CMPE245: Final Exam, Project 2

Note: Your project report should include illustrative plots and discussion about the design of your Kalman filter and results.

The topic of this project is a miniature quadrotor. A simple dynamics of the quadrotor is

$$\begin{aligned}
\ddot{x} &= \{c\} \cdot g(\sin\psi \sin\phi + \cos\psi \cos\phi \sin\theta) & (1)\\
\ddot{y} &= \{c\} \cdot g(\sin\psi \cos\phi \sin\theta - \cos\psi \sin\phi) & (2)\\
\ddot{z} &= \{c\} \cdot g \cos\phi \cos\theta - g & (3)
\end{aligned}$$

where the terms in the braces are unknown values. The parameter g is the gravitational acceleration ($9.81 kgm/s^2$) and θ, ϕ, ψ are the pitch, roll and yaw angles, respectively. A simple feedback control loop, which is based on the model with $c = 1$, is closed. The role of the loop is to control the quadrotor's pitch and roll, which are provided as references for the quadrotor's on-board pitch- and roll-hold controllers with the goal to keep the quadrotor over a certain location. The feedback control is closed based on the noisy measurements

$$x_m(t) = x(t) + n_x(t), \quad y_m(t) = y(t) + n_y(t) \tag{4}$$

$$z_m(t) = z(t) + n_z(t), \quad \psi_m(t) = \psi(t) + n_\psi(t) \tag{5}$$

The loop assumes that the position and velocity measurements are provided. While we take the position measurements (x_m, y_m) from the motion capturing system, the velocities $\dot{x}$ and $\dot{y}$, as well as the yaw angle ψ, have to be estimated. The yaw angle is not a control variable, but its value is necessary for the feedback loop. The yaw angle measurement is noisy and drifts in time, therefore we will model it as

$$\dot{\psi} = \xi_\psi \tag{6}$$

(**Part 1**) Design the extended Kalman filter for the model assuming that the on-board pitch- and roll-hold feedback controllers track the reference values well. Test your filter using the data from *data3D.mat*.

(**Part 2**) In the literature, it can be found that the motor thrust $c = (0.000409s^2 + 0.1405s - 0.099)/c_0$ where $s = 255/6000 \cdot (thrust)$. Include the formula for c in your Kalman filter and find the best value for the parameter c_0 using the average sum of square of the Kalman filter residuals

$$S(c_0) = \frac{1}{N} \sum_{i=1^N} (x_m(k) - x^-(k))^2 + (y_m(k) - y^-(k))^2 + (z_m(k) - z^-(k))^2 \tag{7}$$

(**Part 3**) Consider only the $x - y$ dynamics and measurements from (1)-(5) and redesign the Kalman filter to estimate only x, y and ψ from the measurements that are provided in the files *dataSet1.mat*, *dataSet2.mat* and *dataSet3.mat* and recorded with a sample time of $T_s = 0.1s$. However, the files include corrupted measurements of ψ and your Kalman filter should be able to detect these and be robust to them.