Homework 1 in EL2450 Hybrid and Embedded Control Systems

Javier Cerna
930917
fjch@kth.se

Roubing Li
950911
roubing@kth.se

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Task 1

The gain with value zero is to model the condition when the tap is closed.

Task 2

```
s = tf('s');
uppertank = k_tank / (1+Tau*s); % Transfer function for upper tank
lowertank = gamma_tank / (1 + gamma_tank*Tau*s); % Transfer function
    for upper tank
G = uppertank*lowertank; % Transfer function from input to lower tank
    level
```

Task 3

The reference signal is a step applied at t = 25 s, with gain 10 (step goes from 0 to 10).

According to the model, there is always a constant voltage u_{ss} that is applied to the system, even when the reference error is 0. In order to compensate for this constant voltage in the calculation of (r-y), a constant y_{ss} needs to be applied, so that when (r-y) is equal to 0, u_{ss} won't influence the measurement of y, which will make the condition (r-y) no longer true.

Task 4

```
% Previously defined s=tf('s') to simplify code
[K_pid,Ti,Td,N]=polePlacePID(chi,omega0,zeta,Tau,gamma_tank,k_tank);
F = K_pid * (1 + 1/(Ti*s) + Td*N*s/(s+N));
```

Task 5

χ	ζ	ω_0	T_r	M	T_{set}
0.5	0.7	0.1	8.08	14.5%	45.3
0.5	0.7	0.2	4.96	34.6%	37.0
0.5	0.8	0.2	4.92	31.7%	26.5

Table 1: step response results with different value of parameters

The third pair of parameters give the best configuration, because it is the only one that fulfills all requirements. In particular, it is the only one that results in a settling time less than 30 s.

Task 6

The crossover frequency is 0.362 rad/s. It is defined when the magnitude of the frequency response becomes 1 (equivalent, the bode plot shows a magnitude of 0 dB), where the open loop system is F*G.

Task 7

Comparing with the continuous controller, the controller with ZOH has a larger overshoot, while on the other hand its T_r and T_{set} are smaller. Both controllers have the same parameters as we obtained in Task 5.

Starting from $T_s = 1.005s$, the overshoot $M = 35.1\%$, which is larger than 35.0%, and the step response cannot fulfill the requirements anymore.

Task 8

T_s	T_r	M	T_{set}
0.01	4.86	31.7%	26.4
0.1	4.83	31.8%	26.2
0.2	4.80	32.0%	26.0
0.3	4.76	33.7%	26.3
0.5	4.74	32.6%	25.4
1.005	4.56	35.1%	25.4

Table 2: Task 7, continuous controller with ZOH

T_s	T_r	M	T_{set}
0.01	4.86	31.6%	26.5
0.1	4.93	30.7%	27.6
0.2	5.01	29.8%	28.8
0.3	5.03	28.8%	30.1
0.5	5.08	27.0%	32.0
1.005	5.36	25.3%	53.9

Table 3: Task 8, discretized controller

Task 9

This interval can be determined using the sampling rule of thumb

$$0.05 < T_s\omega_c < 0.14,$$

where $\omega_c = 0.362$ rad/s, thus

$$0.14s < T_s < 0.39s$$

Task 10

Table 3 shows the performance of the discretized controller while changing T_s. It can be seen that a value of $T_s = 0.3$ won't fulfill the controller requirements anymore. This value is below the theoretical 0.39, but one reason of the difference is that the crossover frequency w_c was obtained analyzing the linearized system. The system used in Simulink is the "real" non-linear version and thus can have a different crossover frequency.

Task 11

The system is no longer stable when $T_s = 4s$, and a very serious oscillation can be observed in the step response. This effect can be seen in Figure 1.

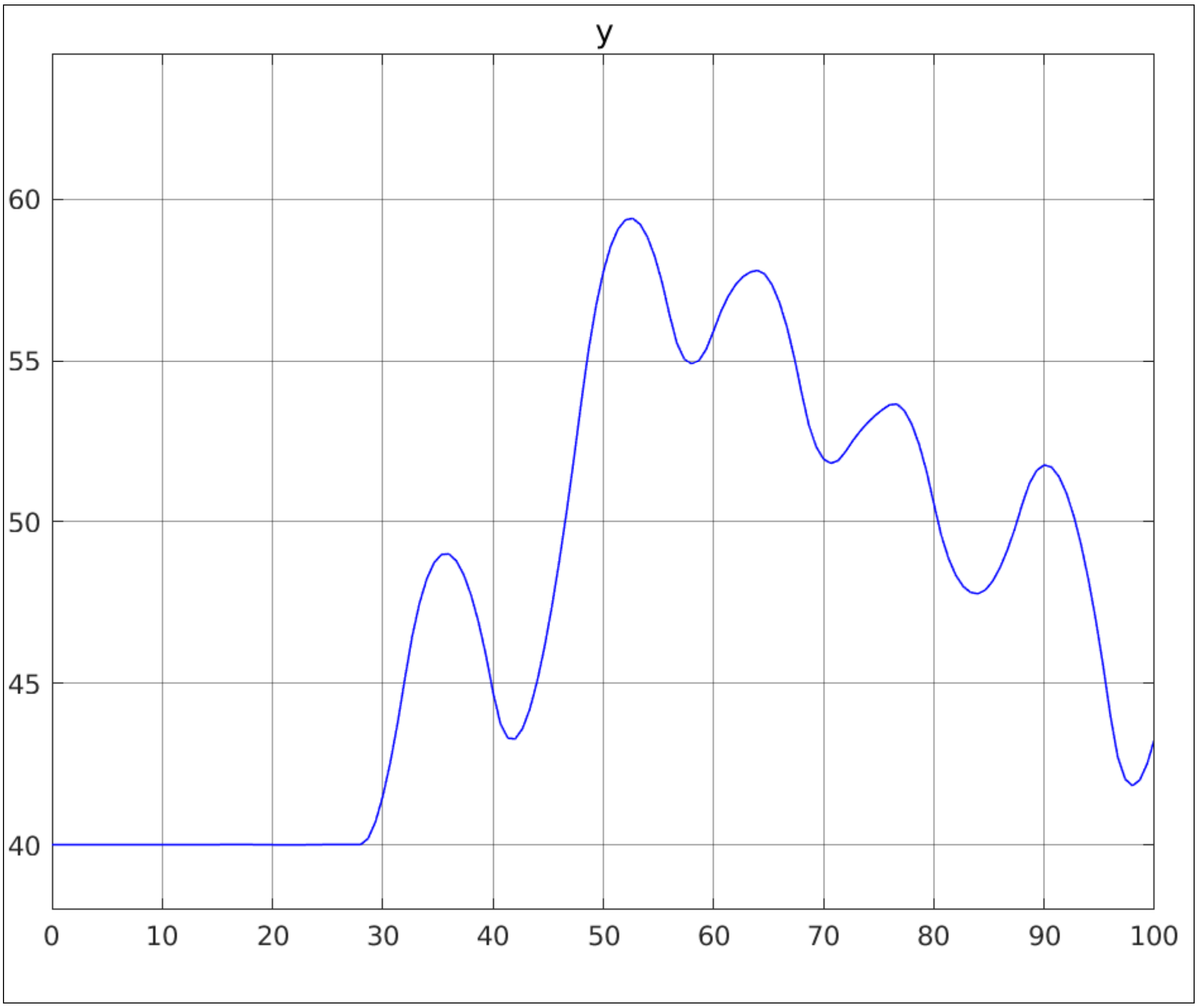

Figure 1: $T_s = 4$ s. Unstable system.

Task 12

In order to determine matrices A and B, we need to apply the inverse Laplace Transform on Equation (1), then we can get

$$\Delta \dot{x}_1(t) = -\frac{1}{\tau}\Delta x_1(t) + \frac{k}{\tau}\Delta u(t),$$

$$\Delta \dot{x}_2(t) = \frac{1}{\tau}\Delta x_1(t) - \frac{1}{\gamma\tau}\Delta x_2(t).$$

Therefore the matrices A and B can be specified as

$$\begin{bmatrix} \Delta \dot{x}_1(t) \\ \Delta \dot{x}_2(t) \end{bmatrix} = \underbrace{\begin{bmatrix} -\frac{1}{\tau} & 0 \\ \frac{1}{\tau} & -\frac{1}{\gamma\tau} \end{bmatrix}}_{A} \begin{bmatrix} \Delta x_1(t) \\ \Delta x_2(t) \end{bmatrix} + \underbrace{\begin{bmatrix} \frac{k}{\tau} \\ 0 \end{bmatrix}}_{B} \Delta u(t).$$

For the discrete-time system, we obtained

$$\Phi = \begin{bmatrix} 0.7249 & 0 \\ 0.2332 & 0.7249 \end{bmatrix}, \Gamma = \begin{bmatrix} 0.6017 \\ 0.0916 \end{bmatrix}.$$

Task 13

The system is observable (or reachable) if the rank of matrix W_o (or W_c) is equal to 2 (full rank), for the second-order system. In this case, both matrices are full rank, which means that the system is both observable and reachable.

Task 14

l_r is used to make the static gain of the system equal to 1 (i.e. at steady state, Y/r should give the value of 1).

Task 15

The state space equation of the observer can be written as

$$\Delta\hat{x}(k+1) = \Phi\Delta\hat{x}(k) + \Gamma\Delta u(k) + K[\Delta y(k) - \Delta\hat{y}(k)],$$

$$\Delta\hat{y}(k) = C\Delta\hat{x}(k).$$

Substituting $\Delta y(k)$ and $(\Delta u(k) = -L\Delta\hat{x}(k) + l_r r(k))$, we got

$$\Delta\hat{x}(k+1) = (\Phi - \Gamma L - KC)\Delta\hat{x}(k) + KC\Delta x(k) + \Gamma l_r r(k). \tag{1}$$

Additionally, we know that

$$\begin{aligned}\Delta x(k+1) &= \Phi\Delta x(k) + \Gamma\Delta u(k) \\ &= \Phi\Delta x(k) - \Gamma L\Delta\hat{x}(k) + \Gamma l_r r(k).\end{aligned} \tag{2}$$

Combining formulae (1) and (2), we can deduce matrices A_a and B_a as

$$\begin{bmatrix} \Delta x(k+1) \\ \Delta\hat{x}(k+1) \end{bmatrix} = \underbrace{\begin{bmatrix} \Phi & -\Gamma L \\ KC & (\Phi - \Gamma L - KC) \end{bmatrix}}_{A_a} \begin{bmatrix} \Delta x(k) \\ \Delta\hat{x}(k) \end{bmatrix} + \underbrace{\begin{bmatrix} \Gamma l_r \\ \Gamma l_r \end{bmatrix}}_{B_a} r(k).$$

Task 16

First of all, we can make $\Delta\tilde{x}(k)$ appear in formula (2) by adding and subtracting term $\Gamma L\Delta x(k)$ at the same time at the right hand side

$$\begin{aligned}\Delta x(k+1) &= \Phi\Delta x(k) - \Gamma L\Delta x(k) + [\Gamma L\Delta x(k) - \Gamma L\Delta\hat{x}(k)] + \Gamma l_r r(k) \\ &= (\Phi - \Gamma L)\Delta x(k) + \Gamma L\Delta\tilde{x}(k) + \Gamma l_r r(k).\end{aligned} \tag{3}$$

Then, by making (2)-(1), we can derive $\Delta\tilde{x}(k+1) as$

$$\begin{aligned}\Delta\tilde{x}(k+1) &= \Delta x(k+1) - \Delta\hat{x}(k+1) \\ &= \Phi[\Delta x(k) - \Delta\hat{x}(k)] - KC[\Delta x(k) - \Delta\hat{x}(k)] \\ &= 0 \cdot \Delta x(k) + (\Phi - KC)\Delta\tilde{x}(k) + 0 \cdot r(k)\end{aligned} \tag{4}$$

The combination of formula (3) and (4) yields the expression of the closed-loop system which is identical with respect to Equation (2).

It can be seen from Equation (2) in the homework statement, that the new augmented "A" matrix is an upper triangular matrix. This means that to calculate the eigenvalues of the system, it is equivalent to calculate the eigenvalues of the first element of the diagonal $(\Phi - \Gamma L)$ and then to calculate the eigenvalues of the second element of the diagonal $(\Phi - KC)$. This in turn means that the values of L won't influence the values of K, and vice versa. This is the separation principle.

Task 17

After cancelling some poles with zeros using MATLAB function "minreal", the continuous-time closed-loop system has 4 poles:

$$p_1 = -0.16 + 0.12j,$$

$$p_2 = -0.16 - 0.12j,$$

$$p_{3,4} = -0.5.$$

Therefore the discrete-time poles can be converted using $z_i = e^{T_s p_i}$, which gives

$$z_1 = 0.4677 + 0.2435j,$$

$$z_2 = 0.4677 - 0.2435j,$$

$$z_{3,4} = 0.1353.$$

In theory, the observer dynamics should be faster than the controller dynamics, thus we placed the discrete-time poles $z_{3,4} = 0.1353$. to the state-feedback gain matrix K. Since the poles are the eigenvalues of $(\Phi - KC)$, we specified the MATLAB code as

```
K = acker(Phi' , C' , discrete_poles_2).';
```

Also considering that the system is observable. A similar procedure is used to calculate the values of L, considering the other pair of eigenvalues.

Finally, the poles of A_a are calculated, and the results are:

$$e_1 = 0.4677 + 0.2435j$$

$$e_2 = 0.4677 - 0.2435j$$

$$e_{3,4} = 0.1353,$$

which are the same as the desired poles.

Task 18

The system with the discrete control design is shown in Figure 2. Compared to Figure 1, it is clear that this system behaves better (first of all, this one is stable). Even if not all the control requirements are met, the system is similar to the desired response, taking into account the fact that the sampling time was 4 s. The difference with Task 11 could be because this discrete controller was designed taking into account the discretized plant (which in turn was derived using T_s), whereas Task 11 was trying to control a continuous plant with a digital controller. The output doesn't reach the same steady state value as the continuous controller.

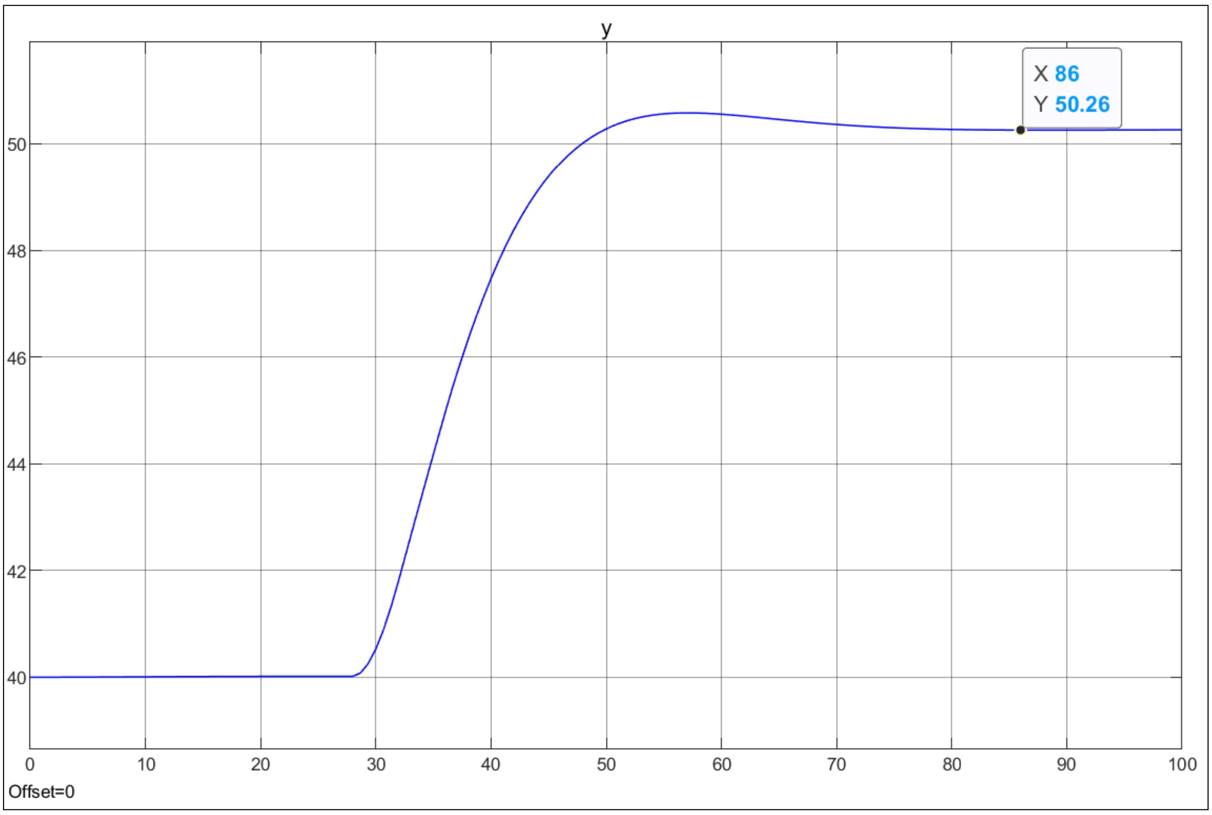

Figure 2: System with discrete control design.

Task 19

To determine the quantization level, one needs to use the number of bits to get all the possible values. This corresponds to the formula: $2^{10} = 1024$. Therefore, the quantization level is 1024.

Additionally, the quantization interval can be calculated as:

$$\Delta = \frac{100}{1024} = 0.098$$

Task 20

The saturation is necessary to give an upper limit to the signal being quantized from the plant (y) and the signal being sent to the controller (u). In this specific case, task 19 specifies that the signal varies between 0 and 100, that's why those values are used.

Task 21

As can be seen in Figure 3, the control performance starts to degrade when the quantizer uses 7 bits. This corresponds to a quantization level of $2^7 = 128$ and a quantization interval of $(100/2^7) = 0.781$.

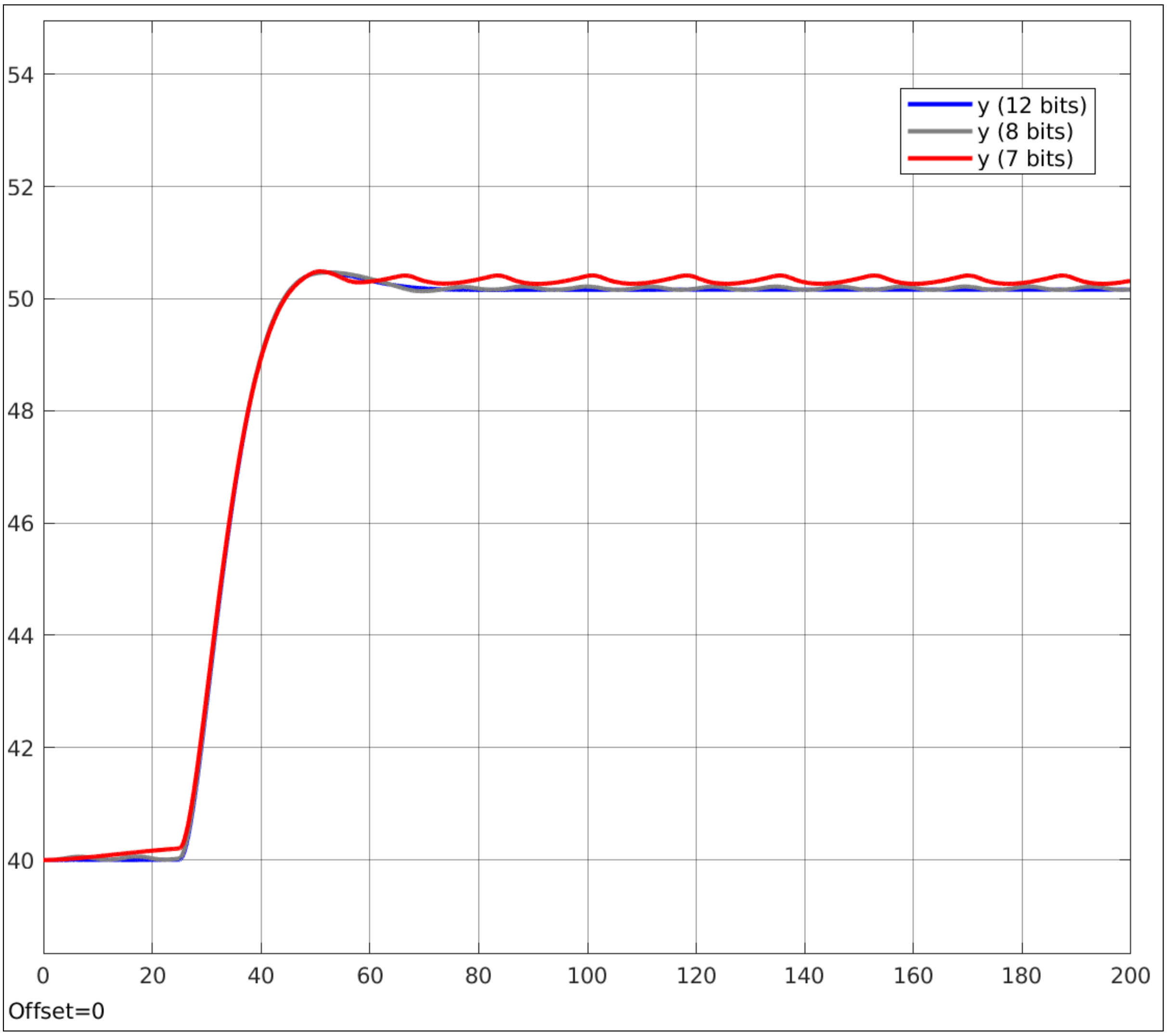

Figure 3: Response for different quantization levels