$Y \sim Bern(\theta) \qquad 0 \leq \theta \leq 1 \qquad y \in \{0,1\}$

$$p(y|\theta) = \theta^y (1-\theta)^{1-y} \qquad p(y|\theta) = f(y) g(\theta) e^{\phi(\theta)^T u(y)}$$

$$= \exp\{\log(\theta^y (1-\theta)^{1-y})\}$$

$$= \exp\{y \log\theta + \log(1-\theta) - y\log(1-\theta)\}$$

$$= \exp\{y \log \frac{\theta}{1-\theta} + \log(1-\theta)\}$$

$f(y) = 1$

$\phi(\theta) = \log(\frac{\theta}{1-\theta}) =$ logit function

$u(y) = y$

$g(\theta) = e^{\log(1-\theta)}$

Poisson: $p(y|\theta) = \frac{1}{y!} \theta^y e^{-\theta} \quad , \theta > 0 \, , \, y \in \mathbb{N}_0 \qquad p(y|\theta) = f(y) g(\theta) e^{\phi(\theta)^T u(y)}$

$$= \frac{1}{y!} e^{y \log\theta} e^{-\theta}$$

$$=$$

$f(y) = \frac{1}{y!} \qquad g(\theta) = e^{-\theta}$

$\phi(\theta) = \log\theta \qquad u(y) = y \qquad t(y) = \sum_i u(y_i)$

$$\boxed{\pi(\theta \mid y)} = \frac{p(y \mid \theta)\,\pi(\theta)}{\int_{\Theta} p(y \mid \theta)\,\pi(\theta)\,d\theta}$$

$$y \mid \theta \sim \text{Binomial}(n, \theta) \qquad p(y \mid \theta) = \binom{n}{y} \theta^{y} (1-\theta)^{n-y}$$

$$\theta \sim \text{Beta}(\alpha, \beta) \qquad \pi(\theta) = \frac{1}{B(\alpha,\beta)} \theta^{\alpha-1} (1-\theta)^{\beta-1}$$

$\alpha, \beta > 0$

$0 \leq \theta \leq 1$

$$\pi(\theta \mid y) = \frac{\binom{n}{y} \theta^{y} (1-\theta)^{n-y} \frac{1}{B(\alpha,\beta)} \theta^{\alpha-1} (1-\theta)^{\beta-1}}{\int_0^1 \quad '' \quad '' \quad d\theta}$$

$$= \frac{\cancel{\frac{\binom{n}{y}}{B(\alpha,\beta)}}\ \theta^{y} (1-\theta)^{n-y}\ \theta^{\alpha-1} (1-\theta)^{\beta-1}}{\cancel{\frac{\binom{n}{y}}{B(\alpha,\beta)}} \int_0^1 \quad '' \quad d\theta}$$

$$= \frac{\theta^{y+\alpha-1} (1-\theta)^{n-y+\beta-1}}{\int_0^1 \quad '' \quad d\theta} \qquad \sim \text{Beta}(y+\alpha, n-y+\beta)$$

$$\frac{B(y+\alpha,\ n-y+\beta)}{B(y+\alpha,\ n-y+\beta)}$$

$$'' = 1$$

$$\theta \mid y \sim \text{Beta}(\underbrace{y+\alpha}, \underbrace{n-y+\beta})$$

$$E[\theta \mid y] = \frac{y+\alpha}{y+\alpha+n-y+\beta} = \frac{y+\alpha}{\alpha+n+\beta}$$

$$\pi(\theta) = \frac{\beta^{\alpha}\,\theta^{\alpha-1}\,e^{-\beta\theta}}{\Gamma(\alpha)} \qquad \alpha,\beta > 0, \quad \theta > 0$$

$$p(y|\theta) = \frac{\theta^{y}\,e^{-\theta}}{y!} \qquad y \in \mathbb{N}_0, \quad \theta > 0$$

$$\pi(\theta|y) \propto \theta^{y+\alpha-1}\,e^{\overset{-\theta(1+\beta)}{-\beta\theta-\theta}}$$

$$\theta|y \sim \text{Gamma}(\alpha+y,\ \beta+1)$$

$$\prod_{i=1}^{n} p(y_i|\theta) \Rightarrow \boxed{\theta|\mathbf{y} \sim \text{Gamma}\left(\alpha+\sum y_i,\ \beta+n\right)}$$

$$\mathbb{E}[\theta|y] = \frac{\alpha+\sum y_i}{\beta+n}$$

$$\pi(\theta) \propto \sqrt{I(\theta)}$$

$$I(\theta) = -\mathbb{E}\left[\frac{\partial^2}{\partial\theta} \log p(y|\theta) \right]$$

$$p(y|\theta) = \theta^{\sum y_i} e^{-n\theta} \cdot \prod^{n} \frac{1}{y_i!}$$

$$\log p(y|\theta) = \sum y_i \log\theta - n\theta - \log \sum y_i!$$

$$\frac{\partial \log p(y|\theta)}{\partial\theta} = \sum y_i \frac{1}{\theta} - n$$

$$\frac{\partial^2 \log p(y|\theta)}{\partial\theta} = -\frac{\sum y_i}{\theta^2}$$

$$\mathbb{E}[\quad] = -\frac{n\theta}{\theta^2} = -\frac{n}{\theta}$$

$$-\mathbb{E}[\sim] \propto \sqrt{\frac{n}{\theta}} = \sqrt{\frac{1}{\theta}} = \theta^{-\frac{1}{2}} \sim \text{Gama}(0.5, 0)$$

$$\boxed{\theta | y \sim \text{Gama}\left(\tfrac{1}{2} + \sum y_i, n\right)}$$