Competitive Programming Midterm, B609/B392, Fall 2022 (90 points)

For each of the problems in this exam you will find test data in the *midterm* directory on the Canvas site. Those data will include both an input file and an output file for each problem. You should write a general algorithm suitable for all possible test cases described in the problem. Your answers should solve all of the test cases given in the input file, though not in a way that is specific to the particular inputs given. Your answers will be graded considering both the results on the test data as well as the code itself.

1. (30 points) You are given n positive integers $a_1, a_2, \ldots, a_n$ where $n \leq 100$ and an integer target, t. You want to form the minimal sum over some subset of the $\{a_i\}$ that exceeds or equals a target, t. You can assume that $t < 10,000$ and $a_i < 10,000$ for $i = 1, \ldots, n$.

   Input The first input line will give the number of test cases. For each test case the first line will give t, the 2nd line will given n, followed by n lines giving the $a_1, \ldots, a_n$.

   Output Print a single line for each test case containing two numbers: the smallest sum greater than or equal to the target, and the number of terms involved in the sum. If there are several possible ways of creating your output sum, your 2nd output number should be the smallest number of terms that can generate the sum.

2. (30 points) You are given $A_0 \subseteq \{0, \ldots, n-1\}$ where $n \leq 10$. You want to generate subsets $A_1, \ldots, A_{2^n}$ so that each subset A_i either adds/subtracts a single element to/from A_{i-1}, and no two of the subsets can be the same.

   Input The input begins with the number of test cases on a single line. A test case is composed of a pair of numbers on a line, n, k, where n is as above and k is the number of elements of A_0. The next line contains the elements of A_0 separated by spaces.

   Output For each test case generate 2^n lines with the ith line, $i = 0, \ldots, 2^n - 1$, giving i, followed by a colon, followed by the elements of A_i in increasing order, separated by spaces. A test case should be followed by a blank line. There may be several possible answers for some test cases, so your output needn't match the output on Canvas, but should match the problem statement.

3. (30 points) Consider the alphabetical listing of all n-length strings of letters from $\{A, \ldots, Z\}$ composed of *distinct* letters. Given such a string you would like to say where it appears in the alphabetic ordering, with the first string numbered as 0.

   Input The input begins with the number of test cases on a single line. Each test case is a single line giving a string with length $n \leq 26$.

   Output For each test case print a single line of output giving the position of the given string in the alphabetically ordered lexicon.