

Project 4 (0.2 points)

- *Input:* non-zero natural numbers k and n with $k \leq n$
- *Output:*
 1. the number of k -dimensional subspaces of the vector space \mathbb{Z}_2^n over \mathbb{Z}_2
 2. a basis of each such subspace (for $1 \leq k \leq n \leq 6$)

Example: The vector space \mathbb{Z}_2^3 over \mathbb{Z}_2 has 8 vectors, namely $(0,0,0)$, $(0,0,1)$, $(0,1,0)$, $(0,1,1)$, $(1,0,0)$, $(1,0,1)$, $(1,1,0)$, $(1,1,1)$. Any 2-dimensional subspace has a basis with two vectors. There are $C_7^2 = 21$ possibilities to choose 2 vectors out of the 8 vectors of \mathbb{Z}_2^3 , but some of them will generate the same subspace. Only 7 choices will generate different subspaces.

- *Input:* $k = 2$, $n = 3$
- *Output:*
 1. the number of 2-dimensional subspaces of the vector space \mathbb{Z}_2^3 over \mathbb{Z}_2 is 7
 2. a basis of each such subspace is:

$((0,0,1),(0,1,0))$	$((0,1,0),(1,0,1))$
$((0,0,1),(1,0,0))$	$((0,1,1),(1,0,0))$
$((0,0,1),(1,1,0))$	
$((0,1,0),(1,0,0))$	$((0,1,1),(1,0,1))$

Note:

- Any (reasonable) programming language may be used.
- The solutions will consist of the source code with comments (do not send executable files!) and at least 5 relevant input and output files, and will be sent to the e-mail address: septimiu.crivei@ubbcluj.ro.
- If necessary, you will be asked to explain your solution.
- The first 25 solutions will be rewarded.
- The final deadline is January 14, 2024.