Homework 1

(Due at 11 AM Thursday, January 19)

Question 1. Let E and F be two events in a sample space for which $P(E) = 1/4, P(F) = 1/2$, and $P(E \cap F) = 1/6$. What is $P(E \cup F)$?

Question 2. Let A and B be two events for which one knows that the probability that at least one of them occurs is 2033/3302. What is the probability that neither A nor B occurs? Hint: use one of DeMorgan's laws: $A^c \cap B^c = (A \cup B)^c$

Question 3. Consider a sample space that is the rectangular region $[0, 1] \times [0, 2]$, i.e., the set of all pairs (x, y) that satisfy $0 \leq x \leq 1$ and $0 \leq y \leq 2$. Consider a "uniform" probability law, under which the probability of an event is half of the area of the event. Find the probability of the following events:

(a) The two components x and y have the same values.

(b) The value, x, of the first component is smaller than or equal to the value, y, of the second component.

(c) The value x^2 is smaller than or equal to the value of y.

Question 4. We toss a coin three times and write down the sample space for this experiment:

$$\Omega = \{HHH, THH, HTH, HHT, TTH, THT, HTT, TTT\}$$

where T stands for tails and H for heads.

(a) Write down the set of outcomes corresponding to each of the following events:

- A : "we throw tails exactly two times."
- B : "we throw tails at least two times."
- C : "tails did not appear *before* a head appeared."
- D : "the first throw results in tails."

(b) What is the probability that event A occurs?

(c) Look back at your answer in part (b). Can your answer be correct? How so? What assumptions did you make to compute $P(A)$?

(d) Can you compute the probability of event B by making the assumption that all outcomes of the experiment are equally likely? If you answered 'yes', please compute it; if you answered 'no', please explain why?

(e) Write down the set of outcomes corresponding to each of the following events:

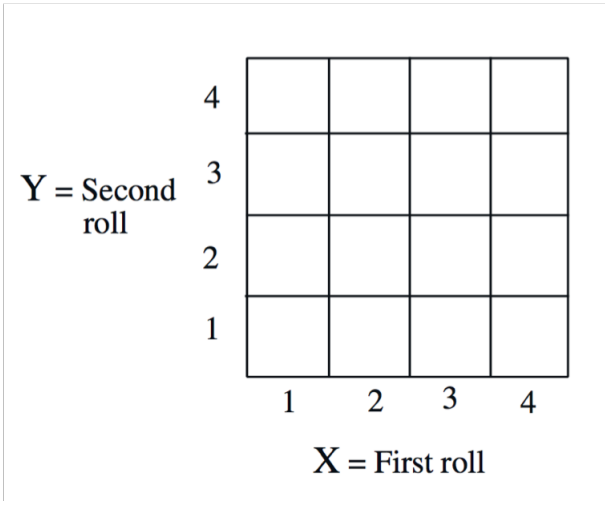
$$A^c, \quad A \cup (C \cap D), \quad A \cap D^c.$$

Question 5. Let A, B, and C be disjoint subsets of the sample space. For each one of the following statements, determine whether it is true or false. Note: "False" means "not guaranteed to be true."

$$P(A) + P(A^c) + P(B) = P(A \cup A^c \cup B) \tag{1}$$
$$P(A) + P(B) \leq 1 \tag{2}$$
$$P(A^c) + P(B) \leq 1 \tag{3}$$
$$P(A \cup B \cup C) \geq P(A \cup B) \tag{4}$$

Question 6. Prove that if events A and B are such that $A \subseteq B$, then $P(A) \leq P(B)$. Do events A and B from question 4 satisfy the conditions on A and B in this problem?

Question 7. You are rolling two tetrahedral dice. Let X and Y stand for the faces of the first and second die, respectively. We discussed the sample space of this experiment in class, and here it is depicted for your convenience again:



Assume that all outcomes of your experiment are eqaully likely.

Let $Z = \min(X, Y)$, let $U = \max(X, Y)$, and let S be the sum of the faces showing, i.e. $S = X + Y$.

(a) What is $P(Z = 3)$?

(b) What is $P(U = 3)$?

(c) What is $P(S \text{ is even })$?

Hint: it may be illuminating to fill in the appropriate squares in this picture as you try to answer these questions.