IF $n > p$ MORE SAMPLES THAN FEATURES

$A \in \mathbb{R}^{n \times p}$ SAMPLES ON ROWS, FEATURES ON COLUMNS

$A = U \Sigma V^T$ $\quad U \in \mathbb{R}^{n \times p}, \Sigma \in \mathbb{R}^{p \times p}, V^T \in \mathbb{R}^{p \times p}$

$\Rightarrow$ V RIGHT SINGULAR VALUES
↳ PRINCIPAL DIRECTIONS

$\Phi = AV$ PRINCIPAL COMPONENTS
(PROJECTION OF DATA ON THE PRINCIPAL DIRECTIONS)

$\Phi \in \mathbb{R}^{n \times p}$

IF $A' \in \mathbb{R}^{p \times n}$, FEATURES ON ROWS, SAMPLES ON COLS

$A' = U' \Sigma' V'^T$ $\quad U' \in \mathbb{R}^{p \times p}, \Sigma' \in \mathbb{R}^{p \times p}, V'^T \in \mathbb{R}^{p \times n}$

$A = A'^T = V' \Sigma' U'^T$

$\Rightarrow$ U' LEFT SINGULAR VALUES
↳ PRINCIPAL DIRECTIONS

$\Phi = U'^T A'$ PRINCIPAL COMPONENTS

$\Phi \in \mathbb{R}^{p \times n}$

IF $n < p$ MORE FEATURES THAN SAMPLES

$A \in \mathbb{R}^{n \times p}$ SAMPLES ON ROWS

$A = U \Sigma V^T, \quad U \in \mathbb{R}^{n \times n}, \Sigma \in \mathbb{R}^{n \times n}, V^T \in \mathbb{R}^{n \times p}$

$\Rightarrow$ V RIGHT SINGULAR VALUES
↳ PRINCIPAL DIRECTIONS

$\Phi = AV$ PRINCIPAL COMPONENTS

$\Phi \in \mathbb{R}^{n \times n}$

$A' \in \mathbb{R}^{p \times n}$ SAMPLES ON COLUMNS

$A' = U' \Sigma' V'^T$ $\quad U' \in \mathbb{R}^{p \times n}, \Sigma' \in \mathbb{R}^{n \times n}, V'^T \in \mathbb{R}^{n \times n}$

$\Rightarrow$ U' LEFT SINGULAR VALUES
↳ PRINCIPAL DIRECTIONS

$\Phi = U'^T A, \quad \Phi \in \mathbb{R}^{n \times n}$ PRINCIPAL COMPONENTS