Explanation Math of Part 2 of day 13 of the advent of code 2020

Robin Camarasa

No Institute Given

A full description of the problem is given at the following **link**. (To access part one you must finish part one first)

In a math way, the problem can be defined as follow:

$$\begin{aligned}
t &\equiv 0 \mod 13 \\
t+3 &\equiv 0 \mod 41 \\
t+13 &\equiv 0 \mod 641 \\
t+25 &\equiv 0 \mod 19 \\
t+30 &\equiv 0 \mod 17 \\
t+42 &\equiv 0 \mod 29 \\
t+44 &\equiv 0 \mod 661 \\
t+50 &\equiv 0 \mod 37 \\
t+67 &\equiv 0 \mod 23
\end{aligned} \tag{1}$$

From these relations one can derive that, $t \equiv 0 \mod 13 \iff \exists q \in \mathbb{Z} | t = 13q$ and that

$$\begin{aligned}
&13q \equiv -30 \equiv -13 \mod 17 \\
&13q \equiv -13 \mod 29 \\
&13q \equiv -13 \mod 37 \\
&13q \equiv -13 \mod 641
\end{aligned} \tag{2}$$

We can modify those relations as follow:

$$\begin{aligned}
13(q+1) &\equiv 0 \mod 17 \\
13(q+1) &\equiv 0 \mod 29 \\
13(q+1) &\equiv 0 \mod 37 \\
13(q+1) &\equiv 0 \mod 641
\end{aligned} \tag{3}$$

13, 17, 29, 37 and 641 being distinct primes implies that:

$$\exists q' \in \mathbb{Z} | q+1 = 17 \times 37 \times 29 \times 641q'$$

.

And $t = 13 \times 17 \times 37 \times 29 \times 641q' - 13$. Looping over the values of t gives the answer in less than 3 seconds on a laptop.