MEEN 644 – Numerical Heat Transfer and Fluid Flow

Spring 2020

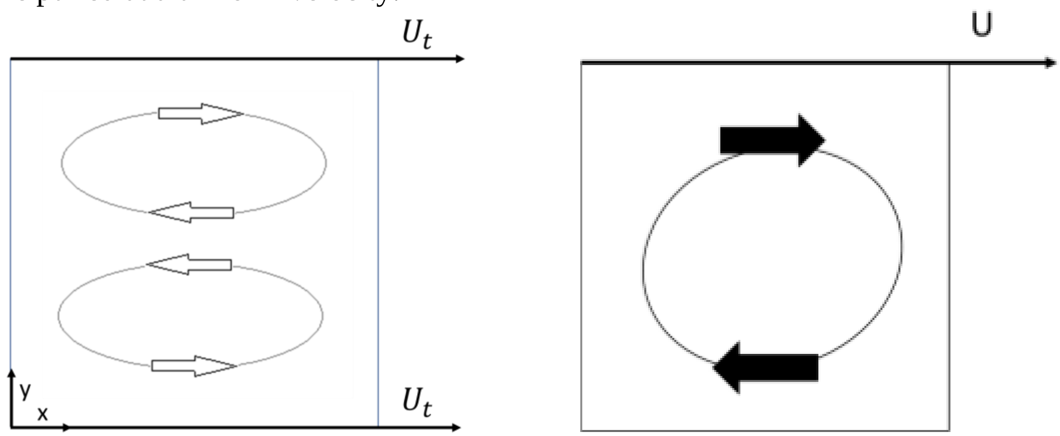
Exam #2

Name ____________________

Instructor: N. K. Anand

Due Date: April 16, 2020 Maximum Points: 100

A viscous fluid (Water) is trapped in a square 2-D cavity of dimension 0.5 m by 0.5 m. One or two walls are pulled at a uniform velocity.



(1) (2)

Write a finite volume-based computer program to predict the 2-D steady laminar flow field for Re = 1,000. Solve the velocity and pressure fields by linking them through the SIMPLE algorithm in a staggered grid. Represent solution to the one-dimensional convection-diffusion problem using the power law scheme.

1) In order to verify your code for symmetry, make calculations using 5 x 5 uniformly sized control volumes (CVs). Top and bottom plates pulled right-hand side at constant velocity at Re = 1,000. Declare convergence when (R_U & $R_V < 10^{-6}$) and ($R_P < 10^{-5}$). Print your velocity and pressure fields up to 5 decimal places. (E.g. 9.12345e-6) **(60 points)**

2) With top plates pulled to right-hand side at constant velocity at Re = 1,000, calculate velocity and pressure fields using 16x16, 32x32, 64x64, and 128x128 CVs.
   a) Plot the centerline U and V velocities for each case (For centerline U, plot @ x=0.25m while for centerline V, plot @y = 0.25m). **(20 Points)**
   b) Compare your solutions of 128x128 CV case with the benchmark solution of Roy et al. (2015) on Table 4 and Table 5. (20 **Points)**

Definition of Residuals:

$$R_U = \frac{\sum_{node}\left|a_e u_e - \sum a_{nb} u_{nb} - b_u - A_e\left(P_P - P_E\right)\right|}{\sum_{node}\left|a_e u_e\right|}$$

$$R_V = \frac{\sum_{node}\left|a_n v_n - \sum a_{nb} v_{nb} - b_v - A_n\left(P_P - P_N\right)\right|}{\sum_{node}\left|a_n v_n\right|}$$

$$R_P = \frac{\sum_{node}\left|\left(\rho_w u_w - \rho_e u_e\right)dy + \left(\rho_s u_s - \rho_n u_n\right)dx\right|}{\rho u_{ref} L_{ref}}$$

Properties of fluid in the cavity:

$$\begin{aligned}
&Water\ @\ 20°C \\
&\rho = 998.3 kg/m^3 \\
&k = 0.609W \\
&\mu = 1.002\times10^{-3} N\cdot s/m^2 \\
&C_p = 4.183 kJ/kg\cdot K
\end{aligned}$$

Notes:

i. Under-relaxation factor suggestion: $\alpha_U = \alpha_V = 0.5$, $\alpha_P = 0.8$
ii. For Reynolds number and calculation for R_P, use cavity height as characteristic length L_{ref} and top velocity U_t as reference velocity u_{ref}.
iii. For the second part, nondimensionalize centerline velocities by dividing your result by u_{ref}.

Reference

Pratanu Roy, N. K. Anand, Diego Donzis, A Parallel Multigrid Finite-Volume Solver on Collocated Grid for Incompressible Navier-Stokes Equations, *Numerical Heat Transfer*, *Par B: Fundamentals*, **67(5)**, 2015