MEEN 644 – Numerical Heat Transfer and Fluid Flow

Spring 2020

Exam #2

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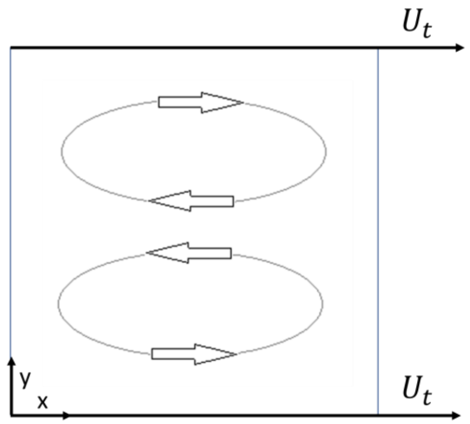
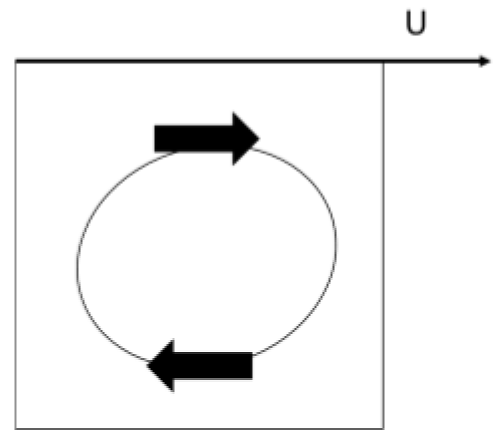
Due Date: April 16, 2020 Maximum Points: 100

A viscous fluid (Water) is trapped in a square 2-D cavity of dimension 0.5 m by 0.5 m. One or two walls are pulled at a uniform velocity.



(1)



(2)

Write a finite volume-based computer program to predict the 2-D steady laminar flow field for Re = 1,000. Solve the velocity and pressure fields by linking them through the SIMPLE algorithm in a staggered grid. Represent solution to the one-dimensional convection-diffusion problem using the power law scheme.

1) In order to verify your code for symmetry, make calculations using 5 x 5 uniformly sized control volumes (CVs). Top and bottom plates pulled right-hand side at constant velocity at Re = 1,000. Declare convergence when (R_U & $R_V < 10^{-6}$) and ($R_P < 10^{-5}$). Print your velocity and pressure fields up to 5 decimal places. (E.g. 9.12345e-6) **(60 points)**

Solution:

The values of the velocities and pressure are tabulated as below:

Table 1 U velocities [m/s] for 5x5 grid size

2.00741E-03	2.00741E-03	2.00741E-03	2.00741E-03	2.00741E-03	2.00741E-03
0.00	9.23042E-05	1.69970E-04	2.05573E-04	1.64756E-04	0.00
0.00	-2.27530E-05	-6.39379E-05	-8.71343E-05	-6.29540E-05	0.00
0.00	-1.39102E-04	-2.12064E-04	-2.36878E-04	-2.03604E-04	0.00
0.00	-2.27531E-05	-6.39380E-05	-8.71344E-05	-6.29540E-05	0.00
0.00	9.23042E-05	1.69970E-04	2.05573E-04	1.64756E-04	0.00
2.00741E-03	2.00741E-03	2.00741E-03	2.00741E-03	2.00741E-03	2.00741E-03

Table 2 V velocities [m/s] for 5x5 grid size

0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	9.23042E-05	7.76658E-05	3.56033E-05	-4.08173E-05	-1.64756E-04	0.00
0.00	6.95511E-05	3.64810E-05	1.24069E-05	-1.66370E-05	-1.01802E-04	0.00
0.00	-6.95512E-05	-3.64810E-05	-1.24069E-05	1.66370E-05	1.01802E-04	0.00
0.00	-9.23042E-05	-7.76658E-05	-3.56033E-05	4.08173E-05	1.64756E-04	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table 3 Pressure [Pa] for 5x5 grid size

0.00	-6.60085E-06	3.76273E-05	5.10205E-05	7.78841E-05	1.39169E-04	0.00
-6.60085E-06	-6.60085E-06	3.76273E-05	5.10205E-05	7.78841E-05	1.39169E-04	1.39169E-04
4.24067E-05	4.24067E-05	3.25790E-05	2.69085E-05	3.07047E-05	6.40303E-05	6.40303E-05
4.78661E-05	4.78661E-05	3.66204E-05	3.17952E-05	4.24411E-05	6.83263E-05	6.83263E-05
4.24067E-05	4.24067E-05	3.25790E-05	2.69085E-05	3.07047E-05	6.40303E-05	6.40303E-05
-6.60084E-06	-6.60084E-06	3.76273E-05	5.10205E-05	7.78841E-05	1.39169E-04	1.39169E-04
0.00	-6.60084E-06	3.76273E-05	5.10205E-05	7.78841E-05	1.39169E-04	0.00

The code is attached in the submission as matlab files. The main file to run is "exam_2.m". The results for the velocity and pressure values for a 5 x 5 grid size are stored in an excel file named "results.xlsx" (sheet 1) after the calculation in the code.

2) With top plates pulled to right-hand side at constant velocity at Re = 1,000, calculate velocity and pressure fields using 16x16, 32x32, 64x64, and 128x128 CVs.

Solution:

The code for the calculation is uploaded.

a) Plot the centerline U and V velocities for each case (For centerline U, plot @ x=0.25m while for centerline V, plot @y = 0.25m). **(20 Points)**

The centerline velocities are plotted in the figure 1 and 2.

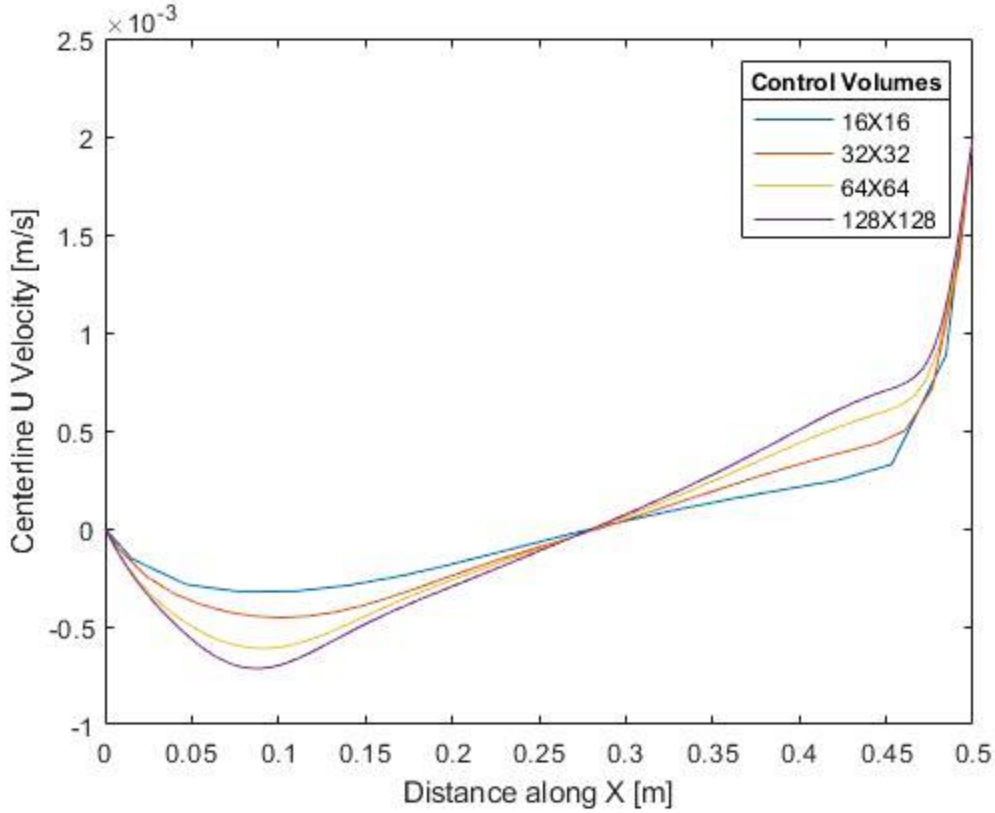


Figure 1 Centerline U velocities for different grid sizes

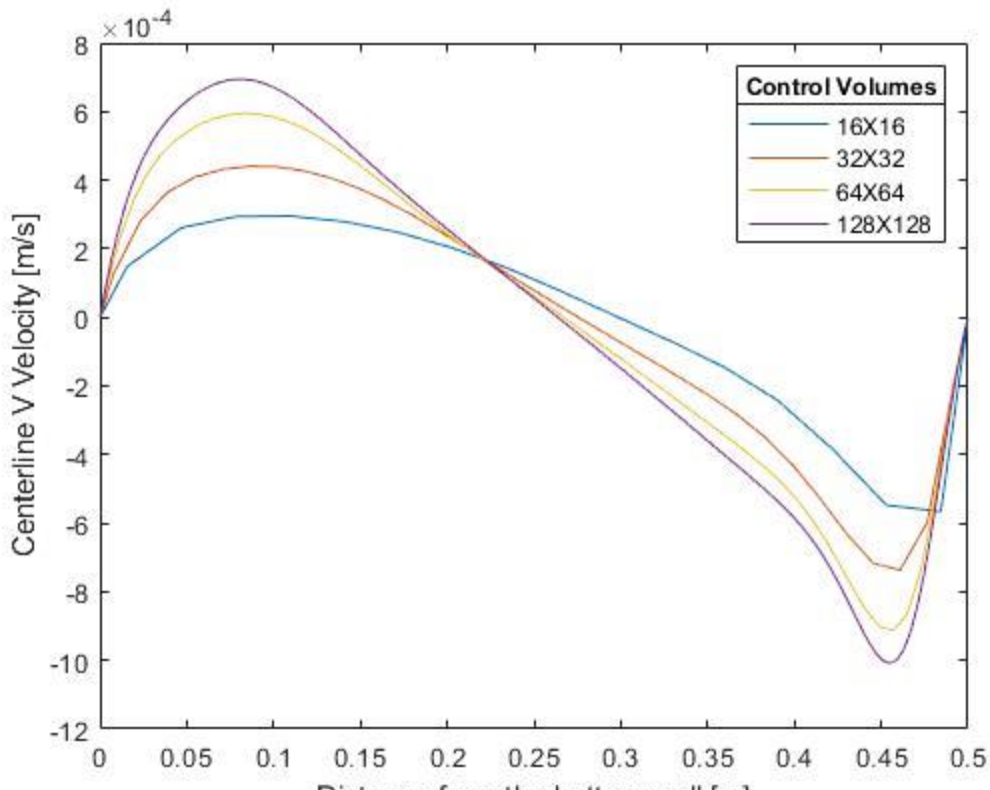


Figure 2 Centerline V velocities for different grid sizes

b) Compare your solutions of 128x128 CV case with the benchmark solution of Roy et al. (2015) on Table 4 and Table 5. (20 **Points)**

The comparison of the computed values with the benchmark solution is presented in the figures 3 and 4. The computed values can be viewed in the excel file results.xlsx in sheet 2.

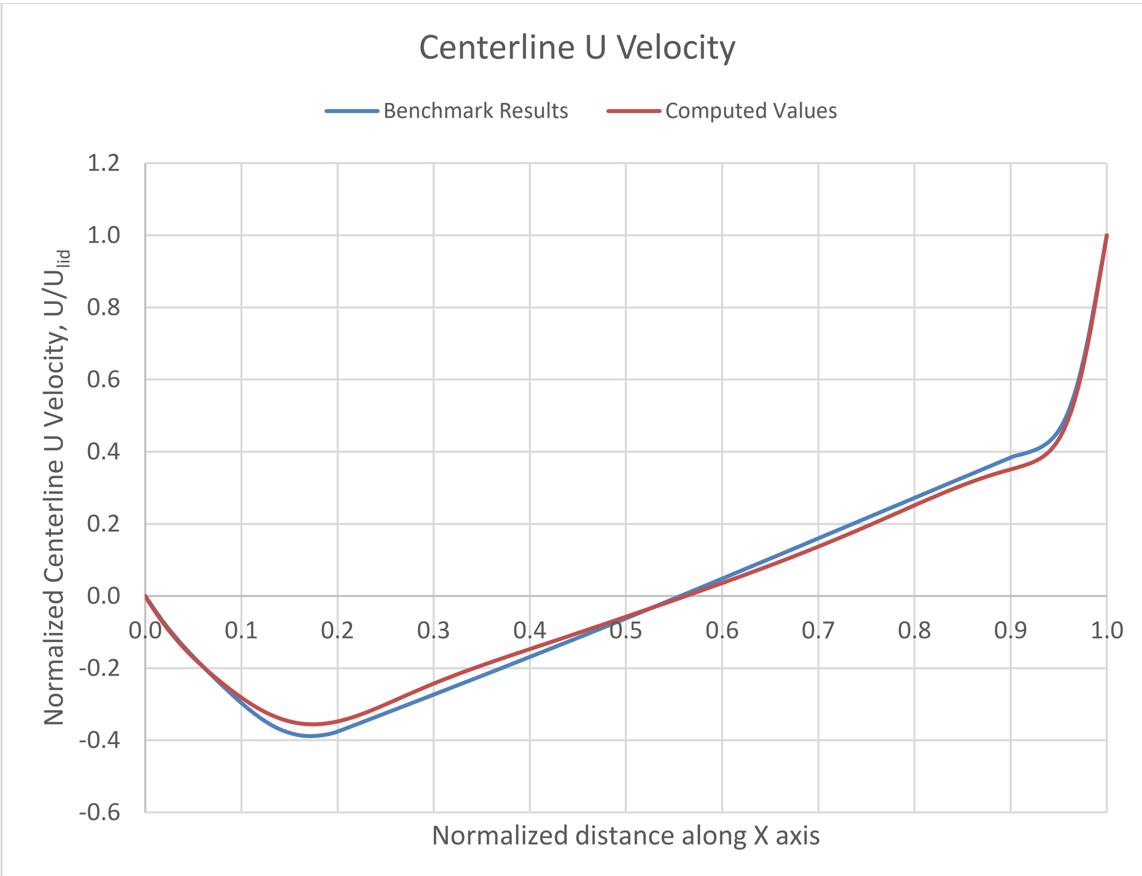


Figure 3 Comparison of computed normalized centerline U velocities with benchmark solution

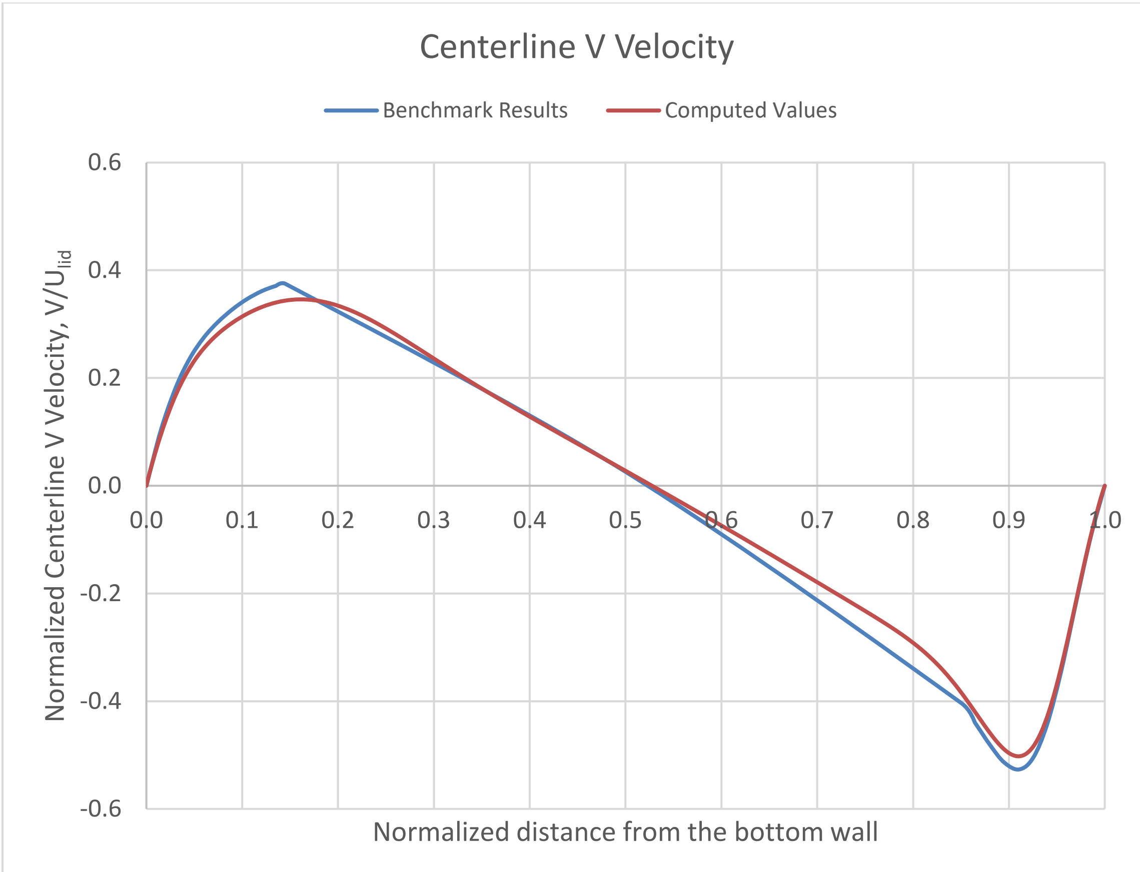


Figure 4 Comparison of computed normalized centerline V velocities with benchmark solution

Definition of Residuals:

$$R_U = \frac{\sum_{node} \left| a_e u_e - \sum a_{nb} u_{nb} - b_u - A_e \left(P_P - P_E \right) \right|}{\sum_{node} \left| a_e u_e \right|}$$

$$R_V = \frac{\sum_{node} \left| a_n v_n - \sum a_{nb} v_{nb} - b_v - A_n \left(P_P - P_N \right) \right|}{\sum_{node} \left| a_n v_n \right|}$$

$$R_P = \frac{\sum_{node} \left| \left(\rho_w u_w - \rho_e u_e \right) dy + \left(\rho_s u_s - \rho_n u_n \right) dx \right|}{\rho u_{ref} L_{ref}}$$

Properties of fluid in the cavity:

$$\begin{aligned} &Water\ @\ 20°C \\ &\rho = 998.3 kg/m^3 \\ &k = 0.609W \\ &\mu = 1.002 \times 10^{-3} N \cdot s/m^2 \\ &C_p = 4.183 kJ/kg \cdot K \end{aligned}$$

Notes:

i. Under-relaxation factor suggestion: $\alpha_U = \alpha_V = 0.5$, $\alpha_P = 0.8$

ii. For Reynolds number and calculation for R_P, use cavity height as characteristic length L_{ref} and top velocity U_t as reference velocity u_{ref}.

iii. For the second part, nondimensionalize centerline velocities by dividing your result by u_{ref}.

Reference

Pratanu Roy, N. K. Anand, Diego Donzis, A Parallel Multigrid Finite-Volume Solver on Collocated Grid for Incompressible Navier-Stokes Equations, *Numerical Heat Transfer*, *Par B: Fundamentals*, **67(5)**, 2015