CMPE145/245: Final Exam, Project 1

The pose of a robot is defined by the coordinates $[x, y, \theta]^T$, where x and y are the robot's coordinates in a rectangular frame, and θ is the orientation angle with $\theta = 0$ lying on the $x - axis$. The robot's posture changes via the on-board control loop, which defines the robot's linear acceleration $\dot{v}$ and angular velocity $\dot{\theta}$. The dynamic model of the robot used in this problem is given in a state space form as:

$$X(t) = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} x \\ y \\ v \\ \theta \end{bmatrix} \quad F(X(t)) = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{bmatrix} = \begin{bmatrix} v(t)\cos\theta(t) \\ v(t)\sin\theta(t) \\ 0 \\ 0 \end{bmatrix} \tag{1}$$

$$\dot{X}(t) = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} x_3 \cos x_4 \\ x_3 \sin x_4 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \xi_v(t) \\ \xi_\theta(t) \end{bmatrix} \tag{2}$$

The knowledge of the control inputs $\dot{v}$ and $\dot{\theta}$ would greatly decrease the error in the pose estimation. In many cases, the control inputs are unknown to an external observer. For this reason, the control variables $\dot{v}$ and $\dot{\theta}$ are modeled with the zero-mean white noises $\xi_v(t)$ and ξ_θ, respectively. The measurements provided by the camera are absolute position measurements x_m and y_m assumed to include the zero-mean Gaussian-distributed noises $w_x \sim N(0, W_x)$ and $w_y \sim N(0, W_y)$. This measurement model is defined as:

$$Z(t) = \begin{bmatrix} x_m(t) \\ y_m(t) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} X(t) + \begin{bmatrix} w_x \\ w_y \end{bmatrix} \tag{3}$$

We utilize stochastic signals as a general model of the unknown control signals. The advantage of modeling the unknown control signals as stochastic signals is that we can exploit the Kalman filter theory to estimate the unknown speed v and angle θ based only on the position measurements x_m and y_m.

Estimate the velocity $v(k)$ and angle $\theta(k)$ of the robot from the movie **"Final.mov"** using the EKF and second order filter (SOF) based on the discrete-time model of (1)-(3). The angle θ should be estimated in the range $(-\pi, \pi]$. The data for your estimations are the coordinates of the robot's center $x_m(k)$ and $y_m(k)$. They are converted in *cm* and provided in **"XYData_cm.csv"** for every 10th frame of the movie. The movie has been recorded with the rate of 30 frames per second. The robot's center was computed as the center of the equilateral triangle formed by the three lights (red and two green). Compare results of the EKF and SOF.

Obviously, the triangle can help you estimate the angle $\theta(k)$ without any filter; let us call that estimation $\alpha(k)$. The angle $\alpha(k)$ is provided in **"HeadingAngle_rad.csv"** and can be considered as a true value of the angle $\theta(k)$.

Compare the value of α with $\hat{\theta}$, which is the angle estimation resulting from your filters. You will likely need to go through the process of filter designs and comparison a couple of times until you find that the filters produce reasonable results.

Write a several-page report in which you will explain your derivations and choices for the initial state, all covariance matrices, and plot representative diagrams, including the data extracted from the movie.

Remark: The data are derived from the movie based on the following: (1) the image resolution is 320x240; (2) the robot's size is about 70mm in diameter; (3) the distance from the front light to the both back lights is 61mm; and (4) the distance between the two back lights is 46mm. While you do not need to extract the data from the movie, you still need these details to figure out what the measurement noise in measuring the robot's position is. You will likely need to try various values for the process noise intensity.