m} \over \omega_n} \cdot \frac{p}{p-1} \Big/ \underset{\text{Beta function}}{B}\left(\frac{p-1}{p}, \frac{2p-3}{p-2}\right)\right]^{\frac{p-2}{2(p-1)}} \cdot \alpha^{-\frac{1}{2(p-1)}} \cdot \left(\frac{p}{p-2}\right)^{\frac{1}{2}} \cdot t^{\alpha}}}$$

where

$$\omega_n = \frac{2\pi^{n/2}}{\Gamma(n/2)}$$

(surface area of unit sphere in $\mathbb{R}^n$)

$\omega_1 = 2$
$\omega_2 = 2\pi$
$\omega_3 = 4\pi$
etc.

and

$$\alpha = \frac{1}{n(p-2)+p}$$

Continuing the computation of mass m in terms of C:

$$m = \int_{|x| < R(t)} u(x,t)\, dx = \dots = \omega_n \cdot \int_0^{R(t)/t^{\alpha}} \left(C - \alpha^{\frac{1}{p-1}} \cdot \frac{(p-2)}{p}\, s^{\frac{p}{p-1}} \right)^{\frac{p-1}{p-2}} \cdot s^{n-1}\, ds$$

$$= \omega_n \cdot \left(\frac{p-1}{p}\right) \cdot \alpha^{-\frac{n}{p}} \cdot \left(\frac{p}{p-2}\right)^{\frac{n(p-1)}{p}} \int_0^C (C-\theta)^{\frac{p-1}{p-2}} \cdot \theta^{\frac{n(p-1)}{p} - 1}\, d\theta$$

$\theta = \alpha^{1/(p-1)} \cdot \frac{(p-2)}{p} \cdot s^{\frac{p}{p-1}}$

$s = \alpha^{-1/p} \cdot \left(\frac{p}{p-2}\right)^{\frac{p-1}{p}} \theta^{\frac{p-1}{p}}$

$ds = \alpha^{-1/p} \cdot \left(\frac{p}{p-2}\right)^{\frac{p-1}{p}} \frac{p-1}{p} \cdot \theta^{-1/p}\, d\theta$

$$= \omega_n \cdot \left(\frac{p-1}{p}\right) \cdot \alpha^{-\frac{n}{p}} \cdot \left(\frac{p}{p-2}\right)^{\frac{n(p-1)}{p}} \cdot C^{(p-1)\cdot\left[\frac{1}{p-2} + \frac{n}{p}\right]} \int_0^1 (1-r)^{\frac{p-1}{p-2}} \cdot r^{\frac{n(p-1)}{p} - 1}\, dr$$

$\theta = C \cdot r$

$d\theta = C \cdot dr$