Homework 7

(Due on: May 27, 9:00PM)

The HW6 figure shows a robot moving left. While moving, it measures each second ($\Delta T = 1$s) the distance from the obstacle behind it (the corner) and the obstacle in front of it (the wall). In this scenario, we consider the corner as the reference point, i.e., with the coordinate 0. As shown in class, in order to simultaneously estimate the robot position and the position of the wall, we use the state vector $\underline{x}$ with the following three components: x_1 is the robot position measured from the corner, x_2 is the robot velocity with the positive direction towards the wall and x_3 is the wall position measured from the corner. The measurement y_1 is the distance from the robot to the corner and y_2 is the distance from the robot to the wall. A linear model describing this scenario is

$$\underline{x}(k+1) = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \underline{x}(k) + \begin{bmatrix} 0 \\ 0.1 \\ 0 \end{bmatrix} w(k) \tag{1}$$

where the Gaussian random variable w(k) has zero mean and variance 1. The observation model is

$$\underline{y}(k) = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & 1 \end{bmatrix} \underline{x}(k) + \underline{\theta}(k) \tag{2}$$

where the covariance matrix of Gaussian measurement noise $\underline{\theta}$ is $diag(10, 10)$. All variables in the above equations are expressed in cm and $cm = s$.

The course webpage (HW6) provides you the sequence of the measurements y_1 and y_2 recorded for $k = 1, 2, ...$ in the fille roboMes.mat. The data are organized so that $y(1, k)$ denotes the measurement $y_1(k)$ and $y(2, k)$ denotes the measurement $y_2(k)$, where $k = 1, 2, 3.....$

Design the Kalman smoother that takes the measurement sequences and produces the estimation sequences of the robot position, its velocity and the distance between the wall and the corner given all available data.

a) Plot on the same diagram the result of the forward Kalman filter and RTS backward iterations for **the robot position**.

b) Plot on the same diagram the result of the forward Kalman filter and RTS backward iterations for **the robot velocity**.

d) Plot on the same diagram the result of the forward Kalman filter and RTS backward iterations for **the wall position**.

e) For the robot position, compare the variance resulting from the Kalman filter and the variance resulting from the smoother.

Comment your results and trends in data. **Note:** Please keep in mind that you can use your Kalman filter code from HW6 with a slight modification, i.e., after removing 0.8 in the equation defining the velocity. Choose your initial condition for $x_1(0)$ and $x_3(0)$, as well as corresponding variances the same way you did for the Kalman filter in HW6. **However, for $x_2(0)$ use the initial condition 0.2 or 1.6 and the variance 1.**