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Doctoral Thesis in Machine Design

Motion Planning and Control of Automated Vehicles in Critical Situations

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Abstract

The road traffic environment is inherently uncertain and unpredictable. An automated vehicle (AV) deployed in such an environment will eventually experience unforeseen critical situations, i.e., situations in which the probability of having an accident is rapidly increased compared to a nominal driving situation. Critical situations can occur for example due to internal faults or performance limitations of the AV, abrupt changes in operational conditions or unexpected behavior from other road users. In such critical situations, the first priority for vehicle motion control is to reduce the risk of imminent accident. If needed, the full physical capacity of the vehicle should be employed to accomplish this. These unique circumstances distinguish automated driving in critical situations from the nominal case.

This work aims to tackle the problem of motion planning and control in such critical situations. We determine a set of characteristics that signify the motion planning and control problem in critical situations, in relation to state of the art algorithms. Further, we incrementally develop a motion planning and control framework, tailored for the particular circumstances of critical situations. In its current form, the framework uses a combination of numerical optimization, trajectory rollout and constraint adaptation, to allow motion planning and control with respect to time-varying actuation capabilities, while realizing a range of behaviors to mitigate accident risk in a range of critical situations.

Results for the research work are generated by exposing the framework to several categories of critical situations in a combination of simulations and full scale vehicle tests. We present the following main findings: (1) Inclusion of risk levels of stopping locations at the local planning level generates satisfactory motion behavior in the evaluated critical situations, enabling a combined assessment of risk of the maneuver and of the stopping location. (2) Traction adaptive motion planning and control improves the capacity of autonomous vehicles to reduce accident risk in critical situations, both when adapting to deteriorated and when adapting to improved traction in a range of tested critical situations. (3) State of the art friction estimation algorithms are insufficient for traction adaptive motion planning in terms of combined requirements on accuracy, availability and foresight. However, fusion of multiple estimation paradigms show potential to yield near-optimal performance.

The combined contributions of this thesis are intended as a step towards further improving accident avoidance performance of automated vehicles and driver assistance systems in critical situations. However, much research work remains to be done in this field. We emphasize the need for further research efforts in terms of experimentally evaluating the impact of motion planning and control concepts on accident avoidance performance in critical situations.

Sammanfattning

Vägtrafikmiljön är oförutsägbar. Autonoma vägfordon i en sådan miljö kommer tids nog att hamna i oförutsedda kritiska situationer, det vill säga situationer där risken för en trafikolycka är markant högre än vid nominell körning. Kritiska situationer kan orsakas av exempelvis interna fel eller prestandabegränsningar hos autonomisystemet, av plötsliga förändringar i operationella förhållanden eller av oförutsett agerande hos medtrafikanter. I kritiska situationer är passagerarkomfort inte längre en prioritet, utan fordonets fullständiga manöverförmåga kan utnyttjas för att minimera olycksrisken. Dessa omständigheter skiljer autonom körning i kritiska situationer från det nominella fallet.

Forskningsinriktningen för denna avhandling är rörelseplanering och styrning av autonoma fordon i kritiska situationer. Vi presenterar en uppsättning egenskaper som kännetecknar detta specifika problem, i relation till ledande algoritmer för rörelseplanering och styrning. Vi presenterar också vår egen stegvis utvecklade metod för att angripa problemet. I sin nuvarande form består metoden av en kombination av optimeringsbaserad och samplingsbaserad trajektorieplanering med tidsvarierande dynamik och bivillkor. Metoden gör det möjligt att representera tidsvarierande dynamik och dynamiska begränsningar hos fordonet (till exempel till följd av varierande vägförhållanden) vid planering av en mängd olika manövertyper som kan minska olycksrisken i kritiska situationer.

Resultaten i forskningsarbetet har genererats genom att testa metoden i ett flertal typer av kritiska situationer som har iscensatts genom en kombination av simuleringsmiljöer och experiment med fullskaliga autonoma testfordon. De huvudsakliga slutsatserna från forskningsarbetet är följande: (1) Att inkludera risknivån hos alternativa stoppositioner på den lokala planeringsnivån genererar tillfredsställande rörelsebeteende vid exempelvis interna fel hos autonomisystemet. Detta möjliggör en sammantagen riskbedömning för manöver och stopposition. (2) Väglagsanpassad rörelseplanering och styrning förbättrar autonoma fordons förmåga att reducera olycksrisk i kritiska situationer, både vid anpassning till försämrade och till förbättrade vägförhållanden. (3) Ledande metoder för skattning av vägfriktion har inte tillfredställande prestanda för väglagsanpassad rörelseplanering och styrning med avseende på kombinerade krav på precision, tillgänglighet och framsynthet, när de används var för sig. Dock är det möjligt att kombinera estimat från olika sensorslag till ett friktionsestimat som ger närmast optimalt rörelsebeteende då det används i kombination med väglagsanpassad rörelseplanering och styrning.

Vår förhoppning är att de sammanlagda forskningsbidragen från denna avhandling kan komma att bidra till fortsatta prestandaförbättringar hos system avsedda att minska olycksrisken i kritiska situationer, både för autonoma fordon och för förarstödsystem. Det finns dock mycket kvar att göra inom detta forskningsfält. Vi vill särskilt framhäva behovet av ytterligare forskningsinitiativ rörande experimentell utvärdering av nya koncept för rörelseplanering och styrning, med avseende på förmågan att minska olycksrisken i kritiska situationer.

Acknowledgements

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I was first introduced to the magical world of mechatronics and robotics during an exchange year at Colorado University Boulder in 2012/13. The combination of math, computers, electronics and mechanics that made machines move by themselves utterly fascinated me back then, and it still does today. Since that time I have done little other than trying to learn as much as possible about how to build machines that can operate by themselves. I wish to thank Prof. Derek Reamon at Colorado University, Prof. Thomas Schön at Uppsala University and Dr. Mohammad Ali, then at Volvo Cars where I did my MSc thesis project, for sequentially and independent of each other inspiring me and motivating me to pursue a PhD in Mechatronics and Automated Driving.

During my time at KTH, I have had the privilege to work on several full scale autonomous vehicle projects which apart from having been loads of fun, gradually thought me some of the craftsmanship and organizational skill that goes into building a large autonomous machine. I would like to thank all the people who participated in those projects: First, thanks to all the participants of the KTH GCDC-2016 team, Stefanos Kokogias, Rui Oliveira, Gonçalo Collares Pereira, Xinhai Zhang, Xinwu Song, Jonathan Fagerström, Chris Tamimi, Henrik Petterson and Jonas Mårtensson for a great adventure, with great friends. Second, thanks to all who continued to work on autonomous functionality for the KTH Research Concept Vehicle, Erik Ward, Gonçalo Collares Pereira, Jonas Krook and Yuchao Li. Third, thanks everyone who participated in finally making the traction adaptive motion planning experiments happen: Arpit Karsolia, Christian Berger, Fredrik von Corswant, Henrik Biswanger and Ola Mattson. This project team worked like a well-oiled machine, sorting out one show-stopping problem after another like nobody's business. Thanks also to the organizations that provided extra funding for the experiments: The SAFER Open Research at Astazero Program and the Integrated Transport Research Lab at KTH. Fourth, thanks to all the participants in the autonomous forwarder project Auto2: Björn Möller, Gustav Sten, Tobias Semberg, Morgan Rossander, Olle Gelin and others. This project opened my eyes to the opportunities and research challenges of autonomous machines for sustainable forestry - towards which I aim to direct my future efforts.

During my time as a PhD student, I have had the privilege to participate in the Wallenberg AI, Autonomous Systems and Software Program (WASP) graduate school, which have provided me with rewarding coursework, amazing study-trips to world-leading robotics labs in academia and industry, and most importantly a vast network of colleagues and friends across the country and the world. Through the WASP program I had the opportunity to do a six month research visit to the Model Predictive Control Lab at UC Berkeley. I would like to express my gratitude toward Prof. Francesco Borrelli for giving me the opportunity, and give a special thanks to Monimoy Bujarbaruah, Ugo Rosolia, George Xiaojing Zhang and Nitin Kapania for welcoming me into the research group and for a wonderful research exchange.

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Lars Svensson,
Mörby, May 10, 2021

List of Papers

The thesis is based on the following four publications:

Paper A: L. Svensson, L. Masson, N. Mohan, E. Ward, A. Pernestål Brenden, L. Feng, M. Törngren, "Safe Stop Trajectory Planning for Highly Automated Vehicles: An Optimal Control Problem Formulation" *2018 IEEE Intelligent Vehicles Symposium (IV)*, pp. 517-522, Changshu, China.

Paper B: L. Svensson, M. Bujarbaruah, N. R. Kapania and M. Törngren, "Adaptive Trajectory Planning and Optimization at Limits of Handling", *2019 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*, pp. 3942-3948, Macau, China.

Paper C: L. Svensson, M. Bujarbaruah, A. Karsolia, C. Berger and M. Törngren, "Traction Adaptive Motion Planning and Control at the Limits of Handling", under review for possible journal publication, 2020.

Paper D: L. Svensson and M. Törngren, "Fusion of Heterogeneous Friction Estimates for Traction Adaptive Motion Planning and Control", submitted for possible conference publication, 2021.

Contributions and responsibilities of the author for each individual appended paper is summarized in Section 3.3. In addition, the author has contributed to writing the following publications during the course of the PhD studies:

S. Annell, A. Gratner and **L. Svensson**, "Probabilistic collision estimation system for autonomous vehicles," *2016 IEEE International Conference on Intelligent Transportation Systems (ITSC)*, pp. 473-478, Rio de Janeiro, Brazil.

G. C. Pereira, **L. Svensson**, P. F. Lima and J. Mårtensson, "Lateral Model Predictive Control for Over-Actuated Autonomous Vehicle", *2017 IEEE Intelligent Vehicles Symposium (IV)*, pp. 310-316, Los Angeles, CA.

S. Kokogias, **L. Svensson**, G. C. Pereira, R. Oliveira, X. Zhang, X. Song, J. Mårtensson, "Development of Platform-Independent System for Cooperative Automated Driving Evaluated in GCDC 2016". *IEEE Transactions on Intelligent Transportation Systems*, vol. 19, no. 4, pp. 1277-1289, April 2018.

J. Krook, **L. Svensson**, Y. Li, L. Feng and M. Fabian, "Design and Formal Verification of a Safe Stop Supervisor for an Automated Vehicle", *2019 International Conference on Robotics and Automation (ICRA)*, pp. 5607-5613, Montreal, QC, Canada.

M. Törngren, X. Zhang, N. Mohan, M. Becker, **L. Svensson**, X. Tao, DJ. Chen, J. Westman, "Architecting Safety Supervisors for High Levels of Automated Driving", *2018 International Conference on Intelligent Transportation Systems (ITSC)*, pp. 1721-1728, Maui, USA.

D. Chen, Z. Yang, **L. Svensson** and L. Feng, ”Optimization based path planning for a two-body articulated vehicle”, *2020 IEEE International Conference on Automation Science and Engineering (CASE)*, pp. 397-403, Hong Kong, China.

M. Parseh, F. Asplund, M. Nybacka, **L. Svensson** and M. Törngren, ”Pre-Crash Vehicle Control and Manoeuvre Planning: A Step Towards Minimizing Collision Severity for Highly Automated Vehicles”, *2019 IEEE International Conference on Vehicular Electronics and Safety (ICVES)*, pp. 1-6, Cairo, Egypt.

M. Parseh, F. Asplund, **L. Svensson**, W. Sinz, E. Tomasch and M. Torngren, ”A Data-Driven Method Towards Minimizing Collision Severity for Highly Automated Vehicles”, accepted for publication in *IEEE Transactions on Intelligent Vehicles*, 2021.

Contents

List of Acronyms

ADAS	Advanced Driver Assistance System
AV	Automated Vehicle
CFTOC	Constrained Finite Time Optimal Control
CPU	Central Processing Unit
DP	Dynamic Programming
GP	Gaussian Process
GPU	Graphics Processing Unit
LQR	Linear Quadratic Regulator
MPC	Model Predictive Control
OCP	Optimal Control Problem
QP	Quadratic Programming
ROS	Robotics Operating System
RRT	Rapidly-exploring Random Tree
RTI (SQP)	Real Time Iteration (Sequential Quadratic Programming)
SAA-RTI	Sampling Augmented Adaptive RTI
SQP	Sequential Quadratic Programming

Part I

Summary of Research Work

Chapter 1

Introduction

The focus of this research work is motion planning and control of automated road vehicles in unforeseen critical situations, i.e., situations in which an accident is probable to occur. This chapter introduces the work by providing a brief background and motivation, outlining the research design and summarizing the research contributions of the thesis.

1.1 Background and Motivation

The big picture motivation behind this research work is to contribute to reducing fatalities and injuries in road traffic. In 2018, 40 million people were injured and 1.35 million died due to road traffic related injuries [1]. To focus efforts in this area, the UN Sustainable Development Goals launched in 2015 include two specific targets associating to traffic safety and sustainable transportation. Target 3.6 seeks to reduce road traffic deaths and injuries by 50% by 2020 and target 11.2 aims to provide access to safe, affordable, accessible and sustainable transport by 2030. Studies conducted in the US indicate that the cause behind the vast majority (94%) of accidents is human error [2], and that 24% occur at poor operational conditions, e.g., fog, rain, sleet, snow, [3]. One widely adopted strategy to reduce such accidents is through partial or full automation of the driving task. Fig. 1.1 introduces the levels of automation as defined in SAE J3016 Surface Vehicle Recommended Practice [4].

Systems at the lower automation levels (1 - Driver Assistance and 2 - Partial Automation) are already widespread in traffic systems throughout the world. Some provide convenience functions like Adaptive Cruise Control (ACC) and some are designed to avoid and/or reduce the severity of accidents. Such Advanced Driver Assistance Systems (ADAS) monitor the traffic scene and the driver during operation, and provides warning or intervenes to assist the driver if particular conditions indicate that there is increased risk of an accident. Forward Collision Warning, Automatic Emergency Braking, Lane Departure Warning, Lane Keeping Assistance,

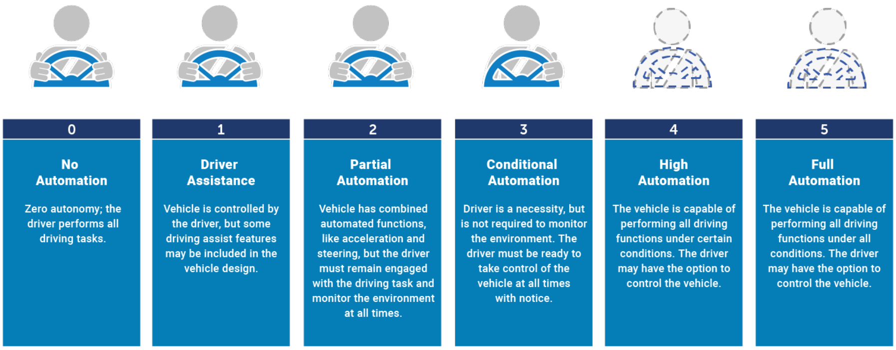


Figure 1.1: SAE J3016 Levels of Automation [4].

and Blind Spot Monitoring are a few of the functions that are currently available. Such features are gradually becoming included among standard features of road vehicles. For example, Automatic Emergency Brake is now required to get the highest safety rating from European New Car Assessment Programme (Euro NCAP), and requirements are likely to continue to increase towards more sophisticated accident avoidance systems and higher levels of automation in the near future [5]. These partial automation technologies are already having an effect on overall road safety, and are estimated to have the potential to reduce the number of fatalities in accidents involving passenger vehicles by 30 % [6] compared to current levels (numbers for United States).

Research and development for higher levels of automated driving (4 - High Automation and 5 Full Automation) have progressed immensely in the past two decades, to the point where several of the leading companies are offering level 4 autonomous taxi services to the general public, albeit in limited geographical areas. At levels 4 and 5, the vehicle is responsible for the complete driving operation. This has the potential of strong positive impact on traffic safety in terms of alleviating accidents caused by human error. Initial deployments of level 4 automated vehicles so far have an excellent safety record [7] [1] and leading manufacturers indicate that their current technology can alleviate the vast majority of fatalities caused by human error. For example, by reconstructing and simulating all fatal accidents in Phoenix Arizona over the past 10 years, one of the leading AV developers claim that their current technology would have avoided 92% of fatalities [9].

At this level of sophistication, the vehicle can operate without human supervi-

[1] This statement relates to the deployment of level 4 systems from, e.g., Waymo One [8]. For example Tesla's recent beta deployment of its Fully Self Driving system is, despite its name and capabilities, currently classified as a Level 2 system, where the driver is ultimately responsible for the safety of operation.

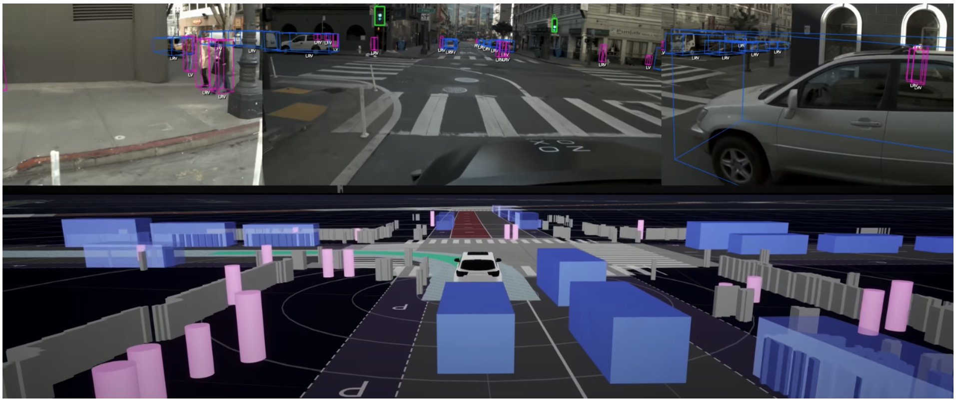


Figure 1.2: Visualization of a semantic scene representation, obtained from sensor data through state of the art perception functionality. In addition, the vehicle's planned future motion is shown in turquoise. Image courtesy of Amazon/Zoox.

sion. This enables cost-efficient services (e.g. cargo delivery without driver, where the driver cost amounts to almost 50% [10]), improved utilization of vehicles, as well as providing access to mobility for persons without the ability to drive themselves [11, 12].

Level 4+ automated vehicles use a range of sensors to register their own state and the state of the surrounding traffic scene, for example radar, lidar, cameras, ultrasonic sensors, inertial measurement units, wheel and steering angle encoders and GPS/GNSS [13]. In addition, prerecorded high definition 3d maps are often used to complement the vehicle's own sensors. Perception software processes all of this information and produces an internal digital representation of the traffic scene in terms of the vehicle's position relative to lanes and drivable area, road infrastructure like stop signs and traffic lights, as well as the predicted motions of all non-static traffic agents such as pedestrians, cyclists and other vehicles. This internal representation forms the basis for motion planning and control of the vehicle. Fig.1.2 shows one example of a visualization of such an internal representation.

Perception in general and prediction of human behavior in particular pose substantial research challenges that remain to be fully addressed [13–16]. Current sensor technologies are subject to fundamental limitations in terms of e.g., weather/light disturbances, occlusions, range and resolution which can only partially be overcome through sensor fusion and clever perception algorithms. Predicting motions of other traffic agents is made extraordinarily difficult due to the inherent unpredictability of humans in the traffic environment. In addition, it is non-trivial to analyze failures of a cyber-physical system as complex as a level 4+ AV. Undesired motion behavior may have a multitude of causes, for example, inadequate training of neural networks, software bugs, system security breaches, mechanical faults/wear, electrical faults or external environment factors [17–19].

Keeping in mind the limitations of current perception technology, the new challenges introduced by vehicle automation, as well as the wide variation of operational conditions that an AV can be exposed to in terms of weather, road conditions etc., it is unlikely that all traffic accidents can be avoided by anticipating them well ahead of time. This would essentially require sacrificing efficiency and availability of the autonomy function, by enforcing overly cautious behavior (e.g., driving very slowly) [20]. This motivates research in automated vehicle operation in *critical situations*.

We use the term *critical situation* to describe a suddenly appearing situation caused by internal or external factors in which the probability of an imminent accident is substantially increased. Our use of term is analog to that of ”hazardous event” in ISO/PAS 21448:2019 ”Road vehicles — Safety of the Intended Functionality” [21]. Such critical situations may originate from a range of sources, for example from unsafe behavior from other road users, rapid changes in operational conditions or internal faults or performance limitations of the AV. The common element is that they all require prompt decisive action in terms of motion planning and control in order to reduce the probability of an accident. Furthermore, we regard a maneuver planned and executed to reduce the probability of accident in a critical situation as an instantiation of the ”dynamic driving task fallback” as described in J3016. The goal such a maneuver is to achieve a ”mimimal risk condition”, i.e., a state in which the risk of an accident is mitigated [4]. In Paper A, we refer to such a maneuver as a ”safe stop maneuver”.

Performance of the automated function in critical situations has a profound impact on the safety of AV occupants and surrounding traffic agents. Arguably, every road accident is preceded by a brief window of time in which a critical situation may be detected, upon which an accident may or may not be avoided through evasive action. Optimal action upon detection can be the difference between a near miss and a fatal accident. This applies to high level automated vehicles as well as ADAS systems. Furthermore, for highly automated vehicles (level 4+), this also has implications for the nominal functionality. The more confident we are that the vehicle is capable of avoiding accidents in certain categories of critical situations, the less conservative the vehicle needs to be in nominal operation, i.e., improving long term efficiency and availability of the autonomy function by e.g. allowing higher speeds, or continued operation in deteriorated weather conditions. Hence, performance in critical situations dictate nominal performance of the AV.

1.2 Research Design

The overall objective of the research work in this thesis has been to contribute to the development of safe automated vehicles by (1) characterizing the motion planning and control problem for critical situations in reference to state of the art motion planning algorithms and (2) evaluate algorithmic design choices with respect to accident mitigation performance in critical situations. In terms of research design, the aim has been to set up evaluation such that conclusions drawn from experiments can be generalized to state of the art production grade AVs, deployed in real traffic. This section presents the scope and delimitations of the research work, outlines the specific research questions that have guided the work, the methodology applied to to tackle the research questions as well as ethical and sustainability considerations.

1.2.1 Research Questions

To provide guidance to the research work, the following research questions were formulated.

1. Which characteristics of the motion planning and control functionality impact the AV's capacity to reduce risk of accident in critical situations?
2. Which motion planning paradigm(s) is/are suitable for realizing these characteristics?
3. To what extent does accurate representation of time-varying actuation capability impact an AVs capacity to reduce risk of accident in critical situations?
4. To what extent does consideration of uncertainty in future actuation capability impact an AVs capacity to reduce risk of accident in critical situations?

Questions 1 and 2 were formulated early on, and are derived directly from the research objective, introduced in the beginning of Section 1.2. The research work associated with all four appended papers were guided by these questions. Question 3 was added around half-way through the PhD program and represents a more in-depth investigation of novel algorithmic concepts in terms of handling variations in operational conditions. This work resulted in Papers B, C and eventually D. Question 4 was initiated during discussions following the work on Paper C, identifying that the impact of estimation uncertainty in the friction estimation functionality would merit further investigation. Paper D is the result of that investigation.

The thesis outline in Section 1.4 includes an overview, Table 1.2, of how the central concepts of the thesis (including research questions) relate to each of the appended papers.

1.2.2 Scope and Delimitations

In terms of AV functionality, the research focus of this thesis has been on motion planning and control. Therefore, we have delimited the scope to critical situations with direct motion planning and control implications. We consider three high level categories of critical situations based on their causes: A: internal system faults or performance limitations, B: rapid changes to operational conditions and C: unsafe behavior from other road users. The categorization is loosely based on [22] and [21].

Some internal system faults and performance limitations have little to no short term effect on the vehicle's motion capacity, for example loss of global localization.

Category/ Cause:	**Possible Effects in terms of Motion Planning & Control:**
A: Internal System Faults or Performance Limitations	• Deteriorated input from perception • Deteriorated actuation capacity† • Altered vehicle dynamics† • Altered mission (safe stop)*
B: Rapid Change in Operational Conditions	• Deteriorated input from perception • Deteriorated actuation capacity* • Altered vehicle dynamics* • Altered mission (safe stop)†
C: Unsafe Behavior from Other Road Users	• Evasive action required*

Table 1.1: Thesis scope in terms of critical situations. The first column presents the three high level categories of critical situations. In the second column, we break each category down in terms of possible effects on motion planning and control. Items marked $(\cdot)^{*}$ are included and evaluated in the thesis, while items marked $(\cdot)^{\dagger}$ are considered in algorithm design, but have not yet been evaluated. Unmarked items are outside the scope of the thesis

Still such a situation has implications on motion planning and control, since the nominal mission needs to be aborted, and the desired motion behavior is that the vehicle should stop at a location outside of active traffic. Other faults or performance limitations may have substantial effect on perception and/or actuation capability, i.e., a fault at any point on the functionality chain from sensors via data processing to actuation, for example loss of a sensor, compute unit or actuator. Critical situations caused by rapid changes of the operational conditions may lead to deteriorated perception capability, e.g., due to rain/snowfall obscuring sensors, deteriorated actuation capacity in terms of reduced traction, or both. Unsafe behavior from other road users may require aggressive evasive action to avoid accident, and therefore have strong implications on motion planning and control functionality. However, such situations do not have any immediate effect on the vehicle's motion capability, unless the vehicle is damaged due to a collision. In that case we categorize such a situation as a system fault after the collision.

The core work of this thesis, the characterization of the motion planning problem in critical situations, as well as the proposed algorithm framework have been done with all three categories in mind (including combinations thereof), with the delimitation that all forms of deteriorated input from perception are excluded.[2] Furthermore, for all three categories, it may be the case that by the time the critical situation is detected, an accident is physically unavoidable. Such cases are out of scope of this work. However, we are approaching this problem in a separate collaborative effort [23, 24].

Due to practical limitations, only a subset of situations were selected for evaluation. Table 1.1, provides an overview of the critical situation categories, their potential effects on motion planning and control functionality, and to which extent they are included in the scope of the thesis.

1.2.3 Research Methodology

This section presents an assessment of the methods considered for tackling the research questions, and justification for the selected options. The research objective warrants synthesis and evaluation of motion planning and control functionality, i.e., comparison of different algorithms and configurations. Since the output of such functionality dictates the motion of the vehicle and subsequantly influences the future inputs, any evaluation must be done in closed loop, i.e., data sets of prerecorded sequences of sensor data, e.g., [25], can not be used for evaluation. That leaves the following method alternatives:

- Simulation studies
- Experimental studies on reduced scale vehicles
- Experimental studies on full scale vehicles on closed circuits
- Experimental studies on full scale vehicles on public road

We proceed by discussing these alternatives with respect to internal and external validity as well as practical limitations. The term internal validity represents the extent to which the results obtained using the method represent the studied phenomenon and avoid being influenced by uncontrolled background factors. The term external validity represents the extent to which the results obtained using the method can be generalized to the research target.

Simulation studies are by far the most flexible and time efficient method choice among the alternatives. In principle, it allows representation of any vehicle deployed in any scenario. Also, internal validity is easily established, by exercising strict experimental control, e.g. ensuring that only the studied variable is altered between two compared runs. However, every simulation study is susceptible to deficiencies in terms of external validity. In order for a simulation study to be externally valid, a number of assumptions must hold. For example, the vehicle model must be sufficiently accurate to represent the real vehicle in relevant operational conditions,

[2] Sensory limitations and failure modes of perception algorithms can be handled either by nominal perception functionality or architecturally through sensor and system redundancy. Such solutions are outside our research focus, motivating this delimitation.

sensor models must be sufficiently representative of real sensors in real conditions etc. This is practically difficult to achieve.

Experimental studies have strong potential for external validity. Since the motion planning and control functions are acting on real sensor data in closed loop with the real vehicle, the study is externally valid if the test vehicle and environment are a good representations of the target vehicle and environment. Reduced scale vehicles are less expensive and more practical to work with compared to full scale. However, the dynamics and tire/road interactions of such test vehicles are not necessarily representative for full scale vehicles. Therefore, for studies like ours where vehicle dynamics play a key roll, full scale experiments are preferable in terms of external validity.

There are a number of practical issues that make experimental studies substantially more time-consuming to prepare compared to simulation studies. First, a real world experiment requires a real time capable implementation of the function to be evaluated as well as all supporting functionality, i.e., basic perception etc. This is not necessary for a simulation study. Second, orchestrating a representative scenario at full scale may be prohibitively expensive and/or unsafe. The safety aspect can be handled by performing experiments on a closed track as opposed to on public roads, provided that an artificially constructed scenario at the test track is representative of the scenario one wishes to generalize conclusions to. If this is the case, closed track tests are preferable in terms of internal validity, since they offer the opportunity to apply strict experimental control of key parameters, which is not always the case on public road.

Performance of motion planning and control functionality is strongly dependent on the quality of the input data provided by perception functionality. For example, if the planner receives a faulty state estimate in terms of its position relative to the road, it will plan erroneous and possibly unsafe motions, even if the motion planning algorithm is without fault. Real implementations of perception functionality are not perfect. Therefore, external validity of motion planning and control experiments hinges on representative perception performance. Considering a state of the art production grade AV, deployed in real traffic as the research target, in the ideal case, experimentation would be performed on precisely such a vehicle, deployed in a realistic physical representation of a range of critical situations. Not having access to such a utopian test vehicle and infrastructure leaves the researcher of motion planning and control functionality with the following methodological choice concerning the perception functionality:

- **Implementation Approach:** Evaluate motion planning and control functionality together with a representative state of the art perception stack. Notable examples: [26, 27].
- **Emulation Approach:** Evaluate motion planning and control functionality together with an emulated state of the art perception stack. Emulation, i.e., replication of function outputs, is obtained using ground truth sensors (only available for test track conditions). Notable examples: [28, 29].

There are virtues and drawbacks of both alternatives, they are not mutually exclusive and to some extent complementary. At first glance, it may seem that experiments performed with the implementation approach will have stronger internal and external validity than those performed with the emulation approach, since the test vehicle functionality more closely resembles the target, in our case a production grade AV deployed in real traffic. However, this is not necessarily always the case. With an implementation approach, there is a risk that the selected perception stack does not have the same performance properties as other algorithms/implementations and therefore the results do not generalize to the target. Also, machine learning based state of the art perception functionality performance is non-deterministic. Therefore stochastic elements may influence motion behavior between runs, possibly biasing results and introducing a threat to the internal validity of the motion planning experiment.

With an emulation approach, experiments are only externally valid if the assumptions made on the emulated perception functions generalize to real state of the art functions. Appropriate emulation parameters have to be selected based on literature review and practical knowledge of perception algorithms and sensors. Selections have to be justified in order for experiments to be valid. However, if this can be achieved for the experiment in question, there are clear benefits in terms of experimental control, flexibility and time-efficiency. For example, performance of emulated functionality can be held fixed between runs, enhancing repeatability of the experiment. This enables a level of experimental control that is not possible with an implementation approach, contributing to the internal validity of the motion planning experiment. Additionally, with an emulation approach, the resulting experimental setup is much more flexible when it comes to varying performance parameters to analyze sensitivities to perception performance parameters of the resulting motion behavior. Finally, and crucially, emulation allows a *much* more time efficient experiment preparation, which may be a necessity for being able to perform the study at all, under the time and resource constraints of academic research.

The implementation and emulation approaches are complementary in the sense that a full implementation may be used to validate or adjust assumptions made for emulation, which in turn provides stronger experimental control and more flexibility in an extended evaluation.

In this work, the method of full scale test vehicles on closed tracks was selected as the primary method of evaluation. The KTH Research Concept Vehicle (RCV), Fig. 1.3a, and The Chalmers REVERE lab's Rhino Truck, Fig. 1.3b were the two primary test vehicles that were used during the project. The RCV was used in the work associated with Paper A [3], and Rhino was used in the work associated with Papers C and D. The choice of full scale experiments was motivated by the stronger case for external validity compared to simulation studies and experimental studies on scaled vehicles. Experimental studies of critical situations on public road

[3]Although not included in the Paper A, the safe stop algorithm was later experimentally validated using the RCV. Parts of the experimental results were presented in [30]

(a) KTH Integrated Transport Research Lab's Research Concept Vehicle at the Arlanda test track

(b) Chalmers Revere Lab's Volvo FH750 test vehicle at the Storaholm low μ test track

Figure 1.3: Experimental platforms that were used during the research work

were excluded for obvious safety/legal reasons. In terms of perception functionality, both implementation and emulation was used at different stages of the work. Also, simulation studies based on experimentally validated models were used extensively in preparation for experiments and as a complement to the experimental evaluation, for example in preliminary evaluation of novel concepts, e.g., in Papers A, B, D. We further discuss the validity implications of our method choices in Section 4.1.

1.2.4 Ethical Considerations

There are obvious ethical implications to designing the motion behavior of automated road vehicles in critical situations. Famously, various instances of the so called ”trolley problem” [31, 32] have been debated in academia as well as in the media. The setup of the trolley problem is that an AV finds itself in a critical situation where it has to choose among several undesirable outcomes. For example, the choices can be either to take no evasive action and collide with a pedestrian - resulting in death/injury of the pedestrian, or to take evasive action, run the car off the road into a brick wall - resulting in death/injury of the vehicle occupants. This is a well-known ethical dilemma and different philosophical/ethical frameworks may justify different decisions. For example, a consequentialist approach would select whichever option that has the least worst consequences according to some metric, a deontological approach would make the choice based on a set of predefined moral rules, and the virtue ethics framework would select the option that best reflects good character traits.

However, the reality of motion planning in critical situations does not offer such a selection among absolute options. A realistic scenario would for example include substantial uncertainties in terms of perception and prediction of a very dynamic scenario, as well as uncertainty with respect to the physical capability of the vehicle. Therefore, the real problem becomes one of decision making under uncertainty, where probability and severity of different outcomes must be considered and weighed against each other. Still, design choices can be made based on ethical frameworks. For example Thornton et al [33, 34] explores various ways to explicitly encode the consequentialist, deontological and virtue ethics frameworks as the cost and constraints that dictate vehicle behavior under an optimization based control framework. Also in our work, the new desired vehicle behavior is connected to ethical frameworks, i.e., we set constraints based on a deontological rule (e.g., do not collide with pedestrian) and cost based on minimizing the risk of negative consequences (e.g., reduce velocity and stop in safe location).

In pursuing the research work of this thesis, the following ethical question has been raised: If it is physically possible (but perhaps difficult) for the AV to avoid harm in a critical scenario, what are the ethical implications of not doing so? As an example, consider a scenario in which a vehicle ahead of the AV brakes unexpectedly and the AV can either only brake (and crash) or execute a combined braking and turning maneuver to try to avoid a crash. In this scenario, the latter option is the only one in which the crash is possibly avoided, on the other hand it has a more uncertain outcome due to the more difficult maneuver, and potential risk of a secondary accident. This closely relates to the liability dimension of AV ethics. If, for example, a company is liable for the actions of the AV, it is likely that the company will prefer the former design, so as to reduce their risk of becoming liable for a secondary accident.

1.3 Research Contributions

The high level research contributions of this thesis are:

- A characterization of the motion planning problem in critical situations in terms of properties that dictate design of motion planning and control functionality.
- A discussion on which state of the art motion planning and control algorithms are suitable for deployment in critical situations and why.

- A proposed framework in terms of architecture and algorithms for motion planning and control, tailored for critical situations.
- Experimental validation of the framework in a set of critical situations.

Specific contributions include

(a) A novel formulation of the safe stop motion planning problem based on optimal control, where the risk level of stopping locations is represented in the cost function.

(b) Development of a trajectory library based algorithm that solves the safe stop motion planning problem in real time for simple critical situations.

(c) Validation of the safe stop motion planning algorithm in simulation and in field experiments.

(d) A novel formulation of a traction adaptive motion planning problem based on a force-input vehicle model and time-varying input constraints to encode local variations in traction.

(e) Development of an optimization based algorithm augmented by trajectory rollout that solves traction adaptive motion planning problem while avoiding local minima.

(f) GPU-acceleration of sampling augmentation procedure (e), that enables real time execution of the algorithm.

(g) Extensive experimental evaluation of the traction adaptive motion planning and control concept.

(h) Justification for the argument that neither camera based nor local state of the art friction estimation techniques in isolation is sufficient for traction adaptive motion planning and control, but fusion of the two complementary paradigms is sufficient.

(i) A proposed GP regression based method for fusion of predictive and local friction estimates that benefits from the virtues of both paradigms.

A more detailed account of the individual contributions of each publication is presented in Chapter 3.

1.4 Thesis Outline

The remaining three chapters of this thesis are organized as follows: Chapter 2 provides an account of related academic works. First, we provide a broad outline of state of the art motion planning and control algorithms for automated vehicles, as well as relevant vehicle representations. Second, we outline and analyse the subset of work that are relevant for motion planning and control in *critical situations*, and point out research gaps. Chapter 3 summarizes the contributions of the thesis at a high level, as well as for each individual appended paper. Chapter 4 presents

Appended Paper:	**Situation Categories:**	**Research Questions:**	**Research Gaps:**	**Specific Contributions:**
Paper A	A	1,2	i	(a),(b),(c)
Paper B	B,C	1,2,3	ii, iii, iv	(d), (e)
Paper C	B,C	1,2,3	ii, iii, iv	(d), (e), (f), (g)
Paper D	B,C	1,2,3,4	v	(h), (i)

Table 1.2: Overview of key parts of the thesis. The table highlights how each paper connects to the categories of critical situations introduced in Section 1.2.2, the research questions introduced in Section 1.2.1, the identified research gaps to be introduced in Section 2.3 and the specific contributions of the thesis listed in Section 1.3.

a discussion on validity, and summarizes conclusions and suggestions for future research directions. In addition, Table 1.2 provides an overview of the connections between key parts of the thesis.

Chapter 2

State of the Art

Research and development of automated road vehicles live at the intersection of multiple traditional research fields [13, 35]. Fig. 2.1 gives an overview of the specific fields involved in motion planning and control of automated vehicles in critical situations. Although we will not cover all of them in detail here, the aim of this chapter is to provide sufficient background of relevant academic fields to communicate where and how the research work of this thesis fits into the large and rapidly evolving landscape of AV research.

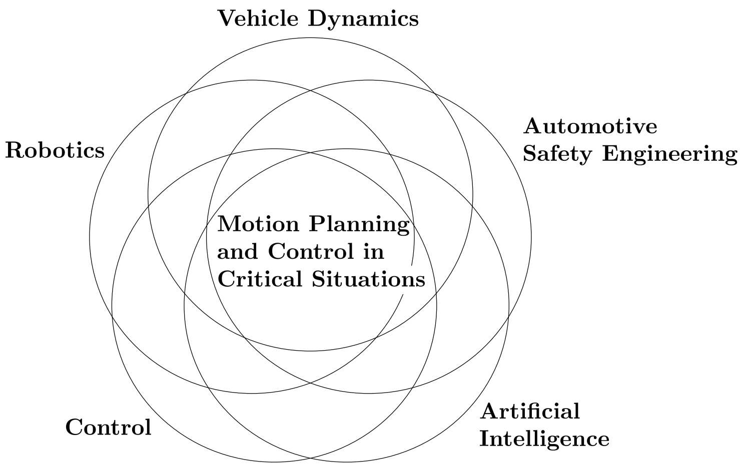


Figure 2.1: Overview of the research fields relevant to motion planning and control in critical situations

All the research work to be described in this chapter takes place against a backdrop of automotive safety engineering, providing context and a guiding frame in the form of standards [4, 21]. Section 2.1 outlines common algorithms for motion planning and control that originate from the fields of robotics, artificial intelligence and control. Section 2.2 outlines the vehicle dynamics modeling concepts that are most relevant for real time motion planning and control. Finally, Section 2.3 includes a more detailed analysis of the research work that is closest related to ours, and identifies research gaps in the existing state of the art literature.

2.1 Motion Planning and Control of Automated Vehicles

Consider a mobile robot, whose motion relevant parameters, e.g., position, orientataion and velocity etc. are collected in a state vector $x_t \in \mathcal{X}$, representing the motion state of the robot at time t. Robot movement, i.e., how the state propagates over time, is described by a discrete time non-linear system $x_{t+1} = f(x_t, u_t)$, where $u_t \in \mathcal{U}$ is a control input vector, containing the control commands of the robot, e.g., steering angle, throttle and brake. The sets $\mathcal{X}$ and $\mathcal{U}$ denote all possible values of the state and control vectors. We further introduce $\mathcal{X}_{\text{free}}$ to represent the subset of $\mathcal{X}$ where the robot is not in collision with obstacles.

The central problem in the field of robot motion planning is to compute a sequence of future states $x_{k|t}$ and control inputs $u_{k|t}$ that take the robot from the current state x_t to or toward some goal state x_f. The sequence should be collision free with respect to static and dynamic obstacles, and feasible with respect to robot dynamics [35–38]. To simplify notation we let $\mathcal{T}_t = \{\{x_{k|t}\}_{k=0}^{N}, \{u_{k|t}\}_{k=0}^{N-1}\}$ represent such a planned trajectory. Note that there are two separate time indices, t denoting the time at which the plan is computed, and k representing the index of future time for which the motion is planned.

The term motion planning is often used as an overarching term encompassing both path planning (purely spatial) and trajectory planning (spatial and temporal), but since this thesis only concerns trajectory planning, the terms motion and trajectory planning are used interchangeably. This section aims at providing an overview of the research field, largely based on recent survey articles [35, 37, 38].

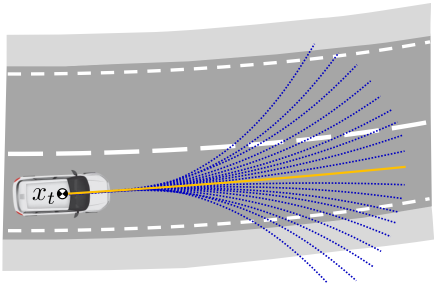
2.1.1 Trajectory Rollout

Trajectory Rollout is an intuitive and practical approach to motion planning. The central concept is the following. At each planning iteration,

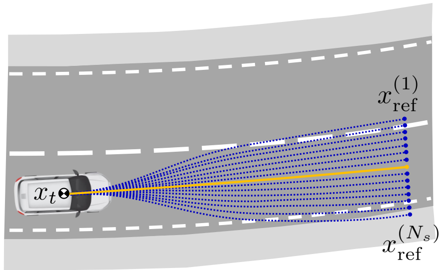
1. Roll out a set $\hat{\mathcal{S}}_t = \bigcup_{i=1}^{N_s} \hat{\mathcal{T}}_t^{(i)}$, $i \in \{1, 2, \ldots, N_s\}$ of candidate trajectories, $\hat{\mathcal{T}}_t^{(i)} = \{\{\hat{x}_{k|t}^{(i)}\}_{k=0}^{N}, \{\hat{u}_{k|t}^{(i)}\}_{k=0}^{N-1}\}$, forward in time from the current state x_t.

2. Prune away candidates resulting in collision, i.e., those where $\hat{x}_{k|t}^{(i)} \notin \mathcal{X}_{\text{free}}$, to obtain a set of collision free candidates $\hat{\mathcal{S}}_{t,\text{free}}$.

3. Among the collision free candidates, evaluate a cost function $J(\cdot)$ and select the lowest cost trajectory $\mathcal{T}_t^{\star} = \arg \min_{\hat{\mathcal{T}}_t \in \hat{\mathcal{S}}_{t,\text{free}}} (J(\hat{\mathcal{T}}_t))$ for execution.



(a) Input space sampling



(b) State space sampling

Figure 2.2: Illustration of trajectory rollout with input space and state space sampling

There are numerous successful motion planning algorithms based on this concept presented in literature under various names, for example the dynamic window algorithm [39], state space sampling and input space sampling [40]. There are three major categories of trajectory rollout methods, distinguished by the way step 1. is performed; namely input sampling, state space sampling, and trajectory library. Next, we briefly describe them individually.

With the input sampling approach, illustrated in Fig. 2.2a, the admissible control input space $\mathcal{U}$ of the model is discretized, and for each discrete input vector, the model of the robot is propagated forward from x_t to generate the set of candidate trajectories $\hat{\mathcal{S}}_t$. Since only admissible inputs are used to propagate the model, all trajectories are dynamically feasible by construction. A negative aspect of input sampling is that in heavily constrained environments, such as road networks, a majority of trajectories will go outside of the road, resulting in an unfavorable trade-off between candidate goal state resolution and computation time [40].

With the state space sampling approach, illustrated in Fig. 2.2b, instead of discretizing the input space $\mathcal{U}$, a discrete set of N_s reference states $x_{\text{ref}}^{(i)}$, $i \in \{1, 2, \ldots, N_s\}$, is defined at positions within the drivable area ahead of the vehicle. Next, a boundary value problem is solved to compute connecting candidate trajectories $\hat{\mathcal{T}}_t^{(i)}$ between the current state x_t and each of the N_s reference states, which constitute $\hat{\mathcal{S}}_t$ in the state space sampling method. This sampling method enables utilization of the structure in the environment by only placing candidate goal states in the permitted parts of the state space, e.g., on the road [40]. Computational tractability of the method hinges on a fast method to obtain solutions to the boundary value problems between the initial state and each candidate goal state. Werling et al. [41] introduce the use of quartic and quintic polynomials to represent the candidate trajectories, for which there exists an analytical (and hence very fast) solution to the boundary value problem. Werling et al. prove that the resulting trajectories minimize jerk for a point mass model, and thus represent smooth, comfortable motion. For more accurate and complex vehicle models, the boundary value problem does not have an analytical solution, making the method computationally intractable. Hence, a downside of the state space sampling method compared to input space sampling is that there is no guarantee that a sampled trajectory is dynamically feasible. Instead, the algorithm includes a procedure that checks dynamic feasibility and removes infeasible candidates.

The trajectory library approach is a successful and widely used real-time approach that allows state space sampling with an arbitrarily accurate model The trick is to move the computationally intensive solving of the boundary value problems offline and store the resulting trajectory set $\hat{\mathcal{S}}_t$ in a library that can be accessed online at a fraction of the computational cost [28, 42, 43]. This enables the use of an arbitrarily complex model, without increasing runtime computational cost. However, this concept assumes that the robot dynamics are position invariant, i.e., constant between different positions. Therefore, the benefit of offline computation of primitives diminishes for systems where the dynamics vary in space. We elaborate further on spatially invariant and non-invariant vehicle models in Chapter 2.2. To some extent, this can be handled by increasing the dimensionality of the trajectory set, i.e., by computing separate sets for each discrete step of a spatially varying parameter. However, the size of the set will grow exponentially with the number of spatially varying parameters, limiting the practicality of this approach.

All three variants of the trajectory rollout approach remain popular choices for a wide range of applications in robotics and automated vehicles, due to their intuitive practicality and because they are inherently suitable for parallelization, which in turn enables computational efficiency through execution on e.g., graphics processing units (GPUs).

2.1.2 Graph Search

Another approach to motion planning is to represent the robot state space $\mathcal{X}$ as a directed graph, built up from vertices connected by edges with an associated cost. A trajectory, i.e., a sequence of edges, that connects the initial state with the goal state can then be obtained using standard graph search algorithms such as Dijkstra's algorithm [44] or A* [45]. Two popular types of graph search based motion planning algorithms are state lattice motion planners [46] and Hybrid A* [47] type algorithms, both of which have been successfully deployed to generate a range of AV motion behaviors [47, 48].

State lattice based motion planning algorithms are based on offline computation of position invariant trajectory pieces called motion primitives, that are connected together as vertices and edges of a directed search graph [46]. The graph is stored in memory and can be efficiently searched at runtime. As the motion primitives are computed offline, computational efficiency is not a priority in this step. Therefore, the primitives can be generated with complex dynamic models, using optimal control [49, 50]. However, just as with the trajectory library approach, Section 2.1.1, this is only viable under the assumption that robot dynamics are position invariant.

The Hybrid A* algorithm [47] is hybrid in the sense that both the input space $\mathcal{U}$ and part of the state space $\mathcal{X}$ are discretized. Each vertex of the graph corresponds to a spatial grid cell and an associated vehicle state. The graph is built by propagating a vehicle model according to a discretized input for a fixed short time (the same principle as input sampling trajectory rollout, Fig. 2.2a). The spatial grid cell that the vehicle arrived in, together with the state after the move, makes up a new vertex of the graph, with a corresponding edge that connects with the previous vertex. If the vehicle ends up in a previously visited cell, the edge corresponding to the higher cost trajectory will be discarded. Due to the input sampling characteristic, the algorithm does not assume position invariance and can accommodate systems with a relatively large number of states [47].

In principle, one can view the state lattice algorithm and Hybrid A* as extensions of trajectory rollout algorithms with graph search (state space sampling and input space sampling respectively). This extension is particularly useful when planning intricate sequential maneuvers over longer times in static environments, e.g., [51]. Such elaborate maneuvering is however rarely required when planning for short time horizons and when reducing risk in critical situations.

2.1.3 Random Sampling

The characterizing concept of random sampling based motion planning algorithms is to find a connection between the initial and goal states by drawing random samples of states $\tilde{x}_t$ from the robot's state space $\mathcal{X}$, attempting to connect the sampled states with dynamically feasible pieces of trajectory, to eventually obtain a connection between the current state x_t and the goal state x_f [36, 52].

The RRT algorithm [53] is an example of a random sampling based motion planning that has been successfully deployed in numerous robotic applications including automated driving [54]. Since its introduction, many modifications and improvements have been made to the RRT algorithm. Karaman et al. introduced the RRT* version [55] that, by introducing a rewiring procedure, i.e., replacing pieces of the tree if a lower cost option becomes available, obtains asymptotic optimality (optimality given a large enough number of iterations). Also, so called "anytime" modifications have been introduced to improve performance in dynamically changing environments [56] by first providing a feasible but suboptimal solution that is subsequently improved.

The random samples in RRT are connected together using a local steering function that solves the boundary value problem associated with connecting pairs of sampled states (equivalent to the problem solved for state space sampling trajectory rollout, Fig. 2.2b). Since the local steering function must be executed for each new sample, computational tractability of the algorithm hinges on a fast solution to the boundary value problem. For rudimentary kinematic representations of a vehicle, analytical solutions to this problem exist [41, 57, 58]. However, this is not the case for more complex vehicle models including e.g., tire dynamics.

2.1.4 Numerical Optimization

Without loss of generality, a motion planning problem as it is described in the introduction of this chapter can be expressed as a constrained finite time optimal control problem (CFTOC):

$$
\begin{array}{rl}
\min\limits_{u_{0|t},\ldots,u_{N-1|t}} & J(\mathcal{T}_t) \\
\text{s.t.,} & x_{k+1|t} = f\left(x_{k|t}, u_{k|t}\right), \\
& u_{k|t} \in \mathbb{U}_{k|t}, \\
& x_{k|t} \in \mathbb{X}_{k|t}, \\
& \forall k \in \{0, 1, \ldots, (N-1)\}, \\
& x_{0|t} = x_t,\ x_{N|t} \in \mathbb{X}_{N|t},
\end{array}
\tag{2.1}
$$

where the combined design of cost function $J(\cdot)$ and constraints, $\mathbb{U}_{k|t} \in \mathcal{U}$, $\mathbb{X}_{k|t} \in \mathcal{X}$ determines the motion behavior of the solution trajectory $\mathcal{T}_t^\star$. Solutions to optimal control problems such as (2.1) are obtained using standard theory [59–62] and tools [63–72] from numerical optimization and optimal control.

The representation of a noisily perceived and dynamically changing traffic scene as hard constraints on the vehicle state, $x_{k|t} \in \mathbb{X}_{k|t}$, may sometimes be too restrictive and lead to infeasibility, i.e., that there is no solution that satisfies all constraints, and therefore no motion plan can be produced. This is commonly mitigated by introducing slack variables, such that state constraint violations are allowed, but heavily penalized [28, 73].

Such a relaxation ensures numerical feasibility of the problem, but still leaves a high-dimensional non-convex optimization problem for which global optimality of the solution cannot be guaranteed, and real time computation is only tractable if the solver can be warm-started with a near optimal initial guess of the solution [74]. Such an initial guess may be obtained either from a random sampling or graph search based planner [47, 50, 74–76], or in the case of periodic replanning applications, by forward shifting the solution trajectory from the previous planning iteration. The latter approach is called the RTI scheme [77–79], and has become tremendously successful in numerous planning and control applications due to its exceptional computational efficiency [26, 28, 41, 67, 80].

The core concept is to locally approximate the non-convex optimization problem (2.1), in the neighborhood of an initial guess, by a Quadratic Program (QP), for which efficient numerical solvers exist [70–72]. The QP approximation must have a quadratic cost, linear model and affine constraints [60]. This is obtained through the following steps.

First, the initial guess $\hat{\mathcal{T}}_t = \{\{\hat{x}_{k|t}\}_{k=0}^{N}, \{\hat{u}_{k|t}\}_{k=0}^{N-1}\}$, is obtained by forward shifting the solution from the previous iteration $\mathcal{T}_{t-1}^\star$, i.e., $\hat{x}_{k|t} = x_{k+1|t-1}^\star$ for $k \in \{1, 2, \ldots, (N-1)\}$ and $\hat{u}_{k|t} = u_{k+1|t-1}^\star$ for $k \in \{1, 2, \ldots, (N-2)\}$. This is not possible for the N-th planned state and $(N-1)$-th planned control input. Instead, the final control input is repeated $\hat{u}_{N-1|t} = u_{N-1|t-1}^\star$ and the final state is obtained by integrating the dynamics one step forward $\hat{x}_{N|t} = f(x_{N-1|t-1}^\star, u_{N-1|t-1}^\star)$.

Second, the nonlinear dynamics are linearized about the initial guess

$$A_{k|t} = \left.\frac{\partial f}{\partial x}\right|_{(\hat{x}_{k|t},\hat{u}_{k|t})}, \ B_{k|t} = \left.\frac{\partial f}{\partial u}\right|_{(\hat{x}_{k|t},\hat{u}_{k|t})}, \forall k \in \{0, 1, \ldots, (N-1)\}.$$

Third, state and input constraints are expressed as sets of linear inequalities, obtained by approximating $\mathbb{X}$ and $\mathbb{U}$ as convex polytopes

$$\mathbb{U}_{\mathbb{P}} = \{u : H_u u \leq h_u\} \quad \mathbb{X}_{\mathbb{P}} = \{x : H_x x \leq h_x\}.$$

Fourth, a quadratic cost function of the form

$$J(\mathcal{T}_t) = x_{N|t}^\top Q_N x_{N|t} + \sum_{k=0}^{N-1} (x_{k|t}^\top Q x_{k|t} + u_{k|t}^\top R u_{k|t}),$$

can be either selected by design, or obtained online from a second order approximation of an arbitrary cost function $J(\cdot)$. Weight matrices $Q_N \succ 0$, $Q \succ 0$ and $R \succ 0$ are tuned to obtain desired motion behavior. The final QP to be solved at each time t can be formulated as

$$\begin{aligned}
\min_{\Delta u_{0|t},\ldots,\Delta u_{N-1|t}} \quad & J(\mathcal{T}_t) = x_{N|t}^\top Q_N x_{N|t} + \sum_{k=0}^{N-1} (x_{k|t}^\top Q x_{k|t} + u_{k|t}^\top R u_{k|t}) \\
\text{s.t.,} \quad & x_{k+1|t} = A_{k|t}(\Delta x_{k|t}) + B_{k|t}(\Delta u_{k|t}) + \hat{x}^\star_{k+1|t}, \text{ with} \\
& [\Delta x_{k|t}, \Delta u_{k|t}] = [x_{k|t} - \hat{x}_{k|t}, u_{k|t} - \hat{u}_{k|t}], \\
& u_{k|t} \in \mathbb{U}_{\mathbb{P}}, \ \forall k \in \{0, 1, \ldots, (N-1)\}, \\
& x_{k|t} \in \mathbb{X}_{\mathbb{P}}, \ \forall k \in \{0, 1, \ldots, N\}, \\
& x_{0|t} = x_t.
\end{aligned} \tag{2.2}$$

Once such a quadratic approximation is obtained, one can choose to solve it until convergence [28, 80, 81] or, to further speed up computation, perform just a single Newton step in the QP optimization procedure [78]. The latter is motivated by the fact that (2.2) is a local approximation of (2.1), therefore it may not be meaningful to solve to convergence. A comparison between the two approaches is provided by Gros et al. [82].

It has been pointed out that the RTI approach is sensitive to local minima [26]. This is a fundamental drawback of the same procedure that enables the outstanding computational efficiency of the method, namely that the original nonlinear CFTOC (2.1) is *locally* approximated by a QP. If the initial guess is far from the global optimum, and (2.1) is non-convex, the optimizer is unlikely to find the globally optimal solution.

2.2 Vehicle Dynamics for Motion Planning and Control

Algorithms for motion planning and control require an internal model of the vehicle's motion capabilities, which is used to represent the dynamics and physical

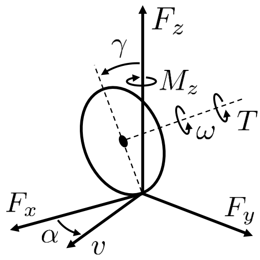


Figure 2.3: Common physical variables used in tire modeling.

limitations of the vehicle, such that planned maneuvers maintain dynamic feasibility. This chapter provides background in the area of vehicle modeling for motion planning and control of AVs. Emphasis is placed on models that represent the vehicle in highly dynamic maneuvers. We start by outlining state of the art models of tire-road interaction and proceed to introducing full vehicle models. General concepts and notation conventions are based on [83].

2.2.1 Modeling Tire-Road Interaction

Wheeled ground vehicle motion is primarily generated by friction forces between the contact patch of the tire and the ground surface [84]. For automated road vehicles, appropriate modeling of the tire-road interaction is of paramount importance for motion planning and control. Any mathematical model of a vehicle includes some representation of this interaction, explicitly in the form of a tire model, or implicitly through an underlying assumption.

A simple way to model this interaction that is useful for many applications is to assume zero tire slip, meaning that the tire motion is always tangential to the tire orientation and the peripheral velocity of the tire is always equal to the forward velocity. This assumption allows a purely kinematic model of the vehicle, which we will revisit in Section 2.2.3.

Real tires however, do slip when the contact patch is exposed to sufficiently high lateral and/or longitudinal forces. The slip can be decomposed into a slip angle α determined by the angular difference between the tire's velocity vector and its orientation, and a longitudinal slip ratio σ_x, determined by the ratio between the peripheral velocity of the tire and the forward velocity of the wheel. Fig. 2.3 shows a graphical representation of variables relevant for tire modeling. Contact forces between road and tire are decomposed in the longitudinal, F_x, lateral, F_y, and normal directions F_z of the tire. Moments M_z and T denote the aligning moment about the z-axis, and the driving torque of the wheel. Variables v, ω and γ denote velocity, angular velocity and camber angle.

The aim of a tire model is to find a mapping from the state of the tire, in terms of e.g. slip angle and slip ratio, to the forces and moments generated from tire-road interaction, i.e., $[\alpha, \sigma_x] \mapsto [F_x, F_y, M_z]$. Tire dynamics are substantially affected by many inter-dependent factors that real-world tires are exposed to on a daily basis, such as variations in vehicle speed, road surface conditions, type and wear of the tire, camber angle etc., that are included in more advanced models. This makes mathematical modeling and prediction of tire dynamics a non-trivial research problem.

The research area of tire dynamics modeling has been active since the 1950s. Early models were derived from first principles and were based on assumptions related to various physical analogues [85, 86]. A more recent generation of models obtained higher accuracy through empirical parametrization of an arbitrary function - not necessarily grounded in physics. This category is exemplified by the so called Magic Formula tire model [87, 88] first introduced in 1987, which has become the industry standard. Other notable examples of tire models are the LuGre model [89, 90] and the Unitire model [91]. In the simplest form of the Magic formula, the longitudinal force F_x, lateral force F_y, or aligning moment M_z, is given by the function

$$y = D \sin\left(C \arctan\left(Bx - E\left(Bx - \arctan\left(Bx \right) \right) \right) \right),$$

with

$$\begin{aligned} Y(X) &= y(x) + S_V, \\ x &= X + S_H. \end{aligned}$$

where X is the input variable (α or σ_x) and Y is the output variable (F_x, F_y or M_z). The stiffness factor B, the shape factor C, the peak value D, the curvature factor E, the vertical shift S_V and the horizontal shift S_H are named, interpretable parameters of the model, determined through regression using experimental data. Fig. 2.4 gives an example of how the magic formula captures tire behavior on various road conditions.

This is the Magic formula in its simplest form, a six parameter model representing a single dimension at a static camber angle and normal load. A multitude of extensions to the model, for example to include combined slip and varying normal load, are presented in [92]. In the extensions, generality is obtained at the price of an increase in the number of parameters. A full magic formula description of a tire may have upwards of 85 parameters [93].

Tire models vary in complexity from a single analytic expression to elaborate finite element analysis schemes. In the context of real time vehicle control and vehicle autonomy, computational complexity of the model also has to be taken into account due to the computational trade-off mentioned in Section 2.1.4. Therefore, using the full Magic formula tire model is computationally intractable for real time motion planning and control. Instead, simplified variants are used. One such example is the linear tire model, here expressed for the lateral dimension.

$$F_y = C_\alpha \alpha,$$

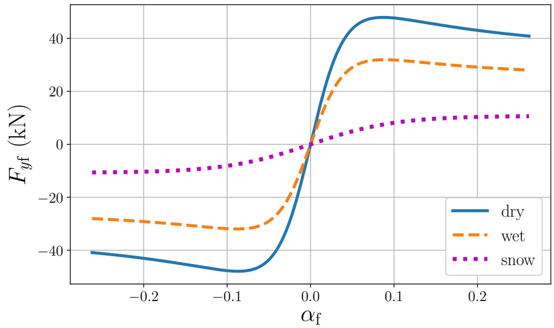


Figure 2.4: Example response of a Magic formula tire model, for three different road surfaces, dry, wet and snow.

where the single parameter C_α is denoted the cornering stiffness, and can be obtained by linearizing a more elaborate model. For example, with the Magic formula, a linearization about the origin can be easily obtained as $C_\alpha = BCD$.

For determining dynamic feasibility in motion planning, a precise mapping from slip to force is not necessarily required. Instead, what is needed is a representation of the maximum tire forces that can be realized at different locations over the prediction horizon. A simple model of the upper limit is the so called friction ellipse [84, 94] expressed as

$$\frac{F_x^2}{F_{x,\max}^2} + \frac{F_y^2}{F_{y,\max}^2} \leq 1, \tag{2.3}$$

with $F_{x,\max} = \mu_x F_z$ and $F_{y,\max} = \mu_y F_z$. Most tire designs can generate higher forces longitudinally than laterally, captured by setting $\mu_x = \mu$ and $\mu_y = k\mu_x$, with $0 < k < 1$. Assuming that $\mu_x = \mu_y = \mu$, the friction ellipse reduces to a friction circle [83].

$$\sqrt{F_x^2 + F_y^2} \leq \mu F_z,$$

This is an elegant representation, however it is not a perfectly accurate description for real tires. For example, Brach et al. [95], show that the real tire force envelope can be substantially larger than the limit provided by the friction ellipse. Also, the real envelope is not elliptic. This suggests that a more permissive representation is warranted for optimal utilization of the available tire force, for example a polygon.

2.2.2 Modeling the Vehicle

Full body four wheel vehicle models are rarely used in the motion planning context. The computational cost associated with such a high-dimensional model results in a

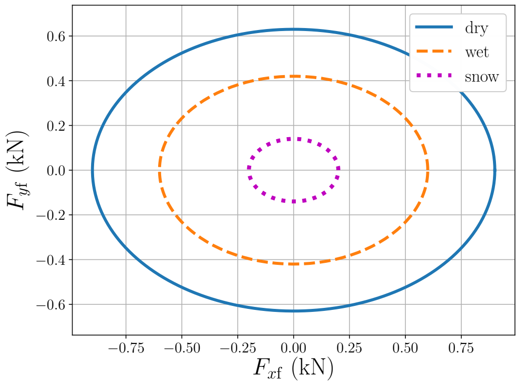


Figure 2.5: Friction ellipse model, Equation 2.3, for the upper bound of tire forces, for three different examples of road surfaces, dry ($\mu = 0.9$), wet ($\mu = 0.6$) and snow ($\mu = 0.3$).

poor trade-off with prediction horizon length and replanning rate, see Chapter 2.1. Instead, variations of the so called bicycle model, sometimes referred to as the single track model, have been a staple of successful approaches to AV motion planning and control [35], including highly dynamic applications [28, 29, 96, 97]. The central assumption of the bicycle model is that the effects of vehicle roll dynamics cancel out between the left and right tire pairs. Thus, the front and the rear tire pairs can be represented by single tire models. The reason behind the success of the bicycle model in this application is that it is accurate enough to represent the dominant dynamics, while being simple enough to conform with the strict computational limitations of real time motion planning and control.

Next, we describe and discuss two variants of bicycle models that are commonly used in AV motion planning and control, characterized by fundamentally different assumptions concerning tire-road interaction.

2.2.3 Kinematic Bicycle Model

The characterizing assumption of the kinematic bicycle model is that tire slip is zero at all times. Thus, the motion of the model is fully determined by its geometry, speed v and steering angle δ, as depicted in Fig. 2.6 [1]. Given an input $u = [\delta, a]$, the

[1] What we show in this section is a selected example out of a wide range of kinematic vehicle models that exist in literature, characterized by the zero slip assumption. However, the implications of the zero tire slip assumption generalizes to all variations of the kinematic bicycle models.

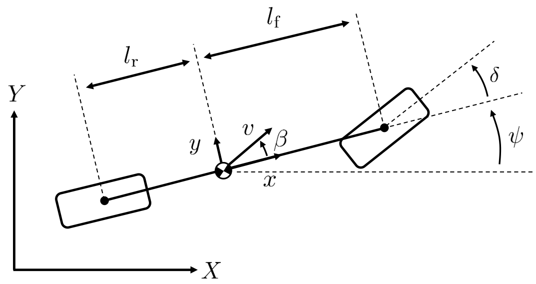


Figure 2.6: Kinematic Bicycle Model.

state $x = [X, Y, \psi, v]$ of the model propagates according to the differential equations

$$\begin{aligned} \dot{X} &= v\cos(\psi + \beta), \\ \dot{Y} &= v\sin(\psi + \beta), \\ \dot{\psi} &= \tan(\delta)\frac{v\cos(\beta)}{l_\mathrm{f} + l_\mathrm{r}}, \\ \dot{v} &= a, \end{aligned} \tag{2.4}$$

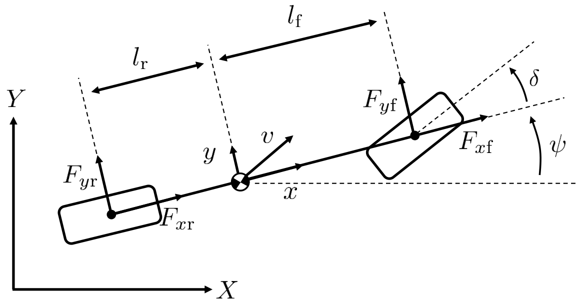
where X, Y and ψ denote the position of the center of mass and the orientation of the chassis. The velocity of the center of mass is denoted v. The control inputs are acceleration a and steering angle δ. The body slip angle β in Fig 2.6 is fully determined by the steering angle and static parameters l_f and l_r

$$\beta = \arctan\left(\frac{l_\mathrm{r}\tan(\delta)}{l_\mathrm{f} + l_\mathrm{r}}\right).$$

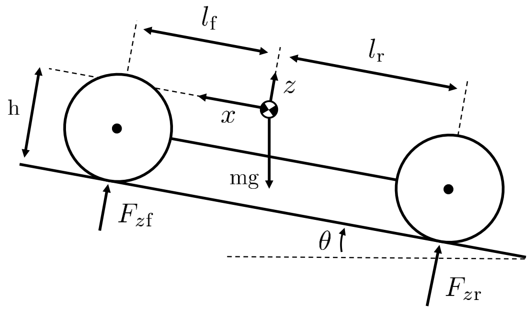
The relatively low complexity of the model (small number of states) and position invariance make it a popular choice in a multitude of robotic applications [98]. It is used in an overwhelming majority of work in the field of motion planning for automated vehicles [26, 35, 37, 74, 81, 99]. The tremendous impact of the kinematic bicycle model indicates that the zero tire slip assumption is valid for most AV applications. However, it is clear that for dynamic use cases, i.e. cases where real tires do slip, the accuracy of the kinematic model deteriorates [100, 101]. An investigation performed by Polack et al. [100] shows that the model is only consistent with vehicle motion for low accelerations, i.e., $a \leq 0.5\mu g$, where μ is the tire-road friction coefficient and g is the gravitational acceleration.

2.2.4 Dynamic Bicycle Model

A dynamic bicycle model enables accurate representation of the vehicle at accelerations above $0.5\mu g$ by including tire slip dynamics. Here we present an example of



(a) Top view



(b) Side view

Figure 2.7: Dynamic bicycle model.

such a model, based on standard literature [83]. The model is derived by means of textbook Newtonian mechanics from the forces generated between the tires and the road. Figures 2.7a and 2.7b outline the kinematic and dynamic properties used in the model. Given an input $u = [\delta, F_{x\mathrm{f}}, F_{x\mathrm{r}}]$ consisting of the steering angle and tractive force inputs at the front and rear tires, the state $x = [X, Y, \psi, \dot{\psi}, v_x, v_y]$ propagates as

$$\begin{aligned}
\dot{X} &= v_x \cos\psi + v_y \sin\psi, \\
\dot{Y} &= v_x \sin\psi - v_y \cos\psi, \\
\ddot{\psi} &= \frac{1}{I_z}\left(l_\mathrm{f} F_{y\mathrm{f}} - l_\mathrm{r} F_{y\mathrm{r}}\right), \\
\dot{v}_x &= \frac{1}{m}\left(F_{x\mathrm{f}} + F_{x\mathrm{r}}\right) - g\sin\theta - R_x - F_\mathrm{aero}, \\
\dot{v}_y &= \frac{1}{m}\left(F_{y\mathrm{f}} + F_{y\mathrm{r}}\right) - v_x\dot{\psi} + g\sin\phi,
\end{aligned} \tag{2.5}$$

with

$$
\begin{aligned}
F_{y\mathrm{f}} &= f(\alpha_f, F_{x\mathrm{f}}, F_{z\mathrm{f}}, \mu), \\
F_{y\mathrm{r}} &= f(\alpha_r, F_{x\mathrm{r}}, F_{z\mathrm{r}}, \mu), \\
F_{z\mathrm{f}} &= \frac{1}{l_\mathrm{f} + l_\mathrm{r}} \left(-m\dot{v}_x h - mgh\sin\theta + mgl_\mathrm{r}\cos\theta \right), \\
F_{z\mathrm{r}} &= \frac{1}{l_\mathrm{f} + l_\mathrm{r}} \left(m\dot{v}_x h + mgh\sin\theta + mgl_\mathrm{f}\cos\theta \right).
\end{aligned}
$$

Just as for our example kinematic model X, Y and ψ denote the position of the center of mass, and orientation of the vehicle. However, contrary to the kinematic model, the longitudinal and lateral accelerations $\dot{v}_x$, $\dot{v}_y$ and yaw acceleration $\ddot{\psi}$ are determined by dynamic properties (forces and inertias), rather than kinematic properties. The lateral forces on the front and rear tires $F_{y\mathrm{f}}$ and $F_{y\mathrm{r}}$ are determined by a tire model $f(\alpha, F_x, F_z, \mu)$. Forces R_x and F_aero represent rolling resistance and aerodynamic drag. The mass of the vehicle is denoted by m and the moment of inertia about the z-axis is denoted by I_z. Parameters l_f, l_r and h specify the position of the center of mass relative to the wheel contact patches. Angles θ and ϕ denote the grade and bank angles of the road and g the gravitational acceleration. From the point of view of real time motion planning and control, there are three practical challenges with using such a dynamic model.

First, several model parameters that have substantial impact on motion capability, for example the friction coefficient μ, the normal loads $F_{z\mathrm{f}}$, $F_{z\mathrm{r}}$ and the grade and bank angles θ, ϕ, are not spatially invariant. This requires the motion planning algorithm to allow a time/space-varying motion model, preventing the use of several approaches, recall Section 2.1.

The second challenge is computational tractability. It is typically not possible to accommodate the full body dynamics including tire models in real time motion planning and control, within an acceptable replanning time. This forces the designer to introduce simplifications in the model based on assumptions related to individual applications.

This leads us to the third practical challenge - coupled dynamics. A common approach to reducing computational complexity is to decouple the lateral and longitudinal dimensions to reduce the overall complexity of the motion planning problem. However, during aggressive maneuvering, lateral, longitudinal and yaw dynamics are tightly coupled [97, 102]. The coupling stems from the upper bound of combined lateral and longitudinal force on each tire, see e.g., Fig. 2.5. This means that optimal utilization of available grip requires *coordinated* planning and control of the three dimensions simultaneously. Hence, simplification of the problem through decoupling is not an option. Nonetheless, previous works show that computational tractability with the full dynamic bicycle model is possible through clever design choices and state of the art motion planning algorithms [27, 28].

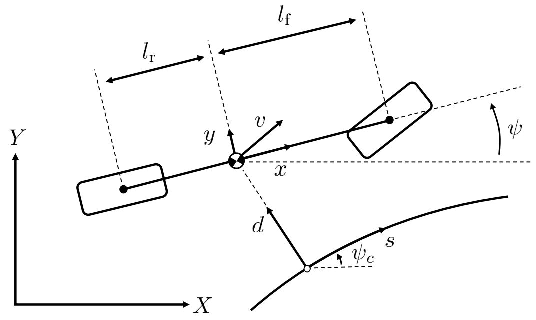


Figure 2.8: Road aligned (Frenet) coordinate frame

2.2.5 Road Aligned Coordinate Frame

For motion planning and control, the position and orientation of the vehicle relative to the road is usually more relevant than its position and orientation in a Cartesian coordinate system. Therefore, it is common to express the vehicle dynamics model in a road aligned coordinate frame, sometimes referred to as a Frenet frame [98, 103]. Fig. 2.8 introduces the position and orientation variables of a road aligned coordinate frame. Here, the s coordinate denotes the progress along the lane center and the d coordinate denotes the lateral deviation from the lane center, increasing to the left. Variables ψ_c and $\kappa_c = \frac{\partial \psi_c}{\partial s}$ denote the tangent angle, and curvature of the lane at a point on the lane center where the vehicle's position is perpendicular to the tangent of the lane. The angle $\Delta\psi = \psi - \psi_c$ represents the relative angle between the vehicle yaw angle and the tangent angle of the lane. The time evolution of s, d and $\Delta\psi$ is derived from geometric relations, and given by

$$
\begin{aligned}
\dot{s} &= \frac{v_x \cos(\Delta\psi) - v_y \sin(\Delta\psi)}{1 - d\kappa_c}, \\
\dot{d} &= v_x \sin(\Delta\psi) + v_y \cos(\Delta\psi), \\
\Delta\dot{\psi} &= \dot{\psi} - \kappa_c \frac{v_x \cos(\Delta\psi) - v_y \sin(\Delta\psi)}{1 - d\kappa_c},
\end{aligned}
$$

This representation of position and orientation in terms of s, d and $\Delta\psi$ can be used to replace X, Y and ψ in both the kinematic model (2.4), and the dynamic model (2.5). In the context of motion planning and control, expressing vehicle dynamics in such a road aligned frame facilitates the encoding of desirable behaviors as costs and constraints. For example, a desire to have the vehicle stay within lane boundaries (which is nontrival to encode using a cartesian coordinate system) can be easily and efficiently encoded as a constraint on d.

2.3 Analysis of Research Gaps in Related Work

The general motion planning problem formulation outlined in Chapter 2.1 can be instantiated for a wide range of AV functionalities/features, for example highway driving, parking etc. The wide range of operational scenarios in which AVs are deployed present substantial variations in the requirements and performance metrics for motion planning and control functionality. For example, in a parking scenario, spatial precision and accurate representation of the vehicle shape are important aspects for obtaining the desired motion behavior, while aspects like replanning rate and tire dynamics are not, due to low speeds and a relatively static environment.

In this chapter, we dive into the specifics of motion planning and control *in critical situations*. We outline related academic works from multiple fields, and provide our own analysis, highlighting research gaps.

2.3.1 Differentiating Stopping Locations in Local Planning

The area of contingency motion planning in the event of faults is much less researched than other aspects of motion planning for automated vehicles [35]. The following proposed methods represent initial steps. Magici et al. [104] formulate a fail-safe motion planning problem using optimal control, with the objective to reach a full stop, under constraints to avoid an over-approximated predicted occupancy of adjacent vehicles [105]. A version of the algorithm presented in [99] is employed to solve the resulting motion planning problem using an RTI scheme. In [106], Pek et al. extend the work in [104], by reducing computational times through decoupling longitudinal and lateral dynamics. Furthermore, the proposed algorithm is compared with a state space sampling, trajectory rollout method, namely Werling et al. [41] and show a performance increase over a range of scenarios from the Commonroad benchmark [107]. A notable non- optimization based example is presented by Salvado et al. [108]. The authors propose a state lattice algorithm based on precomputed motion primitives with the goal of finding a collision free trajectory from an arbitrary initial state to a full stop, in the event of a fault. The lattice based approach enables long term planning all the way to a full stop even from high initial velocities.

The above examples of local motion planning methods are designed to provide collision free fallback solutions without depending on a particular goal location being provided from external functionality. This provides the core safe stop functionality, namely to take the vehicle to a stop, without colliding with obstacles. However, the methods have several common drawbacks. First, there is no differentiation of stopping locations in terms of risk. Therefore, the algorithms are just as likely to guide the vehicle to a stop in the middle of the lane as on the road shoulder, outside active lanes, which would constitute a lower risk stopping location. Second, all three methods employ a kinematic bicycle model to represent vehicle motion, which restricts the algorithms to non-dynamic motion [100]. This does not reflect the potential urgency of action in a critical situation. Next, we outline alternatives for improved vehicle representation in critical situations.

2.3.2 Impact of Vehicle Model Accuracy At The Limit

It is intuitively clear that the planning of motions that are not dynamically feasible, e.g., going through a slippery turn at high speed, may lead to accidents. Therefore, the convention in vehicle motion control is to use conservative representations of the vehicle's physical limitations [83, 109]. This is a sensible strategy for many situations, but not necessarily for all types of critical situations. For example, in a collision avoidance situation, not being able to fully utilize all available actuation capacity due to a conservative representation of dynamic feasibility will impair the vehicle's capacity to avoid accidents.

This motivates research in vehicle motion planning and control at the limits of handling, i.e., near the adhesion limit of the tires. Accurate control in such aggressive maneuvers requires an internal vehicle model that to some extent captures the adhesion limit and the slip dynamics Furthermore, near the limit of tire adhesion, longitudinal, lateral and yaw dynamics are tightly coupled [97]. These properties make motion planning and control at the limits of handling a challenging research problem.

Research in the area has seen substantial progress and increasing interest over the last decade [96,102,110–112]. Many results have been obtained through development toward autonomous vehicle racing, which in many respects is analogous to motion planning in critical situations. Early progress was made for single vehicle racing in static environments, i.e. without other vehicles on the road. In such a setting, a motion plan can be computed offline, ahead of time [113–116], reducing the online problem to trajectory tracking at the limit of handling [96,117,118]. The introduction of unknown and dynamic obstacles (such as other vehicles on the road), requires that both motion planning and control have to be performed online [28,29], to be able to react promptly to updated conditions in the planning problem. This adds strict requirements on the computational efficiency of the motion planning algorithm, highlighting a trade-off between the following properties:

1. Model complexity, i.e., number of states in the model (higher complexity allows higher accuracy).
2. Planning horizon length, i.e. the length of the future time window over which to plan vehicle motions.
3. Planning time, i.e., execution time of the planning and control algorithm.

Increasing either 1 or 2 will lead to an increase in 3, and if planning time becomes excessively long, the vehicle will be unable to react promptly to a dynamically changing environment.

Due to this trade-off, the field of motion planning and control at the limits of handling is dominated by techniques based on numerical optimization, Section 2.1.4, for two reasons. The primary reason is that computational load scales relatively well with the complexity of the vehicle model, compared to alternative methods. This allows the use of a vehicle model that includes the combined lateral, longitudinal, yaw and slip dynamics, which in turn enables the planning of coordinated maneuvers that handle and exploit that these properties are tightly coupled at the limits of handling. Such a model (e.g., the dynamic bicycle model of Section 2.2.4) provides a more accurate representation of the vehicle close to the physical limit [100, 101].

The secondary reason is that solutions computed by a numerical optimization approach are not restricted to a discretized search space, meaning that the planned future state of the vehicle is not constrained by the resolution of the search space, and can therefore find a solution that is closer to the optima. Liniger et al. [28] provide an example highlighting this, by comparing an optimization based scheme with a trajectory rollout scheme based on selection from a precomputed trajectory library, both using the same dynamic vehicle model. The authors conclude that the purely optimization based scheme provides a lower closed loop cost, i.e., higher performance with respect to desired behavior compared to the trajectory library approach.

It is intuitively clear that accuracy in the internal vehicle model (in terms of dynamics and dynamic limitations) of motion planning and control algorithms has an impact on the vehicle's capacity to avoid accidents in critical situations. However, previous research work does not offer explicit experimental evaluation of to what extent this aspect impacts accident avoidance performance in the context of full body real time motion planning and control.

2.3.3 Local Minima of the Motion Planning Problem

It has been identified by several studies that purely optimization based approaches e.g., RTI, sometimes struggle when the planning problem includes discrete decisions [26,28]. As an example, if an RTI approach were to be used in the situation depicted in Fig. 2.9, and the initial guess would be to the left of the pedestrian, then the local solver would not be able to find any solution located to the right of the pedestrian, even if such a solution would be globally optimal with respect to the cost function. The intuition behind the phenomenon is that at each iteration the optimization problem is solved locally around an initial guess (for computational efficiency) and therefore the optimizer is unlikely to find a solution in a different part of the state space, that may contain the globally optimal solution. See Section 2.1.4 for further details on the RTI concept. Liniger et al. [28] mitigate the issue by using dynamic programming to select a high level corridor in which the motion planning problem is convex.

Such a solution works well in the application evaluated in [28], but for critical situations in general, it is non-trivial to select the right corridor. We argue that, since such a discrete decision is dependent on the vehicle's time-varying dynamics and dynamic limitations, it should be made at the local planning level with a high

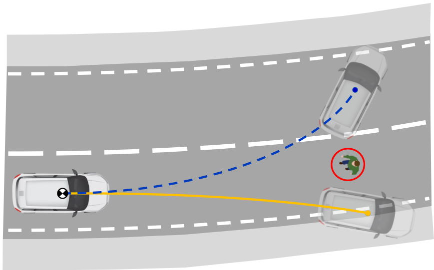


Figure 2.9: Illustration of local optima in relation to motion planning in critical situations. The two planned trajectories, dashed blue and solid orange, correspond to local optima of the motion planning problem, with respect to a value function representing the intended behavior. The solid orange alternative is also the global optimum

fidelity vehicle model, as opposed to at a higher level of abstraction with a lower fidelity vehicle model.

2.3.4 Time-Varying Traction Limits and Tire Dynamics

In Section 2.3.2 we concluded that real time motion planning and control at the limits of tire adhesion is non-trivial to achieve. To further complicate matters, real tire force limits and tire dynamics are not static. Instead, they vary with road/tire conditions and normal load on the tire. Accurately capturing this phenomenon in the context of online motion planning and control is an active research area with strong implications on accident mitigation performance in critical situations.

Several previous works suggest to model the physical limitations of a vehicle in terms of traction by imposing constraints on longitudinal and lateral acceleration. Lima et al. [81], uses a kinematic model under polytopic acceleration constraints derived from the the g-g diagram [119]. The polytopic constraint representation is permissive in terms of design, but near the limit of adhesion (i.e., when the constraints are active) the kinematic model does not provide an accurate representation of the dynamics of the vehicle [100].

Recent work by Subosits et al. [29] proposes a computationally efficient method based on an oriented point mass model expressed in a road aligned frame. The traction limit of the vehicle is represented by an elliptic constraint on lateral and longitudinal accelerations derived from the friction ellipse and predicted normal load. The approach is able to capture the effects of local traction variations and road topography, and the oriented point mass model does not deteriorate in terms of accuracy at high accelerations. The approach is successfully evaluated in a collision avoidance scenario with a full scale test vehicle. Fors et al. [120] use a similar modeling approach for coordinating between braking and steering in critical evasive maneuvers.

A common element among these methods is that the tire force traction boundary is represented as limits on acceleration at the center of mass. Granted, the experimental evaluation of [29] makes clear that such a method captures the dominating physical factors at play, but it remains unclear whether even better results in terms of traction utilization can be obtained by avoiding this simplification. Furthermore, [29] and [120] uses the friction ellipse as a basis for deriving traction constraints. According to Brach et al [95], this is can be an overly conservative representation in some cases.

Alsterda et al. [121] present an alternative approach to the problem of time-varying traction. An optimization based approach, extending on Brown et al. [122], is used for planning lateral motion with *multiple* models of the dynamics. A nominal and a contingency plan are produced in parallel, where the latter assumes a lower value of the tire-road friction coefficient μ, and associated tire model. The approach is validated in simulation for a class of critical situations characterized by rapid traction reduction in a corner. Results show that the method is able to adapt vehicle motion to time-varying traction conditions. Similar contingency planning approaches are presented in [123, 124].

Yet another approach to tackling the problem of time/spatial variations of vehicle dynamics is to employ tools from machine learning. Spielberg et al. [125] highlight the capacity of neural networks to encode a time-varying vehicle model. The network is trained on data collected on multiple road surfaces and deployed in a feedforward-feedback control scheme for path tracking. The neural network scheme outperforms a baseline physics based scheme, particularly where traction conditions vary. For the combined motion planning and control problem, Hewing et al. [126, 127] propose an approach where the dynamics are modeled as a sum of a dynamic bicycle model with nonlinear tire models (simplified Magic Formula) and a Gaussian Process (GP) [128]. The latter is intended to capture residual spatially varying dynamics. The approach was subsequently validated on a full scale test vehicle with impressive racing results [27]. Rosolia et al. [129, 130] propose another data-driven approach that leverages theory from model predictive control to guarantee recursive feasibility and non decreasing performance between each lap. These results validate that combining traditional modeling techniques with data-based online adaptation is an efficient and safe way to adapt the planned motion with respect to local variations in vehicle dynamics.

However, to perform such adaptive motion planning and control, one needs to estimate these changes to the dynamics *ahead* of the vehicle. The listed machine learning based approaches for combined motion planning and control [27, 126, 127, 129, 130] are tailored to racing on closed circuits and build on the assumption that each position on the track is visited multiple times. Data from previous visits to a certain part of the track is used to update dynamics at that particular position, such that in the next lap, local dynamics can be estimated ahead of the vehicle. For an AV deployed in regular traffic, this is not a viable method for obtaining this data, because the AV may not have visited the area recently, hence reliable data

Figure 2.10: Example of road surface classification from forward looking camera. Here, the road surface classes are dry (a), wet (b), snow (c) and gravel (d). Figure included from [132] with permission of the authors.

is unavailable. Thus, the iterative learning approach is generally not applicable to critical situations.

2.3.5 Impact of Friction Estimation Performance

Fortunately, there are other ways to estimate dynamics and dynamic limitations ahead of the vehicle. State of the art techniques for friction estimation are outlined e.g., in [131]. From a motion planning perspective, there are two distinct categories of friction estimation algorithms, namely those that provide a local estimate at the vehicle footprint, and those that provide a predictive estimate ahead of the vehicle.

Local friction estimation techniques [133, 134] use vehicle dynamics models, and data from on-board sensors to estimate the friction coefficient. Such methods have high accuracy but the estimate is only available at the vehicle footprint (as opposed to ahead of the vehicle) and in the presence of tire slip [135]

Predictive friction estimation techniques on the other hand use a forward looking sensor to estimate the friction coefficient ahead of the vehicle. Recent advances in supervised machine learning and computer vision have greatly improved performance of such functionality, mainly through road surface classification from forward looking camera images [132, 136, 137], providing a predictive estimate online at high availability. Fig. 2.10 provides a visualized example of the functionality. However, a drawback of this approach is that only a small number of road surface classes are visually distinguishable (e.g., dry, wet, snow/ice), and therefore the resolution is poor compared to state of the art local friction estimation methods. To the best of our knowledge, the implications of performance limitations in state of the art local and predictive friction estimations have not been evaluated in the context of traction adaptive motion planning and control.

2.3.6 Summary

To summarize this chapter, we have identified five distinct research gaps in the field of motion planning and control in critical situations (one for each preceding subsection):

i. In the case of internal faults or performance limitations, it is beneficial to determine the stopping position at the local planning level, but it is unclear how to represent risk levels of different stopping locations at this level of abstraction.

ii. Over-estimation and under-utilization of the vehicle's dynamic capabilities may impair the vehicle's capacity to avoid accident in critical situations. However, this has not been previously studied in the context of full body motion planning and control in full scale experiments.

iii. Optimization based motion planning schemes are well suited to the computational trade-off associated with motion planning and control in critical situations, but it is unclear how to handle the problem of local minima in the value function.

iv. It has been recognized that vehicle dynamics and dynamic limitations are non-static and can vary along the prediction horizon, but it remains unclear how to represent such time-variations accurately and efficiently in the context of motion planning and control, and how to anticipate future changes from sensor data.

v. It has been recognized that state of the art local and predictive friction estimation techniques have limited performance in terms of accuracy, availability and foresight, but the implications of this when applied to motion planning and control in critical situations have not been previously evaluated.

This thesis is intended as a step towards addressing the identified gaps. Specifically, Paper A is aimed at research gap i., Papers B and C are aimed at research gaps ii., iii., iv., and Paper D is aimed at research gap v. In terms of the categories of critical situations introduced in Section 1.2.2, all the identified research gaps have some impact on all three categories. However, there are particularly strong implications for gap i. in situations with internal system faults or performance limitations, gaps ii., iv. and v. on situations with rapid changes in operational conditions, and gaps ii. and iii. in situations with unsafe behavior from other road users. Relationships between appended papers and key parts of the thesis, including identified research gaps, are summarized in Table 1.2.

Chapter 3

Summary of Contributions

This chapter summarizes the research contributions of the thesis, starting with the high level contributions of characterizing the motion planning and control problem in reference to state of the art motion planning algorithms, followed by a brief summary of our suggested approach. Thereafter follows individual summaries of each of the appended papers.

3.1 Characteristics of Motion Planning and Control in Critical Situations

This thesis and the appended papers Papers A through D represent a sequential learning process in which a list of characteristics of the motion planning and control problem in critical situations was gradually built up, along with knowledge about suitable solution algorithms. At this stage of the research work, we have arrived at the following characterization:

1. **Accuracy of Vehicle Model at the Limit**: The vehicle model needs to accurately represent dynamics and dynamic limitations for maneuvers close to the physical limit.

2. **Time-varying Vehicle Model at the Limit**: The vehicle model needs to account for variations in vehicle dynamics and dynamic limitations.

3. **Foresight in Terms of Motion Capability**: Changes in dynamics and dynamic limitations, e.g., due to local traction changes must be anticipated ahead of the vehicle.

4. **Sufficiently Long Planning Horizon**: A sufficiently long planning horizon (at least 3-5s depending on the scenario) is required to handle the inertia and limited actuation capacity of the vehicle.

5. **Sufficiently Short Planning Time**: A short computation time of the algorithm ($t_s < 100$ms) is required in order to react promptly to sudden changes in the traffic scene.

6. **Discrete Decision Making**: When faced with a discrete decision, e.g., to swerve left or right of a suddenly appearing obstacle, the planner should select the lowest cost reachable option, i.e., avoid getting stuck in local minima.

7. **Risk Levels of Stopping Locations**: If planning to a full stop, differentiation between stopping locations in terms of risk levels needs to be done at the local planning level, in order to make a complete risk assessment of different maneuver alternatives.

8. **Severity Minimization**: If the accident is unavoidable, the goal of motion planning and control should be to utilize the full physical capacity of the vehicle to minimize the severity of the accident.

These characteristics should be collectively considered in the design of motion planning and control functionality for critical situations. Next, we outline our take on algorithm design.

3.2 Proposed Algorithm Design and Implementation

To be able to perform experimental evaluation for the research work, a motion planning and control framework was incrementally developed with the aim to accommodate all of the identified characteristics. In its current form, the framework realizes characteristics 1 through 7, while integration of the work on characteristic 8, severity minimization[1], is left for future efforts. A discussion and motivation of individual design choices is found in Section 4.1.1.

The framework includes elements from all the algorithms described in Papers A through D. The latest software instantiation of the framework is implemented as a ROS network [138], i.e., a set of software nodes written in C++/CUDA and Python, executing concurrently on multicore CPU and GPU. ROS is used to facilitate integration of the planning and control stack with experimental platforms and simulators. For example, in preparing for the field tests of Paper C, parallel ROS-integrations with the test vehicle software stack OpenDLV[2], and our simulation environment based on [139, 140], enabled seamless switching between real and simulated test vehicle, which accelerated development substantially.

The proposed framework encompasses characteristics 1 through 7 in the following ways: The framework uses a dynamic bicycle model with tire force inputs expressed in a road aligned frame (see Appendix B of Paper C for a detailed account

[1]Work in collaboration with Parseh et al. on motion planning and control when facing unavoidable collision, mentioned in Section 1.2.2

[2]Cf. `https://github.com/chalmers-revere/opendlv`

of the model), which allows representation of time-varying tire force limits and tire dynamics at an admissible computational cost. This way, we achieve sufficiently accurate vehicle representation in the algorithm to allow high traction utilization in the presence of time-varying actuation capability (characteristics 1,2). The optimization procedure of the algorithm is based on a state of the art RTI solver framework, [66, 67, 71], which enables a favorable computational trade-off between model complexity, horizon length and computation time (characteristics 1, 4 and 5). Discrete decisions (characteristic 6) and infeasibility issues arising from time-varying input constraints are jointly handled by introducing a sampling augmentation procedure (see Papers B, C). The GPU-accelerated procedure uses forward simulation of the dynamic bicycle model under an LQR controller to produce dynamically feasible candidate trajectories throughout the drivable area. Selection among the set of candidates (including the forward shifted solution from the previous planning iteration), based on a cost function that is shared with the optimization procedure, effectively handles discrete decisions in the planning problem. Since all trajectories in the set are dynamically feasible by construction, local decision making will also take locally varying traction into account. In addition, this feature of the sampling augmentation procedure ensures the availability of a feasible initial guess for the optimization procedure, even at rapid changes to the constraint situation, due to e.g., an obstacle popping up, or a rapid traction reduction. Sufficient accuracy, availability and foresight of dynamics and dynamic limitations (characteristic 3) are obtained by fusing local and predictive (camera based) friction estimates using heteroscedastic gaussian process regression, as per the procedure described in Paper D. The risk level of stopping locations is encoded as a cost on the final state (characteristic 7). The current implementation achieves a worst case planning time (including both sampling augmentation and optimization) around 100ms with the above mentioned vehicle model, and a prediction horizon of 4s on standard compute hardware (a PC with an Intel Core i7-7820HK CPU @ 2.9 GHz and an Nvidia GeForce GTX 1070 GPU).

3.3 Summary and Contributions of Individual Papers

This section summarizes each of the appended papers individually. Recall Table 1.2 for an overview of how research questions, identified research gaps and contributions map to individual papers.

3.3.1 Paper A

L. Svensson, L. Masson, N. Mohan, E. Ward, A. Pernestål Brenden, L. Feng, M. Törngren, "Safe Stop Trajectory Planning for Highly Automated Vehicles: An Optimal Control Problem Formulation" *2018 IEEE Intelligent Vehicles Symposium (IV)*, pp. 517-522, Changshu, China.

This paper concerns the motion planning problem for safe stop maneuvers, i.e., the planning of maneuvers that take the vehicle to the safest reachable stopping location (realizing a minimum risk condition as per the J3016 definition [4]) upon a hazardous event. With a safety supervisor system architecture as a basis [22], we formulate the safe stop motion planning problem as an optimal control problem, without a predefined stopping location. Instead, risk levels of candidate stopping locations are included in the cost function. Furthermore, we propose a trajectory library planning algorithm to solve the problem in real time. The method is validated by comparison to a globally optimal solution computed using dynamic programming and through simulation [3].

The primary contribution (a) of the paper lies in the formulation of the safe stop motion planning problem. With our formulation, the decision on where to stop is made at the local planning level in conjunction with vehicle dynamics and collision avoidance constraints - as opposed to pre-selecting a stopping location at the decision making layer which is subsequently passed to local planning. In theory, solving the problem enables optimal utilization of the vehicle's physical capacity to avoid accidents in critical situations. This is how we propose to tackle research gap i., introduced in Section 2.3. However, solving such a problem over a sufficiently long prediction horizon, under real time computational constraints, with an accurate vehicle model is a difficult task towards which we directed our further research efforts, namely Papers B and C. The secondary contribution (b) is the proposal and development of a trajectory library based planner, as well as its validation, showing that it can solve the problem for simple critical scenarios, i.e., situations with internal faults without immediate impact on perception or actuation capability. The trajectory library approach was selected in order to allow later extension to using the dynamic rather than the kinematic bicycle model, and thereby be able to plan motions up to the limit of handling. We later pivoted away from the trajectory library concept in favor of an optimization based approach, after arriving at the conclusion that the position invariance assumption, which is fundamental for the trajectory library concept, is quite restrictive for maneuvers at the handling limit because of the strong influence of local traction conditions. The third contribution (c) of the paper is the preliminary evaluation of the concept in which we show that the approach generates desired behavior in a particular type of critical situation.

Author's contribution: The author of this thesis is responsible for developing the idea in collaboration with Lola Masson, Naveen Mohan and Martin Törngren, further developing the mathematical problem formulation together with Lei Feng and implementing the method as part of a ROS framework together with Erik Ward, as well as authoring the paper. All co-authors provided feedback on the paper drafts.

[3] Though not included in the paper, a real time capable implementation of the algorithm was later produced, and the algorithm was further validated using the KTH Research Concept Vehicle as a test platform. Parts of the experimental results were presented in [30]

3.3.2 Paper B

L. Svensson, M. Bujarbaruah, N. R. Kapania and M. Törngren, ”Adaptive Trajectory Planning and Optimization at Limits of Handling”, *2019 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*, pp. 3942-3948, Macau, China.

This work tackles the problem of motion planning at the limits of handling, in the presence of locally varying traction conditions, to be employed in critical situations. The problem formulation considers that the physical limitations of a vehicle are subject to local variation, for example in terms of traction. To capture this property, we formulate an optimal control problem with a force-input dynamic vehicle model, under time-varying input constraints. This is how we propose to tackle research gap iv., Section 2.3. Furthermore, we propose a real-time capable solution method based on a combination of trajectory rollout and quadratic programming. Through an extensive simulation study, we show that the proposed adaptive method increases the vehicle's ability to avoid collision in critical scenarios, when compared to a non-adaptive approach.

The first contribution (d) of this paper is a novel problem formulation for traction adaptive motion planning and control. The formulation is based on a force-input vehicle model paired with time-varying input constraints that represent the vehicle's time varying tractive force envelope. The approach is a practically viable way to achieve full utilization of the vehicle's locally varying physical capacity to reduce accident risk in critical situations. The second contribution (e) is the real time capable [4] solution method that combines trajectory rollout and optimization based motion planning. Integrating trajectory rollout in the RTI scheme mitigates two separate issues of a purely optimization based approach: First, the trajectory rollout procedure provides a dynamically feasible initial guess to the optimization, even in the case when the solution from the previous timestep has become infeasible due to the time-varying constraints. Second, it mitigates the sensitivity to local minima and allows discrete decision making by evaluating sampled options throughout the drivable area. This is how we propose to tackle research gap iii., Section 2.3.

Author's contribution: The author produced the original idea, in terms of the problem formulation and the proposed solution method, developed algorithmic aspects in collaboration with Monimoy Bujarbaruah and vehicle dynamics modeling aspects based on discussions with Nitin Kapania, and authored the paper. Monimoy Bujarbaruah contributed substantially to all sections of the paper. All co-authors provided feedback on the paper drafts.

[4] Design choices for the algorithm were made to enable real time execution with a state of the art implementation. However, the preliminary implementation that was used for the simulation study was not real time capable.

3.3.3 Paper C

L. Svensson, M. Bujarbaruah, A. Karsolia, C. Berger and M. Törngren, ”Traction Adaptive Motion Planning and Control at the Limits of Handling”, submitted for possible journal publication, 2020.

This paper is an extension of Paper B and represents a substantial amount of development work with the goal to refine the traction adaptive motion planning and control concept to a point where it could be experimentally evaluated in a realistic setting. Extensions of the algorithmic framework include replacement of the quartic/quintic polynomials in the trajectory rollout step with GPU accelerated forward simulation of the model under a Linear Quadratic Regulator (LQR) and extension of the force input vehicle model to include pitch dynamics. Also, to enable real experiments on a test vehicle, a state of the art C++/CUDA implementation of the algorithm was produced, as well as a low level control interface that translates the force commands from the planner/controller to vehicle controls in terms of steering and acceleration/brake. We proceed with experimentally evaluating the planner in four example critical scenarios and conclude that traction adaptive motion planning and control improves the capacity to avoid accident in all tested critical scenarios, compared to an equivalent scheme with static tire force constraints. The experimental evaluation of the method tackles research gap ii., Section 2.3.

The first contribution (g) of this paper is the extensive experimental evaluation of traction adaptive motion planning and control. The evaluation was performed during a total of 7 vehicle test days at two different test tracks. In comparison with Paper B, this evaluation provides much stronger justification that the conclusions hold for real world conditions. The second contribution (f) is that we show that the proposed algorithmic concept of sampling augmented adaptive RTI is capable of solving optimal control problems with time-varying input constraints, state constraints and dynamics, while avoiding local minima, in real time ($t_s < 100ms$) over a long ($4s$) prediction horizon on regular PC hardware. This makes the algorithm especially suitable for the challenging problem of motion planning in critical situations, as well as many other applications in robotics.

Author's contribution: The author produced the original idea, further developed algorithmic and vehicle dynamics aspects, produced the implementation, integrated the algorithm on the experimental platform, planned, organized and executed the experiments in collaboration with Arpit Karsolia and Christian Berger, and authored the paper. Monimoy Bujarbaruah contributed substantially to all sections of the paper. All co-authors provided feedback on the paper drafts.

3.3.4 Paper D

L. Svensson and M. Törngren, ”Fusion of Heterogeneous Friction Estimates for Traction Adaptive Motion Planning and Control”, submitted for possible conference publication, 2021.

The experimental work on Paper C highlighted that the traction adaptive motion planning concept is sensitive to the accuracy in the predictive friction estimate. Both over-estimation and under-estimation may lead to suboptimal performance in terms of avoiding accidents in critical situations. This motivated an investigation of how realistic performance of state of the art friction estimation techniques would impact accident avoidance performance. In a simulated environment, we emulate three different realistic friction estimation techniques based on state of the art techniques and evaluate performance by comparing with a planner/controller provided with ground truth friction. The first configuration emulates a local friction estimation method of high accuracy, but where the estimate is only available locally at the vehicle footprint, and at tire force utilization above 0.5. The second configuration emulates a camera based method that provides a high availability predictive estimate over the whole prediction horizon, but has lower accuracy. The third configuration is our proposed fusion approach, that merges estimates from the first and second configurations, while preserving estimation uncertainty, using heteroscedastic GP regression. We show that for the two evaluated critical scenarios, the fused estimate performs on par with ground truth, while the two individual friction estimate configurations underperform in either of the two critical scenarios. It is the aim of the simulation based evaluation to tackle research gap v., Section 2.3.

The first contribution (h) of this paper is highlighting that individual paradigms of state of the art friction estimation algorithms (i.e., local or camera based) are insufficient for satisfactory traction adaptive motion planning and control in critical situations. However, fusion of estimates from multiple paradigms can provide a sufficient accuracy, availability and foresight, to generate near optimal motion behavior. This further supports the external validity of the conclusions of Papers C, D. The second contribution (i) is the proposed fusion concept that produces a predictive estimate that is conservative with respect to input data with associated heterogeneous uncertainties. The fused estimate is conservative in the sense that probability of over-estimation is low, while not being overly conservative, such that collision avoidance performance deteriorates.

Author's contribution: The author produced the original idea, developed the algorithm, produced the implementation, integrated the algorithm with existing framework for traction adaptive motion planning and control and authored the paper. Martin Törngren contributed in discussions throughout the work and provided feedback on the paper drafts.

Chapter 4

Discussion and Concluding Remarks

This section concludes the thesis with a discussion on algorithm design choices and on the validity of the performed experiments, followed by a brief summary of conclusions in relation to the research questions and recommendations for future research directions.

4.1 Discussion

First, we present and motivate some of the design choices made for the proposed framework for motion planning and control in critical situations. Thereafter we discuss the validity of the performed experiments first in terms of the similarity of our experimental setup and the research target, followed by specifics on our approach to emulation/implementation and the measures taken to handle non-determinism in the AV perception and control system.

4.1.1 Algorithm Design Choices

From the analysis of related work of Section 2.3, it is clear that optimization based methods, Section 2.1.4, currently outperform most other methods in terms of the central trade-off between model complexity, horizon length and computation time. For time invariant dynamics and dynamic limitations it is possible to achieve similar performance in this regard with trajectory roll-out methods based on precomputed trajectory libraries (used e.g. in Paper A). However, since each trajectory set is computed offline with respect to static operational conditions, the library would soon grow intractably large if one were to consider time-varying dynamics and dynamic limitations, characteristic 2. In contrast, in an optimization based framework, such adaptation can be encoded as time-varying constraints, with potentially negligible addition to computational cost. However, this introduces a new problem

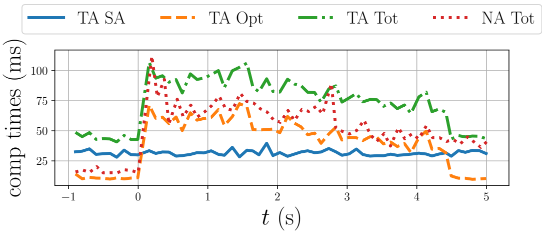


Figure 4.1: Computation times for motion planning and control in a collision avoidance scenario from the field experiments of Paper C. Total time for the traction adaptive configuration (TA Tot) is split into fractions associated with the sampling augmentation procedure (TA SA) and optimization (TA Opt). This is compared against a purely optimization based, non-adaptive scheme (NA Tot).

in terms of the feasibility of the optimization problem. There are well established workarounds for such problems, e.g., softening of the constraints by introducing slack variables [28, 73]. However, this instead introduces further sensitivity to local minima in the value function. This drawback of optimization based methods have additional negative implications with respect to the characteristics of critical situations, e.g., with respect to discrete decision making. This is elaborated in Section 2.1.4. These particular shortcomings are not present in trajectory roll-out methods, indicating that the methods are to some extent complementary with respect to the characteristics of critical situation, which suggests an opportunity for meaningful combination of the two concepts. The sampling augmentation concept, described and evaluated in Papers B, C, represents one example of such a combination, which efficiently alleviates the feasibility and discrete decision making problems of optimization based approaches.

The sampling augmentation procedure is executed at the beginning of each iteration of the motion planning and control algorithm, therefore the average computation time is increased. However, at rapid changes to the constraint configuration (e.g., at traction reduction or sudden obstacle appearance), the sampling augmentation procedure significantly simplifies the subsequent optimization procedure by providing it with a feasible initial guess, thereby reducing optimization time. This reduces worst case computation time of the whole planning algorithm. Fig. 4.1 shows recorded computation times for a collision avoidance run during the field experiments of Paper C. At the critical points in the maneuver, where reaction time has the largest impact on the outcome (e.g., at the first planning iteration following obstacle appearance, $t = 0.1$s in Fig. 4.1), sampling augmentation does not increase planning time compared to a purely optimization based scheme.

The execution time of the algorithm is a major contributor to the reaction time of the vehicle in critical situations. If the sum of computation time for object detection/localization and planning of the evasive maneuver is high, collision avoidance performance will be severely reduced due to a slow time to react. Related works typically present a planning time in the interval 20-200 ms [28,29,41,141]. Because reaction time was not explicitly included in the research scope of our work, a design target of a maximum planning time of 100 ms was selected for the development of our framework. The underlying rationale was to select a target that was comparable to the state of the art, while being practically viable with the desired complexity of vehicle model, and time horizon. A substantial amount of development work was invested to accommodate all steps of the algorithm within this target computation time on the available hardware. Faster is always better when it comes to reaction time. Whether or not 100 ms is a suitable design target for this application is subject to further discussion, which we return to in Section 4.3.

It is evident from the results and conclusions of papers B, C and D that moving from an inaccurate local vehicle model (i.e. one that does not accurately represent the vehicle near the handling limit at all, or one that does not vary along with the time-varying dynamics and dynamic limitations of the real vehicle, or both) to a reasonably accurate one, will drastically improve the level of utilization of the vehicle's capabilities, and in turn its risk reduction performance in critical situations. This raises the question whether an even more accurate model would improve utilization even further? Recent works deploying machine learning techniques to capture and compensate model error online suggest that very accurate vehicle models (and consequently very high traction utilizations) can be obtained by iteratively learning spatially varying dynamics and dynamic limitations [27,130]. However, it remains unclear how such models should make predictions of dynamics and dynamic limitations *ahead* of the vehicle, for positions it has not recently visited, recall Section 2.3.4.

To acheive this, we have opted for an approach where a single key parameter, namely the friction coefficient μ is identified online, ahead of the vehicle, from standard sensor data streams (forward looking camera and inertial sensors). From there we derive changes to our vehicle model from first principle mechanics modeling. This comparatively simple, physics based vehicle model is far from perfectly accurate, but given an accurate friction estimate, it is relatively accurate for a wide range of road conditions. For example, the dynamics model includes a linearized Magic Formula tire model that varies in slope based on the identified friction coefficient. Using this simple tire model strongly contributes to the computational efficiency of the algorithm. The model fares well in capturing the dominating changes to tire dynamics on different surfaces, but loses accuracy as the tire force curve tapers off close to the maximum force utilization. Instead of introducing a more elaborate tire model (that would most likely increase computation time) we counter this by introducing a force utilization factor λ when setting the time-varying tire force constraints. This allows us to limit the force utilization, typically to around 90% of the theoretical maximum, keeping it inside an interval for which the time-varying linear tire model is acceptably accurate. This is highlighted in Figure 11 of Appendix C of Paper C.

This concept is indicative of a guiding principle we employed in designing the framework: We argue that for motion planning and control in critical situations, a system that reliably realizes a high force utilization, e.g. $\sim 90\%$, on a wide range of road surfaces, is preferable to a system that obtains more than 90% utilization on a particular, limited set of road surfaces while obtaining less than 90% on others. That being said, the current version of the framework does have room for improvement in terms of for example tire modeling. We revisit this in Section 4.3.

4.1.2 Validity

In this section we discuss the challenges to internal and external validity encountered during the performed experimental and simulation studies, and the methods used to address them. Recall Section 1.2.3, for a description of the adopted research methodology.

Generalization to Research Target

The research target for this work has been a production grade level 4+ AV deployed in real traffic, being exposed to various critical situations. To represent the target, we performed tests on a full scale heavy duty vehicle, Fig. 1.3b, on closed test tracks for Paper C, and simulated test vehicles and environments for Papers A, B and D.

We argue that our experimental results obtained with a heavy duty vehicle generalize well to any other front wheel steered, four wheeled [1] road vehicle, e.g., a standard passenger vehicle. The vehicle model used in the framework is parameterized according to the test vehicle's physical properties, e.g., its mass, moment of inertia, length, width, position of the center of mass etc. The framework will plan very different motions for different vehicles and road surface conditions, but the virtues of the approach, i.e., dynamic feasibility of planned motions, even at poor local traction conditions and a high utilization of available tire force (90% for the tests in Paper C) is independent of the vehicle type as long as the model parameters are set in accordance with the real vehicle. Effects that are more pronounced with a light vehicle such as normal force variations due to high accelerations are also represented in the framework. Tests on other vehicle types have been performed in simulation with good results, but have not been included in terms of results in the thesis. The road surfaces that were used for testing at the Asta Zero [2] and Stora Holm [3] test tracks have been professionally developed to replicate realistic conditions, representing dry/wet asphalt and snow/ice, for vehicle testing and driving education purposes. All in all, we argue that our test setup is representative of the research target, and our experiments therefore have strong external validity.

[1] The test vehicle used for Paper C has 6 wheels, but the rear axle was lifted throughout the experiments.

[2] https://www.astazero.com/

[3] https://www.storaholm.se/

Due to time and resource constraints, Papers A, B, D have simulation based evaluation instead of experiments. As mentioned in Section 1.2.3 external validity of simulation based evaluation hinges on whether the simulation environment is representative of the target. However, this only needs to hold for factors that are relevant to the outcome of the experiment. By making justified design choices for the simulation environment, a relatively strong argument can be made in regard to the validity of simulation results as well. Two branches of simulation environments with different purposes have been developed during the course of this research work.

- Simplistic MATLAB based simulators for preliminary evaluation of novel concepts
- ROS/Gazebo-based [138, 139] simulators with similar software interfaces as the test vehicles.

The first category was primarily used in Papers A and B, as development and evaluation tools that could be realized comparatively quickly at a point in time where no useful test infrastructure had yet been built up. In the evaluation for Paper A, where the focus is the inclusion of stopping locations in local planning, the motion planning algorithm is designed to generate low acceleration, smooth motions. For such motions, a kinematic vehicle model accurately describes vehicle motion [100, 101]. This allowed the use of a simple custom developed simulation environment based on the kinematic model for the initial evaluation. Paper B on the other hand considers aggressive evasive maneuvers, which disqualified continued use of the kinematic model for evaluation. Instead, we modified the simulator by introducing a dynamic model with tire force inputs. The focus of Paper B was on motion planning with respect to time-varying traction limits rather than how to realize those maximum forces in terms of slip angles and ratios. Therefore, by introducing the assumption that commanded tire forces would be realized if they were inside a permitted boundary determined by the road condition and a simple tire model (friction ellipse), we could greatly simplify the vehicle model in the simulation. This simple setup allowed us to further develop the method and draw initial conclusions on what behavior the motion planner would generate. The motion planning framework was later extended with a control interface, described in detail in Appendix C of Paper C, that translates tire force commands into the acceleration and steering angle commands of the test vehicle. This solution was experimentally validated, strengthening the external validity of both Papers B and C. The second branch of simulators was developed with the primary purpose of facilitating preparations for field experiments. Paper A showcases an embryo of a ROS/Gazebo-based simulator based on [142] intended to have the same interfaces for commands and state information as the real test vehicle. This environment was later replaced by one based on the FSSIM environment [140], which was continuously developed throughout the project. Preparatory vehicle tests for Paper C were performed over a period of several months, where development using simulation, live test and replay of sensor data logs was done interchangeably. During this time, the simulator was further tuned to mimic the behavior of the real vehicle more and more accurately on all tested surfaces. This was later exploited in the evaluation of Paper D. We argue that at the time of the evaluation of Paper D, the simulator was an accurate representation of the test vehicle, which in turn was an accurate representation of the research target. Therefore, we argue that also the study in Paper D has strong externally validity.

In terms of further generalization of results, we note that although the target for this research has been a level 4+ vehicle, many of the results generalize equally well to driver assistance at lower levels of automation, for example in advanced ADAS systems. Traction adaptive motion planning and control (Papers B, C), or predictive friction estimation (Paper D) are just as relevant in such applications, with strong potential for improving safety of road users in the short term.

Implementation versus Emulation

In terms of dealing with the dependency on perception functionality, the implementation approach described in Section 1.2.3 was initially selected. For example, in the experimental evaluation following Paper A, the localization and state estimation of the test vehicle were performed through scan matching [143, 144] in a prerecorded 3D lidar map - a localization technique similar that is commonly used in production grade AVs [13]. During these experiments we learned that by the time we had fine-tuned the localization and state estimation functionality, its performance could have been easily emulated using differential GNSS and IMU, with no apparent effect on motion planning functionality. In subsequent experiments, we generalized this principle of practically grounded emulation to other perception functionality, e.g., object detection and friction estimation.

Object detection was emulated in the field experiments of Paper C. We considered sensor limitations in terms of detection range and occlusions by having virtual obstacles appear suddenly in front of the vehicle. This emulates realistic sensor data in a situation where an obstacle is temporally occluded or for some reason or other, difficult to detect up to a certain point in time. After an object had been detected we assumed perfect tracking of the object so as not to introduce additional stochasticity and therefore maintain experimental control.

In the simulation based evaluation for Paper D, two friction estimation functionalities were emulated. The first was a local estimate, emulated in terms of its availability at tire force utilizations above 50%, and its margin of error with respect to a simulated ground truth value. The second was a predictive estimate, emulated by providing a road surface class associated with a simulated ground truth value. The emulation replicates the output of the friction estimation algorithms at a high level of abstraction, but captures the fundamental properties of the two different paradigms in terms of accuracy, availability and foresight, such that we could evaluate motion planning and control functionality e.g., based on only local friction estimates, only predictive estimates, or fusion of the two.

This principle of justified emulation of selected perception functionality, grounded in the state of the art and practical experience enabled time-efficient (well, reasonably time-efficient) field experiments on novel motion planning and control concepts without loss of internal or external validity of the performed experiments. It should be noted however, that whether or not justified emulation can be applied is strongly dependent on the type of experiment to be performed.

Non-determinism

A software control system as complex as an AV, even with selected parts of its functionality emulated, is always to some extent non-deterministic. Contributing factors for our system include noisy sensor measurements, varying convergence time of optimization solvers, concurrent program execution, non-deterministic task-scheduling in the Linux/ROS based software framework (running on top of a non real-time operating system) etc. This introduces a challenge with respect to experimental control. We tackle this problem by performing multiple repetitions of selected experiments, e.g., the collision avoidance experiment of Paper C. This approach allows the analysis of trends between different configuration in the presence of non-determinism. However, since our experimental study had limited resources, we were not able to perform such repetitions for all experiments, nor a sufficient amount to perform rigorous statistical analysis on the outcome. We choose to employ the strategy only for the high μ collision avoidance case, which due to the short time-scale and high speed was the most susceptible to non-determinism affecting the outcome. For the other cases evaluated in Paper C, we observed during initial testing and tuning that the outcome was largely unaffected by non-determinism in the system, with repeated trials showing very consistent results. Therefore we judged that single runs would suffice for the evaluation.

4.2 Conclusions

In this section we summarize the outcome of the research work in relation to the research questions stated in Section 1.2.1. The high level research questions 1 and 2 directly correspond to the high level contributions stated in Section 1.3. Recall also Table 1.2 for the relations between research questions, identified research gaps and specific contributions. Next, we go through our conclusions relating to each research question individually

> Research Question 1: *Which characteristics of the motion planning and control functionality impact the AV's capacity to reduce risk of accident in critical situations?*

We outline our current characterization of the motion planning and control problem in critical situations in Section 3.1. The list has evolved gradually throughout the project and is tightly coupled to the practical properties of state of the art algorithms. The research field of motion planning for automated vehicles is evolving rapidly and therefore we recognize that further additions may have to be made in relation to emerging algorithms with new virtues and shortcomings. Hence, we do not claim completeness of the current list. Still, we believe it can be useful as a practical reference for researchers and practitioners that design and develop functionality for critical situations.

> Research Question 2: *Which motion planning paradigm(s) is/are suitable for realizing these characteristics?*

We present a review and analysis of state of the art motion planning approaches in relation to critical situations in Section 2.3. Based on this, we gradually developed our own framework for motion planning and control to evaluate a variety of design selections, and to gain further practical knowledge of the methods in question. The conclusion from this work is that combining features from sampling based and optimization based motion planning enables efficient, near-optimal motion planning and control for critical situations. Recall Section 4.1.1, for further elaboration and motivation behind this conclusion.

> Research Question 3: *To what extent does accurate representation of time-varying actuation capability impact an AVs capacity to reduce risk of accident in critical situations?*

Research question 3 highlighted research gaps ii., iii., and iv. and subsequently led to contributions (d), (e), (f) and (g). Experimental results from Paper C show that accurate representation of locally varying dynamics and dynamic limitations have a substantial impact on capacity to avoid accidents in the tested critical situations (low μ turn, collision avoidance at low μ, collision avoidance at high μ). Over-estimation of dynamic limitations leads to the planning of dynamically infeasible motions, which renders the vehicle unable to track the plan and may cause complete loss of control authority. Under-estimation on the other hand leads to poor utilization of available traction, which may needlessly lead to accidents in for example collision avoidance situations. Furthermore, we highlight that the drawbacks of purely optimization based methods with respect to discrete decisions may also needlessly lead to collisions. Our proposed sampling augmented optimization based algorithm is able to adapt to locally varying dynamics and dynamic limitations in real time. We conclude that its favorable behavior in critical situations stems from the following properties: 1. Ensured dynamic feasibility of planned motions, and 2. A high ratio of locally available traction is utilized if necessary to avoid accidents.

> Research Question 4: *To what extent does consideration of uncertainty in future actuation capability impact an AVs capacity to reduce risk of accident in critical situations?*

Research question 4 highlighted research gap v. and subsequently led to contributions (h) and (i). Simulation based results from Paper D show that performance of motion planning and control functionality that take into account time-varying dynamics and dynamic limitations, is indeed sensitive to accuracy, availability and foresight of friction estimation functionality. Through emulation of state of the art local and predictive friction estimation techniques, we show that neither of these techniques in isolation has sufficient performance to yield satisfactory motion behavior in two evaluated critical situations (low μ turn, collision avoidance at high μ). However, we show that this can be alleviated by fusion of the two estimates. The proposed fusion strategy avoids over-estimation of the friction coefficient while not being overly conservative (only 5.6% decrease in tire force utilization compared to providing the planner with the ground truth friction estimate), even at worst case adversarial estimation errors.

4.3 Future Research

The research problem we have approached in this thesis, i.e., motion planning and control in critical situations, is by no means fully solved. All identified research gaps i. through v., require further investigation. The proposed framework shows potential in the tested critical situations, but the overall concept requires much further development and testing before becoming a viable option for production grade AVs.

For the research field in general, we call for further work on evaluating the impact of design choices in motion planning and control on accident avoidance performance. Recent work from Olofsson and Nielsen [145] contributes in this regard by using offline numerical optimization and crash databases to evaluate the potential of optimal motion behavior in terms of saved lives and reduced degree of injuries in accidents. Such studies provide strong motivation and quantitative benchmarks for further development of real time capable concepts for motion planning and control in critical situations. Furthermore, we regard experiments on real test vehicles as a key activity to ensure the relevance of conclusions to real vehicles.

From the current state of the research work presented in this thesis, there is a multitude of interesting avenues for further work. Here follows a selection of specific topics that we judge to be of particular interest.

- **Closing the Loop:** The natural next step from the current state of this research work is full scale tests of the concept of fusing heterogeneous friction estimates, integrated together with state of the art friction estimation algorithms and traction adaptive motion planning. This would result in a closed loop system, that uses a combination of visual cues and responses to control inputs to anticipate future vehicle dynamics and dynamic limitations on variable road surfaces. This is conceptually similar to how expert human drivers operate on variable road surfaces. A positive outcome from such experiments would further validate the practical relevance of Papers B, C and

D, contribute to the closing of research gaps ii., iv., and v. and take traction adaptive motion planning one step closer to deployment.

- **Utilize Machine Learning in Vehicle Modeling:** As we mention in the discussion on algorithm design choices in Section 4.1.1, further improvement in terms of vehicle model accuracy could further improve tire force utilization, and consequently also risk reduction performance in critical situations. Recent advances in machine learning provide multiple interesting opportunities in this regard, for example, the neural network modeling paradigm introduced by Spielberg et al. [125]. We foresee two separate interesting directions for using similar methods with our approach: First, such a modelling paradigm could be used more or less directly to improve the control interface that translates desired tire forces into vehicle control inputs. Second, we learned from the work on Paper D, that the combined sensor data from forward looking camera and onboard sensors contain sufficient information to predict future dynamics and dynamic limitations for near-optimal traction adaptive motion planning and control. A neural network could be used to directly infer future dynamics and dynamic limitations (i.e., over the prediction horizon), from such sensor data, without explicit representation of surface classes or friction coefficient. This could potentially further improve accuracy of the predicted model.

- **Extended Evaluation:** In the simulated and experimental evaluations of this thesis, the proposed framework has been exposed to a selected set of relevant critical situations. A step further toward closing the identified research gaps would be to extend the evaluation to a wider range of critical situations from all three categories stated in Section 1.2.2. It is our intention to release the software framework developed for this work as open source software, in order to facilitate reproducibility and reduce the threshold for further extensions of the framework.

- **Traction Utilization versus Reaction Time:** The trade-off between tire force utilization (through model accuracy) and reaction time, discussed in Section 4.1.1, merits further investigation, especially in terms of relative impact on accident avoidance performance. For example at higher speeds, reaction time becomes increasingly important due to a longer distance travelled per unit time. It is non-trivial to assess its impact on accident avoidance in relation to tire force utilization. It is therefore also non-trivial to prioritize between the two in algorithm design. This merits further investigation. High speed autonomous racing with obstacles would be a suitable setting for such work.

- **Severity Minimization when Accident is Unavoidable:** In particularly challenging critical situations, avoiding collision may be impossible. In such cases, the objective of the AV becomes to minimize the severity of the imminent accident. Although the design of motion behavior in such situations is outside the scope of this thesis, we are tackling the problem in separate work, [23, 24]. There, our approach is to include knowledge of crash severity from historic data, as a severity metric in the cost function of the motion planner. We foresee strong potential in combining these two research branches in the future, in order to allow full utilization of the vehicle's capacity to avoid accident when possible, and in the cases when an accident is inevitable, minimize its severity.

- **Impact on nominal performance:** As mentioned in the introduction, Chapter 1, we argue that performance in critical situations should dictate nominal performance of AVs. For example if we are confident that the fallback functionality is capable of safely handling a wide range of critical situations, we may allow the nominal function to drive e.g, at higher speed or in a wider range of operational conditions. For example, utilizing 50% or 90% of available traction in potential evasive maneuvers could have substantial impact on the admissible nominal velocity of an AV. This would be interesting to study further.

- **Extend to Additional Domains:** We expect potential for the presented algoritmic concepts outside the context of road vehicle control in critical situations, in particular in related applications such as automated racing or mining/construction. For example, with little adaptation and with negligible increase of computational cost one can include additional factors that impact tire normal loads, which in turn determines admissible tire forces. This concept could be used to include for example downforce in a racing context, or load variations for transportation vehicles in e.g., mines or construction sites. In both of these applications, dynamic feasibility at a high tire force utilization level is key for performance and robustness.

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Part II

Publications