

Problem 1: Noise pollution due to cars

Question a)

First check the assumptions of the ANOVA model:

- Normality of the groups:

```
> Ps <- c(
  shapiro.test(noise[fuel == "diesel" & category ==
"commercial"])$p.value,
  shapiro.test(noise[fuel == "diesel" & category ==
"passenger"])$p.value,
  shapiro.test(noise[fuel == "ethanol" & category ==
"commercial"])$p.value,
  shapiro.test(noise[fuel == "ethanol" & category ==
"passenger"])$p.value,
  shapiro.test(noise[fuel == "gasoline" & category ==
"commercial"])$p.value,
  shapiro.test(noise[fuel == "gasoline" & category ==
"passenger"])$p.value
)
Ps
> Ps
0.3574334 0.2930456 0.5325407 0.7372111 0.1878295 0.7897125
```

All p-values are greater than 0.1, so we can't reject at any significant level the normality of the groups.

- Homoscedasticity:

```
> bartlett.test(noise ~ fuel_category)

      Bartlett test of homogeneity of variances

data:  noise by fuel_category
Bartlett's K-squared = 7.968, df = 5, p-value = 0.158
```

The p-value is greater than 0.1, so we can't reject at any significant level the homoscedasticity of the groups.

Now that I have checked the assumptions, I can perform the ANOVA test:

Complete ANOVA model with interactions:

$$X_{ijk} = \mu + \tau_i + \beta_j + \gamma_{ij} + \epsilon_{ijk}$$
 with:

- μ : overall mean

- τ_i : effect of fuel i
- β_j : effect of category j
- γ_{ij} : interaction effect of fuel i and category j
- $\epsilon_{ijk} \sim \mathcal{N}(0, \sigma^2)$: error term

```
> summary(aov(noise ~ fuel + category + fuel:category))
```

	Df	Sum Sq	Mean Sq	F value	Pr(>F)	
fuel	2	375.8	187.91	16.040	7.28e-07	***
category	1	21.8	21.84	1.864	0.1749	
fuel:category	2	63.8	31.91	2.723	0.0699	.
Residuals	114	1335.5	11.72			

From the summary I can see that:

- Test 1:

$H_0: \gamma_i = 0, i = 1, \dots, 6$ vs $H_1: (H_0)^c$

that is:

- H_0 : the effect of the fuel doesn't significantly affect the noise pollution
- H_1 : the effect of the fuel significantly affects the noise pollution

-> the p-value for this test is 0.0699: I reject at 10% the null hypothesis but not at 5% and 1%.

-> we don't have strong evidence that the interaction term has effect

- Test 2: $H_0: \tau_i = 0, i = 1, 2, 3$ vs $H_1: (H_0)^c$

that is

- H_0 : the effect of the fuel doesn't significantly affect the noise pollution
- H_1 : the effect of the fuel significantly affects the noise pollution

-> the p-value for this test is $\sim 7e-07$: I reject at 1% the null hypothesis so I can conclude that the effect of the fuel significantly affects the noise pollution

- Test 3: $H_0: \beta_i = 0, i = 1, 2$ vs $H_1: (H_0)^c$

that is

- H_0 : the effect of the category doesn't significantly affect the noise pollution
- H_1 : the effect of the category significantly affects the noise pollution

-> the p-value for this test is ~ 0.1749 : I reject at any significant level the null hypothesis so I can conclude that the effect of the category doesn't significantly affect the noise pollution

