Homework 5

Due at 11 AM on Thursday, March 2.

Question 1. Consider a random variable X such that

$$p_X(x) = \begin{cases} \frac{x^2}{a}, & \text{for } x \in \{-3, -2, -1, 1, 2, 3\}, \\ 0, & \text{otherwise,} \end{cases}$$

where $a > 0$ is a real parameter.

(a) Find a.

(b) What is the PMF of the random variable $Z = X^2$?

(c) Find $E[X]$.

Question 2. 148 students are registered for CIS 2033 this academic year. The course is taught in 4 different sections, by 4 different professors, and the sections have 40, 33, 25, and 50 students, respectively. One of the students is randomly selected. Let X denote the number of students that were in the same class as this randomly selected student. One of the 4 professors is also randomly selected. Let Y denote the number of students in his class.

(a) Which of $E[X]$ or $E[Y]$ do you think is larger? Give your reasoning in words.

(b) Compute $E[X]$ and $E[Y]$.

Question 3.

(a) Tossing a fair die is an experiment that can result in any integer number from 1 to 6 with equal probabilities. Let X be the number of dots on the top face of a die. Compute $E[X]$ and $Var(X)$.

(b) Every day, the number of network blackouts has a distribution (probability mass function)

x	0	1	2
$P(x)$	0.7	0.2	0.1

A small internet trading company estimates that each network blackout results in a \$500 loss. Compute expectation and variance of this company's daily loss due to blackouts.

(c) There is one error in one of five blocks of a program. To find the error, we test three randomly selected blocks. Let X be the number of errors in these three blocks. Compute $E[X]$ and $Var(X)$.

Question 4.

(a) We throw a coin until a head turns up for the first time. Let p be the probability that a throw results in a head and assume that the outcome of each throw is independent of the previous outcomes. Let X be the number of times we have to throw the coin.

   i. What is $E[X]$?

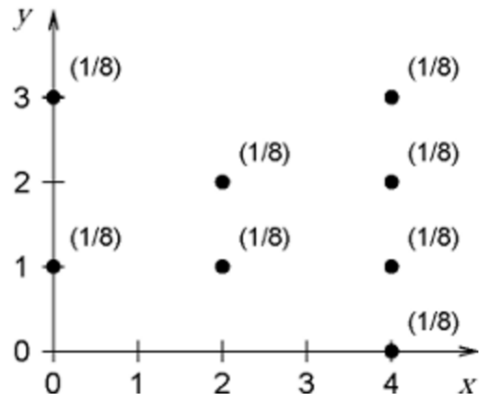
   ii. What is $Var(X)$?

(b) We throw a coin until a head turns up for the second time. Let p be the probability that a throw results in a head and assume that the outcome of each throw is independent of the previous outcomes. Let X be the number of times we have to throw the coin.

   i. Determine $P(X = 2), P(X = 2)$, and $P(X = 4)$.

   ii. Show that $P(X = n) = (n - 1)\, p^2\, (1 - p)^{n-2}$ for $n \geq 2$.

Question 5. Consider a sample space comprised of eight equally likely event points, as shown below:



1. Which value or values of x maximize $E\,[\,Y \mid X = x\,]$?
2. Which value or values of y maximize $Var\,(X \mid Y = y)$?
3. Let $R = min(X, Y)$. Provide a clearly labeled sketch of $p_R(r)$.
4. Let A denote the event $X^2 \geq Y$. Determine numerical values for the quantities $E\,[XY]$ and $E\,[XY \mid A]$

Question 6. *Extra Credit of 2 quiz points. To receive this credit, explain your solution (verbally) to your instructor before the midterm.*

A particular professor is known for his arbitrary grading policies. Each paper receives a grade from the set {A, A-, B+, B, B-, C+}, with equal probability, independently of other papers. How many papers do you expect to hand in before you receive each possible grade at least once?