Optimal Transport Networks in Spatial Equilibrium

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Introduction

- Central questions in international trade and economic geography involve counterfactuals with respect to trade costs
  - Eaton and Kortum (2002), Costinot and Rodriguez-Clare (2013)
- Transport infrastructure is one important determinant of trade costs (e.g., Limao and Venables, 2001)
  - How should infrastructure investments be allocated across regions?
  - What are the aggregate gains?
  - How important are the inefficiencies in infrastructure investments?
- Answering these questions requires identifying the best set of infrastructure investments in a network

This Paper

- **Develop a framework to study optimal transport networks in general equilibrium**

1. **Solve a global optimization over the space of networks**
   - given any primitive fundamentals
   - in a neoclassical trade framework (with labor mobility)
2. **Apply to actual road networks in 25 European countries**
   - how large are the gains from expansion and the losses from misallocation of current networks?
   - how to these effects vary across countries?
   - what are the regional effects?

Key Features

- **Neoclassical Trade Model on a Graph**
  - Infrastructure impacts shipping cost in each link
- Sub-Problems:
  - how to ship goods through the network? ("Optimal Flows")
  - how to build infrastructure? ("Optimal Network")
- **Optimal flows = well known problem from Optimal Transport literature**
  - Numerically very tractable
  - Especially using dual approach (convex optimization in the space of prices)
- **Full problem (Flows+Network+GE) inherits numerical tractability**
  - Map infrastructure investments in each link to equilibrium prices
    - Sidestep direct search in space of networks
  - For sufficiency, add congestion in transport→global optimum

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Literature Background

- Canonical quantitative trade frameworks: Eaton and Kortum (2002), Anderson and van Wincoop (2003)
- Counterfactuals with respect to infrastructure in gravity models: Allen and Arkolakis (2014a), Redding (2016), Nagy (2016), Ahlfeldt, Redding and Sturm (2016), Sotelo (2016),...
- First-order welfare impact of changes in infrastructure: Allen and Arkolakis (2016)
- Optimization or search over networks: Felbermayr and Tarasov (2015), Alder (2016)
- Empirical assessment of actual changes in transport costs: Chandra et al. (2000), Baum-Snow (2007), Feyrer (2009), Donaldson (2012), Duranton et al. (2014), Pascali (2014), Faber (2014), Donaldson and Hornbeck (2016),...
- Here
  - Global optimization over networks, in neoclassical environment, given any primitive fundamentals
  - Related to optimal flow problem on a network (e.g., Chapter 8 of Galichon, 2016)
    - OT literature does not typically embed optimal-transport in GE or optimize over network

Model

Preferences and Technologies

- $\mathcal{J} = \{1, ..., J\}$ locations
  - N traded goods aggregated into c_j
  - 1 non-traded in fixed supply (can make it variable)
  - L_j workers located in j (fixed or mobile)

- Homothetic and concave utility in j,

$$U(c_j, h_j)$$

where

$$c_j L_j = C_j^T \left(C_j^1, \ldots, C_j^N \right)$$

  - $C_j^T(\cdot)$ homogeneous of degree 1 and concave

- Output of sector n in location j is:

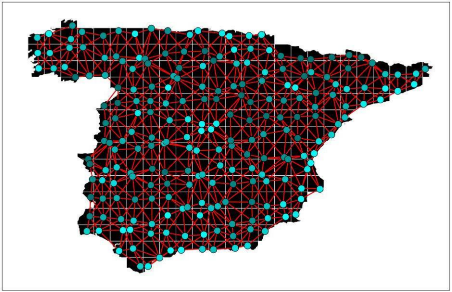
$$Y_j^n = F_j^n \left(L_j^n, \mathbf{V}_j^n, \mathbf{X}_j^n \right)$$

  - $F_j^n(\cdot)$ is either neoclassical or a constant
  - $\mathbf{V}_j^n, \mathbf{X}_j^n$ = other primary factors and intermediate inputs

- Special cases
  - Ricardian model, Armington Specific-factors, Heckscher-Ohlin, Endowment economy, Rosen-Roback...

Underlying Graph

- The locations are arranged on an *undirected* graph
  - $\mathcal{J} = \{1, ..., J\}$ nodes
  - $\mathcal{E}$ edges
- Each location j has a set $\mathcal{N}(j)$ of "neighbors" (directly connected)
  - Shipments flow through neighbors
  - "Neighbors" may be geographically distant
    - ⋆ Fully connected case $\mathcal{N}(j) = \mathcal{J}$ is nested
- Example: square network, $\#\mathcal{N}(j) = 8$



Transport Technology

- Per-unit cost of shipping Q^n_{jk} units of a commodity n from j to $k \in \mathcal{N}(j)$:
  - $\tau_{jk}\left(Q^n_{jk}, I_{jk}\right)$ nominated in units of good itself (iceberg)
  - (alternatively, cross-good congestion: $\tau_{jk}\left(\sum_n Q^n_{jk}, I_{jk}\right)$ nominated in the bundle of traded goods)

- **Decreasing returns to transport:**

$$\frac{\partial \tau_{jk}}{\partial Q^n_{jk}} \geq 0$$

  - "Congestion" in short, may account for travel times, road damage, fixed factors in transport technologies...

- **Returns to infrastructure:**

$$\frac{\partial \tau_{jk}}{\partial I_{jk}} < 0$$

  - On roads: number of lanes, whether road is paved, signals,...
  - **The transport network is defined by** $\{I_{jk}\}_{j \in \mathcal{J}, k \in \mathcal{N}(j)}$

Decentralized Allocation Given the Network

- **Free entry of atomistic traders into shipping each n from o to d for all $(o,d) \in \mathcal{J}^2$**
  - Problem of traders shipping from o to d

$$\min_{r=(j_0,\dots,j_\rho)\in\mathcal{R}_{od}} \underbrace{p_o^n T_{r,0}^n}_{\text{Sourcing Costs}} + \underbrace{\sum_{k=0}^{\rho-1} p_{j_{k+1}} t^n_{j_k j_{k+1}} T^n_{r,k+1}}_{\text{Taxes}}$$

    - $T^n_{r,k}$ is the accumulated iceberg cost from k to d along path r (product of $1+\tau^n_{jk}$)
    - $\mathcal{R}_{od}$ are all possible routes from o to d
    - Corresponds to minimum-cost route problem from gravity literature in the absence of taxes

- **Conservation of flows constraint**

$$C_j^n + \sum_{n'} X_j^{nn'} + \underbrace{\sum_{k\in\mathcal{N}(j)} \left(1+\tau^n_{jk}\right) Q^n_{jk}}_{\text{Exports}} \leq Y_j^n + \underbrace{\sum_{i\in\mathcal{N}(j)} Q^n_{ij}}_{\text{Imports}}$$

- **Remaining features correspond to standard decentralized competitive equilibrium (given I_{jk})**

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Planner's Problem Given Network (Labor Mobility)

Definition

The planner's problem given the infrastructure network is

$$W_0\left(\{I_{jk}\}\right) = \max_{\mathbf{C}_j, \mathbf{L}_j, \mathbf{V}_j^n, \mathbf{X}_j^n, L_j} \max_{Q_{jk}^n} u$$

subject to (i) availability of traded and non-traded goods,

$$c_j L_j \leq C_j^T\left(\mathbf{C}_j\right) \text{ and } h_j L_j \leq H_j \text{ for all } j;$$

(ii) the balanced-flows constraint,

$$C_j^n + \sum_{n'} X_j^{nn'} + \sum_{k \in \mathcal{N}(j)} \left(Q_{jk}^n + \tau_{jk}^n\left(Q_{jk}^n, I_{jk}\right) Q_{jk}^n\right) = F_j^n\left(L_j^n, \mathbf{V}_j^n, \mathbf{X}_j^n\right) + \sum_{i \in \mathcal{N}(j)} Q_{ij}^n \text{ for all } j, n;$$

(iii) free labor mobility, $$L_j u \leq L_j U\left(c_j, h_j\right) \text{ for all } j;$$

(iv) local and aggregate labor-market clearing; and
(v) factor market clearing and non-negativity constraints.

◂ Immobile Labor

Proposition

(Decentralization) Given the network, the welfare theorems hold under Pigovian taxes:

$$1 - t_{jk}^n = \frac{1 + \tau_{jk}}{1 + \tau_{jk}^n + \frac{\partial \tau_{jk}^n}{\partial Q_{jk}^n} Q_{jk}^n}.$$

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Features of Optimal Flows Problem

- Multiplier P_j^n of conservation of flows constraint $=$ price of good n in j in the decentralization

- No-arbitrage conditions ▸ Example

$$\frac{P_k^n}{P_j^n} \leq 1 + \tau_{jk}^n + \frac{\partial \tau_{jk}^n}{\partial Q_{jk}^n} Q_{jk}^n, = \text{ if } Q_{jk}^n > 0$$

- Dual solution coincides with primal ▸ detail
  - Dual $=$ convex optimization with linear constraints in smaller space (just prices)
  - Efficient algorithms are guaranteed to converge to global optimum (Bertsekas, 1998)

Network Building Technology

- Building infrastructure I_{jk} takes up $\delta^I_{jk} I_{jk}$ units of a scarce resource ("asphalt")
  - Building cost δ^I_{jk} may vary across links
    - e.g. due to ruggedness, distance...
  - Asphalt in fixed aggregate supply K
- Bounds:
$$0 \leq \underline{I}_{jk} \leq I_{jk} \leq \bar{I}_{jk} \leq \infty$$
  - E.g. due to existing infrastructure or space restrictions
- Alternatively, can also impose endogenous supply of resources in infrastructure: $\delta^I_{jk} I_{jk} = F^I \left(L^I_j + L^I_k, H^I_j + H^I_k \right)$

Optimization over Transport Network

Definition

The full planner's problem with labor mobility is

$$W = \max_{\{I_{jk}\}} W_0\left(\{I_{jk}\}\right)$$

subject to:
(a) the network building constraint, $\sum_j \sum_{k \in \mathcal{N}(j)} \delta^I_{jk} I_{jk} = K$*; and*
(b) the bounds $\underline{I}_{jk} \leq I_{jk} \leq \overline{I}_{jk}$

- At the global optimum, the optimal network satisfies

$$\underbrace{\mu \delta^I_{jk}}_{\text{Building Cost}} \geq \underbrace{\sum_n P^n_j Q^n_{jk} \left(-\frac{\partial \tau^n_{jk}}{\partial I_{jk}}\right)}_{\text{Gain from Infrastructure}}, \; = \text{ if } I_{jk} > \underline{I}_{jk}$$

  - Reduces numerical search to space of prices → Full problem inherits tractability of optimal flows

Proposition

If the function $Q\tau_{jk}(Q, I)$ is convex in Q and I, the full planner's problem with mobile labor (immobile labor) is a quasiconvex (convex) optimization problem.

- Ensures that our solution is indeed a global optimum for the transport network

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Example: Log-Linear Transport Technology

- Log-linear transport technology:

$$\tau_{jk}\left(Q, I\right) = \delta_{jk}^{\tau} \frac{Q^{\beta}}{I^{\gamma}}$$

  - Global convexity if $\beta > \gamma$

- Optimal network

$$I_{jk}^{*} \propto \left[\frac{1}{\delta_{jk}^{I} \left(\delta_{jk}^{\tau}\right)^{\frac{1}{\beta}}} \left(\sum_{n: P_k^n > P_j^n} P_j^n \left(\frac{P_k^n}{P_j^n} - 1 \right)^{\frac{1+\beta}{\beta}} \right) \right]^{\frac{\beta}{\beta - \gamma}}$$

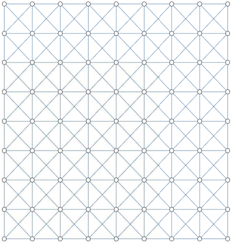
where P_j^n are the equilibrium prices in GE

Additional Properties

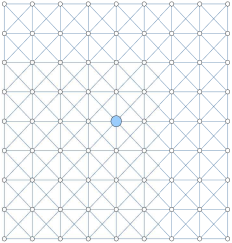
1. Tree property in non-convex cases ▸ Proposition
2. Inefficiencies and externalities in the market allocation ▸ Proposition

Example: One Good on a Regular Geometry

One Traded Good, Endowment Economy, Output 10x Larger at Center, Uniform Fixed Population



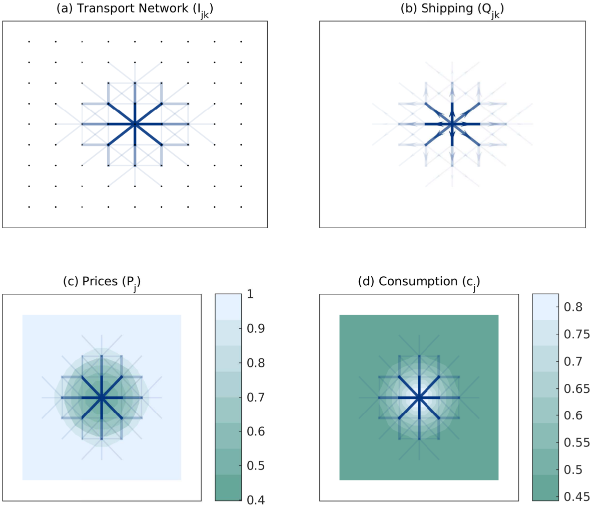
(a) Population



(b) Productivity

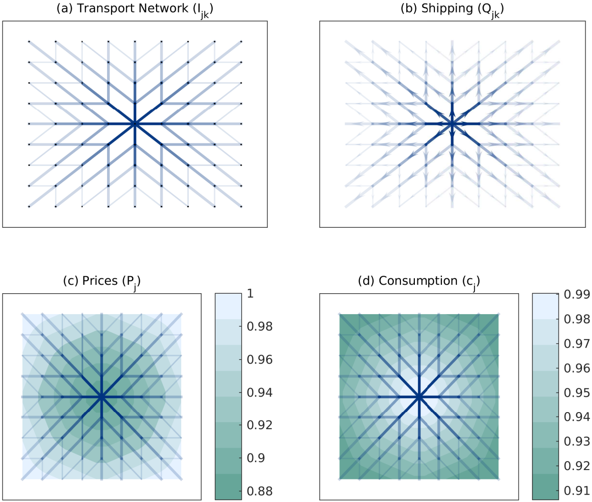
Example: One Good on a Regular Geometry

Optimal Network, $K = 1$



Example: One Good on a Regular Geometry

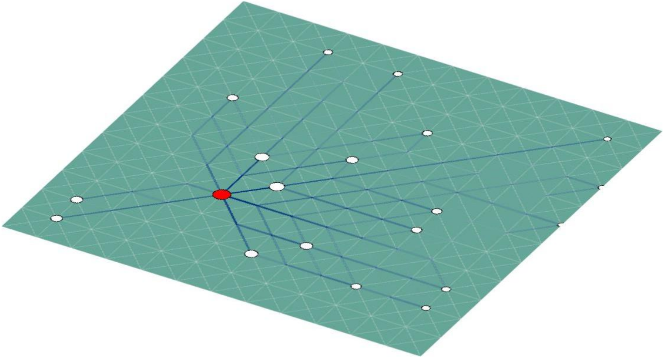
Optimal Network, $K = 100$



Role of Building Costs

20 randomly placed cities

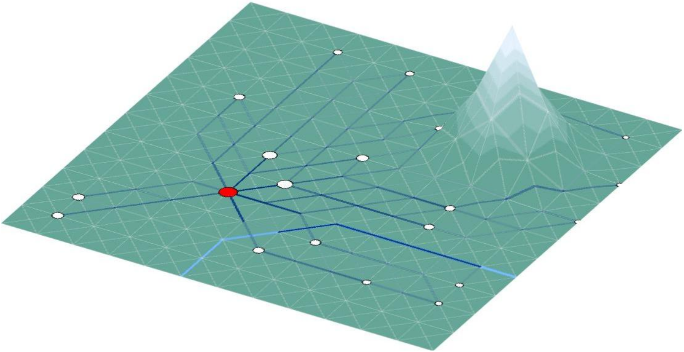
Building Cost: $\delta^{I}_{jk} = \delta_0 \text{Distance}_{jk}^{\delta_1}$



Role of Building Costs

Adding a mountain, a river, bridges, and water transport

Building Cost: $\delta^I_{jk} = \delta_0 \text{Distance}^{\delta_1}_{jk} \left(1 + |\Delta \text{Elevation}|_{jk}\right)^{\delta_2} \delta_3^{\text{CrossingRiver}_{jk}} \delta_4^{\text{AlongRiver}_{jk}}$



Application

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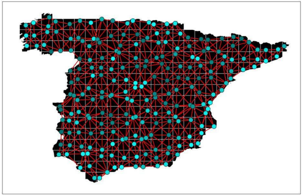
- Questions
  - What are the aggregate gains from optimal expansion of current networks?
  - What would be the regional effects?
  - How important are the inefficiencies in infrastructure investments?
- Study these questions across European countries

Underlying Graph and Observed Infrastructure

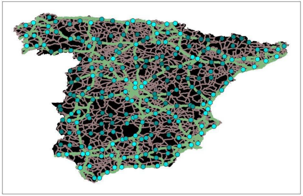
- In 25 European countries we observe, at high spatial resolution
  - Road networks with features of each segment: number of lanes and national/local road (EuroGeographics)
  - Value Added (G-Econ 4.0)
  - Population (GPW)

- Construct the graph $(\mathcal{J}, \mathcal{E})$ and observed road network for each country
  - $\mathcal{J}$: population centroids of 0.5 degree ($\sim$50 km) square cells
  - $\mathcal{E}$: all links among contiguous cells (8 neighbors per node)
  - I_{jk}^{obs}: observed infrastructure between all connected $jk \in \mathcal{E}$

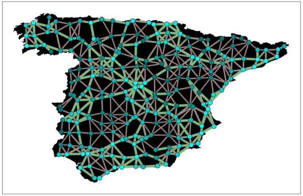
Example: Spain



(a) Underlying Graph



(b) Actual Road Network



(c) Measured Infrastructure I_{jk}^{obs}

▸ table all countries

Calibration

- Production technologies: $Y_j^n = z_j^n L_j^n$
- Transport technologies: $\tau_{jk}^n = \delta_{jk}^{\tau} \frac{\left(Q_{jk}^n\right)^{\beta}}{I_{jk}^{\gamma}}$
  - Geographic frictions $\delta_{jk}^{\tau} = \delta_0^{\tau}\, dist_{jk}$ to match intra-regional share of intra-national trade in Spain
  - Congestion $\beta = 1.24$ to match elasticity of travel times to road use (Wang et al., 2011) Details
  - Returns to infrastructure: $\gamma \in \{0.5\beta, \beta, 1.5\beta\}$
- Preferences: $U(c, h) = c^{\alpha} h^{1-\alpha}$
  - $N = 10$ tradeable sectors with CES demand ($\sigma = 5$)
- Fundamentals: $\{z_j, H_j\}$ such that $\left\{GDP_j^{obs}, L_j^{obs}\right\}$ is the model's outcome given $\left\{I_{jk}^{obs}\right\}$
  - Model fit fit
  - Trade-distance elasticity (not targeted) close to 1 table
- Building costs δ_{jk}^{I} : 2 approaches
  - Assume that observed road networks are optimal→Back out $\delta_{jk}^{I,FOC}$ from FOC's using I_{jk}^{obs}
  - Use estimates from Collier et al. (2016)→Set $\delta_{jk}^{I,GEO}$ as function of distance and ruggedness

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Optimal Expansion and Reallocation

- Optimal expansion
  - Increase K by 50% relative to calibration in every country
  - Build on top of existing network ($\underline{I}_{jk} = I_{jk}^{obs}$)
  - Using both $\delta_{jk}^{I,FOC}$ and $\delta_{jk}^{I,GEO}$
- Optimal reallocation
  - K is equal to calibrated model
  - Build anywhere ($\underline{I}_{jk} = 0$)
  - Using $\delta_{jk}^{I,GEO}$

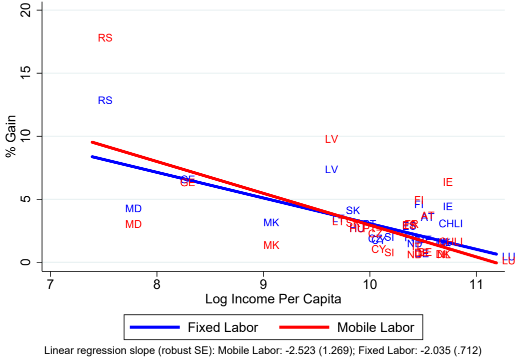
Average Aggregate Effects

Returns to Scale:	$\gamma = 0.5\beta$		$\gamma = \beta$		$\gamma = 1.5\beta$	
Labor:	Fixed	Mobile	Fixed	Mobile	Fixed	Mobile
Optimal Reallocation						
$\delta = \delta^{I,GEO}$	3.2%	3.0%	4.6%	4.8%	5.6%	6.6%
Optimal Expansion						
$\delta = \delta^{I,GEO}$	3.9%	3.5%	5.8%	5.7%	7.2%	7.9%
$\delta = \delta^{I,FOC}$	1.2%	1.0%	3.4%	5.8%	11.3%	12.4%

Each element of the table shows the average welfare gain in the corresponding counterfactual across the 25 countries in our data.

Cross-Country Effects

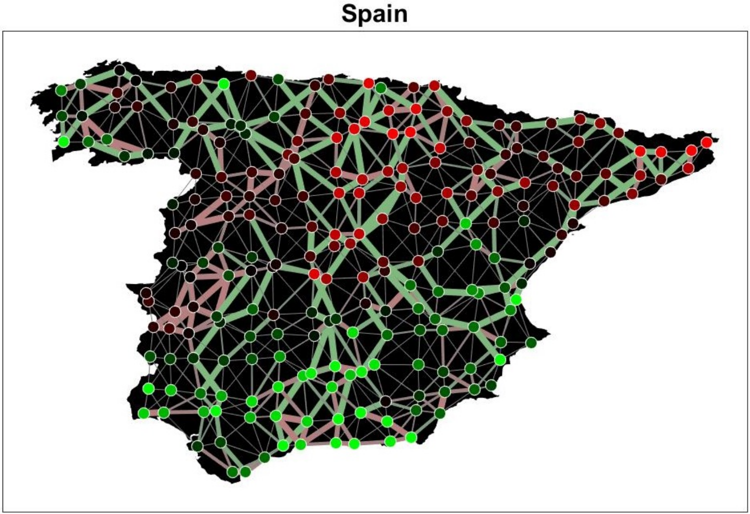
Optimal Reallocation, δ^{GEO}



expansion-GEO expansion-FOC

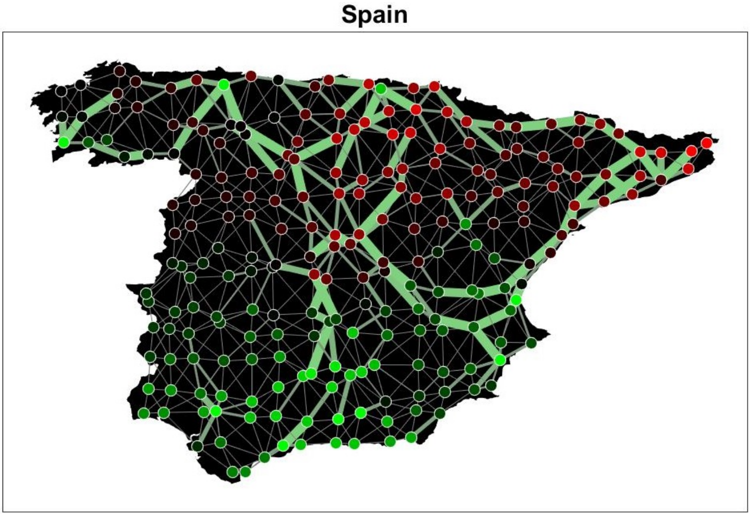
Regional Effects (Spain)

Optimal Reallocation



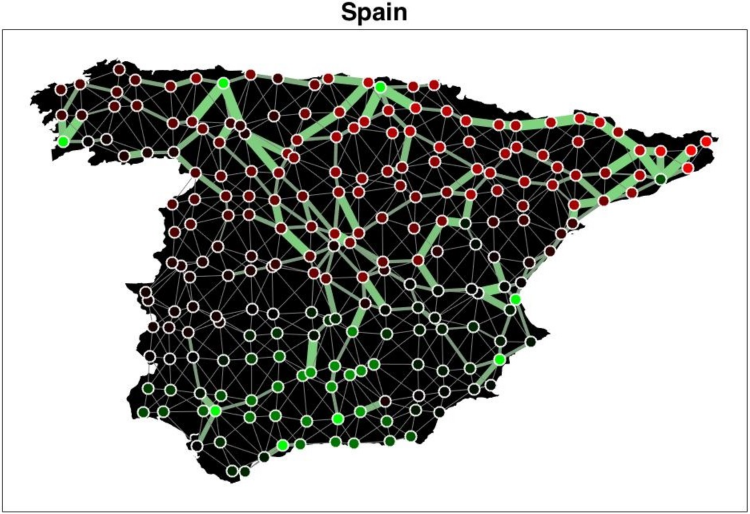
Regional Effects (Spain)

Optimal Expansion



Regional Effects (Spain)

Optimal Expansion, Non-Convex



Where is infrastructure placed?

Dependent variable: Infrastructure growth in the counterfactual

	Reallocation	Expansion ($\delta = \delta^{I,GEO}$)	Expansion ($\delta = \delta^{I,FOC}$)
Population	0.308***	0.104***	0.004
Income per Capita	0.127	0.007	-0.020
Consumption per Capita	0.290**	0.179***	0.130
Infrastructure	-0.362***	-0.195***	-0.067**
Differentiated Producer	0.271***	0.133***	-0.099***
R^2	0.38	0.32	0.38

These regressions pool the outcomes across all locations in the convex case.

Country fixed effects included. SE clustered at the country level.

Which regions grow?

Dependent variable: Employment growth in the counterfactual

	Reallocation	Expansion ($\delta = \delta^{I,GEO}$)	Expansion ($\delta = \delta^{I,FOC}$)
Population	-0.002	-0.001	0.002**
Income per Capita	0.001	-0.002	0.030**
Consumption per Capita	**-0.147*** **	**-0.139*** **	**-0.179*** **
Infrastructure	0.002	0.005***	0.000
Infrastructure Growth	0.013*	0.032**	0.003**
Differentiated Producer	0.013**	0.023***	0.031***
R^2	0.57	0.67	0.90

These regressions pool the outcomes across all locations in the convex case.

Country fixed effects included. SE clustered at the country level.

Conclusion

- We develop and implement a framework to study optimal transport networks
  1. Neoclassical model (with labor mobility) on a graph
  2. Optimal Transport with congestion
  3. Optimal Network

- Application to road networks in Europe
  - Larger gains from optimal expansion and losses from misallocation in poorer economies
  - Optimal expansion of current road networks reduces regional inequalities

- Other potential applications
  - Political economy / competing planners
  - Model-based instruments for empirical work on impact of infrastructure
  - Optimal investments in developing countries
  - Optimal transport of workers
  - Absent forces: agglomeration, dynamics

Planner's Problem (Immobile Labor)

Definition

The planner's problem without labor mobility given the infrastructure network is

$$W_0\left(\{I_{jk}\}\right) = \max_{\mathbf{C}_j, \mathbf{L}_j, \mathbf{V}_j^n, \mathbf{X}_j^n} \max_{Q_{jk}^n} \sum_j \omega_j L_j U\left(c_j, h_j\right)$$

subject to (i) availability of traded and non-traded goods,

$$c_j L_j \leq C_j^T\left(\mathbf{C}_j\right) \text{ and } h_j L_j \leq H_j \text{ for all } j;$$

(ii) the balanced-flows constraint,

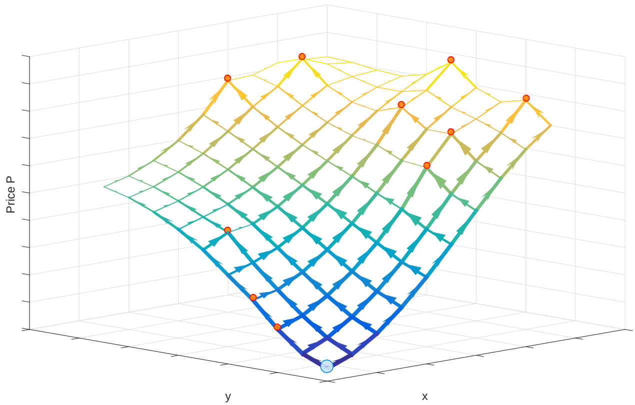
$$C_j^n + \sum_{n'} X_j^{nn'} + \sum_{k \in \mathcal{N}(j)} \left(Q_{jk}^n + \tau_{jk}^n\left(Q_{jk}^n, I_{jk}\right) Q_{jk}^n\right) = F_j^n\left(L_j^n, \mathbf{V}_j^n, \mathbf{X}_j^n\right) + \sum_{i \in \mathcal{N}(j)} Q_{ij}^n \text{ for all } j,\ n;$$

(iii) local labor-market clearing; and

(iv) factor market clearing and non-negativity constraints.

Return

Optimal Flow and Price Field



- The “price field” decentralizes the optimal flows
  - Flows follow the gradient of prices
  - Consumption locations appear as local peaks of the field

Return

Fictitious Planner Problem with Externalities

- We may assume that the market allocation is inefficient, e.g. due to
  - No corrective taxes: $t_{jk}^n = 0$
  - Externalities from population: $F_j^n(\cdot; L_j)$

Definition

The fictitious-planner's problem with externalities given the infrastructure network is

$$W_0\left(\{I_{jk}\}; \overline{L}, \overline{Q}\right) = \max_{\mathbf{c}_j, \mathbf{L}_j, \mathbf{V}_j^n, \mathbf{X}_j^n, L_j} \max_{Q_{jk}^n} u$$

subject to conditions (i), (iii), (iv), (v) from before, and

$$C_j^n + \sum_{n'} X_j^{nn'} + \sum_{k \in \mathcal{N}(j)} \left(Q_{jk}^n + \tau_{jk}^n\left(\overline{Q_{jk}^n}, I_{jk}\right) Q_{jk}^n\right) = F_j^n\left(L_j^n, \mathbf{V}_j^n, \mathbf{X}_j^n; \overline{L}_j\right) + \sum_{i \in \mathcal{N}(j)} Q_{ij}^n \text{ for all } j,\ n.$$

- This problem satisfies the same convexity+duality properties of the standard planner's given the network.
- Letting $\mathbf{L}_0\left(\overline{L}, \overline{Q}\right)$ and $\mathbf{Q}_0\left(\overline{L}, \overline{Q}\right)$ be the solution to the fictitious-planner problem with externalities:

Proposition

(Decentralization with Externalities) $\left(\overline{L}, \overline{Q}\right)$ *corresponds to an inefficient market allocation if and only if* $\overline{L} = \mathbf{L}_0\left(\overline{L}, \overline{Q}\right)$ *and* $\overline{Q} = \mathbf{Q}_0\left(\overline{L}, \overline{Q}\right)$.

Return

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Return

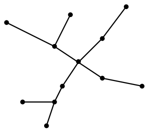
Nonconvex Case: Economies of Scale in Transport

- What if $\gamma > \beta$, i.e., returns to transport technology are increasing?
  - KKT no longer sufficient: local vs. global optimality
- Computations show the network becomes extremely sparse
  - The planner prefers to concentrate flows on few large "highways" (branched transport)

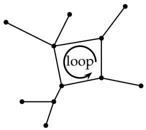
Proposition

*(**Network Shape in Non-Convex Cases**) In the absence of a pre-existing network (i.e., $I^0_{jk} = 0$), if the transport technology is satisfies $\gamma > \beta$ and there is a unique commodity produced in a single location, the optimal transport network is a tree.*

- Intuition: cycles are sub-optimal because it pays off to remove an edge and concentrate flows elsewhere



(a) tree (no loops)

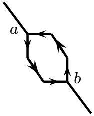


(b) non-tree

return

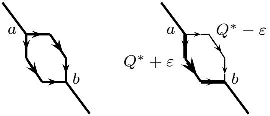
Tree Property with Economies of Scale: Intuition

- Two types of elementary cycles can occur:



Case (i) Case (ii)

- **Argument:**
  - Cycles of type (i) waste resources and can be ruled out
  - With cycles of type (ii), better off to redirect flows to one branch:



(a) Initial (b) After redirection

return

Computation via Duality

- FOCs: nonlinear system of many variables
- Primal

$$\sup_{C,L,Q} \inf_{P} \mathscr{L}\left(C, L, Q; P\right)$$

  - Even if convex, slow to converge (high dimension in C, L, Q)

- Dual

$$\inf_{P} \mathscr{L}\left(C\left(P\right), L\left(P\right), Q\left(P\right); P\right)$$

  - Use FOCs and substitute for C, Q,..., as function of P, then minimize over Lagrange multipliers
  - Convex minimization problem in fewer variables with just non-negativity constraints (just P)

return

Congestion Parameter β

- A well documented empirical relationship in transportation science is the *speed-density* relationship
- Using data from various segments of US roads, Wang et al. (2011) estimate the logistic relationship

$$speed = v_b + \frac{v_f - v_b}{\left(1 + exp\left(\frac{density - k_t}{\theta_1}\right)\right)^{\theta_2}}$$

- Assuming that transport costs are proportional to travel times (inverse of speed) and density is proportional to flows Q, we can identify β in

$$log\,(\text{travel time per km}) = const + \beta \times log\,(\text{density})$$

and find $\beta = 1.2446$

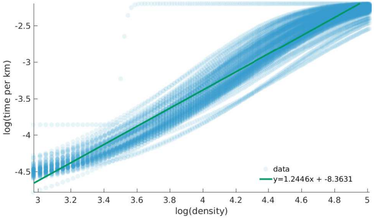


Figure: Pooled estimates of travel times against density

return

Discretization of the Observed Network

- Road network from EuroGeographics data provides the following characteristics:
  - Number of lanes, national/primary/secondary road, paved/unpaved, median, etc.
- For each pair of locations (j,k), we identify the *least cost route* on the observed network using relative user costs from Combes and Lafourcade (2005)
- In the discretized network, we construct an infrastructure index

$$I_{jk} = lanes_{jk} \times \chi_{nat}^{1-nat_{jk}}$$

where

  - $lanes_{jk}$ is the average observed number of lanes on the least cost route from j to k
  - $nat_{jk} \in [0,1]$ is the fraction of that route spent on national roads
  - χ_{nat} is the building and maintenance cost per km. of national roads relative to other types (Doll et al., 2008)

return

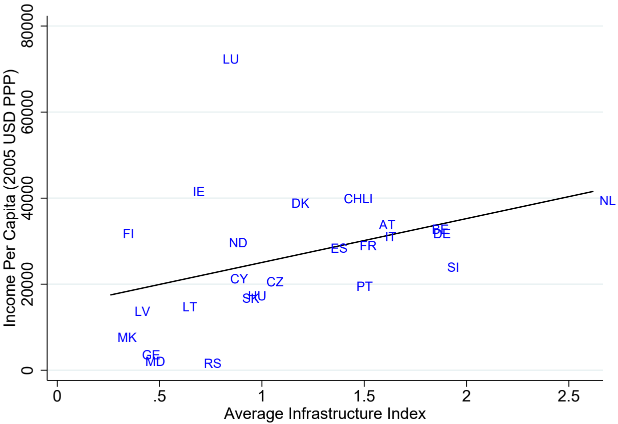
Average Infrastructure Across Countries

Country	Code	Actual Road Network			Discretization		
		Length (Km.)	Number of Segments	Average Lanes per Km.	Number of Cells	Length (Km.)	Average Infrastructure Index
		(1)	(2)	(3)	(4)	(5)	(6)
Austria	AT	17230	6161	2.36	46	9968	1.54
Belgium	BE	19702	10496	2.49	20	3400	1.80
Cyprus	CY	2818	947	2.29	30	3653	0.81
Czech Republic	CZ	28665	10194	2.20	48	10935	0.99
Denmark	DK	11443	4296	2.18	21	4102	1.11
Finland	FI	70394	9221	2.04	73	34262	0.29
France	FR	128822	38699	2.05	276	78405	1.45
Georgia	GE	28682	9009	1.95	32	7895	0.38
Germany	DE	115177	66428	2.42	196	56410	1.80
Hungary	HU	32740	9244	2.10	50	12017	0.90
Ireland	IE	24952	4144	2.10	47	11299	0.63
Italy	IT	77608	44159	2.32	126	32640	1.57
Latvia	LV	11495	2103	2.03	47	10599	0.34
Lithuania	LT	10682	1586	2.39	43	9736	0.58
Luxembourg	LU	1779	866	2.31	8	674	0.78
Macedonia	MK	5578	908	2.15	14	2282	0.26
Moldova	MD	8540	1462	2.21	20	4611	0.40
Netherlands	NL	14333	8387	2.68	20	4263	2.62
Northern Ireland	ND	7087	1888	2.18	12	1714	0.81
Portugal	PT	15034	4933	2.10	43	11000	1.43
Serbia	RS	18992	3656	2.14	45	12261	0.68
Slovakia	SK	11420	2610	2.18	30	6130	0.87
Slovenia	SI	7801	2441	2.19	12	1853	1.87
Spain	ES	101990	18048	2.39	227	68162	1.30
Switzerland	CHLI	14526	11102	2.26	25	5127	1.37

- Infrastructure index:
  - Across countries: correlated with income per capita fig
  - Within countries: 0.7 correlation with travel times (GoogleMaps) across all pairs fig

return

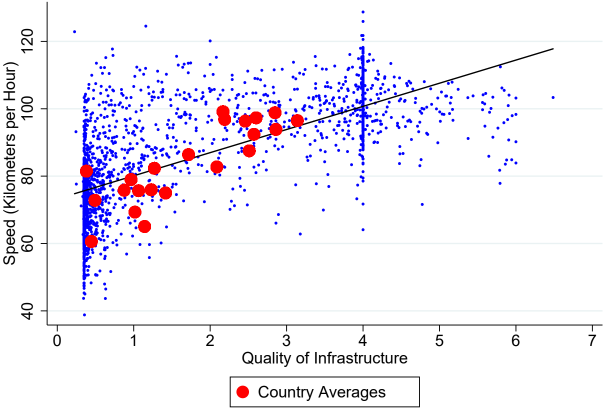
Average Infrastructure and Income Per Capita



Linear regression slope (robust SE): 10200.4 (3603.5)

return

Average Infrastructure and Speed



Regression slope (robust SE): 5.445 (.188). Pools all links. Includes country fixed effects.

return

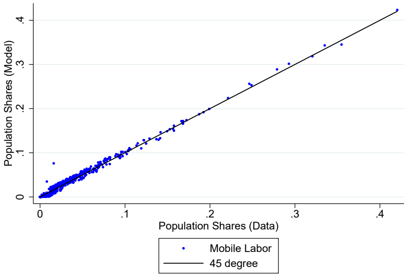
Trade-Distance Elasticity

Value of γ:	0.5β		β		1.5β	
Labor:	Fixed	Mobile	Fixed	Mobile	Fixed	Mobile
Average	-1.08	-1.11	-1.12	-1.19	-1.16	-1.17
Standard deviation	0.17	0.21	0.17	0.26	0.22	0.27

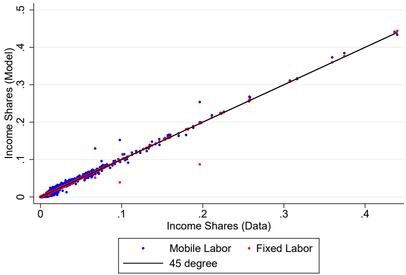
The table reports the average and standard deviation of the trade-distance and elasticity and intra-regional trade share across the 25 countries in our data.

return

Model Fit



Linear regression slope (robust SE): Mobile Labor: 1.025 (.006)

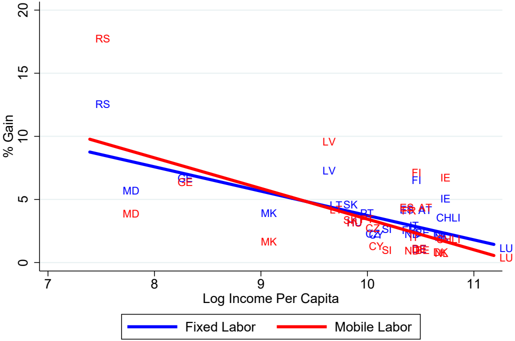


Linear regression slope (robust SE): Mobile Labor: 1.029(.007); Fixed Labor: .992 (.01)

return

Cross-Country Effects

Optimal Expansion, δ^{GEO}

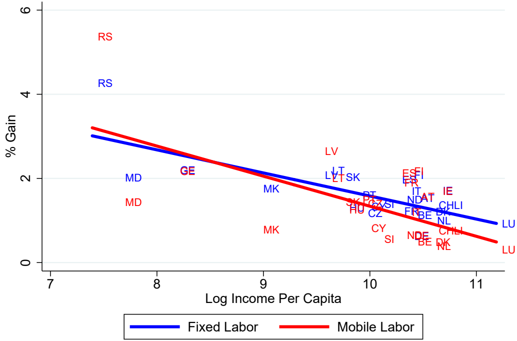


Linear regression slope (robust SE): Mobile Labor: -2.427 (1.209); Fixed Labor: -1.928 (.584)

return

Cross-Country Effects

Optimal Expansion, δ^{FOC}



Linear regression slope (robust SE): Mobile Labor: -.715 (.337); Fixed Labor: -.549 (.191)

return