

Homework 3

(Questions 1, 2, 3, 4 are due at 11 AM on Thursday, February 9;
questions 5, 6 are due on at 11 AM Thursday, February 16, along with
your next homework assignment.)

Question 1. How many different letter arrangements can be formed from the letters PEPPER?

For example, Here is one way of arranging the letters: PPREPE, and here's another: PPEREP – distinct from the first. Note that some permutations of the letters will result in the same letter arrangement (i.e. will "read the same"). We are interested in the number of distinct arrangements.

Question 2.

- (a) From a group of 5 women and 7 men, how many different committees consisting of 2 women and 3 men can be formed?
- (b) What if 2 of the men are feuding and refuse to serve on the committee together?

(Hint: It may be easier to count, first, out of all possible groups of the 3 men - of which there should be $\binom{7}{3}$ - the number of groups that actually do contain the 2 feuding men.)

Question 3.

- (a) An urn contains n balls, one of which is special. If k of these balls are withdrawn one at a time, with each selection being equally likely to be any of the balls that remain at the time, what is the probability that the special ball is chosen?
- (b) An urn contains n balls, out of which exactly m are red. We select k of the balls at random, without replacement (i.e., selected balls are not put back into the urn before the next selection). What is the probability that i of the selected balls are red?

Question 4. A poker hand consists of 5 cards. If the cards have distinct consecutive values and are not all of the same suit, we say that the hand is a straight. For instance, a hand consisting of the five of spades, six of spades, seven of spades, eight of spades, and nine of hearts is a straight. What is the probability that one is dealt a straight?

Question 5. It is known that screws produced by a certain company will be defective with probability .01, independently of each other. The company sells the screws in packages of 10 and offers a money-back guarantee that at most 1 of the 10 screws is defective. What proportion of packages sold must the company replace?

Question 6. Consider the (completely fictitious, of course) situation that you attend, completely unprepared, a multiple-choice exam. It consists of 10 questions, and each question has four alternatives (of which only one is correct). You will pass the exam if you answer six or more questions correctly. You decide to answer each of the questions in a random way, in such a way that the answer of one question is not affected by the answers of the others. What is the probability that you will pass?

Question 7. *Extra Credit of 2 points. To receive this credit, explain your solution to your instructor.*

Eight rooks are placed in distinct squares of an 8-by-8 chessboard, with all possible placements being equally likely. Find the probability that all the rooks are safe from one another, i.e., that there is no row or column with more than one rook.