

# Black-Box Optimization Benchmarking of the Multiobjective Optimizer adaptive IBEA ( $\epsilon$ -indicator) with the COCO platform

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## ABSTRACT

This paper discusses the implementation of a multiobjective evolutionary algorithm (MOEA). We implemented the Multiobjective Optimizer IBEA (indicator-based evolutionary algorithm) with  $\epsilon$ -indicator [9] in Python and benchmarked it using the COCO platform[7]. In addition, we compared the results obtained with this algorithm with those of NSGA-II and Random Search.

## KEYWORDS

Evolutionary algorithm, Benchmarking, Black-box optimization, Bi-objective optimization, Decision Vector

## 1 INTRODUCTION

The adaptive IBEA we implemented is dedicated to the following improvements: it requires no diversity preservation techniques within its population evolution mechanism, and it aims at taking into account arbitrary preference information thanks to its  $\epsilon$  additive binary quality indicators. It operates on a set of candidates in a decision space, which will be modified thanks to selection and variation mechanisms.

Those two last steps remind us of biological evolution: selection in our algorithm could be associated with the competition for reproduction and resources in nature, while variation illustrates the ability of existing living creatures to create new living beings thanks to genetic recombination and mutation. Note that due to the randomness of the variation process, some individuals may not be transformed in the evolutionary mechanism.

Since we are in the case of multiobjective optimization, it is possible that several optimal objective vectors co-exist: they would be trade-offs between the different objectives we are pursuing. As we will discover in the algorithm description, our IBEA could, as any general stochastic search algorithm, be divided into three main elements: a working memory (the population of decision space candidates), a selection module, and a variation module. Among the differences with single-objective optimization, the working memory can here consider several solutions at a time. In single-objective optimizatin no mating selection is required and variation translates into modifying the current solution candidate.

### 1.1 Why binary quality indicators ?

Unary quality measures have been proved to be theoretically limited: they do not allow to determine whether a Pareto set approximation is better than another one. This limitation is still valid for a

finite combination of unary quality measures. Binary quality indicators overcome part of the limitations and can, for certain binary indicators, indicate whether a Pareto approximation set is better than another one. [10]

## 2 DESCRIPTION OF ALGORITHM

### 2.1 Description of the objects associated with the algorithm

The algorithm's inputs consist of decision space vectors  $x^i \in X$ , an objective space  $Z$  and objective functions  $f^j : X \rightarrow Z$ . We suppose that  $X \subseteq \mathbb{R}^l$  with  $l \in \{2, 3, 5, 10, 20, 40\}$  and  $Z \subseteq \mathbb{R}^2$ .

The output of a MOEA is a set of incomparable decision vector, meaning that no member of the output set dominates another one. Domination between decision vectors is defined as follows: a decision vector  $x^1$  is said to dominate another decision vector  $x^2$ , written  $x^1 > x^2$ , if  $f_i(x^1) \leq f_i(x^2) \forall i \in \{1, \dots, n\}$  and  $\exists j \in \{1, \dots, n\}$  for which  $f_j(x^1) < f_j(x^2)$ .

This output will be our Pareto set approximation, and the space of Pareto set approximations will be noted as  $\Omega$ . Our algorithm relies on a binary quality indicator  $I : \Omega \times \Omega \rightarrow \mathbb{R}$  which associates a real number to  $k$  Pareto set approximations.

Our binary quality indicator is defined as the minimum distance by which a Pareto set approximation needs to be translated in each dimension in objective space so that another approximation (image of a decision space vector) is weakly dominated. The weak domination is defined as follows: decision vector  $x^1$  weakly dominates  $x^2$ , written  $x^1 \geq x^2$ , if  $x^1$  dominates  $x^2$  or the corresponding objective vectors are equal [9]. The mathematical translation of this definition consists in the following equation:

$$I_{\epsilon^+}(A, B) = \min_{\epsilon} \{ \forall x^2 \in B \exists x^1 \in A : f_i(x^1) - \epsilon \leq f_i(x^2) \text{ for } i \in \{1, \dots, n\} \} \quad (1)$$

### 2.2 Description of the algorithm adaptive IBEA

The algorithm steps are described in pseudo-code 1. The adaptive version of IBEA answers the potential issue of widely spread binary quality indicators values. Too spread out indicators values complicate the task of determining a correct value for  $K$ , the scaling factor associated with our indicator. By adaptavaly scaling the binary quality indicators values back to a common  $[-1; 1]$  interval, we substantially suppress the need to adapt  $K$  to our different problems.

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**Algorithm 1** Adaptive IBEA

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**Input:**

$\alpha$  (population size)  
 $N$  (maximum number of generations)  
 $K$  (fitness scaling factor)

**Output:**

$A$  (Pareto set approximation)

**Step 1. INITIALIZATION**

generate initial population of size  $\alpha$   
set generation counter  $m$  to 0

**end Step****Step 2. FITNESS ASSIGNMENT**

**for all** objective function  $f_i$  **do**  
    lower bound  $b_i = \min_{x \in P} f_i(x)$

    upper bound  $\bar{b}_i = \max_{x \in P} f_i(x)$

**end for**

**for all** objective function  $f_i$  **do**

$$f'_i(x) = \frac{f_i(x) - b_i}{\bar{b}_i - b_i}$$

**end for**

calculate all indicator values  $I(x^1, x^2)$  with  $f'_i$   
determine max. indicator  $c = \max_{x^1, x^2 \in P} |I(x^1, x^2)|$

**for all**  $x^1 \in P$  **do**

$$F(x^1) = \sum_{x^2 \in P \setminus \{x^1\}} e^{-\frac{|I(\{x^1\}, \{x^2\})|}{ck}}$$

**end for****end Step****Step 3. ENVIRONMENTAL SELECTION**

**while** population  $P \geq \alpha$  **do**

    choose  $x^*$  such that  $F(x^*) \leq F(x)$  for all  $x \in P$ .

    remove  $x^*$  from the population

    update remaining individuals fitness values, ie  $\forall x \in P$ :

$$F(x) = F(x) + e^{-\frac{|I(\{x^*\}, \{x\})|}{ck}}$$

**end while****end Step****Step 4. TERMINATION**

If  $m \geq N$  or another stopping criterion then the output  $A$  is defined as the set of nondominated decision vectors in  $P$

**end Step****Step 5. MATING SELECTION**

Binary tournament selection with replacement on  $P$  in order to fill a temporary mating pool  $P'$

**end Step****Step 6. VARIATION**

Apply recombination and mutation operators to  $P'$

Add the resulting offspring to  $P$

Increment the counter ( $m = m + 1$ ) and go to Step 2

**end Step**

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IBEA can be faster than other algorithms since it only compares pair of decision vectors rather than complete approximation sets.

### 2.3 Details regarding mating selection

Mating selection aims at picking promising solutions for variation and usually is performed in a randomized fashion. Perform binary tournament selection with replacement on  $P$  in order to fill a temporary mating pool (the variable  $P'$  in our implementation). The binary tournament consists in keeping the candidate with the best fitness value from the random pick.

### 2.4 Details regarding mutation

The mutation operator modifies individuals by changing small parts in the associated vectors according to a given mutation rate. The mutation rate determines how much of the considered population we will use to generate new decision space vectors. This process reminds how evolutionary algorithms seem related to biological evolution, mimicking natural hereditary relationships. For mutation we use a polynomial mutation operator.

The operations involved in one single mutation are as follows. A random individual is picked from the population  $P$ . According to a uniform probability pick, the mutation is realized using one mathematical transformation applied to its coordinates in the decision space. Note that the transformation changes from one coordinate to another, since the uniform probability pick is repeated for each. In our case, there are two forms of transformations on coordinates. In the following equations,  $u$  is uniformly picked in  $[0, 1]$ ,  $ind[j]$  the  $j^{th}$  coordinate of the already existing picked individual,  $p_{mut}[j]$  the  $j^{th}$  coordinate of the newly generated individual,  $Up$  the biggest existing coordinate value,  $Lo$  the lowest. The two latters are each defined for each decision space dimension.

- if the uniform probability pick  $\leq 0.5$ :

$$\sigma_L = (2u)^{\frac{1}{\mu+1}} - 1 \quad (2)$$

$$p_{mut}[j] = ind[j] + \sigma_L(ind[j] - Lo) \quad (3)$$

- else:

$$\sigma_R = (2(1-u))^{\frac{1}{\mu+1}} \quad (4)$$

$$p_{mut}[j] = ind[j] + \sigma_R(Up - ind[j]) \quad (5)$$

This is implemented thanks to a loop on individuals, with a nested loop treating each decision space dimension coordinate. Mathematical formulas involving hyperparameters are based on [3].

### 2.5 Details regarding recombination

The recombination operator takes a certain number of parents and creates a predefined number of children by combining parts of the parents. For recombination, we use a simulated binary crossover (SBX) operator. To mimic the stochastic nature of evolution, a crossover probability is associated with this operator. In our case, two parents are selected among the current population. A uniform probability pick in  $[0, 1]$  written  $u$  determines the parameter used in computing the features (decision space coordinates) of the children. In the following equations,  $child0 - 1[j]$  is the  $j^{th}$  coordinate of the generated child decision vector,  $parent0 - 1[j]$  the  $j^{th}$  coordinate of the parent associated with the child.

- if the uniform probability pick  $\leq 0.5$ :

$$\beta_q = (2u)^{\frac{1}{\mu+1}} \quad (6)$$

- else:

$$\beta_q = \left(\frac{1}{2(1-u)}\right)^{\frac{1}{\mu+1}} \quad (7)$$

Thanks to this stochastic parameter we can compute the children's coordinates:

- first child:

$$child0[j] = 0.5((1 + \beta_q)parent0[j] + (1 - \beta_q)parent1[j]) \quad (8)$$

- second child:

$$child1[j] = 0.5((1 - \beta_q)parent0[j] + (1 + \beta_q)parent1[j]) \quad (9)$$

Regarding the hyperparameter  $\mu$ , formulas were taken from [4]. In the article, this parameter is referred to as  $\eta_c$ . A large  $\eta_c$  implies an offspring close to the parents in coordinates. For a smaller  $\eta_c$ , children solutions tend to differ more from their parents. This parameter is therefore essential to controlling the spread of the offspring.

### 3 DESCRIPTION OF IMPLEMENTATION

#### 3.1 Asymptotic analysis

In order to run the algorithm in a acceptable time, we needed to analyze the complexity of each step.

First, let  $P$  be the population size,  $d$  the space's dimension,  $I$  the indicator function,  $do$  the tested function,  $r$  the recombination rate, and  $v$  the mutation rate. Furthermore, at the end of a generation, we have a population size of  $O(P(1 + r + v(1 + r))) = O(P)$ , which doesn't change our analysis.

- step 1 :  $O(P \times d)$  to initialize the population
- step 2 :  $O(P^2 \times d)$ . We need  $P^2$  to calculate every  $I(x_1, x_2)$  for the fitness assignment and compute  $c = \max(I)$
- step 3 : if  $P$  is stored in a hashmap, this takes  $O(P)$
- step 4 : done in  $O(1)$
- step 5 : done in  $O(P)$
- step 6 : done in  $O(P \times d)$

We see that the number of chosen elements is, empirically,  $0.56 \times P$ . In fact, the best elements according to fitness values are chosen multiple times. The  $k^{th}$  element has a probability of  $(2(P - k)) + 1)/(P^2)$  to be chosen.

We consider time complexity of  $I_\epsilon$  the same as  $do$  because they are only computed for sets of a single element,  $I_\epsilon(x_1, x_2)$  which equals to computing a max.

#### 3.2 Code acceleration (non asymptotic)

The following improvements allowed us to be globally 10 times faster. With python's profiling tools, we saw that the function which was called the most was by far  $I_\epsilon$ .

Improve its computation time was quite useful. First, we stored every computed value in a hashmap to avoid having to compute it again later. We also hard-coded a max function instead of using those provided by libraries.

In order to further optimize, implemented a specialized function for only two objectives, because multiobjective functions provided

by the COCO test suite are in fact bi-objective.

The objective functions being externally provided, and knowing nothing about their computation time, we decided to call them the least we could, by storing every computed value in a hashmap.

### 4 CPU TIMING

In order to evaluate the CPU timing of the algorithm, we have run the IBEA algorithm with restarts on the entire bbo-biobj test suite [8] for 2D function evaluations according to [7]. The Python code was run on a Intel(R) Core(TM) i7-7500U CPU @ 2.70GHz with a quad core CPU having 16GB of RAM. The options were, for dimensions 2, 3, 5, 10, 20, batch 1 on 3 running alone, then batch 2 and 3 on 3 running at the same time. For 40 dimensions, we ran it alone on the whole test suite.

Parameters for the algorithm were :

- Population size : 100
- Maximum number of generation : 100
- Scaling factor : 0.05
- Mutation rate : 0.01
- Recombination and mutation mu : 1
- Population initialization in range (-5, 5)

Mean times for function evaluation were as described in table 1.

**Table 1: Mean times for function evaluation**

Dimension \ Batch	2	3	5
Batch 1 on 3	6.0e-04	6.3e-04	8.1e-04
Batch 2 and 3 on 3 run	8.6e-04	8.6e-04	9.1e-04
simultaneously	8.3e-04	8.4e-04	8.9e-04
Dimension \ Batch	10	20	
Batch 1 on 3	8.3e-04	1.1e-03	
Batch 2 and 3 on 3 run	1.1e-03	1.3e-03	
simultaneously	1.0e-03	1.3e-03	
Dimension \ Batch	40		
Whole test suite	4.2e-03		

### 5 DISCUSSION OF THE RESULTS

Results from experiments according to [7], [5] and [1] on the benchmark functions given in [8] are presented in Figures 3, 4 and 5. The experiments were performed with COCO [6], version 2.0, the plots were produced with version 2.0.

#### 5.1 Results of our algorithm

We think that the results of our implementation of the adaptive IBEA algorithm needs not to be taken as it is. There are things that we couldn't do, because of time and equipment constraints, or we just failed to implement it. We couldn't use the suggested parameters of the paper, such as a maximum number of generations of 200. Going further with some parameters like the budget, the size of the initial population or the maximum number of generations would have

probably improved our results. Furthermore, the initial population was randomly generated in the range -5 to 5, for which we failed to find a better solution. Nevertheless, the results of this implementation of an adaptive IBEA algorithm was still good, according to the benchmarking realized with the COCO platform. In the end, compared to the results of other algorithms visible in the coco archives at <http://coco.gforge.inria.fr/ppdata-archive/bbob-biobj/2016-all/>, our results are in the middle group, and comparatively better for higher dimensions than smaller ones.

## 5.2 Comparison with NSGA 2 and Random Search

NSGA-II is a very famous multi-objective optimization algorithm. It has three special characteristics: fast non-dominated sorting approach, fast crowded distance estimation procedure and simple crowded comparison operator. And it has been used successfully in lots of multi-objective optimization problems. We take the data obtained by NSGA-II algorithm from the following link:

[http://coco.gforge.inria.fr/data-archive/bbob-biobj/2016/NSGA-II-MATLAB\\_Auger\\_bbob-biobj.tgz](http://coco.gforge.inria.fr/data-archive/bbob-biobj/2016/NSGA-II-MATLAB_Auger_bbob-biobj.tgz)

The basic idea of the Random search algorithm is to randomly select some points according to certain criteria, then put these points into the functions, and keep the best points from them for the next iteration. It is essentially a violent search mechanism, but when the number of evaluations is greatly increased, it can also achieve good results. We take the data obtained by Random search algorithm from the following link:

[http://coco.gforge.inria.fr/data-archive/bbob-biobj/2016/RANDOM\\_SEARCH-5\\_Auger\\_bbob-biobj.tgz](http://coco.gforge.inria.fr/data-archive/bbob-biobj/2016/RANDOM_SEARCH-5_Auger_bbob-biobj.tgz)

The general results of the NSGA-II, random search algorithm and Adaptive IBEA are as seen in Figure 1 and Figure 2.

Table 2: Comparison of three algorithms

Category Algorithm \	Log10(f-evals/ dimension)	Best for 2-D
Random search	6	0,62
NSGA-II	5	0,76
IBEA	2,4(40D)-3,6(2D)	0,58
Best for 40-D	Best evaluated func- tion	Worst evaluated function
0,08	f13, f53	f37, f46
0,17	f19, f53	f22
0,12	f18, f53	f31, f47

From the above table and Figures 1 and 2, we can see that, in general, NSGA-II is the best algorithm, its evaluation is not too big, but it can get the best results. IBEA is also a good algorithm, it does not need too many evaluations, but it can also get good results. And according to the estimation of COCO, its best value can exceed Random search (if the algorithm does more evaluations). In contrast, the effect of Random search is not very good, it needs too many evaluations to get a good value, and when the dimension

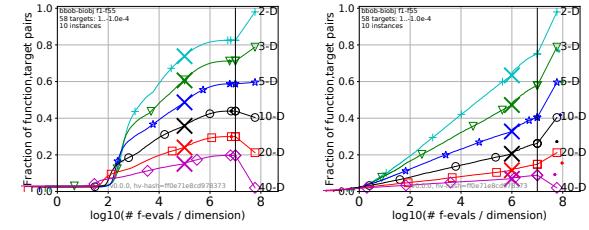


Figure 1: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for the two algorithms for all dimensions. NSGA-II is on the left, random search on the right.

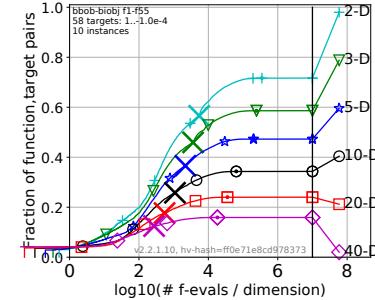


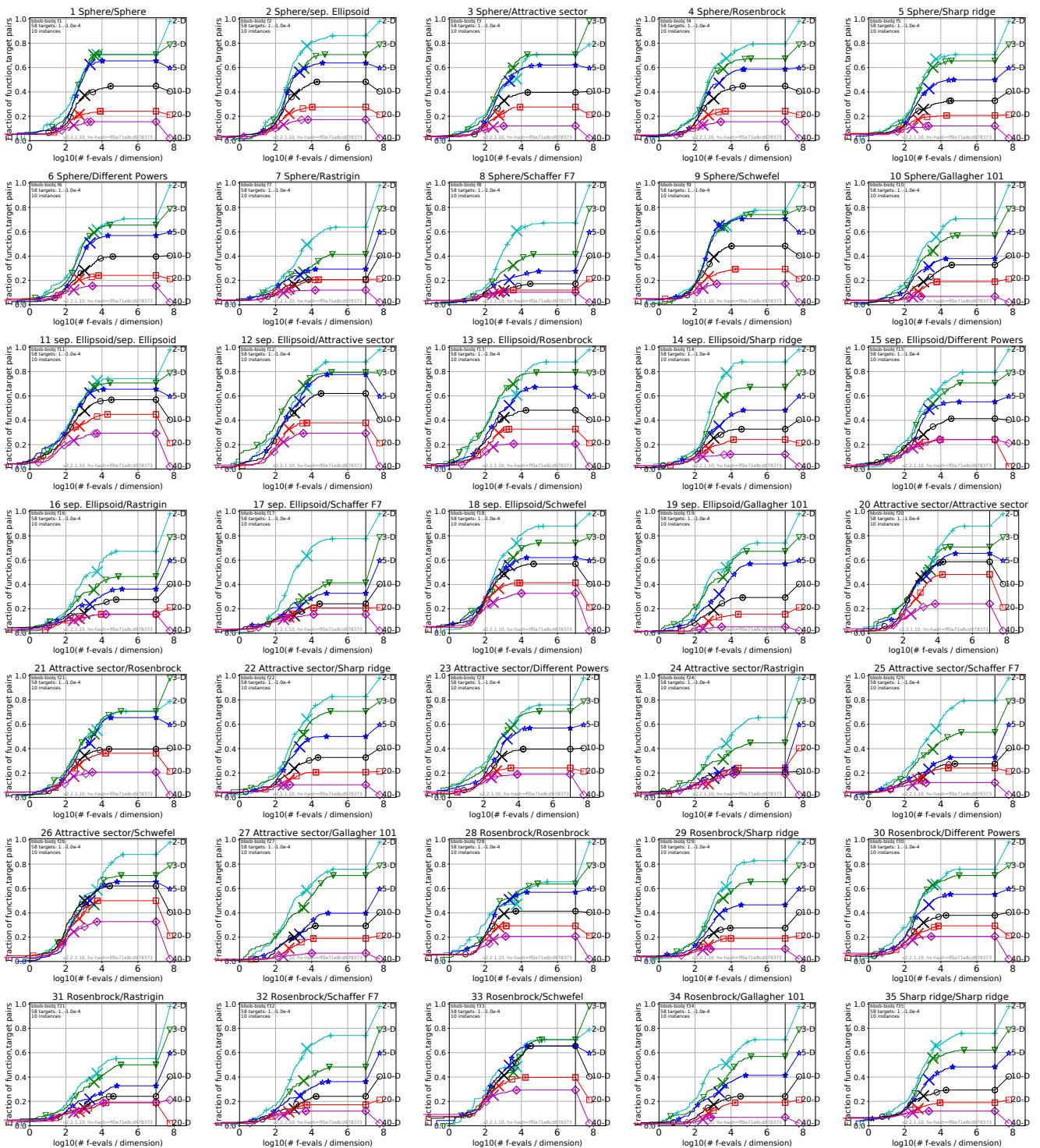
Figure 2: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for all dimensions for Adaptive IBEA.

of input becomes higher, the algorithm is not so effective. From the last two columns of the table, we can see that different algorithms have different optimization capabilities for each function, and f53 is friendly to all the algorithms.

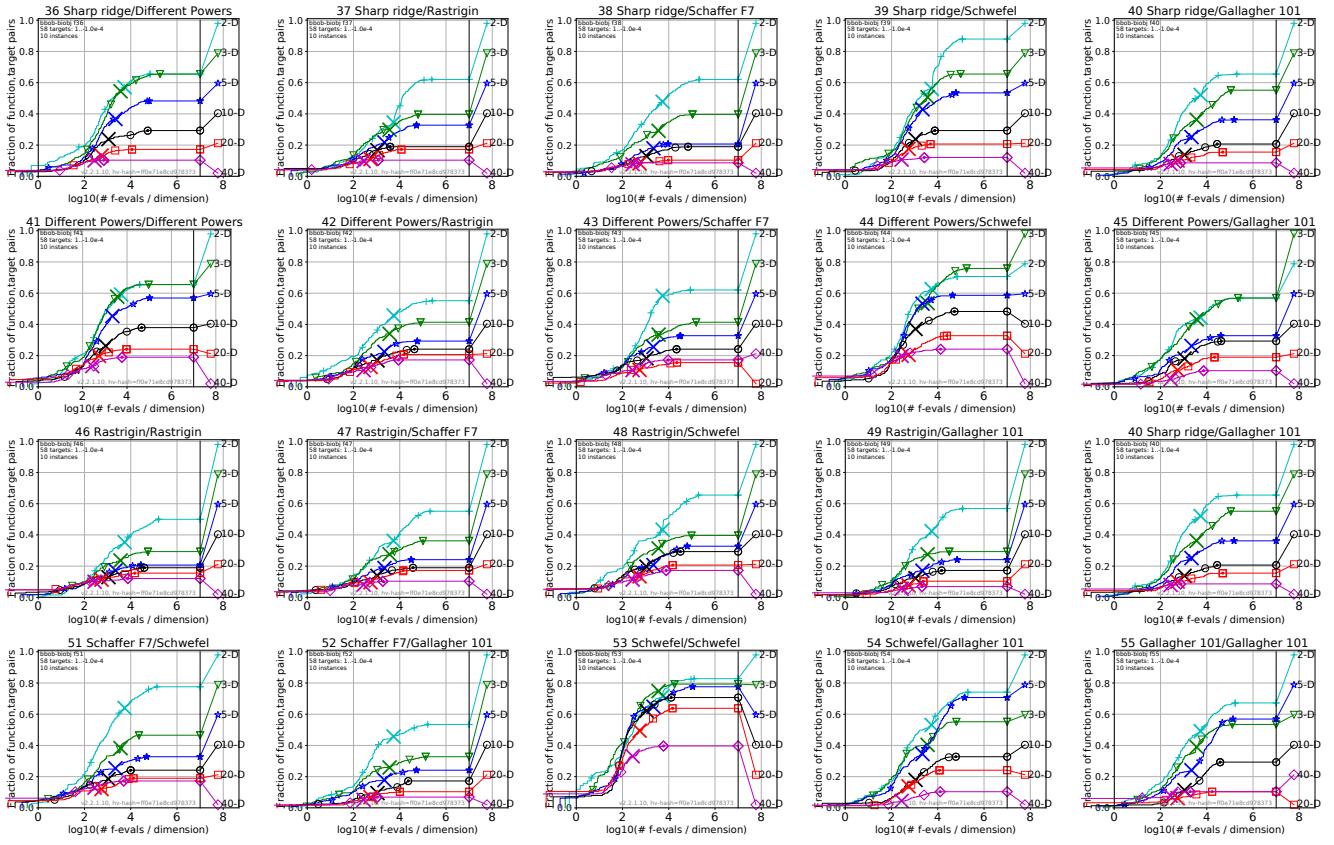
## 6 CONCLUSION

In this paper, an indicator-based evolutionary algorithm (IBEA) specifically with  $\epsilon$  indicator was studied and implemented. Since it can be adapted to the preferences of the user, it does not require any additional diversity preservation mechanism. We benchmarked it with COCO platform, which enables us to compare the algorithm's performance with other well-known multiobjective evolutionary algorithms (MOEAs). In our experiments, we majorly focused on the comparison with NSGA-II and Random search. In the original paper of IBEA, the author revealed that IBEA has been shown to generate significantly better results on six of eight benchmark problems in comparison to NSGA-II and SPEA2. However, we did not have a result as good as theirs. Overall, in our experiment, IBEA outperforms Random search, so a relatively good Pareto set approximations was given by IBEA, but IBEA performs worse than NSGA-II. Nevertheless, this could be biased, since potential factors that may impact performance need to be taken into account. Another way of initializing the original population, other methods of implementing mutation and recombination (other operators are used), or just bigger computing power, and all other hyperparameters are directly impacting the algorithm's performance.

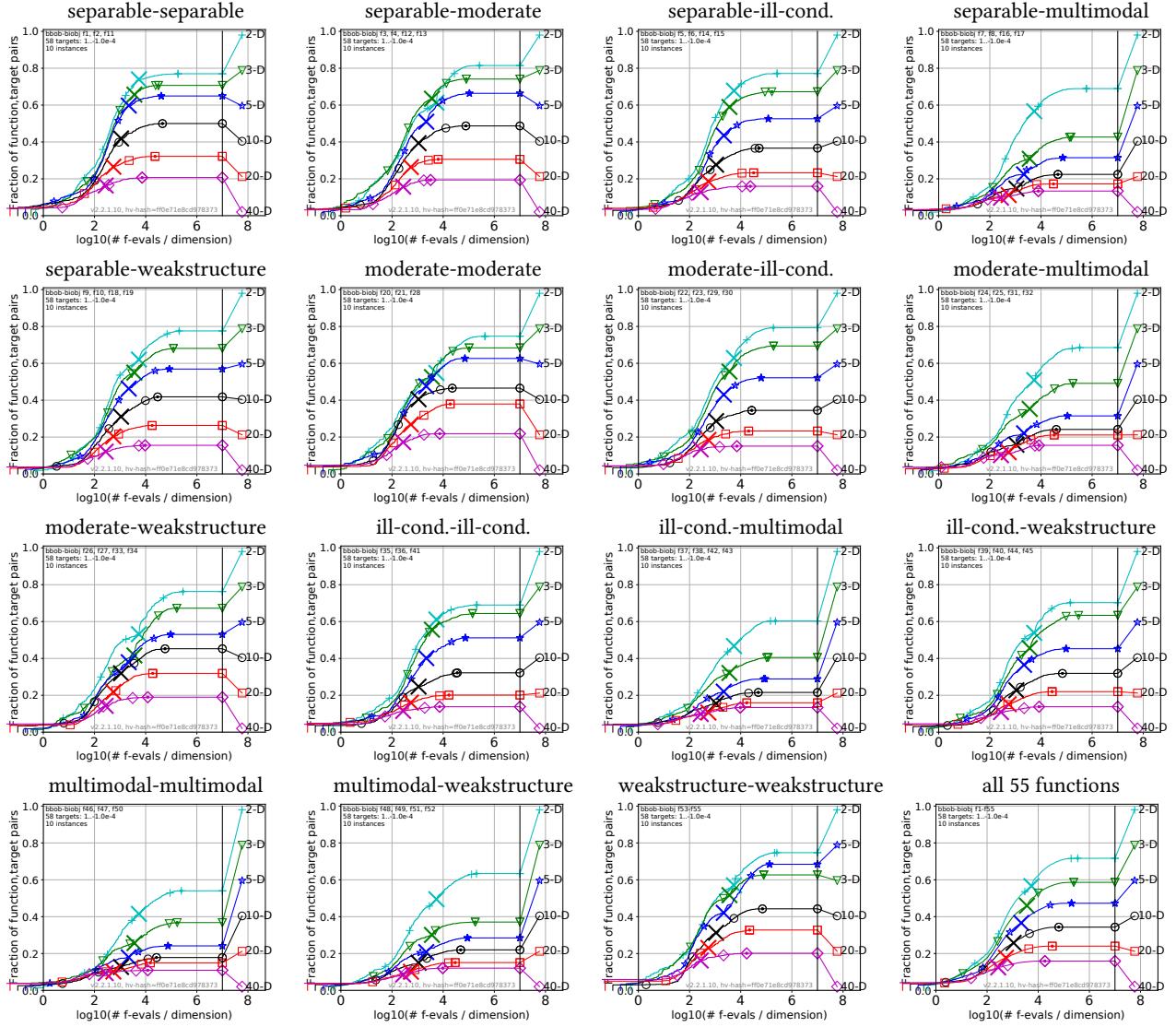
Implementing a well behaved optimisation algorithm is a very empirical and challenging work. It may take a lot of an expert's insight and years of experience. One problem that really confused us is about the choice of mutation index parameter ( $\eta_m$ ). In the paper[2], Deb and Agrawal suggested that a value  $\eta_m \in [20, 100]$  is adequate in most problems that they tried. To our surprise, when we changed  $\eta_m$  from 25 (by default) to 1, our result improved significantly. We do believe that further refinement of these hyperparameters will lead to a much better result.



**Figure 3: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for functions  $f_1$  to  $f_{35}$  in all dimensions.**



**Figure 4: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) as in Fig. 3 but for functions  $f_{36}$  to  $f_{55}$  in all dimensions.**



**Figure 5:** Empirical cumulative distribution of simulated (bootstrapped) runtimes, measured in number of objective function evaluations, divided by dimension (FEvals/DIM) for the 58 targets  $\{-10^{-4}, -10^{-4.2}, -10^{-4.4}, -10^{-4.6}, -10^{-4.8}, -10^{-5}, 0, 10^{-5}, 10^{-4.9}, 10^{-4.8}, \dots, 10^{-0.1}, 10^0\}$  for all function groups and all dimensions. The aggregation over all 55 functions is shown in the last plot.

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