

Actividad

① Resistencia de una viga

$$S(w) = Kw h^2 = Kw(24^2 - w^2) = Kw(576 - w^2)$$

$$S'(w) = K[(576 - w^2) + w(-2w)] = K(576 - w^2 - 2w^2) = K(576 - 3w^2)$$

$$K(576 - 3w^2) = 0$$

$$576 - 3w^2 = 0 \rightarrow 3w^2 = 576 \rightarrow w^2 = 192 \rightarrow w = \sqrt{192} = 8\sqrt{3} = 13.86$$

$$S(0) = K(0)(576 - 0^2) = 0$$

$$S(24) = K(24)(576 - 24^2) = 0$$

$$S(8\sqrt{3}) = K(8\sqrt{3})(576 - (8\sqrt{3})^2) = 3072\sqrt{3} K = \underline{48221.7 \sqrt{3}}$$

$$h = \sqrt{576 - (8\sqrt{3})^2} = 8\sqrt{6} \quad \underline{w = 8\sqrt{3} \quad h = 8\sqrt{6}}$$

② Iluminación

$$\alpha = \tan^{-1}\left(\frac{h}{s}\right) \quad s = \sqrt{s^2 + 4} \quad I = K \frac{\frac{h}{s}}{h^2 + 4} = K \frac{\frac{h}{\sqrt{h^2 + 4}}}{h^2 + 4} = K \frac{h}{(h^2 + 4)^{3/2}}$$

$$I'(h) = K \frac{(h^2 + 4)^{3/2} - 3h^2(h^2 + 4)^{1/2}}{(h^2 + 4)^3}$$

$$I'(h) = 0 \rightarrow (h^2 + 4)^{3/2} - 3h^2(h^2 + 4)^{1/2} \rightarrow (h^2 + 4) = 3h^2 \rightarrow 4 = 2h^2 \rightarrow h^2 = 2 \rightarrow h = \sqrt{2}$$

$$I(h) = 0.096 K$$

③ Fuerza mínima

$$F = \frac{Kw}{\cos(\theta)} + K \sin(\theta) \quad F'(\theta) = \frac{-Kw(-\sin(\theta)) + K \cos(\theta)}{(\cos(\theta) + K \sin(\theta))^2}$$

$$-\sin(\theta) + k \cos(\theta) = 0 \quad k = 0.1$$

$$\rightarrow -\sin(\theta) + (0.1) \cos(\theta) = 0 \rightarrow \tan(\theta) = \frac{\cos(\theta)}{\sin(\theta)} = 10 \quad \theta = \tan^{-1}(10) = \underline{84.29^\circ}$$

$$F(84.29) = \frac{Kw}{\cos(84.29)} + K \sin(84.29) = \underline{5.03 N}$$

④ Volumen de una caja

$$V(x) = (13 - 2x)^2 x = 169x - 52x^2 + 4x^3$$

$$V'(x) = 169 - 104x + 12x^2$$

$$169 - 104x + 12x^2 = 0 \quad x = \frac{-(-104) \pm \sqrt{(-104)^2 - 4(12)(169)}}{2(12)}$$

$$x_1 = 6.5 \quad x_2 = 2.17 \quad x_2 \text{ es el adecuado} \quad 0 < x < 6.5$$

$$V(2.17) = (13 - 2(2.17))^2 (2.17) = \underline{162.74 \text{ u}^3}$$

⑤ Minimización del tiempo

$$t = \frac{\sqrt{9^2 + (15-x)^2}}{4} + \frac{x}{5} \quad t'(x) = \frac{-(15-x)}{4\sqrt{9^2 + (15-x)^2}} + \frac{1}{5}$$

$$\frac{-(15-x)}{4\sqrt{9^2 + (15-x)^2}} + \frac{1}{5} = 0 \quad \text{Usando software nos da } 15-x=12$$
$$\underline{x=3}$$

$$t(3) = \frac{\sqrt{9^2 + (15-3)^2}}{4} + \frac{3}{5} = \underline{4.35}$$