Numerical Methods for Partial Differential Equations A.Y. 2022/2023

Laboratory 01

Finite Element method for the Poisson equation in 1D

Exercise 1.

Let $\Omega = (0, 1)$. Let us consider the Poisson problem

$$\begin{cases} (\mu(x) \ u'(x))' = f(x) & x \in \Omega = (0,1) \\ u(0) = u(1) = 0 \end{cases}$$
 (1a)

with $\mu(x) = 1$ for $x \in \Omega$, and

$$f(x) = \begin{cases} 0 & \text{if } x \le \frac{1}{8} \text{ or } x > \frac{1}{4}, \\ -1 & \text{if } \frac{1}{8} < x \le \frac{1}{4}. \end{cases}$$

- **1.1.** Write the weak formulation of problem (1).
- **1.2.** Write the Galerkin formulation of problem (1).
- 1.3. Write the finite element formulation of problem (1), using piecewise polynomials of degree r. Write the associated linear system, and express the integrals involved by means of numerical quadrature formulas.
- **1.4.** Implement in deal.II a finite element solver for (1), using piecewise polynomials of degree r = 1 and with a number of mesh elements N + 1 = 20.