ADSL equalization

Work in progress.
Two relevant chapters presented.

Abbreviations

ADC	Analog–to–Digital Converter
ADSL	Asymmetric DSL
BER	Bit Error Ratio
CIR	Channel Impulse Response
CNA	Carrier Nulling Algorithm
CO	Central Office
CP	Cyclic Prefix
CPE	Customer Premise Equipment
DAC	Digital–to–Analog Converter
DFT	Discrete Fourier Transform
DMT	Discrete Multi-tone
DSLAM	Digital Subscriber Line Access Multiplexer
xDSL	Digital Subscriber Line
e.g.	exempli gratia: for example
et al.	et alii: and others
EVD	EigenValue Decomposition
FDD	Frequency Division Duplexing
FEQ	Frequency domain EQualizer
FEXT	Far–End CrossTalk
FFT	Fast Fourier Transform
FIR	Finite Impulse Response filter
GSNR	Geometric SNR
HDSL	High data rate DSL
ICI	Inter–Carrier–Interference
IDFT	Inverse Discrete Fourier Transform
i.e.	id est: that is
IFFT	Inverse Fast Fourier Transform
IIR	Infinite Impulse Response filter
ISDN	Integrated Services Digital Network
ISI	Inter–Symbol–Interference
ISP	Internet Service Provider
ITF-EC	Improved Time Frequency Domain EC
ITU	International Telecommunication Union

MA	Margin Adaptive
MAI	Multiple Access Interference
MBR	Maximum Bit-Rate
MDS	Minimum Delay-Spread
MGSNR	Maximum Geometric-SNR
MIMO	Multiple–Input/Multiple–Output
MinISI	Minimum Inter–Symbol–Interference
MISO	Multiple-Input/Single-Output
MMSE	Minimum Mean Square Error
MSE	Mean Square Error
MSSNR	Minimum Shortening SNR
NEXT	Near–End CrossTalk
OFDM	Orthogonal Frequency Division Multiplexing
POTS	Plain Old Telephone Service
P/S	Parallel–to–Serial converter
PSTN	Public Switched Telephone Network
PT-EQ	Per Tone EQualization
QAM	Quadrature Amplitude Modulation
RA	Rate Adaptive
RFI	Radio Frequency Interference
RT	Remote Terminal
RX	Receiver
SDSL	Symmetric DSL
SIMO	Single–Input/Multiple–Output
SISO	Single–Input/Single–Output
SNR	Signal–to–Noise Ratio
S/P	Serial–to–Parallel converter
SVD	Singular Value Decomposition
TEQ	Time domain EQualizer
TIR	Target Impulse Response
TP	Twisted Pair
TX	Transmitter
UEC	Unit Energy Constraint
UNC	Unit Norm Constraint
UTC	Unit Tap Constraint
VDMT	Vectored Discrete Multi-Tone
VDSL	Very high bit rate DSL
ZF	Zero Forcing

Nomenclature

a	scalar a
$\mathbf{a}$	(column) vector $\mathbf{a}$
$\|\|\mathbf{a}\|\|_2$	2-norm of vector $\mathbf{a}$
$\mathbf{A}$	matrix $\mathbf{A}$
$\mathbf{A}^T$	transpose of matrix $\mathbf{A}$
$\mathbf{A}^H$	Hermitian transpose of matrix $\mathbf{A}$
$\mathbf{A}^*$	complex conjugate of matrix $\mathbf{A}$
$\mathbf{A}^{-1}$	inverse of matrix $\mathbf{A}$
$\mathbf{A}^{1/2}$	Cholesky factor of matrix $\mathbf{A}$
$\mathrm{diag}\left\{\mathbf{a}\right\}$	square matrix with elements of vector $\mathbf{a}$ on the main diagonal
$\mathbf{0}$	zero vector
$\mathbf{0}_{1\times n}$	zero row vector of length n
$\mathbf{O}$	zero matrix
$\mathbf{O}_{m\times n}$	$m \times n$ zero matrix
$\mathbf{I}_N$	$N \times N$ identity matrix
$\mathcal{F}_N$	$N \times N$ DFT matrix
$\mathcal{I}_N$	$N \times N$ IDFT matrix
$\mathcal{E}\left\{a\right\}$	expectation of random variable a
$\|a\|$	absolute value of $a \in \mathbb{R}$; modulus of $a \in \mathbb{C}$
$\mathbf{x}_k$	symbol sequence at discrete time k
$\mathbf{x}_{(i)}$	symbol sequence of the i-th user
$\mathbf{X}_i^{(k)}$	complex DMT subsymbol for tone i at symbol period k
$\mathbf{e}_i$	i-th basis vector
$\hat{a}$	estimate of a
$\bar{a}$	averaged variable a

$\mathbb{R}$	set of real numbers
$\mathbb{C}$	set of complex numbers
$\nabla_{\mathbf{a}} J$	gradient of J with respect to $\mathbf{a}$
$\mathbf{A}[i,:]$	i-th row of matrix $\mathbf{A}$
$\mathbf{A}(i,j)$	i-th row and j-th column element of matrix $\mathbf{A}$
$\mathbf{R}_{\Delta}$	matrix $\mathbf{R}$ dependent on variable Δ

Contents

List of Figures

List of Tables

Chapter 1

ADSL equalization

Following the introduction to ADSL equalization given in previous chapter, next sections describe particular algorithms to optimal equalizer design. From time-domain area, there are MMSE based algorithms in the first section and other algorithms based on different cost-functions. The last section presents the equalization in frequency domain, respectively Per-tone equalizer. Comprehensive text concerning ADSL equation can be found in [2].

1.1 MMSE channel shortening

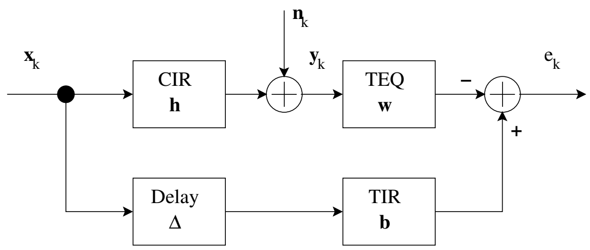

Figure 1.1: System model of TEQ with MMSE optimization

Design of the TEQ based on Minimum Mean Square Error (MMSE) optimization means to shorten the channel impulse response (CIR) to a $(\nu + 1)$-taps target impulse response. According to TEQ design model, depicted on Fig. 1.1, one searches for a TEQ $\mathbf{w}$, TIR $\mathbf{b}$ and system delay Δ such that the MSE cost function (1.1) is minimized.

Error output, e_k, of the system model (Fig. 1.1) with the TEQ of length T, transmitted signal x_k and received signal y_k can be written as:

$$e_k = w_k \star y_k - b_k \star x_k = \mathbf{w}^T \mathbf{y}_k - \mathbf{b}^T \mathbf{x}_k \ ,$$

where

$$\mathbf{x}_k = [x_{k-\Delta} \dots x_{k-\Delta-\nu}]^T \ , \ \mathbf{y}_k = [y_k \dots y_{k-T+1}]^T \ ,$$
$$\mathbf{w} = [w_0 \dots w_{T-1}]^T \ , \ \mathbf{b} = [b_0 \dots b_\nu]^T \ ,$$

where k is the time index and ν is the CP length.

Desired cost function of MSE become:

$$\begin{aligned} J_{MSE} &= \mathcal{E}\left\{||\mathbf{w}^T\mathbf{y}_k - \mathbf{b}^T\mathbf{x}_k||_2^2\right\} \\ &= \mathbf{w}^T\mathbf{R}_{yy}\mathbf{w} + \mathbf{b}^T\mathbf{R}_{xx}\mathbf{b} - 2\mathbf{b}^T\mathbf{R}_{xy}\mathbf{w} \end{aligned} \tag{1.1}$$

where $\mathcal{E}\{\cdot\}$ is statistical expectation operator and $\mathbf{R}_{xx}, \mathbf{R}_{yy}, \mathbf{R}_{xy}$ are signal correlation matrices corresponding to its indices.

Equalizer optimum corresponds with minimum of cost function:

$$\min_{\mathbf{w},\mathbf{b}} J_{MSE}(\mathbf{w}, \mathbf{b}) \ . \tag{1.2}$$

Solving (1.2) needs to be constrained to avoid trivial solution. Different constraint-based MMSE algorithms are described in next subsections.

1.1.1 Unit norm constraint - UNC

Unit norm constraint on TIR

To avoid trivial solution, Unit norm constraint (UNC) implies that the second norm of target impulse response (TIR) is unitary, i.e.:

$$\|\mathbf{b}\|_2^2 = \mathbf{b}^T\mathbf{b} = 1 \ , \tag{1.3}$$

Minimum of the MSE cost function (1.1) could be found using Lagrange multipliers, where the constraint is incorporated:

$$\begin{aligned} &\min_{\mathbf{w},\mathbf{b}} \mathcal{L}(J_{MSE}(\mathbf{w}, \mathbf{b}) \,|\, \mathbf{b}^T\mathbf{b} = 1) \\ &\min_{\mathbf{w},\mathbf{b}} \mathbf{w}^T\mathbf{R}_{yy}\mathbf{w} + \mathbf{b}^T\mathbf{R}_{xx}\mathbf{b} - 2\mathbf{b}^T\mathbf{R}_{xy}\mathbf{w} + \lambda(1 - \mathbf{b}^T\mathbf{b}) \ , \end{aligned} \tag{1.4}$$

where $\mathcal{L}$ is the Lagrangian and λ is a Lagrange multiplier.

To find the minimum value, individual gradients $\mathcal{L}$ with respect to variables are set to be zero:

$$\nabla_{\mathbf{w}}\mathcal{L} = \mathbf{R}_{yy}\mathbf{w} - \mathbf{R}_{yx}\mathbf{b} = 0 \tag{1.5}$$

$$\nabla_{\mathbf{b}}\mathcal{L} = \mathbf{R}_{xx}\mathbf{b} - \mathbf{R}_{xy}\mathbf{w} - \lambda\mathbf{b} = 0 \tag{1.6}$$

$$\nabla_{\lambda}\mathcal{L} = \mathbf{b}^T\mathbf{b} - 1 = 0 \tag{1.7}$$

Coefficients $\mathbf{w}$ are obtained from (1.5)

$$\mathbf{w} = \mathbf{R}_{yy}^{-1}\mathbf{R}_{yx}\mathbf{b} \tag{1.8}$$

and incorporated to (1.6)

$$\underbrace{\left(\mathbf{R}_{xx} - \mathbf{R}_{xy}\mathbf{R}_{yy}^{-1}\mathbf{R}_{yx}\right)}_{\mathbf{R}_\Delta}\mathbf{b} = \lambda\mathbf{b} \ , \quad \mathbf{R}_\Delta\mathbf{b} = \lambda\mathbf{b} \tag{1.9}$$

The optimal solution is finally given by ordinary eigenvalue problem (1.9). It has been proved in [7], that the smallest eigenvalue corresponds to minimal error of MSE problem. Thus the vector of optimal TIR coefficients $\mathbf{b}_{opt}$ is equal to eigenvector of $\mathbf{R}_\Delta$ associated with the smallest eigenvalue λ_{min}.

Optimal TEQ coefficients are obtained according to (1.8):

$$\mathbf{w}_{opt} = \mathbf{R}_{yy}^{-1}\mathbf{R}_{yx}\mathbf{b}_{opt} \tag{1.10}$$

Unit norm constraint on TEQ

Unit norm constraint on TEQ follows same steps as previous constraint on TIR, but the desired optimal coefficients are TEQ, $\mathbf{w}$. Constraint of unit norm for TEQ is defined:

$$\|\mathbf{w}\|_2^2 = \mathbf{w}^T\mathbf{w} = 1 \tag{1.11}$$

From Lagrange multipliers, respectively from gradient $\nabla_{\mathbf{b}}\mathcal{L}$, we obtain TIR coefficients $\mathbf{b}$:

$$\mathbf{b} = \mathbf{R}_{xx}^{-1}\mathbf{R}_{xy}\mathbf{w} \tag{1.12}$$

Incorporating (1.12) into gradient $\nabla_{\mathbf{w}}\mathcal{L}$ leads to eigenvalue problem (1.13). Optimal TEQ coefficients $\mathbf{w}_{opt}$ correspond to eigenvector of $\mathbf{R}_\Delta$ associated with the smallest eigenvalue λ_{min}.

$$\underbrace{\left(\mathbf{R}_{yy} - \mathbf{R}_{yx}\mathbf{R}_{xx}^{-1}\mathbf{R}_{xy}\right)}_{\mathbf{R}_\Delta}\mathbf{w} = \lambda\mathbf{w} \ , \quad \mathbf{R}_\Delta\mathbf{w} = \lambda\mathbf{w} \tag{1.13}$$

Optimal TIR coefficients are given by (1.12):

$$\mathbf{b}_{opt} = \mathbf{R}_{xx}^{-1}\mathbf{R}_{xy}\mathbf{w}_{opt} \tag{1.14}$$

1.1.2 Unit tap constraint - UTC

Unit tap constraint on TEQ

Unit tap constraint coerces one of the filter coefficients to be equal to one. The constraint on TEQ is:

$$\mathbf{e}_i^T\mathbf{w} = 1 \ , \tag{1.15}$$

where e_i is the i-th basis vector of size T.

The cost function minimum with use of Lagrangian become (1.16) and resulting gradients $\nabla\mathcal{L}$ are set to be zero.

$$\min_{\mathbf{w},\mathbf{b}} \mathbf{w}^T\mathbf{R}_{yy}\mathbf{w} + \mathbf{b}^T\mathbf{R}_{xx}\mathbf{b} - 2\mathbf{b}^T\mathbf{R}_{xy}\mathbf{w} + \lambda(1 - \mathbf{e}_i^T\mathbf{w}) \tag{1.16}$$

TIR coefficients $\mathbf{b}$ are easily obtained from gradient $\nabla_{\mathbf{b}}\mathcal{L}$:

$$\mathbf{b} = \mathbf{R}_{xx}^{-1}\mathbf{R}_{xy}\mathbf{w} \tag{1.17}$$

Simplifying zeroed gradients form a close to eigenvalue problem:

$$2\underbrace{\left(\mathbf{R}_{yy} - \mathbf{R}_{yx}\mathbf{R}_{xx}^{-1}\mathbf{R}_{xy}\right)}_{\mathbf{R}_\Delta}\mathbf{w} = \lambda\mathbf{e}_i \tag{1.18}$$

$$2\mathbf{R}_\Delta\mathbf{w} = \lambda\mathbf{e}_i \tag{1.19}$$

Multiplying right side of (1.19) by $\mathbf{e}_i^T\mathbf{w}$, i.e. identity, leads to generalized eigenvalue problem:

$$2\underbrace{\mathbf{R}_\Delta}_{\mathbf{B}}\mathbf{w} = \lambda\underbrace{\mathbf{e}_i\mathbf{e}_i^T}_{\mathbf{A}}\mathbf{w} \tag{1.20}$$

$$\mathbf{B}\mathbf{w} = \lambda\mathbf{A}\mathbf{w} \tag{1.21}$$

Supposing, that $\mathbf{R}_\Delta$ is invertible, (1.20) can be simplified to ordinary eigenvalue problem:

$$\mathbf{B}^{-1}\mathbf{A}\mathbf{w} = \lambda^{-1}\mathbf{w} \tag{1.22}$$

Inverted eigenvalue λ^{-1} is replaced by λ' and optimum is corresponding to the largest eigenvalue instead of the smallest one. Further, when matrices $\mathbf{A}$ and $\mathbf{B}$ are incorporated, solution become (1.24).

$$\mathbf{B}^{-1}\mathbf{A}\mathbf{w} = \lambda'\mathbf{w} \tag{1.23}$$

$$2\,\mathbf{R}_\Delta^{-1}\mathbf{e}_i\mathbf{e}_i^T\mathbf{w} = \lambda'\mathbf{w} \tag{1.24}$$

While the FEQ is the one responsible for scalar correction of tones, multiplying by 2 in (1.24) can be discarded. Functionality of TEQ will not be affected.

Defining $\mathbf{G}$ as (1.25), eigenvalue problem (1.24) become the form (1.26).

$$\mathbf{G} = \mathbf{R}_\Delta^{-1}\mathbf{e}_i\mathbf{e}_i^T \tag{1.25}$$

$$\mathbf{G}\mathbf{w} = \lambda'\mathbf{w} \tag{1.26}$$

Formally, the optimal TEQ coefficients correspond to eigenvector associated with the largest eigenvalue of $\mathbf{G}$, but definition of tap constraint implies, that $\mathbf{G}$ consists only of

i-th column of matrix $\mathbf{R}_\Delta^{-1}$. Thus the desired eigenvector and i-th column $\mathbf{R}_\Delta^{-1}$ span the same subspace.

According to [7] the optimal TEQ coefficients $\mathbf{w}_{opt}$ are given by:

$$\mathbf{w}_{opt} = \frac{\mathbf{R}_\Delta^{-1}\mathbf{e}_{iopt}}{\mathbf{e}_{iopt}^T\mathbf{R}_\Delta^{-1}\mathbf{e}_{iopt}} \ , \tag{1.27}$$

where $i_{opt} = \max\arg_{0<i<T}\{\mathbf{R}_\Delta^{-1}(i,i)\}$.

Note that the desired TEQ is given by i-th column of $\mathbf{R}_\Delta^{-1}$ normalized by element $\mathbf{R}_\Delta^{-1}(i,i)$).

Optimal TIR coefficients are given by (1.17):

$$\mathbf{b}_{opt} = \mathbf{R}_{xx}^{-1}\mathbf{R}_{xy}\mathbf{w}_{opt} \tag{1.28}$$

Unit tap constraint on TIR

Solution follows the previous constraint on TEQ. Briefly described, the constraint is:

$$\mathbf{e}_i^T\mathbf{b} = 1 \tag{1.29}$$

where e_i is the i-th basis vector of size $\nu + 1$.

Matrix $\mathbf{R}_\Delta$ become:

$$\mathbf{R}_\Delta = \mathbf{R}_{xx} - \mathbf{R}_{xy}\mathbf{R}_{yy}^{-1}\mathbf{R}_{yx} \tag{1.30}$$

and obtained expression for TEQ coefficients $\mathbf{w}$ (similarly for $\mathbf{w}_{opt}$) become:

$$\mathbf{w} = \mathbf{R}_{yy}^{-1}\mathbf{R}_{yx}\mathbf{b} \tag{1.31}$$

Optimal TIR coefficients $\mathbf{b}_{opt}$ are given by i-th column and i,i-th element of $\mathbf{R}_\Delta^{-1}$:

$$\mathbf{b}_{opt} = \frac{\mathbf{R}_\Delta^{-1}\mathbf{e}_{iopt}}{\mathbf{e}_{iopt}^T\mathbf{R}_\Delta^{-1}\mathbf{e}_{iopt}} \tag{1.32}$$

where $i_{opt} = \max\arg_{0<i<\nu+1}\{\mathbf{R}_\Delta^{-1}(i,i)\}$.

1.1.3 Unit energy constraints - UEC

Constrains for unit energy involve signal correlations to enumerate energy and there are basically three possible variants:

$$\text{TIR: } \mathbf{b}^T\mathbf{R}_{xx}\mathbf{b} = 1 \tag{1.33}$$

$$\text{TEQ: } \mathbf{w}^T\mathbf{R}_{yy}\mathbf{w} = 1 \tag{1.34}$$

$$\text{TIR + TEQ: } \mathbf{b}^T\mathbf{R}_{xx}\mathbf{b} = 1 \ \text{ a } \ \mathbf{w}^T\mathbf{R}_{yy}\mathbf{w} = 1 \tag{1.35}$$

Unit energy constraint on TIR

Optimum of TIR is searched minimizing cost function (1.1) under the constraint $\mathbf{b}^T\mathbf{R}_{xx}\mathbf{b} = 1$.

Using Lagrange method, the gradients are set to zero and TEQ coefficients are obtained from corresponding cost function gradient $\nabla_{\mathbf{w}}J$:

$$\mathbf{w} = \mathbf{R}_{yy}^{-1}\mathbf{R}_{yx}\mathbf{b} \tag{1.36}$$

Incorporating (1.36) into cost function (1.1) simplifies its minimum to:

$$\min_{\mathbf{b}} J = \min_{\mathbf{b}} \mathbf{b}^T \left(\mathbf{R}_{xx} - \mathbf{R}_{xy}\mathbf{R}_{yy}^{-1}\mathbf{R}_{yx}\right)\mathbf{b} \tag{1.37}$$
$$\text{subject to: } \mathbf{b}^T\mathbf{R}_{xx}\mathbf{b} = 1$$

With effort to avoid generalized eigenvalue problem, Cholesky decomposition is used to simplify the constraint (1.37) and matrix $\mathbf{P}$ is defined (1.40). Minimum of the cost function become:

$$\begin{aligned}\min_{\mathbf{b}} J &= 1 - \max_{\tilde{\mathbf{b}}} \tilde{\mathbf{b}}^T \left(\mathbf{R}_{xx}^{-T/2}\mathbf{R}_{xy}\mathbf{R}_{yy}^{-1}\mathbf{R}_{yx}\mathbf{R}_{xx}^{-1/2}\right)\tilde{\mathbf{b}} \\ &= 1 - \max_{\tilde{\mathbf{b}}} \tilde{\mathbf{b}}^T \left(\mathbf{P}^T\mathbf{P}\right)\tilde{\mathbf{b}}\ , \quad \text{subject to:} \quad \tilde{\mathbf{b}}^T\tilde{\mathbf{b}} = 1\end{aligned} \tag{1.38}$$

$$\tilde{\mathbf{b}} = \mathbf{R}_{xx}^{1/2}\mathbf{b} \tag{1.39}$$

$$\mathbf{P} = \mathbf{R}_{yy}^{-T/2}\mathbf{R}_{yx}\mathbf{R}_{xx}^{-1/2} \tag{1.40}$$

Following approach of previous MMSE constraints, the optimal TIR is given by solution of eigenvalue problem. The optimal transformed coefficients $\tilde{\mathbf{b}}$ are equal to eigenvector associated with the largest eigenvalue of composite matrix $\mathbf{P}^T\mathbf{P}$. The optimal TIR coefficients $\mathbf{b}$ are given by (1.39).

Unit energy constraint on TEQ

Following the energy constraint on TIR, same approach leads to similar minimum of cost function (1.41), where the composite matrix $\mathbf{P}\mathbf{P}^T$ differs slightly.

$$\min_{\mathbf{w}} J = 1 - \max_{\tilde{\mathbf{w}}} \tilde{\mathbf{w}}^T \left(\mathbf{P}\mathbf{P}^T\right)\tilde{\mathbf{w}}\ , \tag{1.41}$$

$$\text{subject to: } \tilde{\mathbf{w}}^T\tilde{\mathbf{w}} = 1 \tag{1.42}$$

where $\tilde{\mathbf{w}}$ is transformed vector $\mathbf{w}$:

$$\tilde{\mathbf{w}} = \mathbf{R}_{yy}^{1/2}\mathbf{w} \tag{1.43}$$

and matrix $\mathbf{P}$ is identical to (1.40):

$$\mathbf{P} = \mathbf{R}_{yy}^{-T/2}\mathbf{R}_{yx}\mathbf{R}_{xx}^{-1/2} \tag{1.44}$$

The optimal transformed coefficients $\tilde{\mathbf{w}}$ are equal to eigenvector associated with the largest eigenvalue of composite matrix $\mathbf{P}\mathbf{P}^T$. The optimal TEQ coefficients $\mathbf{w}$ are given by (1.43).

Unit energy constraints on TIR and TEQ

Considering both of the constraints simultaneously (1.45), solution has to exploit generalized eigenvalue.

$$\mathbf{b}^T \mathbf{R}_{xx} \mathbf{b} = 1 \quad \text{and} \quad \mathbf{w}^T \mathbf{R}_{yy} \mathbf{w} = 1 \ , \tag{1.45}$$

Minimizing MSE cost function with Lagrange method leads to generalized eigenvalue problem in form:

$$\begin{bmatrix} \mathbf{0} & \mathbf{R}_{yx} \\ \mathbf{R}_{xy} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{w} \\ \mathbf{b} \end{bmatrix} = \begin{bmatrix} \mathbf{R}_{yy} & \mathbf{0} \\ \mathbf{0} & \mathbf{R}_{xx} \end{bmatrix} \begin{bmatrix} \mathbf{w} \\ \mathbf{b} \end{bmatrix} (1-\alpha) \ , \tag{1.46}$$

where $(1-\alpha)$ is the generalized eigenvalue.

According to [2], with the constraint on TIR and TEQ, the minimum of MSE cost function (1.1) can be simplified to:

$$J(\mathbf{w}, \mathbf{b}) = 2 - 2\mathbf{b}^T \mathbf{R}_{xy} \mathbf{w} = 2\alpha \tag{1.47}$$

Hence, the TEQ $\mathbf{w}$ and TIR $\mathbf{b}$ are the solution of the generalized eigenvalue problem (1.46), corresponding to the maximal eigenvalue $(1-\alpha)$.

Another approach for optimal equalizer design constrained on TIR and TEQ is based on previous results of TIR $\tilde{\mathbf{b}}$ (1.39) and TEQ $\tilde{\mathbf{w}}$ (1.43). G. Ysebaert [2] proposed a simpler solution, where the optimal TEQ coefficients are given by eigenvector associated with the largest eigenvalue of matrix $\mathbf{Q}$ defined as:

$$\mathbf{Q} = \mathbf{R}_{yy}^{-1} \mathbf{R}_{yx} \mathbf{R}_{xx}^{-1} \mathbf{R}_{xy} \ , \tag{1.48}$$

constrained by:

$$\text{TIR:} \quad \mathbf{Q}\mathbf{b} = \lambda_{max} \mathbf{b} \tag{1.49}$$

and

$$\text{TEQ:} \quad \mathbf{Q}\mathbf{w} = \lambda_{max} \mathbf{w} \tag{1.50}$$

1.2 Maximum shortening SNR (MSSNR)

Algorithm MSSNR is based on shortening channel response (CIR), with effort to suppress symbol interferences (ISI). The goal of this algorithm is to minimize energy outside a window applied to CIR. The window (Fig. 1.2) is rectangle type and its length is equal to length of cyclic prefix $(\nu + 1)$.

Inner part of the window, selected part of channel response resp., can be described as:

$$\mathbf{H}_{win} = \begin{bmatrix} h_\Delta & \cdots & h_{\Delta-T+1} \\ \vdots & \ddots & \vdots \\ h_{\Delta+\nu} & \cdots & h_{\Delta+\nu-T+1} \end{bmatrix} \ , \tag{1.51}$$

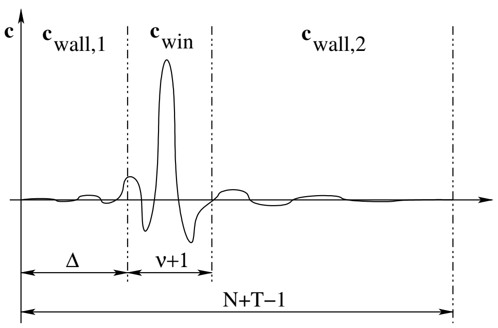

Figure 1.2: MSSNR windowing of CIR.

where Δ is window delay, $\nu+1$ is window length and $N+T+1$ is length of CIR.

Outer parts of the window include the left part:

$$\mathbf{H}_{wall,1} = \begin{bmatrix} h_0 & 0 & \cdots & 0 \\ \vdots & \ddots & & \vdots \\ h_{\Delta-1} & \cdots & & h_{\Delta-T} \end{bmatrix} \tag{1.52}$$

and the right side:

$$\mathbf{H}_{wall,2} = \begin{bmatrix} h_{\Delta+\nu+1} & \cdots & & h_{\Delta+\nu-T+2} \\ \vdots & \ddots & & \vdots \\ 0 & \cdots & 0 & h_{N-1} \end{bmatrix} . \tag{1.53}$$

Thus the outer part of the window is composed to:

$$\mathbf{H}_{wall} = \left[\mathbf{H}_{wall,1}^T \, \mathbf{H}_{wall,2}^T\right]^T . \tag{1.54}$$

Following MSSNR principle, the optimum is given by minimizing energy of outer window:

$$\min_{\mathbf{w}} \mathbf{w}^T \mathbf{H}_{wall}^T \mathbf{H}_{wall} \mathbf{w} \ , \text{ subject to: } \mathbf{w}^T \mathbf{H}_{win}^T \mathbf{H}_{win} \mathbf{w} = 1 . \tag{1.55}$$

With the same effort, energy ratio could be inverted, i.e. maximizing energy of inner window part. The optimum is then given by:

$$\max_{\mathbf{w}} \mathbf{w}^T \mathbf{H}_{win}^T \mathbf{H}_{win} \mathbf{w} \ , \text{ subject to: } \mathbf{w}^T \mathbf{H}_{wall}^T \mathbf{H}_{wall} \mathbf{w} = 1 . \tag{1.56}$$

Since the composite matrix $\mathbf{H}_{wall}^T \mathbf{H}_{wall}$ of ADSL channel is positive semi-definite [2], recommended approach is maximizing, i.e. (1.56). The solution is then given by (1.57). Similarly to problem (**??**), the solution reduces to standard eigenvalue problem, where the optimal TEQ coefficients $\mathbf{w}_{opt}$ corresponds to an eigenvector associated with the largest eigenvalue of $\mathbf{A}^{-1}\mathbf{B}$.

$$\mathbf{w}_{opt} = \arg\max_{\mathbf{w}} \frac{\mathbf{w}^T \mathbf{B} \mathbf{w}}{\mathbf{w}^T \mathbf{A} \mathbf{w}} \ , \tag{1.57}$$

where

$$\mathbf{A} = \mathbf{H}_{wall}^T \mathbf{H}_{wall}$$
$$\mathbf{B} = \mathbf{H}_{win}^T \mathbf{H}_{win}$$

Note that, the received signal is considered as white in spectra, since all frequencies (tones) are treated in the same way.

1.3 Minimum delay spread (MDS)

It has been mentioned that cyclic prefix is not long enough to suppress all the interferences (ISI, ICI). With assumption that the interferences are caused by part of channel response exceeding the CP, an algorithm should minimize this exceeding part (respectively its energy) to suppress the interferences. Furthermore the distance between CP-start and CIR beginning "active" is also significant and should be minimized.

Schur et al. defined a delay-spread measure D (1.58) of the effective channel response, $\mathbf{c} = \mathbf{h} \star \mathbf{w}$, and proposed that minimizing square of the delay-spread D^2 leads to optimal TEQ design.

$$D = \sqrt{\frac{1}{E} \sum_{l=0}^{N+T-2} (l - \bar{l})^2 \left|\mathbf{c}[l]\right|^2} \ , \tag{1.58}$$

where

$$E = \sum_{l=0}^{N+T-2} \left|\mathbf{c}[l]\right|^2 , \qquad \bar{l} = \frac{1}{E} \sum_{l=0}^{N+T-2} l \left|\mathbf{c}[l]\right|^2 \ . \tag{1.59}$$

The solution, minimum of delay-spread, corresponds to generalized eigenvalue problem [2] as described by (**??**), where

$$\mathbf{A}\mathbf{w} = \lambda \mathbf{B}\mathbf{w} \ , \tag{1.60}$$
$$\mathbf{A} = \mathbf{H}^T \mathbf{Q} \mathbf{H} \ , \tag{1.61}$$
$$\mathbf{B} = \mathbf{H}^T \mathbf{H} \ , \tag{1.62}$$

with diagonal weighting matrix $\mathbf{Q} = \mathrm{diag}[(0 - \bar{l})^2 \dots (N + T - 2 - \bar{l})^2]^T$.

Since MDS TEQ design is quite similar to MSSNR TEQ design, all advantages and drawbacks mentioned in section 1.2 also apply here.

1.4 Maximum geometric SNR (MGSNR)

Algorithm of MGSNR was intended to be maximizing transfer capacity. Since the previous algorithms could be considered as an eigenvalue problem described by single Rayleigh quotient, the geometric SNR implies multiple Rayleigh quotients. This multiple approach means iteration over all used and unused symbol tones in particular.

Based on total amount of bits transmitted in DMT symbol defined as:

$$b_{DMT} = \sum_{i \in \mathcal{S}} \log_2 \left(1 + \frac{\mathrm{SNR}_i}{\Gamma_i}\right) = \log_2 \prod_{i \in \mathcal{S}} \left(1 + \frac{\mathrm{SNR}_i}{\Gamma_i}\right) \ , \tag{1.63}$$

the definition of geometric SNR is given by the following expression:

$$\mathrm{GSNR}(\mathbf{w}) = \Gamma \left(\left[\prod_{i \in \mathcal{S}} \left(1 + \frac{\mathrm{SNR}_i}{\Gamma}\right) \right]^{1/Nu} - 1 \right) \ . \tag{1.64}$$

where $\mathcal{S}$ is a set of used tones, SNR_i is SNR of i-th tone and Γ_i is SNR gap (where we assume: $\Gamma_i = \Gamma, \forall_i$).

The total amount of bits (1.63) is then simplified to:

$$b_{\mathrm{GSNR}}(\mathbf{w}) = N_u \log_2 \left(1 + \frac{\mathrm{GSNR}}{\Gamma}\right) \ , \tag{1.65}$$

where N_u is the number of used tones.

Observing (1.65), it is obvious that the transfer capacity depends on GSNR in direct variation sense. Under assumption of higher SNR_i values, the GSNR definition (1.64) can be approximated by:

$$\mathrm{GSNR}(\mathbf{w}) \approx \left(\prod_{i \in \mathcal{S}} \mathrm{SNR}_i \right)^{1/Nu} \ . \tag{1.66}$$

To form a Rayleigh quotients, a transform to frequency domain is proposed in [2]. Then, the TEQ design model, for tone i, in frequency domain is described by:

$$\begin{aligned} B_i &= \mathcal{F}_N[i,:][\mathbf{b}^T \ \mathbf{0}_{1\times(N-\nu-1)}]^T \ , \\ W_i &= \mathcal{F}_N[i,:][\mathbf{w}^T \ \mathbf{0}_{1\times(N-T)}]^T \ , \\ H_i &= \mathcal{F}_N[i,:]\mathbf{h}^T \ . \end{aligned}$$

Per-tone SNR can be simplified to (1.67) using approximation: $B_i \approx H_i W_i$.

$$\mathrm{SNR}_i(\mathbf{w}, \mathbf{b}) = \frac{\sigma_{x,i}^2 \left|H_i\right|^2}{\sigma_{n,i}^2} = \frac{\sigma_{x,i}^2 \left|H_i\right|^2 \left|W_i\right|^2}{\sigma_{n,i}^2 \left|W_i\right|^2} \approx \frac{\sigma_{x,i}^2 \left|B_i\right|^2}{\sigma_{n,i}^2 \left|W_i\right|^2} \tag{1.67}$$

where $\sigma_{x,i}^2$ and $\sigma_{n,i}^2$ are per-tone variances of signal and noise respectively.

Incorporating SNR model (1.67) to simplified GSNR (1.66) leads to approximate relation:

$$\mathrm{GSNR}(\mathbf{w}, \mathbf{b}) \approx \sigma_x^2 \left(\prod_{i \in \mathcal{S}} \frac{\left|B_i\right|^2}{\sigma_{n,i}^2 \left|W_i\right|^2} \right)^{1/N_u} \ , \tag{1.68}$$

assuming that $\sigma_{x,i}^2 = \sigma_x^2, \forall_i$.

Further, the author [2] proposes, that maximizing GSNR (1.67) is equivalent to maximizing cost function:

$$J(\mathbf{b}) = \frac{1}{N_u} \sum_{i \in \mathcal{S}} \ln |B_i|^2 \tag{1.69}$$

$$= \frac{1}{N_u} \sum_{i \in \mathcal{S}} \ln(\mathbf{b}^T \mathbf{Q}_i \mathbf{b}) \ , \tag{1.70}$$

where $\mathbf{Q}_i = \mathcal{F}_N[i, 1 : \nu + 1]^H \mathcal{F}_N[i, 1 : \nu + 1]$.

The optimal TEQ design, maximization of (1.70), can be solved using multiple Rayleigh quotients:

$$\mathbf{b}_{opt} = \arg\max_{\mathbf{b}} \prod_{i \in \mathcal{S}} \frac{\mathbf{b}^T \mathbf{B}_i \mathbf{b}}{\mathbf{b}^T \mathbf{A}_i \mathbf{b}} \ , \quad \text{subject to: } \mathbf{b}^T \mathbf{b} = 1 \tag{1.71}$$

where

$$\mathbf{A}_i = \mathbf{I}_{\nu+1} \ ,$$
$$\mathbf{B}_i = \mathbf{Q}_i \ .$$

The optimal TEQ coefficients can be computed as MMSE TEQ constrained to TIR (1.10), i.e.:

$$\mathbf{w}_{opt} = \mathbf{R}_{yy}^{-1} \mathbf{R}_{yx} \mathbf{b}_{opt} \tag{1.72}$$

1.5 Carrier nulling algorithm (CNA)

Carrier nulling (CNA) is a blind method of equalization proposed for OFDM / DMT systems. This method expects unused tones in symbol, which is common property of OFDM / DMT systems. The CNA expects unused tones to have zero energy at the receiver. When interferences are impairing the symbols, this expectation is also impaired. Thus the algorithm adapt its filter coefficients to achieve expected zero energy at unused tones.

Forcing unused tones to zero (nulling) describes cost function (1.73) as a sum unused tones energies, which has to be minimized afterwards.

$$J = \sum_{i \in \mathcal{S}} \mathcal{E} \left\{ |\mathcal{F}_N[i, :] \mathbf{Y}^k \mathbf{w}|^2 \right\} , \tag{1.73}$$

where $\mathcal{F}_N[i, :]$ represents i-th row of DFT matrix, $\mathcal{S}$ is a set of unused tones and $\mathbf{Y}^k$ are the received symbols. The received signal matrix $\mathbf{Y}$ is the Toeplitz type matrix of size

$T \times N_{FFT}$ and it is composed as:

$$\mathbf{Y} = \begin{bmatrix} y_0 & y_{-1} & \cdots & y_{-T+1} \\ y_1 & y_0 & \cdots & y_{-T+2} \\ \vdots & \vdots & \ddots & \vdots \\ y_{N-1} & y_{N-2} & \cdots & y_{N-T} \end{bmatrix}$$

Extracting matrices $\mathbf{A}$, $\mathbf{B}$ from the expression of cost function (1.73) leads to single Rayleigh quotient problem (**??**), i.e.:

$$\begin{aligned} \mathbf{A} &= \sum_{i \in \mathcal{S}} \mathcal{E} \left\{ (\mathbf{Y}^k)^T \mathcal{F}_N[i,:]^H \mathcal{F}_N[i,:] \mathbf{Y}^k \right\}, \\ \mathbf{B} &= \mathbf{I}_T \end{aligned} \tag{1.74}$$

To avoid trivial solution a constraint has to be involved, e.g.: $\mathbf{w}^T \mathbf{w} = 1$.

1.6 Per-tone equalization

In cooperation with well designed TEQ, a basic type of frequency domain equalizer (FEQ) is usually used. As described in "Introduction to ADSL equalization" (page **??**), such a basic FEQ serves to correct a properly received tones by means of multiplying each tone by complex constant (see Fig. **??**). Complex coefficients, D, of this FEQ are set within initialization of ADSL modem and they are enumerated from channel estimation model, which is determined by known training sequence after TEQ design is done.

Other approach applied to ADSL equalization is focused purely to frequency domain and thus, opposite to time domain, distinguishes and operates on single tones. This per-tone approach fits to DMT tone arrangement in principle and it is considered as more precise than TEQ approach. Furthermore, the main advantage of per-tone approach is that equalizer allows to optimize the bit-rate for each tone separately. Disadvantage of per-tone applications is presumable growth of computational complexity.

Optimal design of Per-tone equalizer (PT-EQ) is based on MMSE optimization for each tone. Basic variant of PT-EQ, derived in the following section, needs to compute one DFT per each tone. This multiple DFT computation is provided by *sliding-window FFT*. In effort to show how to lower the computational complexity, a modified variant of PT-EQ with single FFT is also described in later section.

1.6.1 Per-tone equalizer (PT-EQ)

Per-tone equalizer, its basic design, arises from the approach introduced in this section. Since equalizer algorithms operate in frequency and with separate tones, output of the equalizer, $Z_i^{(k)}$ for tone $i = 1 \dots N$ and k-th symbol, is defined as:

$$[Z_1^k \dots Z_N^k]^T = \mathrm{diag}\{D_1 \dots D_N\} \cdot \underbrace{\mathcal{F}_N(\mathbf{Yw})}_{1\times\mathrm{FFT}} , \tag{1.75}$$

where $D_1 \dots D_N$ are complex correcting coefficients (FEQ), $\mathbf{Y}$ is a signal matrix, Toeplitz type, consisting channel output and $\mathbf{w}$ are EQ coefficients. Matrix $\mathcal{F}_N$ denotes M-point DFT matrix.

Single, i-th, tone then can be expressed:

$$Z_i = D_i \cdot \mathrm{row}_i(\mathcal{F}_N) \cdot (\mathbf{Yw}) . \tag{1.76}$$

Single tone in frequency domain can be viewed as a signal filtered by filter with coefficients equal to corresponding row of DFT matrix $\mathcal{F}_N$. Considering N filters of length T, single tone (1.76) then become:

$$Z_i = \mathrm{row}_i \underbrace{(\mathcal{F}_N \mathbf{Y})}_{T\times\mathrm{FFT}} \cdot \mathbf{w} D_i . \tag{1.77}$$

According to (1.77), for an equalization of one DMT symbol it is necessary to compute the DFT T-times (or else: one DFT per column of signal matrix $\mathbf{Y}$). This DFT computations are provided by sliding-window FFT [8].

This per-tone equalization approach leads to basic design of PT-EQ depicted on Fig. 1.3. According to per-tone equalization principle, each tone has its own T-tap filter (equalizer).

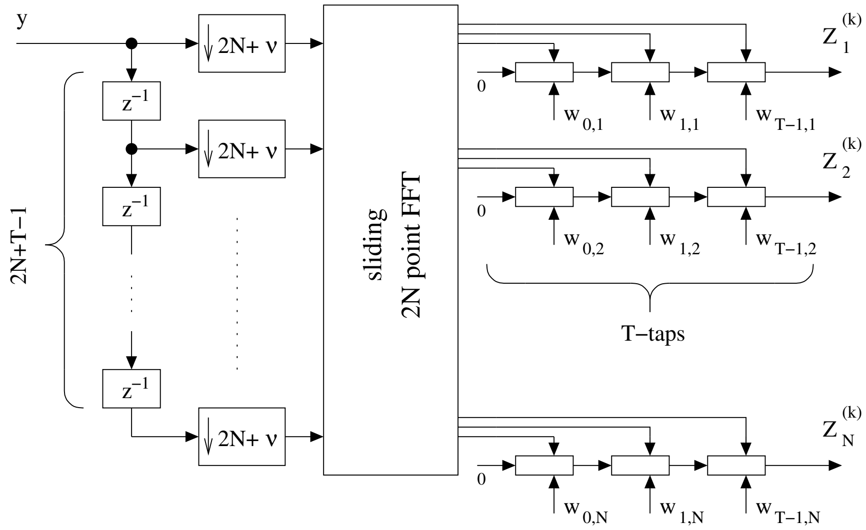

Figure 1.3: Basic design of Per-Tone Equalizer

Optimal coefficients of per-tone EQ can be found by a minimum of mean square error. The cost function to minimize is:

$$\begin{aligned}\min_{\mathbf{w}_i} J(\mathbf{w}_i) &= \min_{\mathbf{w}_i} \mathcal{E}\left\{\left|Z_i^{(k)} - X_i^{(k)}\right|^2\right\} \qquad (1.78)\\ &= \min_{\mathbf{w}_i} \mathcal{E}\left\{\left|\mathbf{w}_i^T \cdot \begin{bmatrix} \mathcal{F}_N[i,:] & 0 & \dots & 0 \\ 0 & \mathcal{F}_N[i,:]) & \ddots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ 0 & \dots & 0 & \mathcal{F}_N[i,:] \end{bmatrix} \cdot \mathbf{y} - X_i^{(k)}\right|^2\right\}\end{aligned}$$

where $\mathcal{F}_N[i,:]$ is the i-th row of DFT matrix.

Extended matrix notation of (1.78) shows again that each per-tone filter is optimized separately. This leads to smooth mathematical relation for achievable bit-rate and thus an optimum is easier to find than for MMSE-TEQ.

MMSE optimized PT-EQ also profits from increasing equalizer order. Opposite to MMSE-TEQ, per-tone variant increases the achievable bit-rate with order of its filters.

1.6.2 Modified PT-EQ

In effort to lower computational complexity of basic PT-EQ, particularly a number of DFT operations, its design should be simplified. Modifications usually proposed replace sliding-window FFT with single FFT as shown on Fig. 1.4. Depicted design exploits the

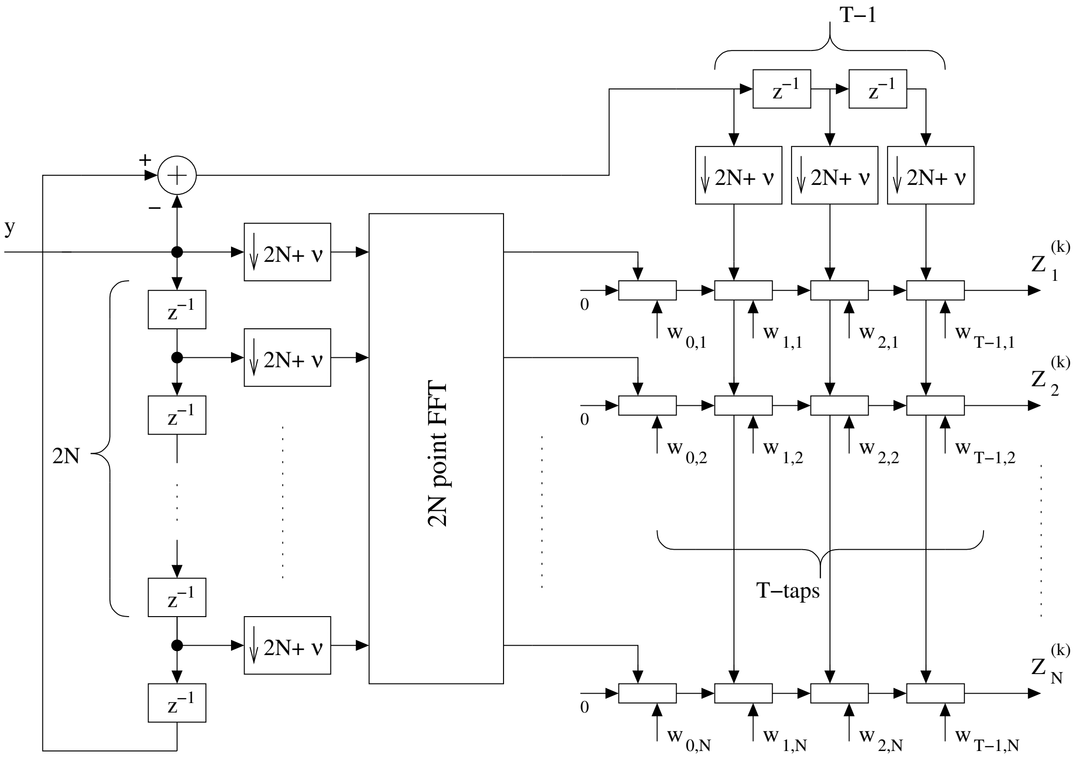

Figure 1.4: Modified PT-EQ, where single FFT used.

property of the received signal matrix $\mathbf{Y}$, i.e. that the $\mathbf{Y}$ is Toeplitz type matrix, and thus sliding-window operation is moved towards to equalizers of tones.

Author of [8] proposes the cost function (1.79) for this altered equalizer, where modifications are included to former cost function (1.78).

$$\min_{\mathbf{v}_i} J(\mathbf{v}_i) = \min_{\mathbf{v}_i} \mathcal{E}\left\{ \left| \mathbf{v}_i^T \cdot \left[\begin{array}{c|c|c} \mathbf{I}_T & \mathbf{0} & \mathbf{I} \\ \hline \mathbf{0} & \multicolumn{2}{c}{\mathcal{F}_N[i,:]} \end{array} \right] \cdot \mathbf{y} - X_i^{(k)} \right|^2 \right\} , \tag{1.79}$$

where the $\mathbf{v}_i^T = [v_{i,T-1} \ \ldots \ v_{i,0}]$ are reverse ordered equalizer coefficients.

Chapter 2

Simulation results

All the equalizer algorithms, we had implemented, were tested within our toolbox. Set of implemented algorithms include: MMSE Unit energy constraint (UEC), MMSE Unit tap constraint (UTC), Maximum Shortening SNR (MSSNR), Minimum Intersymbol Interference (MinISI), Maximum Bitrate (MBR), Maximum Geometric SNR (MGSNR), Minimum Delay Spread (MDS) and Carrier nulling Algorithm (CNA). An example simulation results are presented on next figures. Figures 2.1a and 2.1b show results for reference channel loop "CSA #6". Figures 2.2a and 2.2b show results for reference channel loop "CSA #3". Simulation conditions common for both reference loops were:

- Random user data was given by normally distributed random sequence which were partitioned to 500 of different data symbols.
- Channel noise was shaped type according to noise profile A.
- SNR of transmitted signal and added noise was set in range from 30 to 90dB.
- Each simulation for particular equalizer and SNR was repeated ten-times to reach some statistical independence.
- Bit-loading was computed for each channel loop with the bit-load correction (see below). Each carrier had the capability to carry up to 11 bits, i.e. up to 2048-QAM could be used on each carrier.
- Lower band of twenty carriers was left unused as ISDN/POTS band. Total number of usable carriers was then 235.
- Dimension of FFT in DMT (de)modulator was 512. Length of cyclic prefix was 40 samples long, i.e. each symbol was 552 samples long.
- Lengths of equalizer filter TEQ and target impulse response TIR were set to 32. System delay could vary from 3 to 39 samples.

- Bit-error ratio was enumerated once per each iteration as a fraction of erroneously received bits and total amount of transmitted bits, i.e. defined as:

$$\mathrm{BER}_{iter} = \frac{\sum bits_{\mathrm{Err}}}{bits_{\mathrm{TOTAL}}}$$

  Depicted average ratio aBER was simply averaged BER_{iter} for all iterations N_{iter}:

$$\mathrm{aBER} = \frac{1}{\mathrm{N}_{iter}} \sum_{\mathrm{N}_{iter}} \mathrm{BER}_{iter}$$

Both figures on the left side (2.1a and 2.2a) show a bit-rates which were achieved by RA-water-filling with our customized bit-loading correction for given channel loop. Figures on the right (2.1b and 2.2b) show the graphs of bit-error ratio established for varied SNR.

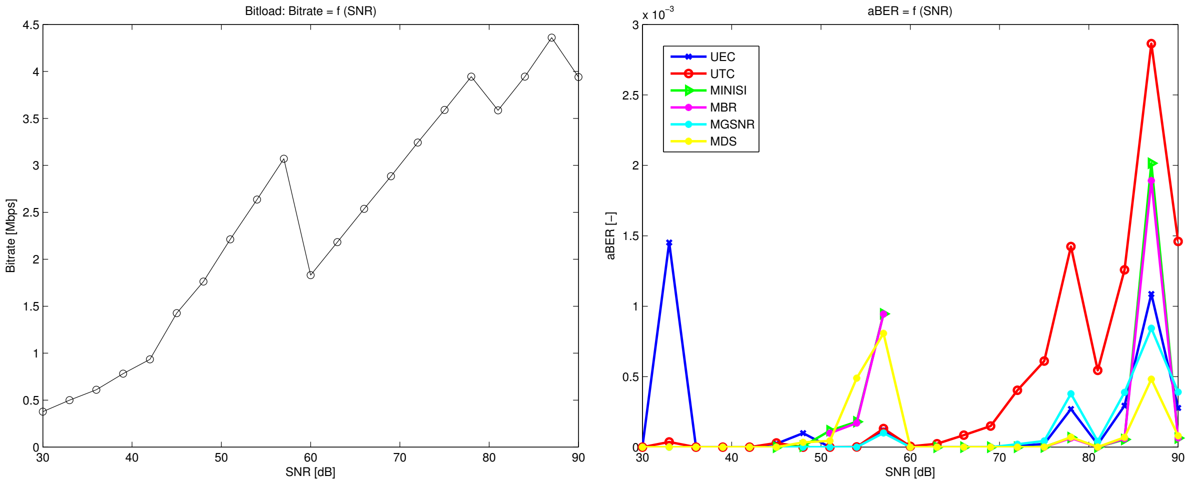

Figure 2.1: Example simulation results for CSA #6: a) achieved bit-rates for given SNR, b) average system bit-error ratios for given SNR

Since the water-filling has tendency to overestimate channel capabilities, we proposed experimental bit-load correction. The correction, simply, lower the bit-load towards to higher SNR. Such a correction holds resulting error ratios in useful range of values. Obviously, lowering the bit-load cause some twists on relation between achieved data-rates and given SNR. The correction is shown on figure 2.3.

In effort to compare water-filling results, we made simulations with two other bit-loading algorithms. Both algorithms have been configured in order to achieve bit-rate approximately 5 Mbps and with respect to ISDN/POTS lower 20 tones have left unused.

The first bit-load has decreasing stairs-like character (fig. 2.4a), maximum number of bits per-tone is $\mathrm{b_n} = 11$, achieved bit-rate is 5.02 Mbps. Other bit-load is simply flat with bit configuration $\mathrm{b_n} = 6$ constant and achieves bit-rate 5.26 Mbps. Resulting error ratios for mentioned bit-loads are shown on figures 2.5a and 2.5b. Stairs-like bit-load respects channel property in way of decreasing bit-load with growing frequency, thus its error ratios resulted lower than the flat bit-load case.

Finally, table 2.1 presents the listing of achieved error ratios for RA-water-filling with correction, decreasing stairs and flat bit-load.

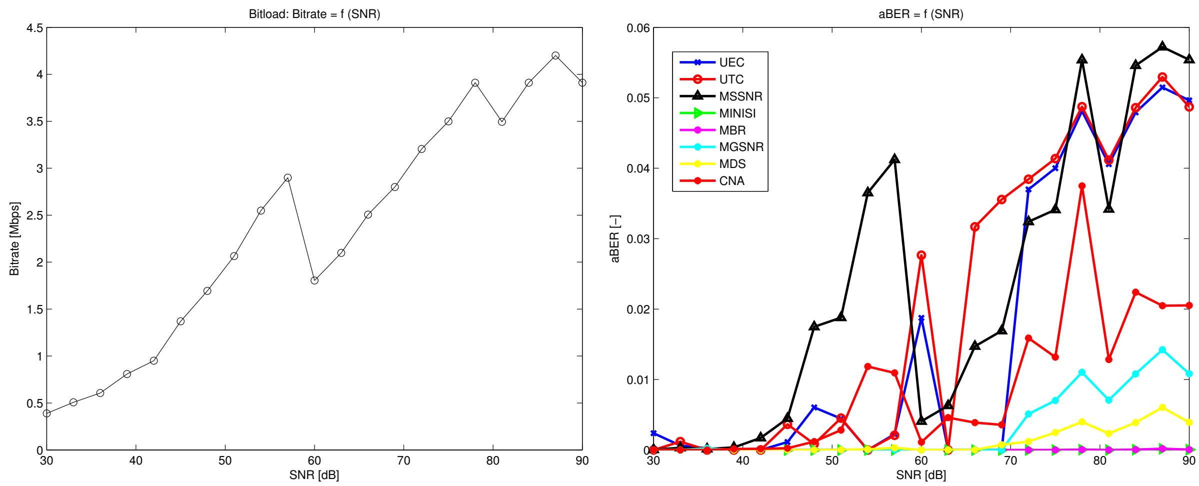

Figure 2.2: Example simulation results for CSA #3: a) achieved bit-rates for given SNR, b) average system bit-error ratios for given SNR

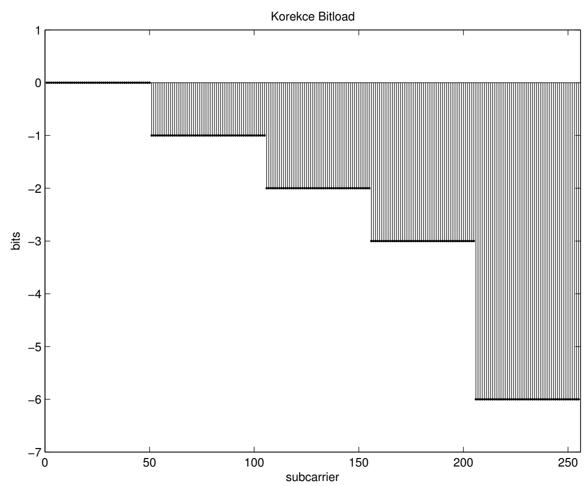

Figure 2.3: Bit-load correction applied on water-filling results

TEQ alg.	Bit-load method					
	Water-filling		Stairs-like		Flat	
	CSA #6	CSA #3	CSA #6	CSA #3	CSA #6	CSA #3
	BER [-]	BER [-]	BER [-]	BER [-]	BER [-]	BER [-]
UEC	$2.7E-04$	0.05	$5.1E-03$	0.06	0.04	0.06
UTC	$1.5E-03$	0.05	$8.5E-03$	0.06	0.04	0.06
MSSNR	0.10	0.06	0.10	0.13	0.08	0.10
MinISI	$6.4E-05$	$4.6E-05$	$4.5E-03$	$1.2E-03$	0.04	0.05
MBR	$6.5E-05$	$4.6E-05$	$4.5E-03$	$1.2E-03$	0.04	0.05
MGSNR	$3.9E-04$	0.01	$3.1E-03$	0.02	0.04	0.05
MDS	$8.0E-05$	$3.9E-03$	$5.9E-03$	0.01	0.04	0.05
CNA	0.03	0.02	0.05	0.05	0.06	0.06

Table 2.1: Comparison of bit-loading algorithms, average error ratios for SNR $=$ 90 dB.

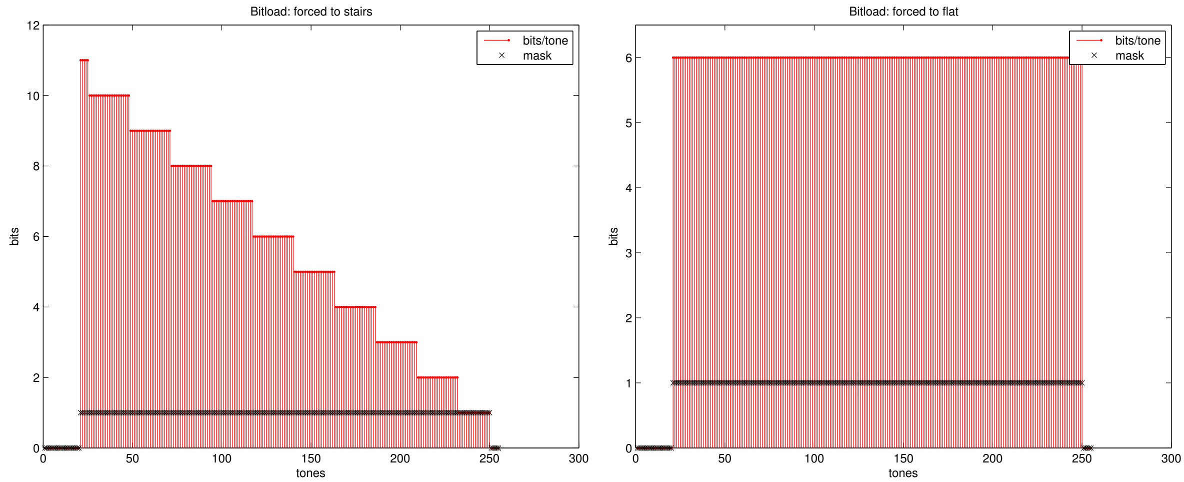

Figure 2.4: Bit-load forced to: a) decreasing stairs: max. $b_n = 11$, b) flat: $b_n = 6$

Conclusions

Bit-loading algorithm has a significant influence on bit-error ratio (BER). Bit-load correction serves as a little aid to achieve better results. Its usage is more suitable than re-evaluation of transmission gap in experimental background. Lowering the bit-load cause some twists on relation between achieved data-rates and given SNR, this side effect can be seen on figures 2.1a and 2.2a.

According to table 2.1., resulted error ratios confirms that the water-filling algorithm finds satisfying optimum for bit-load. Flat bit-load leads to the worst results because it doesn't respect channel's natural property of decreasing capacity with higher frequency.

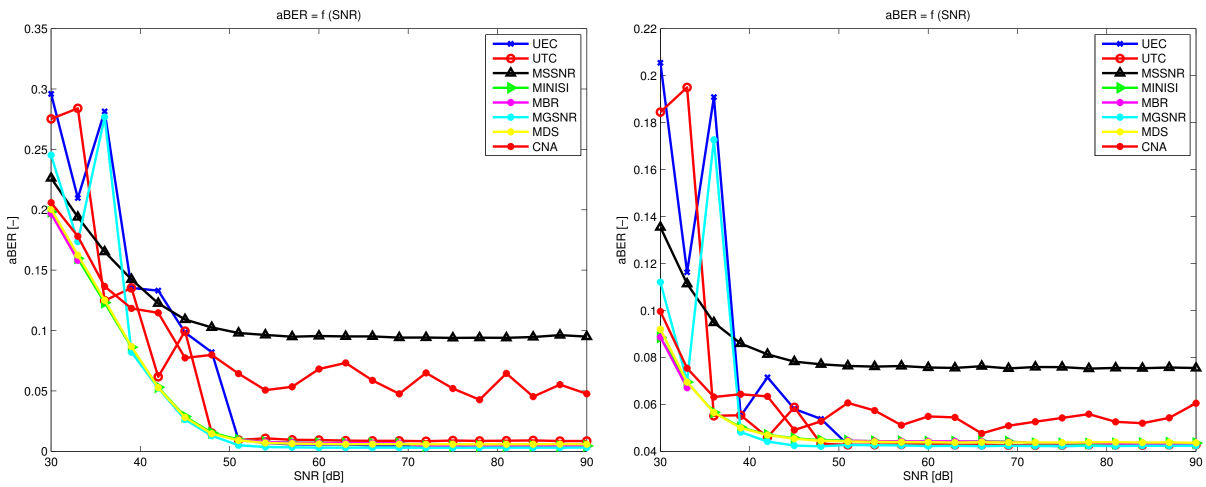

Figure 2.5: Resulting BERs for CSA #6 forced bit-load: a) decreasing stairs: max. $b_n = 11$, b) flat: $b_n = 6$

Bibliography

[1] D. Gesbert, M. Shafi, D. Shiu. From Theory to Practice: An Overview of MIMO Space-TIme Coded Wireless Systems. *IEEE on Selected Areas in Communications*, 21(3):281–302, April 2003.

[2] G. Ysebaert. Equalization and echo Cancellation in DMT-based Systems. SISTA-ESAT K.U. Leuven, Belgium, April 2004.

[3] International Telecommunication Union. ITU-T Recommendation G.992.1: Asymmetric Digital Subscriber Line Transceivers (ADSL). , June 1999.

[4] International Telecommunication Union. ITU-T Recommendation G.992.3: Asymmetric Digital Subscriber Line Transceivers 2 (ADSL2). , July 2002.

[5] J. A. C. Bingham. *ADSL, VDSL and Multicarrier Modulation*. A Wiley-Interscience Publication, John Wiley & Sons, Inc., New York, USA, 2000.

[6] J. M. Cioffi. Advanced Digital Communication classes.
`http://www.stanford.edu/class/ee379c/`.

[7] J. M. Cioffi, N. M. W. Al-Dhahir. Efficiently Computed Reduced-Parameter Input-Aided MMSE Equalizers for ML Detection: A Unified Approach. *IEEE Trans. on Information Theory*, 42(3):903–915, May 1996.

[8] K. Van Acker. Equalization and Echo Cancellation for DMT Modems. SISTA-ESAT K.U. Leuven, Belgium, January 2001.

[9] R. Cendrillon, I. Collings, et al. Advanced Signal Processing for Digital Subscriber Lines. *EURASIP Journal on Applied Sognal Processing*, 2006:, 2006.

[10] R. Cendrillon, M. Moonen, et al. Simplified Power Allocation and TX/RX Structure for MIMO-DSL. *GLOBECOM 2003 Conference proceedings*, pages 1842–1846, 2003.