

$$m\ddot{p} = u - d\dot{p}$$

Set $\dot{p} = v$, we get $m\dot{v} = u - dv$

We discretize by setting $\frac{p_{t+1} - p_t}{\delta t} \simeq v_t$ $m \frac{v_{t+1} - v_t}{\delta t} \simeq u_t - dv_t$

Define $x_t = \begin{pmatrix} p_t \\ v_t \end{pmatrix} = \begin{pmatrix} p_{t,1} - p_{t,3} \\ p_{t,2} - p_{t,4} \\ v_{t,1} \\ v_{t,2} \\ v_{t,3} \\ v_{t,4} \end{pmatrix}$

$$x_{t+1} = \begin{pmatrix} p_{t+1} \\ v_{t+1} \end{pmatrix} = \begin{pmatrix} p_{t,1} + \delta t v_{t,1} - p_{t,3} - \delta t v_{t,3} \\ p_{t,2} + \delta t v_{t,2} - p_{t,4} - \delta t v_{t,4} \\ v_{t,1} + \frac{\delta t}{m} u_{t,4} - \frac{d \delta t}{m} v_{t,1} \\ v_{t,2} + \frac{\delta t}{m} u_{t,3} - \frac{d \delta t}{m} v_{t,2} \\ v_{t,3} + \frac{\delta t}{m} u_{t,2} - \frac{d \delta t}{m} v_{t,3} \\ v_{t,4} + \frac{\delta t}{m} u_{t,1} - \frac{d \delta t}{m} v_{t,4} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & \delta t & 0 & -\delta t & 0 \\ 0 & 1 & 0 & \delta t & 0 & -\delta t \\ 0 & 0 & 1 - \frac{d \delta t}{m} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 - \frac{d \delta t}{m} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 - \frac{d \delta t}{m} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 - \frac{d \delta t}{m} \end{pmatrix} x_t + \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{\delta t}{m} & 0 & 0 & 0 \\ 0 & \frac{\delta t}{m} & 0 & 0 \\ 0 & 0 & \frac{\delta t}{m} & 0 \\ 0 & 0 & 0 & \frac{\delta t}{m} \end{pmatrix} u_t = Ax_t + Bu_t$$

Since Q and R need to be $Q = Q^T$ and $\forall x, x^T Q x > 0$ and $R = R^T, \forall u, u^T R u > 0$

We define Q and R be

$$Q = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad R = \begin{pmatrix} 0.1 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 \\ 0 & 0 & 0.1 & 0 \\ 0 & 0 & 0 & 0.1 \end{pmatrix}$$