Bayesian Statistics - Homework 2

Gabry et al., *Visualization in Bayesian workflow* (2018)

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Visualization in Bayesian workflow

A pipeline for our work

Visualization is an invaluable way of justifying and criticize a statistical model.

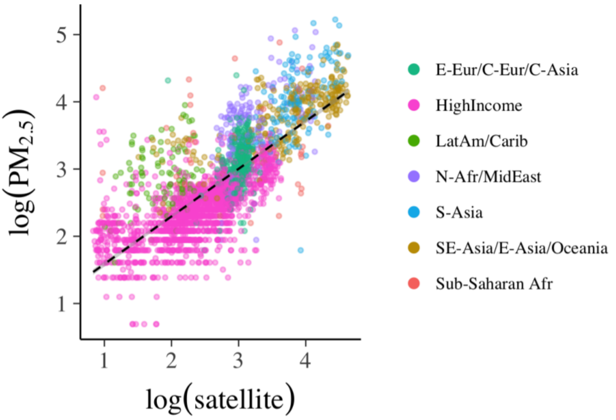
Phases of statistical workflow:

- Set up an initial model
- Model check
- Computational checks for the inference algorithm
- Posterior predictive checks
- Model comparison

Example: Estimate global $PM_{2.5}$ concentration

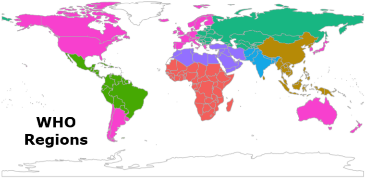
Exploratory data analysis

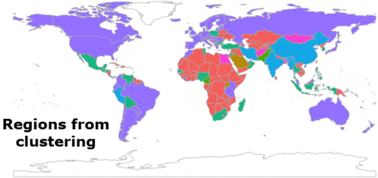
More than just plotting the data

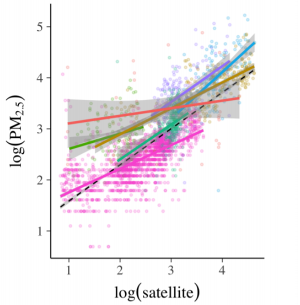


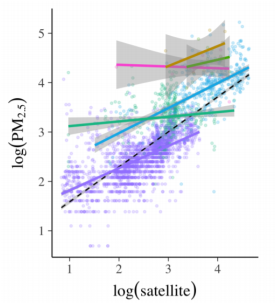
Exploratory data analysis

More than just plotting the data









Exploratory data analysis

More than just plotting the data

- **Model 1**: simple linear regression

$$\log(\mathrm{PM}_{2.5}) \sim \mathcal{N}(\alpha + \beta \log(\mathrm{satellite}), \sigma)$$

- **Model 2**: multilevel model

$$\log(\mathrm{PM}_{2.5,j}) \sim \mathcal{N}(\mu_j, \sigma), \quad \mu_j = \alpha_0 + \alpha_j + (\beta_0 + \beta_j) \log(\mathrm{satellite}_j),$$

  where observations are stratified by WHO super-regions.

- **Model 3**: multilevel model

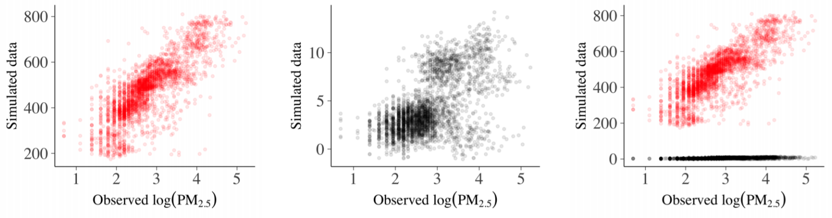
$$\log(\mathrm{PM}_{2.5,j}) \sim \mathcal{N}(\mu_j, \sigma), \quad \mu_j = \alpha_0 + \alpha_j + (\beta_0 + \beta_j) \log(\mathrm{satellite}_j),$$

  where observations are stratified by clustered super-region.

Prior predictive checking

Fake data can be almost as valuable as real data

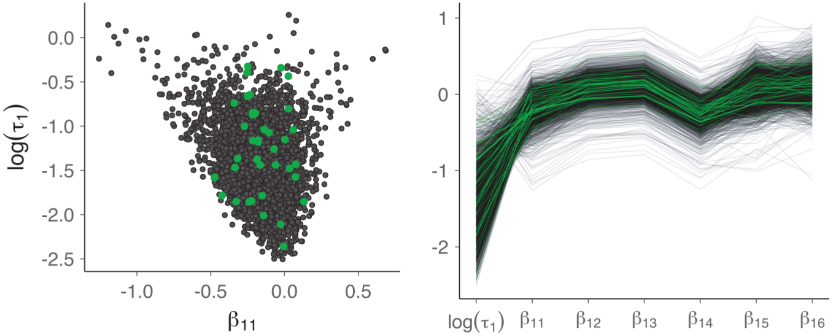
Generative model: $\theta^* \sim p(\theta) \longrightarrow y^* \sim p(y|\theta^*) \Longleftrightarrow y^* \sim p(y)$



(a) Vague priors (b) Weakly informative priors (c) Comparison

MCMC diagnostics

Moving beyond trace plots

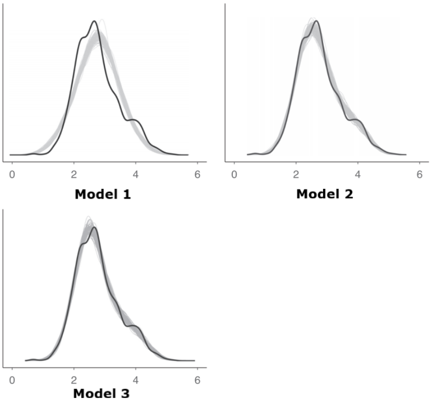


Posterior predictive checks

How did we do?

Posterior predictive distribution: $p(\tilde{y}|y) = \int d\theta \; p(\tilde{y}|\theta)p(\theta|y)$

$$\theta^* \sim p(\theta|y) \longrightarrow \tilde{y}^* \sim p(\tilde{y}|\theta^*) \iff \tilde{y}^* \sim p(\tilde{y}|y)$$

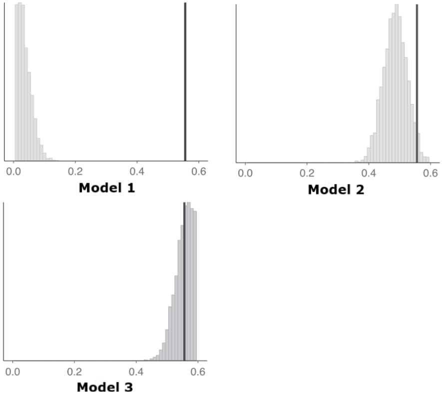


Posterior predictive checks

How did we do?

Posterior predictive distribution: $p(\tilde{y}|y) = \int d\theta \; p(\tilde{y}|\theta)p(\theta|y)$

$$T(\tilde{y}) = \text{skew}(\tilde{y})$$



Model comparison

Looking *when* and *where* a model is better than another

LOO predictive distrubution: $p(y_i|y_{-i})$