Earthquake Forecasting

Dissertation Project 2

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August 2023

```
require(tidyverse)
require(knitr)
require(kableExtra)
require(factoextra)
require(cluster)
require(ETAS.inlabru)
require(ggplot2)
require(dplyr)
require(magrittr)
require(tidyquant)
require(rnaturalearth)
require(terra)
require(sf)
require(ggspatial)
require(rnaturalearthdata)
require(lubridate)

# Increase/decrease num.cores if you have more/fewer cores on your computer.
# future::multisession works on both Windows, MacOS, and Linux
num.cores <- 1
future::plan(future::multisession, workers = num.cores)
INLA::inla.setOption(num.threads = num.cores)
# To deactivate parallelism, run
# future::plan(future::sequential)
# INLA::inla.setOption(num.threads = 1)
```

Copula transformation of the priors

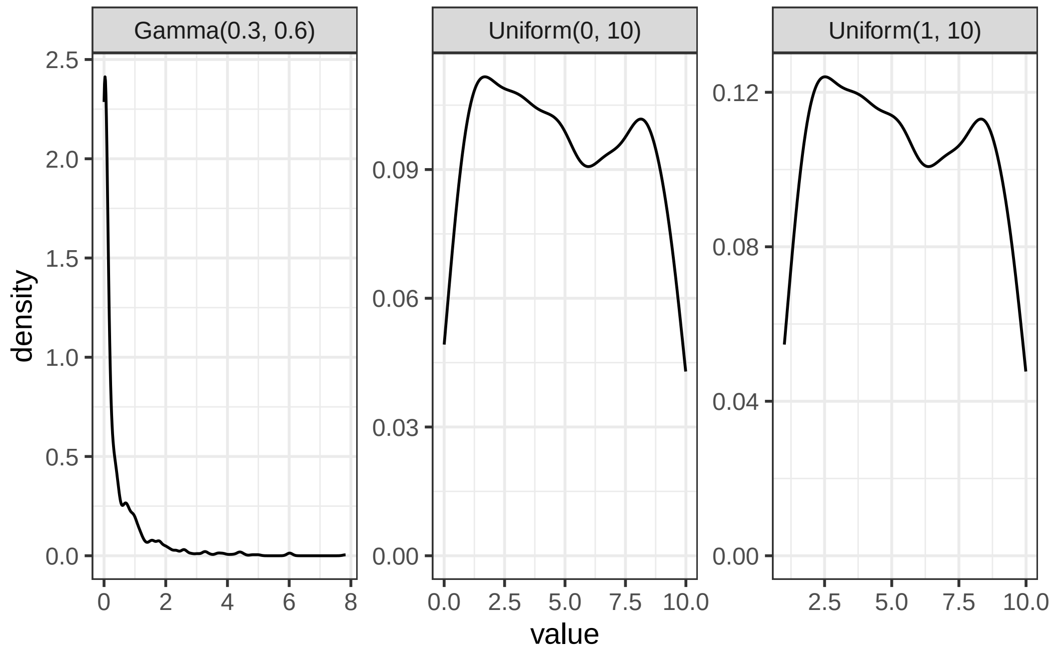
```r
# set copula transformations list
link.f <- list(
  mu = \(x) gamma_t(x, 0.3, 0.6),
  K = \(x) unif_t(x, 0, 10),
  alpha = \(x) unif_t(x, 0, 10),
  c_ = \(x) unif_t(x, 0, 10),
  p = \(x) unif_t(x, 1, 10)
)

# set inverse copula transformations list
inv.link.f <- list(
  mu = \(x) inv_gamma_t(x, 0.3, 0.6),
  K = \(x) inv_unif_t(x, 0, 10),
  alpha = \(x) inv_unif_t(x, 0, 10),
  c_ = \(x) inv_unif_t(x, 0, 10),
  p = \(x) inv_unif_t(x, 1, 10)
)
```

```r
# obtain sample from standard normal distribution
X <- rnorm(1000)
# apply copula transformations
gamma.X <- gamma_t(X, 0.3, 0.6)
unif.X <- unif_t(X, 0, 10)
unif.X.2 <- unif_t(X, 1, 10)

# build data.frame for plotting
df.to.plot <- rbind(
  data.frame(
    value = gamma.X,
    distribution = "Gamma(0.3, 0.6)"
  ),
  data.frame(
    value = unif.X,
    distribution = "Uniform(0, 10)"
  ),
  data.frame(
    value = unif.X.2,
    distribution = "Uniform(1, 10)"
  )
)
```

```r
# plot them
ggplot(df.to.plot, aes(value)) +
  geom_density() +
  theme_bw() +
  facet_wrap(facets = ~ distribution, scales = "free")
```



Italy

```r
# transform time string in Date object
horus$time_date <- as.POSIXct(
  horus$time_string,
  format = "%Y-%m-%dT%H:%M:%OS",
  tz = "UTC"
)
# There may be some incorrectly registered data-times in the original data set,
# that as.POSIXct() can't convert, depending on the system.
# These should ideally be corrected, but for now, we just remove the rows that
# couldn't be converted.
# horus <- na.omit(horus)

# set up parameters for selection
```

```r
start.date <- as.POSIXct("2009-01-01T00:00:00",
                         format = "%Y-%m-%dT%H:%M:%OS")
end.date <- as.POSIXct("2010-01-01T00:00:00", format = "%Y-%m-%dT%H:%M:%OS")
min.longitude <- 10.5
max.longitude <- 16
min.latitude <- 40.5
max.latitude <- 45
M0 <- 2.5

# set up conditions for selection
aquila.sel <- (horus$time_date >= start.date) &
  (horus$time_date < end.date) &
  (horus$lon >= min.longitude) &
  (horus$lon <= max.longitude) &
  (horus$lat >= min.latitude) &
  (horus$lat <= max.latitude) &
  (horus$M >= M0)

# select
aquila <- horus[aquila.sel, ]
```

```r
italy.map <- ne_countries(country = 'Italy', returnclass = "sf",
                          scale = 'medium')

aquila.sf <- st_as_sf(aquila,
                      coords = c("lon", "lat"),
                      crs = st_crs('EPSG:4326'))
```

```r
ggplot() +
  geom_sf(data = aquila.sf[aquila$M > 3,], size = 0.8) +
  geom_sf(data = italy.map, fill = alpha("lightgrey", 0), colour = 'green',
          linewidth = 0.7) +
  geom_sf(data = aquila.sf[aquila$M > 5,], size = 0.9, colour = 'orange') +
  geom_sf(data = aquila.sf[aquila$M > 6,], size = 1, colour = 'red') +
  ggtitle("Map of event locations")
```

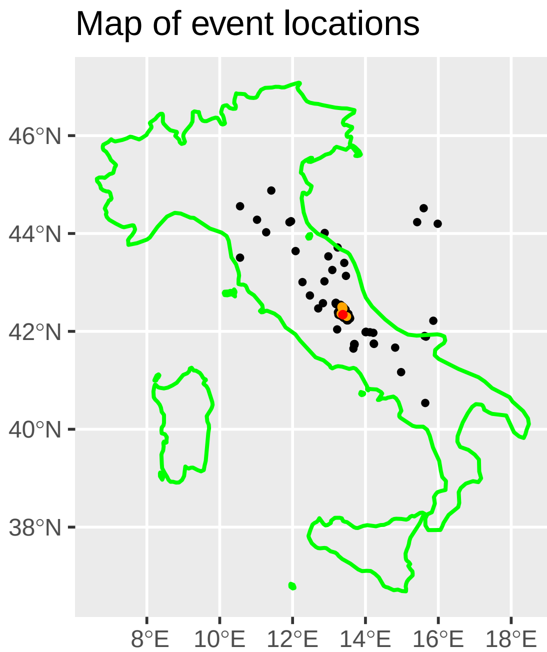


Figure 1: Italian earthquakes in 2009

```
ggplot(aquila, aes(time_date, M)) +
  geom_point() +
  theme_bw()
```

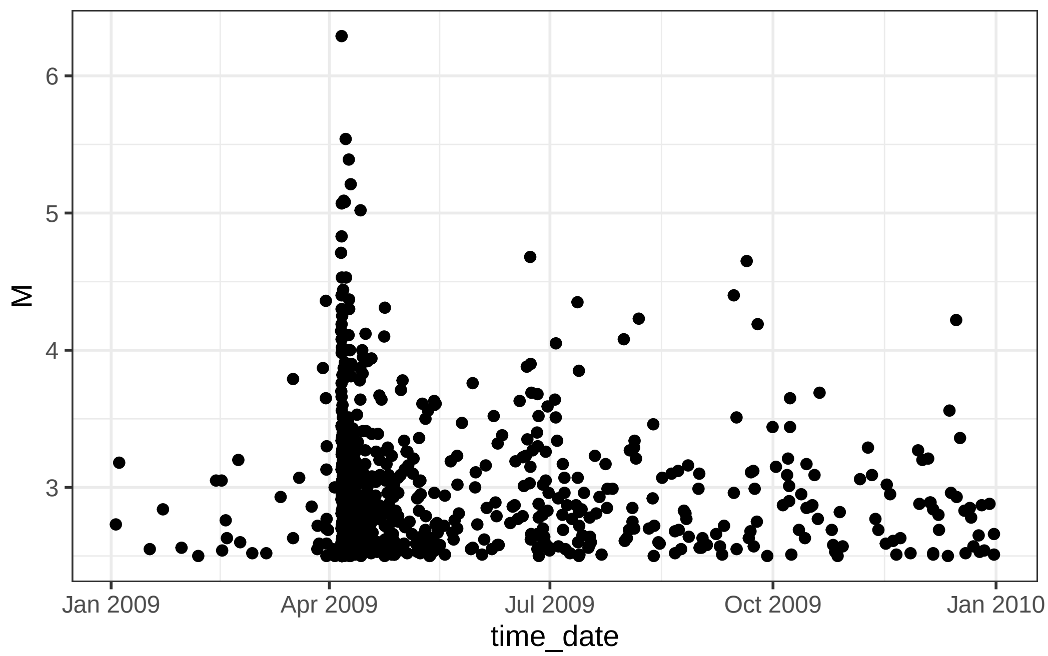


Figure 2: L'Aquila seismic sequence, times versus magnitudes

```
# set up data.frame for model fitting
aquila.bru <- data.frame(
  ts = as.numeric(
    difftime(aquila$time_date, start.date, units = "days")
  ),
  magnitudes = aquila$M,
  idx.p = 1 : nrow(aquila)
)
```

```
# set up list of initial values
th.init <- list(
  th.mu = inv.link.f$mu(0.5),
  th.K = inv.link.f$K(0.1),
  th.alpha = inv.link.f$alpha(1),
  th.c = inv.link.f$c_(0.1),
  th.p = inv.link.f$p(1.1)
)
```

```r
# set starting and time of the time interval used for model fitting.
# In this case, we use the interval covered by the data.
T1 <- 0
T2 <- max(aquila.bru$ts) + 0.2 # Use max(..., na.rm = TRUE) if there may
# still be NAs here
```

```r
# set up list of bru options
bru.opt.list <- list(
  bru_verbose = 3, # type of visual output
  bru_max_iter = 70, # maximum number of iterations
  # bru_method = list(max_step = 0.5),
  bru_initial = th.init # parameters' initial values
)
```

```r
ETAS <- function(data = aquila.bru, m0 = M0, t1 = T1, t2 = T2,
                 ncore = num.cores, Link.f = link.f,
                 Bru.opt.list = bru.opt.list, n.samp = 1000,
                 max.batch = 1000, mag = 4.5, n.breaks = 100,
                 t.end.tri.post = 5, t.end.tri.prior = 10,
                 t.end.omori.post = 5, t.end.omori.prior = 5){

    # maximum likelihood estimator for beta
    beta.p <- 1 / (mean(data$magnitudes) - m0)

    # fit the model
    model.fit <- Temporal.ETAS(
      total.data = data,
      M0 = m0,
      T1 = t1,
      T2 = t2,
      link.functions = Link.f,
      coef.t. = 1,
      delta.t. = 0.1,
      N.max. = 5,
      bru.opt = Bru.opt.list
    )

    # create input list to explore model output
    input_list <- list(
      model.fit = model.fit,
      link.functions = Link.f
```

```r
)

# get marginal posterior information
post.list <- get_posterior_param(input.list = input_list)

# plot marginal posteriors
postplot <- post.list$post.plot

# posterior sampling
post.samp <- post_sampling(
  input.list = input_list,
  n.samp = n.samp,
  max.batch = max.batch,
  ncore = num.cores
)

# taking the averages of the posterior parameter estimates
post.par <- apply(post.samp, 2, mean)

# pair plot
pair.plot <- post_pairs_plot(
  post.samp = post.samp,
  input.list = NULL,
  n.samp = NULL,
  max.batch = max.batch
)
pairplot <- pair.plot$pair.plot

# set additional elements of the list
input_list$T12 <- c(t1, t2)
input_list$M0 <- m0
input_list$catalog.bru <- data

# posterior number of events
N.post <- get_posterior_N(input.list = input_list)
Npostplot <- N.post$post.plot
Npostmean <- N.post$post.df[which.max(N.post$post.df$mean), 1]

# number of large events
large_events <- data[data$magnitudes >= mag,]
Nlarge <- nrow(large_events)
```

```r

  # mean absolute distance of the differences in magnitudes
  diff_mag <- diff(data$magnitudes)
  abs_dist_mag <- mean(abs(diff_mag))

  # mean absolute distance of the inter-arrival time
  interarrival <- diff(data$ts)
  abs_dist_int <- mean(abs(interarrival))

  # check if overdispersion occurs
  m_int_time <- mean(interarrival)
  v_int_time <- var(interarrival)
  overdisp <- m_int_time ^ 2 < v_int_time

  # triggering function plots
  # posterior
  triplotpost <- triggering_fun_plot(
    input.list = input_list,
    post.samp = post.samp,
    n.samp = NULL, magnitude = mag,
    t.end = t.end.tri.post, n.breaks = n.breaks
  )

  # prior
  triplotprior <- triggering_fun_plot_prior(input.list = input_list,
                                            magnitude = mag, n.samp = n.samp,
                                            t.end = t.end.tri.prior)

  # omori plots
  # posterior
  omoripost <- omori_plot_posterior(input.list = input_list,
                                    post.samp = post.samp,
                                    n.samp = NULL, t.end = t.end.omori.post)

  # prior
  omoriprior <- omori_plot_prior(input.list = input_list,
                                 n.samp = n.samp,
                                  t.end = t.end.omori.prior)

  # returns the whole environment
  envir <- as.list(environment())
  return(tibble::lst(envir))
```

```
}
etas <- ETAS()
```

Effect of mis-specifying parameters

```
# # set copula transformations list
# link.f1 <- list(
#   mu = \(x) gamma_t(x, 0.3, 0.6),
#   K = \(x) unif_t(x, 0, 10),
#   alpha = \(x) unif_t(x, 0, 10),
#   c_ = \(x) unif_t(x, 0, 10),
#   p = \(x) unif_t(x, 1, 10)
# )
```

Synthetic catalogues generation

```
mult.synth.ETAS <- function(t1 = NULL, t2 = NULL, n.cat = 1000,
                           ht = NULL){

    # inherits the environment from function `ETAS`
    envir <- etas$envir

    # updates environments if specified by users
    envir$t1 <- ifelse(!is.null(t1), t1, envir$t1)
    envir$t2 <- ifelse(!is.null(t2), t2, envir$t2)

    # Function to generate a synthetic catalogue
    synth.gen <- function(i){
        iteration <- i
        synth <- generate_temporal_ETAS_synthetic(
                    theta = envir$post.par %>% as.list,
                    beta.p = envir$beta.p,
                    M0 = envir$m0, T1 = envir$t1,
                    T2 = envir$t2, Ht = ht, ncore = num.cores)
        return(synth)
    }

    # generates catalogues as list of lists
    multi.synth.cat.list <- lapply(seq_len(n.cat), \(x)
        synth.gen(x))

    # stores catalogues as list of data.frames
```

```
    multi.synth.cat.list.df <- lapply(multi.synth.cat.list,
                                      \(x) do.call(rbind, x))

    # counts the number of events in each catalogue
    Nevents <- lapply(seq_len(n.cat), \(i) nrow(
        multi.synth.cat.list.df[[i]])) %>% unlist

    # counts the number of large events in each catalogue
    mag <- etas$envir$mag
    Nlarge <- lapply(seq_len(n.cat), \(i) sum(
        multi.synth.cat.list.df[[i]]$magnitudes >= mag)) %>% unlist

    # extracts the highest magnitude in each catalogue
    MaxMag <- lapply(seq_len(n.cat), \(i) max(
        multi.synth.cat.list.df[[i]]$magnitudes)) %>% unlist

    # sets catalogue identifier
    multi.synth.cat.list.df <- lapply(seq_len(n.cat),
                                      \(x) cbind(
                                          multi.synth.cat.list.df[[x]],
                                          cat.idx = x,
                                          num_events = Nevents[x],
                                          num_large = Nlarge[x],
                                          max_mag = MaxMag[x]))

    # merges catalogues in unique data.frame
    multi.synth.cat.df <- do.call(rbind, multi.synth.cat.list.df)

    # returns the whole environment
    environ <- as.list(environment())
    return(tibble::lst(environ))
}

mult.synth <- mult.synth.ETAS(ht = NULL)
```

```
# synth.imp <- function(data = etas$envir$data, impose_type = 1, n.cat = 5){
#
#     breaks <- data$magnitudes %>%
#         quantile(probs = seq(0, 1, length.out = 6))
#
#     classes <- data$magnitudes %>%
```

```
#         cut(breaks = breaks)
#
#     samps = switch(impose_type,
#
#                    seq_len(5) %>%
#         lapply(\(i) which(classes == levels(classes)[i]) %>%
#                    sample(2 * i)) %>% unlist,
#
#                    c(5) %>%
#         lapply(\(i) which(classes == levels(classes)[i %/% 2 + 1]) %>%
#                    sample(i * (i + 1))) %>% unlist
#         )
#
#     ht <- data[samps,]
#
#     mult.synth.imp <- mult.synth.ETAS(ht = ht, n.cat = n.cat)
#
#     return(mult.synth.imp)
# }
```

Fitting Models on the Synthetic Catalogues

```
hier_clu_samp <- function(syn = mult.synth){

    # This function aims at performing hierarchical clustering
    # for the synthetic catalogues specified in `syn`, as well as
    # selecting 1 sample in each cluster.

    # Extract the number of events, the number of large events, as well as
    # the highest magnitude in each catalogue.
    # Store them into a data frame.
    multi.synth.cat.list.df <- syn$environ$multi.synth.cat.list.df
    syn.cat.info <- multi.synth.cat.list.df %>% length %>% seq_len %>%
        lapply(\(i) multi.synth.cat.list.df[[i]][1, 5 : 7])
    synth.df <- do.call(rbind, syn.cat.info)

    # Calculate the agglomerative coefficients for different
    # linkage methods, and select the method with the highest
    # agglomerative coefficient.
    link_m <- c('single', 'complete', 'average', 'ward')
    agg_coef <- link_m %>%
```

```
        sapply(\(i) (synth.df %>% agnes(method = i))$ac)
    method <- agg_coef %>% which.max %>% names
    method <- ifelse(!(method == 'ward'), method, 'ward.D2')

    # Determine the optimal number of clusters.
    opt_num <- clusGap(synth.df, FUN = hcut,
                        K.max = 30, B = 20)$Tab[, 3] %>% which.max

    # Add a new column specifying the numbers of clusters.
    synth.final <- synth.df %>%
        cbind(
            cluster = (
                    synth.df %>%
                    dist(method = 'euclidean') %>%
                    hclust(method = method) %>%
                    cutree(k = opt_num)
                )
        )

    # Select 1 sample from each cluster, and return the sample id.
    categories <- as.factor(synth.final$cluster)
    samp.id <- (categories %>% levels %>% length %>% seq_len %>%
        lapply(
            \(i) (categories == levels(categories)[i]) %>%
                which %>% sample(1))) %>% unlist

    return(samp.id)
}
```

```
synth.model <- function(syn = mult.synth,
                        slice = seq(1, 7, by = 1)){

    Nevents <- syn$environ$Nevents
    Nlar <- syn$environ$Nlarge
    MaxMag <- syn$environ$MaxMag %>% round(2)
    samp.id <- hier_clu_samp(syn)

    # modelling
    post <- rep(list(NULL), samp.id %>% length)
    post.par <- matrix(rep(0, (samp.id %>% length) * 5),
                       ncol = 5)
```

```r

    for(i in samp.id %>% length %>% seq_len){
        multi.synth.etas <- ETAS(data =
            syn$environ$multi.synth.cat.list.df[[samp.id[i]]],
                                    t1 = syn$environ$envir$t1,
                                    t2 = syn$environ$envir$t2)
        post[[i]] <- multi.synth.etas$envir$post.list
        post.par[i,] <- multi.synth.etas$envir$post.par

        post[[i]]$post.df$Catalogues <-
        paste('Random Catalogue', i, ':', Nevents[samp.id[i]],
              'Events, with', Nlar[samp.id[i]], 'Large Events, ',
              'and the Highest Magnitude is', MaxMag[samp.id[i]])
    }

    df.true.param <- data.frame(x = etas$envir$post.par,
                                param = names(etas$envir$post.par %>% as.list))

    # bind marginal posterior data.frames
    bind.post.df <- do.call(rbind,
                            lapply(samp.id %>% length %>% seq_len,
                                   \(i) post[[i]]$post.df))

    # plot them
    post.par.plot <- ggplot(bind.post.df,
                            aes(x = x, y = y, colour = Catalogues)) +
      geom_line() +
      facet_wrap(facets = ~ param, scales = "free") +
      xlab("param") +
      ylab("pdf") +
      geom_vline(
        data = df.true.param,
        mapping = aes(xintercept = x), linetype = 2
      )

    # returns the whole environment
    environ <- as.list(environment())
    return(tibble::lst(environ))
}
mult.synth.fit <- lapply(seq_len(3), \(i) synth.model(syn = mult.synth,
                            slice = seq(7 * (i - 1) + 1, 7 * i, by = 1)))
mult.synth.fit[[4]] <- synth.model(syn = mult.synth,
```

```
                             slice = seq(22, sampid %>% length, by = 1))
```

```
synth.fit <- function(samp.each.class = 5,

                      syn = mult.synth,
                      charac_id = 1,
                      bin_id = 1,
                      impose_type = 1){

    if(is.null(syn)){
        syn1 <- synth.imp(impose_type = impose_type)
        syn <- syn1

        # selecting catalogues
        Nevents <- syn$environ$Nevents
        Nlar <- syn$environ$Nlarge
        MaxMag <- syn$environ$MaxMag %>% round(2)

        # we need to bing the synthetics with the observed catalogue
        # for plotting
        cat.df.for.plotting <- rbind(
          syn$environ$multi.synth.cat.df,
          cbind(syn$environ$envir$data[, c("ts", "magnitudes")],
            gen = NA, cat.idx = "observed",
            num_events = nrow(etas$envir$data),
            num_large = etas$envir$Nlarge,
            max_mag = max(etas$envir$data$magnitudes) %>% round(2)
          )
        )

    }else{
        syn1 <- NULL
        # selecting catalogues
        Nevents <- syn$environ$Nevents
        Nlar <- syn$environ$Nlarge
        MaxMag <- syn$environ$MaxMag %>% round(2)

        characteristic <- switch(charac_id, Nevents, Nlar)
        breaks <- switch(charac_id,
                    quantile(characteristic,
                                  probs =
```

```r
                          seq(0, 17 / 20, length.out = 4)[
                              c(bin_id, bin_id + 1)]),
                      quantile(characteristic + .1,
                                  probs =
                          seq(0, 17 / 20, length.out = 4)[
                              c(bin_id, bin_id + 1)])
                      )


        classes <- cut(characteristic, breaks = breaks)
        samp.id <- rep(0, samp.each.class * (classes %>% levels %>% length))
        for(i in classes %>% levels %>% length %>% seq_len){
            samp.id[(samp.each.class * (i - 1) + 1) :
                        (samp.each.class * i)] <-
                sample(which(classes == levels(classes)[i]),
                           samp.each.class)
        }

        # we need to bing the synthetics with the observed catalogue
        # for plotting
        cat.df.for.plotting <- rbind(
          syn$environ$multi.synth.cat.df[
            which(
              syn$environ$multi.synth.cat.df$cat.idx %in% samp.id),
            ],
          cbind(syn$environ$envir$data[, c("ts", "magnitudes")],
            gen = NA, cat.idx = "observed",
            num_events = nrow(etas$envir$data),
            num_large = etas$envir$Nlarge,
            max_mag = max(etas$envir$data$magnitudes) %>% round(2)
          )
        )
    }

    # plot them
    multi.synth.cat.plot <- ggplot(cat.df.for.plotting,
                                   aes(ts, magnitudes)) +
      geom_point(size = 0.5) +
      geom_point(
        data = syn$environ$ht,
        mapping = aes(ts, magnitudes), colour = "black"
```

```r
      ) +
      facet_wrap(facets =
                     vars(cat.idx, num_events, num_large, max_mag),
                 labeller = 'label_both', ncol = 2)

    if(!(is.null(syn1))){
        # modelling
        input <- rep(list(NULL), syn$environ$n.cat)
        post <- rep(list(NULL), syn$environ$n.cat)
        post.par <- matrix(rep(0, (syn$environ$n.cat) * 5),
                           ncol = 5)
        Npost <- rep(list(NULL), syn$environ$n.cat)
        Npostmean <- rep(0, syn$environ$n.cat)
        abs_dist_int <- rep(0, syn$environ$n.cat)
        abs_dist_mag <- rep(0, syn$environ$n.cat)
        overdisp <- rep(0, syn$environ$n.cat)

        for(i in syn$environ$n.cat %>% seq_len){
            multi.synth.etas <- ETAS(data =
                                 syn$environ$multi.synth.cat.df,
                                           t1 = syn$environ$envir$t1,
                                           t2 = syn$environ$envir$t2)
            post[[i]] <- multi.synth.etas$envir$post.list
            post.par[i,] <- multi.synth.etas$envir$post.par
            Npost[[i]] <- multi.synth.etas$envir$N.post
            Npostmean[i] <- multi.synth.etas$envir$Npostmean
            abs_dist_int[i] <- multi.synth.etas$envir$abs_dist_int
            abs_dist_mag[i] <- multi.synth.etas$envir$abs_dist_mag
            overdisp[i] <- multi.synth.etas$envir$overdisp

            post[[i]]$post.df$Catalogues <-
            paste('Random Catalogue', i, ':', Nevents[i],
                  'Events, with', Nlar[i], 'Large Events, ',
                  'and the Highest Magnitude is', MaxMag[i])

            Npost[[i]]$post.df$Catalogues <-
            paste('Random Catalogue', i, ':', Nevents[i],
                  'Events, with', Nlar[i], 'Large Events, ',
                  'and the Highest Magnitude is',
                  MaxMag[i])
        }
```

```
      df.true.param <- data.frame(x = etas$envir$post.par,
                                  param = names(etas$envir$post.par %>% as.list))

      # bind marginal posterior data.frames
      bind.post.df <- do.call(rbind,
                              lapply(syn$environ$n.cat %>% seq_len,
                                     \(i) post[[i]]$post.df))

      # plot them
      post.par.plot <- ggplot(bind.post.df,
                              aes(x = x, y = y, colour = Catalogues)) +
        geom_line() +
        facet_wrap(facets = ~ param, scales = "free") +
        xlab("param") +
        ylab("pdf") +
        geom_vline(
          data = df.true.param,
          mapping = aes(xintercept = x), linetype = 2
        )

      ##
      df.true.N <- data.frame(N = etas$envir$Npostmean, param = 'N')

      # bind marginal posterior data.frames
      bind.post.N.df <- do.call(rbind,
                              lapply(syn$environ$n.cat %>% seq_len,
                                     \(i) Npost[[i]]$post.df))

      # plot them
      post.N.plot <- ggplot(bind.post.N.df,
                            aes(x = N, y = mean, colour = Catalogues)) +
        geom_line() +
        xlab("N") +
        ylab("pdf") +
        geom_vline(
          data = df.true.N,
          mapping = aes(xintercept = N), linetype = 2
        )

    }else{
      # modelling
```

```
        input <- rep(list(NULL), samp.id %>% length)
        post <- rep(list(NULL), samp.id %>% length)
        post.par <- matrix(rep(0, (samp.id %>% length) * 5),
                           ncol = 5)
        Npost <- rep(list(NULL), samp.id %>% length)
        Npostmean <- rep(0, samp.id %>% length)
        abs_dist_int <- rep(0, samp.id %>% length)
        abs_dist_mag <- rep(0, samp.id %>% length)
        overdisp <- rep(0, samp.id %>% length)

        for(i in samp.id %>% length %>% seq_len){
            multi.synth.etas <- ETAS(data =
                syn$environ$multi.synth.cat.list.df[[samp.id[i]]],
                                       t1 = syn$environ$envir$t1,
                                       t2 = syn$environ$envir$t2)
            post[[i]] <- multi.synth.etas$envir$post.list
            post.par[i,] <- multi.synth.etas$envir$post.par
            Npost[[i]] <- multi.synth.etas$envir$N.post
            Npostmean[i] <- multi.synth.etas$envir$Npostmean
            abs_dist_int[i] <- multi.synth.etas$envir$abs_dist_int
            abs_dist_mag[i] <- multi.synth.etas$envir$abs_dist_mag
            overdisp[i] <- multi.synth.etas$envir$overdisp

            post[[i]]$post.df$Catalogues <-
            paste('Random Catalogue', i, ':', Nevents[samp.id[i]],
                  'Events, with', Nlar[samp.id[i]], 'Large Events, ',
                  'and the Highest Magnitude is', MaxMag[samp.id[i]])

            Npost[[i]]$post.df$Catalogues <-
            paste('Random Catalogue', i, ':', Nevents[samp.id[i]],
                  'Events, with', Nlar[samp.id[i]], 'Large Events, ',
                  'and the Highest Magnitude is',
                  MaxMag[samp.id[i]])
        }

        df.true.param <- data.frame(x = etas$envir$post.par,
                                    param = names(etas$envir$post.par %>% as.list))

        # bind marginal posterior data.frames
        bind.post.df <- do.call(rbind,
                                lapply(samp.id %>% length %>% seq_len,
```

```r
                                          \(i) post[[i]]$post.df))

          # plot them
          post.par.plot <- ggplot(bind.post.df,
                                  aes(x = x, y = y, colour = Catalogues)) +
            geom_line() +
            facet_wrap(facets = ~ param, scales = "free") +
            xlab("param") +
            ylab("pdf") +
            geom_vline(
              data = df.true.param,
              mapping = aes(xintercept = x), linetype = 2
            )

          ##
          df.true.N <- data.frame(N = etas$envir$Npostmean, param = 'N')

          # bind marginal posterior data.frames
          bind.post.N.df <- do.call(rbind,
                                  lapply(samp.id %>% length %>% seq_len,
                                         \(i) Npost[[i]]$post.df))

          # plot them
          post.N.plot <- ggplot(bind.post.N.df,
                                aes(x = N, y = mean, colour = Catalogues)) +
            geom_line() +
            xlab("N") +
            ylab("pdf") +
            geom_vline(
              data = df.true.N,
              mapping = aes(xintercept = N), linetype = 2
            )
      }

      # returns the whole environment
      environ <- as.list(environment())
      return(tibble::lst(environ))
  }
  # mult.synth.fit <- lapply(seq_len(6), \(i) synth.fit(syn = mult.synth,
  #                                                    charac_id = (i - 1) %/% 3 + 1,
  #                                                    bin_id = (i - 1) %% 3 + 1))
```

```r
# test <- synth.fit(syn = NULL, impose_type = 1)
```

Analysis on the Behaviours of the Time-between-Events

```r
ECDF.interarrival <- function(j){

    samp.id <- mult.synth.fit[[j]]$environ$samp.id
    Nevents <- mult.synth.fit[[j]]$environ$Nevents
    Nlar <- mult.synth.fit[[j]]$environ$Nlar
    MaxMag <- mult.synth.fit[[j]]$environ$MaxMag

    data.list <- lapply(samp.id %>% length %>% seq_len, \(i) data.frame(
        Time_between_events =
            mult.synth$environ$multi.synth.cat.list.df[[samp.id[i]]]$ts %>%
                sort %>% diff,
        Catalogues = paste('Random Catalogue', i, ':', Nevents[samp.id[i]],
              'Events, with', Nlar[samp.id[i]], 'Large Events, ',
              'and the Highest Magnitude is',
              MaxMag[samp.id[i]])
        )
    )

    data.list[[length(samp.id) + 1]] <- data.frame(
        Time_between_events = etas$envir$interarrival,
        Catalogues = paste('Observed', ':', nrow(etas$envir$data),
              'Events, with', etas$envir$Nlarge, 'Large Events, ',
              'and the Highest Magnitude is',
              max(etas$envir$data$magnitudes) %>% round(2)))

    df <- do.call(rbind, data.list)

    ECDF.plot <- ggplot(df, aes(x = Time_between_events,
                                colour = Catalogues)) +
        stat_ecdf() +
        xlab('Time between Events') +
      ylab('Empirical Cumulative Probability')

    return(ECDF.plot)
}

ecdf_interarrival <- lapply(seq_len(5), \(j) ECDF.interarrival(j))
```

Forecasting

```r
ETAS.forecast <- function(){

    # inherits the environment from function `ETAS`
    envir <- etas$envir

    # express 1 minute in days
    min.in.days <- 1 / (24 * 60)
    # find time of the event with the greatest magnitude
    t.max.mag <- envir$data$ts[which.max(envir$data$magnitudes)]
    # set starting time of the forecasting period
    T1.fore <- t.max.mag + min.in.days
    # set forecast length
    fore.length <- 1
    # set end time of the forecasting period
    T2.fore <- T1.fore + fore.length
    # set known data
    Ht.fore <- envir$data[envir$data$ts < T1.fore, ]

    # produce forecast
    daily.fore <- Temporal.ETAS.forecast(
      post.samp = envir$post.samp, # ETAS parameters posterior samples
      n.cat = nrow(envir$post.samp), # number of synthetic catalogues
      beta.p = envir$beta.p, # magnitude distribution parameter
      M0 = envir$m0, # cutoff magnitude
      T1 = T1.fore, # forecast starting time
      T2 = T2.fore, # forecast end time
      Ht = Ht.fore, # known events
      ncore = num.cores # number of cores
    )

    # find number of events per catalogue
    N.fore <- vapply(
      seq_len(daily.fore$n.cat),
      \(x) sum(daily.fore$fore.df$cat.idx == x), 0
    )
    # find number of observed events in the forecasting period
    N.obs <- sum(envir$data$ts >= T1.fore & envir$data$ts <= T2.fore)
    # plot the distribution
    histfore <- ggplot() +
      geom_histogram(aes(x = N.fore, y = after_stat(density)),
```

```r
                        binwidth = 1) +
      geom_vline(xintercept = N.obs) +
      xlim(100, 500)

    return(tibble::lst(N.fore, N.obs, histfore))
}
# fore <- ETAS.forecast()
```

```r
# save.image(file = 'Dissertation.RData')
```