Rebuttal to second editorial review: overall response

(Below, the reviewer's comments are in black, and my replies to the issues are in blue. I have indicated changes to the manuscript where appropriate.

Short story: I have accommodated all the comments with rewording. The resulting document is, I believe, stronger and more scholarly than before and I recommend it to you.

Detailed rebuttal: Reviewer #1

This short article is a tutorial example of how to use Inkscape and KnotR to beautify 2-dimensional knot diagrams found in the knot tables. The cost function used in optimizing a given diagram tries to minimize the overall bending energy of the knot curve, preserve all shown 2D symmetries, and render all strand crossings as close to 90 degrees as possible. The paper does not discuss the underlying mathematics by which this optimization is achieved, but relies on the power built into the software packages mentioned. Also, the user has to characterize explicitly the desired symmetries; there is no "magic" power that discovers the maximal possible symmetry that a given knot can assume. This exemplified by Figure 7, where the Figure-8-knot is drawn with a single mirror axis, even though this same knot can readily be drawn with two mirror lines; – and also in Figure 9 where a symmetrical and an asymmetrical rendering of the same knot is presented.

This is a good characterisation of the submission, but the comment that there is no "magic" power ascertaining maximal possible symmetry that a given knot may assume is a good one, and one that I had not explicitly realised. I have added a brief discussion of this issue to the caption of Figure 9 that discusses the Perko pair:

OLD text "Two representations of knot 10_{125}, known as the Perko Pair"

NEW text "Two representations of knot 10_{125}, known as the Perko Pair. The software requires the user to specify the symmetry (mirror or rotational) of a knot projecton and has no notion of topological invariance of a knot"

The referee observes that 4_1 has a symmetrical rendering with two mirror lines and this is available in the package, as object `k4_1a`. I have added a description of this to the manuscript.

This paper is clearly useful to someone who specifically wants to clean up a given rendering of a particular knot, but it offers little to readers who are not familiar with Inkscape and KnotR.

In any software, one makes use of previously written work. One rarely writes an operating system from scratch, for example. The knotR package leverages the design capabilities of inkscape in, I think, a useful and informative way; and it further uses the numerical abilities of R in an efficient and natural way.

Detailed comments about the presentation:

Abstract: It should mention the software packages that are used in this paper.

Done. The abstract is now completely rewritten.

P1, line 46: "Knot theory" – ? – Is this an incomplete reference?

Corrected, this was a typo

P2, line 43: "uniform" curvature is not possible, since many knots have inflection points. It is better to ask for 'smoothly changing' curvature, and to limit maximal curvature.

Corrected:

OLD text Curvature to be as uniform as possible

NEW text Curvature to be as smoothly changing as possible, with limited maximal curvature

P2, line 46: What does it mean for symmetry to be "present" in the knot?

OLD text Any symmetry present in the knot should be enforced exactly, and be visually apparent

NEW text Any symmetry desired in the knot should be enforced exactly, and be visually apparent

P3, line 31: Define "visual continuity". Are you referring to tangent continuity, $G1-$, or $G2-$ continuity?

The path does not have G2 continuity because the radius of curvature has a discontiniuity at Bezier nodes. But the change of curvature should be small, I have added a short discussion to clarify the issue.

P3, line 33: The “understrand” has not been designated yet.

fixed

Figure 2: Specify what order Bezier curves are being used.

fixed

P4, line 18: A gentle introduction to “R” would help to make the following easier to understand.

Fixed, a short introductory passage and a reference to R core now added

P4, lines 27-32: What is the meaning of this? - Six points defining two handle pairs? (Why the extreme precision with 8 digits?)

Only the first six lines are shown for brevity, I have changed the text accordingly: “Above we see only the first six lines of the object”

Figure 3: What is the color assignment? - one color for each Bezier segment? How many circles are drawn for each segment? (Say that solid blotches result from densely overlapping circles.)

The original caption was lacking detail.

OLD text “The *path* of (unoptimized) knot 7_6, showing Bezier handles as thin straight lines and circles. The coloured circles have a radius proportional to the curvature (that is, the reciprocal of the radius of curvature) along the strand; note large curvature at loop on left”

NEW text “The *path* of (unoptimized) knot 7_6, showing Bezier handles as thin straight lines and circles. The coloured circles have a radius proportional to the curvature (that is, the reciprocal of the radius of curvature) along the strand. Colouring is arbitrary, one color for each Bezier segment; solid blotches result from densely overlapping circles. Note large curvature at loop on left”

Each segment has one circle per control point (which defaults to $n = 100$) but this is user-settable.

Figure 4: Why is this required? – Does the user have to specify the type of crossing via an ”overunder” object? – (Lines 53-62?)

The `overunderobject` specifies which of two strands at a crossing is the overstrand and which the understand. I have added a definition of this and a brief discussion in the figure caption.

P10, line 17: “... a vertical line of symmetry.” - Better to say that this figure has D_5 symmetry with five mirror lines.

Done.

P11, line 26: It seems that all the functionality needed to deal with links is already present in the software packages mentioned, as long as one can let the program go around more than one single closed loop.

I have spent many many hours pondering how to deal with links. As the referee says, much of the required functionality is (in principle) already present in the software. Each component would have its own badness, and in addition there would be $n(n-1)/2$ inter-component interaction terms measuring features such as closeness between non-intersecting strands of different components. Several problems stood out: firstly, the problem of inter-component intersection points. Currently this is a (symmetrical) Boolean matrix with rows and columns corresponding to strands, and entry (i,j) being whether or not strand i intersects with strand j. The link generalization would be a symmetric matrix of intersection matrices: an object with four indices and entry (i,j,k,l) indicating whether strand j of component i intersects strand k of component l. I found this object to be unwieldy to work with and difficult to manipulate. The second problem was the imposition of symmetry. Sometimes one might desire that some components of a link have a particular kind of symmetry and others to have a different kind (or no!) symmetry. One unexpectedly difficult problem was that one might have a subset of links that individually possess no symmetry but collectively possess mirror-, rotational-, or indeed dihedral- symmetry. Thistlethwaite's L4a1 is problematic: one would expect the two components to be identical except for a 90-degree rotation, and I was unable to implement this in the context of the package. L6a3 is similarly difficult. We would need new badnesses too. Consider L6a4; the smaller component should be not only as circular as possible (?), but also one might desire the overall length to be smaller than that of the other component.

My considered view is that dealing with links would be possible in principle but, due to a number of non-interesting technical reasons (which are not really suitable for inclusion in a scholarly publication), an order of magnitude harder than the single-component links considered in the submission. I'm not saying it's impossible, but just very difficult and, for me, firmly in the category of "further work". I hope that's OK.

Fig. 10: Why is the absence of mirror symmetry pointed out here?

Some knots have dihedral symmetry but not this one. Discussion added.

Fig. 11: Why is this table shown? It is not mentioned in the main text. The figures are "better" than in Figure 1, but still not optimal in the spirit of this paper. "Rolfsen" needs a reference.

Rolfsen now cited, and a discussion of the table now given to place it in context.

References to some of the background material are somewhat random. E.g. For JMA readers, "The Knot Book" by Colin Adams would be a good introductory reference.

Ref [10] Give a more up-to-date access date.

Ref [11] Give URL of Inkscape: `https://inkscape.org/`

Ref [14] Give URL of R Core Team: `https://www.r-project.org/`

All amended as suggested

Detailed rebuttal: Reviewer #2

This lovely article addresses the interesting question of how to produce pleasing and informative images of two dimensional knot diagrams. In addition to the discussion it describes how to use a combination of the software packages Inkscape and R to optimise knots.

At the moment, however the paper is a little too close to simply being a software manual for the software, and needs in particular to describe the techniques used by the software to create the optimised images.

I think this is a perfectly reasonable comment for the original manuscript. I believe the revision is a very much improved document.

In addition there is a rather consistent use of absolute terms for pleasing vs ugly knots. Aesthetics is a complex field, but generally does not provide strong answers.

Absolute terms now rephrased more carefully.

While the choices made in this paper are reasonable, they are simply stated, rather than including a justification for why they are chosen. They certainly only apply to a particular knid of knot diagram. This can be see in the final image showing all the knots with up to 8 crossings. While some of the resulting diagrams are beautiful, I particularly enjoyed `8_10` and `8_17`, some are not so pleasing to me. In particular the algorithm given likes to over-tighten when the line wraps round itself. This can be seen particularly in `8_3`, but also in `8_5`, `8_6` and `8_11`. This is itself a personal preference not an absolute, but a discussion of how to adjust the set-up of the software to make personal aesthetic choices would strengthen the paper.

This is indeed an insightful comment. My original view was that there was a single set of weightings for the various badnesses, that most people would agree upon; and my task as software engineer was to find the projection that optimizes these standard weightings.

Reflecting on the referee's comment, I realise with a shock that my weightings do not reflect some universal ideal. Rather, my choice of weightings are as subjective as any other aspect of the software and, as such, are peculiar to me. This is obvious in retrospect but was not clear to me at all until just now.

One of the benefits of the `knotR` software is that it allows this kind of conversation to occur; I have added a discussion to the manuscript, just before the conclusions section. There, I present projections of 8_3 but with different penalties for angle crossing terms. We see the results and can compare like with like, perhaps coming to a deeper understanding of pleasingness of projections in the process.

The paper also needs to do a far stronger job of placing itself within the relevant literature. Simply showing that a single diagram from wikipedia has weaknesses is not sufficient. The nineteenth century series of papers "On Knots" by Peter Guthrie Tait in the Transactions of the Royal Society of Edinburgh contain far superior images for example. This is not the first paper to look at making pleasing knot diagrams, the software knotplot for example offers many tools to do a similar optimisation to the one discussed here, but is not cited, Laura Taalman and Colin Abrams among others have multiple papers addressing the creation of different knot representations and optimising their form. More broadly the idea of using optimisation in mathematical art has been used widely, the work of Robert Bosch and Craig Kaplan springing immediately to mind.

For these reasons the current paper is not really strong enough for publication. In order to get strong enough I would propose three major pieces of work:

1) Conduct a thorough literature review of work on the creation of knot diagrams and their aesthetics and place this paper into that context as well as the broader context of optimisation in art.

I have added a literature review as suggested that discusses knot diagrams from early Celtic artwork through to mathematica.

The referee makes the perfectly reasonable suggestion of discussing the broader context of optimization of art. I will address this comment colloquially. It is obvious to me, and clearly to the referee as well, that numerical optimization techniques are well-suited to "rounding off rough edges" from a wide range of applications; and further that numerical optimization will produce "nice looking" results. I have cited some of the more prominent workers in the field, whose work is closest to 2D knot projections.

2) Include a discussion of why the aesthetic choices made for the optimisation are pleasing as well as how to adjust the methodology in order to make different personal choices. It might even be interesting to develop some distinct styles of diagrams (by changing the "badness" function, and use the methods to create optimised images for each style.

I have included a figure that compares a knot with two different weightings

3) Give basic information about the specific methods being used for the optimisation rather than just using the function given in R as a black box.

Brief discussion added: the package includes Nelder-Mead by default but the user has the option to use a Newton-type method.

With these changes the paper could be an interesting one, discussing the context of knot diagrams more broadly and providing practical help to help create sets for oneself.