

Ex3

a) Estimates of the coefficients

$$\beta_0 = 9.186768$$

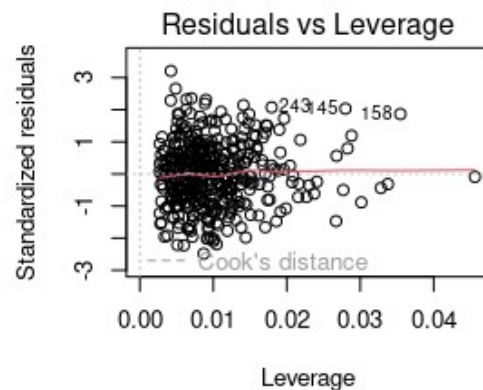
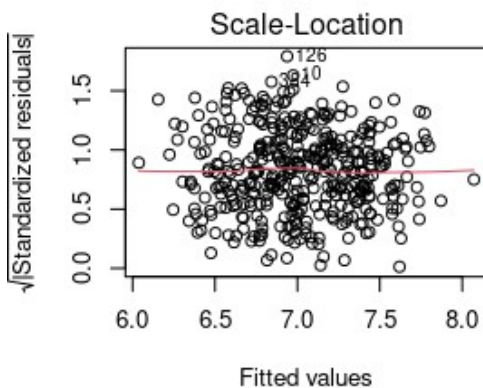
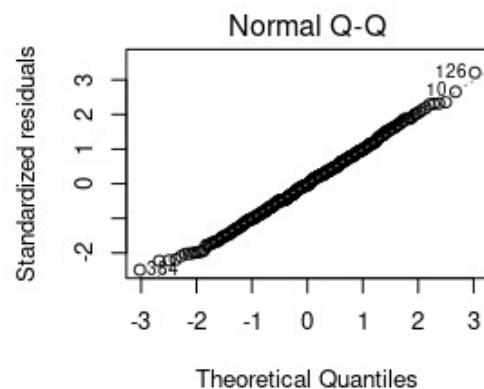
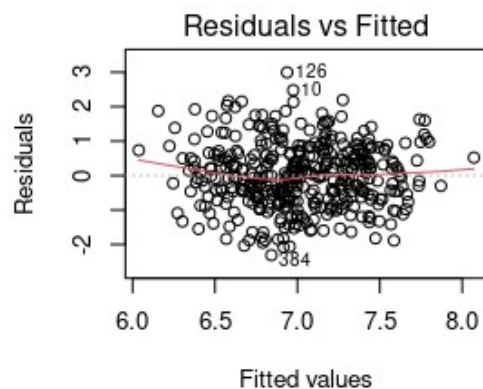
$$\beta_1 = 0.099302$$

$$\beta_2 = 0.072583$$

$$\beta_3 = -0.008890$$

$$\sigma = 0.9350057$$

b) Plot of the residuals



p-value of shapiro test on residuals : 0.6654

Homoscedasticity and normality of the residuals verified (normality needed in order to do inference on the model) *because...*

c)

The test is $H_0: (\beta_1, \beta_2) = (0, 0)$ vs $H_1: (\beta_1, \beta_2) \neq (0, 0)$

p-value is essentially zero, we reject the null hypothesis: loudness and energy can NOT be BOTH discarded from the model

d)

We perform the test $H_0: \beta_2 = 0$ vs $H_1: \beta_2 \neq 0$

p-value is 0.0731, we accept the null hypothesis: we erase from our model the dependency on energy

New estimate for the coefficients

$$\beta_0 = 9.182772$$

$$\beta_1 = 0.170266$$

$$\beta_2 = 0 \text{ (since we removed it from the new model)}$$

$$\beta_3 = -0.008962$$

$$\sigma = 0.9376268$$

e)

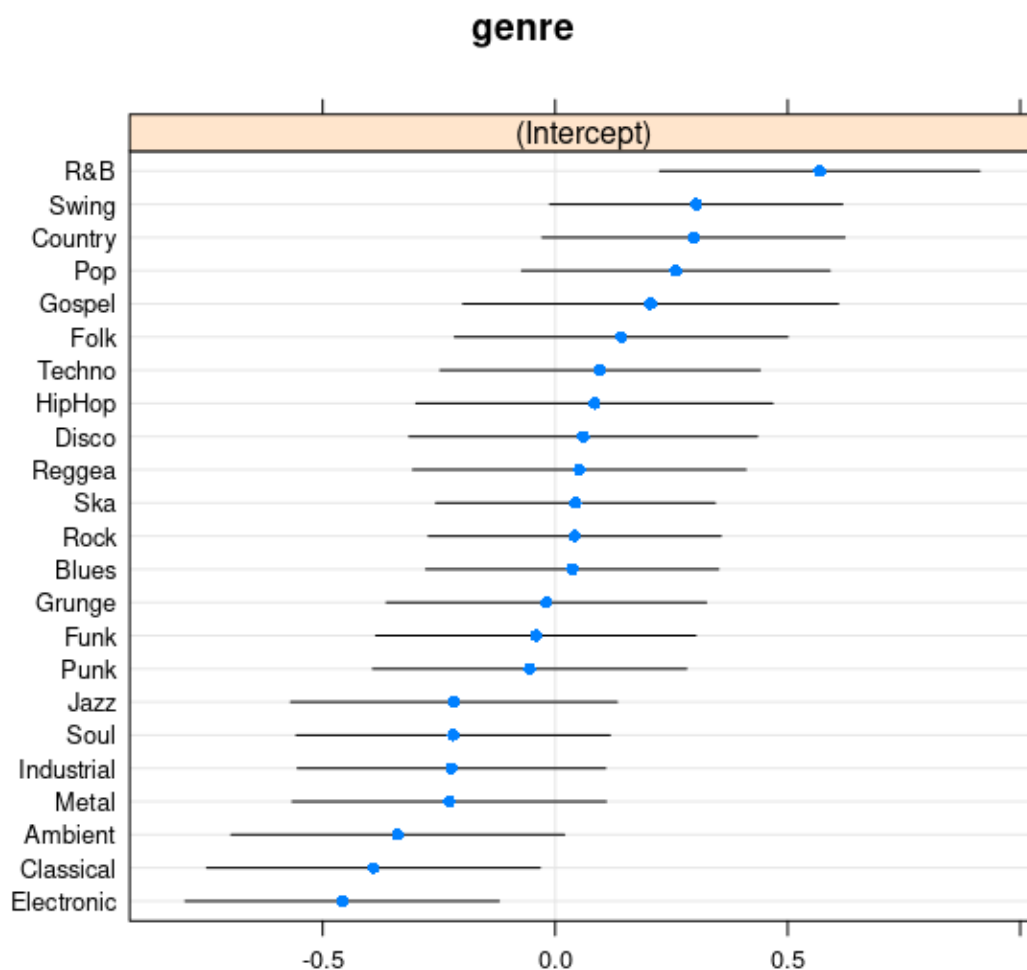
$$\text{MODEL: danceability}_{ij} = \beta_0 + \beta_1 \cdot \text{loudness}_{ij} + \beta_2 \cdot \text{energy}_{ij} + \beta_3 \cdot \text{tempo}_{ij} + b_i + \text{eps}_{ij}$$

$$\text{eps}_{ij} \sim N(0, \sigma^2_{\text{eps}})$$

$$b_i \sim N(0, \sigma^2_b)$$

After fitting the model, and estimate the two variances σ^2_{eps} and σ^2_b , we compute $\text{PVRE}=0.1046363$

f) Dotplot:



Since the fixed effects are the same for each group, the genre associated to the highest danceability is R&B because it has an higher value of random effect on the intercept