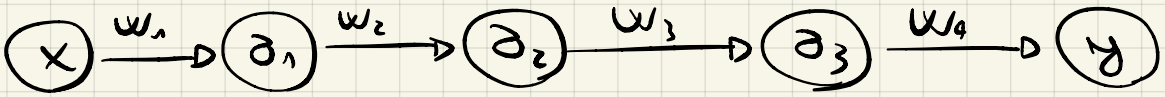

$x \in \mathbb{R}, y \in \mathbb{R}$

$$a_{i+1} = \sigma(w_{i+1} a_i), \quad i = 0, \dots, 3 \qquad (a_0 = x, a_4 = y)$$

$$\frac{\partial y}{\partial x} = \frac{\partial y}{\partial a_3} \cdot \frac{\partial a_3}{\partial a_2} \cdot \frac{\partial a_2}{\partial a_1} \cdot \frac{\partial a_1}{\partial x}$$

$$y = \sigma(w_4 a_3) = \sigma(w_4 \cdot \sigma(w_3 a_2))$$
$$= \sigma(w_4 \sigma(w_3 \sigma(w_2 a_1)))$$
$$= \sigma(w_4 \sigma(w_3 \sigma(w_2 \sigma(w_1 x))))$$

$$\sigma(x) = \frac{1}{1+e^{-x}}$$

$$\sigma'(x) = \frac{d}{dx}(1+e^{-x})^{-1} = -(1+e^{-x})^{-2} \frac{d}{dx}(1+e^{-x})$$

$$= -(1+e^{-x})^{-2} \frac{d}{dx} e^{-x}$$

$$= -(1+e^{-x})^{-2}(-e^{-x})$$

$$= (1+e^{-x})^{-2} e^{-x}$$

$$= \frac{e^{-x}}{(1+e^{-x})^2} = \frac{1}{1+e^{-x}} \cdot \frac{e^{-x}}{1+e^{-x}}$$

$$= \sigma(x) \cdot \frac{e^{-x}}{1+e^{-x}} = \sigma(x) \cdot \frac{1-1+e^{-x}}{1+e^{-x}}$$

$$= \sigma(x) \cdot \frac{-1+1+e^{-x}}{1+e^{-x}}$$

$$= \sigma(x)\left[\frac{-1}{1+e^{-x}} + 1\right]$$

$$= \sigma(x)\left[1 - \frac{1}{1+e^{-x}}\right] = \sigma(x)(1-\sigma(x))$$

$$\sigma'(x) = \sigma(x)(1-\sigma(x))$$

$$y = \sigma(w_4 \sigma(w_3 \sigma(w_2 \sigma(w_1 x))))$$

$$\frac{\partial y}{\partial a_3} = \frac{\partial \sigma(w_4 a_3)}{\partial a_3} = w_4 \sigma(w_4 a_3)(1 - \sigma(w_4 a_3))$$

$$\frac{\partial a_3}{\partial a_2} = \frac{\partial \sigma(w_3 a_2)}{\partial a_2} = w_3 \sigma(w_3 a_2)(1 - \sigma(w_3 a_2))$$

$$\frac{\partial a_2}{\partial a_1} = \frac{\partial \sigma(w_2 a_1)}{\partial a_1} = w_2 \sigma(w_2 a_1)(1 - \sigma(w_2 a_1))$$

$$\frac{\partial a_1}{\partial x} = \frac{\partial \sigma(w_1 x)}{\partial x} = w_1 \sigma(w_1 x)(1 - \sigma(w_1 x))$$

$$\frac{\partial y}{\partial x} = \frac{\partial y}{\partial a_3} \cdot \frac{\partial a_3}{\partial a_2} \cdot \frac{\partial a_2}{\partial a_1} \cdot \frac{\partial a_1}{\partial x}$$

$$= w_1 w_2 w_3 w_4 \sigma(w_1 x)(1 - \sigma(w_1 x)) \sigma(w_2 a_1)(1 - \sigma(w_2 a_1))$$

$$\cdot \sigma(w_3 a_2)(1 - \sigma(w_3 a_2)) \sigma(w_4 a_3)(1 - \sigma(w_4 a_3))$$