Noncommutative and nonassociative three velocity in special relativity: introducing the gyrogroup package

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Abstract

Here I present the **gyrogroup** package for generalized Bradley-Terry models and give examples from two competetive situations: single scull rowing, and the competitive cooking game show MasterChef Australia. A number of natural statistical hypotheses may be tested straightforwardly using the software.

Keywords: Dirichlet distribution, hyperdirichlet distribution, combinatorics, R, multinomial distribution, constrained optimization, Bradley-Terry.

1. Introduction

2. Introduction

"The nonassociativity of Einstein's velocity addition is not widely known" (Ungar 2006).

In this short vignette I will introduce the **gyrogroup** package which gives functionality for manipulating three-velocities in the context of their being a gyrogroup.

3. The package in use

Ungar (2006) shows that the velocity addition law is

$$\mathbf{u} \oplus \mathbf{v} = \frac{1}{1 + \mathbf{u} \cdot \mathbf{v}} \left\{ \mathbf{u} + \frac{\mathbf{v}}{\gamma_{\mathbf{u}}} + \frac{\gamma_{\mathbf{u}} \left(\mathbf{u} \cdot \mathbf{v}\right) \mathbf{u}}{1 + \gamma_{\mathbf{u}}} \right\} \tag{1}$$

where $\gamma_{\mathbf{u}} = \left(1 - \mathbf{u} \cdot \mathbf{u}\right)^{-1/2}$ and we are assuming $c = 1$. Ungar shows that, in general, $\mathbf{u} \oplus \mathbf{v} \neq \mathbf{v} \oplus \mathbf{u}$ and $(\mathbf{u} \oplus \mathbf{v}) \oplus \mathbf{w} \neq \mathbf{u} \oplus (\mathbf{v} \oplus \mathbf{w})$. He also defines the binary operator $\ominus$ as $\mathbf{u} \ominus \mathbf{v} = \mathbf{u} \oplus (-\mathbf{v})$ (and implicitly defines $\ominus \mathbf{u} \oplus \mathbf{v}$ to be $(-\mathbf{u}) \oplus \mathbf{v}$).

If we have

$$\mathrm{gyr}\left[\mathbf{u}, \mathbf{v}\right] \mathbf{x} = -\left(\mathbf{u} \oplus \mathbf{v}\right) \oplus \left(\mathbf{u} \oplus \left(\mathbf{v} \oplus \mathbf{x}\right)\right) \tag{2}$$

Then Ungar shows that

$$\begin{aligned}
\mathrm{gyr}\left[\mathbf{u},\mathbf{v}\right]\mathbf{x}\cdot\mathrm{gyr}\left[\mathbf{u},\mathbf{v}\right]x &= \mathbf{x}\cdot\mathbf{y} && (3)\\
\mathrm{gyr}\left[\mathbf{u},\mathbf{v}\right](\mathbf{x}\oplus\mathbf{y}) &= \mathrm{gyr}\left[\mathbf{u},\mathbf{v}\right]\mathbf{x}\oplus\mathrm{gyr}\left[\mathbf{u},\mathbf{v}\right]\mathbf{y} && (4)\\
(\mathrm{gyr}\left[\mathbf{u},\mathbf{v}\right])^{-1} &= (\mathrm{gyr}\left[\mathbf{v},\mathbf{u}\right]) && (5)\\
\mathbf{u}\oplus\mathbf{v} &= \mathrm{gyr}\left[\mathbf{u},\mathbf{v}\right](\mathbf{v}\oplus\mathbf{u}) && (6)\\
\mathbf{u}\oplus(\mathbf{v}\oplus\mathbf{w}) &= (\mathbf{u}\oplus\mathbf{v})\oplus\mathrm{gyr}\left[\mathbf{u},\mathbf{v}\right]\mathbf{w} && (7)\\
(\mathbf{u}\oplus\mathbf{v})\oplus\mathbf{w} &= \mathbf{u}\oplus(\mathbf{v}\oplus\mathrm{gyr}\left[\mathbf{v},\mathbf{u}\right]\mathbf{w}) && (8)
\end{aligned}$$

Consider the following R session:

```
> library(gyrogroup)
> u <- as.3vel(c(-0.7,+0.2,-0.3))
> v <- as.3vel(c(+0.3,+0.3,+0.4))
> w <- as.3vel(c(+0.4,-0.3,+0.5))
```

Here we have three-vectors `u`, and `v`.

4. Conclusions

Several generalization of Bradley-Terry strengths are appropriate to describe competitive situations in which order statistics are sufficient.

The package is used to calculate maximum likelihood estimates for generalized Bradley-Terry strengths in two competitive situations: Olympic rowing, and *MasterChef Australia*. The estimates for the competitors' strengths are plausible; and several meaningful statistical hypotheses are assessed quantitatively.

References

Ungar AA (2006). "Thomas precession: a kinematic effect of the algebra of Einstein's velocity addition law. Comments on 'Deriving relativistic momentum and energy : II. Three-dimensional case'." *European Journal of Physics*, **27**, L17–L20.

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