Mimicking L'Aquila: Capturing Key Components of the Earthquake Sequence

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Overview

This project aims at mimicking the L'Aquila earthquake sequence using synthetic catalogues, and investigating catalogue characteristics in order to yield a better performance in resembling the real sequence.

L'Aquila Sequence

Map Description

This map illustrates vividly the locations of Italian earthquakes in 2009. Different colours of dots show different magnitudes.

- Black: Richter scale of 3 to 5
- Orange: Richter scale of 5 to 6
- Red: The L'Aquila earthquake, Richter scale of 6.29

Earthquake Characteristics

- Number of events: 1024 (Richter scale of at least $M_0 = 2.5$)
- Number of large events (Richter scale of 4.5 or higher): 14
- Largest event: Richter scale of 6.29

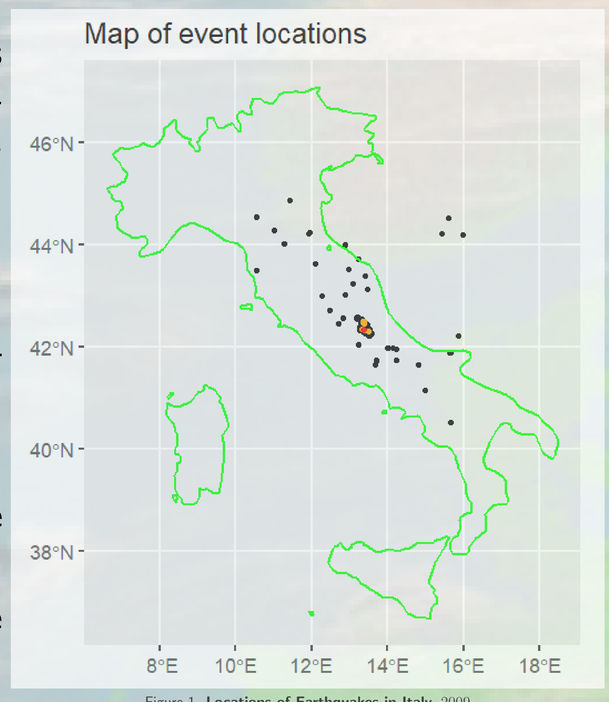

Figure 1. **Locations of Earthquakes in Italy,** 2009

Modelling the Sequence

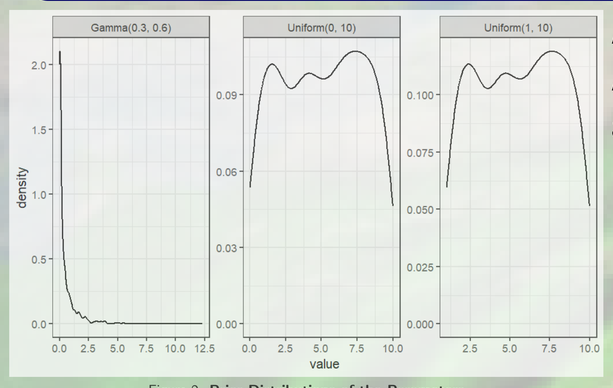

Figure 2. **Prior Distributions of the Parameters**

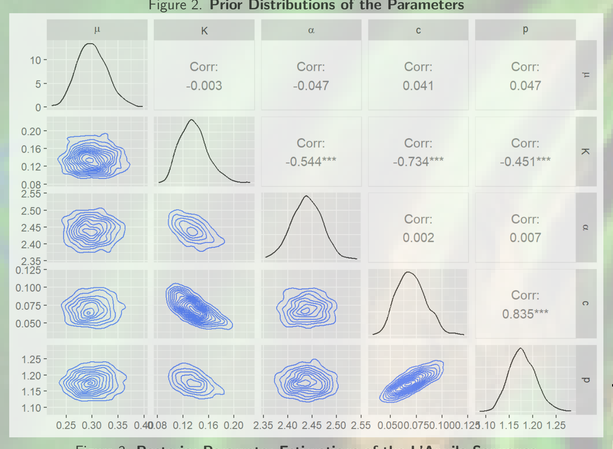

Figure 3. **Posterior Parameter Estimations of the L'Aquila Sequence**

According to [1] [2], the Epidemic Type Aftershock Sequence (ETAS) model is adopted to estimate 5 parameters.

- μ, background rate. Prior distribution: $Gamma(0.3, 0.6)$.
- K, productivity, mean aftershocks. Prior distribution: $U(0, 10)$.
- α, magnitude scales, changes in aftershocks given an earthquake event. Prior distribution: $U(0, 10)$.
- c, time offset, positively correlated to missing events. Prior distribution: $U(0, 10)$.
- p, decaying rate of aftershocks. Prior distribution: $U(1, 10)$.

The posterior distributions of the 5 parameters are then sketched in Figure 3.

- Mis-specifying prior of μ to be $Gamma(5, 1)$, or that of α to be $U(0.99, 1.01)$, no effects on posterior results.
- Mis-specifying prior of K or c to be $U(0.99, 1.01)$, or that of p to be $U(4.9, 5.1)$, significant effects on posterior results.

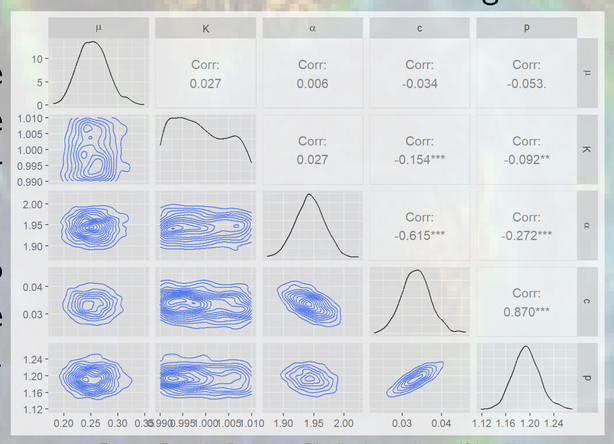

Figure 4. **Posterior Parameter Distributions, mis-specifying** K

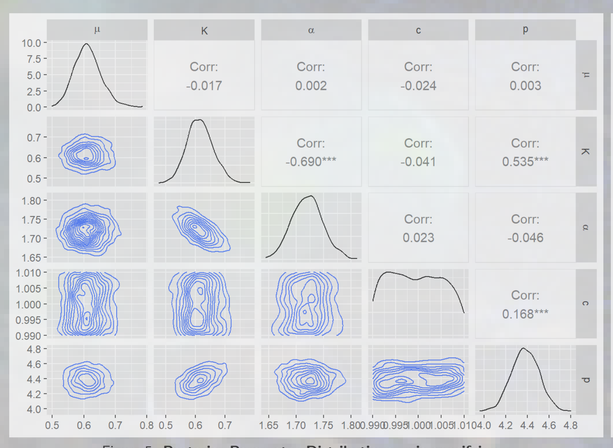

Figure 5. **Posterior Parameter Distributions, mis-specifying** c

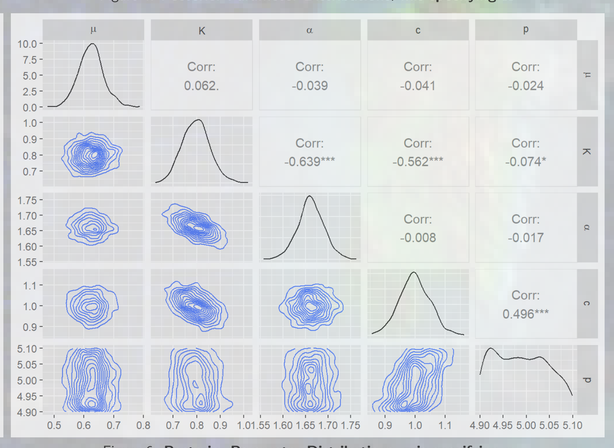

Figure 6. **Posterior Parameter Distributions, mis-specifying** p

Generating and Modelling the Synthetic Catalogues

1000 synthetic catalogues are generated, without imposing history (to avoid generating an unrealistically large amount of catalogues), with parameter values specified in the table.

μ	K	α	c	p	β
0.30	0.14	2.44	0.07	1.18	2.35

Table. **Parameter Values for Synthetic Catalogues**

β: magnitude distribution parameter, $\hat{\beta} = \frac{1}{mean(magnitudes) - M_0}$.

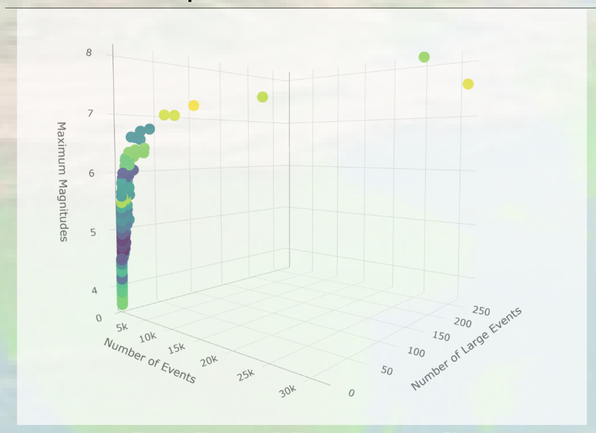

Figure 7. **Clustered** $3D$ **Characteristic Scatter Plot**

Catalogues are recursively grouped into 25 clusters based on their characteristics using hierarchical clustering. Dendrogram and clustered $3D$ characteristic scatter plot are shown.

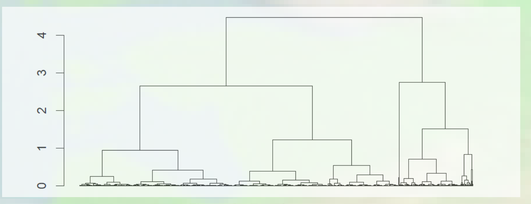

Figure 8. **Dendrogram**

1 catalogue from each cluster is selected, and the ETAS model is applied to these catalogues. 2 of the posterior plots are shown.

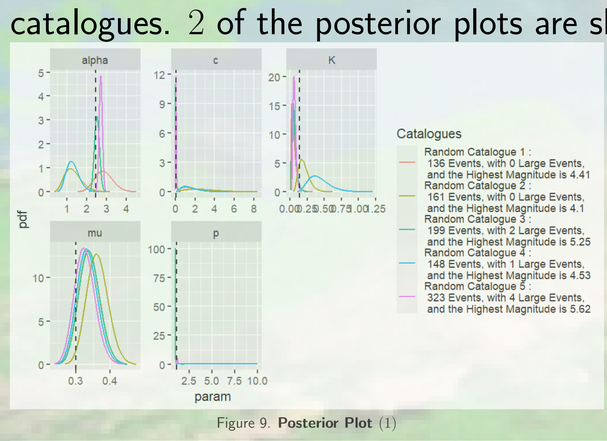

Figure 9. **Posterior Plot** (1)

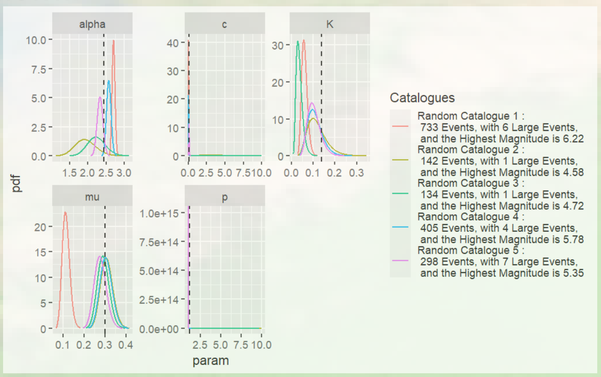

Figure 10. **Posterior Plot** (2)

Organic synthetic catalogues (no history imposed) with 200 to 400 total events, with 3 to 5 large events, and with the largest event of magnitude around 6 would lead to a better performance in estimating the parameters.

Investigating Time-between-Events

Organic Synthetic Catalogues

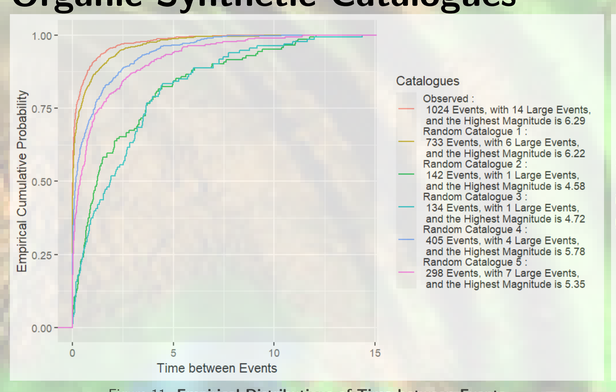

Figure 11. **Empirical Distributions of Time-between-Events**

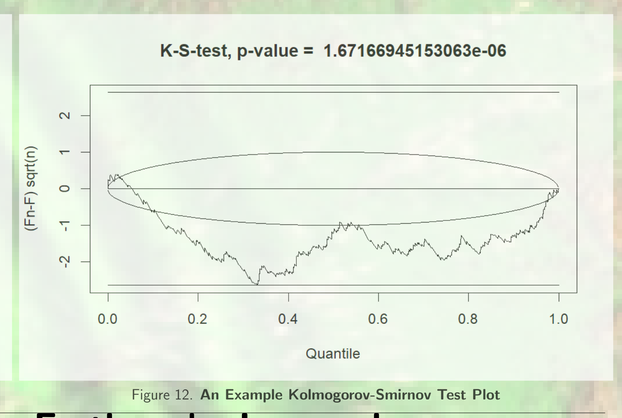

Figure 12. **An Example Kolmogorov-Smirnov Test Plot**

Synthetic Catalogues with L'Aquila Earthquake Imposed

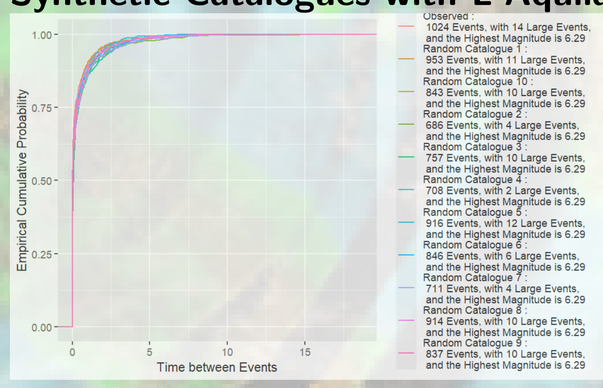

Figure 13. **Empirical Distributions of Time-between-Events**

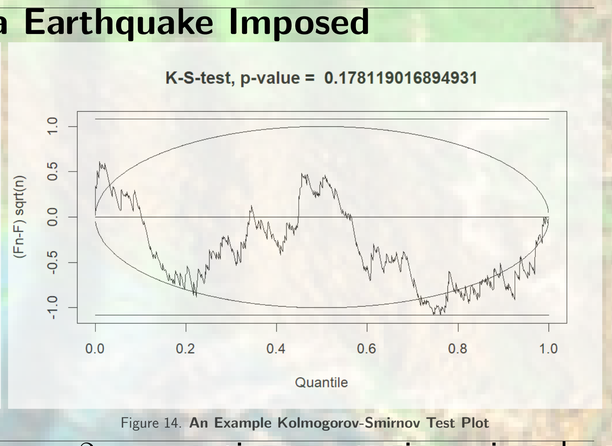

Figure 14. **An Example Kolmogorov-Smirnov Test Plot**

When investigating time differences between 2 consecutive events, imposing the main event to the history will increase the chance of generating catalogues having similar number of events, similar number of large events, as well as the similar highest magnitude with the real L'Aquila sequence, yielding a better performance in simulating the arrival time of the earthquakes. However, this does not necessarily mean that imposing events will lead to better estimates of the 5 parameters specified above, which is the reason why events are only imposed to the history when the inter-arrival periods are to be simulated.

Main References

[1] Francesco Serafini, Finn Lindgren, and Mark Naylor. Approximation of bayesian hawkes process with inlabru. *Environmetrics*, page e2798, 2023.

[2] Mark Naylor, Francesco Serafini, Finn Lindgren, and Ian G Main. Bayesian modeling of the temporal evolution of seismicity using the etas. inlabru package. *Frontiers in Applied Mathematics and Statistics*, 9:1126759, 2023.