Module 5. Bivariate analysis: qualitative - quantitative

Data Science & AI

Sabine De Vreese Stijn Lievens Lieven Smits Bert Van Vreckem

2021–2022

HO
GENT

Contents

HO
GENT

Learning goals

- Apply the t-test for two samples
- Calculate effect size using Cohen's d
- Visualization

HO
GENT

Bivariate analysis: overview

Independent	**Dependent**	**Test/Metric**
Qualitative	Qualitative	χ^2-test Cramér's V
Qualitative	Quantitative	two-sample t-test Cohen's d
Quantitative	Quantitative	— Regression, correlation

HO GENT

Example research questions

- Are male penguins larger than females?
- Do men get a higher salary than women?
- Does a new vaccine protect against a disease?
- Does “retrieval practice” improve learning outcomes (i.e. student grades)?
- …

In these examples, wat is the independent/dependent variable?

HO
GENT

Data visualization

Data visualization

- Chart types for quantitative data
- Grouped by qualitative variable

HO
GENT

Data visualization

- Chart types for quantitative data
- Grouped by qualitative variable

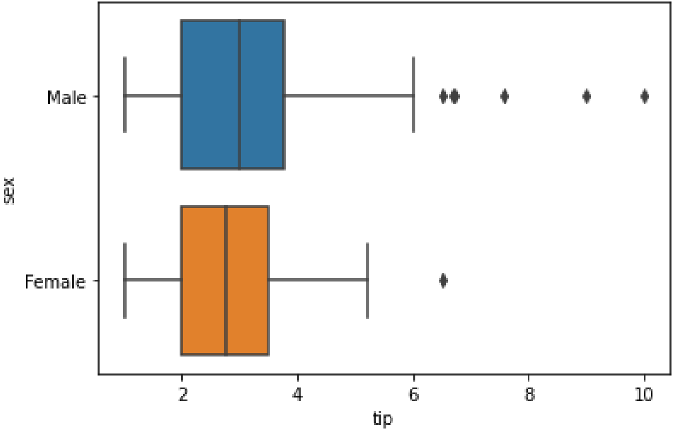
Suitable chart types:

- Grouped boxplot
- Grouped density plot
- Bar chart with error bars
- ...

HO
GENT

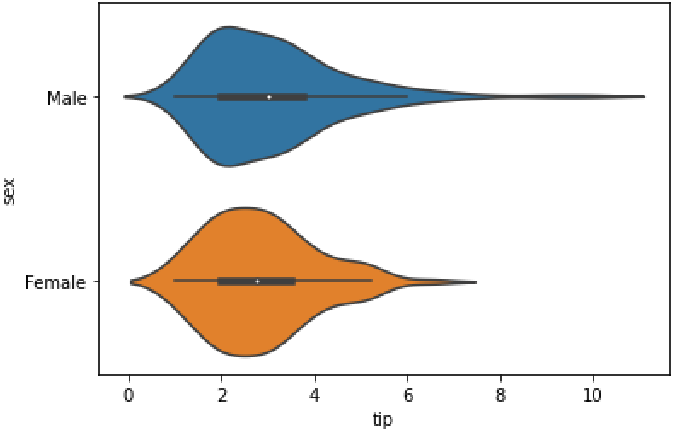
Grouped boxplot

(source code: see `demo-2sample-t.ipynb`)



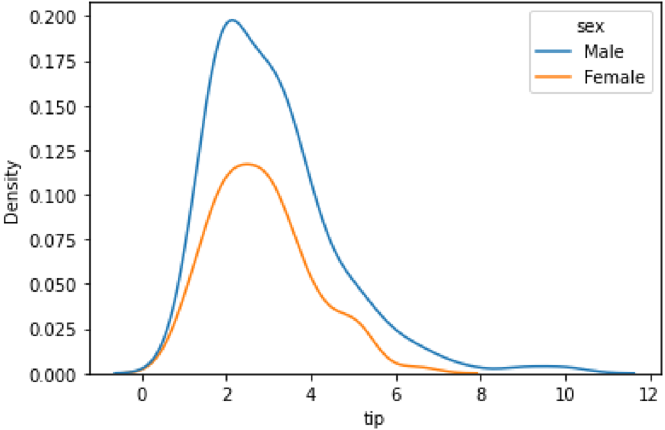
HO
GENT

Grouped violin plot



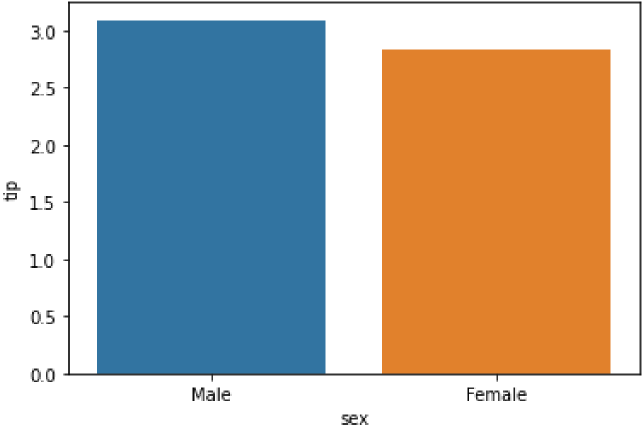
HO
GENT

Grouped density plot



HO
GENT

Beware! Bar chart of group means



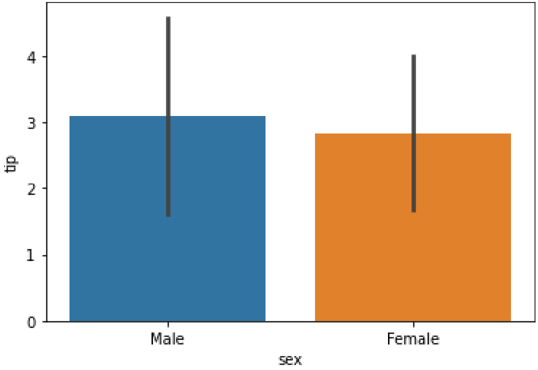
HO
GENT

Beware! Bar chart of group means

HO
GENT

Bar chart with error bars

- Always add error bars!
- Only makes sense for normally distributed data
- Example: 1 standard deviation:



Two-sample t-test

HO
GENT

Comparing two samples

Is the sample mean of two samples significantly different?

- Independent samples
- Paired samples

Independent samples

In a clinical study, the aim is to determine whether a new drug has a delayed (i.e. higher) reaction speed as a side effect

- Control group: 6 participants receive placebo
- Intervention group: 6 participants receive medicine

HO GENT

Independent samples

In a clinical study, the aim is to determine whether a new drug has a delayed (i.e. higher) reaction speed as a side effect

- Control group: 6 participants receive placebo
- Intervention group: 6 participants receive medicine

Next, the reaction speed is measured:

- Control group: 91, 87, 99, 77, 88, 91 ($\overline{x}$ = 88.83)
- Intervention group: 101, 110, 103, 93, 99, 104 ($\overline{y}$ = 101.67)

Are there significant differences between the intervention and control group?

Independent samples

Testing procedure

1. Hypotheses:
    - H_0: $\mu_1 - \mu_2 = 0$
    - H_1: $\mu_1 - \mu_2 < 0$
2. Significance level: $\alpha = 0.05$
3. Test statistic:
    - $\overline{x} - \overline{y} = -12.833$
    - $\overline{x}$ = estimation for μ_1 (control group)
    - $\overline{y}$ = estimation for μ_2 (intervention group)
4. Calculate p

HO GENT

Independent samples

Calculation in Python

```
control = np.array([91, 87, 99, 77, 88, 91])
treatment = np.array([101, 110, 103, 93, 99, 104])
stats.ttest_ind(a=control, b=treatment,
                alternative='less', equal_var=False)
```

Result:

```
Ttest_indResult(statistic=-3.445612673536487,
  pvalue=0.003391230079206901)
```

Independent samples

Testing procedure (continued)

5. $p \approx 0.00339 < \alpha = 0.05$. We reject the null hypothesis. In this sample, there is reason to assume that the drug does indeed have an impact on reaction speed.

HO
GENT

Paired samples

Paired samples

A study examined whether cars that run on petrol with additives have a lower consumption.
For 10 cars, the consumption was measured (expressed in miles per gallon) for both fuel types:

Car	1	2	3	4	5	6	7	8	9	10
Regular petrol	16	20	21	22	23	22	27	25	27	28
With additives	19	22	24	24	25	25	25	26	28	32

HO
GENT

Paired samples

Testing procedure

1. Hypotheses:
   - H_0: $\overline{x - y} = 0$
   - H_1: $\overline{x - y} > 0$
2. Significance level: $\alpha = 0.05$
3. Test statistic:
   - $\overline{x - y}$
   - x = miles per gallon with additives ($\overline{x} = 25.1$)
   - y = miles per gallon regular petrol ($\overline{y} = 23.1$)
4. Calculate p

HO
GENT

Paired samples

Calculation in Python

```
regular = np.array([16, 20, 21, 22, 23, 22, 27, 25, 27, 28])
additives = np.array([19, 22, 24, 24, 25, 25, 26, 26, 28, 32])
stats.ttest_rel(regular, additives, alternative='less')
```

Result:

```
Ttest_relResult(statistic=-4.47213595499958,
    pvalue=0.0007749429558509)
```

HO
GENT

Paired samples

5. $p \approx 0.0007749 < \alpha = 0.05$. We reject the null hypothesis. In this sample, there is reason to assume that the fuel with additives leads to lower fuel consumption.

HO
GENT

Effect size

Effect size

Effect size

The effect size is a metric which expresses how great the difference between two groups is

- Control group vs. intervention group
- Can be used in addition to hypothesis test
- Often used in educational sciences
- There are several definitions, here: *Cohen's d*

HO GENT

Cohen's *d*

$$d = \frac{\overline{x_1} - \overline{x_2}}{s}$$

with $\overline{x_1}, \overline{x_2}$ the sample means
and s a standard deviation of both groups combined:

$$s = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$$

with n_1, n_2 the sample sizes, s_1, s_2 the standard deviation of both groups

HO
GENT

Interpretation Cohen's *d*

$\|d\|$	**Effect Size**
0.01	Very small
0.2	Small
0.5	Average
0.8	Large
1.2	Very large
2.0	Huge

In educational sciences (John Hattie):

- 0,4 = tipping point for desired effects
- effect size $d = 1$: process material from 1y in 6m!

E.g. `https://visible-learning.org/backup-hattie-ranking-256-effects-2017/`

HO GENT

Typical approach research in education

- Research question: Is X a good learning strategy, in other words, does this have a positive effect on final results?
- Control group uses “traditional” approach
- Intervention group uses X
- Followed by an evaluation moment
- Determine scores, calculate *d*

HO
GENT