CMPE 245 Homework1 04/06/2016

Chien-Pin Chen

(a) A simple nonlinear model of pendulum dynamics

Given equation:

$$mL\ddot{x} = -mg\sin x$$

And, above equation can be rewritten as:

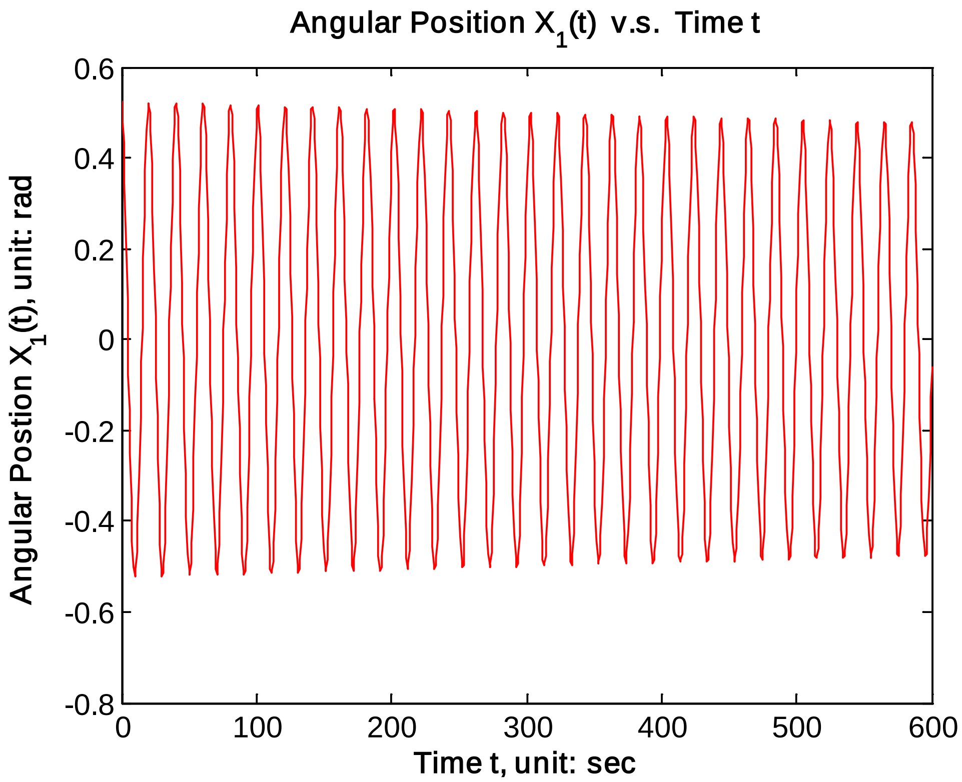
$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -\frac{g}{L}\sin x_1, \quad \text{where g/L=0.1}$$

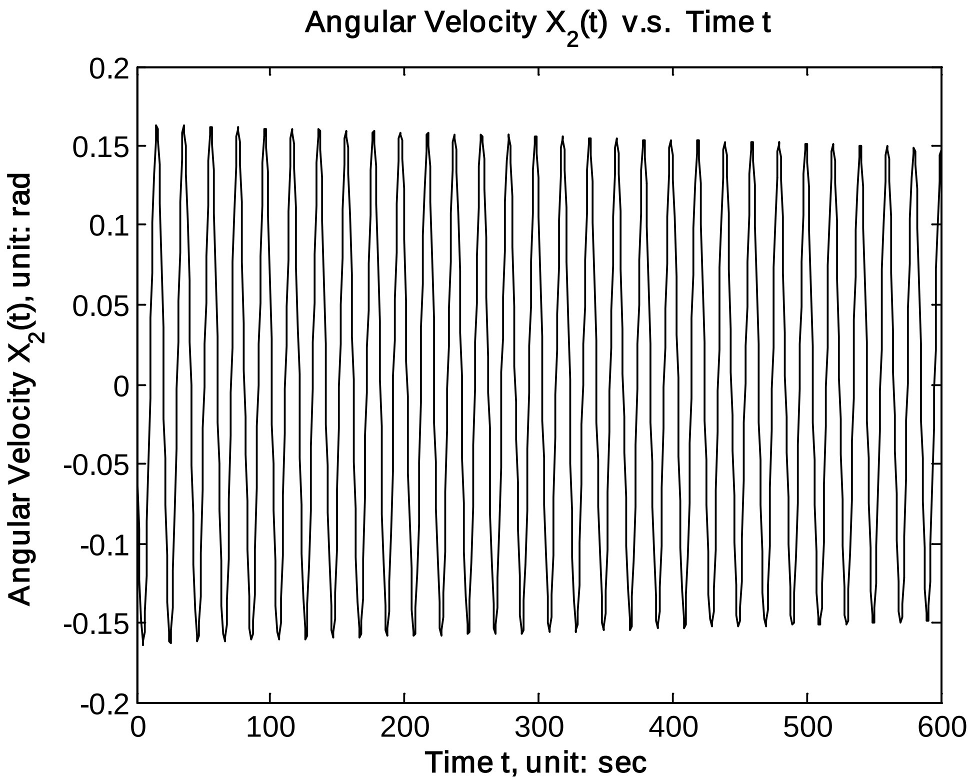
$$\text{Initial Condition: } x_1 = \pi/6,\ x_2 = 0$$

I use MATLAB function ode23 to solve above equations for 600 second. The following are the figure of results:

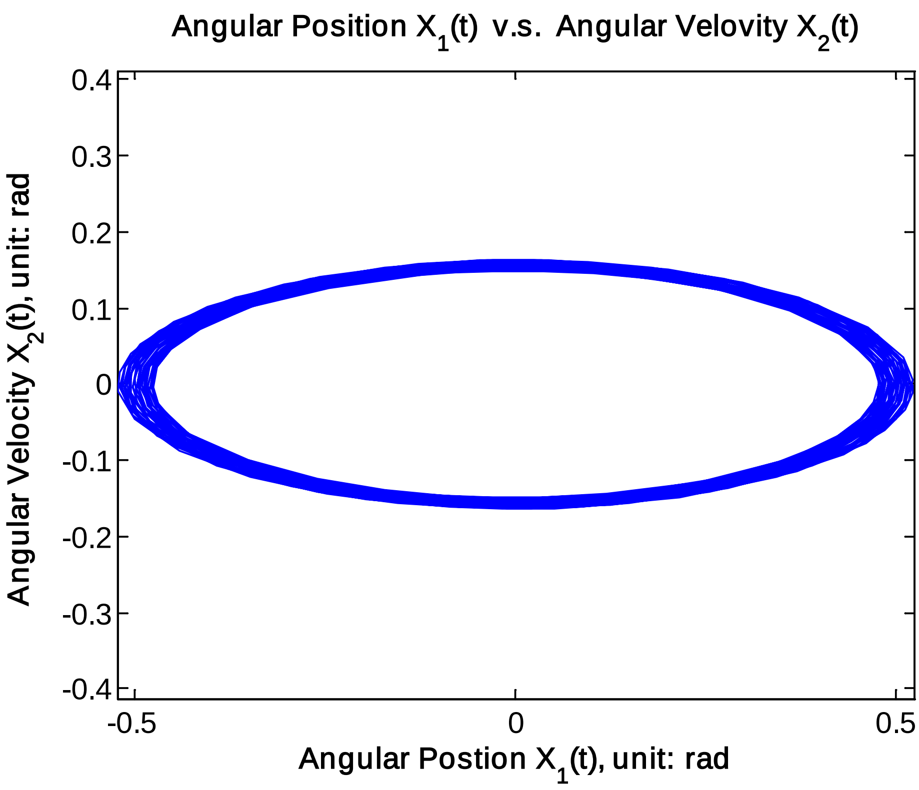
For $x_1(t)$



For $x_2(t)$



For $x_1(t)$ vs. $x_2(t)$



From the result of x1(t) and x2(t), the pendulum just periodical oscillate, but the amplitude of oscillation will become smaller if the time of simulation is long enough. From the result of x1(t) vs. x2(t), I find that x2 (angular velocity) will have maximum value when x1 (angular position) is zero. And, x1 (angular position) will have maximum value when x1 (angular velocity) is zero.

(b) Previous model and add an external force f(t)

Then original equations become:

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -\frac{g}{L}\sin x_1 + f(t), \quad \text{where g/L=0.1}$$

$$\text{Initial Condition: } x_1 = \pi/6,\ x_2 = 0$$

And, the probability density function of time intervals (describe when external force f(t)) is:

$$p(t_k - t_{k-1}) = \lambda e^{-\lambda(t_k - t_{k-1})}, \quad \lambda = 0.02, t_k > t_{k-1}, t_0 = 0$$

f(t) is random external force, which is described as:

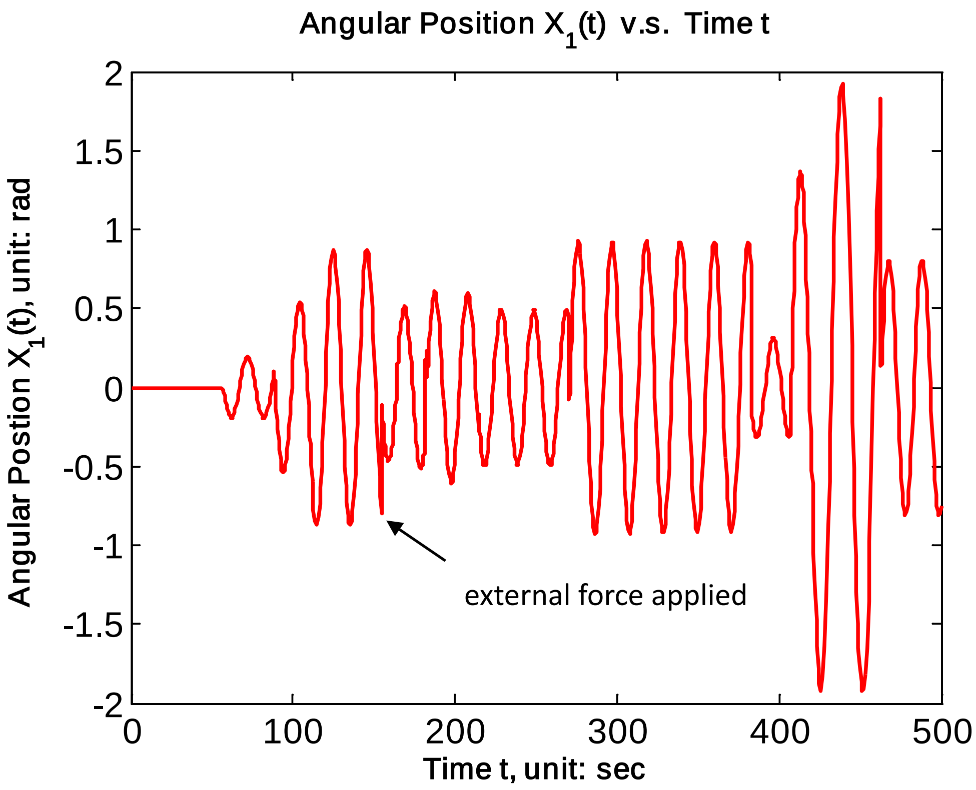
$$f(t) = \sum_k f(t_k)\delta(t - t_k), t_k$$

where

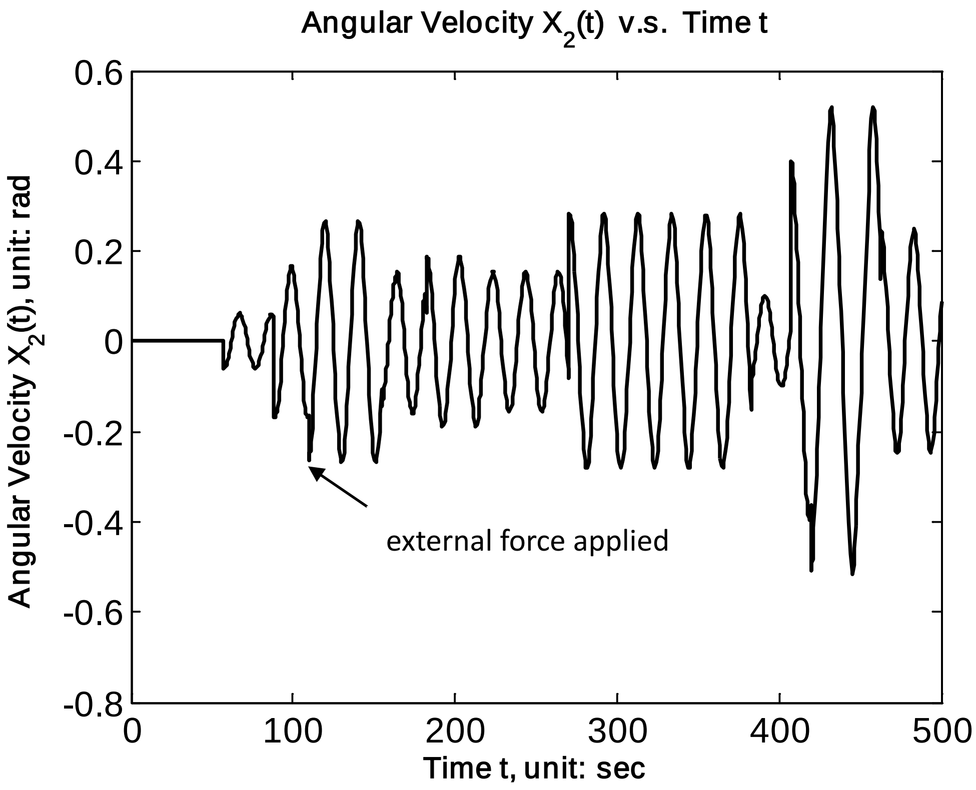
$$f(t_k) = \begin{cases} 0, & t \neq t_k \\ \varepsilon \sim \mathrm{N}(0, \sigma = 0.2), & t = t_k \end{cases}$$

I use MATLAB function ode45 to solve above equations for 500 second.
The following are the figure of results:

For $x_1(t)$

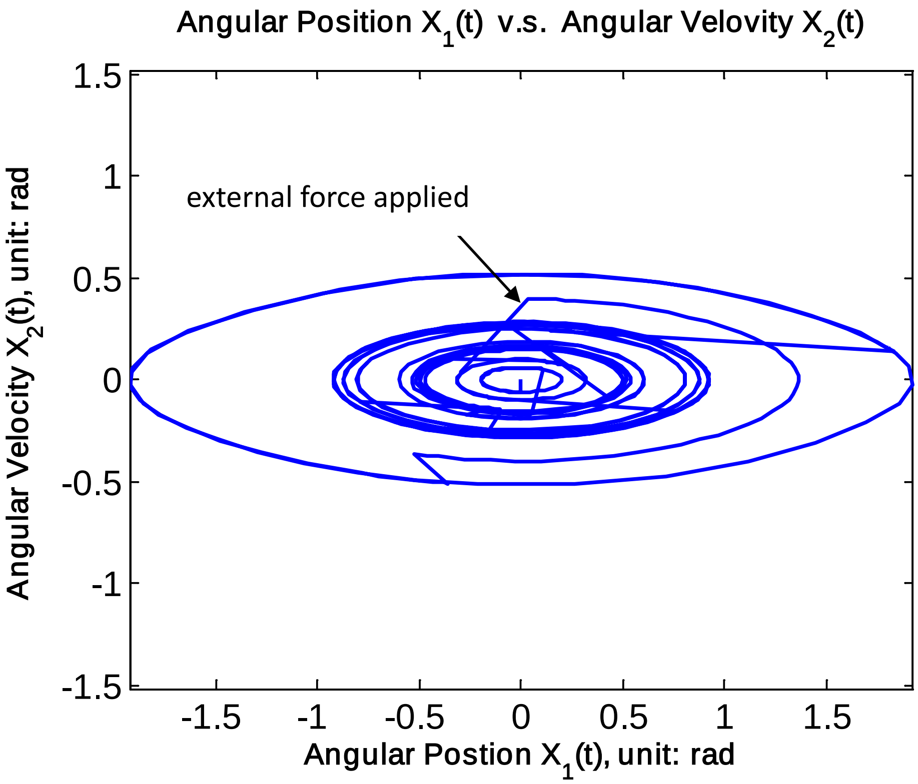


For $x_2(t)$



From the result of x1(t) and x2(t), the pendulum will oscillate periodically when there is no external force hit on it. When external force is applied on the pendulum the amplitude will change depend on the value of external force.

For $x_1(t)$ vs. $x_2(t)$



From the result of x1 vs. x2, when external force occurred, there are straight lines show up in the plot.