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| T1 | _____ | | | F1 |
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| T4 | _____ | | | F4 |
| | | Problem Chosen | | |
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2015 Mathematical Contest in Modeling (MCM) Summary Sheet
 (Attach a copy of this page to each copy of your solution paper.)

Abstract

Ebola has killed more 8000 people in Africa in 2014. Nowadays, a new medication which can stop Ebola and cure patients was announced by the world medical association. However, how to construct the delivery system and distribute drugs is also a challenge. We regard 14 districts of the whole Sierra Leone from the official site of World Health Organization (WHO) (57 weeks) as the object of study because it is the severest country in the world. The process of modeling is presented as following.

Firstly, the mortality tends to be higher due to the Ebola. In order to solve the problem of drug delivery, we apply improved susceptible and infected (SI) model to introduce the spread of Ebola in Sierra Leone.

Secondly, we proposed an indicator called severity of Ebola (SE) to measure this severity in every district. The spread of the flow in the tube resembles the spread of Ebola disease in people due to the similar disordered. Therefore, the spread factors which effect the degree of disordered include local people, infected people and areas merge into the certain parameter (SE).

Thirdly, in order to obtain an optimize delivery system, we use the hierarchical cluster method to cluster the districts. In the process of the clustering, we define an abstract distance according to the geographical distance and the values of SE. And then we can obtain the coordinates and scales of delivery centers to delivery drugs.

Lastly, we use the Logistics Model to study the manufacture of newly developed medicine and develop a strategy to distribute medicine. And then, we consider four different strategies including SE, population, the number of infected people and the possibility of getting infected at the outbreak time of the virus. Moreover, we simulate these strategies by a model called susceptible, infected and removed (SIR) model. As a result, the SE strategy reveals the best conclusion due to its fast decrease rate of infected people. After that, we calculate the dosage of each delivery center at the stable time of the virus according to the need of the decrease rate of the infected people, the SE strategy and pesticide effect.

In a word, the strategy based on the SE model is a better way to control the Ebola virus and it could be followed in any other countries. Besides that, if we can consider the pesticide effect and the different periods of the morbidity, we will get a more intensive result.

Divide and conquer the steps to eradicate Ebola

Team # 37545

February 10, 2015

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Key Words: Ebola; Medicine delivery; SE Modal; Sierra Leone

1 Introduction

The tendency of the spread can be figured out by a mathematical model called SI model. To measure the severity of Ebola, an indicator should be defined which contains factors as much as possible. While establishing the feasible delivery systems and locating the delivery, it is significant to place the delivery centers which are the core of delivery systems. After assuming the speed of manufacturing of the drug, a distribution plan of delivering medication could be drafted according to the production of drugs. Also, this distribution plan is related to the geographical position of delivery centers.

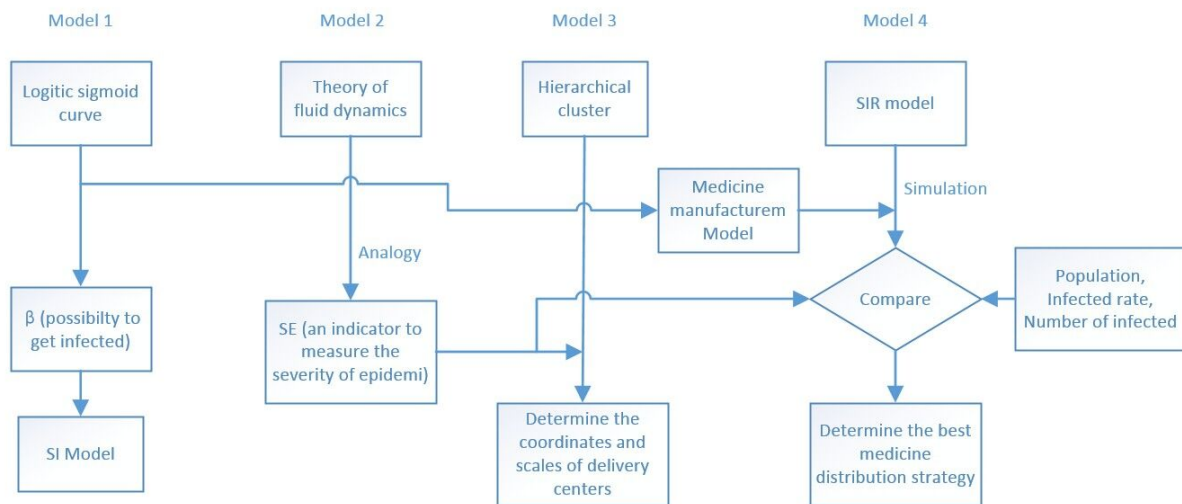


Figure 1: Flow Chart

2 Restatement of the problem

With the research of the Ebola deepened, a new medication has been developed to control the spread of Ebola. So how to distribute medication, place the delivery center as well as manage other critical factors attaches great importance to optimize the eradication of Ebola. Besides that, to establish a useful model, we need to consider the speed of manufacturing and demand of the medicine.

In order to make our model more realistic and sensible, all factors just mentioned should be settled down. As a result, we need to dig out a mathematical model to simulate the spread of Ebola. Exact coordinates $d(i,j)$, population N , the number of infected I , regional area A of every districts in Serra Leone. Besides that, some important messages about the medication should also been figured out, such as the function which reflects the conjunction between drug production and the time of progression t . Certainly, other factors should be considered.

3 Problem analysis

To solve this problems, we should propose a general model simulating the delivery system and the plan of distributing medicine. We plan to complete the establishment of the model into 4 steps.

Firstly, the spread of Ebola is a significant factor we need to consider. What we should do is to find a way to figure out the spread of disease and the tendency of Ebola. Besides that, our model should be able to predict the development of the disease which could help us work out the next models.

Secondly, we need to define a parameter to measure the severity of the disease spreading. There are some measurements already existed that could do the same thing, for example, the infected rate and the number of infected. However, using the prevalence rate to measure the severity is too limited, it can be measured by number of common factors.

Thirdly, we are supposed to model to solve the problems about delivery, such as establishing the delivery system, determining locations of delivery. In our model, we introduced a concept called delivery centers as the center of delivery system whose exact positions and scales should be figured out. The criterion to elect delivery centers should be based on the parameter we defined in the second model.

Finally, after placing the delivery centers, we could establish a model about medicine. We should take speed of manufacturing the drugs but also the distribution plan of drug based on the quantity of the medicine needed in every area into consideration. We are also going to figure out the effect of the medicine after distributing.

4 Assumption

- **The possibility of an individual from a certain area to be infected will not change in a period of time.**
The possibility that individual get Ebola is determined by many factors. The most important factor is the local health situation. However, it is obvious that the situation could not change frequently. As a result, we assume this possibility is stable in a period of time.
- **The model has taken many factors into account.**
- **Ebola transmit only among people. People will not get infected by animals, food, air etc.**
- **The total population of Sierra Leone remains unchanged.**
- **The virus will not mutate.**
- **The positions of delivery centers are not influenced by geographical factors.**
It is hard for us to get the current local geographical conditions. As a result, we assume that the delivery centers could be built at every coordinates we work out.
- **The largest delivery centers are able to send medicine to five districts, and the smallest delivery centers are the center for only one district.**
Because the characteristics of hierarchical cluster method, the process of clustering will not stop until there is only one cluster left. That means we could get only one delivery center every time we calculate. It is obviously unreasonable. Thence, we made the assumptions above.
- **The relationship between medicine production and time is in line with logistic.**
The problem mentions that the medication has just been developed. The initial yield could be fairly low while it will grow continuously over time.

5 Notations

Table 1: Symbol Table Constants

| Symbol | Definition | Units |
|------------------------|---|---------------|
| N | the amount of local people | number |
| I | the amount of infected people | number |
| t | the development time of disease | time |
| i | the radio of infected people | unitless |
| A | the area | area |
| S | the amount of healthy people | number |
| s | the radio of healthy people | unitless |
| SE | the severity of the Ebola disease | unitless |
| β | probability of an individual to get infected in unit time | unitless |
| τ | the function speed of the health center | time |
| u | the velocity of flow | length/time |
| ν | the viscosity of flow | poise |
| l | the equivalent diameter of the tube | length |
| ρ | the density of flow | mass/volume |
| μ | the other factors | unitless |
| $\sigma_{contact}$ | the frequency of the contacts among people | number/person |
| $\sigma_{facilities}$ | the extent of the cleanness | number/area |
| $\sigma_{policies}$ | the strength of the isolation | unitless |
| $P_{i,...,j} = (x, y)$ | The coordinate of the districts | unitless |
| SE_i | the severity of the Ebola disease in a city | unitless |
| $d(i, j)$ | the distance between two countries | length |
| D_a | the cluster distance | length |

6 Our model

6.1 Model 1 Ebola Spreading Model

We established a model based on the SI epidemic model[1]. We take current methods of dealing with Ebola into consideration and make the parameter β a variable of time. We use this model to explain the trend of Ebola confirmed cases in Sierra Leone and estimate the β of different cities. Moreover, our model also reveals the possibility of the resurgence of Ebola.

6.1.1 SI epidemic model

In SI epidemic model, $S(t)$ represent the number of individuals which are not infected in time t, $I(t)$ represent the number of individual already infected with epidemic. Because Ebola has a high death rate, individuals recover from Ebola are not taken into consideration in our model. Common SI epidemic model can be represent in the following equations:

$$\frac{di}{dt} = \beta * i * (i - i), i(0) = i_0$$

β denotes the probability for an individuals to have Ebola, which is regarded to be same among people in a certain district. When β is a constant value, i behave as a logistic sigmoid function

In our model. However, considering the effect of medical treatment and policy, β is a variable about time.

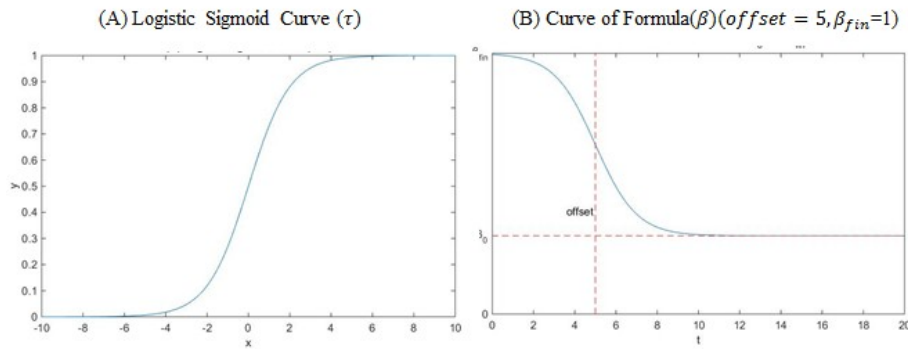


Figure 2: Curve of Logistic Sigmoid and Formula(β)

We assume the trend ϵ is similar to logistic sigmoid curve as shown in Fig(2)(A). The equation of logistic sigmoid is:

$$y = \frac{1}{1 + e^{-\frac{x}{\tau}}}$$

We represent β as:

$$\beta = (\beta_0 - \beta_{min} \frac{1}{1 + e^{-\frac{t - offset}{\tau}}}) + \beta_{min}$$

We assume $offset = 5$. when $t = 0$

$$\beta = (\beta_0 - \beta_{min} \frac{1}{1 + e^{-5}}) + \beta_{min} = 0.993 * (\beta_0 - \beta_{min} + \beta_{min} \approx \beta_0)$$

When Ebola starts out breaking, people lack efficient methods to resist the spread of Ebola, so β_0 is of a quite high level. When infected people are found in a certain area, government and some organization will highly focus on this issue and give their support to infected area. With new method introduced and temporary medical center built in infectious area, β goes down. When epidemic break out in rural area, it's quite hard to transport instrument medicine into it and poor living condition may make situation worse. So β will decrease slowly during this period. With efficient transportation route established and local resident changing their poor hygiene habits, β will decrease in a higher speed. However, it's hard to suppress β to 0 by simply applying isolation and treatment. We assume that β will convergent in a constant value. In Ebola case, isolation is proved to be a very successful method, so β_{fin} can be a quite small value. When β_{fin} is nearly 0, the trend of infected people will be seemly a horizontal line. As shown in Fig(3). Although it will still increase very slowly, the spread of epidemic is under control.

It's interesting to find that when β_{min} fall into a certain range. The trend of spreading can be a little "deceive". In Fig(3)(B) in t_1 , the increase rate of infected people are very small, it seems that epidemic is under control. But our model indicate a resurgence of infected in the nearly future.[2]

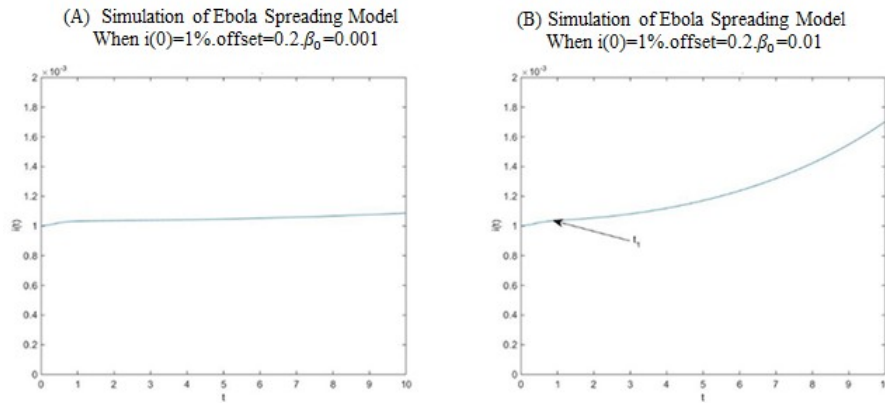


Figure 3: Simulation of Ebola Spreading Model with Different Parameter

6.1.2 Model Testing

We use the data of Sierra Leone to test our model. As shown in the data from WHO, the spread of Ebola can be restrained by the isolation. When the disease break out at the beginning, the local government and the WHO take action to isolate the infected people from health people. At first, this method played a great role in controlling the development of patients. We assume that $\beta_{fin} = 0, \tau = 1$.

It is hard to estimate the β_0 of each city, which is related to a lot of different factor in each district like hygienic condition, population density and the living habit of local. We use the newest data of infected rate to estimate β_0 . Which is shown in Tab(2)

| | | | | | | |
|-----------|---------|---------|-----------|----------|-----------|--------------------|
| Bo | Bombali | Bonthe | Freetown | Kailahun | Kambia | Kenema |
| 0.8 | 0.95 | 0.05 | 1 | 0.45 | 0.4 | 0.75 |
| Koinadugu | Kono | Moyamba | Port Loko | Pujehun | Tonkolili | Western Rural Area |
| 0.65 | 0.7 | 0.75 | 1 | 0.3 | 0.85 | 0.95 |

Table 2: β_0

We find that districts with high density like Freetown and Western Rural Area have a quite high value of β , which is accordant with the result of our analysis.

As indicated in Fig(4), our model has its accuracy in some degree. However, the current β value may not be 0, so there are still possibilities of the outbreak of Ebola.

6.2 Model 2 Virus Flow Model

Virus flow model is concluded from the hydrodynamics. In the following, we briefly illustrate the concept of the development of flow in hydrodynamics. Then, we discuss the relationship between our virus flow model and the theory of hydrodynamics. Finally, we establish the virus flow model.

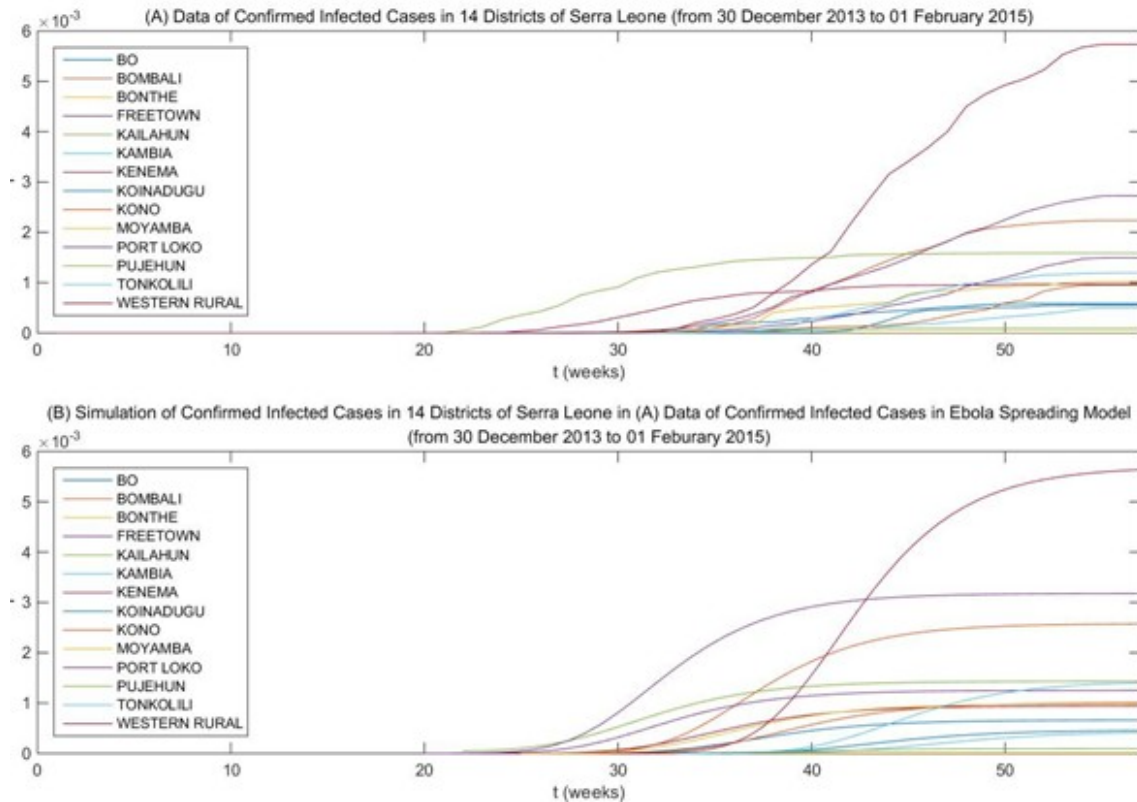


Figure 4: Contrast between Data and Simulation

6.2.1 The theory in hydrodynamics

There are two major conditions in the flow of fluid. [3]One is laminar flow, another one is turbulent flow (Figure 5a and b). when the gas flow passes through a circular tube, the flow condition will be changed from laminar flow to turbulence one. With the development of the flow, the laminar one is much more ordered than the turbulence one, which means the molecules of the gas in turbulence flow collide more intensely than those are in the laminar flow. In fact, most of flow is turbulent.

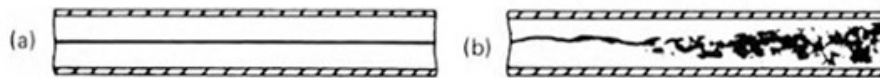


Figure 5: Two conditions of flow. This figure is from John Ward-Smith Mechanics of Fluids 2005

Although the conditions of flow can be roughly divided into those two types, we also want to know the exact extent of disorder. In hydrodynamics, a method called nondimensionalization is used to describe it. A scientist named Reynolds found that the degree of disorder is related to the character of flow. After a large number of experiments, Reynolds established a dimensionless number (Re) to describe the degree of disorder in a quantitative way. The formula is as following:

$$Re = \frac{\rho u l}{\mu}$$

where u is the velocity of the flow, ρ is the density of the gas, μ is dynamic viscosity of the gas, l is the characteristic length.

The size of Re can represent the degree of disorder in flow. If Re becomes larger, the flow becomes more chaotic.

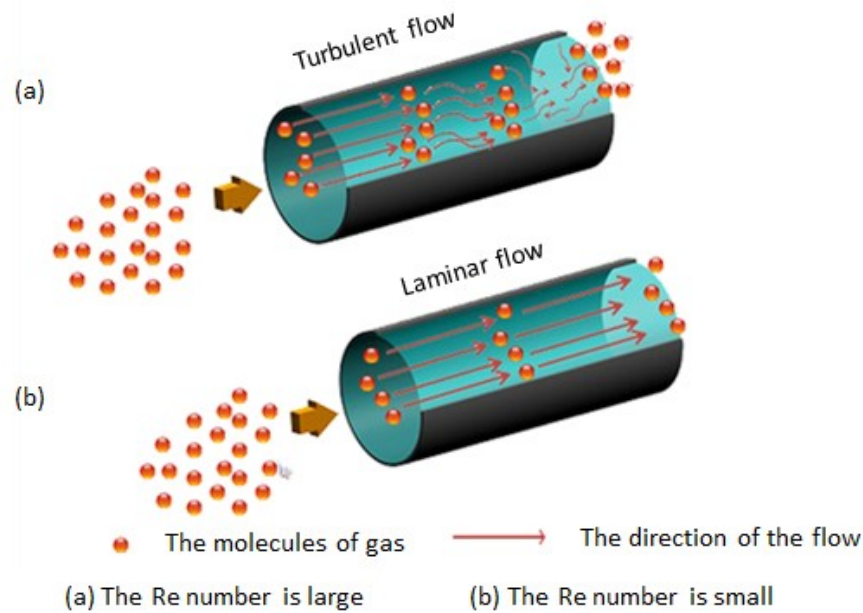


Figure 6: The connection between Re number and the flow conditions

As shown in Fig(6), we assume that the severity of illness is parallel to the flow in a tube. We analyze the process of the flow and the process of the virus and then prove that our analogy is feasible. The analyze process is following.

The Ebola virus cannot be transported by the air.(reference) The transmission between people's body fluid and blood is the only way to spread the virus. Thus, by analyzing the behaviors of people, we can know the method of the spread. The virus flow and the gas flow have three major similar characters:

- The gas is always behaved in groups which called micelle, while Human beings are social animals and like living in groups too. [4] The virus can only be transmitted with the people. So, the virus molecules also behave in groups
- As we all know, the density or viscosity of the gas is much lower than that of fluid. [3][4] As for the virus, there are more than $10 \wedge 31$ virus particles in the ocean which we can calculate that the density of virus is $1.38 * 10^{17}$ per cubic meter. [5] As a result, the amount of the water molecules in ocean is $4.658 * 10^{45}$, which means the density of virus is also less than the water molecules.
- The gas flow cannot be described by a continuity equation due to the large distance among different gas molecules and the intermittent movement of gas flow . [3] Similarly, virus can only be transmitted by either air or the contact between people.

So, the virus molecules are similar to the fluid molecules. The crowd of people can be parallel to the area, which the flow sweeps through. As the flow develops, the quantity of flow becomes larger. So, the velocity of flow becomes faster. According to the formula, we know that the larger Re number means the more disordered flow. This phenomenon resembles the spread of Ebola disease. As the Ebola disease develops, it will infect more people and the virus will generate themselves. So the velocity of virus will become faster.

6.2.2 The virus flow model

We want to get an amount from the comprehensive formula. This amount can measure the severity of illness and how urgent this district need medication. So, based on the analogy we have discussed above, we can set up the comprehensive formula in a way, which is similar to the Re number.

This formula is a comprehensive standard to measure the severity of illness. And we can also use this formula to assess a district's urgency to get medication. So, we must take these following factors into account:

- The amount of the local people N
- The amount of the infected people I
- The increase velocity of the infected people $\frac{dI}{dt}$
- The area of this district S .
- **Other factors.** We take the other factors into account, which can influence the amount of the comprehensive formula. These factors include the local culture, the healthy conditions, the fitness of local people and so on. We determine three factors which can influence the severity of Ebola.

(1) $\sigma_{contact}$. This parameter measure the frequency of the contacts among people. We use the number of contacts per person to measure the frequency.

(2) $\sigma_{facilities}$. This parameter concerns about the health facilities in the district. We make this parameter connect with the area. The number of health facilities per area can represent the extent of the cleanness.

(3) σ_{policy} . Aimed at eradicate Ebola, some local governments have formulated some policies to isolate the infected people. We use σ_{policy} to measure the strength of the isolation.

So, we can use a parameter to measure the above three factors. We have

$$\mu = \sigma_{contact} * \sigma_{facilities} * \sigma_{policy}$$

The fundamental formula is following

$$Re = \frac{\rho ul}{\mu}$$

Then, because the density of gas molecule can be parallel to the density of local infected people, we have

$$\rho = \frac{iN}{S}$$

In a short time, we assume that the amount of local people remain changeless. The equivalent diameter of the tube is also constant. So, we have

$$l = N$$

We can use $\frac{dI}{dt}$ to measure the velocity of the virus spread, we have already know that

$$\frac{dI}{dt} = \beta i(1 - i)$$

So, we have

$$\mu = \beta i(1 - i)$$

The Viscosity Coefficient of the flow, which is a constant during the spread of gas, is only in connection with the gas attribute. The physical meaning of Viscosity Coefficient is that it shows the degree of resistance, which the tube impose to the gas flow. We have discussed that there are also many other factors, which will influence the spread condition of virus. We can put all these factors into a parameter ϵ . This parameter can take the place of ϵ in the Reynolds formula. So, we have

$$v = \mu$$

We put the above equations into simultaneous equations, then, we have

$$SE = \frac{N \star \frac{iN}{S} \star \beta \star i \star (1 - i)}{\mu}$$

When we simplify this equation, we have

$$SE = \frac{N^2 i^2 \beta (1 - i)}{\sigma_{contact} \sigma_{facilities} \sigma_{policy}}$$

6.2.3 Testing of Model 2

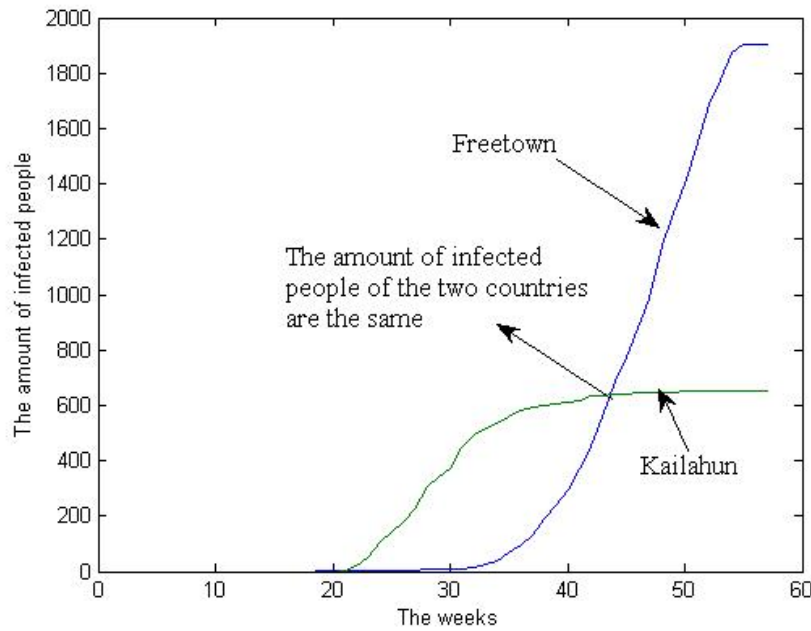


Figure 7: The analysis of the Model 2

We can learn from the above figure that the amount of infected people cannot reflect the severity of local disease. At the day of 43, Freetown and Kailahun have the same number of infected people, but the future development of disease in these two cities is very different. However, the SE number can measure the tendency of infected people's development.[4] We put the data and parameters of these two cities into Matlab and we get that $S_{Freetown} > S_{Kailahun}$.

6.3 Model 3 Delivery centers planning Model

6.3.1 The Hierarchical clustering

[5] In order to determine the position and the scale of the delivery center, we use the hierarchical clustering method (reference) with agglomerative strategies. The steps of hierarchical clustering method are described as below [6]

- (1) Supposing each sample is a cluster
- (2) Each time merge the lowest distance between two clusters
- (3) Add the new cluster which merged in the step (2) into cluster set, and delete two clusters just mentioned
- (4) repeat step (2)(3) until there is only one cluster in the cluster set

In this part, fourteen districts in Sierra Leone are regarded as basic clusters with a specific SE and we use the hierarchical clustering method to cluster them. It should be noted that the statistics is elected from a long period which could make our model more precise. [7]

Firstly, to decide which clusters should be combined, we need to explore a measure of dissimilarity between sets of observations. In most cases, it is accomplished by use of an appropriate metric and a linkage criterion which specifies the distances of different nodes. As a result, we define a distance called infectious distance based on the model we built in part 2.

$$D_a = d / (SE_i + 0.0001) * (SE_j + 0.0001)$$

Through the calculation, we get a matrix with one row and ninety-one columns. After that, convert it into a symmetric matrix. It contains the D_a between any two districts.

Secondly, we use the method to cluster fourteen cities in Sierra Leone. The dendrogram for clustering is shown in Figure 8.

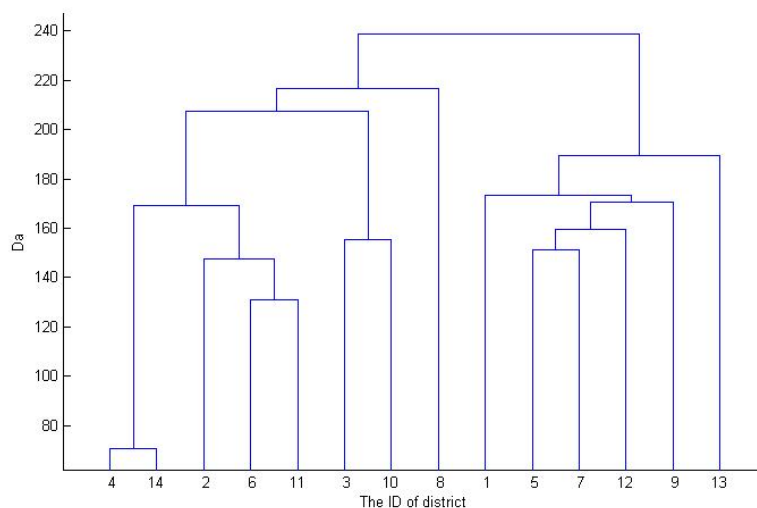


Figure 8: The dendrogram of cluster

Thirdly, we cut the dendrogram to make it meaningful. We cluster fourteen cities into five sets and define the scales of delivery centers with the number of one set contained. The division value we set is 180. The result of clustering is shown in Table (3)

The result can be presented in a geographical map(see Figure 9)



Figure 9: The coordinates of the delivery centers

Finally, we need to determine the exact position of the delivery center in each area. The coordinate of delivery centers can be calculated :

$$P(i....j) = \frac{\sum SE_i * P_i}{\sum SE_i}$$

Through the calculation we get the specific coordinates of five delivery centers.

| The Id of district | District Name | The ID of district | District Name |
|--------------------|---------------|--------------------|---------------|
| 1 | BO | 8 | KOINADUGU |
| 2 | BOMBALL | 9 | KONO |
| 3 | BONTHE | 10 | MOYAMBA |
| 4 | FREETOWN | 11 | PORT LOKO |
| 5 | KAILAHUN | 12 | PUJEHUN |
| 6 | KAMBIA | 13 | TONKOLILI |
| 7 | KENEMA | 14 | WESTERN RURAL |

Table 3: Distinct's name

| The level of center | The combination ID | Coordinates |
|---------------------|--------------------|------------------|
| 1 | 8 | (9.50°N,11.50°W) |
| 1 | 13 | (8.67°N,11.67°W) |
| 2 | 3,10 | (8.37°N,11.30°W) |
| 5 | 4,14,2,6,11 | (8.70°N,12.67°W) |
| 5 | 1,5,7,12,9 | (8.30°N,11.10°W) |

Table 4: The Coordinates and scales of delivery centers

6.3.2 The testing of Model

After we get the exact coordinates of delivery centers, we could compare them with the real positions of treatment centers in Sierra Leone. Because the treatment centers have a great demand of medicine, if the treatment centers and delivery centers are placed in the same place, it means that our model is precise. The result is shown in Figure

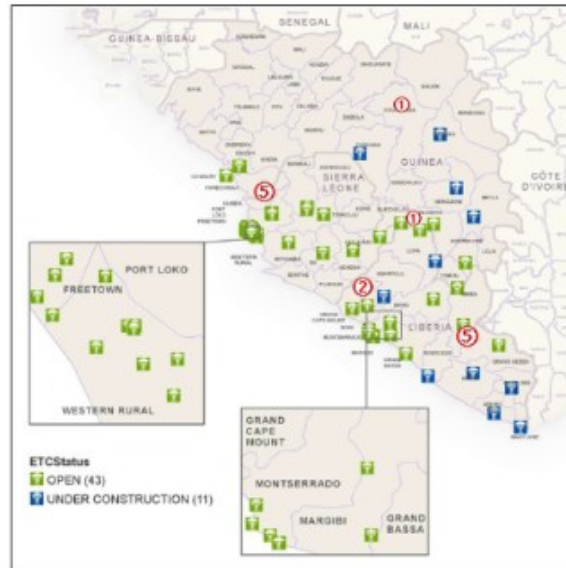


Figure 10: The coordinates of the delivery centers

6.4 Model 4 Medicine Distribution and Medicine Quantities Estimation

We use Logistics Model to study the manufacture of newly developed medicine and develop a strategy to distribute medicine, according to indicator released in Model 2. Then we use SIR model to simulate the trend of infected when applied different medicine distribution strategy. Finally, given the product rate and the effect of a medicine, we can estimate the quantities of medicine we needed.

6.4.1 SIR (the full name)model

When a medicine which can cure the epidemic disease efficiently is available, the number of people who recovered from disease increase so the SI epidemic model 'the full name' cannot suit this situation, the SIR model is more accurate for this case.

SIR model can be represent as

$$\begin{cases} \frac{dS}{dt} = -\lambda S(1-p)I \\ \frac{dI}{dt} = \lambda S(1-p)I - \mu I \\ \frac{dR}{dt} = \mu I \\ S(t) + I(t) + R(t) = 1 \end{cases}$$

In this model, p denote the degree of isolation, which we assume to be constant. Parameter μ represent the rate of recovery, which is related to the efficacy of medicine, the quantity of medicine and the severity of epidemic in an area. We assume:

$$\mu = k_{medicine} \frac{Value of Medicine}{Population}$$

$k_{medicine}$ is a constant related to the characteristic of medicine.

6.4.2 Medicine Manufacture Model

There are several process in the introduction of a new medicine. After it is developed in laboratory and ready to come into use. Manufacturer have to organize a new production, materials supply chain line. It will take a time period before this medicine can be large-scale manufactured. In our model, we assume that the speed of newly-developed medicine manufacture is accordant with Logistic curve as shown in Fig(9). Denote the total quantities of our medicine as $M(t)$.

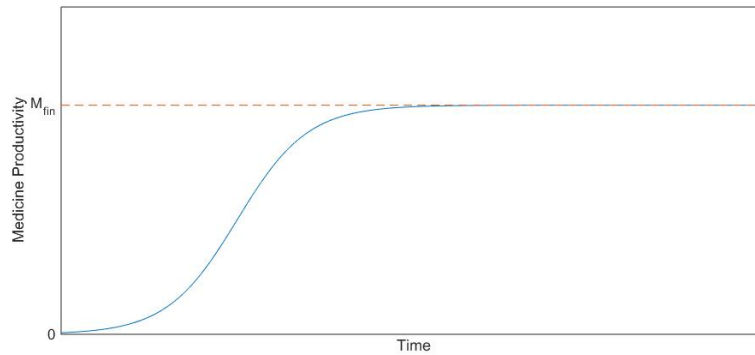


Figure 11: Medicine Manufacture Model

$$M(t) = M_{fin} \frac{1}{1 + e^{-\frac{t - offset}{\tau_{medicine}}}}$$

We assume $offset = 5 * \tau_{medicine}$, which is similar to our model of ϵ . When $t = 0$.

M_{fin} represent the target productivity which is decided by manufacturer which should be accordant with the quantity of medicine needed.

At the bAs the value of medicine is limited.

We had to elaborate a strategy to distribution medicine into different district considering the epidemic situation.

6.4.3 Distribution Strategy Based On Different Indicator

The strategy of medicine distribution should be based on the situation of different area. However, it's hard to estimate the severity of epidemic accurately. So we had to select an indicator to represent the overall situation best.

(1) Basic distribution model

We are going to distribute those medicine to different areas $A_1, A_2, A_3, \dots, A_N$. The indicator we use is $Indicator_i$. In our model, we decide the medicine distribution for each areas is :

$$m_i = \frac{\text{Indicator}_i}{\sum_{n=1}^N \text{Indicator}_n} M(t)$$

Combining the SIR model with the distribution model, we get:

$$\mu = \frac{k_{\text{medicine}} M_{\text{fin}}}{\text{Population}} \frac{\text{Indicator}_i}{\sum_{n=1}^N \text{Indicator}_n} \frac{1}{1 + e^{-\frac{t - \text{offset}}{\tau_{\text{medicine}}}}}$$

(2) Select the indicator

There are a lot of indicator can reveal the severity of disease in a district, which can be used to make decision in medicine distribution. However it is hard to measure the overall situation fully.

In our model, we used indicator SE, which raised in model 3. In SIR model

$$SE = \frac{N^2 i^2 \beta_s}{\sigma_{\text{contact}} \sigma_{\text{facilities}} \sigma_{\text{policy}} S}$$

SE can reflect the influence of N, i, β and $\frac{di}{dt}$ and the factor about the overall situation of an area. If we use the indicator which can be directly measured like population or infected rate, some aspect of the target area cannot be taken into consideration.

6.4.4 Model Testing

We use the data of Sierra Leone to test our model. The data of the newly developed medicine is hard to obtain. We assume $\tau_{\text{medicine}} = 1$ week. So the model is decided by $\frac{k_{\text{medicine}} M_{\text{fin}}}{\text{Population}}$ and β . We have already got the estimated value of β_0 of different districts. So we can estimate the value of β .

We assume the medicine studied by us come up into use in 29 September 2014. Because the total value of medicine in the beginning is very small which can be ignored. So the trend predicted by our model should be accordance with the data we already have. With k already known, we use the data obtained about the later trend to estimate $\frac{k_{\text{medicine}} M_{\text{fin}}}{\text{Population}}$.

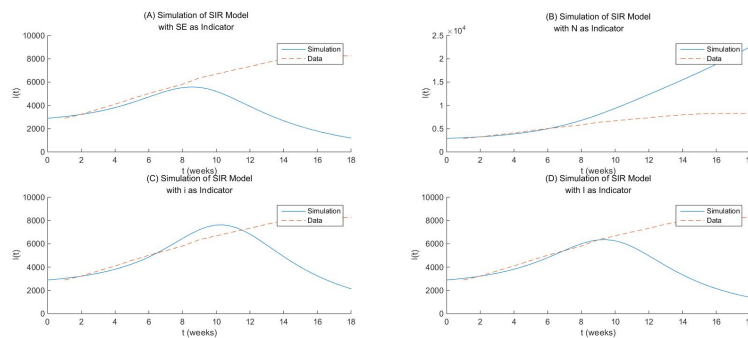


Figure 12: Contrast of Simulation of Different Distribution Strategy

As shown in Fig(12)(ABCD). We get the result of simulation of trends of the amount of infected people I with different distribution strategy. The Fig(12)(A) is about using the indicator

SE proposed in model 2. It has the lowest I. In Fig(12)(B), we use the population as the indicator to make the strategy. Which ends in a quite bad result. Distributing medicine according to the population cannot reflect the severity of epidemic so it is not a good strategy.

In Fig(12)(B)C we distribute medicine in consideration of the number of the infected $I(t)$ and the infected rate $i(t)$. Those two indicator can present the severity of epidemic in some degree. However, they cannot reflect the population and the overall situation of an area. So they also don't perform as well as strategy A.

Then we can use our model to predict the trend of epidemic situation after we start to distribute our medicine with the strategy according to indicator SE in 01 February 2015.

Due to the new policy or isolation method applied. We can predict k values decrease a lot. We assume k to be 0. The result of prediction is shown in Fig(12). We can also get $k_{medicine} M_{fin}$. In most cases, $k_{medicine}$ can be measured by experiment in laboratory, so we can further predict M_{fin} , which represent the quantities of medicine we need.

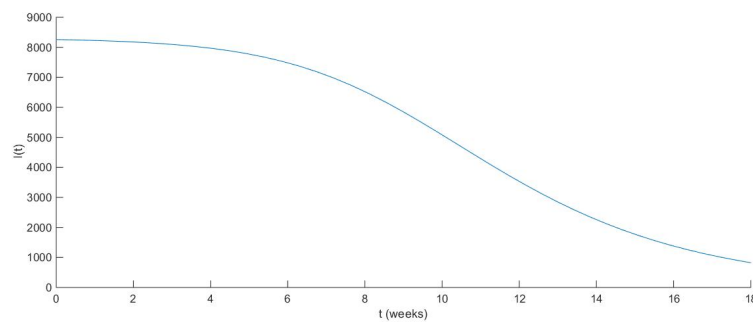


Figure 13: Prediction of $I(t)$ When After Using An Assumed Medicine with SE Distribution Strategy

7 Conclusions

7.1 Model 1 Ebola Spreading Model

The spread of the Ebola is in line with the SI epidemic model.

In addition, the existence of treatment center could decrease the value of β and stabilize the infected rate in a specific area. With our Ebola Spreading Model, we could predict the development of the disease. It indicates that there is the possibility of another outbreak.

Besides that, our model explains the phenomenon that the disaster center moved from the south to the west of Sierra Leone.

7.2 Model 2 Virus Flow Model

In Model 2, we set up a virus flow model, which is similar to the gas flow model in hydrodynamic.

In hydrodynamic, the Re number represents the degree of disorder in the gas flow. So, we can get a SE number, which is similar to Re number, to measure the severity of disease.

SE number has taken many factors into account. Using SE number to measure the severity of disease can get a much better effects. In the last part of Model 2, we use the examples of Freetown and Kailahun to prove the Superiority of our model.

7.3 Model 3 Delivery centers planning Model

By using the hierarchical cluster, we determine five coordinates of delivery centers in Sierra Leone(see Fig.8). Also, this model can be applied in any other countries to calculate the delivery centers.

7.4 Model 4 Medicine distribution Strategy and Medicine Quantities Estimation

We firstly develop a model to learn the trend of production rate of a newly developed medicine after it come into use. Then estimate different medicine distribution strategy using SIR model and data from 14 districts of Serra Leone, we find distributing medicine according to the indicator SE can lead a better result than distributing medicine in accordant with population, number of infected or infected rate. Finally we propose a new method to estimate the quantities of medicine needed.

In addition to the above-mentioned conclusions, our four models could also be seemed as steps to find out a proper solution to eradicate Ebola which can be applied in any other country.

8 Overview: Strengths and weaknesses of the Model

We present a model which can measure the severity of disease. The model use a number called SE number to assess the degree of disease. Based on the amount of SE number, we can set up another model to distribute the medication.

- **The model connects the disease severity with the theory in hydrodynamics.**
We use an analogy to describe the characteristic of the spread of virus and establish a comprehensive formula. This analogy can be proved feasible. Thus, our formula has its theory evidence. And, the physical meaning between the two things can be also corresponded with each other.
- **The model has taken many factors into account.**
We consider the traffic conditions, the local culture, the body health and so on. So, the model is an exact one.
- **The model can predict the future tendency of the disease.**
We compare real data and our model. We found that the model curve can accord to the reality.
- **The model can be used in a larger scale.**
Although we build our model based on a certain district, Sierra Leone, the method to analyze this country can also be used to analyze other countries. The parameters can be used commonly.
- **The model have given a delivery system to help distribute the meditation.**
If the condition of virus changes, then we can also use our model and system to optimize the medical distribution. The delivery system is not constant, it can develop as the condition of disease changes.

At the same time, our model have also some weaknesses:

- **The model's parameters can hardly be gotten.** We use some parameters which are blur parameters. So, the outcome will have some error.
- **We don't know the exact production process of the medication, so it is difficult for the model to predict future tendency.**
- **In the hydromechanics model, we ignore the influence of flow boundary.** The flow boundary means that the flow in the tube does not fully correspond to the Re number. At first, the flow is laminar, and then, it become turbulent. As the flow develops, it finally become laminar again because of the influence of boundary.

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LETTER

Ebola virus disease (EVD), which is an international health problem, has killed more 8000 people in Africa in 2014. These days, a new medication has developed. What we need to do is establishing a general model to simulate the spread of Ebola as well as to put forward a method to construct the delivery system and distribute drugs. However, there are so many factors affecting the establishment of model. As a result, we convert this complex model into four basic models. These four models are also four steps which we could follow in eradicating or restricting the development of Ebola.

The first step is to obtain the spread of Ebola. If we could predict the tendency of Ebola development, our strategy to be determined will be more effective. We find that this tendency can be imitated by a mathematical model. Therefore, we solve the first problem properly. The second step is to measure the severity of Ebola in every district. This measurement can help us determine a method to delivery drugs. For example, if the value of a district is extremely high, we are supposed to delivery more drugs to this district. The advantage of this measurement is also being proved by contrasting.

Then, according to the severity of Ebola in every district and the geographical distance between two districts, we can divide all districts into some clusters. In this model, we are going to place the delivery centers which are cores of clusters. Drugs could be allocated through these centers.

The last step, we assume that the speed of the productivity is suited to an existed model. And then, we consider four different strategies. Moreover, we simulate these strategies by another model. As a result, the strategy reveals the best conclusion due to its fast decrease rate of infected people in the 14 districts of the whole Sierra Leone. After that, we calculate the dosage of each delivery center at the stable time of the virus according to the need of the decrease rate of the infected people.

In general, we get the exact coordinates and scales of delivery centers. Besides that, we figure out the tendency of the quantities of infected people, we proved that our distributing method is better than any other method by contrasting.