

# Homework 03: Binary Heaps

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## Exercise 1

By modifying the code written during the last lessons, provide an array-based implementation of binary heaps which avoids to swap the elements in the array `A`.

(*Hint*: use two arrays, `key_pos` and `rev_pos`, of natural numbers reporting the position of the key of a node and the node corresponding to a given position, respectively)

This new two-array based representation of the binary heap has been implemented inside this folder. I take as reference the same implementation done during the Homework 02.

Inside the struct `binheap_type` I add two new members: `key_pos` and `rev_pos`. The first stores in its  $i$ -th position the index inside the node's array of the  $i$ -th node, while the second stores in the  $i$ -th position the index in the heap order of the  $i$ -th element of the node's array. With this new implementation the functions `extract_min`, `decrease_key`, `heapify` and `print_heap` don't modify the array of nodes `A` and all the operations related to the nodes involve just `key_pos`.

## Exercise 2

Consider the next algorithm:

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`Ex2(A) :`

```
D ← build(A)
while do ← is_empty(A)
    extract_min(A)
end while
```

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where `A` is an array. Compute the time-complexity of the algorithm when:

- `build`, `is_empty`  $\in \Theta(1)$ , `extract_min`  $\in \Theta(|D|)$ ;
- `build`  $\in \Theta(|A|)$ , `is_empty`  $\in \Theta(1)$ , `extract_min`  $\in O(\log |D|)$ ;

In the first case, calling the function **build** will costs  $\Theta(1)$ , then the **while**-loop statement will be called  $|D|$  times since **extract\_min** will remove at every iteration only one node. However, this last function costs  $\Theta(|D|)$ , so the cost of performing this function will be  $\Theta(|D|^2)$ . More formally,

$$T(|D|) = \Theta(1) + \sum_{i=1}^{|D|} (\Theta(1) + \Theta(|D|)) = \Theta(1) + \Theta(|D|) + \Theta(|D|^2) \in \Theta(|D|^2)$$

In the second case, assuming that  $|A| = n = |D|$ , the **while**-loop statement will be called  $n$  times, but now **extract\_min** is  $aO(\log n)$ , then the overall cost of this part will raise to  $O(n \log n)$ :

$$T(n) = \Theta(n) + \sum_{i=1}^n (\Theta(1) + O(\log n)) = \Theta(n) + O(n \log n) \in O(n \log n)$$