1b

$$\operatorname{div}\left(|\nabla u|^{p-2}\nabla u\right) = - t^{-(\alpha+\beta)(p-1)-\alpha} \cdot \Big[-(p-1)\left(-w'(\xi)\right)^{p-2}\cdot w''(\xi) + (n-1)\left(-w'(\xi)\right)^{p-1}\xi^{-1}\Big]$$

$$u_t = - t^{-(\beta+1)}\left[\beta w(\xi) + \alpha\xi w'(\xi)\right]$$

$r = |\underline{x}|_2$

$\xi = r/t^{\alpha}$

$$u_t = \operatorname{div}\left(|\nabla u|^{p-2}\nabla u\right)$$

$\Rightarrow$

$$t^{-(\alpha+\beta)(p-1)-\alpha}\left[-(p-1)\left(-w'(\xi)\right)^{p-2}w''(\xi) + (n-1)\left(-w'(\xi)\right)^{p-1}\xi^{-1}\right]$$

$$= t^{-\beta-1}\left[\beta w(\xi) + \alpha\xi w'(\xi)\right]$$

$(\forall \xi, \forall t)$

$\Rightarrow$

$$(\alpha+\beta)(p-1)+\alpha = \beta+1$$

ie: $\boxed{\alpha p + \beta\cdot(p-2) = 1}$

final results:

$$\alpha = \frac{1}{n(p-2)+p}$$

$$\beta = \frac{n}{n(p-2)+p} = n\alpha$$

and

$$-(p-1)\left(-w'(\xi)\right)^{p-2}w''(\xi) + (n-1)\left(-w'(\xi)\right)^{p-1}\xi^{-1} =$$

ie:

$$= \beta w(\xi) + \alpha\xi w'(\xi)$$

$$\frac{d}{d\xi}\left(-w'(\xi)\right)^{p-1} + (n-1)\xi^{-1}\cdot\left(-w'(\xi)\right)^{p-1} = \beta w(\xi) + \alpha\xi w'(\xi)$$

$\Rightarrow$

$$\xi^{n-1}\frac{d}{d\xi}\left(-w'(\xi)\right)^{p-1} + (n-1)\xi^{n-2}\left(-w'(\xi)\right)^{p-1} = \beta\cdot\xi^{n-1}w(\xi) + \alpha\xi^{n}w'(\xi)$$

ie:

take $\boxed{\beta = \alpha n}$

$$\frac{d}{d\xi}\left[\xi^{n-1}\left(-w'(\xi)\right)^{p-1}\right] = \beta\xi^{n-1}w(\xi) + \alpha\xi^{n}w'(\xi) =$$

$$= \alpha\cdot n\xi^{n-1}w(\xi) + \alpha\xi^{n}w'(\xi)$$

$$= \alpha\cdot\left[\frac{d}{d\xi}\left(\xi^{n}w(\xi)\right)\right]$$

Hence:

$$u(\underline{x},t) = t^{-n\alpha} \cdot w\left(\frac{r}{t^\alpha}\right) = t^{-n\alpha} \cdot \left(C - \alpha^{\frac{1}{p-1}} \frac{p-2}{p} \left(\frac{|x|}{t^\alpha}\right)^{\frac{p}{p-1}} \right)_+^{\frac{p-1}{p-2}}$$

where

$$\alpha = \frac{1}{n(p-2)+p}$$

Let

$$R(t) = C^{\frac{p-1}{p}} \alpha^{-\frac{1}{p}} \cdot \left(\frac{p}{p-2}\right)^{\frac{p-1}{p}} \cdot t^\alpha = \lambda \cdot t^\alpha$$

THEN solution mass is

solution mass

$$m = \int_{|\underline{x}|<R(t)} u(\underline{x},t)\, d\underline{x} = \omega_n \int_0^{R(t)} t^{-n\alpha} w\left(\frac{r}{t^\alpha}\right) r^{n-1}\, dr =$$

$s = r/t^\alpha$ $\quad dr = t^{\alpha} ds$ $\quad r = t^\alpha s$

$$= \omega_n \cdot \int_0^{R(t)/t^\alpha} t^{-n\alpha} \cdot w(s)\, t^{\alpha n - \alpha}\, s^{n-1}\, t^{\alpha}\, ds$$

$$= \omega_n \cdot \int_0^{R(t)/t^\alpha} w(s)\, s^{n-1}\, ds$$

$$= \omega_n \int_0^{\lambda} \left(C - \alpha^{\frac{1}{p-1}} \frac{p-2}{p} s^{\frac{p}{p-1}} \right)^{\frac{p-1}{p-2}} s^{n-1}\, ds =$$

$$\boxed{\lambda = C^{\frac{p-1}{p}} \cdot \alpha^{-\frac{1}{p}} \cdot \left(\frac{p}{p-2}\right)^{\frac{p-1}{p}}}$$

$$\boxed{\omega_n = \frac{2 \cdot \pi^{n/2}}{\Gamma(n/2)}}$$

$$\boxed{\alpha = \frac{1}{n(p-2)+p}}$$

$\theta = \alpha^{\frac{1}{p-1}} \frac{p-2}{p} s^{\frac{p}{p-1}}$ $\quad s = \alpha^{-\frac{1}{p}} \cdot \left(\frac{p}{p-2}\right)^{\frac{p-1}{p}} \cdot \theta^{\frac{p-1}{p}}$

$$= \omega_n \cdot \int_0^{C} (C-\theta)^{\frac{p-1}{p-2}} \frac{p-1}{p} \alpha^{-\frac{1}{p}} \left(\frac{p}{p-2}\right)^{\frac{p-1}{p}} \cdot \theta^{-\frac{1}{p}} \cdot \alpha^{-\frac{1}{p}(n-1)} \cdot \left(\frac{p}{p-2}\right)^{(n-1)\frac{(p-1)}{p}} \cdot \theta^{(n-1)(p-1)/p}\, d\theta =$$

$s = \theta / C$ $\quad d\theta = C\, ds$

$$= \omega_n \cdot \int_0^1 C^{\frac{p-1}{p-2}} (1-s)^{\frac{p-1}{p-2}} \cdot \frac{p-1}{p} \cdot \alpha^{-\frac{1}{p}} \cdot \left(\frac{p}{p-2}\right)^{\frac{p-1}{p}} \cdot C^{-\frac{1}{p}} s^{-\frac{1}{p}} \cdot C\, ds$$

$$= \omega_n \cdot C^{\frac{2(p-1)^2}{p(p-2)}} \frac{p-1}{p} \alpha^{-\frac{1}{p}} \cdot \left(\frac{p}{p-2}\right)^{\frac{p-1}{p}} \cdot \boxed{\int_0^1 s^{-1/p} \cdot (1-s)^{\frac{p-1}{p-2}}\, ds} = B\left(\frac{p-1}{p}, \frac{2p-3}{p-2}\right)$$

$$C = \left[\frac{m}{\omega_n} \frac{p}{p-1} \alpha^{\frac{1}{p}} \cdot \left(\frac{p-2}{p}\right)^{\frac{p-1}{p}} \Big/ B\left(\frac{p-1}{p}, \frac{2p-3}{p-2}\right) \right]^{\frac{p(p-2)}{2(p-1)^2}}$$

where m is the solution mass.

i.e.:

$$m = \omega_n \cdot \left(\frac{p-1}{p}\right) \cdot \alpha^{-\frac{n}{p}} \cdot \left(\frac{p}{p-2}\right)^{\frac{n(p-1)}{p}} \cdot C^{\frac{(p-1)(p+n(p-2))}{p\cdot(p-2)}} \cdot \mathrm{beta}\left(\frac{n(p-1)}{p}, \frac{2p-3}{p-2}\right)$$

função beta de Euler

$$\alpha = \frac{1}{p+n(p-2)}$$

ω_n = area of unit sphere in $\mathbb{R}^n$
($\omega_1 = 2$, $\omega_2 = 2\pi$, $\omega_3 = 4\pi$, etc.)

This gives:

$$C = \left[\frac{m}{\omega_n} \cdot \frac{p}{p-1} \cdot \left(\frac{1}{p+n(p-2)}\right)^{\frac{n}{p}} \cdot \left(\frac{p-2}{p}\right)^{\frac{n(p-1)}{p}} \Big/ B\left(\frac{n(p-1)}{p}, \frac{2p-3}{p-2}\right) \right]^{\frac{p\cdot(p-2)}{(p-1)(p+n(p-2))}}$$

Recall that we have

$$R(t) = C^{\frac{p-1}{p}} \cdot (p+n(p-2))^{\frac{1}{p}} \cdot \left(\frac{p}{p-2}\right)^{\frac{p-1}{p}} \cdot t^{\frac{1}{p+n(p-2)}} \qquad \forall\, t>0$$

Therefore:
time T to reach a distance R is given by:

$$T = \left(R \cdot (p+n(p-2))^{-\frac{1}{p}} \cdot \left(\frac{p-2}{p}\right)^{\frac{p-1}{p}} \cdot C^{-\frac{p-1}{p}} \right)^{p+n\cdot(p-2)}$$

the constant C
is a function of the mass m
and the parameters $n, p>2$
$n \geq 1$