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Homework 8: Differential Equations

- Make sure that you scripts run without error in order to get credit. Do not hesitate to ask for help if needed!
- Check the output of the Autograder for any issue that should be fixed. In case of "Unexpected error", email me so I can take a look as it may be an issue with the Autograder rather than your code.
- Take ownership of your learning! Remember that you are responsible for the work you turn in. Simply copying somebody else's answers, copying from the Internet, using AI to generate your code, sharing your code (or part of your code) in any way, or copying it from someone else will be considered academic dishonesty. Please, contact me if you have any questions about collaborations.

Problem 1 Air drag on a vertical motion

Consider an object moving vertically along the y-axis, subject to the gravitational force and to air drag. For "regular-sized" object moving at "normal" speed, the force of air drag can be model as being quadratic in the object's speed, in a direction opposite the velocity of the object.

For our object moving along the vertical direction, with a position given by $y(t)$, Newton's equation can then be written as:

$$m\frac{\mathrm{d}^2y}{\mathrm{d}t^2} = -mg - \alpha \cdot s \cdot \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2, \tag{1}$$

where α sets the strength of the drag force and s represents the sign of $\frac{\mathrm{d}y}{\mathrm{d}t}$, so the force is in the direction opposite the velocity $v_y \equiv \frac{\mathrm{d}y}{\mathrm{d}t}$.

While this is a second-order differential equation for $y(t)$, we can rewrite it as a first-order differential equation for the velocity $v_y(t)$:

$$m\frac{\mathrm{d}v_y}{\mathrm{d}t} = -mg - \alpha \cdot s \cdot (v_y(t))^2\,. \tag{2}$$

This can then be solved for $v_y(t)$ using the method discussed in this section.

For this exercise, we are principally interested in the velocity as we want to study the terminal velocity of the object (Remember that the terminal velocity of the final velocity reached by an object falling with air drag when the drag force is equal and opposite to the gravitational force, so the net force is zero).

Your tasks for this exercise are the following (using a mass $m = 1\,\mathrm{kg}$ and $\alpha = 0.5$):

- Write a program that solves equation (2) for $v_y(t)$ from $t = 0$ to such a time when the terminal velocity has been reached. Write to file not only t and $v_y(t)$, but also the drag force $F_d(t)$ (last term in Eq. (2); you might want to have a function that calculates that force since you'll need it in multiple places.)

- Make plots showing $v_y(t)$ as well as the drag and gravitational forces as a function of time for 3 values of the initial velocity $v_y(0)$: 1) the object starts from rest; 2) the object initially moves upward at $10\,\mathrm{m/s}$; and 3) the object initially moves downward at $10\,\mathrm{m/s}$.
  What do you observe? Does it make sense to you? Make sure you understand what you observe and have the python code that makes the plots write an explanation of what you observe to the screen.

Problem 2 Position vs time with air drag

Use the data from problem 1 (i.e. read it from the data file you created) to calculate the position vs time function of the object moving vertically with air drag, for each of the three cases from problem 1, namely with a positive, negative, or zero initial speed. Remember that $v(t) = \frac{\mathrm{d}x}{\mathrm{d}t}$, so $x(t)$ is the solution of a differential equation with initial condition $x(0) = x_0$.

You can pick any value for x_0, and plot all three curves on the same plot (with appropriate legend) over a time range sufficient to clearly shows the part where the object has reached terminal velocity, but also shows the initial accelerated motion.

What to submit: For each problem, submit your `Python` code and the figure(s) you created. There is no need to include the data files as those will be re-created by the problem 1 code.