## Homework 1

Instructor: David Dobor

(Due at 11 AM Thursday, January 26)

**Question 1.** Let E and F be two events in a sample space for which P(E) = 1/4, P(F) = 1/2, and  $P(E \cap F) = 1/6$ . What is  $P(E \cup F)$ ?

**Question 2.** Let A and B be two events for which one knows that the probability that at least one of them occurs is 2033/3302. What is the probability that neither A nor B occurs? Hint: use one of DeMorgan's laws:  $A^c \cap B^c = (A \cup B)^c$ 

**Question 3.** Consider a sample space that is the rectangular region  $[0,1] \times [0,2]$ , i.e., the set of all pairs (x,y) that satisfy  $0 \le x \le 1$  and  $0 \le y \le 2$ . Consider a "uniform" probability law, under which the probability of an event is half of the area of the event. Find the probability of the following events:

- (a) The two components x and y have the same values.
- (b) The value, x, of the first component is smaller than or equal to the value, y, of the second component.
- (c) The value  $x^2$  is smaller than or equal to the value of y.

**Question 4.** We toss a coin three times and write down the sample space for this experiment:

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$$\Omega = \{HHH, THH, HTH, HHT, TTH, THT, HTT, TTT\}$$

where *T* stands for tails and *H* for heads.

- (a) Write down the set of outcomes corresponding to each of the following events:
  - *A* : "we throw tails exactly two times."
  - *B* : "we throw tails at least two times."
  - C: "tails did not appear before a head appeared."
  - *D* : "the first throw results in tails."
- (b) What is the probability that event *A* occurs?
- (c) Look back at your answer in part (b). Can your answer be correct? How so? What assumptions did you make to compute P(A)?
- (d) Can you compute the probability of event *B* by making the assumption that all outcomes of the experiment are equally likely? If you answered 'yes', please compute it; if you answered 'no', please explain why?
- (e) Write down the set of outcomes corresponding to each of the following events:

$$A^c$$
,  $A \cup (C \cap D)$ ,  $A \cap D^c$ .

**Question 5.** Let *A*, *B*, and *C* be disjoint subsets of the sample space. For each one of the following statements, determine whether it is true or false. Note: "False" means "not guaranteed to be true."

$$P(A) + P(A^c) + P(B) = P(A \cup A^c \cup B)$$
 (1)

$$P(A) + P(B) \le 1 \tag{2}$$

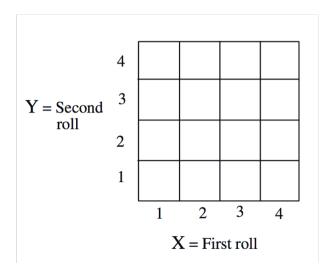
$$P(A^c) + P(B) \le 1 \tag{3}$$

$$P(A \cup B \cup C) \ge P(A \cup B) \tag{4}$$

**Question 6.** Prove that if events A and B are such that  $A \subseteq B$ , then  $P(A) \leq P(B)$ . Do events A and B from question 4 satisfy the conditions on A and B in this problem?

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**Question 7.** You are rolling two tetrahedral dice. Let *X* and *Y* stand for the faces of the first and second die, respectively. We discussed the sample space of this experiment in class, and here it is depicted for your convenience again:



Assume that all outcomes of your experiment are eqaully likely.

Let  $Z = \min(X, Y)$ , let  $U = \max(X, Y)$ , and let S be the sum of the faces showing, i.e. S = X + Y.

- (a) What is P(Z=3)?
- (b) What is P(U=3)?
- (c) What is P(S is even )?

*Hint:* it may be illuminating to fill in the appropriate squares in this picture as you try to answer these questions.