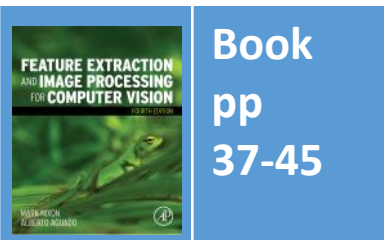


# Lecture 2 Image Formation

# COMP3204 & COMP6223 Computer Vision

# What is inside an image?



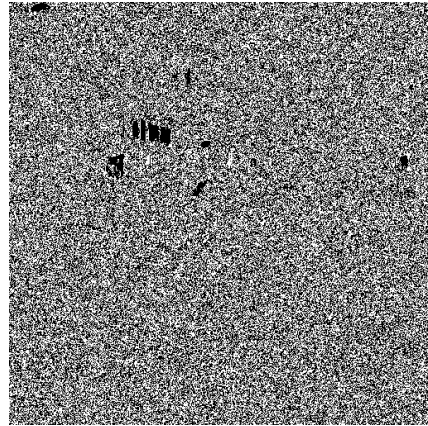
**Department of  
Electronics and  
Computer Science**

UNIVERSITY OF  
**Southampton**  
School of Electronics  
and Computer Science

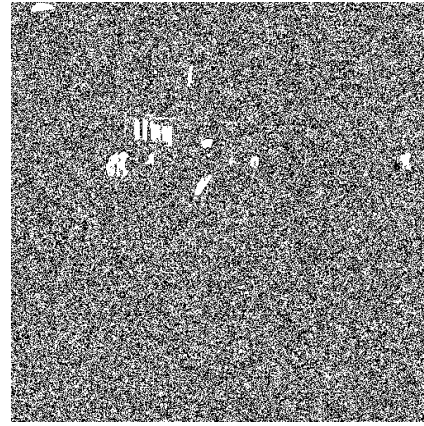
# Decomposing an image into its bits



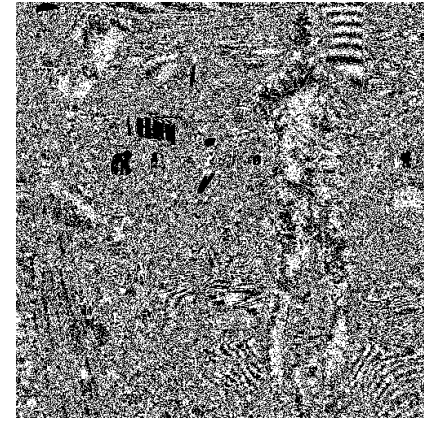
(a) original image



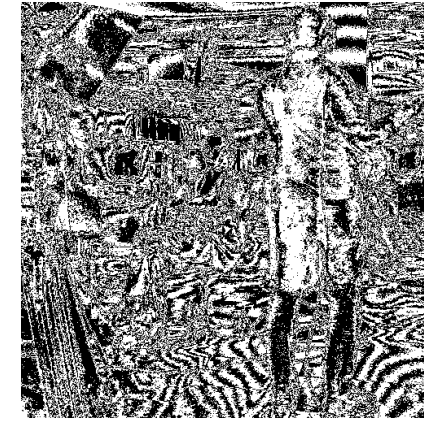
(b) bit 0 (LSB)



(c) bit 1



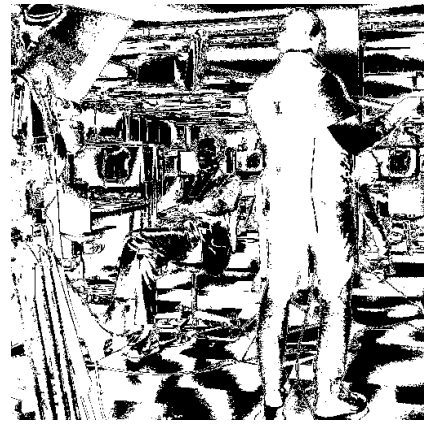
(d) bit 2



(e) bit 3



(f) bit 4



(g) bit 5



(h) bit 6



(i) bit 7 (MSB)





# Effects of differing image resolution



(a)  $64 \times 64$



(b)  $128 \times 128$



(c)  $256 \times 256$



# Jean Baptiste Joseph Fourier

- Any periodic function is the result of adding up sine and cosine waves of different frequencies
- Sceptical? Yeah, so were Lagrange and Laplace. Good company eh?
- “Fourier’s treatise is one of the very few scientific books that can never be rendered antiquated by the progress of science”  
James Clerk Maxwell 1878
- Fourier 10 Laplace 0 ...



# What are 2D waves?

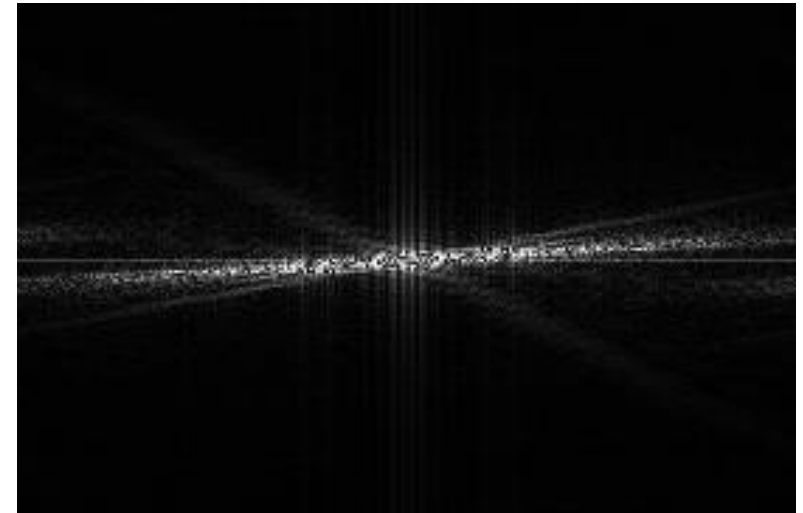
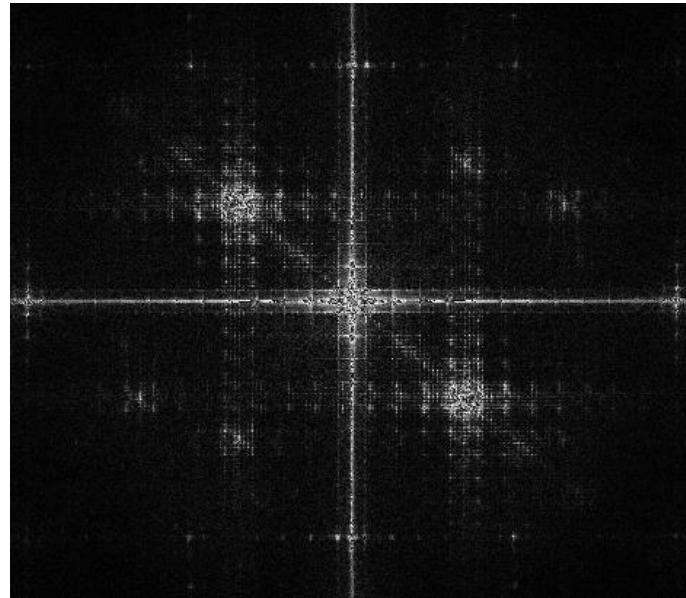
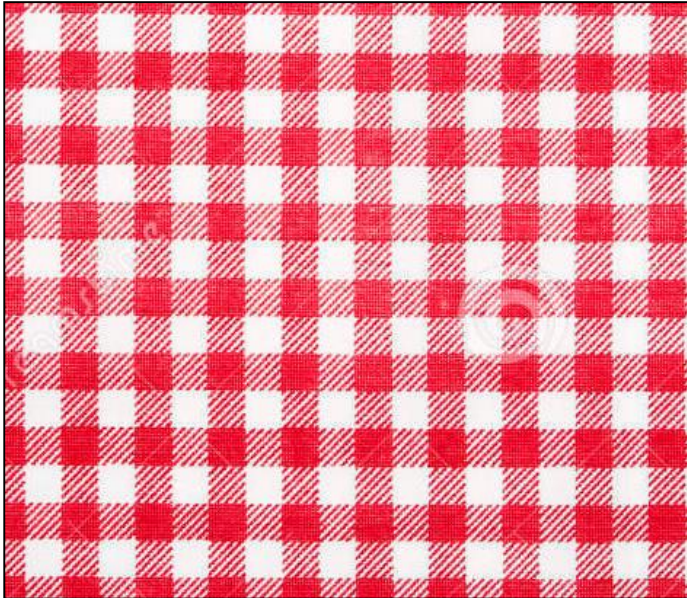
2D waves are along x and y axes simultaneously





and in terms of frequency

- N.b. colour immaterial (just for visuals)




# Step up Monsieur Fourier...

$$Fp(\omega) = \mathfrak{F}(p(t)) = \int_{-\infty}^{\infty} p(t)e^{-j\omega t} dt$$

where:  $Fp(\omega)$  is the Fourier transform, and  $\mathfrak{F}$  denotes the Fourier transform process;

$\omega$  is the **angular** frequency,  $\omega = 2\pi f$  measured in **radians/s** (where the frequency  $f$  is the reciprocal of time  $t$ ,  $f = 1/t$ );

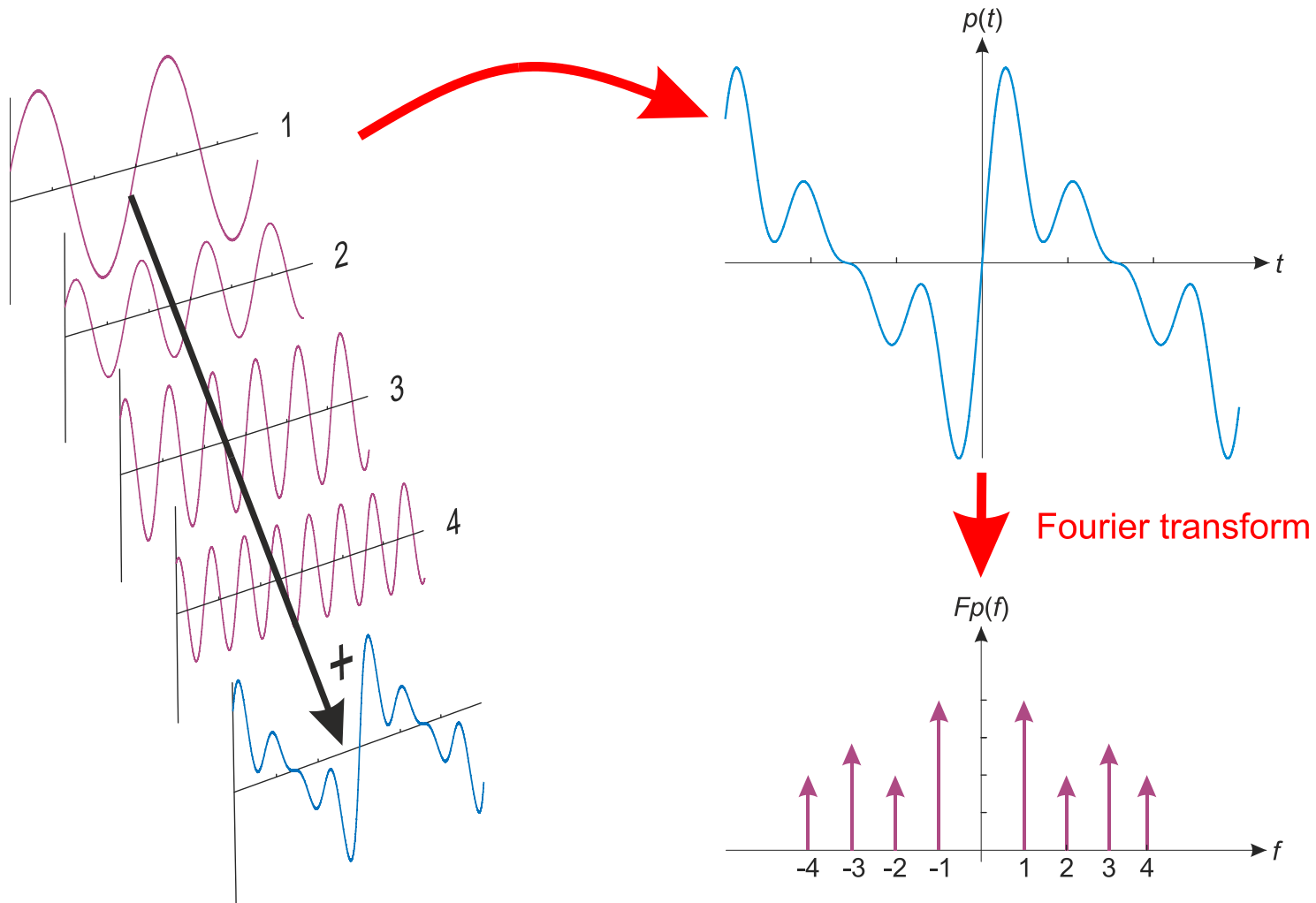
  $j$  is the complex variable  $j = \sqrt{-1}$  (electronic engineers prefer  $j$  to  $i$  since they cannot confuse it with the symbol for current; perhaps they don't want to be mistaken for mathematicians who use  $i = \sqrt{-1}$ );

$p(t)$  is a **continuous** signal (varying continuously with time); and

$e^{-j\omega t} = \cos(\omega t) - j\sin(\omega t)$  gives the frequency components in  $p(t)$ .



# What does the Fourier transform do?





Zut alors! On doit applique ca

- Pulse 
$$p(t) = \begin{cases} A & \text{if } -T/2 \leq t \leq T/2 \\ 0 & \text{otherwise} \end{cases}$$

- Use Fourier  $Fp(\omega) = \int_{-T/2}^{T/2} Ae^{-j\omega t} dt$

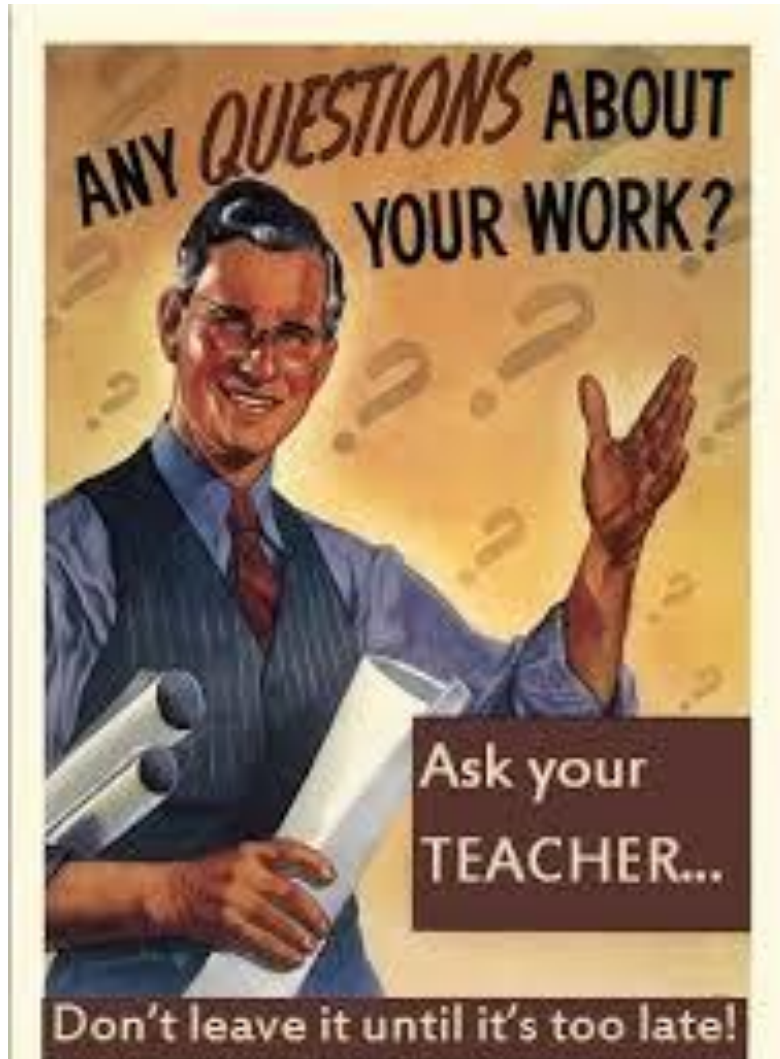
- Evaluate integral  $Fp(\omega) = -\frac{Ae^{-j\omega T/2} - Ae^{j\omega T/2}}{j\omega}$

- And get result

$$Fp(\omega) = \begin{cases} \frac{2A}{\omega} \sin\left(\frac{\omega T}{2}\right) & \text{if } \omega \neq 0 \\ AT & \text{if } \omega = 0 \end{cases}$$

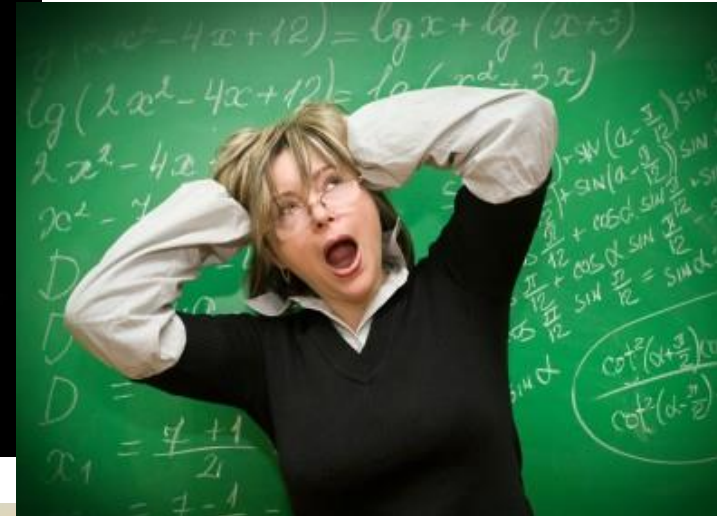


# Google “are you frightened of maths”

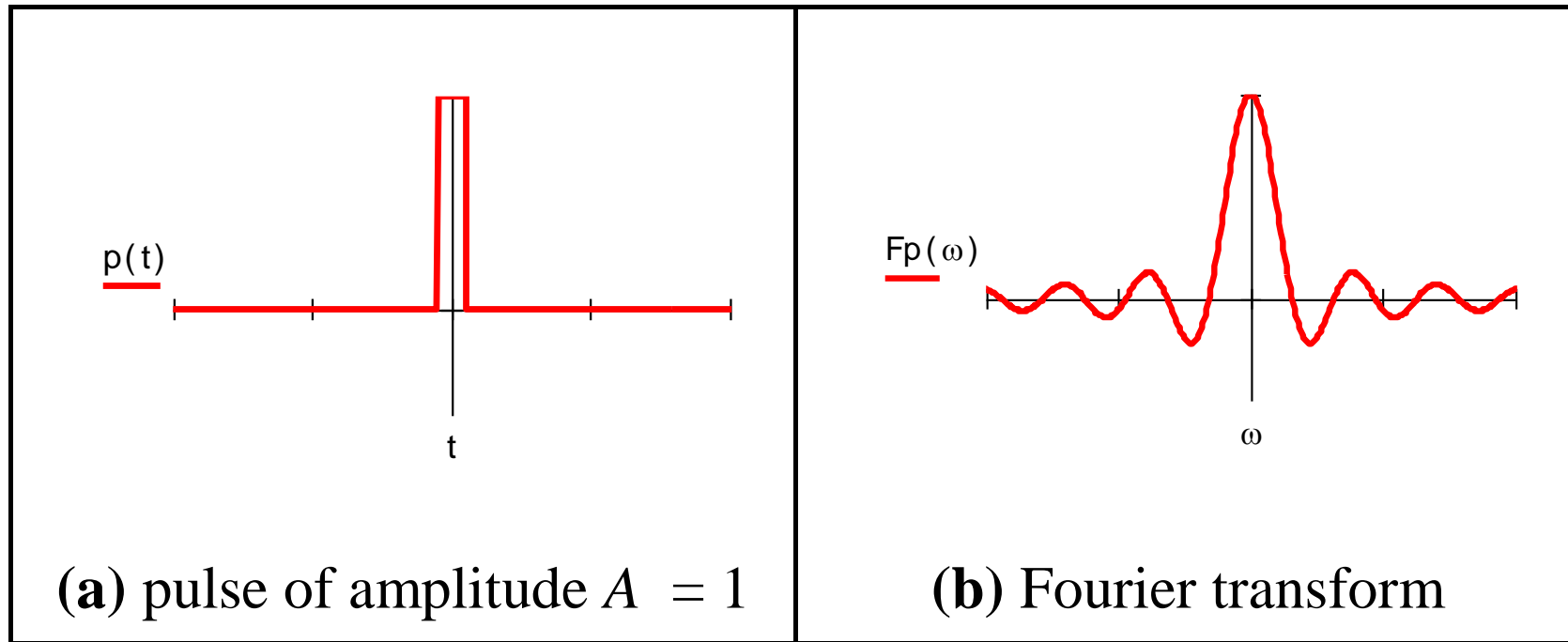


$2+2=\text{fish}$   
 $3+3=\text{eight}$   
 $7+7=\text{triangle}$

Only smart people would get this.

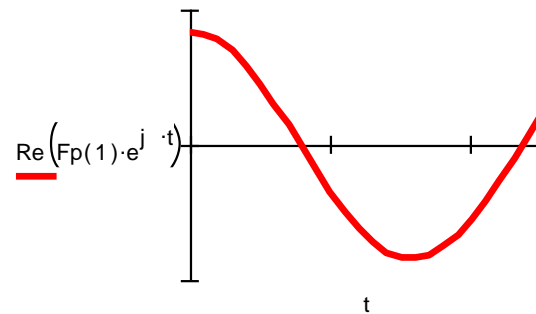
A black rectangular box containing white text. The text lists three incorrect mathematical equations: "2+2=fish", "3+3=eight", and "7+7=triangle". Below these, it says "Only smart people would get this."

# A pulse and its Fourier transform

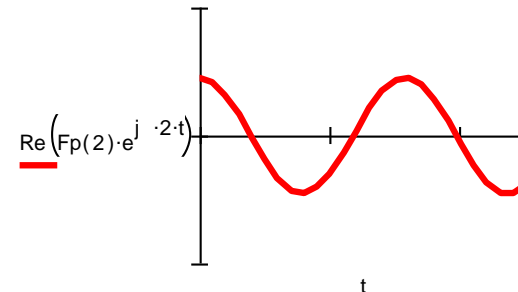




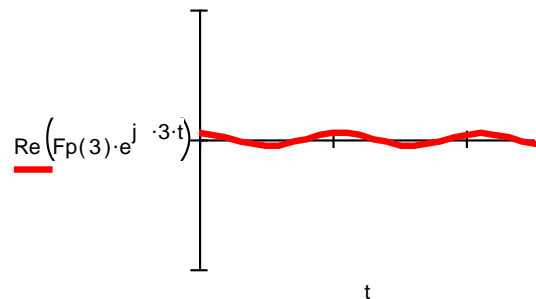
# Reconstructing a signal from its Fourier transform



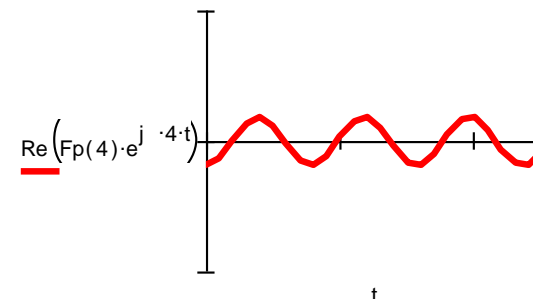
(a) contribution for  $\omega = 1$



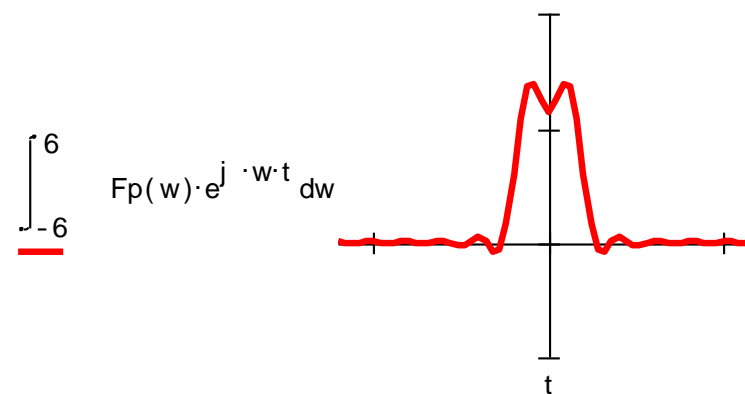
(b) contribution for  $\omega = 2$



(c) contribution for  $\omega = 3$



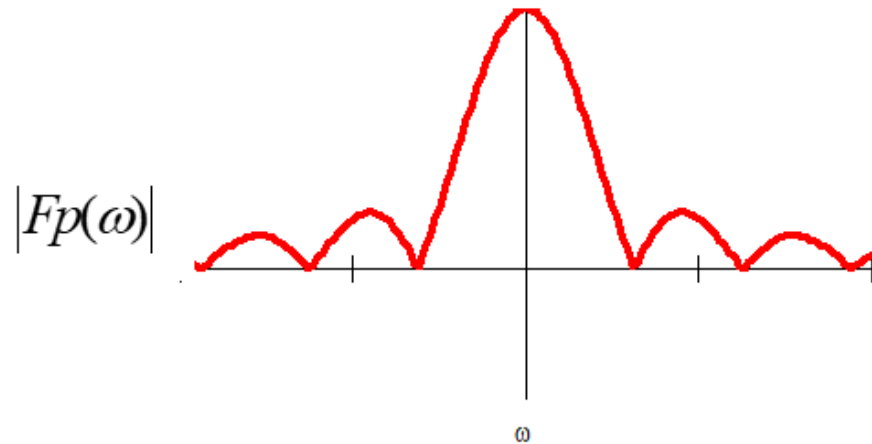
(d) contribution for  $\omega = 4$



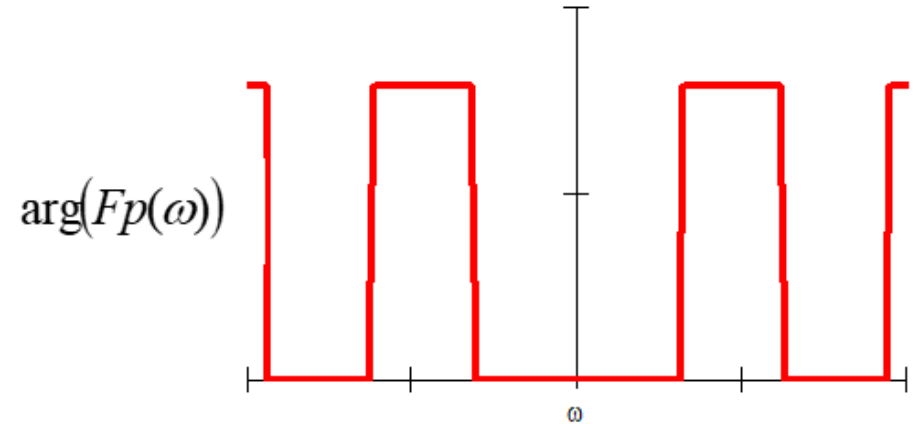
(e) reconstruction by integration

# Magnitude and phase of Fourier transform of a pulse

$$Fp(\omega) = \int_{-\infty}^{\infty} p(t)e^{-j\omega t} dt = \text{Re}(Fp(\omega)) + j \text{Im}(Fp(\omega))$$



(a) magnitude



(b) phase

$$|Fp(\omega)| = \sqrt{\text{Re}(Fp(\omega))^2 + \text{Im}(Fp(\omega))^2}$$

$$\arg(Fp(\omega)) = \tan^{-1}\left(\frac{\text{Im}(Fp(\omega))}{\text{Re}(Fp(\omega))}\right)$$

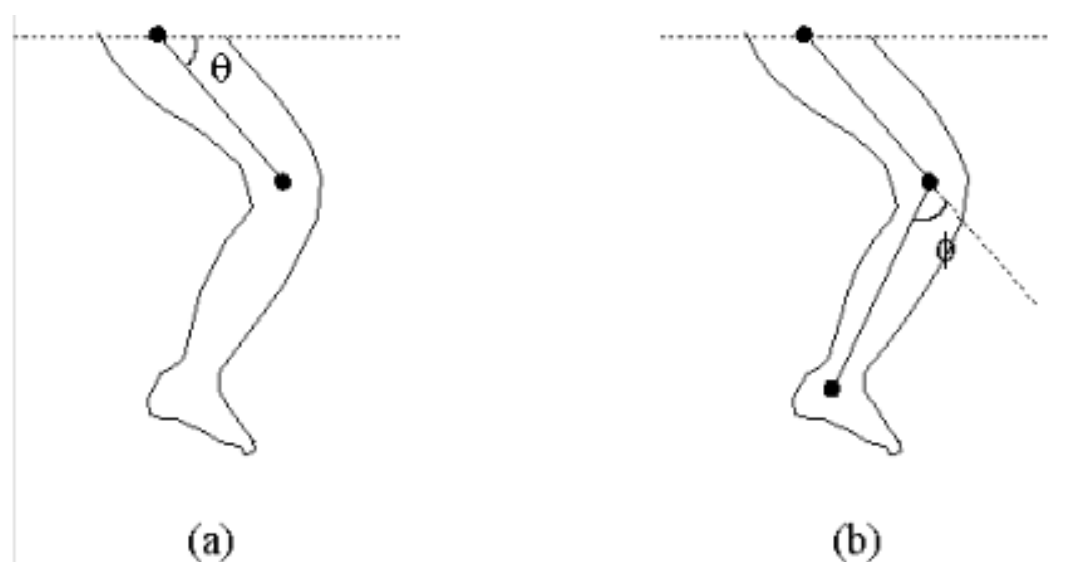


# Using Gait as a Biometric, via Phase-Weighted Magnitude Spectra

David Cunado, Mark S. Nixon and John N. Carter

Department of Electronics and Computer Science, University of Southampton,  
Highfield, Southampton SO17 1BJ, England.

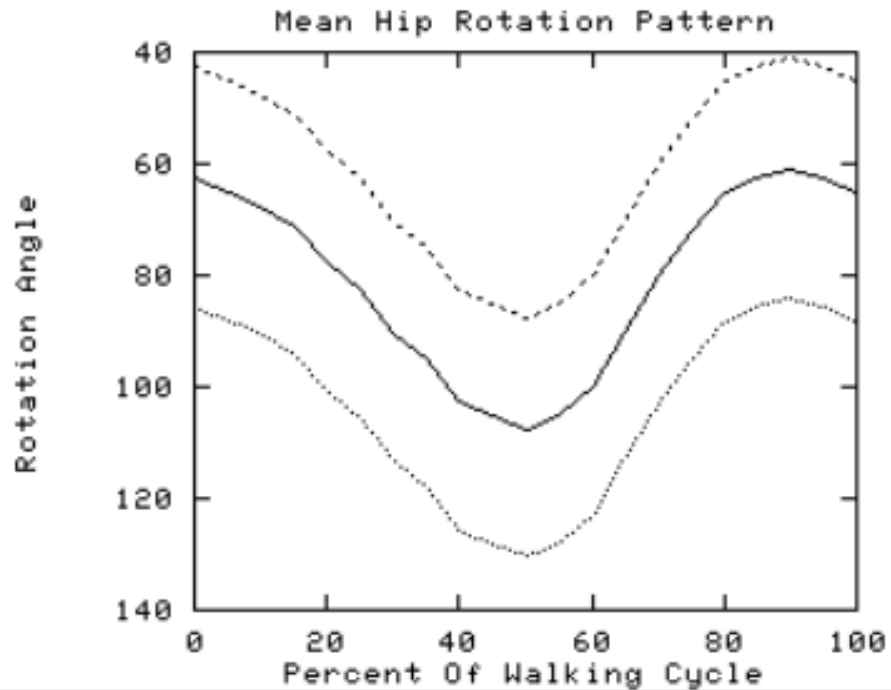
Email: `dc95r@ecs.soton.ac.uk` and `msn@ecs.soton.ac.uk`



**Fig. 1.** (a) Hip and (b) Knee rotation angles.



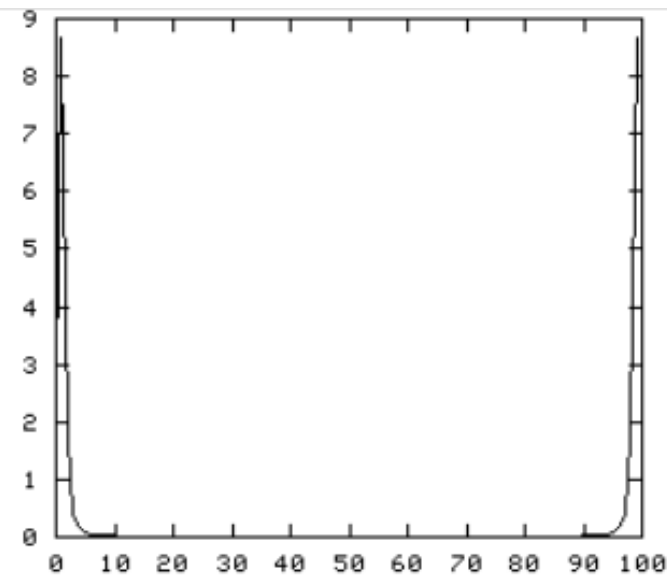
## Gait patterns (angle of swinging leg)



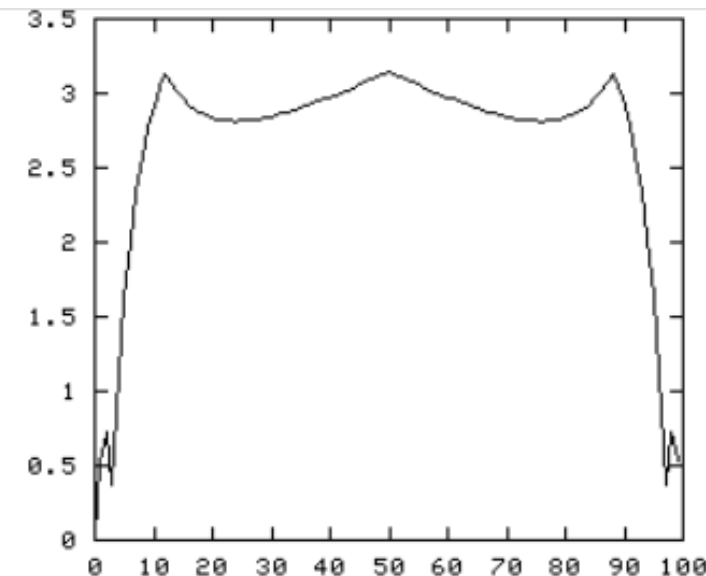
**Fig. 2.** Variation in Hip Rotation.



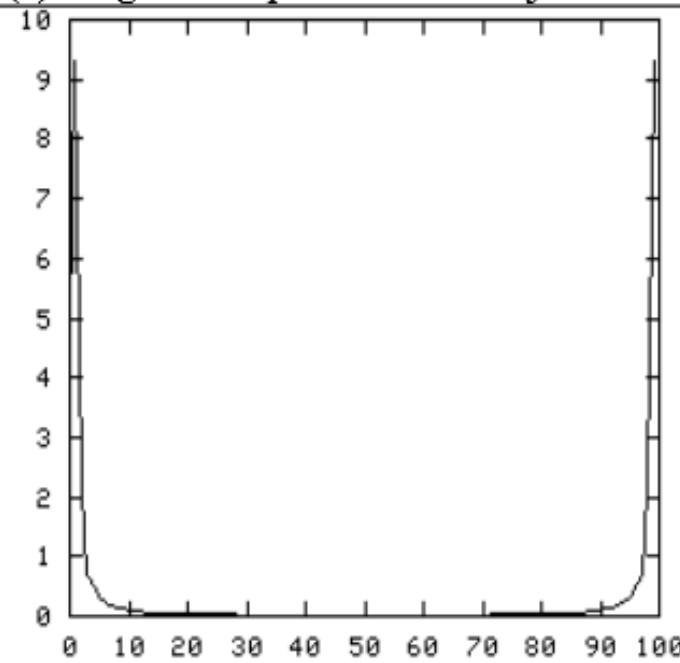
**Fig. 3.** Example Image of Walking Subject.



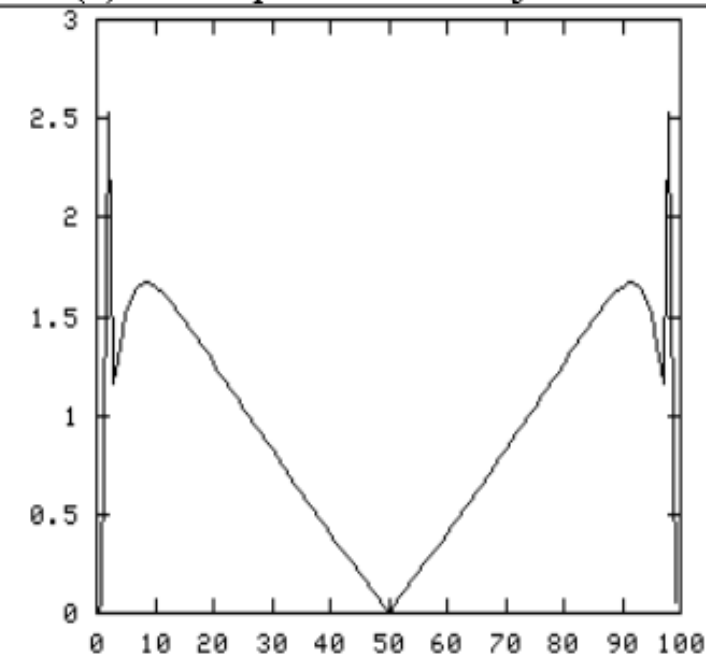
(a) Magnitude spectrum for subject 2.



(b) Phase spectrum for subject 2.



(c) Magnitude spectrum for subject 5.



(d) Phase spectrum for subject 5.

**Fig. 6.** Phase and Magnitude Gait Spectra.

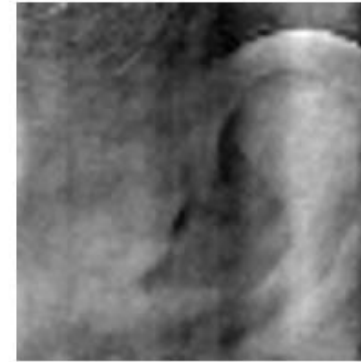
# Illustrating the importance of phase



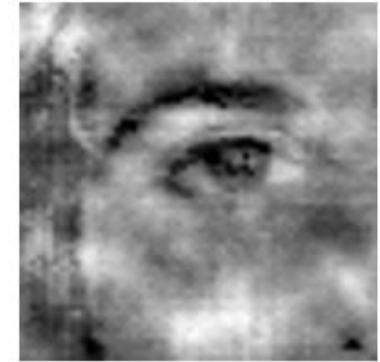
**(a)** eye image



**(b)** ear image



**(c)** reconstruction  
from magnitude(eye)  
and phase(ear)



**(d)** reconstruction from  
magnitude(ear) and  
phase(eye)